Measuring Productivity

OECD Manual

MEASUREMENT OF AGGREGATE AND INDUSTRY-LEVEL PRODUCTIVITY GROWTH

OECD

Measuring Productivity

MEASUREMENT OF AGGREGATE AND INDUSTRY-LEVEL PRODUCTIVITY GROWTH

OECD Manual

OECD

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

Pursuant to Article 1 of the Convention signed in Paris on 14th December 1960, and which came into force on 30th September 1961, the Organisation for Economic Co-operation and Development (OECD) shall promote policies designed:

- to achieve the highest sustainable economic growth and employment and a rising standard of living in Member countries, while maintaining financial stability, and thus to contribute to the development of the world economy;
- to contribute to sound economic expansion in Member as well as non-member countries in the process of economic development; and
- to contribute to the expansion of world trade on a multilateral, non-discriminatory basis in accordance with international obligations.

The original Member countries of the OECD are Austria, Belgium, Canada, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The following countries became Members subsequently through accession at the dates indicated hereafter: Japan (28th April 1964), Finland (28th January 1969), Australia (7th June 1971), New Zealand (29th May 1973), Mexico (18th May 1994), the Czech Republic (21st December 1995), Hungary (7th May 1996), Poland (22nd November 1996), Korea (12th December 1996) and the Slovak Republic (14th December 2000). The Commission of the European Communities takes part in the work of the OECD (Article 13 of the OECD Convention).

Publié en français sous le titre :
MESURER LA PRODUCTIVITÉ
Mesurer la croissance de la productivité par secteur et pour l'ensemble de l'économie

FOREWORD

Measures of productivity growth constitute core indicators for the analysis of economic growth. However, there are many different approaches to productivity measurement and their calculation and interpretation requires careful consideration, in particular when undertaking international comparisons. The *OECD Productivity Manual* is the first comprehensive guide to the various productivity measures aimed at statisticians, researchers and analysts involved in constructing industry-level productivity indicators.

The *Manual* presents the theoretical foundations to productivity measurement, and discusses implementation and measurement issues. The text is accompanied by empirical examples from OECD countries and by numerical examples to enhance its readability. The *Manual* also offers a brief discussion of the interpretation and use of productivity measures.

This manual is a joint product between the OECD Directorate for Science, Technology and Industry and the OECD Statistics Directorate. It has been authored by Paul Schreyer to whom comments and questions should be addressed. However, the manual would not have been possible without the active advice and review process of the Statistical Working Party of the OECD Industry Committee and an informal expert group (see Annex 7 for list of participants), both chaired by Edwin Dean (formerly of the United States Bureau of Labor Statistics). The report is published on the responsibility of the Secretary-General of the OECD.

Enrico Giovannini
Chief Statistician, OECD

Risaburo Nezu
Director, OECD Directorate
for Science, Technology and Industry

TABLE OF CONTENTS

Boxes

1. INTRODUCTION

1.1. Objectives

1. The main objectives of this manual are to:

- Provide an accessible guide to productivity measurement for those involved in constructing and interpreting productivity measures, in particular statistical offices, other relevant government agencies and productivity researchers.
- Improve international harmonisation: although there is no strong prescriptive element in the manual, it contains indications about desirable properties of productivity measures. Hence, when countries have a choice in constructing new measures or developing a system of indicators, the manual may provide guidance.
- Identify desirable characteristics of productivity measures by reference to a coherent framework that links economic theory and index number theory. Desirable properties have to be assessed against the reality of data availability or the costs of producing statistics. Broad trends can often be discerned with tools that do not live up to full theoretical standards as long as they are interpreted with the necessary caution. However, the user has to be aware of simplifications that occur in the practice of productivity measurement.

1.2. Coverage and structure of the manual

2. The manual is focused in four ways:

- First, the manual focuses on measures of productivity growth rather than on the international comparison of productivity levels. Although there may be few conceptual differences between growth and level comparisons (the former compares different points in time, the latter different points in space), there are practical differences between the two. In particular, productivity level comparisons between industries have to address the tricky issue of currency conversion.[1] Productivity growth measurement avoids this question and constitutes a useful starting point, given its frequent use in analysis and policy formulation.
- Second, the manual focuses on the measurement of productivity at the industry level. This is a natural choice given that much of the underlying methodology relies on the theory of production and on the assumption that there are similar production activities across units of observation (firms or establishments). Because industries are defined as “a group of establishments engaged in the same, or similar, kinds of activity” (Commission of the European Communities, OECD, IMF, United Nations, World Bank, 1993, *System of National Accounts 1993*, paragraph 5.40 – SNA 93), the industry level is an appropriate level

---

[1] See van Ark (1996) for a discussion of the main issues.

of analysis. At the same time, an important part of the manual is also devoted to issues of aggregation across industries and the link to economy-wide or sector-wide measures of productivity growth.

- Third, the manual does not cover productivity measures of production activities beyond the production boundary of the System of National Accounts, in particular households' production. Within the SNA production boundary, emphasis is given to productivity measures of those industries that are characterised by a large share of market producers, leaving aside those activities where non-market producers dominate in many OECD countries.[2] These activities pose specific problems of productivity measurement, due to the difficulty or impossibility of observing and/or defining market prices or output.[3] Reference will be made when appropriate but an in-depth treatment of the output measurement in each of these industries would go beyond the scope of the present manual.[4]

- Fourth, the manual focuses on non-parametric methods of productivity measurement. This choice has been made because the manual's primary audience is statistical offices and other, regular producers of productivity series. Econometric methods, as opposed to non-parametric approaches to productivity measurement are a tool that is much more frequently used in the context of individual, academic research projects.

3. This manual is organised as follows. Chapter 2 starts out with an overview of those productivity measures that fall within the scope of the manual, as defined above. Chapter 3 then discusses measurement of output, followed by the measurement of labour input (Chapter 4), capital inputs (Chapter 5) and intermediate inputs (Chapter 6). Chapter 7 deals with index numbers, Chapter 8 with issues of aggregation. Chapter 9 is a short implementation guide. Chapter 10 addresses the interpretation and use of productivity measures and provides a synopsis of the different measures. Six annexes formulate many of the statements in the main text in a more rigorous way, and so provide a bridge to the more academically oriented literature.

---

[2] As explained in the *System of National Accounts 1993*, paragraph 5.41: "An industry […] consists of a group of establishments engaged in the same type of productive activity, whether the institutional units to which they belong are market producers or not. […] For example, the health industry in a particular country may consist of a group of establishments, some of which are market producers while others are non-market producers that provide their services free or at prices that are not economically significant". Within the institutional classification of the SNA, market producers comprise non-financial corporations, financial corporations and households to the extent that they are engaged – as unincorporated enterprises – in the production of market goods and services.

[3] Practices of deflation of output and value added of non-market activities are described in OECD (1996b). A more recent discussion can be found in Eurostat (2001), *Handbook on Price and Volume Measures in National Accounts*. When market prices are missing or when observed prices are not meaningful, techniques of data envelopment analysis (DEA) can play a useful role. Brief reference to DEA is made in Section 6.3, but a fuller treatment is beyond the scope of this manual.

[4] The following activities are considered to possess a large share of non-market producers (ISIC Rev. 3, division 75-99): public administration and defence and compulsory social security, education, health and social work, sewage and refuse disposal, sanitation and similar activities, activities of membership organisations, private households with employed persons, extra-territorial organisations and bodies.

4. Each of the main chapters on measuring output, inputs and on index numbers and aggregation starts out with an overview of the main concepts and conclusions, and with reference to those parts of the document that provide greater in-depth treatment of individual issues. It is hoped that this facilitates access and increases readability of the manual.

2. OVERVIEW OF PRODUCTIVITY MEASURES

2.1. Purposes of productivity measurement

5. Productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use. While there is no disagreement on this general notion, a look at the productivity literature and its various applications reveals very quickly that there is neither a unique purpose for, nor a single measure of, productivity. The objectives of productivity measurement include:

- *Technology.* A frequently stated objective of measuring productivity growth is to trace technical change. Technology has been described as "the currently known ways of converting resources into outputs desired by the economy" (Griliches, 1987) and appears either in its disembodied form (such as new blueprints, scientific results, new organisational techniques) or embodied in new products (advances in the design and quality of new vintages of capital goods and intermediate inputs). In spite of the frequent explicit or implicit association of productivity measures with technical change, the link is not straightforward.

- *Efficiency.* The quest for identifying changes in efficiency is conceptually different from identifying technical change. Full efficiency in an engineering sense means that a production process has achieved the maximum amount of output that is physically achievable with current technology, and given a fixed amount of inputs (Diewert and Lawrence, 1999). Technical efficiency gains are thus a movement towards "best practice", or the elimination of technical and organisational inefficiencies. Not every form of technical efficiency makes, however, economic sense, and this is captured by the notion of allocative efficiency, which implies profit-maximising behaviour on the side of the firm.[5] One notes that when productivity measurement concerns the industry level, efficiency gains can either be due to improved efficiency in individual establishments that make up the industry or to a shift of production towards more efficient establishments.

- *Real cost savings.* A pragmatic way to describe the essence of measured productivity change. Although it is conceptually possible to isolate different types of efficiency changes, technical change and economies of scale, this remains a difficult task in practice. Productivity is typically measured residually and this residual captures not only the above-mentioned factors but also changes in capacity utilisation, learning-by-doing and measurement errors of all kinds. Harberger (1998) re-stated the point that there is a myriad of sources behind productivity growth and labelled it the real cost savings. In this sense, productivity measurement in practice could be seen as a quest to identify real cost savings in production.

---

[5] The distinction and identification of technical change and efficiency change is at the heart of "data envelopment analysis" – a mathematical programming approach towards productivity measurement that was pioneered by Rolf Färe. For a survey of DEA methodologies, see Seiford and Thrall (1990) and Charnes *et al.* (1994). Diewert and Mendoza (1995) also discuss the DEA approach and compare it to the more traditional index number and econometric approaches. A recent application can be found in Ball *et al.* (2001).

- *Benchmarking production processes.* In the field of business economics, comparisons of productivity measures for specific production processes can help to identify inefficiencies. Typically, the relevant productivity measures are expressed in physical units (e.g. cars per day, passenger-miles per person) and highly specific. This fulfils the purpose of factory-to-factory comparisons, but has the disadvantage that the resulting productivity measures are difficult to combine or aggregate.[6]

- *Living standards.*[7] Measurement of productivity is a key element towards assessing standards of living. A simple example is per capita income, probably the most common measure of living standards: income per person in an economy varies directly with one measure of labour productivity, value added per hour worked. In this sense, measuring labour productivity helps to better understand the development of living standards. Another example is the long-term trend in multifactor productivity (MFP). This indicator is useful in assessing an economy's underlying productive capacity ("potential output"), itself an important measure of the growth possibilities of economies and of inflationary pressures.

2.2. Main types of productivity measures

6. There are many different productivity measures. The choice between them depends on the purpose of productivity measurement and, in many instances, on the availability of data. Broadly, productivity measures can be classified as single factor productivity measures (relating a measure of output to a single measure of input) or multifactor productivity measures (relating a measure of output to a bundle of inputs). Another distinction, of particular relevance at the industry or firm level is between productivity measures that relate some measure of gross output to one or several inputs and those which use a value-added concept to capture movements of output.

7. Table 1 uses these criteria to enumerate the main productivity measures. The list is incomplete insofar as single productivity measures can also be defined over intermediate inputs and labour-capital multifactor productivity can, in principle, be evaluated on the basis of gross output. However, in the interest of simplicity, Table 1 was restricted to the most frequently used productivity measures. These are measures of labour and capital productivity, and multifactor productivity measures (MFP), either in the form of capital-labour MFP, based on a value-added concept of output, or in the form of capital-labour-energy-materials MFP (KLEMS), based on a concept of gross output. Among those measures, value-added based labour productivity is the single most frequently computed productivity statistic, followed by capital-labour MFP and KLEMS MFP.

---

[6] For an example of such an approach, see Baily (1993).

[7] A more extensive discussion of productivity and living standards can be found in Baumol *et al.* (1992).

Table 1. **Overview of main productivity measures**

Type of output measure	***Type of input measure***			
	Labour	***Capital***	***Capital and labour***	***Capital, labour and intermediate inputs (energy, materials, services)***
Gross output	Labour productivity (based on gross output)	Capital productivity (based on gross output)	Capital-labour MFP (based on gross output)	KLEMS multifactor productivity
Value added	Labour productivity (based on value added)	Capital productivity (based on value added)	Capital-labour MFP (based on value added)	-
	Single factor productivity measures		***Multifactor productivity (MFP) measures***	

8. These measures are not independent of each other. For example, it is possible to identify various driving forces behind labour productivity growth, one of which is the rate of MFP change. This and other links between productivity measures can be established with the help of the economic theory of production.

9. Once productivity measures are conceptualised on the basis of economic theory, there are several ways to go about their empirical implementation. From a broad methodological viewpoint, parametric approaches can be distinguished from non-parametric ones. In the first case, econometric techniques are applied to estimate parameters of a production function and so obtain direct measures of productivity growth. In the second case, properties of a production function and results from the economic theory of production are used to identify empirical measures that provide a satisfactory approximation to the unknown “true” and economically defined index number. The growth accounting approach to productivity measurement is a prominent example for non-parametric techniques.

2.3. A short guide to some productivity measures

10. The following pages review the five most widely used productivity concepts. They point out major advantages and drawbacks and briefly interpret each measure. For a further discussion, see also Chapter 10.

Labour productivity, based on gross output	
Definition	$\frac{Quantity\ index\ of\ gross\ output}{Quantity\ index\ of\ labour\ input}$
Interpretation	Shows the time profile of how productively labour is used to generate gross output. Labour productivity changes reflect the joint influence of changes in capital, intermediate inputs, as well as technical, organisational and efficiency change within and between firms, the influence of economies of scale, varying degrees of capacity utilisation and measurement errors. Labour productivity only partially reflects the productivity of labour in terms of the personal capacities of workers or the intensity of their effort. The ratio between output and labour input depends to a large degree on the presence of other inputs, as indicated above. When measured as *gross* output per unit of labour input, labour productivity growth also depends on how the ratio of intermediate inputs to labour changes. A process of outsourcing, for example, implies substitution of primary factors of production, including labour, for intermediate inputs. Gross-output based labour productivity rises as a consequence of outsourcing and falls when in-house production replaces purchases of intermediate inputs. Obviously, this does not reflect a change in the individual characteristics of the workforce, nor does it necessarily reflect a shift in technology or efficiency. Although *some* efficiency gain should be expected as a consequence of input substitution, it cannot be captured by the measured change in labour productivity. MFP measures are required for this purpose. Because labour productivity measures reflect the combined effects of changes in capital inputs, intermediate inputs and overall productivity, they do not leave out any direct effects of technical change, be they embodied or disembodied. The former operates via capital goods and intermediate inputs and so affects labour productivity; the latter generally enhances production possibilities for a given set of inputs and so also affects labour productivity.
Purpose	Gross-output based labour productivity traces the labour requirements per unit of (physical) output. It reflects the change in the input coefficient of labour by industry and can help in the analysis of labour requirements by industry.
Advantages	Ease of measurement and readability. In particular, the gross-output measure requires only prices indices on gross output, not on intermediate inputs as is the case for the value-added based measure.
Drawbacks and limitations	Labour productivity is a partial productivity measure and reflects the joint influence of a host of factors. It is easily misinterpreted as technical change or as the productivity of the individuals in the labour force.

Labour productivity, based on value added	
Definition	$\frac{\text{Quantity index of value added}}{\text{Quantity index of labour input}}$
Interpretation	Shows the time profile of how productively labour is used to generate value added. Labour productivity changes reflect the joint influence of changes in capital, as well as technical, organisational and efficiency change within and between firms, the influence of economies of scale, varying degrees of capacity utilisation and measurement errors. Labour productivity only partially reflects the productivity of labour in terms of the personal capacities of workers or the intensity of their effort. The ratio between output and labour input depends to a large degree on the presence of other inputs, as mentioned above. In comparison with labour productivity based on gross output, the growth rate of value-added productivity is less dependent on any change in the ratio between intermediate inputs and labour, or the degree of vertical integration. For example, when outsourcing takes place, labour is replaced by intermediate inputs. This leads to a fall in value added as well as a fall in labour input. The first effect raises measured labour productivity; the second effect reduces it. Thus, value-added based labour productivity measures tend to be less sensitive to processes of substitution between materials plus services and labour than gross-output based measures. Because labour productivity measures reflect the combined effects of changes in capital inputs, intermediate inputs and overall productivity, they do not leave out any direct effects of technical change, be they embodied or disembodied. The latter operates via capital goods and intermediate inputs and so affects labour productivity; the former generally enhances production possibilities for a given set of inputs and so also affects labour productivity.
Purpose	Analysis of micro-macro links, such as the industry contribution to economy-wide labour productivity and economic growth. At the aggregate level, value-added based labour productivity forms a direct link to a widely used measure of living standards, income per capita. Productivity translates directly into living standards, by adjusting for changing working hours, unemployment, labour force participation rates and demographic changes. From a policy perspective, value-added based labour productivity is important as a reference statistic in wage bargaining.
Advantages	Ease of measurement and readability.
Drawbacks and limitations	Labour productivity is a partial productivity measure and reflects the joint influence of a host of factors. It is easily misinterpreted as technical change or as the productivity of the individuals in the labour force. Also, value-added measures based on a double-deflation procedure with fixed-weight Laspeyres indices suffer from several theoretical and practical drawbacks.

Capital-labour MFP based on value added	
Definition	$\dfrac{\text{Quantity index of value added}}{\text{Quantity index of combined labour and capital input}}$ Quantity index of combined labour and capital input = Quantity index of (different types of) labour and capital, each weighted with its current-price share in total value added.
Interpretation	Capital-labour MFP indices show the time profile of how productively combined labour and capital inputs are used to generate value added. Conceptually, capital-labour productivity is not, in general, an accurate measure of technical change. It is, however, an indicator of an industry's capacity to contribute to economy-wide growth of income per unit of primary input. In practice, the measure reflects the combined effects of disembodied technical change, economies of scale, efficiency change, variations in capacity utilisation and measurement errors. When the capital input measure is an aggregator of detailed types of assets, each weighted by their respective user cost, and based on capital goods prices that reflect quality change, the effects of embodied technical change are picked up by the capital input term, and only disembodied technical change affects MFP.
Purpose	Analysis of micro-macro links, such as the industry contribution to economy-wide MFP growth and living standards, analysis of structural change.
Advantages	Ease of aggregation across industries, simple conceptual link of industry-level MFP and aggregate MFP growth. Data directly available from national accounts.
Drawbacks and limitations	Not a good measure of technology shifts at the industry or firm level. When based on value added that has been double-deflated with a fixed weight Laspeyres quantity index, the measure suffers from the conceptual and empirical drawbacks of this concept.

Capital productivity, based on value added	
Definition	$\frac{\text{Quantity index of value added}}{\text{Quantity index of capital input}}$
Interpretation	The capital productivity index shows the time profile of how productively capital is used to generate value added. Capital productivity reflects the joint influence of labour, intermediate inputs, technical change, efficiency change, economies of scale, capacity utilisation and measurement errors. Like labour productivity, capital productivity measures can be based on a gross-output or a value-added concept. The same reasoning as for labour productivity applies between gross-output and value-added based measures in the case of outsourcing and changing vertical integration: value-added based capital productivity measures tend to be less sensitive to processes of substitution between intermediate inputs and capital than gross-output based measures. When capital input is measured in its theoretically preferred form, *i.e.* as a flow of services adjusted for changes in the quality of investment goods, the capital measure translates embodied technical change (rising or falling quality of capital goods) into a larger or smaller flow of constant-quality capital services. Thus, rising quality of capital goods implies a larger amount of capital services. For the same rate of output growth, this implies a fall in capital productivity. Capital productivity has to be distinguished from the rate of return on capital. The former is a physical, partial productivity measure; the latter is an income measure that relates capital income to the value of the capital stock.
Purpose	Changes in capital productivity indicate the extent to which output growth can be achieved with lower welfare costs in the form of foregone consumption.
Advantages	Ease of readability.
Drawback and limits	Capital productivity is a partial productivity measure and reflects the joint influence of a host of factors. There is sometimes confusion between rates of return on capital and capital productivity.

KLEMS Multifactor productivity	
Definition	$\dfrac{\text{Quantity index of gross output}}{\text{Quantity index of combined inputs}}$ Quantity index of combined inputs = Quantity index of (different types of) labour, capital, energy, services, each weighted with its current-price share in total gross output.
Interpretation	Shows the time profile of how productively combined inputs are used to generate gross output. Conceptually, the KLEMS productivity measure captures disembodied technical change. In practice, it reflects also efficiency change, economies of scale, variations in capacity utilisation and measurement errors. When capital and intermediate input measures are aggregators of detailed types of assets and products, each weighted by their respective share in total cost, and based on prices that reflect quality change, the effects of embodied technical change are picked up by the capital and intermediate inputs terms, and only disembodied technical change enters the MFP measure.
Purpose	Analysis of industry-level and sectoral technical change.
Advantages	Conceptually, KLEMS-MFP is the most appropriate tool to measure technical change by industry as the role of intermediate inputs in production is fully acknowledged; "Domar" aggregation of KLEMS-MFP across industries provides an accurate picture of the contributions of industries to aggregate MFP change.
Drawback and limitations	Significant data requirements, in particular timely availability of input-output tables that are consistent with national accounts; Inter-industry links and aggregation across industries more difficult to communicate than in the case of value-added based MFP measures.

2.4. Growth accounting and main assumptions underlying the conceptual framework

11. The economic theory of productivity measurement goes back to the work of Jan Tinbergen (1942) and independently, to Robert Solow (1957). They formulated productivity measures in a production function context and linked them to the analysis of economic growth. The field has developed considerably since, in particular following major contributions by Dale Jorgenson, Zvi Griliches and Erwin Diewert. Today, the production theoretical approach to productivity measurement offers a consistent and well-founded approach that integrates the theory of the firm, index number theory and national accounts.

12. This manual largely adopts the index number approach in a production theoretic framework. This "growth accounting" technique examines how much of an observed rate of change of an industry's output can be explained by the rate of change of combined inputs. Thus, the growth accounting approach evaluates multifactor productivity (MFP) growth residually.

13. To construct an index of an industry's output, different types of outputs have to be weighted with their share in total output. To construct an index of combined inputs, the rates of change of different inputs (labour, capital, intermediate inputs) have to be weighted appropriately. Production theory tells us that, under some simplifying assumptions, factor income shares should be used as weights. These income shares (for example the share of employee compensation in total cost) approximate production elasticities or the effects of a 1% change in individual inputs on output. For

every period under consideration, income shares are re-calculated and combined with the rates of change of factor inputs to obtain an index of combined inputs. Alternatively, an econometric approach could be chosen (see Box 1).

Box 1. **The econometric approach to productivity measurement**

The econometric approach to productivity measurement is only based on observations of volume outputs and inputs. It avoids postulating a relationship between production elasticities and income shares, which may or may not correspond to reality, and indeed puts researchers in a position of testing these relationships. Further possibilities arise with econometric techniques: allowance can be made for adjustment cost (the possibility that changes in factor inputs are increasingly costly the faster they are implemented) and variations in capacity utilisation. Furthermore, it is possible to investigate forms of technical change other than the Hicks-neutral formulation implied by the index number based approach; and there is no *a priori* requirement to assume constant returns to scale of production functions. The literature about the econometric approach is large, and examples of integrated, general models can be found in Morrison (1986) or Nadiri and Prucha (2001).

All these possibilities come at a cost, however. Fully-fledged models raise complex econometric issues and sometimes put a question mark on the robustness of results. Often, researchers are constrained by the sample size of observations, and have again to revert to *a priori* restrictions (for example constant returns to scale) to increase the degrees of freedom for estimation. From the point of view of statistical offices concerned with the publication of regular productivity statistics, complex econometric approaches bear little attractiveness because: *i)* updating involves full re-estimation of (systems of) equations; *ii)* methodologies are often difficult to communicate to a broad spectrum of users of productivity statistics; and *iii)* significant data requirements tend to reduce the timeliness of results.

Hulten (2001) points out that there is no reason why the econometric and the index number approach should be viewed as competitors; he quotes examples of synergism that proved particularly productive. Synergies arise in particular when econometric methods are used to further explain the productivity residual, thereby reducing the ignorance about the “measure of our ignorance”.

Overall, econometric approaches are a tool that is best suited for academically oriented, single studies of productivity growth. Their potential richness and testable set-up make them a valuable complement to the non-parametric, index number methods that are the recommended tool for periodic productivity statistics.

14. However, in its simpler form, the growth accounting framework has to rely on several simplifying assumptions. These include in particular:

- Production processes can be represented by production or transformation functions at various levels of the economy. Production functions relate maximum producible output to sets of available inputs.
- Producers behave efficiently, *i.e.* they minimise costs and/or maximise revenues.
- Markets are competitive, and market participants are price-takers who can only adjust quantities but not individually act on market prices.

15. These conditions are not necessarily met in practice, but provide a reasonable approximation to many markets. Also, in many cases, productivity analysis has developed methods to deal with situations where one or several of these conditions do not prevail. Usually, however, this requires more complex methodology or enhanced data requirements. A case in point is the measurement of output and productivity in non-market activities, such as government, where markets may not be competitive or producers may not be efficient. (Chapter 7 sketches some methodologies for productivity measurement that might apply in such instances.)

16. However, if the above conditions hold approximately, they permit construction of productivity measures on the basis of price and quantity observations only which are frequently available in OECD countries. This is an advantage over econometric methods where larger data sets must be available.

2.5. Some conclusions

2.5.1. Use and interpretation of productivity measures[8]

- Labour productivity is a useful measure: it relates to the single most important factor of production, is intuitively appealing and relatively easy to measure. Also, labour productivity is a key determinant of living standards, measured as per capita income, and from this perspective is of significant policy relevance. However, it only partially reflects the productivity of labour in terms of the personal capacities of workers or the intensity of their efforts. Labour productivity reflects how efficiently labour is combined with other factors of production, how many of these other inputs are available per worker and how rapidly embodied and disembodied technical change proceed. This makes labour productivity a good starting point for the analysis of some of these factors. One way of carrying out further analysis is to turn to multifactor productivity (MFP) measures.

- Multifactor productivity measurement helps disentangle the direct growth contributions of labour, capital, intermediate inputs and technology. This is an important tool for reviewing past growth patterns and for assessing the potential for future economic growth.

- However, one has to be aware that *not all technical change translates into MFP growth.* An important distinction concerns the difference between embodied and disembodied technological change. The former represents advances in the design and quality of new vintages of capital and intermediate inputs and its effects are attributed to the respective factor as long as the factor is remunerated accordingly. Disembodied technical change comes “costless”, for example in the form of general knowledge, blueprints, network effects or spillovers from other factors of production including better management and organisational change. The distinction is important from a viewpoint of analysis and policy relevance.

- Further, in empirical studies, measured *MFP growth is not necessarily caused by technological change:* other non-technology factors will also be picked up by the residual. Such factors include adjustment costs, scale and cyclical effects, pure changes in efficiency and measurement errors.

- *MFP measures tend to understate the eventual importance of productivity change in stimulating the growth of output.* In static models of production such as the one used in this manual, capital is an exogenous input. In a dynamic context, this is not the case and feedback effects exist between productivity change and capital: suppose that technical change allows more output to be produced per person. The static MFP residual measures just this effect of technical change. However, additional output per person may lead to additional savings and investment, and to a rise in the capital-labour ratio. Then, a traditional growth accounting measure would identify this induced effect as a growth contribution of capital, although it can be traced back to an initial shift in technology. Thus, the MFP residual correctly measures the shift in production possibilities but does not capture the induced effects of technology on growth (Rymes, 1971; Hulten, 2001).

---

[8] For a more extensive discussion, see Chapter 10.

- *Accounting is not explaining the underlying causes of growth.* Growth accounting and productivity measurement identifies the relative importance of different proximate sources of growth. At the same time, it has to be complemented by institutional, historical and case studies if one wants to explore the underlying causes of growth, innovation and productivity change.

2.5.2. Challenges for statisticians

17. From the perspective of productivity measurement, there are at least four areas with a specific need for further research and development of data and statistics:

- *Price indices for output measures* by industry, in particular for high-technology industries and difficult-to-measure but economically important services such as the financial sector, health care and education.

- Measurement of *hours worked* by industry, as labour is the single most important factor of production. Currently, there are many problems associated with the accurate measurement of hours worked, in particular when disaggregated by industry. Specific challenges in this context include successfully combining information from the two main statistical sources, enterprise and household surveys, and measuring labour input and compensation of self-employed persons. A cross-classification of hours worked by productivity-relevant *characteristics of the workforce* (education, experience, skills, etc.) would also be highly desirable.

- The quality of existing measures of *capital input* typically suffers from an insufficient empirical basis. For example, there are too few and often outdated empirical studies to determine the service lives of assets and their age-efficiency and age-price profile. More generally, capital measures for productivity analysis (capital services) should be set up consistently with capital measures for asset balance sheets (wealth stocks), and consumption of fixed capital in the national accounts.

- *Input-output tables* are sometimes missing or dated, and not always integrated with national accounts. The development of a consistent set of supply, use and industry-by-industry tables and their full integration with national accounts at current and constant prices is an important element in deriving reliable productivity measures.

3. OUTPUT

Overview: measures of output

Gross-output based productivity measures capture disembodied technical change.

For an individual firm or industry, measures of gross output, combined with labour, capital and intermediate inputs, correspond directly to a specific model of a production function with "neutral" or "output-augmenting" technical change. When multifactor productivity measures are based on such a gross-output concept, MFP growth approximates the rate of neutral, disembodied technical change.

Alternatively, MFP measures could be based on a value-added concept where value added is considered a firm's output and only primary inputs are taken as a firm's input. Value-added based productivity measures reflect an industry's capacity to contribute to economy-wide income and final demand. In this sense, they are valid complements to gross-output based measures.

Value-added based productivity: meaningful in its own right…

At the aggregate level of the economy, gross-output and value-added based measures converge when gross-output measures are defined as sectoral output. Sectoral output is a measure of production corrected for deliveries within a given sector. From this perspective also, gross-output and value-added based measures are complements.

…and often more easily available.

A useful strategy in the development of productivity measures is to start with aggregate value-added based productivity measures: the necessary data tends to be relatively easily available and the choice between gross output and value added makes less difference than at the detailed industry level.

☛ More on the choice between gross output and value added in Section 3.1.

☛ Further discussion on double-deflation and alternative quantity indices of value added in Section 3.3.1.

☛ Sectoral output is defined in Section 3.1.3.

The preferred source: national accounts.

National accounts constitute the preferred statistical source for productivity measurement. The utility of national accounts for productivity analysis can be greatly enhanced when they are set up jointly and consistently with an input-output framework.

☛ More on sources in Section 3.2.

☛ More on input-output tables in Chapter 6.

The quality of price indices is vital for productivity measurement…

Price indices to deflate current-price series of inputs and outputs play a major role in productivity measurement. It is, for example, important that price statistics on inputs are generated independently from price series for outputs. Input-based indicators that are used to deflate output series generate an obvious bias in productivity measures: (labour) productivity growth will either be zero by construction or will reflect any assumptions about productivity growth made by statisticians. Occurrences of input-based extrapolation are concentrated in activities where market output prices are difficult to observe. Difficult-to-measure industries include increasingly important activities such as banking, insurance, and distribution.

…but often difficult to achieve.

While independence of input and output measures is important, so is their correspondence – preferably, they should be based on the same statistical sources. In practice, this is not always the case, and there is a risk of using unmatched input and output data for productivity measurement.

☞ More on extrapolation in Section 3.3.2.

Another issue: treatment of quality change and new products.

The rapid development of information and communication technology products has brought to centre-stage two long-standing questions of price measurement: how to deal with quality changes of existing goods and how to account for new products in price indices. There is no easy solution to these questions, although some countries have used "hedonic" approaches to better capture quality change in computer prices. Different methodologies can yield quite different profiles of price and quantity indices, and so reduce international comparability of measures of output and productivity.

☞ More on quality change in Section 3.3.3.

3.1. Gross-output and value-added based productivity

3.1.1. Definitions

18. Multifactor productivity measures can be computed for different representations of the production process. One such representation is a measure of gross output in relation to primary and intermediate inputs. Another representation relates value added to primary inputs. Whether one of these measures should be preferred over the other has been an issue of considerable debate. Before shedding further light on this topic, it is useful to clarify terms and to show the links to the system of national accounts.

19. Consider Table 2 below which shows a simplified production and generation of income account of an economic unit (a firm, industry, or sector). We call *gross output* the goods or services that are produced within a producer unit and that become available for use outside the unit. This is a gross measure in the sense that it represents the value of sales and net additions to inventories without, however, allowing for purchases of intermediate inputs. When purchases of intermediate inputs are deducted from gross output, one obtains a measure of *value added*. In this sense, value added is a net measure. It may not be considered a net measure in the sense that it includes the value of depreciation or consumption of fixed capital. However, throughout this manual, value added and gross output are understood to include the value of consumption of fixed capital.

20. On the income side, value added corresponds to the income generated by primary factors of production, labour and capital plus any net taxes on production. *Primary inputs* are those factors of production that are treated as exogenous in the framework of production analysis. In a static framework such as the one underlying this manual, primary inputs comprise capital and labour. In a

dynamic framework, capital becomes an endogenous factor of production, but the treatment of this case is beyond the scope of the present document. *Intermediate inputs* are those factors of production that are endogenous to the present framework, *i.e.* those goods and services that are produced and transformed or used up by the production process within an accounting period.

Table 2. **Combined production and generation of income account**[1]

Uses	Resources
Intermediate consumption (purchases of intermediate inputs)	Output Market output For intermediate consumption For final consumption Output for own final use
Gross value added	
Consumption of fixed capital Compensation of employees Other taxes less subsidies on production Operating surplus	

1. Details on the income components of gross value added are found in the generation of income account; the other elements in the table are found in the production account of corporations (SNA 93).

3.1.2. Production functions, gross output and value added

21. To discuss the different approaches towards productivity measures, it is useful to refer to a production function. A production function relates the maximum quantity of gross output (*Q*) that can be produced by all inputs, primary ones (*X*), *i.e.* labour and capital, and intermediate ones (*M*). The function also contains a parameter *A(t)* that captures disembodied technological shifts. Disembodied technical change can be the result of research and development that leads to improved production processes, or technical change can be the consequence of learning-by-doing, or imitation. It is called "disembodied" because it is not physically tied to any specific factor of production. Rather, it affects inputs proportionally. This form of technical change is also called "Hicks-neutral" and is "output augmenting" when it raises the maximum output that can be produced with a given level of primary and intermediate inputs, and without changing the relationship between different inputs. Under this assumption, the production function can be represented as:

$$Q = H(A, X, M) = A(t) \cdot F(X, M) \qquad (1)$$

22. It is easy to see that the level of technology in (1) can be presented as the ratio of output over combined primary and intermediate inputs: $A(t) = \frac{Q}{F(X,M)}$. In terms of rates of change, MFP growth is positive when the rate of change in gross output exceeds the rate of change in all combined measured inputs. Put differently, a valid measure of technical change is the rate at which the production function shifts over time, or $\frac{\partial \ln H}{\partial t}$. When technology is Hicks-neutral, this shift just equals the rate of change of the technology parameter: $\frac{\partial \ln H}{\partial t} = \frac{\partial \ln A}{\partial t}$.

23. Because the technology parameter cannot be observed directly, MFP growth is derived as the difference between the rate of growth of a Divisia index of output and a Divisia index of inputs, as shown below. The Divisia index of inputs is made up of the logarithmic rates of change of primary and intermediate inputs, weighted with their respective share (s_X, s_M) in overall outlays for inputs:

$$\text{\% change of gross-output based MFP} = \frac{\partial \ln H}{\partial t} = \frac{\partial \ln A}{\partial t} = \frac{d \ln Q}{dt} - s_X \frac{d \ln X}{dt} - s_M \frac{d \ln M}{dt} \qquad (2)$$

24. Alternatively, one could define a *value-added function*. A value-added function presents the maximum amount of current-price value added that can be produced, given a set of primary inputs and given prices of intermediate inputs and output. Such a value-added function is an equivalent (“dual”) representation of the technology described by a production function. For the present purpose, call the value-added function $G = G(A(t), X, P_M, P)$. Dependence of the value-added function on intermediate input prices P_M and on gross-output prices P signals that producers adjust the level of intermediate inputs when relative prices change. Just as the measure of technical change for the production function was defined as the shift of that function over time, productivity change could be defined as a shift of the value-added function, *i.e.* as the relative increase in value added that is associated with technical change. Parallel to the earlier statement regarding the production function, this can be formulated as $\frac{\partial \ln G}{\partial t}$. Again, this change cannot be directly observed but it can be shown that it corresponds to the difference between the growth rate of the Divisia volume index of value added[9] (called *VA*) and the growth rate of the Divisia index of primary inputs.

$$\text{\% change of value-added based MFP} = \frac{\partial \ln G}{\partial t} = \frac{d \ln VA}{dt} - \frac{d \ln X}{dt} \qquad (3)$$

25. This is a common way of measuring MFP based on value added. It turns out that there is a direct relation between the gross-output and the value-added productivity measure (Bruno, 1978). Specifically, the rate of change of value-added based MFP equals the rate of change of gross-output based MFP, multiplied by the inverse of the nominal share of value added in gross output:

$$\frac{\partial \ln G}{\partial t} = \frac{1}{s_{VA}} \cdot \frac{\partial \ln A}{\partial t} \quad with \;\; s_{VA} = \frac{G}{P \cdot Q} \qquad (4)$$

---

[9] There is of course no physical quantity that corresponds to value added. But it is always possible to define a volume index of value added as $\frac{d \ln VA}{dt} \equiv \frac{1}{s_{VA}} \left(\frac{d \ln Q}{dt} - s_M \frac{d \ln M}{dt} \right)$ where s_{VA} is the share of value added in gross output and s_M is the share of intermediate inputs in gross output. One notes, however, that this volume index may depend on the level of primary inputs, for example if the share s_{VA} depends on *X*. This could be interpreted as an undesirable property because it makes the measurement of output (volume of value added in this case) dependent on the measure of input (capital and labour in this case). To qualify as a measure of output that is truly independent of inputs, the underlying production function has to be separable in primary and intermediate inputs. The required separability conditions (see Goldman and Uzawa, 1964) can be quite restrictive but the right choice of index number formulae can partly overcome this problem.

Table 3. **Value-added and gross-output based productivity measures: an example**

Machinery and equipment industry, Finland
Averages of annual rates of change

	1990-98	1990-94	1994-98
Gross output (deflated)	10.1%	4.2%	16.0%
Value added (deflated)	9.5%	3.3%	15.8%
Labour input (total hours)	1.6%	-3.7%	6.9%
Capital input (gross capital stock)	3.0%	1.5%	4.5%
Intermediate inputs (deflated expenditure)	10.4%	4.8%	16.1%
Share of value added in gross output (current prices)	37.0%	38.9%	33.4%
Gross-output based productivity (KLEMS MFP)	2.7%	2.1%	3.3%
Value-added based productivity (capital-labour MFP)	7.8%	5.7%	9.8%

Note: The productivity estimates in this table are averages of annual data. As such, they cannot be exactly reproduced from the averages of the input-output data also presented in this table.
Source: OECD, STAN database.

26. Because the share of value added in gross output is smaller than or equal to unity, value-added based MFP growth for a particular industry will be systematically higher than gross-output based MFP measure for the same industry. Table 3 provides an empirical example from the Finnish machinery and equipment industry. The differences between value added and KLEMS MFP are quite large (corresponding to the inverted share of value added in gross output): over the 1990s, KLEMS-type MFP grew by 2.7% on average, value-added based MFP by 7.8%. This does not constitute a bias, but calls for an interpretation that is different from the gross-output based productivity measure. Several points are noteworthy here.

27. ***Value-added shares may not be constant***. The scaling factor $\frac{1}{s_{VA}}$ that links the two productivity measures is not in general constant over time. The numerator of the s_{VA} ratio, nominal value added, depends on the level of primary inputs and relative prices, as does current-price gross output, the denominator of this share. A constant rate of MFP growth measured on a gross-output basis could thus be perfectly consistent with an accelerating or decelerating rate of MFP growth measured on a value-added basis. This may be important, given that productivity analysts are often interested in the *acceleration* or *deceleration* of productivity growth, as for example in the case of the “productivity slowdown” in the years following 1973. The example from the Finnish machinery and equipment industry (Table 3) underlines this point. Between the first and the second half of the 1990s, the share of value added in current-price gross output dropped from 38.9% to 33.4%. A drop in the share of value added implies a rise in the scaling factor $\frac{1}{s_{VA}}$. Consequently, gross-output based productivity growth and value-added based productivity growth accelerate at different speeds. The first measure rises from 2.1% to 3.3% per year between the first and the second half of the 1990s, or by 1.2 percentage points. The value-added measure rises from 5.7% to 9.8% – that is, by 4.1 percentage points and significantly faster than the gross-output measure.

