Systemic Cascades in Financial Networks

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New Analytical Tools and Techniques for Economic Policymaking
New Approaches to Economic Challenge
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Systemic Risk

Definition

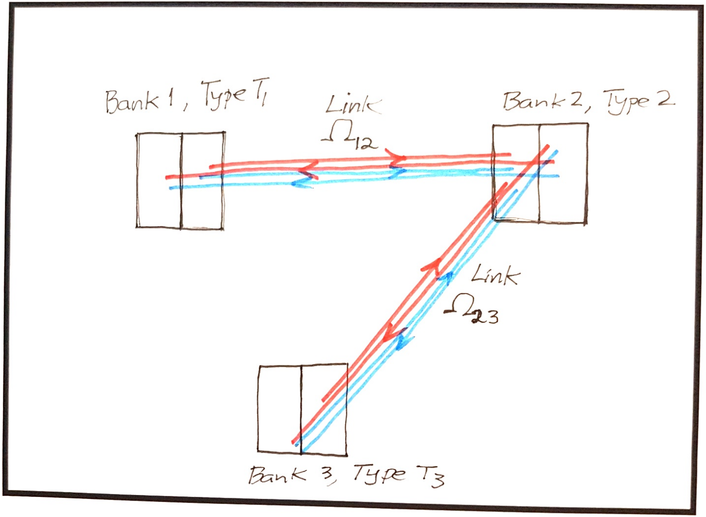
systemic risk (SR): risk that default or stress of one or more financial institutions ("banks") will trigger default or stress of further banks, leading to large scale cascades of system failures and large negative impact on external economy.

Main Aim

1. To crystallize a basic modelling structure for systemic risk.
2. To ensure mathematical tractability, scalability and reality for actual finance network specifications.
3. To learn, to inform the debate, to find/test new heuristics for FSR.

What is the System?

System as a network of nodes v (“banks”) with balance sheets B interconnected by contractual obligations Ω_{vw} (“links”).



How Complex is the System?

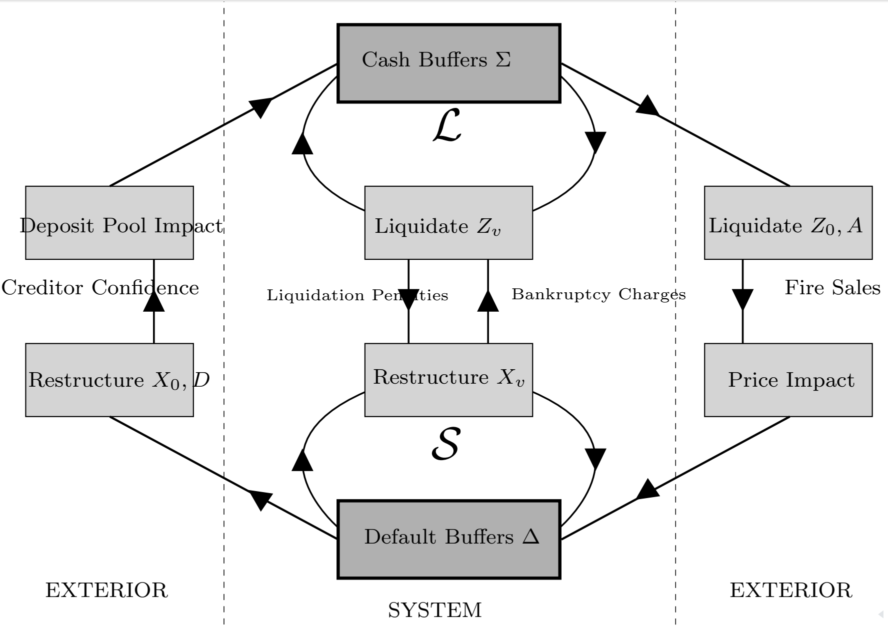
1. “node” or “bank”: diverse collection of regulated and unregulated hierarchical institutions.
2. “link” or “exposure”: complex portfolio of complex bilateral financial contracts, changing intra-daily. Global banking systems are strongly connected.
3. Bank strategies: highly dynamic; the result of decision making distributed across all the subdivisions of the bank; made under great uncertainty.
4. “banks are special” (Corrigan, 1982): they play a critical systemic role at the heart of the much larger macroeconomy.
5. System is opaque, controlled by central banks/regulators, strongly tied to external economy.

Stylized Bank Balance Sheet

Assets	Liabilities
inter-bank assets $\bar{Z}$	inter-bank debt $\bar{X}$
external illiquid assets $\bar{A}$	external debt $\bar{D}$
external liquid assets $\bar{C}$ $\sim$ liquidity buffer $\bar{\Sigma}$	equity $\bar{E}$ $\sim$ solvency buffer $\bar{\Delta}$

Table: A stylized balance sheet.

