Scaling theory and the structure of diverse systems

New analytic tools and techniques for economic policymaking

Chris Kempes
16/4/2019

Santa Fe Institute
• An overview of scaling theory from the biological sciences

• Examples in the social sciences

• Outlook
The Challenge of Biology
The Challenge of Biology

- Advantage: well-defined cost function of organism fitness
- Disadvantage: complicated function of the environment and ecology
Systematic Trends in Biology

White and Seymour, The Journal of Experimental Biology, 2005
Scaling, Constraints, and Mechanism

West, Brown, and Enquist, Science 1997
Detailed Predictions

Observations

Predictions

Shifts In Structure

Metabolic Rate vs. Volume

After Delong et al., PNAS, 2010
Shifts In Structure

Specific Growth Rate (per day)

Unicellular prokaryotes
Unicellular eukaryotes
Multicellular organisms

\[ G = \frac{1}{b(1 - \alpha)} \ln \frac{\gamma_0}{1 - e^{(1-\alpha)(1 - \gamma_0)}} \]

Kempes et al., PNAS, 2012
data from Delong et al., 2010
Shifts In Structure

Dynamics of Populations

[Graph showing the relationship between recovery rate $\rho$, starvation rate $\sigma$, and the probability of extinction $Pr(\text{Ext})$ for different values of $M$.]

Yeakel, Kempes, Redner, Nature Communications, 2018
Steady-states of Populations

Yeakel, Kempes, Redner, Nature Communications, 2018
Steady-states of Populations

\[ M_{\text{max}} = 6.54 \times 10^7 (g) \]
\[ M_{\text{max}} = 1.5 \times 10^7 (g) \]
\[ M_{\text{max}} = 1.74 \times 10^7 (g) \]

Yeakel, Kempes, Redner, Nature Communications, 2018
Social Systems
Social Systems

\[ \beta = 1.12 \quad R^2 = 0.97 \]

\[ \beta = 0.093 \quad R^2 = 0.80 \]

Bettencourt et al, PNAS, 2007
Social Systems

\[ \beta = 1.12 \quad R^2 = 0.97 \]

Bettencourt et al, PNAS, 2007

Bettencourt, Science, 2013
## Social Systems

### Table 1. Scaling exponents for urban indicators vs. city size

<table>
<thead>
<tr>
<th>Indicator</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>Adj-$R^2$</th>
<th>Observations</th>
<th>Country-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>New patents</td>
<td>1.27</td>
<td>[1.25, 1.29]</td>
<td>0.72</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Inventors</td>
<td>1.25</td>
<td>[1.22, 1.27]</td>
<td>0.76</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Private R&amp;D employment</td>
<td>1.34</td>
<td>[1.29, 1.39]</td>
<td>0.92</td>
<td>266</td>
<td>U.S. 2002</td>
</tr>
<tr>
<td>&quot;Supercreative&quot; employment</td>
<td>1.15</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
</tr>
<tr>
<td>R&amp;D establishments</td>
<td>1.19</td>
<td>[1.14, 1.22]</td>
<td>0.77</td>
<td>287</td>
<td>U.S. 1997</td>
</tr>
<tr>
<td>R&amp;D employment</td>
<td>1.26</td>
<td>[1.18, 1.43]</td>
<td>0.93</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Total wages</td>
<td>1.12</td>
<td>[1.09, 1.13]</td>
<td>0.96</td>
<td>361</td>
<td>U.S. 2002</td>
</tr>
<tr>
<td>Total bank deposits</td>
<td>1.08</td>
<td>[1.03, 1.11]</td>
<td>0.91</td>
<td>267</td>
<td>U.S. 1996</td>
</tr>
<tr>
<td>GDP</td>
<td>1.15</td>
<td>[1.06, 1.23]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>GDP</td>
<td>1.26</td>
<td>[1.09, 1.46]</td>
<td>0.64</td>
<td>196</td>
<td>EU 1999–2003</td>
</tr>
<tr>
<td>GDP</td>
<td>1.13</td>
<td>[1.03, 1.23]</td>
<td>0.94</td>
<td>37</td>
<td>Germany 2003</td>
</tr>
<tr>
<td>Total electrical consumption</td>
<td>1.07</td>
<td>[1.03, 1.11]</td>
<td>0.88</td>
<td>392</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>New AIDS cases</td>
<td>1.23</td>
<td>[1.18, 1.29]</td>
<td>0.76</td>
<td>93</td>
<td>U.S. 2002–2003</td>
</tr>
<tr>
<td>Serious crimes</td>
<td>1.16</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
</tr>
<tr>
<td>Total housing</td>
<td>1.00</td>
<td>[0.99, 1.01]</td>
<td>0.99</td>
<td>316</td>
<td>U.S. 1990</td>
</tr>
<tr>
<td>Total employment</td>
<td>1.01</td>
<td>[0.99, 1.02]</td>
<td>0.98</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Household electrical consumption</td>
<td>1.00</td>
<td>[0.94, 1.06]</td>
<td>0.88</td>
<td>377</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Household electrical consumption</td>
<td>1.05</td>
<td>[0.89, 1.22]</td>
<td>0.91</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Household water consumption</td>
<td>1.01</td>
<td>[0.89, 1.11]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Gasoline stations</td>
<td>0.77</td>
<td>[0.74, 0.81]</td>
<td>0.93</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Gasoline sales</td>
<td>0.79</td>
<td>[0.73, 0.80]</td>
<td>0.94</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Length of electrical cables</td>
<td>0.87</td>
<td>[0.82, 0.92]</td>
<td>0.75</td>
<td>380</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Road surface</td>
<td>0.83</td>
<td>[0.74, 0.92]</td>
<td>0.87</td>
<td>29</td>
<td>Germany 2002</td>
</tr>
</tbody>
</table>