28. Different forms of technical change. For the production technology (1) with Hicks neutrality, the gross-output based productivity measure is a valid representation of disembodied technical change. This is not the case for the associated value-added based measure which depends also on the share of value added in gross output, and thus on the time paths of inputs, outputs, prices as well as the level of technology in the period under consideration. Rather than technical change itself, the value-added

based measure reflects an industry's capacity to translate technical change into income and into a contribution to final demand.

29. Note, however, that this interpretation of the gross-output and value-added based productivity statistics rests entirely on the assumption that the production function (1) is a valid representation of the production processes. Suppose that technical change does not affect all factors of production symmetrically ("output augmenting") but only operates on primary inputs ("primary input augmenting"). In this case, the value-added based measure becomes the independent and valid measure of technical change and the gross-output based measure loses its significance. Such a set-up requires that firms choose their input combinations in two stages: in a first stage, it is decided how to combine value added and intermediate inputs; in a second stage, a labour/capital mix is determined to generate value added.

30. The question arises as to which of the two formulations of technology, if any, commands sufficient empirical support. Generally, the hypothesis whereby technology affects only primary inputs has not held up to empirical verification. This makes it difficult to defend the value-added based productivity measure as an independent representation of disembodied technical change. However, the output-augmenting formulation of technical change, as represented by equation (1), has also not always been supported by econometric studies. This suggests a more complex working of technical change, with several, combined influences – one that affects all factors of production simultaneously ("output augmenting"), and others that affect individual factors of production separately ("labour, capital or intermediate input-augmenting"). Under such a general formulation it may well be that there is no independent productivity measure at all. Fortunately, the right choice of index number formulae can be of help here.

31. *Index numbers.* So far, the discussion has been conducted in continuous time (with Divisia indices). In practice, observations come in discrete intervals, and the statistician has to make choices about index number formulae so as to approximate the Divisia indices empirically. Later on in this manual (Chapter 7), it will be argued that "superlative" index numbers such as the Fisher Ideal or the Törnqvist index exhibit a number of advantageous features. One of these features is that, under certain conditions,[10] they provide a reasonable *approximation* to an independent measure of technical change even when technologies in practice do not show the simple, output-augmenting layout of equation (1).

32. *An example.* A numerical example is useful in this context. Consider the basic data in Table 4, which presents a simplified use table for two industries. Data are expressed in current prices, with the exception of employment that is given in hours worked. To keep things simple, only one primary factor, labour, is considered. Consequently, labour income equals value added in the present example. The data for the two time periods is set so as to reflect a process of outsourcing. Industry 1 uses products from industry 2 as an intermediate input. Between the two time periods, the price of product 2 declines relative to labour input, and industry 1 substitutes some of its labour input for the relatively cheaper intermediate inputs from industry 2. The converse holds for industry 2 that uses fewer intermediate inputs and more employment in period t_1 than in t_0. Given this set-up, it is now possible to compute value added and gross-output based productivity measures. Each measure is calculated both with a Törnqvist and a Laspeyres index number formula. Details regarding the calculation of productivity indices can be found in Chapter 9 (Implementation Guide).

---

[10] Diewert (1980, 1983) and Diewert and Morrison (1986) use superlative index numbers and approximate measures of technical change even when the underlying production function is not strictly Hicks neutral.

Table 4. **Numerical example: use tables for two industries**

t_0	Industry 1	Industry 2
Commodity		
1	0	10
2	6	0
Labour income	5	7
Gross output	11	17
Price index of gross output	1.00	1.00
Employment (hours)	10	8

t_1	Industry 1	Industry 2
Commodity		
1	0	8
2	7	0
Labour income	4	7.5
Gross output	11	15.5
Price index of gross output	1.01	0.98
Employment (hours)	7	9

33. Several observations can now be made regarding the productivity measures in Table 5. First, note that gross-output based MFP in industry 1 grows by 3.3%, whereas value-added based productivity grows by 8.0%, or more than twice as fast. If the gross-output measure reflects technical change, the rapid rise in the value-added based measure is due to outsourcing and not to an acceleration of technical change. However, the 8.0% productivity growth is an accurate reflection of that industry's increased capacity to translate technical change into a contribution to overall income and final demand. A different way of putting this same observation is that gross-output based productivity measures are less sensitive to the degree of outsourcing.

34. Second, just the opposite is true for *labour* productivity measures: on a gross-output basis, labour productivity of industry 1 increases by 34.7%, and that of industry 2 declines by 19%. The steep productivity rise in industry 1 reflects the fact that less labour is used and more intermediate inputs, but there is hardly a decline in gross output, so that gross output per hour worked rises very rapidly. Thus, when there is substitution between primary and intermediate inputs, this results in a change in labour productivity measured under a gross-output concept: gross output is unaffected and for each unit of labour there is now a larger amount of intermediate input. When labour productivity measures are based on value added, such a substitution reduces *both* labour input *and* value added and so reduces the sensitivity of labour productivity measures to the degree of vertical integration. Therefore, gross-output based labour productivity measures are more sensitive to the degree of vertical integration and outsourcing than value-added based labour productivity measures.[11]

---

[11] In the present numerical example, value-added based MFP growth equals value-added based labour productivity growth as there is only one primary input, labour. In practice, this is of course not the case.

Table 5. **Numerical example *(cont'd.)*: various productivity measures for two industries**

	Industry			Industry	
	1	2		1	2
Gross output			***Intermediate inputs***		
Value index	1.00	0.91	Value index	1.17	0.80
Price index	1.01	0.98	Price index	0.98	1.01
Indirect quantity index	0.99	0.93	Indirect quantity index	1.19	0.79
Level of gross output at constant t_0 prices	10.9	15.8	Level of intermediate inputs at constant t_0 prices	7.14	7.92
Index of gross output at constant t_0 prices	0.99	0.93	Index of intermediate inputs at constant t_0 prices	1.19	0.79
Labour input			***Value added***		
Index of employment	0.70	1.13	Index of value added at current prices	0.80	1.07
			Price index of value added	1.05	0.94
Share of value added in gross output			Index of deflated value added	0.76	1.13
t_0	0.45	0.41			
t_1	0.36	0.48	Level of value added at constant t0 prices	3.75	7.90
Average	0.41	0.45	Index of value added at constant t0 prices	0.75	1.13
Gross-output based MFP growth			***Value-added based productivity growth***		
Tornqvist index of combined labour and intermediate inputs	0.96	0.93	Tornqvist index of combined labour and intermediate inputs	0.96	0.93
Laspeyres index of combined labour and intermediate inputs	0.97	0.93	Laspeyres index of combined labour and intermediate inputs	0.97	0.93
Tornqvist index of productivity growth	1.03	1.01	Tornqvist index of productivity growth	1.08	1.01
Tornqvist index: % change	3.3%	0.5%	Tornqvist index: % change	8.0%	0.9%
Laspeyres index of productivity growth	1.02	1.00	Laspeyres index of productivity growth	1.07	1.00
Laspeyres index: % change	2.3%	0.1%	Laspeyres index: % change	6.9%	0.3%
Addendum:					
Gross-output based labour productivity			***Value-added based labour productivity***		
Tornqvist index	1.41	0.83	Tornqvist index	1.08	1.01
Tornqvist index: % change	34.7%	-19.0%	Tornqvist index: % change	8.0%	0.9%
Laspeyres index	1.41	0.83	Laspeyres index	1.07	1.00
Laspeyres index: %change	34.7%	-19.0%	Laspeyres index: %change	6.9%	0.3%

35. Third, the present example shows the sizeable differences between index numbers. Gross-output based MFP in industry 1, calculated with a Törnqvist index, rises by 3.3%, whereas the Laspeyres-type calculation produces a mere 2.3% change. For industry 2, the Törnqvist case of 0.5% compares with a Laspeyres-type measure of 0.1%. Differences in the results based on different index number formulae are also sizeable for value-added based productivity measures.[12]

---

[12] There are no differences for gross-output based labour productivity measures because a single, homogenous output and a single type of labour input appear in the numerical example. Otherwise, the measures would not coincide.

36. In conclusion, it would appear that gross-output and value-added based MFP measures are useful complements. When technical progress affects all factors of production proportionally, the former is a better measure of technical change. Empirically, it is important to base productivity calculations on superlative index number formulae because they provide approximations to independent measures of outputs, inputs and technical change. Generally, gross-output based *MFP measures* are less sensitive to situations of outsourcing, *i.e.* to changes in the degree of vertical integration between industries. Value-added based MFP measures vary with the degree of outsourcing and provide an indication of the importance of the productivity improvement for the economy as a whole. They indicate how much extra delivery to final demand per unit of primary inputs an industry generates. When it comes to *labour productivity*, value-added based measures are less sensitive to changes in the degree of vertical integration than gross-output based measures. Practical aspects also come into play. Measures of value added are often more easily available than measures of gross output although in principle, gross-output measures are necessary to derive value-added data in the first place. Consistent sets of gross-output measures require dealing with intra-industry flows of intermediate products, which may be difficult empirically (see Section 3.1.3).

3.1.3. Intra-industry flows of products

37. When a gross-output concept is adopted for productivity measurement at the industry level, the question arises how to treat those transactions that occur within industries, *i.e.* intra-industry deliveries of intermediate inputs. It is not difficult to see that the inclusion of intra-industry flows of intermediate products adds identically to both the input and output side of an industry production function [as in (1) where both Q and M change with the inclusion or exclusion of intra-industry deliveries]. This is a form of double-counting and, in principle, output and intermediate inputs can be made larger and larger by basing industry aggregates on increasingly smaller statistical units: an industry-output measure based on establishments would be larger than one based on firms and one based on firms larger than one based on groups, etc.[13] The exclusion of intra-industry deliveries circumvents these problems. Industry-level output measures that exclude intra-industry deliveries have been labelled *sectoral output* (Gollop, 1979; Gullickson and Harper, 1999b).

38. Conceptually, adoption of such measures of sectoral output (and the corresponding measures of sector input) amounts to a process of *integration* of different units or industries – as one moves up the hierarchy of the activity classification, larger and larger units are formed and treated as if they were single firms. At every level of aggregation, only flows out of or into the sector are considered. Sectoral output is also consistent with the notion of "output" in the SNA 93 that defines it as those goods and services that become available for use *outside* the establishment (industry). At the level of the entire economy, measures of sector output and of value added converge, although not entirely in the presence of imported intermediate inputs. The sector output concept offers one possibility for consistent aggregation of gross-output based MFP growth across industries.

39. One implication of using a concept of sectoral output is, however, that growth rates of components cannot be compared to their aggregate. As discussed in greater detail in Chapter 8 on aggregation, productivity measures for aggregates are built up as weighted *sums* (but not *averages*) from their components. Thus, a 1% growth of MFP in all individual industries may lead to a 1.5% increase of the (integrated) total economy. This reflects the fact that the new aggregate cumulates productivity gains from intra-industry deliveries. Under these circumstances, it is difficult to compare

---

[13] Strictly speaking, this statement is only correct when all firms of a group of firms are classified in the same industry and when all establishments of a firm are classified in the same industry. This is not the case in practice but the basic point of dependency on the choice of units remains.

productivity growth of component industries with that of the aggregate. Value-added based productivity measurement is a way to avoid dealing with intermediate inputs in the process of aggregation. Current-price values of value added can simply be summed up across different units, without regard to any inter-industry flows of inputs. Quantity indices of value added can be aggregated by forming weighted *averages*, with weights adding to unity. Value-added based productivity measures of aggregates are also weighted averages of their components and can be compared across levels of aggregation.

3.2. Depreciation

40. Another point of debate in the 1970s and 1980s was whether output should be measured net or gross of *depreciation*. Depreciation measures the loss of the market value of a capital good between consecutive periods. One notes that this gross/net distinction of output relates to depreciation and not to the treatment of intermediate inputs. Denison (1974) advocated a concept of output net of economic depreciation on the grounds that it traces improvements in welfare more closely than output measures that are gross of depreciation. A group of researchers, including Dale Jorgenson and Zvi Griliches, on the other hand, argued that output must be measured gross of depreciation if it is to conform to the logic of production theory. Hulten (1973) provided a theoretical underpinning for the Jorgenson/Griliches approach. Today, a large majority of productivity research uses output measures gross of depreciation.[14]

3.3. Quantity measures of output

41. Different methodologies to obtain quantity series of output can significantly shape the outcome of productivity measurement. Quantity indices of output are normally obtained by dividing a current-price series or index of output by an appropriate price index (deflation). Only in a minority of instances[15] are quantity measures derived by direct observation of volume output series. Measurement of volume output is therefore tantamount to constructing price indices – a task whose fuller description far exceeds the scope of the present manual. We refer to the *Eurostat Handbook on Price and Volume Measures in National Accounts* (Eurostat, 2001) for a more in-depth treatment of these issues. The following sections will focus on the more general issues of single and double deflation and address the treatment of quality change in products in Box 2. Closely related to the calculation of price indices are matters of index number formulae – a topic dealt with in Chapter 7.

---

[14] One of the reasons for advocating an output measure gross of depreciation has been the need for a consistent treatment of capital input as a flow of capital services (see Chapter 5). The price of capital services (user costs) includes a depreciation component, and if depreciation is part of an input measure, it should also be part of the output measure. However, in a comment on a draft of this manual, Erwin Diewert points out that the user cost term could be split into two parts: depreciation – which could then appear as an intermediate input; and the net real return to capital (nominal interest less capital gains or losses) – which could be considered the primary input cost of capital. The overall quantities of capital services would remain unchanged and a measure of output net of depreciation would be compatible with a measure of capital services and user costs.

[15] For a discussion regarding the United States, see Eldridge (1999).

3.3.1. *Deflation of value added*

42. Deflation of gross output is conceptually straightforward. An index of the nominal value of output is divided by an output price index to yield an (indirect) volume or quantity index of gross output. Deflation becomes somewhat more complicated when output measures are based on value added. As indicated earlier in this chapter, production theory leads the way to consistently defined price and quantity indices of value added. Specifically, the volume change in value added can be defined[16] as an average of the volume change of gross output ($\frac{\partial \ln Q}{\partial t}$) and the volume change of intermediate inputs ($\frac{\partial \ln M}{\partial t}$). The volume change of intermediate inputs is weighted by the share of intermediate inputs in gross output ($\frac{P_M M}{PQ}$) and the entire expression is multiplied by the inverted share of value added in gross output ($\frac{PQ}{P_{VA}VA}$). This is shown in expression[17] (5).

$$\frac{d\ln VA}{dt}=\frac{PQ}{P_{VA}VA}\left(\frac{d\ln Q}{dt}-\frac{P_M M}{PQ}\frac{d\ln M}{dt}\right) \tag{5}$$

43. Because the volume change for value added combines the volume change for gross output and intermediate inputs, it constitutes a general-form double deflation. However, to turn Divisia indices into operational measures, they have to be approximated empirically. One procedure is double-deflation in a more narrow sense, where the volume measure of value added is obtained by *subtracting* a constant-price value of intermediate inputs from a constant-price value of gross output. This corresponds to an approximation of the Divisia index by a fixed-weight Laspeyres quantity index. In this case, expression (5) is measured as in equation (6) where all variables are expressed in prices of a specific base year:

$$\frac{\Delta VA_t}{VA_{t-1}}=\frac{Q_{t-1}}{VA_{t-1}}\left(\frac{\Delta Q_t}{Q_{t-1}}-\frac{M_{t-1}}{Q_{t-1}}\frac{\Delta M_{t-1}}{M_{t-1}}\right) \tag{6}$$

44. This form of double deflation requires that constant price values of intermediate inputs can be subtracted from constant price values of gross output[18] ($VA_t = Q_t - M_t$) – and this is only possible with Laspeyres quantity indices (or Paasche price indices). As mentioned earlier (Section 3.1.2), the use of fixed-weight Laspeyres quantity indices raises a number of problems, and implicitly poses restrictive assumptions on the underlying production technology. The situation is different when the empirical approximation to the Divisia quantity index is based on superlative index numbers such as the Törnqvist index (see Chapter 9, Implementation sheet 3).

---

[16] As pointed out earlier, it is always possible to construct this volume index of value added, which, conceptually, constitutes a measure of output. Depending on the form of the underlying production function, this output index may or may not be independent of primary inputs.

[17] Alternatively, a price index of value added could be defined and then used to deflate the current price value. In continuous time, the two approaches yield the same result. In empirical approximations, this need not be the case.

[18] For an alternative measure of real value added, see Durand (1994).

45. *Possibility of negative values*. Another issue is the occasional occurrence of negative value-added figures when double deflation operates with Laspeyres quantity indices. Nothing ensures that the subtraction of constant-price intermediate inputs from constant-price gross output yields a positive number. The SNA 93 notes that negative real value added can occur when relative prices change: "a process of production which is efficient at one set of prices, may not be very efficient at another set of relative prices. If the other set of prices is very different, the inefficiency of the process may reveal itself in a very conspicuous form, namely negative value added". Despite this insight, the fact remains that such data are difficult to interpret and use in a context of productivity measurement. In these circumstances, a different accounting method should be used to estimate an aggregate like value added, such as the methods based on "superlative" index numbers.[19]

46. *Sensitivity to value-added shares in gross output*. A third issue associated with double deflation is the sensitivity of the growth rates of value added with respect to the rate of change of gross output or intermediate input when the share of intermediate inputs in gross output is large.[20] For a small value of the share of value added in gross output ($\frac{P_{VA}VA}{PQ}$), the inverse of this expression becomes very large and may give rise to significant variations in the rate of change of deflated value added, even if the rates of change of gross output and intermediate inputs vary only by little. For example, a 2% rise in gross output and a 1% rise in intermediate inputs leads to an 11% rise in value added, if the initial share of value added in gross output is 10%. Value added grows by only 7.7% if its share of gross output is 15% while a 7% share of value added raises value-added growth to over 15%. The likelihood of the occurrence of small value-added shares rises with the level of industry detail and disappears at the aggregate level of the economy.

47. Methods of single deflation use a single price index to deflate current-price series of value added. The price index used for single deflation is a gross-output price index, a consumer price index, or its relevant components. It is not difficult to show that as the wedge between double-deflated and single-deflated value added increases, the less stable the share of intermediate inputs in gross output becomes. In general, single deflation constitutes an inferior alternative to double deflation, especially when the latter uses a chained or superlative index number formula.

3.3.2. The need for independent estimates

48. An important point for the validity of productivity measures is that price and quantity indices of output should be constructed independently of price and quantity indices of inputs. Such dependence occurs, for example, when quantity indices of outputs are based on extrapolation of some input series. Extrapolation relates to applying quantity indicators to carry forward and backward real value-added series. Such quantity indicators may be inputs to the industry under consideration, in particular observations on employment. For example, data on the total production for an industry sometimes comprise estimates for those establishments which have not been included in the survey that serves as the main statistical source for the national accounts estimate. Estimates for those missing establishments are sometimes made by applying productivity assumptions to employment statistics from Labour Force Surveys. Input-based extrapolation is more frequent and quantitatively more important for service industries than for other parts of the economy (see OECD, 1996b, for a survey of methods in OECD countries).

---

[19] To be precise, the occurrence of negative values from double deflation is an indication that the additive form of the production function that double deflation implies is not consistent with the data. If this is the case, the specification error implicit in double deflation yields negative values.

[20] This point was raised by Hill (1971).

49. In other instances, output-related measures are used to extrapolate real value added. Though often imperfect, it is apparent that the implied bias for productivity measurement is less severe than in the case of input-based extrapolation. For example, Eldridge (1999) reports that, in the United States, the quantity indicator for auto insurance expenditure is the deflated value of premiums, where deflation itself is based on a component index of the CPI. In other instances, physical output data are used as the quantity indicator: the United States quantity indicator for brokerage charges is based primarily on BEA estimates of orders derived from volume data from the Security and Exchange Commission and trade sources (Eldridge, 1999).

50. From the perspective of productivity measurement, the independence of statistics on inputs and outputs is key. Input-based indicators that are used to deflate output series generate an obvious bias in productivity measures: (labour) productivity growth will either be zero by construction or will reflect any assumption about productivity growth made by statisticians. Occurrences of input-based extrapolation are concentrated in activities where market output prices are difficult to observe. This may create a case for excluding from productivity measurement those industries that are characterised by a large share of non-market producers, thereby avoiding potential biases in output measurement.

51. Genuinely new items within a product group are normally linked into the sample of observations some time after their occurrence on the market. However, in technologically dynamic industries, new products' prices often fall very rapidly and before they are linked into the sample. A price index will then not pick up the initial fall in prices. Immediate introduction of new items, on the other hand, poses the problem of reservation prices, *i.e.* the imputation of hypothetical prices for the new items in the preceding period when they were still unavailable. One method to obtain such hypothetical prices, both for new goods and for existing ones, is the hedonic method (see Box 2).

3.3.3. Quality change and new products

52. The rapid development of information and communication technology products has brought to centre-stage two long-standing questions of price measurement: how to deal with quality changes of existing goods and how to account for new goods in price indices.[21] The distinction between these two issues is blurred because it is unclear where to draw the borderline between a "truly" new good and a new variety of an existing good.[22]

53. Economically, new varieties of existing goods as well as quality improvement of existing goods can be considered as special cases of "new goods" if this notion is defined broadly enough: the emergence of new varieties of existing goods is a case of horizontal differentiation, quality improvement a case of vertical differentiation (where the product with inferior quality may or may not disappear) and the emergence of entirely new goods spans a new dimension in product space. There is a continuum from simple varieties of existing goods (such as producing the same car in an additional colour), over their quality improvement (such as supplying more powerful computers) to entirely new products (DVD players, mobile phones).

---

[21] See the *OECD Handbook on the Quality Adjustment of Price Indices for ICT Products* (OECD, forthcoming) and the *Eurostat Handbook on Price and Volume Measures in National Accounts* (Eurostat, 2001).

[22] For an overview, see Bresnahan and Gordon (1996).

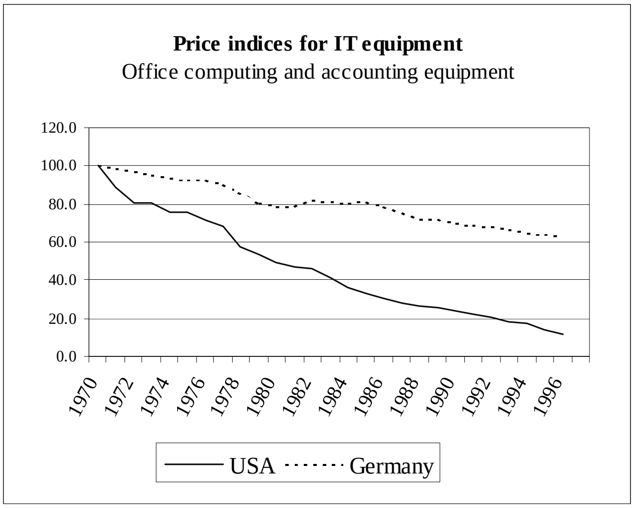
Box 2. **Hedonic price indices**

The dramatic improvements in the quality and technical capacity of many information and communication technology products constitute a significant challenge for statisticians. The traditional comparison of identical models between two periods may not capture price movements that take place for all models, in particular those that newly enter the market. When non-identical models are compared it is often difficult to separate price and quality change.

The hedonic approach is one of the tools for quality-adjustment.* Essentially, it redefines goods in terms of their characteristics so that modified or new models do not open up a new product category but simply represent a new combination of characteristics. Thus, to some extent, the shift to characteristics does away with the question of how to deal with new goods; at least as long as new goods do not incorporate fundamentally new characteristics. In the case of computers, for example, typical characteristics are speed, memory size and so forth. Empirically, a hedonic function is estimated, relating observations of prices of computer "boxes" to their respective characteristics. One of the uses of the hedonic function is to estimate so-called "reservation prices" of new models, *i.e*. an indication of how much a new model would have cost in a previous period had it been available. Alternatively, price changes can be obtained directly from hedonic regressions.

Despite its interesting features, the systematic uptake of hedonic methods in national price statistics has so far been confined to a small number of countries and products. One of the reasons** is that the construction of hedonic deflators tends to be costly for statistical offices because a sizeable amount of primary data must be gathered, evaluated by specialists and treated using a comparatively resource-demanding econometric methodology. This gives rise to fixed costs that weigh more heavily in the statistics budget of small countries than larger ones. Reservations against hedonic price indices also exist when they are used in the context of fixed-weight price indices. The substitution bias implicit in fixed-weight price indices is compounded when there are large changes of relative prices such as the ones induced by rapidly falling computer prices. This bias is minimised when price or quantity indices are based on index numbers with flexible weights, such as the Fisher ideal index or the Törnqvist index (see Section 7.1).

To illustrate, consider the graph below which plots the United States (investment) price index for office, computing and accounting machinery (based on hedonic methods) against the closest equivalent component of the German producer price index (not based on hedonic methods). Differences are striking and show that international comparisons of output and productivity measures in information technology industries have to be interpreted very cautiously.



* See Triplett (1989) for a comprehensive treatment.
** OECD (forthcoming), *OECD Handbook on the Quality Adjustment of Price Indices for ICT Products*.

54. Typically, statistical agencies derive price indices for products by observing price changes of items in a representative sample. New products, quality change and new variants are common phenomena in the observation of price changes of items and statistical offices have well-established procedures to deal with them.[23]

55. A frequent occurrence is the need for substitution of particular items that are replaced by new models. The new model or variety is compared to the old one, and a judgement is made to which extent any price difference between the two should be considered a change in quality or a change in prices. However, if quality improvements are larger than the observed price difference, quality changes will be under-valued and price changes overstated. This can only be avoided through the explicit imputation of a quality-adjusted price (derived, for instance, from hedonic methods) for the replacement item. Restricting the sample to models that are identical between two periods can isolate pure price changes of these established models but fails to be representative for an entire product group if the established models' price changes fail to duplicate the price changes of new models – a situation that is frequently encountered in markets for information technology products.

56. A related problem is the judgement about the nature of new items: are they highly substitutable with existing ones or are they actually a new type of item within a product class? In the first case, they should be treated jointly with another item; in the second case, a separate sub-category should be opened for them. This choice can have a sizeable influence on the resulting price index as was shown by Berndt *et al.* (1996) for the case of branded and generic pharmaceuticals.

57. The treatment of quality change has far-reaching consequences for productivity measurement. One obvious impact is on the volume measures of output where understatement of quality change leads to an understatement of output and productivity growth. Moreover, measures of real inputs – capital input or intermediate inputs – are also implicated. Understatement of quality change in these products implies an understatement of real inputs and an overstatement of productivity growth. There is no straightforward answer to the eventual effects on industry-level productivity measures and a more complete assessment requires analysis based on input-output techniques.[24]

58. Finally, it should be pointed out that treatment of quality change in the price indices of individual outputs and inputs cannot be dissociated from the choice of index numbers. Dean *et al.* (1996) examine how different forms of index numbers carry over to the assessment of trends in outputs, inputs and productivity in the United States, and conclude that effects are substantial. A more extensive discussion of index numbers can be found in Chapter 7.

3.4. Statistical sources and statistical units

59. The prime statistical source for industry-level measures of output is the production side of national accounts. Conceptually embedded in a wealth and income accounting system, it provides a link to other statistical tools, in particular input-output tables. It is broken down by activity, following classifications such as ISIC (at the international level), NACE (in the European Union), NAICS (in NAFTA countries), or national systems of industry classification. Nonetheless, in practice, production accounts can suffer from certain shortcomings. For examples, some countries construct industry-level value added top-down, *i.e.* starting from an aggregate measure of GDP that is based on the income or

---

[23] For example, Lowe (1996) provides an overview of how quality change is handled in the Canadian National Accounts.

[24] See Gullickson and Harper (1999a, 1999b) for a general treatment or Triplett (1996a) for the computer industry.

expenditure side of national accounts. Also, the quality of the concordance of industry-level measures of output and input can be uncertain, because primary sources for data on production may be different from primary sources on employment, investment, or intermediate inputs.

60. The direct use of censuses and annual surveys has also been suggested as an alternative source of input and output data (Gersbach and van Ark, 1994). Surveys such as the structural business surveys in EU countries constitute a single data source for both inputs and outputs. Although surveys are themselves the major source for national accounts estimates, their direct use ensures that observations on inputs and outputs relate to exactly the same population of firms or establishments. Also, business surveys typically offer significantly greater industry detail than those used and published in national accounts. Of course, survey data may suffer from the fact that its coverage of statistical units within each industry is sometimes incomplete and potentially biased when underlying samples comprise only firms or establishments with certain characteristics, such as minimum size. Nonetheless, survey data are, if not a substitute, a valuable complement for the user of national accounts.

61. Productivity measures, in particular by industry, are sensitive to the choice of statistical units. The choice of the statistical unit relates to the question of partitioning enterprises into smaller and more homogenous units with a view to forming industry groupings of similar activities. In this context, the SNA 93[25] defines enterprises, establishments, local units and kind-of-activity units. The latter stand for a part of an enterprise, which engages only in one kind of productive activity or in which the principal productive activity accounts for most of the value added. Each enterprise must, by definition, consist of one or more kind-of-activity units. Each of these units must be more homogenous with respect to output, cost structure and technology than the enterprise as a whole. Local units relate to the parts of an enterprise that engage in productive activity from one particular location. The establishment combines the kind-of-activity and the locality dimension as that part of an enterprise that is situated in a single location and in which only a single productive activity is carried out or in which the principal productive activity accounts for most of value added. Establishments are designed to be units that provide data suitable for analyses of production in which the technology of production plays an important role.[26] This makes the establishment and the kind-of-activity unit a particularly suitable choice for productivity analysis.

62. However, not all countries' national accounts are based on establishments or kind-of-activity units. When accounts are based on enterprises, this may limit international comparability with productivity statistics that are based on establishments: for any given industry or activity class, establishment-based data represent a more homogenous group of units than enterprise-based data because the latter comprise both primary and secondary activities of units. No statements can be made about size and direction of the difference between establishment- and enterprise-based productivity statistics and international comparisons have to be interpreted with the necessary caution.

---

[25] *System of National Accounts 1993,* paras. 5.17-5.47.

[26] *System of National Accounts 1993,* para. 5.23.

4. LABOUR INPUT

Overview: labour input measures

The quantity of labour input in production is best measured by hours worked…

Labour remains the single most important input to many production processes. From a perspective of production analysis, and ignoring quality differences for the moment, labour input is most appropriately measured as the total number of hours worked. Simple head-counts of employed persons will hide changes in average hours worked, caused by the evolution of part-time work or the effect of variations in overtime, absence from work or shifts in normal hours. However, a number of statistical issues arise regarding the measurement of hours actually worked. One of them is the best use of available statistical sources, in particular establishment and household surveys. Consequently, the quality of hours-worked estimates, and their degree of international comparability, are not always clear.

Notwithstanding some of the measurement issues, it is recommended that hours actually worked be the statistical variable used to measure labour input, as opposed to simple head counts of employed persons. Hours paid and full-time equivalent persons can provide reasonable alternatives. Significant differences in country practices for calculating hours worked and full-time equivalent persons persist, and raise issues of international comparability.

☛ More on the choice of units for measuring labour input in Section 4.1.

☛ More on alternative statistical sources in Section 4.2.

☛ More on measurement issues associated with hours worked in Section 4.3.

…and its price by average compensation per hour.

Conceptually, labour income and labour shares should reflect the compensation paid to labour from a producer's point of view, *i.e.* including supplements to wages and salaries such as employers' contributions to social security payments. However, as with "hours actually worked", a seemingly straightforward concept gives rise to many conceptual and empirical questions when it actually comes to measuring it. This concerns issues such as the treatment of non-wage parts of income for employees (*e.g.* stock options) or the treatment of the self-employed.

Specifically, the calculation of labour shares in income has to allow for the fact that labour income of self-employed persons is only a part of the "mixed income" assigned to them in the national accounts. A simple procedure to adjust labour shares is based on the assumption that the self-employed earn the same average compensation as employees in the same industry.

☛ More on the measurement of compensation in Section 4.4.

Desirable but difficult: differentiation of labour input by skill category.

Because a worker's contribution to the production process consists of his/her "raw" labour (or physical presence) and services from his/her human capital, one hour worked by one person does not necessarily constitute the same amount of labour input as one hour worked by another person. There may be differences in skills, education, health and professional experience that lead to large differences in the contribution of different types of labour. A differentiation of labour input by type of skills is particularly desirable if one wants to capture the effects of a changing quality of labour on the growth of output and productivity. Explicit differentiation is, however, data- and research-intensive. As a minimum, time series of hours worked, broken down by one differentiating characteristic have to be available, alongside corresponding statistics for average compensation, broken down by the same characteristic. Measurement problems are compounded when explicit differentiation of labour input by industry is sought. Aggregation of undifferentiated labour input across detailed industries can provide some form of implicit differentiation.

☞ More on differentiation between different types of labour in Section 4.5.

4.1. Choice of units

63. In the spirit of production theory, and disregarding quality differences for the moment, labour input for an industry is most appropriately measured as the number of hours actually worked. The simplest, though least recommended, measure of labour input is a head count of employee jobs. Such a measure neither reflects changes in the average work time per employee nor changes in multiple job holdings and the role of self-employed persons (nor in the quality of labour).

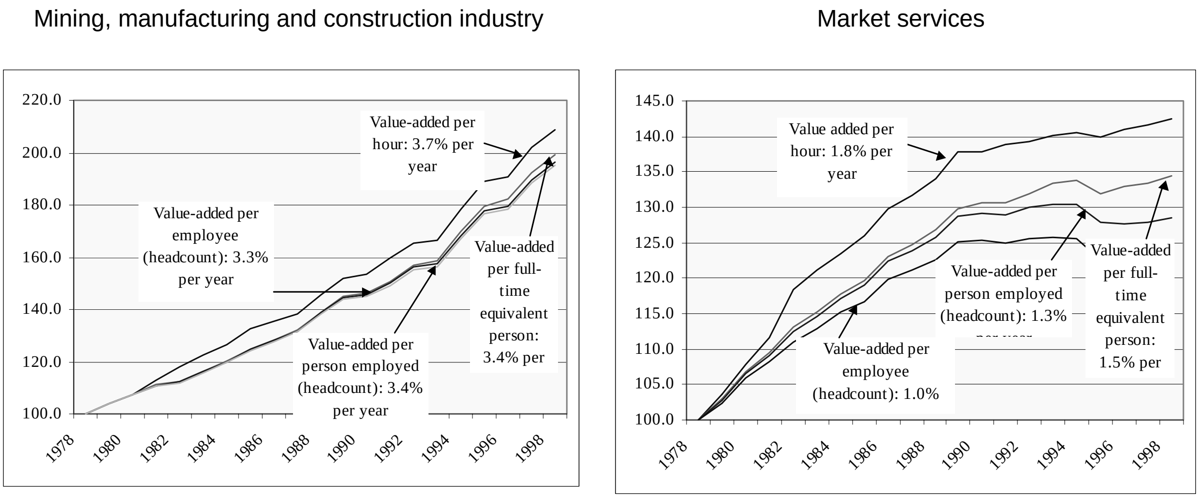
64. A first refinement to this measure is its extension to total employment, comprising both wage and salary earners, and the self employed (including contributing family members). A second refinement is the conversion from simple job (or person) counts to estimates of total "hours actually worked". Rates of change of the number of persons employed differ from the rates of change of total hours worked when the number of average hours worked per person shifts over time. Such shifts may be due to a move towards more paid vacations, shorter "normal" hours for full-time workers and greater use of part-time work. These developments have taken place in many OECD countries and underline the importance of choosing "hours actually worked" as the variable for labour input in productivity measurement because it bears a closer relation to the amount of productive services provided by workers than simple head counts.

65. An example of the impact on labour productivity measures of choosing different measures for employment is given in Figure 1 below. For France, for the period 1987-98, labour productivity indices were calculated using total hours, the number of full-time equivalent persons, the number of employed persons (head counts) and the number of employees (head counts). Results are presented for industry (comprising mining, manufacturing and construction) and for market services. Not surprisingly, the productivity measures based on total hours rise significantly faster than those based on other employment measures. In industry, correcting for part-time employment hardly changes the productivity series. This is quite different for the service sector where part-time employment plays a more important role. Even more pronounced are the effects of including or excluding the self-employed in the service sector, as reflected by the differences in productivity estimates based on total employment and based on the number of employees only.

66. Full-time equivalent jobs (or persons) is another variable sometimes used for measuring labour input. By definition, full-time equivalent employment is the number of total hours worked divided by average annual hours actually worked in full-time jobs. Conceptually, then, in full-time

equivalent measures, part-time employed persons are counted with a smaller weight than are persons working full time. Consequently, the full-time equivalent measure should avoid the bias arising from a shifting share of part-time employment in the workforce but will not adjust for changes in the number of hours which constitutes a full-time job, *e.g.* as a consequence of changes in legislation or collective agreements. In addition, methodologies underlying the construction of full-time equivalent persons (or jobs) are not always transparent and may vary internationally. For example, crude estimates are sometimes made whereby the number of part-time jobs (themselves often defined as all jobs with less than normal working hours) is simply counted as half a full-time job.

Figure 1. **Labour productivity[1] based on different measures of employment in France**



Mining, manufacturing and construction industry

Market services

1. Output is measured as a quantity index of value added.
Source: INSEE.

4.2. Statistical sources[27]

67. There are several statistical sources for measures of labour input, including *household-based labour force surveys* (LFS) and *establishment or firm-based surveys* (ES). LFS are typically conducted from a socio-economic perspective to provide reliable information about personal characteristics of the labour force, such as educational attainment, age, or the occurrence of multiple job holdings, as well as information about the jobs (*e.g.* occupation and type of contract). Also, LFS have the advantage of full coverage of the economy, although for purposes of domestic productivity measurement, some adjustment may be needed in cases where the number of cross-border workers or workers in institutional households is important.

68. *Establishment-based or enterprise surveys* are conducted from a production perspective, and describe labour as an input factor. One distinguishing feature of establishment surveys is that they gather information on jobs rather than on persons employed, thus persons who have jobs in more than

---

[27] Most of the information presented here draws on work carried out by the OECD Directorate for Education, Employment, Labour and Social Affairs. In particular, use is made of a 1998 document prepared for the Working Party on Employment and Unemployment Statistics on *Annual Hours of Work: Definitional and Comparability Issues* and of the notes to the annual *OECD Employment Outlook.*

one establishment will be counted more than once. Another feature is that ES will often only cover a subset of all establishments in an industry, normally those above a certain size limit. If establishments included in the survey have systematically higher productivity levels than those excluded, productivity estimates based on ES will inadequately reflect the effects of the size composition in an industry. Along with the number of jobs, information is normally also gathered on labour compensation.

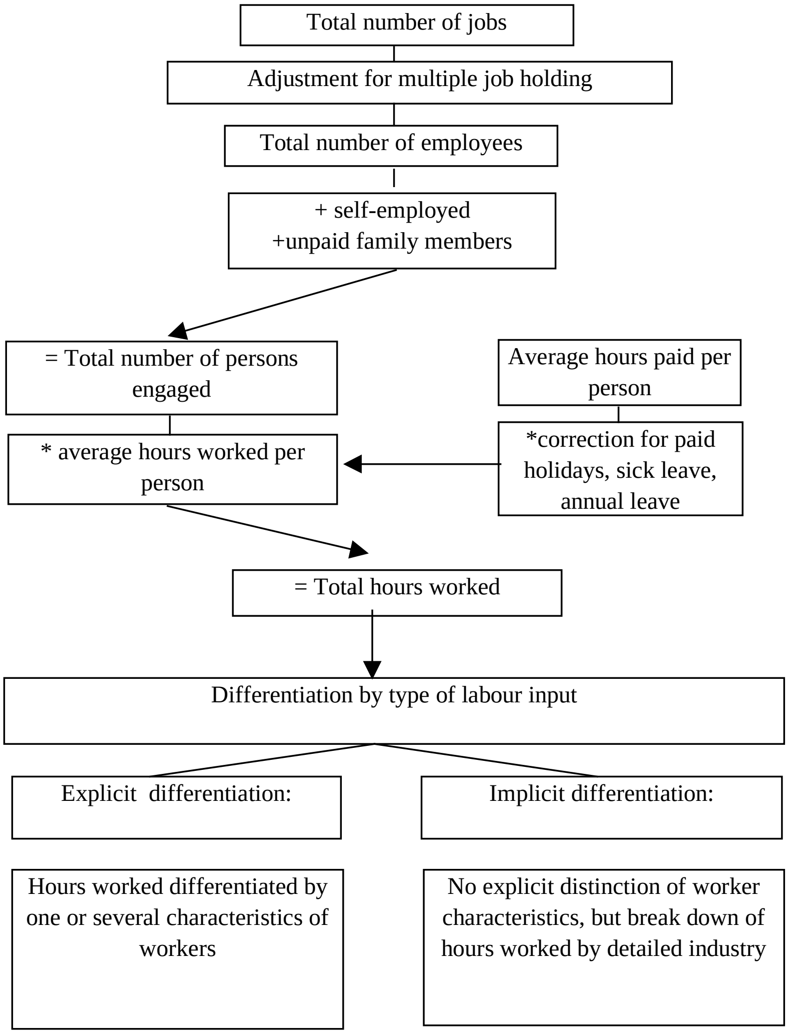
69. The respective strengths and weaknesses of LFS and ES as statistical sources must be carefully evaluated with the use of these statistics for estimating a measure of "hours actually worked" for an industry. The advantage of LFS-based statistics are better coverage of all establishments and jobs and the better and more complete information about productivity-relevant characteristics of the workers, such as age, experience, educational attainment and occupation. This also includes more accurate information about the hours actually worked during the reference period for the survey. Depending upon the frequency and scheduling of the LFS during the period for which productivity estimates are to be produced, it may be necessary to adjust the direct estimates to take into account, for example vacation periods, if they are not included in the surveys.