Systemic Risk Schematic



Global Financial System: Modeling Ingredients for IRFN

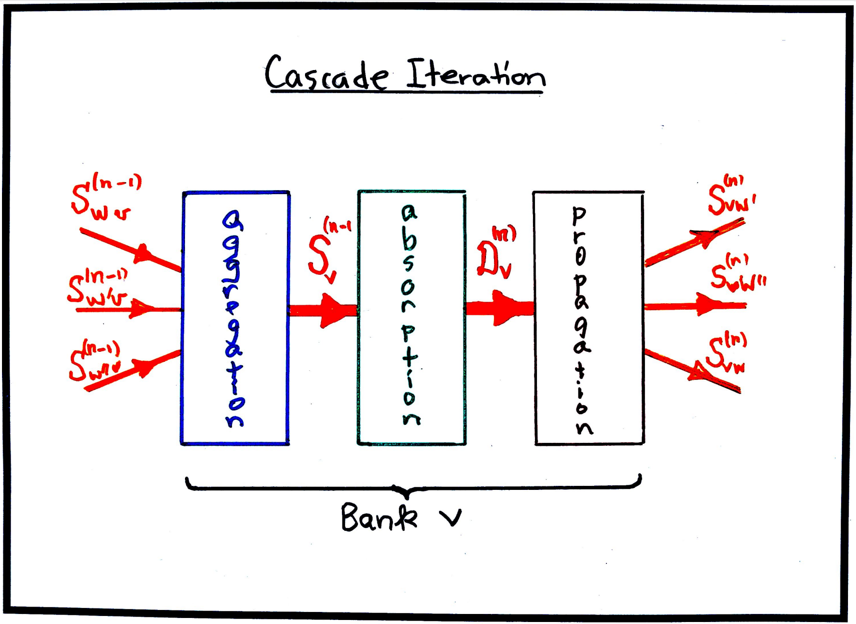
Inhomogeneous Random Financial Networks:

1. Missing data? randomize!
2. Bank types $\mathcal{T} = \{1, 2 \ldots, T\}$ (attributes such as country, size, category);
3. Number of banks $\bar{N} = \sum_{t \in \mathcal{T}} \bar{N}_t$; Label bank $v \in \{1, \ldots, \bar{N}\}$ with type T_v.
4. Skeleton random graph: Indicators $I_{vw} \in \{0, 1\}$ for a "large exposure" between v and w.
5. Balance sheets ($M_B = 3$ "sectors"): $B_v \in \mathbb{R}^6_+$
6. Exposures ($M_\Omega = 2$ kinds of links): $\Omega_{vw} \in \mathbb{R}^4_+$.
7. Cascade Mechanism: stylized crisis management behaviour of every bank.

Static Cascade Paradigm

1. Simulate the initial state of system $(T_v, I_{vw}, B_v, \Omega_{vw})$ immediately after a crisis trigger.
2. Iterate the cascade step for $n = 0, 1, \dots$ until convergence criterion is achieved.
3. Large $N = k\bar{N}$ limit of IRFN: often simplifies. Roughly speaking, take k scaled copies of each bank, let $k \to \infty$, use Poisson limit theorems.

Cascade Iteration



Cascade Iteration Step n

Suppose $S_{wv}^{(n-1)} \in \mathbb{R}_+^{M_\Omega}$ is the cumulative shock transmitted from w to v up to the end of step $n-1$.

1. Shock aggregation: total shock hitting v is $S_v^{(n-1)} = \sum_w S_{wv}^{(n-1)}$.
2. Shock absorption: Determine new stress state $\mathcal{D}_v^{(n)}$ of each bank, as result of shock $S_v^{(n-1)}$.
3. Shock propagation: For each w, define $S_{vw}^{(n)} = I_{vw}\Omega_{vw} \otimes \mathcal{D}_v^{(n)}$

Default Cascade

Impacted default buffer at step $n-1$ is $\Delta_v^{(n-1)}$:

1. Insolvency level: $\mathcal{D}_v^{(n-1)} = \min(1, \frac{\max(-\Delta_v^{(n-1)},0)}{\lambda \bar{X}_v}) \in [0,1]$. λ gives fractional recovery given default.
2. Shock propagation: $S_{wv}^{(n-1)} = I_{wv} \bar{\Omega}_{wv} \mathcal{D}_w^{(n-1)}$
3. Shock aggregation: $S_v^{(n-1)} := \sum_{w \neq v} S_{wv}^{(n-1)}$
4. Shock absorption, impacted default buffer: $\Delta_v^{(n)} = \Delta_v^{(0)} - S_v^{(n-1)}$

Conditional Buffer Characteristic Function

In large N limit $\hat{f}^{(n)}_\Delta(k|T) = \mathbb{E}(e^{ik\Delta^{(n)}_v}|T_v = T)$ satisfies iteration

$$\hat{f}^{(n)}_\Delta(k|T) = \hat{f}^{(0)}_\Delta(k|T)\exp[\sum_{T'}\int_{-\infty}^{\infty} R(k,k'|T,T')\hat{f}^{(n-1)}_\Delta(k'|T')\mathrm{d}k]$$

or in matrix notation, simply

$$\hat{\mathbf{f}}^{(n)}_\Delta = \mathrm{diag}(\hat{\mathbf{f}}^{(0)}_\Delta) * \exp[\mathbf{R} * \hat{\mathbf{f}}^{(n-1)}_\Delta] \ .$$

Here function $\mathbf{R} = R(k,k'|T,T')$ captures all relevant features of network structure, while $\hat{f}^{(0)}_\Delta(k|T)$ captures initial trigger shock.

Remarks about IRFNs

1. Bank types, link values, balance sheets can all have diverse financial interpretations.
2. Models can capture multiple channels of direct and indirect contagion.
3. Network correlations arise two ways: static correlation (from the initial state of network); dynamic correlation (from cycles in the skeleton, from cascade mechanism).
4. Mean field theory/Locally tree-like independence: in sparse IRFN models, sometimes correlations wash out, and there are analytical formulas valid for $N = \infty$.
5. Hybrid IRFNs also work. For example, model 24 banks in Canada with "expert knowledge", couple to coarse-grained/large N IRFN model of ROW ("rest of the world").

Thanks

- TRH 2018: “Bank Panics and Fire Sales, Insolvency and Illiquidity” `arxiv.org/abs/1711.05289`
- IRFN paper is forthcoming