Bettencourt et al, PNAS, 2007
Social Systems

Yang et al., arXiv:1712.00476

Vicky Yang

Yang et al., arXiv:1712.00476
Effects of Institutional Structure
United States Higher Education System

Community colleges
Non-profit private colleges
State colleges
Private research universities
Public research universities

Ryan Taylor
Xiaofan Liang
United States Higher Education System

Taylor and Liang et al., In prep
### United States Higher Education System

<table>
<thead>
<tr>
<th>Variable</th>
<th>Public Research Universities</th>
<th>Private Research Universities</th>
<th>State Colleges</th>
<th>Community Colleges</th>
<th>Non-profit Private Colleges</th>
<th>Professional Schools</th>
<th>For-profit Colleges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching expenditure</td>
<td>$1.2 \pm 0.1$</td>
<td>$1.44 \pm 0.18$</td>
<td>$0.9 \pm 0.04$</td>
<td>$0.81 \pm 0.02$</td>
<td>$0.99 \pm 0.02$</td>
<td>$0.97 \pm 0.03$</td>
<td>$0.93 \pm 0.04$</td>
</tr>
<tr>
<td>Tuition revenue</td>
<td>$1.18 \pm 0.09$</td>
<td>$1.2 \pm 0.09$</td>
<td>$1.04 \pm 0.06$</td>
<td>$0.89 \pm 0.03$</td>
<td>$1.15 \pm 0.02$</td>
<td>$1.09 \pm 0.03$</td>
<td>$0.99 \pm 0.03$</td>
</tr>
<tr>
<td>Research expenditure</td>
<td>$1.52 \pm 0.31$</td>
<td>$1.75 \pm 0.61$</td>
<td>$0.89 \pm 0.27$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Research revenue</td>
<td>$1.29 \pm 0.23$</td>
<td>$1.94 \pm 0.49$</td>
<td>$0.65 \pm 0.1$</td>
<td>$0.71 \pm 0.05$</td>
<td>$0.85 \pm 0.08$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Maintenance</td>
<td>$1.07 \pm 0.11$</td>
<td>$1.33 \pm 0.18$</td>
<td>$0.8 \pm 0.05$</td>
<td>$0.88 \pm 0.02$</td>
<td>$0.89 \pm 0.02$</td>
<td>-</td>
<td>$0.75^* \pm 0.15$</td>
</tr>
<tr>
<td>Total faculty</td>
<td>$1.16 \pm 0.09$</td>
<td>$1.18 \pm 0.14$</td>
<td>$0.88 \pm 0.04$</td>
<td>$0.84 \pm 0.02$</td>
<td>$0.89 \pm 0.02$</td>
<td>$0.76 \pm 0.02$</td>
<td>$0.83 \pm 0.04$</td>
</tr>
<tr>
<td>Faculty pay</td>
<td>$1.2 \pm 0.1$</td>
<td>$1.4 \pm 0.17$</td>
<td>$0.91 \pm 0.04$</td>
<td>$0.82 \pm 0.02$</td>
<td>$0.98 \pm 0.02$</td>
<td>-</td>
<td>$0.92^* \pm 0.24$</td>
</tr>
<tr>
<td>FSA completions</td>
<td>$1.09 \pm 0.07$</td>
<td>$1.09 \pm 0.09$</td>
<td>$1.11 \pm 0.05$</td>
<td>$1.03 \pm 0.03$</td>
<td>$0.99 \pm 0.04$</td>
<td>$0.96^* \pm 0.06$</td>
<td>$1.06^* \pm 0.09$</td>
</tr>
<tr>
<td>FTFT completions</td>
<td>$1.24 \pm 0.06$</td>
<td>$1.17 \pm 0.04$</td>
<td>$1.11 \pm 0.04$</td>
<td>$0.79 \pm 0.04$</td>
<td>$1.09 \pm 0.02$</td>
<td>$1.02^* \pm 0.02$</td>
<td>$0.96^* \pm 0.05$</td>
</tr>
<tr>
<td>Mid-career earnings</td>
<td>$1.09 \pm 0.11$</td>
<td>$1.16 \pm 0.15$</td>
<td>$1 \pm 0.03$</td>
<td>$0.97 \pm 0.02$</td>
<td>$1.18^* \pm 0.04$</td>
<td>$0.96^* \pm 0.1$</td>
<td>$0.95 \pm 0.05$</td>
</tr>
</tbody>
</table>

Taylor and Liang et al., *In prep*