70. In addition to the concordance of units for the production and employment variables, the main advantages of the ES are linked to the fact that in most cases the information about the units are provided on the basis of written audited records for the whole reference period for which the productivity estimates are made, normally a calendar year. When the accounting period is linked to the fiscal year, which may be different from the statistical reference period, adjustments are necessary. Other, sometimes complicated ,adjustments may have to be made because the records which firms keep reflect the information that they themselves and the regulatory and fiscal authorities consider important and which do not necessarily correspond to the information required for valid and reliable statistics. For example, information on unpaid overtime is not easily obtained from firms' records, which tend to record "hours paid" rather than hours actually worked, and thus may include periods of paid vacation and other paid absences. Another potentially weak point of ES is their dependence on business registers whose quality shapes the quality of the business survey, and tend to vary between industries. Finally, ES rarely include the self-employed even in those establishments which are covered and often omit certain sectors of the economy (*e.g.* agriculture).

71. It is also important to be aware of differences between the coding of "industry" in LFS and ES. In LFS, industry coding is either done entirely on the basis of information given by the respondent about the type of product, service or function provided by his/her place of work, or it is partly obtained through a link to the business register based on information about the name and address of the place of work. In an ES, the coding is directly based on information from the reporting unit about its main product, services and functions and their relative importance. Alternatively, coding in ES uses similar information from the business register to determine the correct industry code. Thus, ES tend to provide better statistics on the industry distribution of employment, although the quality of this information depends directly on the quality of the business register and the coverage of units.

72. In a number of OECD countries, statistical offices combine the different sources to obtain industry-level information on employment for national accounts. From the perspective of the productivity analyst, it follows that national accounts data on employment that makes use of the comparative advantages of different statistical sources should generally be preferred over employment data from any single source.

Figure 2. **Measures of labour input**



Source: OECD.

4.3. Measuring hours worked

73. *Normal hours.* When starting from statistics provided by establishment surveys, one may want to use as a starting point the concept of “normal hours”. In ES, this corresponds to negotiated or contractual hours for staff that normally are not entitled to paid overtime, and paid hours which may include overtime hours for those that are entitled to such pay. In LFS, questions are usually asked about actual hours worked during the reference period and about usual or normal hours of work. Uncertainties about the exact nature of data on both “actual” and “normal” hours arise especially for persons who are not paid on an hourly basis and who may or may not include as “normal hours” regularly occurring hours of overtime, whether paid or unpaid, in their responses. Conceptually, normal hours in labour force surveys typically include regularly occurring hours of overtime. Where evidence exists on “normal hours” from both establishment and household surveys, the latter tend to exceed the former, consistent with the inclusion of regular overtime in labour force statistics.

74. *Overtime and absences*. In a second step, variations from standard hours are assessed to obtain measures of actual hours worked. In establishment surveys, reporting on hours typically shows paid hours of work and paid hours of absence. The difference between those two will, in general, correspond to actual hours of work only for workers paid on an hourly basis. The situation may be less clear for salaried workers, supervisory personnel and managers for whom the paid hours reported may correspond to a conventional norm which may not represent the actual hours worked by persons in this group.

75. The most important forms of absences from work are holidays, sick leave and vacation time. Country practices vary in the estimation of these items, partly reflecting differences in their status and prevalence in different industries. For example, vacation and holiday time may be estimated on the basis of specifications in legislation or wage agreements, or may actually be derived from information obtained in labour force surveys. Labour force surveys potentially pick up extra hours worked by managers and professionals that are over and above the conventional hours. At the same time, there is evidence, from time use surveys, that persons who report working long hours tend to overestimate their working time. In the aggregate this may be offset to some extent, but not necessarily in the same industry, by a tendency for respondents to LFS to under-estimate part-week absences, in particular those due to illness and holidays.

76. In sum, it is difficult to draw any firm conclusions about the quality and international comparability of the statistics on annual hours actually worked. It has to be kept in mind, however, that possible biases have to change over time in order to affect measures of labour input growth. Efforts by statistical offices to combine information from both establishment and household surveys, with a view to making use of the most reliable aspects of each of the surveys, appears to be a most promising avenue towards improving the quality and consistency of data on labour input.

4.4. Labour compensation and labour shares

77. With “hours worked” as the quantity measure of labour services, average compensation per hour will be its price component. Average hourly compensation corresponds to the wage rate from a producer’s point of view, provided it includes all supplements to wages and salaries. The generation of income accounts in the national accounts provide data on the compensation of employees, and (residually) on capital income where the latter includes mixed income of the self-employed, consumption of fixed capital and the operating surplus of the corporate sector, net of taxes on production minus subsidies. Several issues arise in the measurement of labour compensation.

78. *Allocating mixed income*. The residual capital income in the generation of income account has two components: one is the net operating surplus of corporate and quasi-corporate enterprises and as such undoubtedly part of capital income, the other is “mixed income” or the income that accrues to unincorporated enterprises owned by members of households, *i.e.* to self-employed persons.

> *“In a few cases it may be possible to estimate the wage or salary element implicitly included with mixed income, but there is usually not enough information available about the number of hours worked or appropriate rates of remuneration for values to be imputed systematically. Thus, mixed income contains an unknown element of remuneration for work done by the owner of the enterprise […] as well as the surplus accruing from production. The element of remuneration could be predominant in some cases.” (System of National Accounts, 1993, paragraph 7.85)*

79. Thus, the treatment of income generated by self-employed persons constitutes an issue in the calculation of labour and capital shares in total income. Clearly, part of “mixed income” is

remuneration of labour and should be allocated to the labour share to be consistent with the recommendation that labour input quantities should comprise hours worked by self-employed persons.

80. A common way to deal with this point is to assume that the average compensation per hour of a self-employed person equals that of a wage earner.[28] It is then possible to adjust the share of labour in total income (total value added) as follows:

$$\textit{Adjusted labour share} = \textit{initial labour share} * (\textit{Employees} + \textit{Self-employed})/\textit{Employees} \qquad (7)$$

81. Alternatively, a common rate of return could be assumed to the capital of corporate and quasi-corporate firms and the capital of the self-employed. This allows capital income of the self-employed to be computed and labour income to be adjusted residually. The two methods do not necessarily yield the same results and on theoretical grounds it is difficult to recommend one over the other.[29] On practical grounds, however, the first method (assuming a common average compensation between self-employed and wage and salary earners) is simpler to implement than the second method, which requires the computation of a rate of return. The calculation of rates of return is itself not free of ambiguity and may, in some cases, be unnecessary.[30] A combination of imputing wage rates and rates of return is used by some countries. For example, the Australian Bureau of Statistics imputes both labour and capital components and then applies a pro-rata adjustment to ensure that the two components sum to gross mixed income. In any case, strong assumptions are used that may not be justified. Over the longer term, improved estimates can only be obtained from surveys of the self-employed or from census questions that specifically target the self-employed.

82. *Training expenses*. Distinguishing labour from capital income is but one of a number of other measurement issues associated with the measurement of labour compensation. One such issue concerns training expenses that constitute a form of investment in human capital. The acquisition of knowledge, skills and qualifications increases the productive potential of the individual concerned and is a source of future economic benefit to them and to their employer. Different from physical assets, however, investment in training does not lead to the acquisition of assets by an employer that can be easily identified, quantified and valued for balance-sheet purposes. Thus, the SNA 93 states that they continue to be classified as intermediate consumption, even though it is recognised that they may bring future benefits.[31]

83. *Non-wage portion of compensation*. The non-wage portion of labour compensation, in particular social contributions payable by employers, is another element that is often difficult to include adequately in labour compensation. The SNA 93 specifies that employers' social contributions are part of the compensation of employees. Two types of contributions are distinguished: employers' actual contributions and imputed contributions. Actual contributions are those payments made by

---

[28] Whether this an accurate procedure remains open: long hours and below-average remuneration are often found among small business owners, implying a lower average compensation than those for employees.

[29] The US Bureau of Labor Statistics uses both methods and reconciles them at a later stage (Bureau of Labor Statistics, 1983).

[30] When capital input measures do not discriminate between different types of assets and when production is based on a constant returns to scale function, the contribution of capital to output growth and MFP can be evaluated without the need to calculate a rate of return on capital (see Chapter 5 on capital input).

[31] *System of National Accounts 1993*, paragraph 1.51.

employers to social security funds, insurance enterprises and the like for the benefit of their employees. Accordingly, employees should be treated as being remunerated by an amount equal to the value of these contributions.[32] When employers provide social benefits themselves directly without involving an insurance enterprise or autonomous pension fund, the SNA 93[33] recommends imputing an amount to the remuneration of employees. This imputation should be equal to the amount of social contributions that would be needed to secure the *de facto* entitlements to the social benefits that employees accumulate. In practice, it may be difficult to decide how large such imputed contributions should be. According to the SNA 93, the only practical alternative to obtaining direct estimates by firms may be to use the unfunded social benefits payable by the enterprise during the same accounting period as an estimate of the imputed remuneration of this type.

84. As a final example for an emerging measurement issue, consider the increasingly frequent practice of firms of offering *stock options* to their employees. The right to exercise an option is given to employees as part of a compensation package and as such constitutes compensation in exchange of labour input provided by employees. At the same time, there is an entrepreneurial component because the employee bears the risk of not gaining any benefit from the option. Thus, even if statisticians knew the extent and market value of stock options with sufficient reliability, it would be next to impossible to separate the pure wage component from the entrepreneurial component. The SNA 93 provides indications about the treatment of options and financial derivatives in the financial accounts of the economy,[34] but makes no recommendations about possible imputations to the compensation of employees. However, the consequences for productivity estimates can be illustrated by the following example. Suppose that a large number of employees accept stock options in lieu of fixed salaried income. Everything else equal, and given usual accounting practices,[35] this implies a lower share of labour income in total income in the economy. Because the measured contribution of labour to output growth is the product of this labour share multiplied by rate of change of labour input, there is a potential downward bias to the contribution of labour to output growth. More generally, no fully satisfactory solution has yet been found for the treatment of stock options in national accounts, nor is the current view of labour compensation as simply the price of labour services in current production satisfactory, and the discussion continues.

4.5. Accounting for different types of labour input

85. Labour input reflects the time, effort and skills of the workforce. While data on hours worked capture the time dimension, they does not reflect the skill dimension. When total hours worked are the simple sum of all hours of all workers, no account is taken of the heterogeneity of labour. In the context of productivity measurement, Jorgenson *et al.* (1987), Denison (1985) and the US Bureau of Labor Statistics have tackled this issue:

> *"Labour productivity measures have traditionally defined labour input as the sum of all hours worked by employees, proprietors and unpaid workers. As a result, an hour worked by a highly experienced surgeon and an hour worked by a newly hired teenager at a fast food restaurant are treated as equal amounts of labour. It does not matter who was actually*

---

[32] *System of National Accounts 1993,* para. 7.44.

[33] *System of National Accounts 1993,* para. 7.45.

[34] *System of National Accounts 1993*, para. 11.38.

[35] Some countries, for example Canada, include the value of stock options in labour compensation. Stock options are valued at the time they are exercised. Even so, there is an issue of timing: should the value be recorded when stock options are exercised or when they are granted?

> *working or what kind of job workers held. All workers are treated as if they were identical" (Bureau of Labor Statistics, 1993)*

86. For the estimation of productivity changes, the question is whether, over time, the composition of the labour force changes, *i.e.* whether there is an increase or decrease in the average quality of labour input. By most measures, there has been a steady increase in the quality of labour (OECD, 1998a). An increase in the average quality of labour implies that a quality-adjusted measure of labour input would rise faster than an unadjusted measure of labour input. Successful quality adjustment is tantamount to measuring labour in constant-quality units. Measuring constant-quality labour input is interesting from several perspectives.

87. First, it provides a more accurate indication of the contribution of labour to production. This has implications for productivity measures and growth accounting. One recalls that MFP measures the residual growth in output that cannot be explained by the rate of change in the services of labour, capital and intermediate inputs. Growth accounting is a different way of looking at the same equation: growth in output is attributed to labour, capital, intermediate inputs and residual changes in MFP. When quality-adjusted measures of labour input are used in growth accounting instead of unadjusted hours worked, a larger share of output growth will be attributed to the factor "labour" instead of the residual factor "productivity growth". In other words, substituting quality-adjusted labour input measures for simple ones can shift the appreciation of the sources of growth, from externalities or spillovers captured by the productivity residual to the effects of investment in human capital (see also Section 10.1).

88. Second, a comparison of an adjusted and unadjusted measure of labour input yields a measure of the corresponding compositional or quality change of labour input. This can usefully be interpreted as one aspect in the formation of human capital. As such it is a step towards measuring one important aspect of the effects of "intangible investment".

89. In the literature and in statistical practice there have been different approaches to explicit differentiation of labour input. Differences between these approaches are closely linked to how "skills" are measured. One possibility is to assume a direct relation between skills and occupations, to rank occupations by their skill intensity and then use information on the occupational distribution of hours worked to derive differentiated measures of labour input. This is, for example the approach taken by Lavoie and Roy (1998) for the case of Canada, or by OECD (1998a) for a broader number of OECD countries.

90. The assumption of capturing all relevant differences in skills by looking at occupations may, however, not be correct. Other differentiating characteristics such as age, health or educational attainment can reasonably be considered significant traits. Jorgenson *et al.* (1987) have used as many as five characteristics (age, education, class of workers, occupation and gender) to cross-classify labour input by detailed industry. Because the different characteristics are correlated, the resulting labour composition measure reflects both the direct contributions of these characteristics to output growth and the interaction effects between them.

91. Another possibility is to use a small number of differentiating characteristics but choose them so as to minimise the correlation between them. This is the approach adopted by the US Bureau of Labor Statistics: hours worked are cross-classified only by educational attainment and work experience. Further, there is no differentiation by industry. This reduces interaction effects between variables and facilitates identification of independent sources of the change in labour quality (Bureau of Labor Statistics, 1993).

92. Regardless of whether there is one or a number of differentiating traits, hours of highly skilled persons and hours worked of unskilled persons cannot simply be added to obtain an aggregate measure of labour input – they have to be weighted by their respective relative productivity to account for differences in skills. The theory of the firm stipulates that, under certain conditions (the firm is a price-taker on labour markets and aims at minimising its total costs), labour of a certain type will be hired up to the point where the cost of an additional hour of labour is just equal to the additional revenue that using this labour generates (see Annex 3 for a more technical derivation). This equality implies that, for a measure of total labour input, the individual labour inputs of different quality can be weighted with the respective relative wage rate, or more specifically, with the share that each type of labour occupies in total labour compensation.

93. Thus, the growth rate of total, quality-adjusted labour input L is measured as in (8), where L_i stands for a particular type of labour, and where v_i is the share that labour type *i* occupies in total labour compensation:

$$\frac{d\ln L}{dt}=\sum_{i=1}^{M}v_i\frac{d\ln L_i}{dt} \tag{8}$$

94. Note that even when only a simple trait such as occupation is chosen to differentiate labour input, information requirements are severe: data are needed that distribute the number of total hours worked across different occupations, by individual industry and by individual year. In addition, quantity measures of labour input (hours worked) have to be accompanied by price measures (relative average compensation) to construct weights for aggregation. Such rich data sets are normally both difficult and costly to collect and therefore not readily available in practice.

95. In this case, implicit differentiation can provide one, although incomplete substitute. Implicit differentiation arises when labour input (simple hours worked) is measured by detailed industry without, however, distinguishing between different types of labour within each industry. When the rate of change in hours worked by industry is aggregated to the economy-wide level and when each industry's share in total labour compensation is the aggregation weight, these weights will be comparatively large for industries that pay above-average wages and relatively small for industries with below-average wages. Assuming that above-average wages reflect above-average skills of the workforce, some of the quality change of labour input is taken into account.

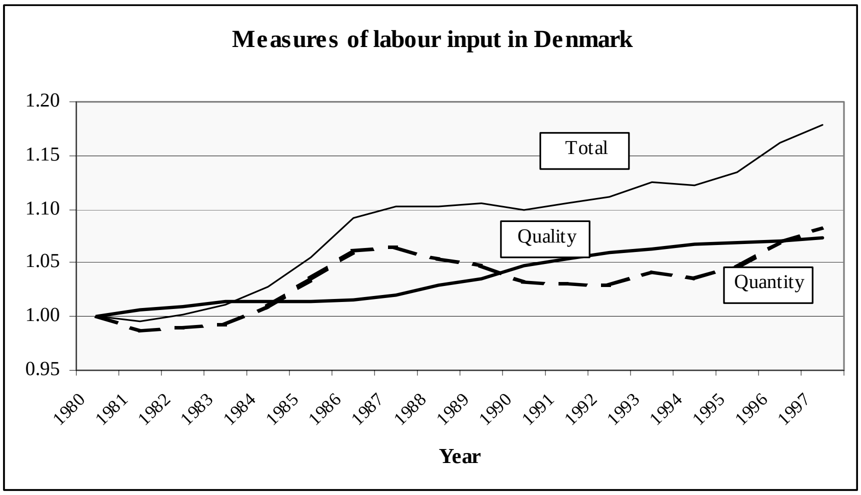
Box 3. **Quality adjustment of labour input in Denmark**

Productivity researchers in the Danish Ministry of Trade and Industry have constructed measures of quality-adjusted labour input, very much in line with the methodology outlined in this manual. The underlying data concern all Danish individuals. They are collected on the first of November each year and concern this date. Income information, however, originates from tax registrations and excludes capital income.

The data permits to cross-classify individuals by 124 industries; by gender; by eight age groups (0-15, 16-24, 25-34, 35-44, 45-54, 55-64, 65-75, 75+); by 29 types of educational attainment; by four employment classes (self-employed, employer, co-working family member, employee), and by hours worked. The latter were established on the basis of pension payments, with nine categories covering full-time and different lengths of part-time employment. No information on overtime or length of the working week was available from this source.

In a first step, individuals were aggregated into full-time equivalent persons, using the information on hours worked. At present, the information on hours worked is not considered sufficiently reliable to establish series on total hours. The aggregation and cross-classification gives rise to roughly 90 000 non-zero cells for each year covering the period 1980-97. The industry classification changed in 1993 such that a number of new industries appeared from 1993 onwards that had previously been recorded as part of other industries. This gives rise to a break in the series that make use of the industry dimension. In the figure below, this break has, however, been removed. In a second step, self-employed, employers and co-working household members were assigned the same wages as employees within the same industry, age, sex, and education class. This permitted construction of the necessary weights for aggregation. The rate of change of the (quality-adjusted) labour input, was computed using a share-weighted average of the rates of change of each type of labour input, *i.e.* for each cell of the cross classification by six criteria.

The graph below shows results in the form of three indices: *a)* quantity of labour input – this is the time profile of the simple sum of full-time equivalent employment; *b)* total labour input – this is the time profile of the quality-adjusted measure of labour input; *c)* quality of labour input – this is the time profile of the differential effect between the total and the quantity measure of labour input. It represents the compositional change in the use of labour and its re-allocation between industries. As for other countries, this effect is positive and comparatively strong (0.42% per year). It accounts for about as much of total labour input growth as the quantity effect (0.47% per year). Thus, labour productivity based on the quality adjusted measure grew by about 0.4% per year less than labour productivity based on simple full-time equivalents. This is not negligible, given a rate of GDP growth in Denmark of about 2% over the same period 1980-97.



Source: Fosgerau *et al.* (2000).

5. CAPITAL INPUT

5.1. Introduction

96. In a production process, labour, capital and intermediate inputs are combined to produce one or several outputs. Conceptually, there are many facets of capital input that bear a direct analogy to measures of labour input (Table 6). Capital goods that are purchased or rented by a firm are seen as carriers of capital services that constitute the actual input in the production process. Similarly, employees hired for a certain period can be seen as carriers of stocks of human capital and therefore repositories of labour services. Differences between labour and capital arise because producers usually own capital goods. When the capital good "delivers" services to its owner, no market transaction is recorded. The measurement of these implicit transactions – whose quantities are the services drawn from the capital stock during a period and whose prices are the user costs or rental prices of capital – is one of the challenges of capital measurement for the productivity analyst. We also note that there has been a longstanding academic debate about the fundamental nature of capital and its role in production. One approach, also adopted in this manual, is centred around prices and volumes of capital services. Another approach considers as fundamental the services not of the capital good, but of "waiting", *i.e.* the act of foregoing today's consumption in favour of building up capital goods and future consumption (see Rymes, 1971, for a discussion).

97. Because of a sometimes-conflicting use of terminology and concepts, the main notions are laid out first. They build on work by Hulten (1990 and 1996), Triplett (1996b and 1998), Jorgenson (1996), Biorn *et al.* (1989) and the *OECD Manual on Capital Stock Measurement* where more detail can be found on specific aspects of capital measurement.

Table 6. **Labour and capital inputs**

	Labour input	**Capital input**
Stock measures	Human capital	Physical capital
Services to production from input factors:		
Quantity	Labour services, measured as total person hours worked	Capital services, measured as total machine hours (typically, assumed to be in fixed proportion to capital stock)
Prices	Compensation per hour	User cost of capital per unit of capital service
Differentiation	By industry and by type of labour input	By industry and by type of capital asset
Factor cost or factor income	Compensation per hour * total hours	User costs * productive capital services
Aggregation weights	Industry-specific and labour quality-specific shares in total compensation	Industry-specific and asset-specific shares of user costs of capital

5.2. Overview

Overview: capital input measures for a single asset type

The quantity of capital input in production is measured by capital services…

For any given type of asset, there is a flow of productive services from the cumulative stock of past investments. This flow of productive services is called *capital services* of an asset type and is the appropriate measure of capital input for production and productivity analysis. Conceptually, capital services reflect a quantity, or physical concept, not to be confused with the value, or price concept of capital. To illustrate, take the example of an office building. Service flows of an office building are the protection against rain, the comfort and storage services that the building provides to personnel during a given period.

Because flows of the quantity of capital services are not usually directly observable, they have to be approximated by assuming that service flows are in proportion to the stock of assets after each vintage has been converted into standard “efficiency” units. The so-computed stock is referred to as the *“productive stock”* of a given type of asset. Thus, the importance of capital stock measures in productivity analysis derives from the fact that they offer a practical tool to estimate flows of capital services – were the latter directly observable, there would be no need to measure capital stocks. In terms of the above example, the assumption is that the services per year of an office building are fixed. If a second building of the same type is bought, this is tantamount to saying that services have doubled, although only the number of buildings has been observed. Note an immediate practical difficulty in defining the physical unit of capital services of an office building: do they arise only when there are actually office workers around? Or is the flow of services a permanent one? A case can probably be made for both, and is of relevance in the context of the discussion on capital utilisation.

…and their price by the user costs of capital.

The price of capital services is measured as their *rental price.* If there were complete markets for capital services, rental prices could be directly observed. In the case of the office building, rental prices do indeed exist and are observable on the market. This is, however, not the case for many other capital goods that are owned and for which rental prices have to be imputed. The implicit rent that capital good owners “pay” themselves gives rise to the terminology *user costs of capital*.

☛ More about the calculation of user costs in Section 5.4.

☛ For a more technical exposition, see Annex 4.

Productive and net (wealth) stock of a particular asset

Productive services of an asset are typically taken as a proportion of the *productive stock of a particular asset*. The productive stock should reflect the productive capacity of capital and is thus appropriate to gauge the quantity of capital services in production analysis. By contrast, the *net* or *wealth capital stock* is the current market valuation of an industry’s (a country’s) productive capital. One of the purposes of the wealth stock is measuring (economic) depreciation or the loss in value of an asset as it ages. Total depreciation across all vintages of an asset is exactly the amount by which the value of the net capital stock of an asset declines as an effect of ageing. However, it is not the appropriate tool to capture the quantity side of capital services.

☛ More about the calculation of the productive capital stock in Section 5.3 and in Annex 3.

☛ More about the calculation of the net capital stock in Section 5.4.1 and in Annex 3.

Decay and depreciation

Depreciation measures the loss in *value* of a capital good as it ages. It is therefore associated with the net (wealth) capital stock and has to be distinguished from *decay* or *efficiency decline* that reflects the loss of productive services that can be drawn from a capital good. Efficiency decline or decay is associated with the productive capital stock. Patterns of depreciation pertain to the age-price profile of an asset, and patterns of decay to its age-efficiency profile.

☛ More about the definition of depreciation and its link to the SNA in Section 5.4.1.

Age-price profile

The loss in value of a capital good as it ages is shown in its *age-price profile* or the pattern of relative prices for different vintages of the same (homogenous) capital good. How steeply the price of a capital good falls as it ages depends on several factors, including the rate of loss of productive capacity and the remaining service life. Obsolescence is another source for the loss of value of an old asset because a newly introduced asset of the same class contains improvements in productiveness or efficiency (Triplett, 1998). Note that obsolescence affects the value of a used asset but not necessarily its productive characteristics. The market value of a five-year old truck is much lower than that of a new one, because the older truck has suffered from wear and tear and because its remaining service life is five years less than that of the new vehicle.

☛ More about the empirical evidence on age-price profiles in the *OECD Manual on Capital Measurement*.

☛ More about age-price profiles, depreciation and the net stock in Section 5.4.1.

Age-efficiency profile

The loss in productive capacity of a capital good over time is shown in its *age-efficiency profile* or the rate at which the physical contributions of a capital good to production decline over time, as a result of wear and tear. The age-price profile and age-efficiency profile of a specific type of capital good are not necessarily identical, but they are related. Thus, they cannot be defined independently of each other.

A one-year old truck may have lost 20% of its market value but it has not necessarily lost 20% of its capacity to ship goods from one place to another. Indeed, the trucking services of a one-year old vehicle are probably nearly identical to those of a new one. Nonetheless, a change in service life or a different rate of efficiency loss will necessarily influence the value of existing assets. This illustrates the link between the age-price and age-efficiency patterns.

Retirement pattern

A retirement pattern describes how assets are withdrawn from service (scrapped, discarded). Typically, a retirement pattern is a distribution around the expected or mean service life. One notes the difference between retirement pattern and age-efficiency or age-price patterns: the former determines simply the survival probability of a particular capital good while the latter depicts the productive capacities or the market value, *conditional on survival*. Each truck in a fleet of identical vehicles of the same age has the same expected service life. In practice, some of the trucks will be retired or scrapped before the expected service life, others later. This phenomenon is described by the retirement pattern.

Relationship with "gross stock" and "net stock"

The *"gross capital stock"* is the cumulative flow of investments, corrected for the retirement pattern. It constitutes thus an intermediate step in the calculation of the productive stock that takes account of the withdrawal of assets but does not correct the assets in operation for their loss in productive capacity. Alternatively, gross capital stocks can be considered a special case of the productive stock, where the age-efficiency profile follows a pattern where an asset's productive capacity remains fully intact until the end of its service life (sometimes called "one-hoss-shay"). As indicated above, the *"net capital stock"* is synonymous with the wealth capital stock. "Wealth stock" is sometimes considered a more precise terminology, however, because there are other forms of "net" stock, in particular the productive stock which is the gross stock "net" of efficiency declines in productive assets.

Valuation of stocks and their interpretation

Three valuations of capital stocks exist: at historical prices, at constant prices, and at current prices, but only the latter two are of relevance in productivity measurement.

Productive capital stock measures at constant prices are quantity-type estimates for each asset. If all assets were truly homogenous, this measure would be expressed in physical units. That is rarely the case, however, and capital stock measures at real prices are then expressed as quantity indices (or in prices of a fixed base year, if a Laspeyres-type index is used).

Productive capital stock measures at current prices express these quantity measures in prices of each year. They are to be interpreted as the expenditure on new investment that would be necessary to produce the same amount of output as the productive stock at constant prices is able to produce. This interpretation explains the synonymous use of the notion "capital stock *at replacement costs*".

The *net (wealth) capital stock at current prices* has a natural interpretation as the market value of assets in a given year, expressed in prices of that year. It is typically assumed that the market value of an asset equals the discounted future revenues that the asset is expected to generate.

The *net (wealth) capital stock at constant prices* equals the market value of an asset, expressed in prices of a base year, or as a volume index. However, because net stocks are inherently value measures, a "quantity" interpretation is not very helpful. The quantity ("physical") interpretation of net stocks at constant prices is only meaningful when age-price and age-quantity profiles coincide, in which case the net and the productive capital stock are identical. Otherwise, net measures at constant prices are best interpreted as a market value of assets, relative to the price index of an investment good.

☞ More about the calculation of the productive and net stocks with different valuations in Section 5.3 and in Annex 3.

Overview: aggregation across different types of capital goods

Total capital input is a weighted average of inputs from individual assets.

Because many different types of capital goods are used in production, an aggregate measure of the capital stock or of capital services must be constructed. For net (wealth) stocks this is a straightforward matter of summing estimates for different types of assets. In so doing, market prices serve as aggregation weights. The situation is different in productivity analysis. Typically, each type of asset is associated with a specific flow of capital services and strict proportionality is assumed between capital services and capital stocks at the level of individual assets. This ratio is not the same, however, for different kinds of assets, so that the aggregate stock and the flows covering different kinds of assets must diverge. A single measure cannot serve both purposes, except when there is only one single homogenous capital good (Hill, 1999a).

Weights can be based on user cost…

Jorgenson (1963) and Jorgenson and Griliches (1967) were the first to develop aggregate capital service measures that take the heterogeneity of assets into account. They defined the flow of quantities of capital services individually for each type of asset, and then applied asset-specific user costs as weights to aggregate across services from the different types of assets. User costs are prices for capital services and, under competitive markets and equilibrium conditions, these prices reflect the marginal productivity of the different assets. User cost weights thus provide a means to effectively incorporate differences in the productive contribution of heterogeneous investments as the composition of investment and capital changes.

☞ More about user costs in Section 5.4.

☞ For a more technical exposition, see Annex 4.

…or on market prices.

An aggregate measure of the capital *stock*, on the other hand, uses *market prices* of individual assets to weight its components. Consequently, the difference between an aggregate index of capital services and an aggregate index of a capital stock can be seen in the sets of weights – the former is based on user costs, the latter on the market prices of the assets. In statistical practice, aggregate capital stock is often computed by summing up the stocks of individual assets, each measured at prices of a given base year. Over time, the quantity index of the capital stock will represent a Laspeyres-type index of aggregate capital, with market prices of a base year as weights.

☛ More about aggregation with different weights in Section 5.5.

Index number formulae

This leads to a final distinction between empirical capital measures, the choice of the index number formula in aggregation. The economics literature[36] recommends the use of index numbers with flexible weights. Such index number formulae include the Fisher ideal index, and the Törnqvist index. The Laspeyres formula, on the other hand, has often been formulated as a fixed-weight index number.[37] The advantage of flexible forms is that they use a set of weights (user costs or market prices) that remains up to date and hence reflects relative prices on which producer's decisions are based. In times of rapid changes in relative prices, the weights in fixed-weight Laspeyres indexes can quickly become obsolete and so give rise to biased quantity indexes.

☛ More about index numbers in Section 7.2.

Three dimensions to categorise capital measures

Overall, then, there are at least three dimensions by which capital measures can be categorised (Table 7):

- The form of the age-efficiency or age-price function for each type of asset.
- The nature of the weights that are used to aggregate across different types of assets.
- The index number formula by which this aggregation takes place.

---

[36] See Diewert (1987a). For a discussion of index numbers and capital measurement in the United States Bureau of Labour Statistics, see Dean *et al.* (1996).

[37] Increasingly, and following recommendations of the *System of National Accounts 1993*, chain Laspeyres indices are used in official statistics. Chain Laspeyres indices update price weights on a yearly basis and often result in measures that are close to Fisher or Törnqvist indices.

Overview: capital utilisation

Utilisation of capital goods is rarely uniform over several periods…

…but standard capital measures make this assumption.

There are many reasons why the rate of utilisation of capital, or more generally, the rate of utilisation of capacity of a firm varies over time: a change in demand conditions, seasonal variations, interruptions in the supply of intermediate products or a breakdown of machinery are all examples of factors that lead to variations in the flow of capital services drawn from a stock of assets. And yet, it is frequently assumed (for want of better information on utilisation rates) that the flow of services is a *constant* proportion of the capital stock. This is one of the reasons for the pro-cyclical behaviour of productivity series: variations in output are reflected in the data series, but the corresponding variations in the utilisation of capital (and labour) inputs are inadequately captured. If machine hours were measured, adjustments could be made. However, in practice, the required data do not exist and consequently, swings in demand and output are picked up by the residual productivity measure.

There have been several attempts to deal with this issue, but a generally accepted solution – if desirable – has yet to crystallise. In practice, statistical offices make no attempt to adjust their standard productivity measures for changes in the rate of capital and capacity utilisation.

☛ More about capital utilisation in Section 5.6.

Table 7. **Overview of capital measures**

	Type of age-efficiency or age-price profile					
	One-hoss-shay (O) or hyperbolic (H)		Straight-line		Geometric	
	User cost weights	Market prices as weights	User cost weights	Market prices as weights	User cost weights	Market prices as weights
Fixed-weight index number		Typical “gross stock” measure in OECD countries (O) Statistics Canada’s net capital stock measure with hyperbolic depreciation profile		Typical “net” capital stock measure in OECD countries	Statistics Canada’s MFP capital input measure	
Flexible weight index number **(*e.g.* Fisher, Törnqvist indices)**	US Bureau of Labor Statistics’ Capital services measure (H) Australian Bureau of Statistics’ capital service measure (H)	Australian Bureau of Statistics’ net capital stock measure (age-price profile based on hyperbolic age-efficiency profile)			Jorgenson (1989) measure of capital services	US Bureau of Economic Analysis’ *Fixed Reproducible Tangible Wealth* measure

Box 4. **Capital measures in the United States**

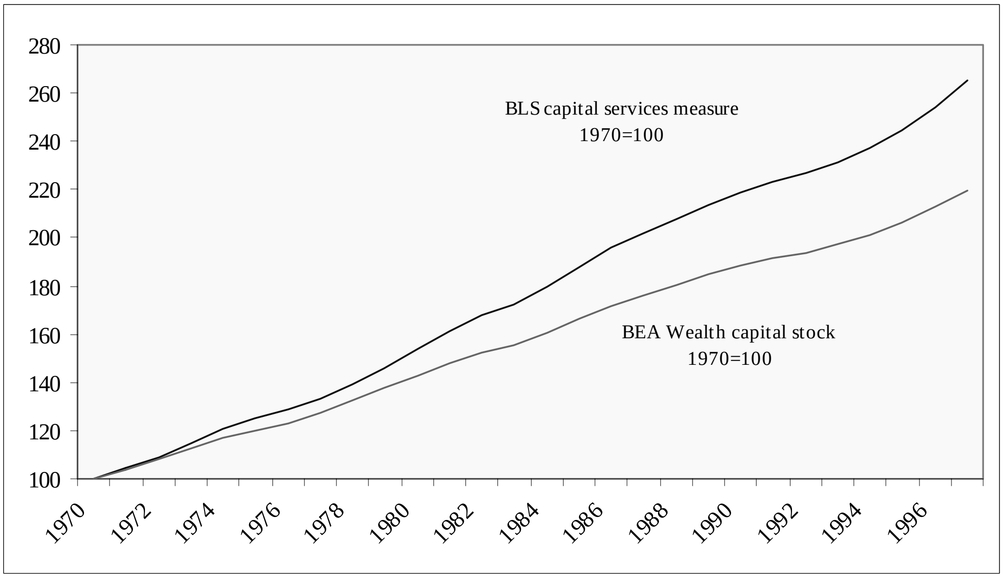
Two official sources for capital measures are currently available in the United States:

- The Bureau of Economic Analysis' series on *Fixed Reproducible Tangible Wealth.*
- The Bureau of Labor Statistics' series on *Capital Services* in the context of its *Annual Multifactor Productivity* statistics.

BEA produces a *wealth* capital stock that represents the value of fixed reproducible assets. These stock estimates and their associated estimates of depreciation are used in studies of national income, product and wealth. In terms of the three dimensions identified above, BEA's net stock in most cases uses: *i)* a geometric age-price profile; *ii)* market prices as aggregation weights; and *iii)* a Fisher index number formula in aggregation across assets. There is no explicit retirement function in BEA's estimates. The geometric depreciation rates applied to each type of asset are based on empirical estimates that reflect both the loss in value of an individual asset and the pattern of retirement across all identical assets in a particular cohort.

BLS' capital services measure is based on: *i)* a hyperbolic age-efficiency function (assuming that efficiency losses of an individual asset are relatively slow at the beginning of its service life and accelerate thereafter), combined with a normally-distributed retirement function; *ii)* user costs of capital as aggregation weights; and *iii)* a Törnqvist index number formula in aggregation across assets.

In light of the differences in the treatment of individual assets (geometric *vs.* hyperbolic) and in aggregation (market prices *vs.* user costs), it is not surprising that the BLS capital services series grows at a different rate than the BEA wealth stock (see graph). For a more extensive discussion of methodology, see Bureau of Labor Statistics *Handbook of Methods* at http://stats.bls.gov/opub/hom/homtoc.htm, and Bureau of Economic Analysis *Fixed Reproducible Tangible Wealth* on http://www.bea.doc.gov/bea/mp.htm.



Other capital measures have come out of academia, for example Jorgenson (1989). He uses a geometric age-efficiency/age-price profile and aggregation across assets is carried out with flexible weight (Törnqvist) indices, and based on user-cost weights.

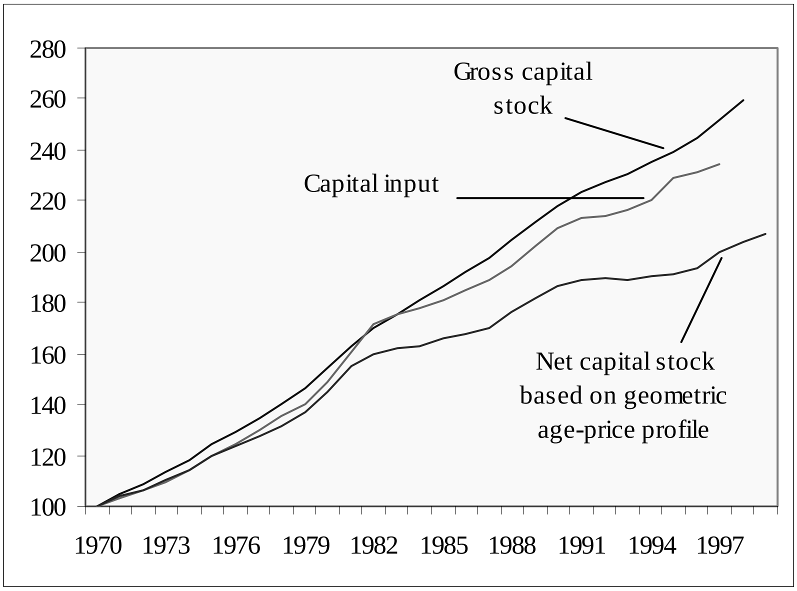
Box 5. **Capital measures in Canada**

Two official sources for capital measures are currently available in Canada:

- Several measures of the business sector capital stock are produced by Statistics Canada's *Fixed Capital Flows and Stocks* statistics. Apart from the gross stock measure (reflecting a one-hoss-shay pattern) three different measures of the net stock reflect different assumptions about age-efficiency patterns: a hyperbolic pattern, a straight-line pattern and a modified geometric pattern. All three measures have in common that aggregation is based on market prices and that the underlying index number is of a Laspeyres-type, with 1992 market prices as weights.

- In its MFP series for the business sector, Statistics Canada publishes a capital input series with the following characteristics: for each industry, capital stock measures are based on the Fixed Capital Flows and Stocks statistics, and more specifically, on the data reflecting a geometric age-efficiency (or age-price) pattern. Thus, at the level of each industry, aggregation across different assets uses 1992 market prices as weights and a Laspeyres quantity index. However, the aggregation across industries to a measure of capital input for the total business sector is (implicitly) based on user cost weights and on a Fisher-type index for aggregation.

A comparison of the rate of change of Canada's gross capital stock (the measure based on a one-hoss-shay age-efficiency profile and on an aggregation based on 1992 prices) and its measure of capital input for the period 1970-97 shows that differences are relatively small. These small differences reflect, however, two partly offsetting effects. First, the capital input measures are based on a geometric age-efficiency pattern and hence rise by much less than the one-hoss-shay type. Second, capital input measures of different industries are aggregated with a Fisher index number formula and with user cost weights. For the period under consideration, this aggregation yields faster growth than the aggregation with fixed-weight 1992 market prices. For more comprehensive information and source data, see Statistics Canada's publications on http://www.statcan.ca/start.html.



Box 6. **Capital measures in Australia**

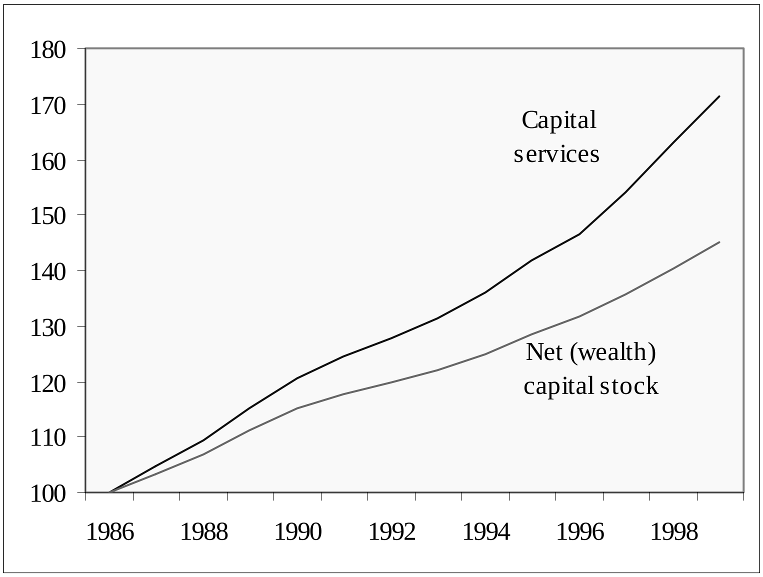
The Australian Bureau of Statistics publishes two distinct and complementary capital measures. The Australian methodology stands out in that it ensures full consistency between the different measures:

- A measure of capital services, as part of ABS' multifactor productivity series.
- An end-year net capital stock, as part of the Australian System of National Accounts.

Conceptually, ABS' measure of capital services matches closely that of the United States' Bureau of Labor Statistics (BLS). For each asset, capital services are taken as proportional to the productive capital stock, itself based on hyperbolic age-efficiency functions and a symmetric retirement function. ABS distinguishes six types of machinery and equipment, buildings and structures, four types of intangible assets, inventories and land. Aggregation across asset types is based on user cost shares, and on a flexible weight (Törnqvist) index.

The net capital stock is a measure of wealth, and based on age-price profiles for each type of asset that are derived from (and hence fully consistent with) the age-efficiency profiles used in the construction of the productive capital stock and capital services. The derived age-price functions are all of a convex form. As appropriate for a measure of wealth, aggregation uses market prices as weights and is carried out with a flexible weight index number. Measures are presented at current prices and at constant prices ("chain volume measures").

In light of the differences between the form of the age-efficiency function and the age-price function and given the differences in aggregation (market prices *vs.* user costs), the capital services series grows at a rate that is different from that of the wealth stock series. For a more extensive discussion, see Section 23 of Australian Bureau of Statistics, *Australian National Accounts: Concepts, Sources and Methods*.



5.3. Measurement of the productive stock and of capital services

98. In the absence of directly observable flows of capital services, these are approximated as a proportion of the productive capital stock. Thus, construction of the productive stock for each type of asset constitutes the first step towards measuring the quantity of capital services. This is done with the perpetual inventory method, described below. The step-by-step description relates to a particular type of asset and is accompanied by a numerical example. A more formal presentation can be found in Annex 4.

99. For a particular asset, four basic ingredients are required to construct an empirical measure of its productive capital stock:

- A time series of *investment expenditure* on the asset. Although theory is based on the notion of a homogenous asset, in practice, a particular "type" of asset will always consist of several different assets, and so exhibit a certain degree of heterogeneity. Such heterogeneity should be minimised, however, by applying as narrow a definition as possible of what constitutes an asset type. In this context it is important to note that heterogeneity can also arise over time, in particular in the form of quality change of an asset. Capital theory tells us (Hulten, 1990) that the practice of aggregating investment of different vintages should take into account the changes in quality with a view to aggregating across different years in a common "efficiency" unit. For example, technological progress embodied in information technology capital goods should be reflected in the productive capital stock of computers. When price indices of investment goods are carefully constructed, they are the appropriate tools to convert capital expenditure on different vintages into a common efficiency-weighted unit.

- Thus, the second key ingredient for the computation of productive capital stocks is *producer price indices* of investment goods to deflate investment expenditure series and to obtain constant-quality volume measures of vintage investment. The numerical example presented below (Table 8) uses two assets, A and B and five periods of observation, from year 0 to year 4. For simplicity, it is assumed that current price investment expenditure equals 100 in all periods and for both assets.

Table 8. **Numerical example: investment data**

	Year	Investment expenditure at current prices	Price index of investment good	Investment expenditure at constant prices
Asset A	0	100.0	1.00	100.0
	1	100.0	1.05	95.1
	2	100.0	1.11	90.5
	3	100.0	1.16	86.1
	4	100.0	1.22	81.9
Asset B	0	100.0	1.00	100.0
	1	100.0	0.82	122.1
	2	100.0	0.67	149.2
	3	100.0	0.55	182.2
	4	100.0	0.45	222.6

- The third piece of information is *retirement patterns* to account for discarded assets. Constructing retirement patterns involves a decision about the service life of different assets and an assumption about the distribution around this service life. Many empirical issues arise in the choice of the retirement pattern and the reader is referred to the OECD Manual on Capital Stock Measurement for an in-depth discussion. To keep things simple, a normal distribution around average expected service life is chosen here. Average service lives are (unrealistically) taken as three years for asset A and two years for asset B. This gives rise to the distribution shown in Table 9: after three years (asset A) and two years (asset B), half of all capital goods of a particular cohort have been retired.

Table 9. **Numerical example *(cont'd.)*: retirement pattern**

Retirement function (normal distribution)		
Share of cohort investment remaining after year	Asset A	Asset B
0	1.00	1.00
1	1.00	0.98
2	0.91	0.50
3	0.50	0.02
4	0.09	0.00
5	0.00	0.00

- In many OECD countries, gross capital stock series are routinely constructed. For individual assets, only the retirement pattern and investment and asset price series are needed to construct a gross stock measure. For an individual asset, the gross capital stock can thus be considered an intermediate step towards deriving a productive capital stock. Alternatively, the gross capital stock is interpreted as a special type of productive capital stock where all assets exhibit a one-hoss-shay efficiency profile: as long as assets exist, their productive capacity remains fully intact and drops to zero when they are retired. For a given type of asset, the use of gross capital stocks in productivity analysis has to be assessed from this perspective. To the extent that a one-hoss-shay pattern is plausible, the gross capital stock measure of an asset provides a reasonable substitute for the productive capital stock. It has, for example, been argued that the efficiency profile of computers can usefully be described with a one-hoss-shay pattern. For other assets, decay is a fact, and using the gross capital stock in productivity analysis would therefore over-estimate the quantity of services drawn from a particular asset. Table 10 shows the gross stock calculation for asset A: each cohort of real investment expenditure is multiplied by the retirement function in Table 9. For example, investment in year 1 amounts to 95.1 units. After two years (i.e. in year 3), 91% of this investment are still in operation, corresponding to 86.4 units.

- Fourth, *age-efficiency patterns* have to be defined to account for the loss of productive capacity of capital goods as they age. Age-efficiency patterns map the decline in productive services from an asset as it ages. They are designed to capture the effects of wear and tear. Age-efficiency patterns in combination with retirement patterns reflect deterioration, the combined effects of wear and tear and retirement. A plausible age/efficiency pattern to many kinds of assets is a hyperbolic pattern where productive services of a capital good fall slowly in earlier periods and at an increasing rate in later periods. The US Bureau of Labor Statistics uses such a hyperbolic pattern in its construction of a productive capital stock. Other authors, for example Jorgenson (1989), have extensively used a geometric age-efficiency pattern. The key advantage of a geometric pattern is its analytical tractability. Its use greatly facilitates the computation and treatment of depreciation, and, for a single, homogenous asset, the distinction between net (wealth) and productive capital stock disappears. For the present numerical example, a hyperbolic age-efficiency function has been used (Table 12).

Table 10. **Numerical example *(cont'd.)*: gross stock asset A**

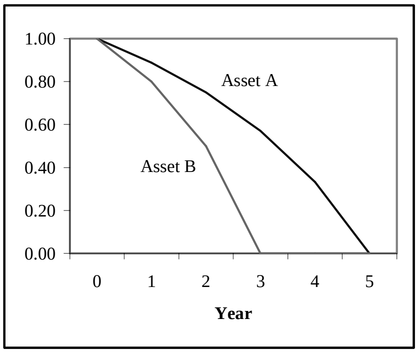
Investment at constant prices		Of which remaining at the end of year				
Year	Amount	0	1	2	3	4
0	100.00	100.0	99.6	90.9	50.0	9.1
1	95.12	-	95.1	94.8	86.4	47.6
2	90.48	-	-	90.5	90.1	82.2
3	86.07	-	-	-	86.1	85.7
4	81.87	-	-	-	-	81.9
Gross capital stock at constant prices		100.0	194.7	276.1	312.7	306.5
% change			66.7%	34.9%	12.4%	-2.0%
Gross capital stock at current prices		100.0	204.7	305.2	363.2	374.4

Table 11. **Numerical example *(cont'd.)*: gross stock asset B**

Investment at constant prices		Of which remaining at the end of year				
Year	Amount	0	1	2	3	4
0	100.00	100.0	97.7	50.0	2.3	0.0
1	122.14	-	122.1	119.4	61.1	2.8
2	149.18	-	-	149.2	145.8	74.6
3	182.21	-	-	-	182.2	178.1
4	222.55	-	-	-	-	222.5
Gross capital stock at constant prices		100.0	219.9	318.5	391.3	478.0
% change			78.8%	37.1%	20.6%	20.0%
Gross capital stock at current prices		100.0	180.0	213.5	214.8	214.8

Table 12. **Numerical example *(cont'd.)*: age-efficiency profile**

Share of productive capacity remaining at the beginning of year	Asset A	Asset B
0	1.00	1.00
1	0.89	0.80
2	0.75	0.50
3	0.57	0.00
4	0.33	-
5	0.00	-



- It should be restated here that the age-efficiency profile of a capital good is in general not identical with its age-price profile. The age-efficiency profile of a particular asset may decline slowly in early years of the asset's service life and faster towards the end; yet the age-price profile may show an inverted shape with fast rates of price declines in early years that level off towards the end of the service life. Only in the case of geometric decay does the age-price profile have the same shape as the age-efficiency profile. This greatly facilitates its analytical tractability. However, a geometrically declining age-efficiency profile is of little intuitive appeal. Yet, it has been pointed out by Wykoff (1989) and Hulten (1990) that if – for a given vintage – there are sub-cohorts of the otherwise homogenous capital good with different retirement dates, the average efficiency decline across sub-cohorts may well take a geometric shape that is different from that for each individual asset.

Table 13. **Numerical example *(cont'd.)*: productive stock of asset A**

Investment at constant prices		Of which remaining at the end of year				
Year	Amount	0	1	2	3	4
0	100.00	100.0	88.5	68.2	28.6	3.0
1	95.12	-	95.1	84.2	64.8	27.2
2	90.48	-	-	90.5	80.1	61.7
3	86.07	-	-	-	86.1	76.2
4	81.87	-	-	-	-	81.9
Productive stock at constant prices		100.0	183.7	242.9	259.6	250.0
% change			60.8%	27.9%	6.7%	-3.8%
Productive stock at current prices		100.0	193.1	268.4	301.6	305.3

Table 14. **Numerical example *(cont'd.)*: productive stock of asset B**

Investment at constant prices		Of which remaining in year				
Year	Amount	0	1	2	3	4
0	100.00	100.0	78.2	25.0	0.0	0.0
1	122.14	-	122.1	95.5	30.5	0.0
2	149.18	-	-	149.2	116.6	37.3
3	182.21	-	-	-	182.2	142.5
4	222.55	-	-	-	-	222.5
Productive stock at constant prices		100.0	200.3	269.7	329.4	402.3
	% change		69.5%	29.7%	20.0%	20.0%
Productive stock at current prices		100.0	164.0	180.8	180.8	180.8

- To derive the productive stock for each asset, each cell in the matrix for the gross capital stock computation is multiplied by the age-efficiency profile. For the present numerical example, the result is shown in Table 13. For example, real investment in year 1 amounted to 95.1 units. After two years (i.e. at the end of year 3), the productive capacity of this cohort has shrunk to 64.8 units, the combined effects of retirement (only 91% remain as shown in Table 9) and of efficiency losses (reduced to 75%, taken from Table 12). Similar, a productive stock can be computed for asset B (Table 14).

5.4. Measurement of user costs

100. Capital services have been described as the flow of services from capital goods into the production process. Associated with the quantity component of capital services comes a price component, the user cost or rental price of capital as formulated by Jorgenson (1963), and in an early form by Walras (1874).[38] User costs represent the amount of rent that would have been charged in order to cover costs of *q* dollars' worth of an asset (Bureau of Labor Statistics, 1983). In their simplest form, they are expressed as:

$$\mu_t = q_t \cdot (r_t + d_t) - (q_t - q_{t-1}) .$$

101. In this expression, the user cost of capital of an asset, μ_t, is the per-period cost of using the services of the asset. q_t designates the market price of a new asset, d_t is the rate of depreciation and r_t is some measure of the cost of financial capital such as the market rate of interest.

102. The first term of the user cost expression, $q_t \cdot (r_t + d_t)$, measures the cost of financing the asset. It comprises $q_t r_t$, the interest payment if a loan was taken out to acquire the asset or the opportunity cost of employing capital elsewhere than in production if the acquisition of the asset was financed from equity capital. r_t has also been called "internal rate of return" or "net rate of return". To

---

[38] This early reference was pointed out by Erwin Diewert who quotes W. Jaffé (1954), translation of L. Walras (1874), *Elements of Pure Economics,* Homewood, Illinois, R.D. For an explicit user cost formula, see p. 269.

the interest cost has to be added $q_t d_t$, the cost of depreciation or the loss in value of the machine because it ages. The loss in value reflects physical decay or efficiency loss of the asset, but also the fact that its expected service life has declined by one period.

103. The second term of the user cost expression $(q_t - q_{t-1})$ measures capital gains or losses, or re-valuation of an asset. This is the change in value that corresponds to a rise or fall in the price of that asset, independent of the effects of ageing. Independence of ageing effects arise because $(q_t - q_{t-1})$ is a comparison of prices of *new* capital goods in two different periods. Thus, any difference that may be observed cannot be due to wear and tear of the asset, but only due to other effects such as general price changes or obsolescence. There are several possibilities for the empirical implementation of this term, each with potentially different results. This is further discussed in Section 5.4.2.

104. Note the implicit definition of "depreciation" in this manual, which is confined to the change in value because of ageing. Thus, a one-year depreciation rate corresponds to comparing the price of a one-year old asset in year *t* to the price of a new asset in the same year *t*. This notion of "depreciation" excludes the change in value of an asset that is due to a change in its market price from one period to the next. The latter effect, which may be due to obsolescence, is incorporated in the capital gains/loss term discussed below. Hill (2000) points out that the SNA 93 uses a broader notion of "depreciation" to describe the entire "foreseen" or "normal" change in value of an asset between the beginning and the end of an accounting period:[39] "…the decline, during the course of the accounting period, in the current value of the stock of fixed assets owned and used by a producer as a result of physical deterioration, normal obsolescence or normal accidental damage." This type of depreciation is also called "consumption of fixed capital". A useful way to distinguish the two concepts is to label the more narrowly defined depreciation (as used in this manual) as "cross-section depreciation" and to use the term "time-series depreciation" when both ageing and capital gains/losses are included.

105. It should be noted that the formula $\mu_t = q_t \cdot (r + d_t) - (q_t - q_{t-1})$ abstracts from all effects of taxation. The Australian Bureau of Statistics and the United States Bureau of Labor Statistics, for example, augment the user cost expression to incorporate effects of corporate income taxes, tax depreciation allowances, investment tax credits and indirect taxes.[40] It is apparent that the inclusion of tax variables comes at a non-negligible cost. For a full implementation, historical and current tax laws have to be monitored, by type of asset and by industry, along with assumptions of assets' tax lives and estimates of business income tax rates.

5.4.1. Age-price profiles, net stock and depreciation

106. Given these concepts, how are rates of depreciation, and net rates of return actually measured? The following paragraphs provide again a step-by-step description of implementation.

- Age-price profile. There are at least three possible ways to obtain information about age-price profiles. First, there are econometric studies of used asset markets such as Hulten and Wykoff (1980). Unfortunately, such studies are scarce and, by their very nature, cannot capture price profiles of capital goods for which there are no second-hand markets. A second possibility is to assume a certain price/age pattern. In the case of a geometrically declining

---

[39] *System of National Accounts 1993*, para. 6.179.

[40] For details, see Bureau of Labor Statistics (1983), p. 49.

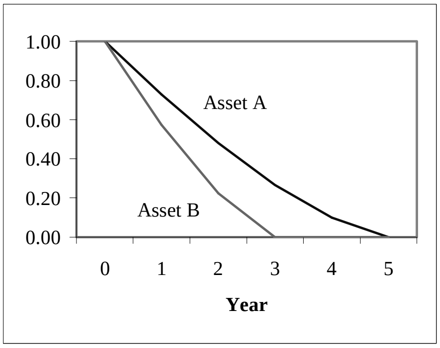
pattern, the (constant) rate of depreciation can be computed,[41] given a measure for the asset's service life. A third possibility is to formulate an hypothesis about the form of the age-efficiency profile and then infer the age-price profile.

- If age-price profiles are empirically determined or assumed, then there is an implication for the corresponding age-efficiency profile. In the case of a strictly geometric age-price profile, the age-efficiency profile will take the same geometric form. For other versions of age-efficiency profiles, the relationship is more complicated and, even in a simplified form requires assumptions on interest rates. By this link, every age-efficiency profile gives rise to a specific age-price profile and parameters have to be chosen so as to ensure consistency. This includes the frequently encountered case of a linear age-price profile or straight-line depreciation, which is not normally consistent with a linear age-efficiency profile and vice versa. One notes also that the combined retirement and age-efficiency pattern may have a form that is similar to a geometric age-price profile. For example, the French statistical office combines lognormal retirement functions and straight-line depreciation patterns. This results in figures that are close to those obtained when assuming geometric patterns of depreciation for the overall group of assets (Mairesse, 1972).

- If the third possibility is adopted, an hypothesis is first adopted about the form of the age-efficiency profiles and the age-price profiles are then inferred from the age-efficiency pattern. Of course, this implies replacing one set of assumptions (about age-price profiles) by another one (about age-efficiency profiles). The example presented here follows this approach (Table 15). Age-price profiles are derived from the age-efficiency profiles with the help of a simplified version of a fundamental asset market equilibrium condition. This condition, which states that the price of an asset equals the discounted stream of its future services, has a long history in economics (Hill, 1999b) with its origins going back at least to Hotelling (1925). A more comprehensive presentation of this condition and of its simplified version to compute the age-price profile is given in Annex 4.

Table 15. **Numerical example *(cont'd)*: age-price profile of assets**

Value of an asset of age x compared to a new asset	Asset A	Asset B
0	1.00	1.00
1	0.73	0.57
2	0.48	0.22
3	0.27	0.00
4	0.10	0.00
5	0.00	0.00



- Depreciation rates. Given an age-price profile for a particular asset, its net stock can be constructed. The procedure is symmetric to the one for the productive stock: starting from the gross stock measure, the age-price profile is applied to the various vintages of investment. For the present numerical example, the outcomes for assets A and B are given

---

[41] This is commonly done with the method of the "double declining balance", where the rate of depreciation is evaluated as 2 divided by the service life.

below (Table 16 and Table 17). In a first instance, the net (wealth) stock is expressed at constant prices, and then inflated to yield market values of assets at current prices. When expressed in real terms, the year-to-year change of the net (wealth) stock has two components: additions to the net (wealth) stock due to new investment, and deductions from it due to depreciation. Because depreciation is expressed in real terms here, it only reflects the effects of ageing, and does not pick up any re-valuation effects. This is the same concept of depreciation as the one underlying the depreciation term $d_{i,t}$ in the user cost equation. The ratio of the level of depreciation to the net (wealth) stock can therefore be used to yield an estimate for the rate of depreciation, or $d_{i,t}$ in the user cost term. Depreciation rates in the numerical example are quite high due to the short life spans that were assumed for the two assets. The tables also show a line entry "consumption of fixed capital" to point to the link with the SNA93 production and wealth accounts.

Table 16. **Numerical example *(cont'd)*: net (wealth) stock and depreciation rates of asset A**

Investment at constant prices		Of which remaining (real) value at the end of year				
Year	Amount	0	1	2	3	4
0	100.0	100.0	72.6	43.7	13.3	0.9
1	95.1	-	95.1	69.1	41.6	12.7
2	90.5	-	-	90.5	65.7	39.6
3	86.1	-	-	-	86.1	62.5
4	81.9	-	-	-	-	81.9
Wealth stock at constant prices		100.0	167.7	203.3	206.7	197.5
Change between adjacent years			67.7	35.5	3.4	-9.2
= investment at constant prices			95.1	90.5	86.1	81.9
minus depreciation at constant prices			27.4	54.9	82.6	91.0
Depreciation rate*			0.16	0.27	0.40	0.46
Wealth stock at current prices		100.0	176.3	224.6	240.1	241.3
Change between adjacent years			76.3	48.3	15.5	1.1
= Investment at current prices			100.0	100.0	100.0	100.0
minus consumption of fixed capital			23.7	51.7	84.5	98.9

*Depreciation rate = depreciation at constant prices/wealth stock at constant prices

Table 17. **Numerical example *(cont'd)*: net (wealth) stock and depreciation rates of asset B**

Investment at constant prices		Of which remaining (real) value at the end of year				
Year	Amount	0	1	2	3	4
0	100.00	100.0	56.1	11.2	0.0	0.0
1	122.14	-	122.1	68.5	13.7	0.0
2	149.18	-	-	149.2	83.7	16.7
3	182.21	-	-	-	182.2	102.2
4	222.55	-	-	-	-	222.5
Wealth stock at constant prices		100.0	178.2	228.9	279.6	341.5
Change between adjacent years			78.2	50.7	50.7	61.9
= investment at constant prices			122.1	149.2	182.2	222.6
minus depreciation at constant prices			43.9	98.5	131.5	160.7
Depreciation rate*			0.25	0.43	0.47	0.47
Wealth stock at current prices		100.0	145.9	153.4	153.4	153.4
Change between adjacent years			45.9	7.5	0.0	0.0
= Investment at current prices			100.0	100.0	100.0	100.0
minus consumption of fixed capital			54.1	92.5	100.0	100.0

*Depreciation rate = depreciation at constant prices/wealth stock at constant prices

5.4.2. *Nominal rate of return and capital gains/losses*

107. Theory provides no specific guidance as to the measurement of the nominal rate of return, *r*. Depending on a firm's financing pattern, *r* could be measured as the interest rate at which a firm can raise funds or it could be measured as a return on government bonds, *i.e.* the risk-less opportunity cost of investment, if investment is financed from retained earnings. For purposes of productivity measurement, such distinctions are rarely made,[42] and there are two basic approaches towards measuring the rate of return.

108. This first approach, in the spirit of Hall and Jorgenson (1967) estimates the internal rate of return with the help of an accounting identity: total non-labour income is the difference between value added and labour compensation. Non-labour income is equated with capital income. Given this estimate for the total value of capital services, in conjunction with a measure of the capital stock, of depreciation and of capital gains, the rate of return can be computed residually. More specifically, suppose there is only one type of asset, and let μK^P be capital income. Capital income is calculated residually as the difference between gross value added and labour compensation.[43] It equals the value of capital services, or the product of the user cost of capital (called μ here) times the quantity of capital services, that are proportional to the productive capital stock at constant prices, K^P. Given capital income $\mu_t K_t^P$, as well as a measure of depreciation and the price index for capital goods, *r* can

---

[42] In general, financing and investment decisions of firms are interdependent and capturing these links implies modelling efforts that are clearly beyond the scope of non-parametric productivity measurement. See Nickel (1978) for a discussion of issues.

[43] In practice, this is less straightforward. In particular, mixed income of self-employed persons has to be allocated between labour and capital. See Section 4.4 for a discussion.

be computed from the relationship $\mu_t K_t^P = q_t(r_t + d_t - \frac{q_t - q_{t-1}}{q_t}) \cdot K_t^P$. This way, the rate of return is computed as an endogenous variable.

109. A different way of interpreting the same relationship is to call $(r_t + d_t - \frac{q_t - q_{t-1}}{q_t}) = GRR_t$ a gross rate of return and to state that capital income (user costs times the productive real stock) equals the gross return on the current price productive stock, where gross return is computed as the gross rate of return applied to the current price capital stock: $\mu_t K_t^P = GRR_t \cdot q_t K_t^P$. If there are several types of assets, it is assumed that *r* is the same across assets and a similar procedure applies.[44]

110. To illustrate, consider Table 18. It starts with a total capital income of 300 currency units at current prices, taken from the production account of the national accounts. Following the procedure outlined in the preceding paragraph, an *ex post* internal or net rate of return is computed that is equal across all assets. When the rate of depreciation is added to, and the rate of capital gains/losses is deducted from, the internal rate of return, the gross rate of return to the capital stock follows. Multiplication of the gross rate with the current price productive stock yields capital income by asset. By construction, the sum across assets equals exactly the total capital income (300 units in the present example). It was also shown earlier that capital income equals the value of the real productive stock multiplied by its service price, or user cost of capital. Hence, an asset's share in total capital income is also its share in total user costs.

Table 18. **Numerical example *(cont'd.)*: user costs and capital income**

Year	Capital income	Net (internal) rate of return	Gross rate of return		Productive stock at current prices		Capital income		
			Asset A	Asset B	Asset A	Asset B	Asset A	Asset B	Total
0	300.0								
1	300.0	0.6	0.7	1.0	193.1	164.0	132.7	167.3	300.0
2	300.0	0.3	0.5	0.9	268.4	180.8	135.0	165.0	300.0
3	300.0	0.2	0.5	0.8	301.6	180.8	151.3	148.7	300.0
4	300.0	0.1	0.5	0.8	305.3	180.8	159.0	141.0	300.0

111. While this approach is quite common and easy to implement, it requires that the underlying production function exhibit constant returns to scale, that markets are competitive and that the expected rate of return equal the *ex post*, realised rate of return. A practical problem can also arise when capital income in the national accounts (gross operating surplus) is small and rates of return turn negative.

112. A second possibility is to use some exogenous value for the rate of return, for example interest rates for government bonds or as indicated in the *OECD Manual on Capital Stock Measurement,* as an average of the borrowing and lending rates faced by producers. Such an external measure can be a plausible alternative to the internal measure outlined above. One of its consequences is that the external rate fully defines the user cost term. Consequently, the total value of capital

---

[44] For several assets *i,* the rate of return is determined from the equation

$$PQ - wL = \mu K_t^P = \sum_i (r + d_{i,t} - \frac{q_{i,t} - q_{i,t-1}}{q_{i,t}}) q_{i,t} K_{i,t}^P .$$

services (user costs times capital stock) is in general not equal to non-labour income as determined in the national accounts. This discrepancy can either be interpreted as a “surprise term”, capturing differences between expected and actual user costs, as an expression of non-constant returns to scale, of market imperfections or of a combination of these causes. Identification of these causes requires econometric analysis – typically outside the realm of standard productivity statistics.

113. Harper *et al.* (1990) carried out a thorough investigation into the different ways of estimating rates of return and their impact on user cost measures. They compared effects from using external rates of return with those based on the capital income identity and find significant differences in the resulting user costs. Overall, and based on several performance measures (such as the share of negative rental prices, or the volatility of measures) as well as on theoretical considerations, the authors express a certain preference for the approach based on the income identity. However, no strong conclusion has been reached on the matter and much speaks for solutions that are governed by data availability.

114. *Capital gains/losses*. Related to the choice of the rate of return is the measurement of the capital gains or losses and the overall asset inflation. The most common approach consists in approximating expected asset price inflation by the observed change in the price index for (new) assets of type *i*. The implication is that asset price changes have been perfectly anticipated by agents. Alternatively, expressions for expected asset price changes could be formed, for example by using smoothed series. The advantage of using smoothed series is the reduction in volatility of the user cost terms. Another solution is to use constant real rates of return, combining the nominal rate and capital gains. This eliminates the possibility of negative user costs but remains somewhat arbitrary. Yet another option is to leave the capital gains term out of the user cost expression. This is only a justifiable procedure when the internal rate of return is calculated endogenously. Harper *et al.* (1990) show that, in the case of several assets, this procedure is equivalent to assuming that average capital gains (across all assets) are employed in the asset-specific user cost equations.

5.5. Aggregation across assets

115. To this point, capital measures have been discussed at the level of individual asset types, and the question arises how to aggregate them with a view to obtaining an overall measure of capital services or wealth. In aggregating, two choices have to be made. The first one relates to the nature of aggregation weights. More specifically, such weights can be the share of individual assets in total capital income (in which case weights reflect user costs), or weights could be the share of individual assets in the constant or current value of the capital stock (in which case weights reflect constant or current market prices). The second choice relates to the specific index number formula that is used for aggregation.

116. *Aggregate capital services*. Production theory is very clear on the appropriate aggregation procedure to obtain total capital services: it should be based on user cost weights and carried out with a superlative index number. Employing user cost terms for aggregation amounts to giving greater weight to those assets that depreciate quickly, compared to the weights that would result from a direct aggregation, or simple addition of the asset-specific stocks. The rationale for placing more weight on rapidly depreciating assets is that investors must collect more rents on a dollar’s worth of short-lived assets to compensate for their higher depreciation costs (Dean and Harper, 2001). The rationale for a superlative index number such as the Törnqvist or Fisher index derives from its property as an approximation to general functional forms of the production function (see Chapter 7 on index numbers).

117. For example, given observations on the productive capital stock for different types of assets, and given a set of user cost weights, a Törnqvist index of total capital services is given by the expression below. As earlier, $K_{i,t}^{P}$ stands for the productive stock of asset type *i*, and $\mu_{i,t}$ for the user cost of this asset. This formulation is, for example, used in the capital service measure as derived by the US Bureau of Labor Statistics and the Australian Bureau of Statistics (see also placement in Table 7). With the same type of data, it is also possible to construct a Fisher index of aggregate capital services and experience has shown that results will be very similar:

$$Tornqvist\ quantity\ index\ of\ aggregate\ capital\ services = \prod_i \left(\frac{K_{i,t}^{P}}{K_{i,t-1}^{P}} \right)^{\bar{v}_i} ,$$

$$\text{where } \bar{v}_i = 0.5(v_{i,t} + v_{i,t-1}) \text{ and } v_{i,t} = \frac{\mu_{i,t} K_{i,t}^{P}}{\sum_i \mu_{i,t} K_{i,t}^{P}} .$$

118. *Aggregate gross capital stock*. For purposes of comparison, suppose that aggregation takes place by summing up the gross stocks at constant prices of individual assets. The result is simply the standard gross capital stock measure at constant prices. By necessity, this quantity index is of the fixed-weight Laspeyres type; otherwise simple summation across real values would not be feasible. In this index, weights are given by the shares in period $t-1$ of each asset in the total value of all assets, expressed in prices of period *t-1*:

$$Laspeyres\ quantity\ index\ of\ aggregate\ gross\ capital\ stock = \sum_i w_{i,t-1} \frac{K_{i,t}^{G}}{K_{i,t-1}^{G}}, \text{ where } w_{i,t-1} = \frac{K_{i,t-1}^{G}}{\sum_i K_{i,t-1}^{G}} .$$

119. The difference between the quantity index of the gross stock and that of capital services reflects several effects: first, the gross capital stock is based on a one-hoss-shay assumption, the productive stock takes into account decay of the capital good. If only this effect were present, the gross stock measure would evolve more rapidly than the capital service measure. But there are other differences. One arises from the differences in index numbers. The gross stock series is often based on a Laspeyres-type index with prices of a fixed base year whereas the capital service series is typically based on flexible-weight index numbers. For years following the base year, the fixed-weight Laspeyres index will tend to rise faster than the flexible-weight index. For years preceding the base year, the converse holds. The difference is the substitution bias of fixed-weight indices. There is another difference associated with weights. In the case of capital services, these are based on user costs whereas gross stocks are aggregated with weights that reflect market prices. The resulting difference in capital series reflects the compositional change of assets. Generally, the choice of user costs over market prices tends to lead to a more rapid growth in the capital service series. For the United States, for example, Dean *et al.* (1996) find that over the period 1948-94, the average annual rate of increase in the input-to-stock ratio was 0.9%. As the authors point out, this reflects an increased annual rate of service flows from the "average" asset and resulted from a shift in the composition of assets towards assets with higher estimated rental prices. This trend is considerably influenced by a long-term trend towards shorter-lived asset types (such as computers) which yield their services over a shorter life span and have higher annual user costs.

120. Table 19 spells out the results for the numerical example. To construct the aggregate index of capital services, year-to-year changes in each asset's productive stock are weighted with the current user cost shares. This measure of capital services can then be compared with other capital measures, as

described above. Table 20 draws the link between a simple additive, constant price gross capital stock and the capital service index. In so doing, the various effects are identified: the impact of using an age-efficiency profile different from the one-hoss-shay assumption (negative effect), the effect due to the choice of index numbers (negative) and the composition effect (positive). In the numerical example, the negative effects outweigh the positive ones, leading to a more rapid growth of the simple gross capital stock measure over the capital services index. In practice, this is not necessarily the case, and the net effect may well be positive.

Table 19. **Numerical example *(cont'd)*: index of aggregate capital services**

	Productive capital stock at constant prices						**Tornqvist index of aggregate capital services**	
	Level		Index between adjacent years		Share in total user costs		Between adjacent years	Year 0 = 1
Year	Asset A	Asset B	Asset A	Asset B	Asset A	Asset B		
0	100.0	100.0						1.00
1	183.7	200.3	1.84	2.00	0.44	0.56	1.93	1.93
2	242.9	269.7	1.32	1.35	0.45	0.55	1.34	2.57
3	259.6	329.4	1.07	1.22	0.50	0.50	1.15	2.95
4	250.0	402.3	0.96	1.22	0.53	0.47	1.08	3.19

Table 20. **Numerical example *(cont'd.)*: comparison of capital measures**

	Productive stock by asset, aggregation with current acquisition prices						**Tornqvist index of capital services, aggregation with user cost weights**
	Laspeyres quantity index, gross capital stock	**Effect of age-efficiency pattern**	**Laspeyres quantity index**	**Substitution bias**	**Tornqvist quantity index**	**Composi-tion effect**	
Year			Index				
0	1.00	0.00	1.00	0.00	1.00	0.00	1.00
1	2.07	-0.15	1.92	-0.01	1.91	0.02	1.93
2	2.97	-0.41	2.56	-0.02	2.55	0.03	2.57
3	3.52	-0.58	2.94	-0.08	2.87	0.08	2.95
4	3.92	-0.66	3.26	-0.24	3.02	0.17	3.19

121. In summary, then, quantity indices of capital services are the appropriate measure of capital input in production analysis. When, in empirical research, only measures of gross stocks are available, the differences between the two series have to be kept in mind when interpreting results. However, when both measures exist, their comparison gives rise to a meaningful interpretation as the rate of compositional change of capital assets.

5.6. Capital utilisation

122. Capital services in production analysis are assumed to be proportional to the capital stock. If the factor of proportionality does not change over time, the growth rate of capital services is identical to the rate of growth of the capital stock. This is clearly an unrealistic assumption, given the

documented variations in the rates of capacity utilisation of capital stocks. One of the consequences of the constant proportionality assumption is that measured multifactor productivity rates exhibit strong pro-cyclical swings: variations in output are reflected in the data series, but the corresponding variations in the utilisation of capital (and labour) inputs are inadequately captured. Consequently, swings in demand and output are picked up by the residual productivity measure.[45]

123. The simplest way to minimise the influence of cyclical effects on measures of capital and productivity is to examine productivity growth between similar points in the business cycle. In other words, periods are selected for which, on average, capacity utilisation is close to normal. An obvious drawback of this peak-to-peak adjustment method is the fact that only entire business cycles can be described. This reduces the timeliness of the information on productivity growth in between peaks. It also poses a problem with international data when countries' business cycles are not synchronised, as cross-country comparisons would be based on different time periods.

124. Another way to correct this problem is to use some external measure of capacity utilisation to adjust capital stock series. This avenue has been followed by several productivity analysts, who correct estimates of multifactor productivity using electricity consumption as a proxy for capital utilisation. This idea, which dates back to Foss (1963), suffers, however, from an instability of the relationship between electricity consumption and capacity utilisation over time (Denison, 1969). Other authors have used capacity utilisation indices based, for example, on industry surveys. It is not always clear, however, that such indices relate exclusively or mainly to capital input – they are typically measures of *capacity* utilisation as opposed to measures of *capital* utilisation.[46] On theoretical grounds, the direct adjustment of capital input measures is at variance with the equilibrium model underlying the basic growth accounting approach.

125. It has also been unclear how changes in capacity utilisation should be reflected in the user costs of capital. Berndt and Fuss (1986) resolved this issue by treating capital as a quasi-fixed input, *i.e.* as a factor of production that cannot be instantly adjusted in response to output shocks. The authors then go on to show that in such a situation, income accruing to quasi-fixed capital is correctly measured residually, or *ex post*. This amounts to an implicit adjustment of the capital weights in productivity measurement and, in principle, makes an adjustment of capital quantities superfluous. The Berndt-Fuss results thus provide a theoretical justification for this widely used procedure in productivity analysis (Hulten, 1986). Berndt and Fuss also derive revised productivity estimates without, however, fully removing the pro-cyclical component of the productivity residual in practice.[47]

126. Finally, there is a large body of research using econometric techniques to identify variations in utilisation of capital and, possibly, distinguish them from changes in product mark-ups or varying returns to scale. Examples of such work include Beaulieu and Mattey (1998) or Basu and Fernald (2001). None of these approaches is, however, in the realm of the non-parametric approach to productivity measurement that provides the frame for the present manual.

127. A different way of interpreting variations in capital utilisation is to argue that it is not only a measurement issue. After all, a machine incurs certain costs independent of whether it is used or not. Examples include the costs of financing (such as interest payments) or the costs associated with

---

[45] Apart from variable utilisation, the literature has identified several other reasons for the pro-cyclicality of productivity measures but a treatment of these points is beyond the scope of this manual. For a recent study on the subject, see Basu and Fernald (2001).

[46] See Beaulieu and Mattey (1998) for this distinction as well as for an overview of the different strands of literature on capacity utilisation.

[47] Also, strictly speaking, the Berndt-Fuss approach is only valid for a single homogenous asset.

technical obsolescence. Thus, if MFP is to reflect real cost changes, cyclical terms are clearly part of the productivity measure: in times of recession, user costs of capital are spread over a smaller number of actual machine hours and consequently, real cost savings are limited. In times of cyclical upswings, the same user costs are spread over a larger number of machine hours and lead to more rapid real cost savings.

128. Put differently, adjusting capital stock series for their rate of utilisation is correct if one seeks to measure productivity in the sense of technical change or outward shifts of a production function.[48] If productivity measures strive to identify real cost changes, the occurrence of fluctuations seems not out of place: in periods of recession, capital is under-utilised because demand has shifted unexpectedly and real unit costs of production can rise or fall at a slower rate than in periods of boom when equipment is fully utilised. A formal elaboration along similar lines can be found in Hulten (1986), who shows how measured productivity growth can be decomposed into a true multifactor productivity residual and a capacity utilisation term.

5.7. Scope of capital investments

129. The SNA 93 describes[49] gross fixed capital formation as the acquisition less disposal of fixed assets. Fixed assets are produced assets (mostly machinery, equipment, buildings or other structures, but also including some intangible assets) that are used repeatedly or continuously in production over several accounting periods. In the context of productivity measurement, several issues arise.

130. The first issue is the treatment of *residential investment* and, consequently, residential capital. The system of national accounts stipulates that the activity of gross fixed capital formation is restricted to institutional units in their capacity as producers. Thus, there is no gross fixed capital formation of households, unless they are producers. One consequence is that persons who own the dwellings in which they live are treated as unincorporated enterprises that produce housing services that are consumed by the household to which the owner belongs. For practical purposes, it seems preferable to exclude the formation of owner-occupied residential capital from the range of assets that are usefully included in productivity analysis. At the same time, the imputed income generated by the fictitious unincorporated enterprises should also be excluded from the production side of the accounts, *i.e.* all or part of “Real estate activities with own or leased property” (group 701 in ISIC Rev. 3).

131. At the same time, the part of residential investment that is carried out by specialised producers of market services should be included in measures of capital input. Broadly, this corresponds to “Real estate activities on a fee or contract basis” (group 702 in ISIC Rev. 3). In practice, and for purposes of aggregate productivity measurement, the distinction between residential capital that corresponds to an actual production process and owner-occupied residential capital that corresponds to a fictitious production process may be difficult. In this case, a decision has to be made whether to entirely include or entirely exclude residential investment from measures of capital formation and capital input.

132. A second issue concerns activities that are at the *borderline between intermediate consumption and capital formation*. Some of these are highly relevant for productivity analysis. For

---

[48] As Griliches (1990) points out: “[procyclical] fluctuations in ‘productivity’ do not make sense if we want to interpret them as a measure of the growth in the level of technology or the state of economically valuable knowledge of an economy. The US economy did not ‘forget’ 4% of its technology between 1974 and 1975.”

[49] *System of National Accounts 1993*, para. 1.49.

example, expenditures by enterprises on staff training or research and development are intermediate inputs in the current period, but with the expectation of higher productivity or a greater range of production possibilities in the future, in much the same way as expenditure on machinery, equipment, buildings and other structures. However, expenditures on training and research and development do not lead to the acquisition of assets that can be easily identified, quantified and valued for balance-sheet purposes. Such expenditures continue to be classified as intermediate consumption, therefore, even though it is recognised that they may bring future benefits.[50] The economics literature has explored the role of such "intangible" assets and, for example, constructed stocks of R&D capital, very much in parallel to stocks of physical, "tangible" capital.[51] While useful, such efforts remain in the realm of research and are not presently recommended as standard practice for productivity measurement.

133. Another example with more immediate practical implications concerns the treatment of software: SNA 93 has extended the asset boundary to include intangible assets in software, formerly part of intermediate inputs, and several countries have started to implement this extension. Effects on gross fixed capital measures can be sizeable: in the Netherlands, for example, newly introduced software raised 1997 investment in producer durable equipment by about 5%. Changes to the asset boundary in the national accounts will affect productivity measurement through their impact on capital stock measures and on the level and industry distribution of value added.

134. No mention has been made of *inventories* and *land* as factors of production. Neither is part of gross fixed capital formation and therefore often neglected in capital and productivity measures. This can bias measured rates of return industries where inventories are large, such as retail and wholesale industries. Including inventories in capital measures is quite difficult from an empirical perspective, however, because the quality of data on changes in inventory in national accounts is often poor. Land is treated as a non-produced asset in the national accounts and not part of capital formation. Only acquisitions that lead to major improvements in the quantity, quality or productivity of land are treated as gross fixed capital formation, for example reclamation of land from the sea or clearance of forest to enable land to be used for the first time. However, as Diewert (2000) points out, the quantity of land in use by any particular firm or industry may change over time, and its price may change over time, causing shifts in user costs of capital. As with inventories, there are many empirical problems with the treatment of land as a factor of production. This explains why they have rarely been included in productivity analysis.

---

[50] *System of National Accounts 1993*, para. 1.51.

[51] See, for example, Stoneman (ed.) (1996).

6. INTERMEDIATE INPUT AND VALUATION

Overview: intermediate inputs

Information on intermediate inputs is required for most productivity measures…

Measurement of value-added based productivity or gross-output based productivity requires information on prices and quantities of the flow of intermediate inputs bought by a firm, industry or sector. Even at the level of the entire economy, imported intermediate inputs constitute an important data element for productivity measurement.

Yet, in many statistical systems, availability of a full set of intermediate input price and quantity indices is far from guaranteed. The main tool towards achieving this objective is the development and the maintenance of input-output tables.

…input-output tables are key in this respect.

Consistent KLEMS but also value-added calculations require that input-output tables are available to statisticians and researchers, as does the tackling of additional analytical issues. Although input-output tables are costly to produce and to maintain, their value as a tool for analysis is difficult to overstate and goes far beyond productivity analysis. Input-output tables are also increasingly used as a tool for the construction of consistent national accounts. Such developments are highly welcome and, indeed, recommended in the SNA 93.

Overview: valuation

Valuation should reflect the price that is most relevant for the producer's decision making.

Input-output tables are also instrumental in valuing outputs and inputs consistently. Valuation concerns the treatment of taxes and subsidies on products and on production. From the perspective of productivity measurement, the choice of valuation should reflect the price that is most relevant for the producer's decision making; regarding both inputs and outputs. Therefore, it is suggested that output measures are best valued at basic prices, *i.e.* excluding net taxes on products, whereas inputs are best valued at purchaser's prices, *i.e.* including net taxes on products as well as trade and transport margins.

6.1. Input-output tables

135. Multifactor productivity measurement – be it in the form of KLEMS or in the form of value-added based measures of productivity – requires information on the flows of intermediate inputs: explicitly, as a factor of production in KLEMS; or implicitly, as a building block to construct measures of value added. In KLEMS measures, energy, materials and services are broken out separately. This ensures a consistent treatment of intermediate and primary inputs: quantity indices of intermediate products are weighted with their current-price share in total inputs, allowing for substitution effects between different inputs.

136. The level of aggregation at which intermediate inputs are identified is primarily governed by the availability of price and quantity series for intermediate inputs. Generally, the most detailed level

of aggregation should be used. Input-output tables are ideal tools for such purposes. They provide a consistent accounting tool where individual cells of matrices show the flow of different intermediate products to individual industries. Ideally, there is also an industry-product specific time series of price indices. The table below illustrates the use of industry-by-industry input-output tables. Each column depicts the deliveries of intermediate products from industry *j* to industry *i*, X_{ij}, as well as primary inputs labour and capital. Table 21 shows – in schematic form[52] – the current-price deliveries of intermediate products.

137. When input-output tables are integrated with the system of national accounts, they are provide powerful tool for obtaining measures of value added and productivity.[53] In the context of KLEMS productivity measures, they are an indispensable source for the identification, measurement and weighting of intermediate inputs. In the same context, they are also required to measure *sectoral output, i.e.* gross output by industry net of intra-industry deliveries (see Section 3.1.3).

138. An input-output framework for productivity measurement raises two major practical questions:

- *Availability and timeliness of input-output tables*. Not every country's statistical system features input-output tables and where this tool is available, there is a significant time lag (often three to ten years) between the year of observation and the publication of the tables. In addition, benchmark tables are not established on a yearly basis, making it difficult to construct annual time series observations for intermediate inputs. Intermediate years between benchmark tables and recent years have to be estimated with interpolation methods such as the RAS procedure. Alternatively, it is possible to base productivity computations exclusively on available benchmark tables, calculating average annual rates of change of inputs, outputs and productivity between available years. In any event, the establishment and maintenance of input-output tables is costly for statistical offices.

- *Consistency with other statistical sources.* A second point of concern is the consistency of input-output tables with other statistical sources, in particular national accounts. In principle, consistency should prevail, and the link between input-output tables and national accounts is described in detail in the SNA 93. In a number of countries (*e.g.* Australia, Canada, France, United Kingdom, the Netherlands, Denmark), this integration exists but in other countries these statistical tools are only partly integrated. Inconsistency can lead to different sets of productivity figures, or biased results if sources are mixed.

---

[52] A more comprehensive illustration would build on both "use" and "make" matrices of input-output tables. "Use" matrices show the flow of intermediate deliveries, classified by type of *product* to individual industries. "Make" matrices show which industries produce which products. "Make" and "use" matrices can be combined to yield an industry-by-industry presentation such as in Table 21.

[53] For an integrated approach, see Gullickson and Harper (1999b).

Table 21. **Intermediate inputs in a simplified input-output table**

		To industry:				
From industry:		*1*	*2*	*3*	*:*	*N*
1		*0*	$P^1 X^{12}$	$P^1 X^{13}$	*:*	$P^1 X^{1N}$
2		$P^2 X^{21}$	*0*	$P^2 X^{22}$	*:*	$P^2 X^{2N}$
3		$P^3 X^{31}$	$P^3 X^{32}$	*0*	*:*	$P^3 X^{3N}$
:		*:*	*:*	*:*	*0*	*:*
N		$P^N X^{N1}$	$P^N X^{N2}$	$P^N X^{N3}$	*:*	*0*
Sum of above = intermediate inputs at basic prices (but including trade margins and transport charges)		$\sum P^k X^{k1}$	$\sum P^k X^{k2}$	$\sum P^k X^{k3}$	:	$\sum P^k X^{kN}$
+Taxes (including non-deductible VAT) minus subsidies on intermediate products						
Sum of above = intermediate inputs at purchaser's prices ($\hat{P}$)		$\sum \hat{P}^k X^{k1}$	$\sum \hat{P}^k X^{k2}$	$\sum \hat{P}^k X^{k3}$	:	$\sum \hat{P}^k X^{kN}$
Value added at basic prices	Taxes minus subsidies on production	T^1	T^2	T^3	:	T^N
	Consumption of fixed capital and net operating surplus	$\mu^1 K^1$	$\mu^2 K^2$	$\mu^3 K^3$	:	$\mu^N K^N$
	Labour compensation	$w^1 L^1$	$w^2 L^2$	$w^3 L^3$		$w^N L^N$
Gross output at basic prices		$P^1 Q^1$	$P^2 Q^2$	$P^3 Q^3$		$P^N Q^N$

6.2. Valuation

139. Valuation concerns the decision to include or exclude taxes, subsidies and transport costs in the price of outputs and inputs. From the perspective of productivity measurement, the choice of valuation should reflect the price that is most relevant for the producer's decision making; regarding both inputs and outputs. This raises issues of data availability in input-output tables and of consistency between different national accounts aggregates that enter productivity calculations.

140. The SNA 93 distinguishes between valuations at basic prices, at producer's prices and at purchaser's prices. The basic price is intended to measure the amount actually retained by the producer. It therefore excludes taxes payable, but includes subsidies receivable, as a consequence of

production or sale. It excludes any transport charges invoiced separately by the producer.[54] Because the basic price measures the amount retained by the producer, it is the price most relevant for his decision taking. Consequently, the SNA 93 states that valuation at basic prices is the preferred method for the valuation of output, especially when a system of VAT, or similar deductible tax, is in operation.[55]

141. Another valuation of output is at producer's prices. Contrary to basic prices, this valuation includes taxes on products and excludes subsidies on products. It thus moves the valuation from a measurement of the amount actually retained by the producer (the basic price) to an amount at which transactions take place or at which a transaction is concluded.

142. The third valuation is at purchaser's prices. It measures the amount actually expensed by the purchaser to take possession of a particular good or service at a specific place and time. Thus, purchaser's prices are the prices relevant for purchasing decisions. Consequently, for expenditure by enterprises on goods or services intended to be used for intermediate consumption, the SNA 93 recommends valuation at purchaser's prices.

Seller's perspective: preferred valuation of output ↓
Basic price (excluding deductible VAT)
+ Taxes on product, except VAT - Subsidy on products
= Producer's price
+ Non-deductible VAT + Transport charges and trade margins paid by the purchaser
= Purchaser's price
↑ Buyer's perspective: preferred valuation of inputs

143. The economics literature frequently uses a valuation of output at factor cost for productivity measurement. Valuation at factor cost is not a concept used explicitly in the SNA 93. However, valuation at factor cost is readily related to the SNA concept of basic prices, the difference being "other taxes less subsidies on production". As stated in the SNA 93,[56] these consist mostly of current taxes or subsidies on the labour or capital employed in the enterprise, such as payroll taxes or current taxes on vehicles or buildings. Such taxes or subsidies on production cannot be eliminated from input and output prices and thus, despite its traditional name, gross value added at factor cost is not strictly a

---

[54] When output is recorded at basic prices, any tax (subsidy) on the product actually payable on the output is treated as if it were paid (received) by the purchaser directly to the government instead of being an integral part of the price paid to the producer.

[55] *System of National Accounts 1993*, para. 6.218.

[56] *System of National Accounts 1993*, paras. 6.229-6.230.

measure of value added or output but essentially a measure of income. However, factor cost valuation is a useful substitute for valuation at basic prices. Another traditional concept of valuation, that of recording transactions at "market prices", is close to valuation at purchasers' prices, the main difference being the treatment of VAT or similar deductible taxes. Hence, valuation at market prices may be a reasonable substitute for purchaser's price valuation of input measures.

144. Whereas valuation of gross output at basic prices and of intermediate inputs at purchaser's prices is the correct approach from the perspective of an individual industry, the presence of taxes and subsidies raises several accounting issues:

- First, net taxes exist not only on products but also on production. While the latter play no role in the distinction between basic and purchaser's prices, they are part of gross value added and somehow have to be allocated to primary factors of production. Where possible, this should be done with regard to the nature of these taxes and subsidies. For example, payroll taxes are related to labour, taxes on motor vehicles are specific to capital. When such identification is not possible, net taxes on production have to be allocated proportionally to labour and capital income. This is further described in implementation sheet 8 in Chapter 9. Alternatively, the factor cost definition of value added could be used. This avoids the often-arbitrary apportionment of net taxes on production to labour and capital but implies foregoing full consistency between the accounting framework and productivity measures.

- Second, the presence of taxes complicates the accounting relationships between industry-level and aggregate productivity. For example, it can be seen from Table 21 that an industry's gross output at basic prices equals value added at basic prices and intermediate consumption at purchaser's prices. At the same time, gross output at basic prices equals deliveries to other industries at basic prices and deliveries to final demand at basic prices. The implication is that, at the aggregate level, final demand at basic prices equals value added at basic prices plus net taxes on the products used in intermediate consumption. To preserve consistency with an aggregate transformation function where the total value of final demand is allocated among factors of production, a re-definition of the valuation of final demand is required that amounts to subtracting net taxes on products from the final demand expression at basic prices.[57] Alternatively, the original valuation of final demand at basic prices is maintained and net taxes on products are simply excluded from the variables that enter productivity measurement.

- The same issue regarding the treatment of net product taxes arises when sectoral output (and input) measures are formed. This is further explained in Section 8.1 under the topic of aggregation.

---

[57] For a similar treatment, see Ezaki and Jorgenson (1995).

7. INDEX NUMBERS

Overview: index numbers

The SNA 93 recommends chain index number formulae.

Productivity is usually measured as a quantity index of output over a quantity index of inputs. Indices are required because the heterogeneity of goods and services does not permit simply adding up units of different types of commodities. However, results of index aggregation are in general sensitive to the choice of a specific index number formula and formulae should therefore be chosen on conceptual and on practical grounds.

A first choice that must be made for comparisons over several periods is whether to compare two periods directly (say, between period 0 and period 2) or indirectly (in which case the change between period 0 and 2 is derived from the change between period 0 and 1, combined with that from period 1 to 2). The economics literature as well as the SNA 93 are quite unanimous in this respect: for inter-temporal comparisons, changes over longer periods should be obtained by chaining: *i.e.* by linking the year-to-year movements.

✒ More on chain and direct comparisons in Section 7.1.

When tested against a number of criteria, the Fisher and the Törnqvist index come out first…

A second choice pertains to the specific index number formula. The most widely used index number formulae are the Laspeyres and Paasche indices (the former uses base-period weights, the latter current-period weights), the Fisher index (a geometric average of the Laspeyres and Paasche indices) and the Törnqvist index (a weighted geometric average of its components).

To help decide between different index number formulae, a series of intuitively meaningful criteria, or tests, have been developed in the index number literature. The different formulae can be checked against these criteria. Another approach makes use of economic theory to derive formulae that approximate theoretically defined indices. In particular, one assesses whether a particular index number formula can be derived from an economically meaningful relationship, such as a production or cost function.

…and the choice between these two seems to be of little empirical consequence.

From such considerations, the Fisher and the Törnqvist index come out first on most criteria. Empirically, it turns out that the choice between these two index numbers matters little and can thus be left to the individual researcher.

✒ More on the properties of the different index numbers in Section 7.2.

Digression: the importance of assumptions about producer behaviour.

In practice, many productivity indices rely implicitly or explicitly on simplifying assumptions about production technology and producer behaviour. For example, it is commonly assumed that production processes are subject to constant returns to scale and that production takes place efficiently. Relaxing these assumptions opens a wide array of possible alternative productivity indices. Unless there are sufficient empirical observations to employ econometric or similar techniques, it is not possible to choose between the different approaches. However, if one accepts the hypothesis of efficient producers, it lends further support to the use of Törnqvist or Fisher type index numbers.

✒ More on alternative productivity indices in Section 7.3.

7.1. Chained and direct comparisons

145. An important distinction between index numbers draws on whether they establish chained or direct comparisons. In a time-series context, whenever price or quantity indices of two non-adjacent periods have to be compared, the question arises as to which period should be chosen as a basis for comparison. Options include choosing the first or the last observation as the base, or using the chain principle. The chain principle makes use of the natural order provided by the march of time: for every index for period *t*, period *t-1* provides the base. In a time-series context, *i.e.* for the measurement of the rates of change of outputs, inputs and productivity, there is a strong preference in the literature in favour of chained indices.[58]

146. One main advantage of chained indices is that they minimise the substitution bias that is potentially present in direct comparisons, for example in fixed-based (Laspeyres-type) indices. When relative prices of individual inputs or outputs that make up the index, change, fixed-weighted indices tend to place too much weight on goods or services for which relative prices have fallen and too little emphasis on items for which relative prices have risen. Chain weighted indices, on the other hand, successively reduce the weight of items whose relative prices fall and increase the weight of items whose relative prices rise. Hence, they are much less prone to a substitution bias than fixed-weight indices. The wedge between fixed and chain-weighted indices became specifically visible with the rising importance of information technology products, in particular computers. Computer prices have fallen very rapidly relative to other goods while their volumes have been growing more rapidly than other goods and services, on average. Therefore, price or quantity indices that include computers as one item can move quite differently, according to whether they are chained or fixed-weight series. A similar point could be made for direct comparisons with Paasche-type indices that are based on weights of the current year. In what follows, the differences between chain and direct comparisons will be discussed with respect to the Laspeyres-type formula, as the most frequently encountered (volume) index number in the national accounts.

147. Coelli *et al.* (1998) also point out that since the chain index involves only comparisons with consecutive periods, the index is measuring smaller changes. Therefore, some of the approximations involved in the derivation of theoretically meaningful productivity indices are more likely to hold. Another advantage (Diewert, 1978) is that the Laspeyres-Paasche spread is likely to be small, indicating that the choice between individual index number formulae is less consequential than in the case of fixed-base indices.

148. Szulc (1983) discusses some of the drawbacks of chain indices, notably their behaviour when prices “bounce”, due to cyclical or seasonal variations, in the interval from the base to the current period. In particular, it can be shown that even if there are identical prices and quantities in the base and current period, a chain index may turn out to take a value different from unity whereas a fixed-weight index is (trivially) equal to one. At the same time, Szulc (1983) observes that such bouncing is unlikely to be important in practice, in particular with respect to aggregates composed of numerous commodities, as is typically the case in industry-level measures of inputs and outputs.

149. Chain quantity indices are also attractive in that they use those prices as weights that are actually relevant for quantity decisions of economic agents. With fixed-weight Laspeyres indices, the

---

[58] The case for chained index numbers is less clear in a cross-section context, such as the comparison of output, input or productivity *levels* between countries. In particular, there is no natural order between countries, akin to the course of time in a time-series context. Milana and Fujikawa (1996) provide a discussion of index numbers in international productivity level comparisons. For a more general discussion of the relative merits of fixed and chain-weighted indices, see Szulc (1983) and Hill (1988).

years of the price structure and the year at which quantities are observed can be far apart and so combine price weights with unrelated observations on quantities. This can yield results of modest analytical value such as negative constant-price value added under double deflation.

150. A frequently mentioned disadvantage of chain indices lies in their lack of additive consistency: non-additivity means that when the current values in the reference year are extrapolated backwards or forwards using a chain index, the extrapolated values of the components of some aggregate do not sum identically to the extrapolated value of the aggregate. From a perspective of measuring productivity growth, the lack of additivity is inconsequential. It may come into play in the context of macroeconomic models that contain interdependent variables that are linked by accounting relationships between their constant-price levels.

151. A practical disadvantage of fixed-weight indices arises when they are linked. Periodically, base-years for fixed-weight indices are changed and in many countries, Laspeyres indices based on the new price structure are only constructed for years from the new base year onwards. Indices of years preceding the base year are obtained by “linking in” the series based on the previous base year. Linking is carried out by multiplying the old index through with an adjustment factor that corresponds to the ratio of the old index and the new index in an overlapping year – typically this is the new base year. Then a decision must be taken about the level of aggregation at which linking takes place. One possibility is to link at the aggregate level and to apply the same linking factor to constant price values at all lower levels of aggregation. The magnitude of the adjustment factor could be quite large when considering rapid changes in prices and shifts in price relatives. This procedure preserves additivity and keeps aggregate growth rates unchanged when data are expressed in constant-price levels of the new base year. However, the procedure forces the aggregate adjustment factors to all levels of the economy, although industry-specific adjustment factors might well be different. Consequently, this approach preserves additive consistency but distorts the linked comparisons at a detailed level. A second approach uses adjustment factors at the component level, and so preserves the validity of linked comparisons at the detailed level of aggregation. By adding up constant dollar values of components, additivity can be preserved but at the cost of changing the headline growth rates for years prior to the new base year. A third option consists of linking at every level of aggregation, thereby preserving the validity of comparisons for every component, but foregoing additivity. The SNA 93[59] expresses a preference for the latter approach, given its transparency and given that the main purpose of linking is to obtain good measures of price and volume change. None of the solutions is, however, entirely satisfactory, and this supplies an additional argument in favour of chain indices.

---

[59] *System of National Accounts 1993*, para. 16.39.

Box 7. **Chain and fixed-weight index numbers in national accounts**

The introduction of chain-weighted indices in OECD Member countries' national accounts can have marked implications on the time path of growth rates, in particular in years where the base year of a fixed-weight index is remote from the reference year and/or in periods when there are marked shifts in relative prices. Two examples of comparisons between output measures based on fixed and chain-weighted indices support this statement.

In *Australia*, the latest fixed-weight index uses the constant prices of 1989/90 to calculate data covering the period since 1984/85. These fixed-weight data can be compared with a chained (Fisher) index. The comparison shows that, while differences between the fixed-weight and the chained index are comparatively modest for years close to the base year, they increase for reference years that are further away from the base year. Also, differences between index numbers widen as one considers individual components of GDP. For example, volume growth of gross fixed capital formation between 1986 and 1987 is 2.2% based on a fixed-weighted index and 4.1% based on a chained index.

Statistics Netherlands introduced chain-weighted volume indices into its annual national accounts from the reporting year 1981 onwards. Broadly, results confirm the observations made for Australia, including increased discrepancy between index numbers at lower levels of aggregation as the example of the chemical industry shows.

Australia

	GDP			Gross fixed capital formation		
Fiscal year	Fixed 1989/90 weights (Laspeyres index)	Annual weights (Fisher index)	Difference	Fixed 1989/90 weights (Laspeyres index)	Annual weights (Fisher index)	Difference
1985-86	4.5	4.6	-0.1	-1.4	0.2	-1.6
1986-87	2.4	2.7	-0.3	2.2	4.1	-1.9
1989-90	3.3	3.2	0.1	-4.7	-4.8	0.1
1993-94	3.8	3.7	0.1	8.2	6.0	2.2
1994-95	3.7	4.0	-0.3	21.4	19.4	2.0

Netherlands

	GDP			Value-added, petro-chemical industry		
Year	Fixed 1986 weights (Laspeyres index)	Annual weights (Laspeyres index)	Difference	Fixed 1986 weights (Laspeyres index)	Annual weights (Laspeyres index)	Difference
1987	1.4	1.4	0.0	1.2	1.2	0.0
1988	3.4	2.6	0.8	-0.6	5.6	-6.2
1989	4.8	4.7	0.1	9.6	4.0	5.6
1990	4.2	4.1	0.1	9.0	6.7	2.3
1991	2.3	2.3	0.0	-2.8	-8.2	5.4
1992	2.0	2.0	0.0	-2.8	-1.6	-1.2
1993	1.3	0.8	0.5	5.1	2.7	2.4

Source: Australian Bureau of Statistics (1997), *Development of Annually Re-weighted Chain Volume Indexes in Australia's National Accounts*, paper presented at the 1997 OECD/Eurostat meeting of National Accountants.
De Boer, Sake, Jan van Dalen and Piet Verbiest (1997), *The Use of Chain Indices in the Netherlands: Statistics Netherlands*, paper presented at the 1997 OECD/Eurostat meeting of National Accountants.

152. Table 22 surveys the practice concerning index numbers in OECD countries. It shows that an increasing number of countries have implemented chain-weighted indices, following the recommendation in SNA 93 or in ESA 1995. However, when countries move to chain indices, they differ in the degree to which accounts are established retroactively under the new methodology. Also, the finest level of detail at which volume aggregates are formed varies greatly between countries. This

reduces comparability because index numbers are generally sensitive to the level of detail from which they are built up. In addition, countries' practices diverge as to whether volume GDP data is constructed from the demand side, supply side or on the basis of input-output tables.

Table 22. **Index numbers in national accounts**

Country	Price base	Number of bases since 1970
Australia	Annually-weighted	-
Belgium	Fixed-weighted	3
Canada	Annually-weighted and fixed-weighted	-
Denmark	Annually-weighted	5
Finland	Fixed-weighted	5
France	Annually-weighted and fixed-weighted	2
Germany	Fixed-weighted	5
Greece	Annually-weighted	-
Ireland	Fixed-weighted	5
Italy	Fixed-weighted	4
Netherlands	Annually-weighted	-
Norway	Annually-weighted	-
Portugal	Annually-weighted	-
Sweden	Annually-weighted	5
United Kingdom	Fixed-weighted	5
United States	Annually-weighted	-

Source: Report of Eurostat Taskforce on Volume Measures, paper presented at the 1997 OECD/Eurostat meeting of National Accountants and updated country information.

7.2. Choice of index number formula

153. The above discussion suggested the use of chain index numbers in productivity analysis. This still leaves open the question of the specific index number formula, of which the most frequent are the Laspeyres, Paasche, Fisher and Törnqvist indices. The choice between these need not be arbitrary since economic and index number theory provide guidance on the matter.

154. A strong argument can be made in favour of the Törnqvist and Fisher indices, and indeed, of a whole family of "superlative" index numbers. Diewert (1976) rationalised their use on the grounds that they could be directly derived from so-called "flexible aggregators". Flexible aggregators are functional forms that provide a second-order approximation to an arbitrary, twice differentiable linear homogenous function. Such functions include a wide range of utility, production, cost or revenue functions. For example, the Törnqvist index can be directly derived (is "exact") for the translog flexible functional form – a widely used specification in empirical economics. Similar propositions can be derived for Irving Fisher's (1922) ideal index number. The latter is exact for a quadratic functional form (see Box 8).

Box 8. **Superlative indices of inputs and outputs**

“Superlative” index numbers were developed as part of the economic approach to index numbers. Under this approach, the microeconomic theory of producers or consumers serves as a rationale for choosing between index numbers. Diewert (1976) introduced the notion of “flexible aggregators”. These are functional forms that provide a second-order approximation to an arbitrary, twice differentiable linear homogenous function. Flexible aggregators can be interpreted as functional forms that cover a wide range of utility, production, distance, cost or revenue functions. Furthermore, Diewert calls index numbers “exact” when they can be directly derived from a particular flexible aggregator. For example, the Törnqvist index is exact for the translog flexible functional form – a widely used specification in empirical economics. Thus, if one accepts a translog form as an approximation to a production function, and uses standard assumptions about producer behaviour, the Törnqvist quantity index provides an exact formulation for inputs and outputs. An index that is exact for a flexible functional form is called “superlative”.

However, Diewert also showed that the translog flexible aggregator is only a special case of a broader family of functional forms, the quadratic mean of order *r* aggregator function f_r defined by:

$$f_r = \left(\sum_i \sum_j a_{ij} X_i^{r/2} X_j^{r/2} \right)^{(1/r)} \quad (r \neq 0).$$

For example, for *r=2*, f_r is a quadratic function for which the Fisher ideal index is “exact”. In this case, the components X_i could, for example, be interpreted as factor inputs and f_r as a production function. The resulting Fisher quantity index of inputs, index X^F between period *t* and period *t-1* is defined as:

$$X^F = \left[X^L \cdot X^P \right]^{\frac{1}{2}}.$$

where $X^L = \sum_i v_{i,t-1} \dfrac{X_{i,t}}{X_{i,t-1}}$ is a Laspeyres index (with base year *t-1);* $X^P = \dfrac{1}{\sum_i v_{i,t} / \dfrac{X_{i,t}}{X_{i,t-1}}}$ is a Paasche index and

v_i represents the share of input *i* in the total value of inputs.

Alternatively, if *r* tends to zero, f_r takes the form of a translog function for which the Törnqvist index is exact. If the components X_i are interpreted as factor inputs and f_r as a production function, the resulting Törnqvist quantity index of inputs between period *t* and period *t-1* is given by:

$$X^T = \prod_{i=1}^{N} \left(X_{i,t} / X_{i,t-1} \right)^{\frac{1}{2}\left(v_{i,t} + v_{i,t-1}\right)},$$

where v_i represents the share of input *i* in the total value of inputs.

155. These relationships can be applied for various price or quantity indices. For example, if one postulates that production technologies can be reasonably represented by a translog specification, then, under standard assumptions about producer behaviour, the Törnqvist quantity index provides an exact formulation for an output quantity index.[60] Or, if one postulates that technologies are characterised by translog cost functions, and under standard assumptions about producer behaviour, the Törnqvist input price index is exact.

---

[60] This was shown by Caves, Christensen and Diewert (1982). Balk (1998) generalised the result.

156. Diewert (1992) examined the relative merits of the Törnqvist and Fisher indices in the measurement of inputs, outputs and productivity. He concludes that there is an equally strong economic justification for Törnqvist and Fisher indices but that the Törnqvist index does not pass all the (axiomatic) tests passed by the Fisher index. Also, the Fisher index has greater intuitive appeal because it combines the well-known Laspeyres and Paasche formulae. At the same time, the translog form of production and cost functions has been a widely used and tested tool in econometric analysis. This favours the Törnqvist formulation. Overall, it appears that there is no strong reason to prefer one form to the other – even more so as empirical results from the two functional forms tend to be very similar.[61]

157. One notes in passing that using a Laspeyres or Paasche index to calculate output (or input) indices implies an underlying fixed-coefficient technology for the production structure – clearly a strongly simplifying assumption because it excludes the possibility of substitution between inputs or outputs, and implies constant marginal products throughout. At best, these indices provide bounds for the true underlying quantity or price indices.

7.3. A digression: from Malmquist to Törnqvist

158. *Malmquist indices.* Implicitly or explicitly, many of the statements about index number properties have so far relied on two important assumptions about firms' behaviour and technology: *i)* firms or establishments are economically efficient; and *ii)* technologies exhibit global constant returns to scale. Economic efficiency has two distinct components, "allocative" efficiency and "technical" efficiency (Farrell, 1957). Technical efficiency is the ability of a firm to obtain maximum output from a given set of inputs ("output technical efficiency") or to use minimum inputs for a given set of outputs ("input technical efficiency"). Allocative efficiency relates to a firm's ability to use inputs in optimal proportions, given a set of input prices, or to produce outputs in optimal proportions, given a set of output prices. Constant returns to scale occur when a proportional increase in all inputs results in the same proportional increase in output.

159. It is plausible that there are inefficiencies in firms' operations. The analysis of these inefficiencies, however, may be complicated. For example, production function presentations of technologies typically imply that firms are operating technically efficiently, and to allow for inefficiencies, they have to be replaced by their generalised counterpart, "distance functions".[62] Distance functions are an important tool in index number theory, and form the basis for Malmquist indices of prices, quantities and productivity. Consider the Malmquist quantity index as an example.

160. The *Malmquist (1953)* quantity index is based on the concept of a *distance function.* An output distance function describes the factor by which the production of all output quantities could be increased while still remaining within the feasible production possibility set for a given input level. Similarly, an input distance function indicates by how much input use can be reduced for a given output level and within the production possibilities. In this general formulation, a distance function is very much an engineering-type relationship. In its most general form, it requires neither assumptions about efficient producer behaviour nor about constant returns to scale technology. This property makes it a very versatile tool that is also suited for the measurement of non-market input, output and productivity.

---

[61] See, for example, the empirical results in Diewert (1978).

[62] See Coelli *et al.* (1998) for an introduction, and Balk (1998) for a more advanced treatment.

161. Somewhat more precisely,[63] an output distance function $D_o^t(Q^t, X^t)$ indicates the output technical efficiency of an observed production process where a vector of input quantities in period *t*, X^t, produces output quantities, Q^t under a technology (the production possibilities) prevailing at time *t.* The distance function takes a value of one when production is technically efficient, *i.e.* when the output vector Q^t is the maximum output achievable with inputs X^t. Malmquist (1953) defined an output quantity index as $Q_o^t \equiv \frac{D_o^t(Q^t, X)}{D_o^t(Q^{t-1}, X)}$ where X is an arbitrary reference vector of inputs. It is a measure of the "distance" between Q^t and Q^{t-1} and reduces to the ratio $\frac{Q^t}{Q^{t-1}}$ when there is only one output. Note that the specific form of the distance function is generally unknown. Also, the Malmquist quantity index as presented here depends on the reference technology in year *t* and on the vector of inputs. Conceivably, an output index could also be defined for the distance functions at *t-1*, *i.e.* for a different reference technology. It would then be formulated as $Q_o^{t-1} \equiv \frac{D_o^{t-1}(Q^t, X)}{D_o^{t-1}(Q^{t-1}, X)}$. Because there is no obvious reason to choose one over the other, it is common to take a geometric average of the two indices: $Q_o = \left(Q_o^t Q_o^{t-1}\right)^{0.5}$. Further, there are many possible input vectors *X* and hence a whole family of possible output quantity indices, conditional on reference technology and reference inputs.

162. How is it possible to empirically implement a Malmquist quantity index, if all one has are price and/or quantity observations for two or more periods?

163. Suppose that only quantity observations are available for inputs and outputs. This could, for example, be the case for non-market activities where either no price exists or where an existing price has little economic meaning, as in the case of subsidised health or education services.[64] Output (input, and productivity) indices can nonetheless be established but require econometric or linear programming techniques, such as those present in *Data Envelopment Analysis*. With these tools, underlying technologies and efficiency frontiers can be identified and measured productivity growth can be split up into efficiency changes and into shifts in the technology frontier. The price of applying these techniques is that a sufficient number of observations must be available in each period.

164. In the context of market activities, economic theory helps implementation. If one accepts cost minimising or revenue maximising behaviour, *i.e.* efficiency, on behalf of firms, the search for the right empirical formulation is significantly facilitated. For example, it can be shown[65] that the Laspeyres output quantity index is a lower bound for the Malmquist output quantity index defined over the technology of period *t-1*, and the Paasche output quantity index is an upper bound for the Malmquist output quantity index defined over the technology of period *t*. The geometric mean of the Laspeyres and Paasche indices (*i.e.* the Fisher Ideal Index) then constitutes an approximation to the Malmquist quantity index Q_o defined above.

165. Alternatively, an assumption can be made about the functional form of the distance function. One common functional form is the translog output distance function. Under this characterisation, the

---

[63] For a full discussion, see, for example, Balk (1998).

[64] See Coelli *et al.* (1998) for examples of application.

[65] Balk (1998).

Törnqvist output quantity index is an exact representation[66] of the Malmquist output quantity index Q_o. Other plausible functional forms exist, for example quadratic forms, which give rise to the Fisher index as an exact representation of the Malmquist quantity index. These are two applications of the theory of *exact* and *superlative* index numbers (see Box 8). By a similar reasoning, it is possible to show that the Törnqvist input quantity index is an exact representation of the Malmquist input quantity index $Q_i = (Q_i^t Q_i^{t-1})^{0.5}$. Thus, the Törnqvist productivity index is a valid representation of the ratio of a Malmquist output quantity over a Malmquist input quantity index. This is an important relation because it provides another underpinning for the use of Törnqvist (or other superlative) index numbers:

$$\frac{Tornqvist\ \ output\, quantity\ \ index}{Tornqvist\ \ input\ \ quantity\, index} = \frac{Malmquist\, output\ \ quantity\ \ index}{Malmquist\, input\ \ quantity\ \ index} = \frac{Q_o}{Q_i} = \frac{(Q_o^t Q_o^{t-1})^{0.5}}{(Q_i^t Q_i^{t-1})^{0.5}}.$$

166. *Malmquist productivity index*. Measuring productivity as a ratio of an output to an input index (the “Moorsteen-Hicks approach”) is a common, but by no means the only, way to define productivity. Other possibilities exist, notably different versions of the Malmquist productivity index. In the most general case, where one allows for inefficiencies and non-constant returns to scale, one such productivity index could be computed as the ratio of output distance functions, for example: $\frac{D_o^t(Q^t, X^t)}{D_o^{t-1}(Q^{t-1}, X^{t-1})} \frac{D_o^{t-1}(Q^{t-1}, X^{t-1})}{D_o^t(Q^{t-1}, X^{t-1})}$. The first part of this expression shows changes in efficiency between the two periods, the second part shows technical change (for a given set of inputs and outputs, what is the maximum production achievable in period *t* as opposed to period *t-1*?). Other combinations are possible, for example a measure of technical change with respect to the reference period *t*, rather than *t-1*.

167. It is equally justifiable to define productivity measures with respect to input distance functions, and, without further restrictions on technology, there is no guarantee that input-related productivity measures yield the same result as output-related ones. The equivalence of input and output-related measures is only ensured under constant returns to scale of the production technology, and herein lies much of the attractiveness of this simplifying assumption.

168. Even under the assumption of constant returns to scale, there is again the question of how to implement the Malmquist productivity measure, expressed as distance functions, in practice. As before, there are essentially two choices: if sufficient observations are available to the researcher, econometric or linear programming techniques can be applied to estimate underlying production frontiers. Alternatively, one accepts efficient behaviour on the side of producers as well as a translog (or other general quadratic) functional form of distance functions. With constant returns to scale, the Malmquist productivity index then turns out to be identical to the Törnqvist or the Fisher productivity indices.

---

[66] Caves *et al.* (1982).

169. This brings the more general Malmquist approach back to the operational Fisher and Törnqvist indices. It also shows that under the simplifying assumptions prevailing throughout this manual (in particular, constant returns to scale and efficient producer behaviour), the various approaches towards productivity measurement converge. However, it also points to the methodologies that can be adopted if these assumptions are not deemed realistic, as in the case of non-market activities. The relevant methods are well established but typically require a larger amount of empirical observations to be put to work.

8. AGGREGATING PRODUCTIVITY GROWTH ACROSS INDUSTRIES

Overview: aggregation of productivity measures

From individual industries to the aggregate economy.

The relation between industry-level and aggregate productivity measures is of significant interest to analysts and policy-makers because it establishes a link between the micro and macro levels of the economy and helps answer questions about, for example, the contribution of individual industries to overall productivity growth.

✒ More on aggregation and integration in Section 8.1.

In the presence of intermediate inputs, aggregation…

To form an overall picture, simple weighted averages of industry-level productivity growth could be formed but such aggregation does not necessarily account for the links that exist between industries. In particular, they are connected via flows of intermediate products. Forming productivity measures at higher levels of aggregation often entails *integration* of individual industries into increasingly larger units. In the process of integration, intra-industry deliveries are netted out to obtain consistent higher-level productivity measures. This concept of sectoral outputs and inputs treats every level of aggregation as if it were a single unit of production, with its specific technology and productivity pattern. At the level of the entire economy, sectoral output coincides with final demand, and a consistent link is established between industry-level and aggregate observations on productivity growth.

…leads to the use of "Domar" weights.

Rates of growth of industry-level KLEMS MFP measures can be summed to form their aggregate counterparts. Summation weights are given by each industry's gross output relative to economy-wide value added. The sum of these weights exceeds unity, and signals that productivity gains of the integrated economy exceed the average productivity gains across industries. This occurs because flows of intermediate inputs between industries contribute to aggregate productivity by allowing productivity gains in successive industries to augment one another.

✒ More on Domar weights in Section 8.2.

✒ More on aggregation of capital-labour MFP in Section 8.3.

✒ For a more technical exposition, see Annex 5.

8.1. Integration, aggregation and intermediate inputs

170. Aggregation relates to forming consistent totals from components. In the context of productivity measurement, aggregation concerns the relationship between industry-level (or firm-level) productivity measures and their counterparts at the macroeconomic level. Consistent aggregation is, for example, necessary to answer questions about the contribution of individual industries to overall productivity growth. An important consideration is whether or not to take existing upstream and downstream relationships between industries into account in the process of

aggregation.[67] If these relationships are taken into account, aggregation is accompanied by *integration.* Integration is the process whereby several industries or units are combined so that they form a new industry or unit whose output consists only of deliveries outside the new unit and whose inputs comprise only the inputs from outside the new unit. Thus, intra-industry flows are netted out in the process of integration and the newly formed aggregate is treated as a single, integrated unit. If inputs and outputs of different industries are simply added without netting out intra-industry flows, aggregation proceeds without integration.

171. By way of example, suppose that there are two firms and let firm 1 (a leather factory) only produce intermediate inputs for firm 2 (a shoe producer). Firm 2 itself produces only final output. Simple aggregation of the flows of outputs and inputs is still possible, but is not the right procedure to obtain measures of output and input of the shoe and leather industry *as a whole*. There is double counting of outputs and inputs because of the intermediate flows between the leather and the shoe producer. In the process of integration, these flows are netted out. Thus, the output of the integrated shoe and leather industry consists only of the shoes produced, and integrated intermediate inputs consist only of the purchases of the leather industry and non-leather purchases of the shoe industry.[68]

172. This integration leads directly to the notion of *sectoral output* introduced in Chapter 3. Sectoral output was defined as an industry's gross output net of all intra-industry deliveries. Similarly, sectoral input was defined as an industry's intermediate inputs net of all purchases from within that industry. In terms of the present example, integration has led to forming measures of sectoral output and input of the integrated shoe and leather industry. Further integration with other industries is possible, and as before, this involves netting out intermediate inputs between industries that are merged. This process of integration can continue to the point of the total economy. In the case of a closed economy, sectoral output at the most aggregate level is identical to total value added, and sectoral input equals total primary inputs as all intermediate flows become intra-industry flows and disappear from the computations.

8.2. Domar weights: aggregation of KLEMS measures

173. The process of vertical aggregation has important implications for productivity measurement. Assume that both the shoe and the leather producers' MFP growth is 1%. The simple (weighted) average of the shoe and leather producers' MFP growth will be 1%. However, productivity

---

[67] Some researchers, for example Durand (1996), stress this distinction between "integration" and "aggregation". The former relates to combining individual units to a larger unit, or to a single activity; the latter to forming weighted averages of the individual units, without however modifying these units. The consequence is that aggregation does not necessarily imply integration. Thus, aggregate productivity measures could be a simple weighted average of industry-level productivity measures. Although this involves double counting, it has analytical meaning. Alternatively, integration proceeds with aggregation, implying that the sub-components are consolidated into a single new aggregate unit. This is the avenue pursued in the present manual.

[68] Note another aggregation issue here: given input or output price or quantity indices for single establishments or firms, under which conditions is it possible to combine them so that the industry-wide observations can be treated as if they came from a single unit? This is far from obvious. For example, Fisher and Shell (1998) point out that "an industry as a whole will typically face upward-sloping curves for input factors, whereas the individual firm takes no account when making their decisions. As a result, […] it follows that [an] industry-wide production-theoretic output price index cannot be built up from the firm-level indexes." The problem is absent in the case where the industry even as a whole is a small buyer or supplier. Such an assumption is realistic at a very low level of aggregation, and/or when industries are fully open to international competition. It will be maintained throughout this manual.

growth of the integrated shoe and leather industry will be more than 1%, because the shoe producer's productivity gains cumulate with those of the leather producer as the former buys inputs from the latter.

174. The link between aggregate and industry-level productivity measures was explored by Domar (1961) and further developed by Hulten (1978). Domar showed that economy-wide rates of MFP changes can be expressed as a weighted sum of industry-specific MFP growth where the weights are the ratio of each sub-industry's gross output to its value added (for a more explicit derivation see Annex 6: Aggregation of Output, Inputs and Productivity). This ratio is always greater than or equal to unity, implying that, in the presence of intermediate inputs, aggregate productivity growth will exceed the weighted average of the growth of its component industries. This is tantamount to saying that "Domar weights" reflect the combined effects of productivity growth within individual industries and the induced effects on those downstream industries that benefit from more efficiently produced intermediate inputs.[69]

Table 23. **Numerical example: basic data for individual industries**

t_0	Industry (commodity)		Final demand	Gross output	Price index
Industry (commodity)	1	2			
1	0	10	5	15	1.00
2	2	0	14	16	1.00
Value added, of which:	13	6	19		
Labour income	10	4			1.00
Capital income	3	2			1.00
Gross output	15	16		31	

t_1	Industry (commodity)		Final demand	Gross output	Price index
Industry (commodity)	1	2			
1	0	11	4	15	1.01
2	3	0	12	15	0.98
Value added, of which:	12	4	16		
Labour income	10	3			1.02
Capital income	2	1			1.03
Gross output	15	15		30	

175. To demonstrate some of the steps involved in aggregation, consider a numerical example with basic data presented in Table 23. There are two industries, and for simplicity, each industry produces exactly one commodity. The economy is closed, and intra-industry flows have been netted out, but there are inter-industry deliveries of intermediate products. For example, in period t_0, industry 1 supplies inputs of value 10 to industry 2. Current-price value added is the difference between gross output and purchases of intermediate inputs, and amounts to 13 currency units in industry 1 and in the base period. Along with current-price input-output flows comes a set of deflators for each industry's output and for primary inputs labour and capital services. With the information in

---

[69] In this sense, Hulten (1978) distinguishes between productivity change *originating in* a sector and the *impact* of productivity change *on* the sector. Similarly, in Durand's (1996) framework, Domar weights combine the effects of (narrowly defined) aggregation and vertical integration.

Table 23, it is possible to obtain gross-output based productivity measures for individual industries as well as the Domar weights to aggregate the industry-level MFP indices.

176. Before following this route, consider Table 24. It shows again the input-output flows of the stylised economy, but after vertically integrating the two industries. The entire economy is now treated as a single unit. In line with the concept of sectoral output, commodity flows between industry 1 and 2 have been netted out because they now constitute intra-industry deliveries. Consequently, the value of gross output in the integrated economy is smaller than the sum of each industry's gross output in the decentralised economy, although value added and total income have remained unchanged. It is now possible to obtain measures of MFP growth for the integrated industries (in the second column of Table 25). It can then be demonstrated that this result coincides with the aggregate measure based on applying Domar weights to individual industries' MFP indices.

Table 24. **Numerical example *(cont'd.)*: basic data for integrated industries**

t_0	Industry			Final demand	Gross output	Price index
Industry	1	2	1&2			
1	0	0		5	5	1.00
2	0	0		14	14	1.00
Value added, of			19	19		
Labour			14			1.00
Capital			5			1.00
Gross			19		19	

t_1	Industry			Final demand	Gross output	Price index
Industry	1	2	1&2			
1	0	0		4	4	1.01
2	0	0		12	12	0.98
Value added, of			16	16		
Labour			13			1.02
Capital			3			1.03
Gross			16		16	

Table 25. **Numerical example *(cont'd.)*: MFP calculations**

Indices relate to base year t_0=1

Individual industries			**Integrated industries 1 and 2**	
		Output		
	Industry (commodity)			
	1	2		
			Share in final demand (average over 2 periods)	
Indirect quantity index of gross output	0.990	0.957	Commodity 1	0.26
Indirect quantity index of intermediate inputs	1.531	1.089	Commodity 2	0.74
			Indirect quantity indices	
			Commodity 1	0.792
			Commodity 2	0.875
			Tornqvist quantity index of final demand	0.853
		Indirect quantity index of primary inputs		
Labour	0.980	0.735	Labour	0.910
Capital	0.647	0.485	Capital	0.583
Tornqvist quantity index of primary inputs	0.903	0.651	Tornqvist quantity index of primary inputs	0.823
		Share in current-price value added (average over 2 periods)		
Labour	0.80	0.71	Labour	0.77
Capital	0.20	0.29	Capital	0.23
		Share in current price gross output (average over 2 periods)		
Value added	0.83	0.32	Value added	1.00
Intermediate inputs	0.17	0.68	Intermediate inputs	0.00
		Tornqvist MFP index		
By industry, gross-output based	1.004	1.036		
Domar weights	0.86	0.89		
Aggregate economy		1.036	***Integrated industries 1 and 2***	1.036

177. Consider now the computation of gross-output based MFP measures for each industry, as shown in the first column of Table 25. Several steps are required:

- First, from the available price indices and the changes in current-price values of commodity flows, implicit quantity indices are obtained for the gross output of both industries and for each industry's intermediate inputs. Thus, industry 1's purchases of intermediate inputs from industry 2 can be described by a quantity index, obtained by dividing the value change in purchases by the price index for commodity 2:(3/2)/0.98=1.531. Computation of quantity indices for labour and capital input proceeds along similar lines. Given the respective shares of labour and capital in current price value added, a Törnqvist quantity index of primary inputs can be formed. In the example, quantities of labour and capital both fell in industry 2, its weighted geometric average, the Törnqvist index of primary inputs, dropped to 0.651.

- Second, division of the quantity index of gross output by the geometrically-weighted average of primary and intermediate inputs yields the KLEMS MFP measure for each industry: by way of the numerical example, productivity growth rises by 0.4 % in industry 1 and by 3.5% in industry 2.

- Third, Domar weights are computed as the ratio between each industry's gross output and economy-wide value added. The weights sum up to more than unity (0.86+0.89=1.75), and applying them to each industry's MFP index yields an aggregate index of productivity change of 1.036, *i.e.* a 3.6% rise in economy-wide MFP. This result can be compared with the MFP calculation for the integrated industry, carried out directly in the right column of Table 23. The integrated industry produces a composite output, and a quantity index of this output is built up using current-price shares in deliveries to final demand. Akin to the individual industries, an index of aggregate primary inputs can be computed, based on value-added shares of labour and capital. For the integrated industry, there are no intermediate inputs. As expected, and in line with the more formal derivation in Annex 6, the aggregated industry's productivity grows at 3.6% – *i.e.* at the same rate as the one obtained by applying Domar weights to industry-level MFPs.

8.3. Weighted averages: aggregation of value-added based productivity

178. A different way of dealing with intermediate inputs is to base productivity measurement on a value-added concept. This approach has been discussed in Section 3.1. A value-added based measure of productivity growth is computed as the difference between the rate of growth of deflated value added and the rate of growth of primary inputs. It was also pointed out earlier that, although not an accurate measure of disembodied technical change, value-added based productivity could be interpreted as an industry's capacity to contribute to economy-wide productivity and final demand. This characteristic comes out quite clearly in the process of aggregation. Current-price industry-level value added sums to aggregate value added and this aggregation is unaffected by intermediate inputs. It is therefore straightforward to aggregate industry-level productivity growth to an economy-wide measure. Aggregation weights are simply each industry's current price share in total value added and, unlike Domar weights, these aggregation weights sum to unity. One accepts, however, that the components of this weighted average, value-added based productivity growth by industry, are not in general a valid representation of disembodied technical change in the individual industries.

179. Value-added based MFP calculations are carried out in Table 26. In line with the description in Section 3.1.2, a first step consists in computing a value-added deflator. This is obtained by subtracting a price change of intermediate inputs from a price change of gross output, the first one weighted with the inverse share of value added in gross output, the second one with the ratio of intermediate inputs over value added [see equation (5)]. In the present example, the price index of value added takes a value of 1.016 in industry 1 and 0.917 in industry 2. Division of the current price index of value added by its price index yields the deflated (quantity) index of value added. Further division by the quantity index of primary inputs produces the value-added based productivity index (1.006 and 1.116 in industries 1 and 2, respectively).

180. It was shown earlier that there is a direct link between an industry's value-added based MFP growth and its gross-output based MFP growth. More specifically, the former differs from the latter by a factor that equals the ratio of an industry's gross output over its value added. In the numerical example above, these ratios are 0.83 and 0.32 in the two industries. Then, the gross-output based MFP growth of industry 1 can directly be derived from the value-added based MFP growth by multiplying

100*ln(1.006)*0.833=0.52%. Similarly, the gross-output based MFP growth in industry 2 is obtained as 100*ln(1.116)*0.32=3.5%. These growth rates are very close to the gross-output based MFP measures calculated in Table 26 (last panel). Differences arise exclusively because the empirical approximation of the continuous Divisia index by a discrete Törnqvist index is not quite complete.

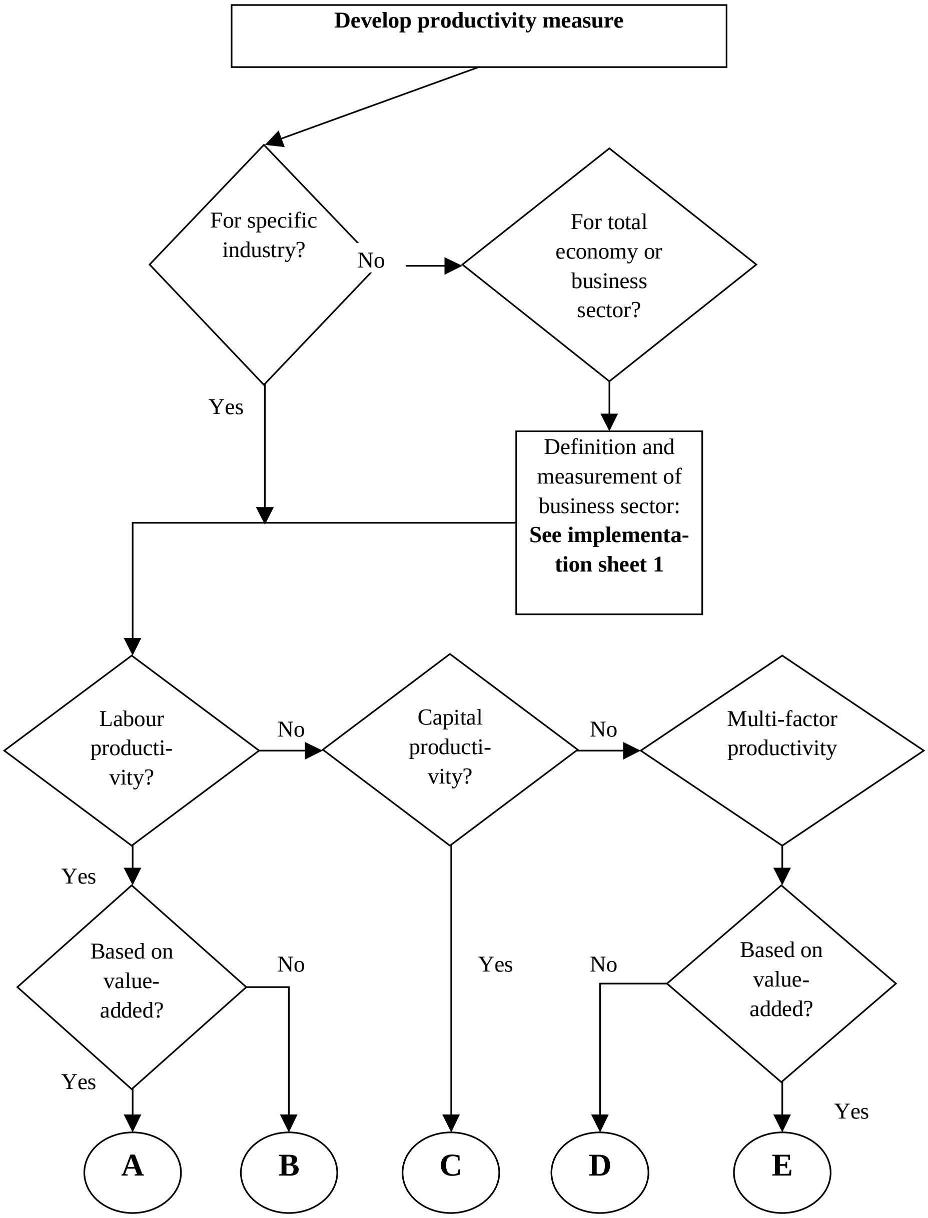
Table 26. **Numerical example *(cont'd.)*: Value-added based MFP**

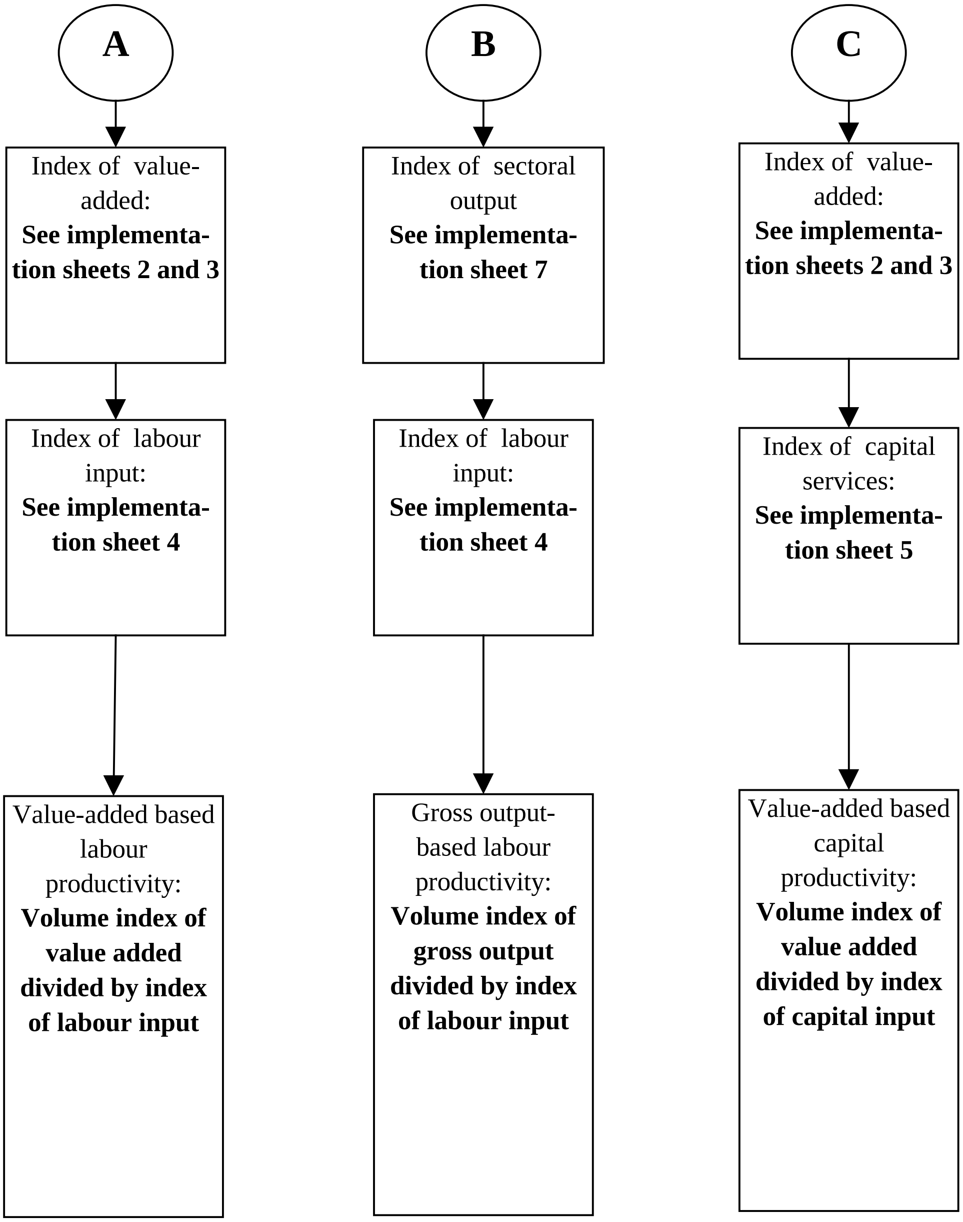
Indices relate to base year t_0

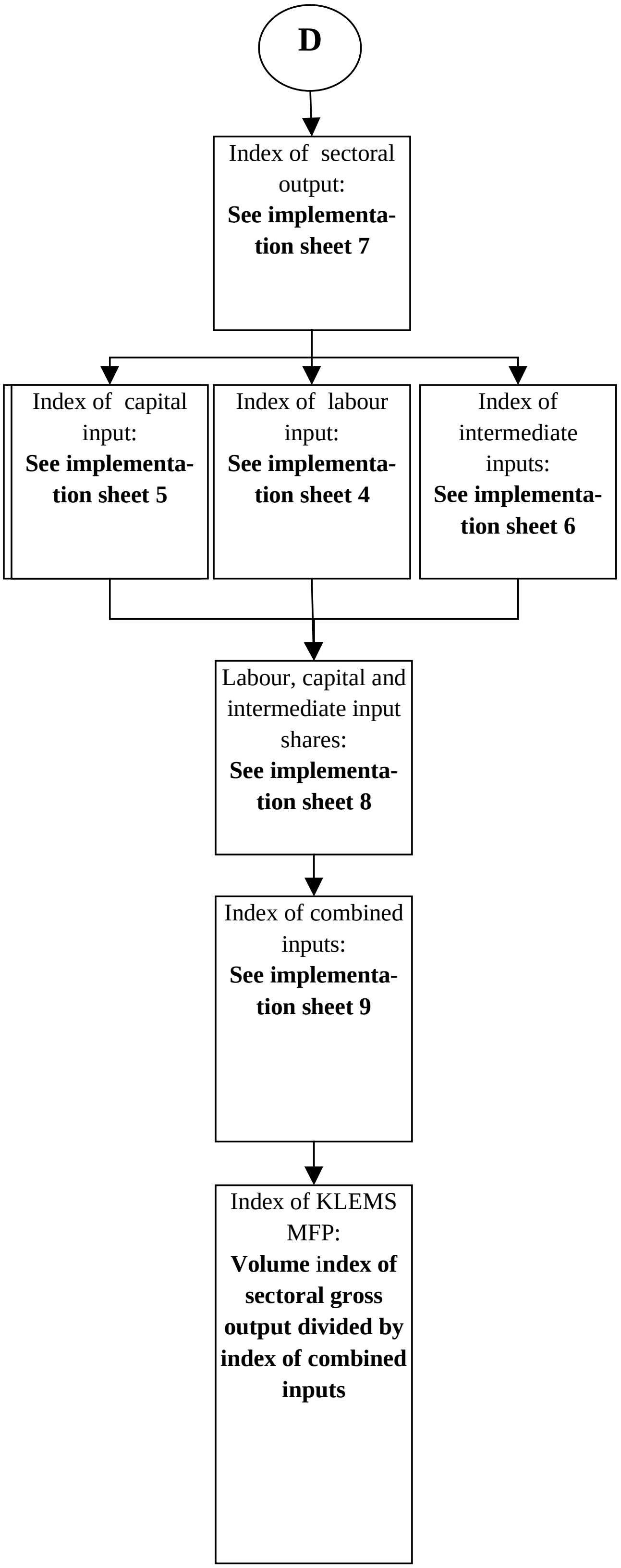
	Industry	
	1	2
Price index of gross output	1.010	0.980
Price index of intermediate inputs	0.980	1.010
Share of value added in gross output		
t_0	0.87	0.38
t_1	0.80	0.27
Average over two periods	0.83	0.32
Tornqvist price index of value added	1.016	0.917
Value index of value-added	0.923	0.667
Tornqvist index of deflated value added	0.908	0.727
Tornqvist quantity index of primary inputs	0.903	0.651
Value-added based Tornqvist productivity index	1.006	1.116
Value-added based Tornqvist productivity index: rate of change	0.6%	11.0%
Gross-output based Tornqvist MFP index	1.005	1.036
Gross-output based Tornqvist MFP: rate of change	0.52%	3.53%

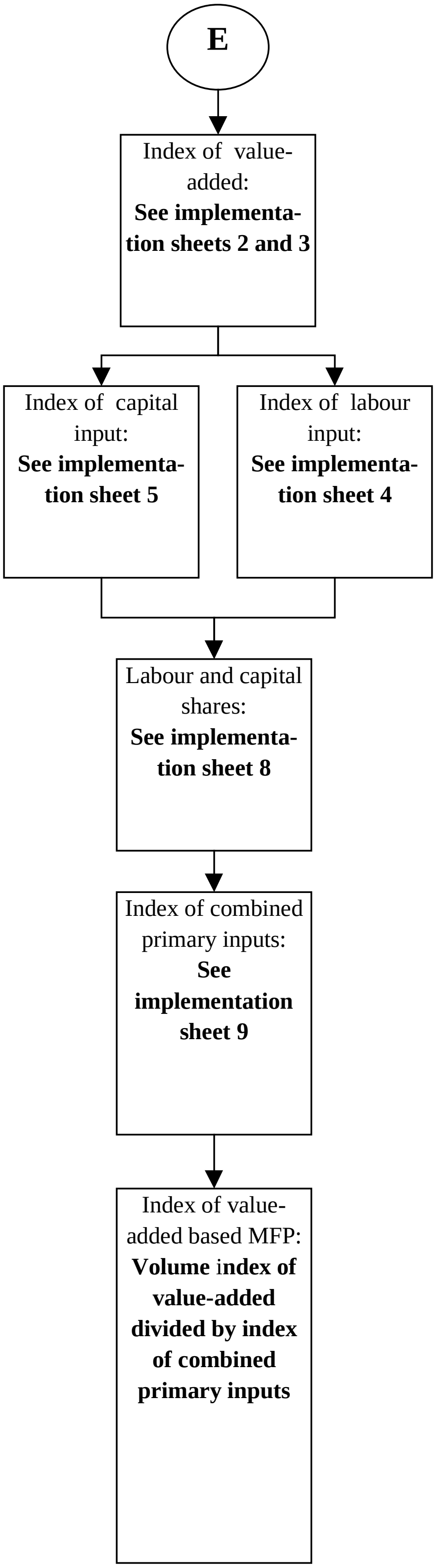
9. IMPLEMENTATION GUIDE

181. The following pages provide a short guide to implementing the most frequently used productivity measures. Presentation follows a flow diagram, with more detailed explanations in a sequence of "implementation sheets".




A
Index of value-added: See implementation sheets 2 and 3
Index of labour input: See implementation sheet 4
Value-added based labour productivity: Volume index of value added divided by index of labour input
B
Index of sectoral output See implementation sheet 7
Index of labour input: See implementation sheet 4
Gross output-based labour productivity: Volume index of gross output divided by index of labour input
C
Index of value-added: See implementation sheets 2 and 3
Index of capital services: See implementation sheet 5
Value-added based capital productivity: Volume index of value added divided by index of capital input


D
Index of sectoral output:
See implementation sheet 7
Index of capital input:
See implementation sheet 5
Index of labour input:
See implementation sheet 4
Index of intermediate inputs:
See implementation sheet 6
Labour, capital and intermediate input shares:
See implementation sheet 8
Index of combined inputs:
See implementation sheet 9
Index of KLEMS MFP:
Volume index of sectoral gross output divided by index of combined inputs


E
Index of value-added:
See implementation sheets 2 and 3
Index of capital input:
See implementation sheet 5
Index of labour input:
See implementation sheet 4
Labour and capital shares:
See implementation sheet 8
Index of combined primary inputs:
See implementation sheet 9
Index of value-added based MFP:
Volume index of value-added divided by index of combined primary inputs

Implementation sheet 1: Definitions of the "business sector"

Data situation	Method
Case 1: Activity-based definition	The "business sector" is not an internationally defined notion. However, productivity measures are often calculated for this aggregate, on the ground that it reflects productive activities in a market environment where competitive conditions are more likely to prevail than in other parts of the economy. One way to define the business sector is by way of activities. Given observations on outputs and inputs by industry (*i.e.* in an activity-based classification such as ISIC or NAICS), the business sector could be defined as a grouping of all industries except Public administration, defence and compulsory social security (ISIC Rev. 3 Category L), and Private households (ISIC Rev. 3 Category P). Depending on the country, other service activities such as Education (Category M), health (Category N) and parts or all of Other community, social and personal service activities (Category O) could also be excluded. Often, owner-occupied dwellings (part of the real estate industry) are also excluded from the business sector in productivity studies. ***Comment**: Even if service industries such as health and education are dominated by market producers, there are often significant problems involved in measuring volume output in these industries. This can make productivity indicators unreliable or meaningless. For this reason, the exclusion of categories M, N, O and P in the definition of the business sector constitutes a pragmatic solution. It also helps international comparisons, which are otherwise hampered by the different institutional arrangements in industries such as health and education. Of course, if it is possible to distinguish between establishments that are market producers and those that are not, an activity-based grouping of market producers would be a preferred option for productivity analysis.*
Case 2: Institutional definition	An alternative way of defining the "business sector" is by way of institutional sectors and institutional units. In this context, the "business sector" could be defined as the "corporate sector", and measured like all financial and non-financial corporations, including quasi-corporations, as defined in the SNA 93. A "top-down" computation of the corporate sector would then consist of the total economy minus the general government sector, the households sector and all non-profit institutions, whether they operate in the corporate sector or not. ***Comment**: This institutional definition of the business sector is close to that used by the United States' Bureau of Labor Statistics. It has the advantage of more closely targeting market producers than an activity-based approach. Its drawback is that there is no direct link to industry-level productivity measures which follow an activity-based approach.*

Implementation sheet 2: Measuring aggregate constant price value added

Data situation	Method
Case 1: Industry-level data are available at current and constant prices	Aggregate (*e.g.* business sector) value added at constant prices should be calculated “bottom-up”, (reflecting the lowest level of aggregation for which data are available) and based on a superlative index number. The Törnqvist index is a correct and computationally simple index number formula for this purpose. Steps required: a) Calculation of annual indices of constant price value added $\left(\frac{VA_t^j}{VA_{t-1}^j}\right)$ for each industry *j* (see implementation sheet 3). ***Comment****: Industry-level value added at constant prices is often based on index numbers that are not superlative, in particular the Laspeyres formula. In many cases, the implied bias will have to be accepted by users of national accounts statistics.* b) Calculation of annual series of current price shares of each industry (s_t^j) in total value added: $s_t^j = \frac{P_{VA,t}^j VA_t^j}{\sum_j P_{VA,t}^j VA_t^j}$. c) Calculation of annual Törnqvist indices of constant price value added for the desired (business sector) aggregate: $\frac{VA_t^B}{VA_{t-1}^B} = \prod_j \left(\frac{VA_t^j}{VA_{t-1}^j}\right)^{\frac{1}{2}(s_t^j + s_{t-1}^j)}$. ***Comment****: The resulting index of business sector constant price value added may be different from the “official” index directly taken from the national accounts if the latter is based on a different index number formula such as the Laspeyres formula. When national accounts are based on a superlative index (such as the Fisher index number in the case of the United States), the Törnqvist aggregation will closely approximate the official index.*
Case 2: Industry-level data not available	Business sector constant price value added is taken directly from the national accounts source, by way of a “top-down” approach. ***Comment:*** *While simple, and by definition consistent with official data, this procedure may imply a substitution bias if accounts are based on fixed-weight Laspeyres quantity indices.*

Implementation sheet 3: Measuring constant price value added by industry

Data situation	Method
Case 1: Price indices and current price observations for gross output and intermediate inputs are available for individual industries	Preferred method, computation of a volume series for value added based on a superlative index number. Steps required: a) Calculation of price indices of gross output and intermediate inputs for each industry. Price indices of intermediate inputs are weighted averages of price indices of intermediate products used by a given industry, with current-price cost shares of each input as weights. In the case of a Törnqvist index, the weights are geometric, and averaged across the comparison periods. Similarly, a price index of gross output should be constructed as a weighted average of the price indices of different products produced by an industry. In the case of the Törnqvist index, geometric weights represent the current price share of each product in total output, averaged across comparison periods. b) Calculation of indirect volume indices of gross output ($\frac{Q_t^j}{Q_{t-1}^j}$) and of intermediate inputs ($\frac{M_t^j}{M_{t-1}^j}$) for each industry *j*, by dividing the current-price index of gross output and the current-price index of intermediate inputs by their respective price indices. c) Calculation of annual series of current-price shares of value added in gross output for each industry ($s_{VA,t}^j$) in total value added: $s_{VA,t}^j = \frac{P_t^j Q_t^j - P_{M,t}^j M_t^j}{P_t^j Q_t^j}$. In this expression, $P_t^j Q_t^j$ is the current-price value of industry *j*'s gross output and $P_{M,t}^j M_t^j$ the current-price value of all its intermediate inputs, domestic and imported. The share of intermediate inputs in gross output is given by $s_{M,t}^j = 1 - s_{VA,t}^j$.

Implementation sheet 3 (cont'd.): Measuring constant price value added by industry	
	d) Calculation of annual Törnqvist volume indices of value added for industry *j*: $\frac{VA_t^j}{VA_{t-1}^j}=\left(\frac{Q_t^j}{Q_{t-1}^j}\cdot\left(\frac{M_t^j}{M_{t-1}^j}\right)^{-\frac{1}{2}(s_{M,t}^j+s_{M,t-1}^j)}\right)^{\frac{1}{\frac{1}{2}(s_{VA,t}^j+s_{VA,t-1}^j)}}$. ***Comment****: The resulting volume index of value added constitutes the Törnqvist version of double-deflation. It is in general different from the more narrowly defined double-deflation with a Laspeyres index. In that case, constant-price value added is also a difference between the constant price index of gross output and the constant price index of intermediate inputs, but weights do not enter as geometric weights and are expressed in prices of the base period. The Törnqvist formulation uses geometric weights , expressed in current prices.*
Case 2: Price and volume series intermediate inputs not available	It occurs that price indices for intermediate inputs are not available. In this case, statistical offices estimate constant-price value-added series by extrapolating reference year estimates of current-price value added using a volume indicator of output such as an index of production. This has implications for the quality and interpretation of the constant-price value-added series but the latter can still serve as a useful measure of output in productivity calculations. When constant-price value-added series are based on extrapolation of volume inputs (*e.g.* employment), they cannot be used in productivity analysis.

Implementation sheet 4: Measuring labour input	
Data situation	Method
Case 1: Data on hours worked are available by industry	Hours worked are the preferred measure for the quantity of labour input. Hours worked by industry (L_t^j) are measured as the product of employment and average hours worked ($L_t^j = E_t^j \cdot h_t^j$). Where time series of such data exist, they are readily expressed as an index of labour input for industry *j*, $\frac{L_t^j}{L_{t-1}^j} = \frac{E_t^j}{E_{t-1}^j} \Big/ \frac{h_t^j}{h_{t-1}^j}$. Aggregation of hours worked across industries often proceeds by simple addition of hours worked in individual industries. A preferred way of constructing an index of aggregate hours worked is to attach weights to the industry-specific index. These weights should reflect each industry's share in total compensation at current prices. In the case of a Törnqvist index, the resulting aggregate index of labour input is given by: $$\frac{L_t}{L_{t-1}} = \prod_j \left(\frac{L_t^j}{L_{t-1}^j} \right)^{\frac{1}{2}(s_{L,t}^j + s_{L,t-1}^j)} .$$ ***Comments****:* *a) Hours worked by industry are difficult to come by, especially for individual industries. In many countries, information on average hours per employed person exists only for major aggregates or the entire economy. Even where industry-level data are available, their international comparability is often hampered by differences in countries' methodologies to derive average hours per person.* *b) This implementation guide does not dwell on the issue of differentiating labour input by different types of labour quality. This reflects practical considerations, not a judgement on the importance of the issue. Differentiating labour input by skills requires substantive investment in data and methodology. A useful and extensive reference is BLS (1993).*
Case 2: Only data on full-time equivalent employment is available by industry	Usage of the number of full-time equivalent (FTE) employed persons constitutes a second-best approach to the measurement of labour input. Aggregation across industries should follow the method outlined above, *i.e.* as a geometrically-weighted average of industry-specific indices of FTEs with each industry's current-price share in total compensation as weights. ***Comment:*** *FTE data are more frequently available than hours worked. As in the case of hours worked, there may be significant variations between countries in their practices to compute FTE series. As a consequence, international comparisons have to be interpreted with care.*
Case 3: Only data on numbers employed are available.	Numbers employed constitute the least preferable measure of labour input because they neither reflect shifts in the composition of part- and full-time work or changes in the average number of hours worked by full-time employees. ***Comment:*** *These shortcomings imply a possible bias in labour input measures. However, as such, their international comparability is likely to be better than that for hours worked or FTE persons.*

Implementation sheet 5: Measuring capital input	
Data situation	Method
Case 1: Data on capital services are already available	Capital services are the preferred measure for capital input. As a rule, time series of capital services do not exist as statistics independent from productivity measurement. They have to be constructed for this purpose and a step-by-step guide can be found in Chapter 5 of the manual. ***Comments****:* *a) A satisfactory measure of capital services requires investment series by type of asset. Moreover, the accuracy of the capital services measure increases with a rising degree of asset detail. By implication, industry-level measures of capital services require data on investment by industry, cross-classified by type of asset. This type of information is rarely available. A useful and important first step is therefore to compile capital service indices at the level of the entire economy or the entire business sector.* *b) It is important to note, however, that the data requirements for capital services measures hardly exceed those for other measures of capital, such as the gross and net (wealth) stock. Ideally, all three capital measures are established jointly and consistently – each with their specific purpose.*
Case 2: Only the gross and/or the net stock of capital are available	Measures of gross and net stocks of capital clearly constitute an inferior measure of capital input compared to capital service series. Both measures have routinely been used in productivity studies but are likely to produce a biased measure of the contribution of capital to growth. ***Comment:*** *Empirically, indices of gross and net capital stocks tend to rise less rapidly than measures of capital services. The implication is a tendency to understate capital's contribution to output growth and to overstate the residual MFP index.*

Implementation sheet 6: Measuring intermediate inputs	
Data situation	Method
Case 1: Use tables and price indices for individual products are available	The statistical source for intermediate inputs is a use table, specifying the flow of products used in different industries. Constant price data for intermediate inputs are obtained by deflating current-price expenditure on intermediate inputs by an industry-specific price index for intermediate inputs. This price index ($\frac{P^{j}_{M,t}}{P^{j}_{M,t-1}}$) is a weighted average of the price indices of individual products *k* consumed by industry *j* ($\frac{P^{j,k}_{M,t}}{P^{j,k}_{M,t-1}}$). Weights should reflect the share of each input *k* in industry *j*'s total expenditure for intermediate inputs at current prices ($s^{j,k}_{M,t}$). In the present manual, the use of a Törnqvist index is recommended. It is defined as $\frac{P^{j}_{M,t}}{P^{j}_{M,t-1}} = \prod_{k} \left(\frac{P^{j,k}_{M,t}}{P^{j,k}_{M,t-1}} \right)^{\frac{1}{2}\left(s^{j,k}_{M,t}+s^{j,k}_{M,t-1}\right)}$. ***Comments****: The level of detail at which individual inputs can be distinguished for each industry varies between countries. This concerns current price data as well as price indices. As a general rule, the greatest possible product detail should be used to compute price or volume indices of intermediate inputs.*

Implementation sheet 7: Measuring sectoral output	
Data situation	Method
Case 1: Industry-by-industry symmetric input-output tables are available with imports identified separately	Sectoral output is a measure of an industry's gross output but excluding intra-industry deliveries. For example, manufacturing sectoral output excludes shipments from one manufacturing establishment to another. Sectoral output constitutes the preferred measure of gross output for KLEMS MFP indices at different levels of aggregation. To remove intra-industry deliveries from total gross output, information on these intra-industry shipments has to be available. This requires availability of a symmetric, industry-by-industry input-output table. ***Comments****:* *a) For aggregate sectors, such as the total business sector, sectoral output and value added converge, if imported intermediate inputs are relatively unimportant. This may not be the case for small open economies where imports of intermediate inputs can be significant.* *b) To identify intra-industry deliveries, industry-by-industry symmetric input output tables are required on an annual basis. In the statistical practice, such symmetric tables have to be derived from supply and use tables. This derivation requires several, often restrictive assumptions. If these assumptions are not met in reality, the resulting industry-by-industry tables may be of mixed quality. This generates a trade-off in comparison with input and output measures gross of intra-industry deliveries where such assumptions are unnecessary.*
Case 2: No industry-by industry symmetric input-output tables are available	In the absence of input-output tables by industry, intra-industry transactions cannot be identified. In this case, gross-output data inclusive of intra-industry deliveries presents an alternative. This information is typically available from national accounts statistics and/or from supply-use tables. ***Comment:*** *Netting out intra-industry flows amounts to treating the entire industry or sector as a single producing unit. If intra-industry flows are not netted out, no such "integration" is implied and the output measure reads as a weighted average of gross output of all individual units present in the industry.*

Implementation sheet 8: Measuring factor shares

Purpose	Method
Value-added based MFP measures	The following simplified methodology is recommended to compute shares of labour and capital in gross value added. a) From the production account or supply-use framework, one distinguishes the following component of gross value added: • Compensation of employees (W); • Other taxes less subsidies on production and imports (T); • Gross mixed income (I); • Gross operating surplus (GOS). b) To split gross mixed income into a labour and capital component, it is recommended to assign the same average compensation to self-employed as to employees. Calling the labour component of gross mixed income I_L, this implies $I_L = \frac{W}{Employees} \cdot Selfemployed$. Employees and self-employed are either measured in numbers of full-time equivalents or in terms of hours worked. Given I_L, the capital part of mixed income is calculated residually: $I_K = I - I_L$. c) Net taxes on production and imports are allocated proportionately to labour and capital. For this purpose, call t_L the share of labour in net taxes and $1 - t_L$ the share of capital. Proportional allocation implies that $t_L = \frac{W + I_L}{W + I + GOS}$. d) The share of labour in gross value added is then given by: $\frac{W + I_L + t_L \cdot T}{W + I + T + GOS}$; The share of capital in gross value added is $\frac{W + I_K + (1 - t_L) \cdot T}{W + I + T + GOS}$. ***Comment**: The procedure described here is specifically simple. It can be improved in two ways. First, the split of mixed income could start with two estimates, one for the labour and one for the capital portion with subsequent reconciliation. Second, taxes and subsidies on production could be at least partly allocated to labour and capital by their nature. For example, land tax or motor vehicle registration are capital-related taxes, taxes on payrolls or fringe benefits are labour-related.*

Implementation sheet 8 (cont'd.): Measuring factor shares

Purpose	Method
Gross-output based MFP measures	For a gross-output based MFP measure, factor shares are computed with regard to total costs, not value added. This requires an extension of the calculations above. Letting *M* be the expenditure on intermediate inputs at purchaser's prices (*i.e.* including taxes minus subsidies on products), factor shares are now given by the following relationships: Labour share = $\frac{W + I_L + t_L \cdot T}{W + I + T + GOS + M}$; Capital share = $\frac{W + I_L + t_L \cdot T}{W + I + T + GOS + M}$; Share of intermediate inputs = $\frac{M}{W + I + T + GOS + M}$.

Implementation sheet 9: Index of combined inputs	
Purpose	Method
Gross-output based MFP measures	In the case of a gross-output based MFP measure, the recommended procedure in this manual for an index of combined labour, capital and intermediate inputs is to use a Törnqvist formula: $Quantity\ index\ of\ combined\ inputs\ in\ industry\ j = \left(\frac{L_t^j}{L_{t-1}^j}\right)^{\bar{s}_L^j} \cdot \left(\frac{K_t^j}{K_{t-1}^j}\right)^{\bar{s}_K^j} \cdot \left(\frac{M_t^j}{M_{t-1}^j}\right)^{\bar{s}_M^j}$, where: L_t^j = labour input in industry *k* and period *t*; K_t^j = capital input in industry *k* and period *t*; M_t^j = intermediate inputs in industry *k* and period *t*; $\bar{s}_L^j = \frac{1}{2}(s_{L,t}^j + s_{L,t-1}^j);\ s_{L,t}^j =$ labour share in total cost; $\bar{s}_K^j = \frac{1}{2}(s_{K,t}^j + s_{K,t-1}^j);\ s_{K,t}^j =$ capital share in total cost; $\bar{s}_M^j = \frac{1}{2}(s_{M,t}^j + s_{M,t-1}^j);\ s_{M,t}^j =$ intermediate input share in total cost.
Value-added based MFP measure	In the case of a value-added based MFP measure, the recommended procedure in this manual for an index of combined labour and capital inputs is to use a Törnqvist formula: $Quantity\ index\ of\ combined\ inputs\ in\ industry\ j = \left(\frac{L_t^j}{L_{t-1}^j}\right)^{\tilde{\bar{s}}_L^j} \cdot \left(\frac{K_t^j}{K_{t-1}^j}\right)^{\tilde{\bar{s}}_K^j}$ where: L_t^j = labour input in industry k and period t; K_t^j = capital input in industry k and period t; $\tilde{\bar{s}}_L^j = \frac{1}{2}(\tilde{s}_{L,t}^j + \tilde{s}_{L,t-1}^j);\ \tilde{s}_{L,t}^j =$ labour share in gross value added; $\tilde{\bar{s}}_K^j = \frac{1}{2}(\tilde{s}_{K,t}^j + \tilde{s}_{K,t-1}^j);\ \tilde{s}_{K,t}^j =$ capital share in gross value added.

10. INTERPRETATION OF PRODUCTIVITY MEASURES

10.1. Technology and productivity measures: some links

182. Technological change drives long-term economic growth and improved standards of living. This is a well-documented observation (OECD, 1998a, 2000). Frequently, however, the measurement of technical change is reduced to observing the rate of MFP growth, whereas closer inspection, both from a conceptual and from an empirical angle, shows that MFP is not necessarily technology, nor does technological change exclusively translate into changes in MFP. Consider these two points in turn.

183. *Technological change does not necessarily translate into MFP growth.* Economic theory and empirical work have accorded great importance to the distinction between embodied and disembodied technology. Embodied technological changes are advances in the design and quality of new vintages of capital and intermediate products: machinery and equipment embody the fruits of research performed by the capital goods-producing industry, and other sectors obtain access to the outcome of this research through the purchase of new capital equipment or intermediate goods. Disembodied technical change, on the other hand, relates to the advances in science, to blueprints and formulae and to the diffusion of knowledge of how things are done, including better management and organisational change. The distinction is important because the diffusion of embodied technical change is dependent on market transactions: investment in the improved capital or intermediate good will be undertaken until its marginal contribution to revenue generation just equals its user cost, itself dependent on the market price of the capital good. The diffusion of disembodied technical change is not necessarily associated with market transactions: information may circulate freely and its use by one person does not normally restrict its use by another one.

184. The distinction between embodied and disembodied technical change can have significant implications for analysis and for the formulation of technology policies.[70] To explore the link to productivity measurement, it is useful to re-state the growth-accounting approach: the rate of change of output, $\frac{d\ln Q}{dt}$ is a weighted average of the rate of growth of labour input $\frac{d\ln L}{dt}$, the rate of growth of capital input, $\frac{d\ln K}{dt}$, intermediate inputs, $\frac{d\ln M}{dt}$, and technical change, designated $\frac{d\ln A}{dt}$. Weights correspond to the current-price shares of each factor in total costs, and sum to unity.

$$\frac{d\ln Q}{dt} = s_L \frac{d\ln L}{dt} + s_K \frac{d\ln K}{dt} + s_M \frac{d\ln M}{dt} + \frac{d\ln A}{dt} \qquad (9)$$

---

[70] See OECD (1998b) for a discussion.

185. When capital inputs are measured as indicated by production theory, with a differentiation between types of assets, and based on price indices of capital goods that reflect the improvement in quality and design between vintages, the capital term $\frac{d\ln K}{dt}$ captures both changes in the quantity and in the quality of capital as an input to production. A similar point can be made for labour input, and for intermediate inputs. When $\frac{d\ln L}{dt}$ is a measure based on different types of labour input, differentiated by skills and aggregated with the skill-specific share in total wages, the contribution of labour to output growth captures both changes in the quantity of labour (simple hours worked) and changes in the skill composition of labour. It follows that when labour and capital are carefully measured, taking into account their heterogeneity and quality change, the effects of embodied technical change (in capital and intermediate inputs) and of improved human capital (in labour) should be fully reflected in the measured contributions of each factor of production. It also follows that the MFP term A does not reflect the effects of embodiment. It does, however, capture the effects of disembodied technological change.

186. In the above framework, the MFP term also picks up *spillover effects* from capital, labour and intermediate inputs. Economically, spillovers are costless additions to overall productivity generated by certain types of physical or human capital employed in production. They are costless, because they occur in addition to the remunerated contribution of factor inputs to production – remuneration being captured by the income share of labour and capital.

187. Conceptually, and following the distinction drawn by Jorgenson (1995a), the productivity term A reflects all those effects on output growth that are *not* investment, where investment is understood as the commitment of current resources in the expectation of future returns, implying that these returns can be internalised by the investor.

188. More often than not, data and resource constraints do not permit a careful differentiation and full coverage of all labour and capital inputs. As a consequence, some of the embodiment effects of technological change and some or all of the changes in the skill composition of labour input are picked up by the MFP residual. To see this, it is useful to disentangle the quality and quantity effects of labour and capital inputs. The compositional effect or quality component of labour input can be computed as the difference between the growth in total labour input and the growth of unadjusted hours: $\frac{d\ln L^Q}{dt} = \frac{d\ln L}{dt} - \frac{d\ln L^U}{dt}$. Along the same lines, capital input K can be split up into a quality change term K^q and into a term that reflects the quantity of capital, unadjusted for differences in its design between different vintages, K^u. This is inserted into the above growth decomposition and yields the following expression:

$$\frac{d\ln Q}{dt} = s_L\frac{d\ln L^U}{dt} + s_k\frac{d\ln K^U}{dt} + s_k\frac{d\ln M^U}{dt} + \frac{d\ln A^Q}{dt} \qquad (10)$$

$$where\ \frac{d\ln A^Q}{dt} = s_L\frac{d\ln L^Q}{dt} + s_K\frac{d\ln K^Q}{dt} + \frac{d\ln A}{dt}$$

189. It becomes apparent that the use of unadjusted labour and capital input measures in the calculation of the productivity residual leads to a situation where the new MFP term A^Q captures the effects of both embodied and disembodied technical change.[71] Thus, the correct interpretation of the

---

[71] Denison (1989), in his discussion of the treatment of quality change in capital goods, describes an approach following Rymes (1971), where capital input is measured in terms of the consumption

productivity term with respect to technological change requires knowledge about the methodology used to compute time series of capital and labour input.

190. *Empirically, MFP growth is not necessarily caused by technological change.* The preceding paragraphs argued that all facets of technological change are not necessarily captured by the MFP residual. Even where the residual reflects all technology shifts, other non-technology factors will also bear on measured MFP. Such factors include adjustment costs, economies of scale, cyclical effects, changes in efficiency and measurement errors. This is confirmed by econometric studies that link MFP growth to technology variables, in particular research and development and patents[72] or those that explicitly control for adjustment costs or allow for non-constant returns to scale. Research and development expenditure, for example, tends to show a statistically significant relationship to productivity growth, but only explains a relatively small part of the overall annual movements in MFP. This indicates the presence of other factors.

191. Strictly speaking, however, the presence of such factors invalidates the assumptions underlying the simple growth-accounting model. For example, the standard MFP computation assumes constant returns to scale in production and is therefore badly suited to dealing with the empirical occurrence of non-constant returns to scale. Similarly, pure changes in efficiency (as opposed to shifts in the technology frontier) are common empirical phenomena. Yet, the standard MFP calculation assumes that every establishment operates technically efficiently, thereby excluding the gains from elimination of inefficiencies.

192. *MFP measures tend to understate the eventual importance of productivity change in stimulating the growth of output*. This reflects the fact that in growth-accounting models such as the ones used in this manual, capital is considered an exogenous input to the production process. In a dynamic context, this is no longer the case, and further considerations about the role of technology and its eventual effects on growth enter the picture. In a dynamic context, a feedback effect exists between productivity change and capital: suppose that technical change allows more output to be produced per person. The static MFP residual measures just this effect of technical change. Typically, however, additional output per person will lead to additional savings and investment, and to a rise in the capital-labour ratio. A traditional growth-accounting measure would identify this induced effect as a growth contribution of capital, although it can be traced back to an initial shift in technology. Thus, the MFP residual correctly measures the shift in production possibilities but does not capture the induced effects of technology on growth (Rymes, 1971; Hulten, 2001).

10.2. Productivity growth as cost reduction

193. Productivity and growth-accounting measures described in this manual have been discussed with the help of production functions and quantity measures of inputs and outputs. There exists an equivalent, and intuitively appealing "dual" approach to express advances in productivity as shifts of a cost function.[73] A cost function shows the minimum input cost of producing a certain level of output, given a set of input prices. Under relatively weak regularity conditions, cost functions can be derived

---

foregone to provide the amount of savings needed to permit capital accumulation. In practice, this would imply the use of a private consumption deflator in the perpetual inventory method. The effect is to relegate all advances in knowledge (embodied and disembodied) explicitly to the productivity residual. See also Durand (1996) on this point.

[72] Griliches (1996b) provides an overview.

[73] Key properties of cost functions were first derived by Shephard (1953) – many other developments followed. For an overview, see Diewert (1987b).

from production functions, and vice versa – there is duality. To illustrate this point, one expresses a simple cost function C as $C = B \cdot Q \cdot c(w_1, w_2, ..., w_N)$, where C is total cost that varies as a function of the level of output, Q, of unit costs c (themselves dependent on input prices w_i) and of a parameter B. This parameter plays a role similar to the productivity parameter A in the production function $Q = A \cdot F(X_1, X_2, ... X_N)$. It can indeed be shown that $\frac{d \ln A}{dt} = -\frac{d \ln B}{dt}$. Thus, the MFP productivity residual can be measured either as the residual growth rate of output not explained by the growth rate of inputs or as the residual growth rate of average costs not explained by change in input prices:[74]

$$\frac{d \ln C}{dt} - \frac{d \ln Q}{dt} = \sum_i s_i \frac{d \ln w_i}{dt} - \frac{d \ln A}{dt} \qquad (11)$$

194. The expression in (11) states that the rate of growth of average costs equals the rate of growth of aggregate input prices, reduced by advances in multifactor productivity. A slightly different formulation is that productivity growth equals the diminution in total costs that is neither explained by a fall in output nor by substitution of inputs that have become relatively more expensive for those whose relative price has fallen.

195. This formulation of MFP in terms of average costs lends a richer interpretation to technological change. It is intuitively plausible that total and average costs can be reduced by many means including technological innovations in an engineering sense but also by organisational innovations, learning-by-doing, and managerial efforts.

196. The cost approach also shows how average cost can decline as a consequence of embodied technological change only: suppose that one of the inputs (*e.g.* computer services) exhibits falling prices (user costs) relative to other inputs as a consequence of (embodied) technical change. Most likely, a substitution process will take place where computer services replace other factors of production. The ensuing decrease in aggregate input prices leads to a fall in average costs, even if disembodied technology does not grow at all ($\frac{d \ln A}{dt} = 0$).

197. Finally, the above formulation points to another representation and measurement of disembodied technical change; namely, the difference between the growth rate of output prices and that of input prices. In a competitive market, prices evolve in line with marginal cost. Further, under constant returns to scale, average costs of production equal marginal costs and consequently, the rate of change of average costs of production equals the rate of change of the market price of output, or the expression on the left-hand side of equation (11). At the same time, the share-weighted average of the price changes of all inputs is an input price index, and its rate of change appears as the first expression on the right-hand side of (11). It is then easy to see that the rate of disembodied technical change is the difference between the rate of change of the input price index and the output price index:

---

[74] 74. In expression (11), s_i are the shares of each input in total costs. Each input share is equal to the elasticity of total costs with respect to the price of the corresponding input: $\frac{\partial C}{\partial w_i} \frac{w_i}{C} = \frac{w_i X_i}{C} = s_i$. This follows from Shephard's lemma ($\frac{\partial C}{\partial w_i} = X_i$) and from the assumption of cost-minimising behaviour.

$$\frac{d\ln A}{dt} = \frac{d\ln P_I}{dt} - \frac{d\ln P}{dt} \quad where \quad \frac{d\ln P_I}{dt} \equiv \sum_i s_i \frac{d\ln w_i}{dt};\ \frac{d\ln P}{dt} = \frac{d\ln C}{dt} - \frac{d\ln Q}{dt} \tag{12}$$

10.3. Productivity measures over the business cycle

198. In the discussion on capital and capacity utilisation in Section 5.6, allusion was made to the pro-cyclicality of many productivity measures: productivity growth tends to accelerate during periods of economic expansion and decelerate during periods of recession. One explanation given earlier was one of measurement: while variations in volume output tend to be relatively accurately reflected in economic statistics, variations in the rate of utilisation of inputs are at best partially picked up in data series. In particular, the rate of utilisation of capital equipment, *i.e.* the measurement of machine hours, is rarely accomplished. Labour input, if measured by hours actually worked, is better suited to reflect the changing rate of utilisation of manpower, but remains an imperfect measure. Consequently, a higher rate of capacity utilisation in periods of expansion is accompanied by output measures that may show rapid growth whereas input measures may remain stable or grow less rapidly. The result is a rise in measured productivity growth. The converse holds for periods of recession.

199. However, even if capacity utilisation were accurately measured, the standard productivity model is not easily fitted to the realities of the business cycle. Because much of the economic and index number theory relies on long-term, equilibrium relationships, with little or no unforeseen events for economic actors, the economic model of productivity measurement is easier to implement and to interpret during periods of continued, and moderate expansion than during rapidly changing phases of the business cycle. This has implications for the interpretation of productivity measures. In particular, it means that year-to-year changes in productivity growth should not be interpreted *prima facie* as shifts in disembodied technology. For this purpose it is preferable to examine productivity growth patterns over longer periods of time – and best between years that mark the same position in the business cycle.

200. Short-term productivity measures are nonetheless useful, even when they mirror the business cycle. Although they should not be interpreted as reflections of technical change, short-term productivity data can, for example, shed light on the relationship between short-term changes in output, employment and average weekly hours. Econometric analysis of high-frequency productivity measures over the business cycle can yield useful insights. For example, the decomposition of labour productivity into cyclical and trend components has important implications for macroeconomic analysis. Historical decompositions allow the dating of business cycle peaks and troughs, while "real-time" decompositions make it possible to judge the current phase of the cycle, increasing the reliability of economic predictions. The measurement of trend productivity and output can be used to compute "gaps" which contribute to the understanding of the fiscal policy stance and, when interpreted as deviations from potential, are expected to determine many important macroeconomic variables such as wage and price inflation (Nicoletti and Reichlin, 1993).

201. There are a number of econometric techniques to deal with disentangling short-term from long-term movements in productivity growth. They may be pure time-series and filtering techniques or involve more explicit modelling approaches. For example, Guellec and Pottelsberghe (2001) use an error correction mechanism in their analysis of the determinants of MFP growth. This permits both a quantification of the relative importance of the determinants of productivity growth (research and development efforts, domestic and abroad, etc.) and a separation of long-term and short-term productivity shifts. Morrison (1986) provides another econometric treatment to quantify productivity growth in the presence of adjustment costs and disequilibria.

10.4. Industry and firm-level productivity growth

202. The main focus of this manual has been productivity measurement at the industry and aggregate level of the economy. Industries and branches are themselves made up of individual firms and establishments and new micro-level databases have greatly enhanced the possibilities for empirical research to better understand individual units' productivity performance carries over to what is observed at the industry level. Several important conclusions have arisen from a signficant body of studies (Haltiwanger, 2000; Bartelsman and Doms, 2000). First, there are large differences in productivity performance between individual units. Second, there is a continuous and large-scale re-allocation of outputs and inputs between producers, including within industries. Third, this re-allocation contributes significantly to aggregate productivity growth. For example, Haltiwanger reports that for the United States manufacturing sector, roughly half of multifactor productivity growth over the course of a decade can be accounted for by the re-allocation of outputs and inputs away from less productive to more productive businesses.

203. These findings do not invalidate the theory of productivity measurement in this manual whereby an industry, a sector or even the entire economy is essentially treated as if it were a single firm. Rather, such treatment adds to the understanding and interpretation of measured productivity growth. For example, it points to one mechanism by which an industry *as a whole* implements technical change: if new technology is mainly adopted by new establishments, productivity growth occurs with entry and exit, and this requires re-allocation. Technical advances at the industry level are then associated with the diffusion of new technology among establishments rather than with a simultaneous shift in the production frontier of an existing set of businesses. This provides an extra interpretation of changes in the productivity residual at the industry level. Micro-level data studies with their focus on firm dynamics, entry and exit and re-allocation of resources also form a natural link to the question of how innovation and "creative destruction" (see section below) translate into industry-level productivity growth. Nonetheless, micro-level approaches cannot replace the more aggregate type of productivity measurement. This has to do with data quality at the firm or establishment level (*e.g.* for capital input), the timeliness and exhaustiveness of the available data sets. However, micro-level studies undoubtedly enhance our understanding about some of the underlying drivers and dynamics of productivity growth.

10.5. Innovation and productivity measurement

204. Growth accounting and most other approaches to measuring productivity are firmly rooted in a standard neo-classical equilibrium concept. Equilibrium conditions are very important because they help to guide measurement of parameters that would otherwise be difficult to identify. An obvious example is the use of cost shares instead of output elasticities – the former are observable, the latter are not, but theory shows that, in competitive equilibrium, one must equal the other.

205. Although its usefulness is generally recognised, it has been argued that an equilibrium approach sits uneasily with the notion of innovation and productivity growth. Evolutionary economists (*e.g.* Dosi, 1988; Nelson and Winter, 1982; Nelson, 1981), in the tradition of Schumpeter, argue that innovation and technical change occur as a consequence of information asymmetries and market imperfections. In a quite fundamental sense, innovations and information asymmetries are one and the same phenomenon. Indeed, such asymmetries can scarcely be termed market imperfections when they are necessary conditions for any technical change to occur in a market economy (Metcalfe, 1996). The point made by evolutionary economists is that equilibrium concepts may be the wrong tools to approach the measurement of productivity change, because if there truly was equilibrium, there would be no incentive to search, research and innovate, and there would be no productivity growth.

206. Such criticism has to be taken seriously in the interpretation and use of productivity measures. An important lesson from this debate is that *accounting is not explaining the underlying causes of growth:* Griliches (1997) makes a related point:

> *"We can take productivity growth calculation and allocate it in great detail to the various missed components, reducing thereby the role of the "unallocated" residual. But this, while very instructive and valuable, only shifts the problem to a new set of questions: why was there all this investment in human capital? Will it continue? Where did the improvements in capital equipment come from? […] Real explanations will come from understanding the sources of scientific and technological advances and from identifying the incentives and circumstances that brought them about and that facilitated their implementation and diffusion. Explanation must come from comprehending the historical detail."*

207. This does not invalidate the usefulness of the equilibrium approach towards productivity measurement, but it does alert us to some of its limits. What emerges as a conclusion is the complementarity of approaches: (growth) accounting and productivity measurement allows one to quantify – in a systematic and consistent way – the proximate sources of growth. It has explanatory power in that it captures the workings of supply of, demand for and substitution between categories of measurable inputs. At the same time, growth accounting has to be complemented by institutional, historical and case studies if one wants to explore some of the underlying causes of growth, innovation and productivity change.

Annex 1

GLOSSARY

Glossary of main terms and their usage

Age-efficiency profile	Shows the loss in productive capacity of a capital good over its life time, or the rate at which the physical contribution of a particular asset declines over time, as an effect of wear and tear.
Age-price profile	Shows the loss in value of a capital good as it ages, or the pattern of relative prices for different vintages of the same (homogenous) capital good.
Basic price	The basic price is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy received, on that unit as a consequence of its production or sale; it excludes any transport charges invoiced separately by the producer.
Capital-labour MFP	Productivity measure that relates value added to primary (capital and labour) inputs.
Capital services	The flow of productive services provided by an asset that is employed in production. Capital services reflect a (physical) quantity, not to be confused with the value, or price, concept of capital. Capital services are the appropriate measure of capital input in production analysis.
Consumption of fixed capital	Loss in value of an asset over an accounting period. CFC comprises the effects of ageing and the effects of capital gains or losses (foreseen obsolescence).
Depreciation	Loss in value of an asset due to ageing. This definition (which is common in the productivity literature) is different from the definition of depreciation, or consumption of fixed capital (CFC) as defined by the 1993 System of National Accounts. Depreciation in the SNA comprises both the loss in value due to ageing and the effects of foreseen obsolescence. In the productivity literature, value changes due to obsolescence are reflected in the capital gains/loss term of the user cost formula.
Domar weights	Weights to combine industry-level, gross-output based MFP (KLEMS) to higher-level aggregates. Domar weights are special in that they do not normally add to one. This reflects the combined effects of integration and aggregation.
Double deflation (broadly defined)	Procedure to obtain a measure of deflated value added that is based on a price index that combines the price index of gross output with the price index of intermediate inputs.
Double deflation (narrowly defined)	Procedure to obtain a volume measure of value added by subtracting intermediate inputs at constant prices from gross output at constant prices. Subtraction of levels implies that constant prices reflect Laspeyres-type quantity indices.

Efficiency	Degree to which a production process reflects “best practice”, either in an engineering sense (“technical efficiency”) or in an economic sense (“allocative efficiency”). Full technical efficiency characterises a production process where the maximum possible output has been achieved, given a fixed set of inputs and given a certain technology. Full allocative efficiency prevails when the input-output combination is cost-minimising and/or profit-maximising.
Gross capital stock	The cumulative flow of volume investments, corrected for retirement. In the gross stock, assets are treated *as new* until they are retired: it is assumed that they retain their full productive capacity until removed from the stock.
Gross output	Synonym for output.
Gross-output based MFP	Synonym for KLEMS MFP.
Integration	Statistical process whereby units or industries are combined to a larger unit on the basis of the flows of intermediate deliveries that may exist between the individual units. Output of the new, integrated unit is net of intra-industry flows and represents only deliveries outside of the unit. Similarly, input of the integrated unit is net of intra-industry deliveries. Integration is a necessary step in the computation of sectoral output measures.
Intermediate inputs	Those factors of production that are produced and transformed or used up by the production process within an accounting period.
KLEMS MFP	Productivity measure that relates gross output to primary (capital and labour) and intermediate inputs (energy, other intermediate goods, services).
Multifactor productivity (MFP)	Relates a change in output to several types of inputs. MFP is often measured residually as that change in output that cannot be accounted for by the change in combined inputs.
Net (wealth) capital stock	Current market valuation of an industry’s or an economy’s productive stock.
Output	Goods or services that are produced within a producer unit and that become available for use outside the unit plus any goods and services for own final use.
Primary inputs	Those factors of production that are treated as exogenous in the framework of production analysis. In a static framework such as the one underlying this manual, primary inputs comprise capital and labour.
Producer’s price	The producer’s price is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any VAT, or similar deductible tax, invoiced to the purchaser; it excludes any transport charges invoiced separately by the producer.
Production	Activity in which an enterprise or establishment uses inputs to produce output.
Production function	The maximum set of output(s) that can be produced with a given set of inputs. Use of a production function implies technical efficiency. Synonym for production frontier, the technically efficiency part of a feasible production set, the set of all input-output combinations that are feasible (but not necessarily efficient).
Productive capital stock	The stock of a particular, homogenous, asset expressed in “efficiency” units. The importance of the productive stock derives from the fact that it offers a practical tool to estimate capital services. Typically, the latter are assumed to be proportional to the former.

Productivity change	Conceptually, the combined effects of changes in technical efficiency, allocative efficiency, disembodied technical change and economies of scale. When measured residually, additional factors bear on the residual, in particular the rate of capacity utilisation and measurement errors.
Purchaser's price	The amount paid by the purchaser, excluding any deductible VAT or similar deductible tax, in order to take delivery of a unit of a good or service at the time and place required by the purchaser; the purchaser's price of a good includes any transport charges paid separately by the purchaser to take delivery at the required time and place.
Returns to scale	The rate by which output changes if all inputs are changed by the same factor. Constant returns to scale: a *k*-fold change in all inputs leads to a *k*-fold change in output. Under increasing returns to scale, the change in output is more than *k*-fold, under decreasing returns to scale; it is less than *k*-fold.
Sectoral output	Output of an industry at a given level of aggregation that only reflects deliveries outside of the industry. Sector output is the output of an integrated industry.
Separability	Mathematical property of a (production) function. In productivity analysis, it is sometimes assumed that production functions are *weakly separable* between primary and intermediate inputs. This means that the marginal rate of substitution between any pair of primary inputs is independent of the amount of intermediate inputs used. In other words, the demand for any two primary inputs depends only on the price ratio of primary inputs, and is independent from prices of intermediate inputs.
Single-factor productivity	Synonym for partial productivity measure. It relates output to one particular type of input.
Superlative indices	Price or quantity indices that are "exact" for a flexible aggregator. A flexible aggregator is a second-order approximation to an arbitrary production, cost, utility or distance function. Exactness implies that a particular index number can be directly derived from a specific flexible aggregator.
Technology	The state of knowledge concerning ways of converting resources into outputs.
Technical change, disembodied	The shift in the production function (production frontier) over time. Disembodied technical change is not incorporated in a specific factor of production.
Technical change, embodied	Improvements in the design or quality of new capital goods or intermediate inputs.
Total factor productivity	Synonym for multifactor productivity (MFP). This manual uses the MFP acronym to signal a certain modesty with respect to the capacity of capturing *all* factors' contribution to output growth.
User cost of capital	The price per unit of capital service. Often used as synonymous to *rental price.*
Value-added function	Indicates the maximum amount of current-price value added that an establishment or industry can generate, given a set of prices for its output and for its intermediate inputs, and given a certain quantity of primary inputs.

Annex 2

LINKS AND REFERENCES TO NATIONAL PRODUCTIVITY STATISTICS

Institution	Publication or link
Australian Bureau of Statistics	Value-added based labour productivity Capital-labour MFP Capital productivity http://www.abs.gov.au
Statistics Canada	Value-added based labour productivity Value-added based MFP Gross-output based MFP with exclusion of intra-industry transaction Gross-output based MFP including intra-industry transactions http://www.statcan.ca/start.htm
Statistical Office of the Czech Republic	Gross-output based labour productivity by activity for mining, manufacturing and utility industries *Statistical Yearbook of the Czech Republic*
Statistics Finland	Value-added based labour productivity: *National Accounts of Finland* Labour productivity, capital productivity and MFP: annual publication *Tuottavuuskatsaus* (Productivity Report – Finnish language only)
Statistisches Bundesamt (Germany)	Value-added based labour productivity (per employed person and per hour worked) by detailed mining and manufacturing industry Value-added based labour productivity (per employed person and per hour worked) by major activity http://statistik-bund.de/zeitreih/def.htm
Korea Productivity Centre	Gross-output and value-added based labour productivity index: (quarterly publication Productivity Review); Analysis of value added for Korean companies: (yearly publication) http://www.kpc.or.kr/

Hungarian Central Statistical Office	Gross-output based labour productivity published in: Monthly Report of Industry Monthly Bulletin of Statistics Statistical Yearbook *Yearbook of Industry and Construction Statistics*.
Central Statistical Office of Poland	Gross-output based labour productivity in selected primary industries: *Statistical Yearbook*
Office of National Statistics (United Kingdom)	Value-added based labour productivity for total economy, "production" industries, and manufacturing industry *Economic Trends*
United States Bureau of Labor Statistics	Gross-output based labour productivity Value-added based labour productivity Gross-output based MFP Capital productivity International comparisons of manufacturing productivity http://stats.bls.gov/iprdata1.htm

Annex 3

PRODUCTIVITY MEASUREMENT IN A GROWTH-ACCOUNTING FRAMEWORK

1. The growth-accounting model

1. The theoretical framework for the growth-accounting approach is rooted in the economic theory of production. An explicit derivation of the expressions that enter the growth-accounting framework has two advantages: it spells out some of the assumptions that enter MFP calculations and it ensures consistency for empirical work. For example, production theory provides a clear indication about how to value capital input and its contribution to output growth. Theory provides also guidance to the consistent treatment of different types of labour and capital inputs, their aggregation and the appropriate choice of index numbers.

2. The standard model presented here is based on the seminal work by Tinbergen (1942) and Solow (1957) and its development, in particular by Zvi Griliches, Dale Jorgenson, and Erwin Diewert.[75] The standard growth-accounting model is based on the microeconomic theory of production[76] and rests on a number of assumptions stated below. Most of these assumptions can be relaxed and the economics literature has gone a long way in doing so. At times, such generalisations come at the price of considerable complexity that is beyond the scope and purpose of this manual.

- There exists a production technology that can be represented by a production function, relating gross output, *Q*, to primary inputs labour *L* and capital services *K* as well as intermediate inputs such as material, services or energy (*M*).

- The production function exhibits constant returns to scale.

- Neither labour nor capital inputs are necessarily homogenous. There are *N* different types (qualities) of labour, $L_1, L_2, ...L_N$; *M* different types of capital services $K_1, K_2, ...K_M$, and *R* different types of intermediate inputs $M_1, M_2, ...M_P$:

$$Q = H(L_1, L_2, ...L_N, K_1, K_2, ...K_M, M_1, M_2, ...M_R, t) \qquad (A.1)$$

- Productivity changes are of a Hicks-neutral type, *i.e.* they correspond to an outward shift of the production function, captured by a parameter *A* and (A.1) becomes:

$$Q = A \cdot F(L_1, L_2, ...L_N, K_1, K_2, ...K_M, M_1, M_2, ...M_P) \qquad (A.2)$$

---

[75] See, for example, Jorgenson and Griliches (1967), Jorgenson (1995a, 1995b), Diewert (1976).

[76] See, for example, Varian (1984), or Nadiri (1998).

- For any desired level of output, the firm minimises costs of inputs, subject to the production technology shown above. Factor input markets are competitive, so that the firm takes factor prices as given and adjusts quantities of factor inputs to minimise costs.

- Labour and intermediate inputs can be hired at any moment at the market rates $w_i \ (i=1,2,...N)$ for labour and $p_{m_i} \ (i=1,2,...,R)$ for intermediate inputs.

- Provision of capital services requires investment in the different types of capital or the hiring of a capital good for one period. Every investment adds to the productive capital stock from which capital services are derived. Because there is a decline in capital services yielded by an individual capital good as it ages, or a full loss of its services because it retires, these types of efficiency losses or deterioration must be taken into account. Deterioration, a required net rate of return and the losses or gains from a change in the market price of capital goods together constitute the user cost of capital (see Annex 4 for a more complete treatment). The user cost of capital (μ) is the price for capital services that flow from the productive capital stock of each type of capital asset.

- There are no adjustment costs associated with investment. Alternatively, all adjustment costs are strictly proportional to the volume of investment.

3. This set-up gives rise to a standard optimisation problem, where labour, intermediate inputs and capital services are chosen so as to minimise total costs:

$$Min\, C = \sum_{i=1}^{N} w_i L_i + \sum_{i=1}^{M} \mu_i K_i + \sum_{i=1}^{R} p_{M_i} M_i \qquad \text{(A.3)}$$

subject to:

$$Q = A \cdot F(L_1, L_2, ... L_N, K_1, K_2, ... K_M, M_1, M_2, ... M_R)$$

4. The solution to this problem produces the usual optimality conditions, whereby:

$$P \cdot A \frac{\partial F}{\partial L_i} = w_i ; i = 1,2,...N \qquad \text{(A.4)}$$

: labour is hired until the marginal revenues it generates equal the wage rate *w* where *P* stands for the price of output;

$$P \cdot A \frac{\partial F}{\partial K_i} = \mu_i ; i = 1,2,...M$$

: investment in capital is undertaken (capital assets are hired) until the marginal revenues it generates equal the user costs of capital μ_i;[77]

$$P \cdot A \frac{\partial F}{\partial M_i} = p_{M_i} ; \ i = 1,2,...R$$

: intermediate inputs are purchased up to the point where the marginal revenues they generate equal the purchasing price for intermediate inputs, p_{M_i}.

---

[77] See Annex 5 for a derivation and description of the user cost term.

5. As a next step, the production function is differentiated with respect to time. This yields the following expression:

$$\frac{d\ln Q}{dt}=\sum_{i=1}^{N}\frac{F_{L_i}L_i}{F}\frac{d\ln L_i}{dt}+\sum_{i=1}^{M}\frac{F_{K_i}K_i}{F}\frac{d\ln K_i}{dt}+\sum_{i=1}^{R}\frac{F_{M_i}M_i}{F}\frac{d\ln M_i}{dt}+\frac{d\ln A}{dt} \tag{A.5}$$

6. It shows that the rate of growth of output is a weighted average of the rates of growth of the various inputs, and of the multifactor productivity term, *A*. The weights attached to each input are the output elasticities for each factor of production, *i.e.* the percentage change in output that can be attributed to a 1% addition of that input. Output elasticities cannot be directly observed, however, and econometric techniques are necessary to yield the corresponding parameters. Alternatively, in non-parametric approaches, the optimality conditions (A.4) are employed to obtain observable measures. For example, when the optimality condition for input of labour type *i*, $PAF_{L_i}=w_i$ is inserted into $\frac{F_{L_i}L_i}{F}$, this gives $\frac{F_{L_i}L_i}{F}=\frac{w_iL_i}{PAF}=\frac{w_iL_i}{PQ}$, or the share of labour type i in the total value of production. The same relation can be used for other factors and gives rise to the next expression:

$$\frac{d\ln Q}{dt}=\sum_{i=1}^{N}\frac{w_iL_i}{PQ}\frac{d\ln L_i}{dt}+\sum_{i=1}^{M}\frac{\mu_iK_i}{PQ}\frac{d\ln K_i}{dt}+\sum_{i=1}^{R}\frac{p_{M_i}M_i}{PQ}\frac{d\ln M_i}{dt}+\frac{d\ln A}{dt} \tag{A.6}$$

7. Expression (A.6) is then turned around to compute, as a residual, the rate of change of MFP:

$$\frac{d\ln A}{dt}=\frac{d\ln Q}{dt}-\sum_{i=1}^{N}\frac{w_iL_i}{PQ}\frac{d\ln L_i}{dt}-\sum_{i=1}^{M}\frac{\mu_iK_i}{PQ}\frac{d\ln K_i}{dt}-\sum_{i=1}^{R}\frac{p_{M_i}M_i}{PQ}\frac{d\ln M_i}{dt} \tag{A.7}$$

2. Measures of the compositional change of inputs

8. The basic model presented above allows for multiple types of labour, capital and intermediate inputs. This takes account of the fact that a working hour of labour or capital is not a homogenous input but varies, for example, with the experience or educational background of the employee or with the specific type of capital asset. In general, each type of labour, capital or intermediate input has its specific marginal productivity and is therefore remunerated accordingly. Consistent growth rates of aggregate labour, capital and intermediate inputs can be computed by weighting the rate of change of each type of input by its marginal productivity or its share in total outlays for labour, capital or intermediate inputs:

$$\frac{d\ln L}{dt}=\sum_{i=1}^{N}\frac{w_iL_i}{wL}\frac{d\ln L_i}{dt};\; wL=\sum_{i=1}^{N}w_iL_i\;; \tag{A.8}$$

$$\frac{d\ln K}{dt}=\sum_{i=1}^{M}\frac{\mu_iK_i}{\mu K}\frac{d\ln K_i}{dt};\; \mu K=\sum_{i=1}^{M}\mu_iK_i$$

$$\frac{d\ln M}{dt}=\sum_{i=1}^{R}\frac{p_{M_i}M_i}{p_M M}\frac{d\ln M_i}{dt};\ p_M M=\sum_{i=1}^{R}p_{M_i}M_i$$

9. Alternatively, consider an un-weighted index of labour, capital or intermediate inputs, such as $L'=\sum_{i=1}^{N}L_i$, a simple sum of undifferentiated hours worked. Its rate of change is given by $\frac{d\ln L'}{dt}=\sum_{i=1}^{N}\frac{L_i}{L'}\frac{d\ln L_i}{dt}$. The difference between the weighted and un-weighted measure of labour input is readily interpreted as an index of compositional change of the employed labour force (see Bureau of Labor Statistics, 1993, for an application). Similarly, it is possible to produce measures of compositional change of capital and intermediate inputs. Finally, the notation in (A.8) leads to a simplified presentation of the growth-accounting equation (A.7):

$$\frac{d\ln Q}{dt}=s_L\frac{d\ln L}{dt}+s_K\frac{d\ln K}{dt}+s_M\frac{d\ln M}{dt}+\frac{d\ln A}{dt}, \tag{A.9}$$

10. In (A.9), $s_L=\frac{wL}{PQ}, s_K=\frac{\mu K}{PQ}, s_M=\frac{p_M M}{PQ}$ are the current-price shares of labour, capital and intermediate inputs in gross output. Under constant returns to scale, these income shares add to unity.

Annex 4

CAPITAL STOCK MEASURES

1. This section presents a more formal derivation of the productive capital stock and capital services, as described in Section 5.3. It refers to a single type of asset, in theory a truly homogenous capital. Because capital services are difficult to observe directly, it is assumed that capital services are proportional to the productive stock of asset *i*, $K_{i,t}^{G}$. The productive stock for a particular, homogenous, asset is constructed with the perpetual inventory method, which consists of cumulating past investment expenditure. Weights are attached to each vintage investment to reflect the decline in productive efficiency, and the retirement of investment cohorts:

$$K_{i,t}^{P} = \sum_{\tau=0}^{T} h_{i,\tau} \cdot F_{i,\tau} \frac{IN_{i,t-\tau}}{q_{i,t-\tau,0}} \qquad \text{(A.10)}$$

2. $IN_{i,t}$ is nominal investment expenditure on asset type *i* at time *t*. Real investment is obtained by division by an investment price index $q_{i,t,0}$ where subscripts indicate a price index for asset *i* of age zero (a new asset) in year t. $F_{i,\tau}$ is a *retirement function* that spells out the share of assets of age τ that are still in service. $F_{i,\tau}$ is declining and takes values between unity (when all assets are in existence) and zero (when all assets of a particular vintage have been retired). *T* is the maximum service life of asset type *i*, *i.e.* the amount of years after which all capital goods of a particular vintage are retired. This is equivalent to stating that $F_{i,T} = 0$.

3. h_{τ} is an *age-efficiency profile*, tracing the loss in productive efficiency as an asset ages. h_{τ} is declining and takes values between unity (when an assets is new) and zero (when it has lost its entire productive capacity). h_{τ} exceeds $h_{\tau+1}$ by an amount that depends on the specific age profile of asset efficiency. Profiles used in the literature include a linear declining balance, a hyperbolic profile and a geometric one. Implicit in the above formulation is the assumption that the capital stock is measured at the beginning of a period. Hence, only investment during the preceding year and before contributes to the size of the stock.

4. From (A.10), a direct link can be made to the *gross capital stock*. The difference between productive and gross capital stock lies in the effects of wear and tear, captured by h_{τ}. The gross capital stock is simply a special case of the productive capital stock where no allowance is made for wear and tear – only the retirement of assets is taken into account:

$$K_{i,t}^{G} = \sum_{\tau=0}^{T} F_{i,\tau} \frac{IN_{i,t-\tau-1}}{q_{i,t-\tau-1,0}} \quad \text{(A.11)}$$

5. Both the productive and gross capital stocks presented above are valued at *constant prices* (in the terminology introduced in overview Section 5.1) and so reflect a quantity concept. Multiplication of the expressions above with the market price of a new capital good at time *t* ($q_{i,t,0}$) yields a measure of the productive stock (of the gross stock) at *current prices*.

6. *Net (wealth) capital stock.* The net (wealth) capital stock of a particular asset is the value of all vintages to their owner. Valuation takes place via $q_{i,t,\tau}$ the market price at time *t* for an asset of age τ:

$$K_{i,t}^{N} = \sum_{\tau=0}^{T} q_{i,t,\tau} \cdot F_{i,\tau} \frac{IN_{i,t-\tau}}{q_{i,t-\tau,0}} \quad \text{(A.12)}$$

7. In (A.12), the net (wealth) capital stock is expressed in *current prices*. One notes that the ratio of the prices for a τ year old asset and a new asset, $\frac{q_{i,t,\tau}}{q_{i,t,0}}$ is precisely the age-price profile as described in Section 5.1. In practice, this age-price profile is often taken as time-invariant. Expressing such a time-invariant age-price profile as $z_{i,\tau} = \frac{q_{i,t,\tau}}{q_{i,t,0}}$, one obtains:

$$K_{i,t}^{N} = \sum_{\tau=0}^{T} q_{i,t,0} \cdot z_{i,\tau} \cdot F_{i,\tau} \frac{IN_{i,t-\tau}}{q_{i,t-\tau,0}} \quad \text{(A.13)}$$

8. This net capital stock at current prices can easily be expressed in *constant prices*, by dividing through the current market price for new assets, $q_{i,t,0}$. A comparison of the resulting expression with (A.10) demonstrates that the productive stock at constant prices and the net (wealth) stock at constant prices will only be identical if $h_{i,\tau} = z_{i,\tau}$, in other words, if the age-efficiency profile and the age-profile coincide.

Annex 5

USER COSTS

1. A key relationship in capital theory states that the value (market price) of an asset depends on the expected stream of revenues that the asset provides over its service life. More specifically, in a well-functioning market, and in equilibrium, the market price of an asset of age *s* at time *t* ($q_{i,t,s}$) should equal the discounted sum of future revenues generated by the asset. But at any point in time, cost-minimisation will lead to equality between this marginal revenue from a particular asset and its marginal cost. Marginal costs of an asset of age *s* in period *t* are the user costs or rental prices of this asset: $\mu_{i,t,s}$. The price of an asset at time *t* can then be written as:

$$q_{i,t,s} = \sum_{\tau=0}^{T} \mu_{i,t+\tau,\tau+s} / (1+r)^{\tau+1} \tag{A.14}$$

2. Note another relationship, that between user costs and the age-efficiency profile of an asset. At any point in time, the ratio between the user costs of two vintages of the same asset corresponds to the relative productive efficiencies of the two vintages, weighted by their survival probability. This relation arises from the implicit assumption that different vintages of the same asset type are perfect substitutes for each other – if this is the case, they are distinguishable only by their relative efficiency and their rental prices must exactly reflect this:

$$\frac{\mu_{i,t,s}}{\mu_{i,t,0}} = h_{i,s} F_{i,s} \tag{A.15}$$

3. With this relation between relative user costs, the asset market equilibrium condition (A.14) can then be written in terms of the user costs of a new asset, and the age-efficiency profile, because $\mu_{i,t,s} = \mu_{i,t,0} \cdot h_{i,s} \cdot F_{i,s}$:

$$q_{i,t,s} = \sum_{\tau=0}^{T} \mu_{i,t+\tau,0} h_{i,\tau+s} F_{i,\tau+s} / (1+r)^{\tau+1} \tag{A.16}$$

4. Expression (A.16) can be solved for the user cost term:

$$\mu_{i,t,s} = q_{i,t,s} r + (q_{i,t,s} - q_{i,t,s+1}) - (q_{i,t+1,s+1} - q_{i,t,s+1}) \tag{A.17}$$

5. (A.17) is a common formulation for the user costs of an asset of age *s* at time *t* (Hulten, 1990 or 1996). It has three distinct components: *(a)* the net return on investment $q_{i,t,s}r$, or the financing costs of the asset if *r* stands for the borrowing rate; *(b)* a depreciation[78] component that captures the effects of ageing or the difference between the value of an *s*-year old asset ($q_{i,t,s}$) and the value of an *s+1*-year old asset ($q_{i,t,s+1}$) *in the same year t; (c)* a capital gains or losses term capturing general movements of the capital goods price, *i.e.* the difference between an *s*-year old asset in year *t+1*, ($q_{i,t+1,s}$) and an *s*-year old asset in year *t*, ($q_{i,t,s}$).

6. It is useful to re-formulate the user cost expression in terms of the rate of depreciation and the rate of capital gains/losses. To this purpose, define the rate of depreciation as $d_{i,t,s} = 1 - \frac{q_{i,t,s+1}}{q_{i,t,s}}$ and the rate of capital gains/losses as $\rho_{i,t} = \frac{q_{i,t+1,s+1}}{q_{i,t,s}} - 1$. The user cost term above can then be expressed as:

$$\mu_{i,t,s} = q_{i,t,s}\left(r + d_{i,t,s} - \rho_{i,t} + d_{i,t,s}\rho_{i,t}\right) \qquad \text{(A.18)}$$

7. For a new asset ($s = 0$), this is nearly identical to the user cost expression described in Section 5.4. The one difference lies in the interaction term between depreciation rate and capital gains, $d_{i,t,s}\rho_{i,t}$, which arises here due to the specific assumptions about the timing of investment (beginning or end of the period) and the timing of its effectiveness as a capital good (immediately operational or only at the next period).

8. *Calculating the rate of depreciation*. For practical purposes, $\rho_{i,t}$ is directly measured as the rate of change of the price index of new capital goods but the rate of depreciation has to be calculated. One possibility is to use the change in the net (wealth) stock at constant prices to derive a consistent rate of depreciation.

9. The change in the net (wealth) capital stock at current prices from one period to the next, before adding new investment, presents the change in the value of existing assets. This change in value between two periods (or between the beginning and the end of one period), reflects both depreciation (the loss in value due to ageing) and re-valuation (or capital gains/losses, *i.e.* the change in value due to price changes of the asset that are not associated with the ageing of the capital stock). To control for re-valuation effects, and to single out depreciation, the change in value of the net stock at constant prices is considered. In line with the definition of the net (wealth) stock in expression (A.12), the net (wealth) stock at the beginning of period *t* in prices of period *t* is given by $K_{i,t}^{N}$. The net (wealth) stock one period earlier, again at current prices, is $K_{i,t-1}^{N}$. A comparison in constant prices requires that $K_{i,t-1}^{N}$ be expressed in prices of period *t* (or vice versa). The change in the net (wealth) stock at constant prices of year *t* is then given by:

---

[78] Here, “depreciation” is used in line with conventions in the productivity literature. This is different to the national accounts convention that includes the capital gains/loss term.

$$K_{i,t}^{N} - K_{i,t-1}^{N}\frac{q_{i,t,0}}{q_{i,t-1,0}} = IN_{i,t,0} + q_{i,t,0}D_{i,t} \tag{A.19}$$

where $D_{i,t} = \sum_{\tau=0}\left(z_{i,\tau}F_{i,\tau} - z_{i,\tau+1}F_{i,\tau+1}\right)\frac{IN_{i,t-\tau-1}}{q_{i,t-\tau-1}}$

10. This change in the net (wealth) stock has two components: the value of new investment during *t*, $IN_{i,t,0}$ and the real depreciation term, $D_{i,t}$. The rate of depreciation, the term that forms part of the user cost expression, is then calculated as the ratio between real depreciation and the real net (wealth) stock, or $d_{i,t} = \frac{D_{i,t}}{K_{i,t}^{N}}$.

11. The estimate for the depreciation rate $d_{i,t}$ is one that reflects the average depreciation, across all vintages of the asset. It is thus influenced by the vintage composition of the asset and may therefore vary over time. Alternatively, and exactly following the theoretical set-up [as developed in equation (A.18)], the rate of depreciation in the user cost term for a new asset is exclusively governed by the ratio in value of a new and a one-year old capital good. This ratio can be directly taken from the first element of the age-price profile, $z_{i,1}$ (see below) and this provides an alternative way of obtaining a rate of depreciation in the user cost term.

12. *From age-efficiency to age-price profiles.* A relatively straightforward way to obtain age-price profiles from a set of age-efficiency profiles is to make use of a simplified version of the asset price equilibrium condition (A.16). Recalling that the age-price profile of an asset was given by $z_{i,\tau} = \frac{q_{i,t,\tau}}{q_{i,t,0}}$, and inserting the asset price equilibrium condition yields:

$$z_{i,s} = \frac{q_{i,t,s}}{q_{i,t,0}} = \frac{\sum_{\tau}\mu_{i,t+\tau,0}h_{i,\tau+s}F_{i,\tau+s}/(1+r)^{-(1+\tau)}}{\sum_{\tau}\mu_{i,t+\tau,0}h_{i,\tau}F_{i,\tau}/(1+r)^{-(1+\tau)}} \tag{A.20}$$

13. At this point, the simplifying assumption is made that $\mu_{i,t+\tau,0}$, the nominal future income of a new asset follows a path of constant growth at the rate β: $\mu_{t+\tau,0} = \mu_{t+\tau-1,0}(1+\beta)$. Successive use of this relation shows that the future nominal income τ periods ahead, $\mu_{t+\tau,0}$ is equal to the nominal income in period *t*, multiplied by the rate $(1+\beta)^{\tau}$: $\mu_{t+\tau,0} = \mu_{t,0}(1+\beta)^{\tau}$. This relation is now used in equation (A.20):

$$z_{i,s}=\frac{q_{i,t,s}}{q_{i,t,0}}=\frac{\sum_{\tau}\mu_{i,t,0}(1+\beta)^{\tau}h_{i,\tau+s}F_{i,\tau+s}/(1+r)^{-(1+\tau)}}{\sum_{\tau}\mu_{i,t,0}(1+\beta)^{\tau}h_{i,\tau}F_{i,\tau}/(1+r)^{-(1+\tau)}}= \tag{A.21}$$

$$=\frac{\mu_{i,t,0}(1+\beta)^{-1}\sum_{\tau}h_{i,\tau+s}F_{i,\tau+s}(\frac{1+r}{1+\beta})^{-(1+\tau)}}{\mu_{i,t,0}(1+\beta)^{-1}\sum_{\tau}h_{i,\tau}F_{i,\tau}(\frac{1+r}{1+\beta})^{-(1+\tau)}}$$

14. The first terms in the numerator and denominator cancel out of (A.21). Given assumptions about the age-efficiency profile $h_{i,\tau}$, the retirement profile $F_{i,\tau}$, the discount rate *r*, and β, it is possible to derive an age-price profile $z_{i,s}$. Rather than setting individual parameters for *r* and β, it is practical to choose a "real" discount rate, $\frac{1+r}{1+\beta}$. In empirical applications, this rate has frequently been set at 4 %. In this case, the final expression for deriving the age price profile is:

$$z_{i,s}=\frac{q_{i,t,s}}{q_{i,t,0}}=\frac{\sum_{\tau}h_{i,\tau+s}F_{i,\tau+s}(1.04)^{-(1+\tau)}}{\sum_{\tau}h_{i,\tau}F_{i,\tau}(1.04)^{-(1+\tau)}} \tag{A.22}$$

Annex 6

AGGREGATION OF OUTPUT, INPUTS AND PRODUCTIVITY

1. So far, the discussion of the theoretical model has been conducted at the level of an individual (representative) firm or industry. However, there is considerable interest in constructing measures of productivity growth for more aggregate sectors such as total manufacturing or for the entire economy. Several approaches towards aggregation exist, which may or may not yield identical results at the aggregate level, depending, for example, on whether an open or a closed economy is considered. As was explained in the main text, aggregation is understood here as a process where increasingly larger parts of the economy are lumped together in a process of integration. Thus, at every level of aggregation, productivity measures are computed based on the flows of outputs from the relevant sector to the rest of the economy and on flows of inputs from outside into the relevant sector. Treating each level of aggregation as an integrated unit of production implies affirming the existence of some production possibility function at each level of aggregation, including at the level of the entire economy. In what follows, exposition will be limited to aggregation of industry-level productivity measures to the level of the entire economy, although it would be possible to generalise the description to cover aggregation at some intermediate level.

1. Domar aggregation

2. One approach towards constructing the link between aggregate and industry-level measures was explored by Domar (1961) and further elaborated by Hulten (1978). They start with KLEMS-type industry-level measure of productivity and ask how they can be combined to yield an economy-wide equivalent. For an economy-wide equivalent, one has to postulate the existence of an economy-wide production possibility function that relates available primary inputs to total final output, *i.e.* to deliveries to final demand. This production possibility frontier can be represented as a function *H* with arguments *FD* (an index of deliveries to final demand), *X* (an index of combined primary inputs labour and capital services), M_M (an index of imported intermediate inputs), and the parameter *A*, to indicate shifts of the function over time:

$$H(FD, X, M_M, A) = 0 \qquad \text{(A.23)}$$

3. Aggregate productivity change is defined as a shift of the aggregate production possibility frontier over time, or the rate of change of *A*. In competitive equilibrium, it can be measured as the difference between the rate of change in total final demand and the rate of change in primary and imported intermediate inputs.

$$\frac{d\ln A}{dt} = \frac{d\ln FD}{dt} - \frac{P_x X}{P_{FD}FD}\frac{d\ln X}{dt} - \frac{P_{MM}M_M}{P_{FD}FD}\frac{d\ln M_M}{dt} \qquad \text{(A.24)}$$

4. In this expression, $\frac{P_x X}{P_{FD} FD}$ is the current-price share of primary inputs in total final demand, and $\frac{P_{MM} M_M}{P_{FD} FD}$ the share of imported intermediate inputs. These shares sum to one because at the level of the total economy, deliveries to final demand are equal to total income or factor payments, part of which accrue to domestic primary inputs and part of which accrue to foreign suppliers of intermediate products. The aggregate rate of change in final demand and in primary and intermediate inputs are themselves weighted averages of deliveries to final demand from different industries and of primary and imported inputs used in individual industries. The precise relation will be spelled out presently.

5. At the industry level, one starts from the familiar production function $Q^j = A^j F^j(X^j, M^j, M_M^j)$, that links industry-level gross output to the primary inputs labour and capital used by industry *j* (as above, these primary inputs are combined in the quantity index X^j), to domestically produced intermediate inputs M^j and to imported intermediate inputs, M_M^j. Note that domestically produced inputs M^j, do not figure in the production possibility function of the entire economy – they disappear at that level of aggregation because they represent intra-industry flows that are netted out in a process of vertical integration. Given competitive markets and constant returns to scale technologies, industry-level productivity growth is given by:

$$\frac{d\ln A^j}{dt} = \frac{d\ln Q^j}{dt} - \frac{P_x^j X^j}{P^j Q^J}\frac{d\ln X^j}{dt} - \frac{P_M^j M^j}{P^j Q^J}\frac{d\ln M^j}{dt} - \frac{P_{MM}^j M_M^j}{P^j Q^J}\frac{d\ln M_M^j}{dt} \qquad \text{(A.25)}$$

6. Furthermore, at the level of each industry, two accounting identities hold. One breaks down the total value of industry output into deliveries to other industries and into deliveries to final demand. For simplicity but without much loss of generality, it is assumed here that each industry produces exactly one final demand commodity and that it charges the same output price to all buyers. The other identity states that the value of gross output is used to pay for primary and intermediate inputs, domestic and foreign. In national accounts' terms, the two identities represent an industry's production account. Some additional notation is needed to write down these two identities. In expression (A.26), Q^{kj} stands for industry *j*'s deliveries of its product to industry *k* and FD^j marks industry *j*'s deliveries to final consumers.

$$P^j Q^j = \sum_k P^j Q^{kj} + P^j FD^j \qquad \text{(A.26)}$$

$$P^j Q^j = P_X^j X^j + P_M^j M^j + P_{MM}^j M_M^j$$

7. Based on the first accounting relationship in (A.26), it is possible to define an expression for the rate of change of the final demand deliveries of industry *j*, as spelled out in (A.27). The rate of growth in final demand deliveries by industry *j* is the difference between the rate of growth of total gross output and the rate of change of a quantity index of deliveries to other industries, scaled by the ratio of the value of gross output to final demand deliveries:

$$\frac{d\ln FD^j}{dt}=\frac{P^jQ^j}{P^jFD^j}\left(\frac{d\ln Q^j}{dt}-\sum_k \frac{P^jQ^{kj}}{P^jQ^j}\frac{d\ln Q^{kj}}{dt}\right) \tag{A.27}$$

8. Given the relations at the economy-wide and at the industry level, it is of interest to link industry-specific productivity growth measures in (A.25) with those for the entire economy in (A.24). To set up this relationship, one observes that: *i)* aggregate final demand can be expressed as a weighted average of industries' deliveries to final demand, as shown in the first expression in (A.28); *ii)* aggregate primary input use can be expressed as a weighted average of each industry's primary input use; and *iii)* aggregate imported intermediate inputs can be expressed as a weighted average of each industry's imported intermediate inputs:

$$\frac{d\ln FD}{dt}=\sum_j \frac{P^jFD^j}{P_{FD}FD}\frac{d\ln FD^j}{dt} \tag{A.28}$$

$$\frac{d\ln X}{dt}=\sum_j \frac{P_X^jX^j}{P_XX}\frac{d\ln X^j}{dt}$$

$$\frac{d\ln M_M}{dt}=\sum_j \frac{P_{MM}^JM_M^J}{P_{MM}M_M}\frac{d\ln M_M^j}{dt}$$

9. The expressions for aggregate final demand, primary and imported intermediate inputs in (A.28) can now be inserted into the expression for economy-wide productivity growth (A.24). After some manipulations, this yields:

$$\frac{d\ln A}{dt}= \tag{A.29}$$

$$=\sum_j \frac{P^jQ^j}{P\cdot FD}\left(\frac{d\ln Q^j}{dt}-\sum_k \frac{P^jQ^{kj}}{P^jQ^j}\frac{d\ln Q^{kj}}{dt}\right)-\frac{P_xX}{P\cdot FD}\sum_j \frac{P_x^jX^j}{P_xX}\frac{d\ln X^j}{dt}-\frac{P_{MM}M_M}{P\cdot FD}\sum_j \frac{P_{MM}^jM_M^j}{P_{MM}M_M}\frac{d\ln M_M^j}{dt}$$

10. Each delivery of industry *k* to industry *j* is also recorded as an intermediate input by industry *j*: $Q^{kj}=M^{jk}$, and consequently $\frac{d\ln Q^{kj}}{dt}=\frac{d\ln M^{jk}}{dt}$. It is then possible to form the following expression:

$$\sum_j\sum_k \frac{P^jQ^{kj}}{P\cdot FD}\frac{d\ln Q^{kj}}{dt}=\sum_k\sum_j \frac{P^jM^{jk}}{P\cdot FD}\frac{d\ln M^{jk}}{dt} \tag{A.30}$$

11. Observing (A.30), (A.29) can be re-written as in (A.31):

$$\frac{d\ln A}{dt}=\sum_j \frac{P^jQ^j}{P\cdot FD}\left(\frac{d\ln Q^j}{dt}-\frac{P_M^jM^j}{P^jQ^j}\frac{d\ln M^j}{dt}-\frac{P_x^jX^j}{P^jQ^j}\frac{d\ln X^j}{dt}-\frac{P_{MM}^jM_M^j}{P^jQ^j}\frac{d\ln M_M^j}{dt}\right) \tag{A.31}$$

12. A comparison with (A.25) shows that the aggregate shift in technology can be represented as a weighted sum of industry-level productivity change. Each industry's productivity change is weighted by the ratio of its gross output to overall final demand, as shown in the first expression in (A.32). In a closed economy, total value of final demand is equal to total value added (itself always equal to total income of primary factors of production) and in this case the aggregation procedure can be written as in the second expression in (A.32). This is Domar's aggregation formula:

$$\frac{d\ln A}{dt}=\sum_j \frac{P^jQ^j}{P\cdot FD}\frac{d\ln A^j}{dt} \tag{A.32}$$

$$\frac{d\ln A}{dt}=\sum_j \frac{P^jQ^j}{P_X X}\frac{d\ln A^j}{dt}$$

13. Several points are noteworthy here:[79]

- Industry-level productivity measures are of a KLEMS type, linking gross output to primary and intermediate inputs. Implicitly, intra-industry flows of intermediate inputs have been netted out. The same concept applies to the aggregate level where gross output takes the form of deliveries to final demand, and where inputs consist of primary domestic and intermediate imported inputs only. All intra-economy flows of domestically produced output are netted out.

- A direct consequence of this process of integration is that weights in (A.32) do not sum to unity. They exceed unity, implying that productivity growth amounts to more than a weighted average of industry-level productivity growth. As explained in the text (Section 8.2), this reflects the fact that productivity gains in the production of intermediate inputs do not only have an "own" effect but in addition they lead to reduced input prices in downstream industries, and effects cumulate.

2. Aggregation of capital-labour MFP

14. A different way of linking industry-level and aggregate measures of productivity growth rests on a value-added concept of productivity measurement. Before showing the links between industry and aggregate level, it is helpful to briefly recall definition and interpretation of value-added measures. The same notation as in the section above applies, and in addition $P_{VA}^jVA^j$ denotes the current-price (index) of value added, composed of a price index P_{VA}^j and a quantity index VA^j. Note first the accounting identity that defines value added at current prices as the difference between gross output (P^jQ^j) and intermediate inputs – domestically produced ($P_M^jM^j$) and imported ($P_{MM}^jM_M^j$).

---

[79] For an exposition in relation to the United States productivity measures, see Gullickson and Harper (1999a).

Alternatively, current-price value added corresponds to total income of primary factors ($P_x^j X^j$), as shown in the second line of (A.33):

$$P_{VA}^j VA^j = P^j Q^j - P_M^j M^j - P_{MM}^j M_M^j \tag{A.33}$$

15. Following Diewert (1978) or Lau (1976), a nominal value-added function *G* can be defined that reflects the maximum amount of value added producible given a set of prices and a set of primary inputs. The value-added maximising amounts of intermediate inputs are obtained by differentiating (A.33) with respect to M^j and M_M^j. This yields $P^j \frac{\partial Q^j}{\partial M^j} = P_M^j$ and $P^j \frac{\partial Q^j}{\partial M_M^j} = P_{MM}^j$: marginal revenues from domestic and imported intermediate inputs equal their respective marginal costs. Cost-minimising amounts of intermediate inputs are then given by $M^{*j} = M^{*j}(X^j, P^j, P_M^j, A^j)$ and $M_M^{*j} = M_M^{*j}(X^j, P^j, P_M^j, A^j)$. The value-added function *G* is then given by:

$$P_{VA}^j VA^j = G^j(X^j, P^j, P_M^j, A^j) = P^j Q^j - P_M^j M^{*j} - P_{MM}^j M_M^{*j} \tag{A.34}$$

16. Next, the value-added function is differentiated totally with respect to time. In terms of logarithmic rates of change, one obtains the expression below:

$$\frac{d\ln G^j}{dt} = \frac{d\ln P_{VA}^j}{dt} + \frac{d\ln VA^j}{dt} = \tag{A.35}$$

$$= \frac{P^j Q^j}{G^j}\left(\frac{d\ln Q^j}{dt} + \frac{d\ln P^j}{dt} - \frac{P_M^j M^j}{P^j Q^j}\left(\frac{d\ln P_M^j}{dt} + \frac{d\ln M^j}{dt}\right) - \frac{P_{MM}^j M_M^j}{P^j Q^j}\left(\frac{d\ln P_{MM}^j}{dt} + \frac{d\ln MM^j}{dt}\right)\right)$$

17. (A.35) provides a decomposition of the rate of change of nominal value added. This decomposition has a price and a quantity component. A natural choice for the rate of change of the Divisia price index of value added are the combined movements of output prices and intermediate input prices, as shown in (A.36):

$$\frac{d\ln P_{VA}^j}{dt} = \frac{P^j Q^j}{G^j}\frac{d\ln P^j}{dt} - \frac{P_M^j M^j}{G^j}\frac{d\ln P_M^j}{dt} - \frac{P_{MM}^j M_M^j}{G^j}\frac{d\ln P_{MM}^j}{dt} \tag{A.36}$$

18. Dual to the price index of value added,[80] there is an implicit quantity index, obtained by deflating the nominal value-added function *G* with its price index, as given by expression (A.37):

---

[80] Note that these price and volume measures of value added were derived from the Hicks-neutral ("output augmenting") version of a production function without any additional assumptions about production technology. In this sense, price and volume indices of value added always exist. There is, however, no guarantee that these index numbers are "path-independent". Path-independence means that the price index of value added depends exclusively on the price paths of its two components, the price index of gross output and the price index of intermediate inputs. A path-dependent price index depends on other variables as well and may therefore not be a unique price index. See Sato (1976) for a discussion, but also Lau (1976) and Diewert (1980).

$$\frac{d\ln VA^j}{dt} = \frac{d\ln G^j}{dt} - \frac{d\ln P_{VA}^j}{dt} = \tag{A.37}$$
$$= \frac{P^j Q^j}{G^j}\frac{d\ln Q^j}{dt} - \frac{P_M^j M^j}{G^j}\frac{d\ln M^j}{dt} - \frac{P_{MM}^j M_M^j}{G^j}\frac{d\ln M_M^j}{dt}$$

19. The volume rate of change of value added can be interpreted as a quantity index that traces changes in the quantity of gross output that are not due to changes in (domestically produced or imported) intermediate inputs. Consequently, real value added captures those shifts in gross output that are caused by changes in primary inputs and by overall shifts of the production function. Real value added is of interest because – in a closed economy – it describes precisely the contribution of an industry to final demand deliveries to consumers and investors.

20. Next, turn to productivity measurement based on value added and define capital-labour MFP growth as the difference between the rate of change in the quantity index of value added and the rate of change in the quantity index of primary inputs. We mark the capital-labour productivity residual $\tilde{A}^j$ with a tilde to distinguish it from its counterpart in (A.25) that is based on a gross-output concept:[81]

$$\frac{d\ln \tilde{A}^j}{dt} = \frac{d\ln VA^j}{dt} - \frac{d\ln X^j}{dt} \tag{A.38}$$

22. A direct relationship exists with the gross-output based productivity measure.[82] This is readily seen through inserting the real-value-added index (A.37) into expression (A.38). The two measures are linked by a term that corresponds to the inverse share of current price value added in gross output, as shown in (A.39). Because value added can never exceed gross output, the value-added share is always less than one, and its inverse always larger than or equal to one. Consequently, a value-added based productivity measure will always be at least as large as a gross-output based measure:

$$\frac{d\ln \tilde{A}^j}{dt} = \frac{d\ln VA^j}{dt} - \frac{d\ln X^j}{dt} = \frac{P^j Q^j}{P_{VA}^j VA^j}\left(\frac{P_X^j X^j}{P^j Q^j}\frac{d\ln X^j}{dt} + \frac{d\ln A^j}{dt}\right) - \frac{d\ln X^j}{dt} = \frac{P^j Q^j}{P_{VA}^j VA^j}\frac{d\ln A^j}{dt} \tag{A.39}$$

---

[81] To interpret (A.38) as a path-independent measure of technological change, one has to assume that there exists a value-added function that links technological change exclusively to real value added and primary inputs: $V^j = V^j(X^j, \tilde{A}^j)$. This relation exists only if the industry-level production function is separable in primary and intermediate inputs such that $Q^j = H^j(V^j(X^j, \tilde{A}^j), M^j, M_M^j)$. A production function is weakly separable with respect to a partition between value added and intermediate inputs if the marginal rate of substitution between any pair of primary inputs is independent of the quantities of intermediate inputs. For a formal discussion, see Goldman and Uzawa (1964). Because separability constitutes a quite restrictive assumption that has repeatedly been rejected in empirical tests, it is likely that value-added based productivity measures at the industry level will produce an inaccurate picture of technological shifts, if technical progress concerns all types of inputs in the production process.

[82] See Bruno (1978) for a more complete discussion of the value-added function and productivity measurement.

given that

$$G^j = P_{VA}^j VA^j = P_X^j X^j \,.$$

23. This leads back to the issue of aggregation. Relation (A.39) is important because it permits establishing the link between aggregate and industry-level productivity growth. Combining (A.39) and (A.32) shows that shifts in the aggregate production possibility function can either be obtained as a weighted sum of KLEMS productivity measures (with "Domar" weights) or as a weighted average of value-added based productivity measures (with value-added shares as weights). The last statement is true in a closed economy. In an open economy, the result has to be adjusted by the ratio of value added over final demand to account for the fact that some intermediate inputs have been imported from abroad, as indicated in (A.40):

$$\frac{d\ln A}{dt} = \sum_j \frac{P^j Q^j}{P \cdot FD} \frac{d\ln A^j}{dt} = \sum_j \frac{P_{VA}^j VA^j}{P \cdot FD} \frac{d\ln \tilde{A}^j}{dt} = \frac{P_{VA} VA}{P \cdot FD} \sum_j \frac{P_{VA}^j VA^j}{P_{VA} VA} \frac{d\ln \tilde{A}^j}{dt} \qquad \text{(A.40)}$$

3. Aggregation of value-added based labour productivity

25. Parallel to the aggregation of MFP growth by industry, value-added based measures of labour productivity by industry can be aggregated. The link between the aggregate and the industry-level rate of labour productivity growth permits to assess the contribution of different industries to changes of output per hour at the macroeconomic level. No specific effort is made here, however, to link micro-level labour productivity growth to shifts in a macroeconomic production possibility frontier.

25. The value-added based measure of labour productivity by industry (π^j) is given by the relation $\frac{d\ln \pi^j}{dt} = \frac{d\ln VA^j}{dt} - \frac{d\ln L^j}{dt}$. As earlier, $\hat{VA}^j$ stands for the rate of change of real value added in industry *j* and $\hat{L}^j$ for the rate of change of labour input. As before, the aggregate rate of change in value added is a share-weighted average of the industry-specific rate of change of value added where weights reflect the current-price share of each industry in value added:

$$\frac{d\ln VA}{dt} = \sum_j s_{VA}^j \cdot \frac{d\ln VA^j}{dt}, \text{ where } s_{VA}^j = \frac{P_{VA}^j VA^j}{P_{VA} VA}, \; P_{VA} VA = \sum_j P_{VA}^j VA^j \qquad \text{(A.41)}$$

26. On the input side, aggregation of industry-level labour input is achieved by weighting the growth rates of hours worked by industry with each industry's share in total labour compensation:

$$\frac{d\ln L}{dt} = \sum_j s_L^j \cdot \frac{d\ln L^j}{dt}, \text{ where } s_L^j = \frac{w^j L^j}{wL}, \; wL = \sum_j w^j L^j \qquad \text{(A.42)}$$

27. A common practice in forming aggregates of labour input is to simply add up hours worked across industries. This is a special case of the procedure above, which applies when wage rates across

industries are identical. In this case, $s_L^j = \frac{L^j}{L}$ and the growth rate of total labour is isomorphic to the case where hours are simply added up.

28. Aggregate labour productivity growth is defined as the difference between aggregate growth in value added and aggregate growth in labour input:

$$\frac{d\ln\Pi}{dt} = \sum_j \left(s_{VA}^j \frac{d\ln VA^j}{dt} - s_L^j \frac{d\ln L^j}{dt}\right) \tag{A.43}$$

29. An industry's contribution to aggregate labour productivity growth is $s_{VA}^j \frac{d\ln VA^j}{dt} - s_L^j \frac{d\ln L^j}{dt}$, or the difference between its contribution to total value added and to total labour input. One notes that if $s_{VA}^j = s_L^j$, total labour productivity growth is a simple weighted average of industry-specific labour productivity growth. Yet another way to represent (A.44) is by decomposing it into a weighted average of industry-specific productivity growth and a re-allocation term *R*:

$$\frac{d\ln\Pi}{dt} = \sum_j s_j^{VA} \left(\frac{d\ln VA^j}{dt} - \frac{d\ln L^j}{dt}\right) + R \ \text{with}\ R = \sum_j (s_{VA}^j - s_L^j) \frac{d\ln L^j}{dt} \tag{A.44}$$

30. The re-allocation term will be positive if an expanding industry (*i.e.* one with an increase in labour input) holds a share in output that exceeds its share in labour compensation. The latter is tantamount to saying that this industry enjoys a higher than average *level* of labour productivity. A shift of resources to sectors with higher levels of productivity implies an increase in aggregate productivity growth.

Annex 7

ACKNOWLEDGEMENTS

This manual has significantly benefited from the discussions in the Statistical Working Party of the OECD Industry Committee. Moreover, many useful insights and suggestions were provided by an informal steering group of experts, by way of written comments and during a meeting in June 2000. All errors and omissions remain, of course at the responsibility of the OECD Secretariat. The following persons participated in the steering group of experts and/or provided comments on the draft manual:

Mr. Edwin Dean (Chair)	George Washington University, Washington, DC, and former Associate Commissoner at the United States Bureau of Labor Statistics
Mr. Charles Aspden	Australian Bureau of Statistics
Mr. Bart van Ark	University of Groningen, Netherlands
Ms. Pirkko Aulin-Ahmavaara	Statistics Finland
Mr. Erwin Diewert	University of British Columbia, Canada
Mr. René Durand	Industry Canada and former Statistics Canada
Mr. Mogens Fosergau	Ministry of Industry, Denmark
Mr. Gabriel Gamez	National Statistical Office, Switzerland
Mr. Charles Hulten	University of Maryland, United States
Mr. Eivind Hoffmann	Bureau of Statistics, International Labour Office
Mr. Jacques Mairesse	INSEE, France
Mr. Joachim Recktenwald, Ms. Isabelle Maquet-Engsted, Mr. John Verrinder	Eurostat
Mr. Carlo Milana	Institute for Economic Studies and Analysis, Italy
Mr. Nick Oulten	Bank of England, United Kingdom
Mr. Prasada Rao	University of New England, Australia
Mr. Anders Sorensen	Ministry of Industry, Denmark
Mr. Jack Triplett	Brookings Institution, United States

REFERENCES AND BIBLIOGRAPHY

Ark, Bart van (1996), "Issues in Measurement and International Comparison of Productivity – An

Australian Bureau of Statistics (1997*), Development of Annually Re-weighted Chain Volume Indexes in Australia's National Accounts*, paper presented at the 1997 OECD/Eurostat meeting of National Accountants.

Baily, Martin (1993), "Competition, Regulation, and Efficiency in Service Industries", *Brookings Papers on Economic Activity*, pp. 71-159.

Balk, Bert M. (1998), *Industrial Price, Quantity and Productivity Indices, The Micro-economic Theory and an Application*, Kluwer Academic Publishers.

Ball, V. Eldon, Rolf Färe, Shawna Grosskopf and Richard Nehring (2001), "Productivity of the U.S. Agricultural Sector: The Case of Undesirable Outputs", in Hulten, Dean and Harper (eds.).

Bartelsman, Eric and Mark Doms (2000), "Understanding Productivity: Lessons from Longitudinal Microdata", *Journal of Economic Literature,* Vol. 38, No. 3, September, pp. 569-594.

Basu, Susanto and John G. Fernald (2001), "Why is Productivity Procyclical? Why do we Care?", in Hulten, Dean and Harper (eds.).

Baumol, William J., Sue Anne Batey Blackman, and Edward N. Wolff (1992), *Productivity and American Leadership: The Long View*, MIT Press.

Beaulieu, Joseph J. and Joe Mattey (1998), "The Workweek of Capital and Capital Utilisation in Manufacturing", *Journal of Productivity Analysis*, Vol. 10.

Berndt, Ernst R. and Melvyn A. Fuss (1986), "Productivity Measurement with Adjustments for Variations in Capacity Utilisation and Other Forms of Temporary Equilibria", *Journal of Econometrics* 33.

Berndt, Ernst R., Ian Cockburn and Zvi Griliches (1996), "Pharmaceutical Innovations and Market Dynamics: Tracking Effects on Price Indexes for Antidepressant Drugs", *Brookings Papers on Economic Activity*.

Biorn, Erik, Erling Holmoy and Oystein Olsen (1989), "Gross and Net Capital, and the Form of the Survival Function: Theory and Some Norwegian Evidence", *Review of Income and Wealth*, Series 35.

Bresnahan, Timothy F. and Robert J. Gordon (eds.) (1996), *The Economics of New Goods*, Studies in Income and Wealth, University of Chicago Press.

Bruno, Michael (1978), "Duality, Intermediate Inputs and Value Added", in Fuss, Melvyn and Daniel McFadden (eds.), *Production Economics: A Dual Approach to Theory and Applications*, North Holland.

Bureau of Labor Statistics (1983), *Trends in Multifactor Productivity, 1948-81*, Bulletin 2178.

Bureau of Labor Statistics (1993), *Labor Composition and U.S. Productivity Growth, 1948-90,* US Government Printing Office.

Caves, Douglas W., Laurits R. Christensen, and W. Erwin Diewert (1982), "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity", *Econometrica*, pp. 1393-1413.

Charnes, A., W.W. Cooper, A. Y. Lewin and L.M. Seiford (1994), *Data Envelopment Analysis: Theory, Methodology and Application*, Kluwer.

Coelli, Tim, D.S. Prasada Rao and George E. Battese (1998), *An Introduction to Efficiency and Productivity Analysis,* Kluwer Academic Publishers.

Commission of the European Communities, OECD, IMF, United Nations, World Bank (1993), *System of National Accounts 1993,* Brussels/Luxembourg, New York, Paris, Washington, DC.

De Boer, Sake, Jan van Dalen and Piet Verbiest (1997), *The Use of Chain Indices in the Netherlands: Statistics Netherlands*, paper presented at the 1997 OECD/Eurostat meeting of National Accountants.

Dean, Edwin R., and Michael J. Harper (2001), "The BLS Productivity Measurement Program", in Hulten, Dean and Harper (eds.).

Dean, Edwin R., Michael J. Harper and Mark S. Sherwood (1996), "Productivity Measurement with Changing-weight Indices of Outputs and Inputs", in OECD (1996a).

Denison, Edward F. (1969), "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches", *Survey of Current Business* 49.

Denison, Edward F. (1974), *Accounting for United States Economic Growth 1929-69*, The Brookings Institution, Washington, DC.

Denison, Edward F. (1985), *Trends in American Economic Growth 1929-82*, The Brookings Institution, Washington, DC.

Denison, Edward F. (1989), *Estimates of Productivity Change by Industry: An Evaluation and an Alternative*, The Brookings Institution, Washington, DC.

Diewert, Erwin W. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics*, pp. 115-45.

Diewert, Erwin W. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, pp. 883-900.

Diewert, Erwin W. (1980), “Aggregation Problems in the Measurement of Capital”, in Dan Usher (ed.), *The Measurement of Capital*, University of Chicago Press.

Diewert, Erwin W. (1983), “The Theory of Output Price Index and the Measurement of Real Output Change”, in Erwin W. Diewert and Claude Montmarquette (eds.), *Price Level Measurement*, Statistics Canada.

Diewert, Erwin W. (1987a), “Index Numbers”, in J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: A Dictionary of Economics*.

Diewert, Erwin W. (1987b): “Cost Functions”, in J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: A Dictionary of Economics*.

Diewert, Erwin W. (1992), “Fisher Ideal Output, Input, and Productivity Indices Revisited”, *The Journal of Productivity Analysis*, Vol. 3.

Diewert, Erwin W. (2000), *The Challenge of Total Factor Productivity Measurement*, International Productivity Monitor.

Diewert, Erwin W. and Denis Lawrence (1999), “Measuring New Zealand’s Productivity”, Treasury Working Paper 99/5, http://www.treasury.govt.nz/workingpapers/99-5.htm.

Diewert, Erwin W. and M.N.F. Mendoza (1995), “Data Envelopment Analysis: A Practical Alternative?”, The University of British Columbia, *Department of Economics Discussion Paper* No. DP 95-30.

Diewert, Erwin W. and Catherine J. Morrison (1986), “Adjusting Output and Productivity Indexes for Changes in the Terms of Trade”, *The Economic Journal 96*, pp. 659-679.

Domar, Evsey (1961), “On the Measurement of Technological Change”, *Economic Journal* 71.

Dosi, Giovanni (1988), “Sources, Procedures and Micro-economic Effects of Innovation”, *Journal of Economic Literature* 36.

Durand, René (1994), “An Alternative to Double Deflation for Measuring Real Industry Value Added”, *Review of Income and Wealth*, Vol. 40.

Durand, René (1996), “Canadian Input-Output-based Multifactor Productivity Accounts”, *Economic Systems Research*, Vol. 8.

Eldridge, Lucy P. (1999), “How Price Indexes Affect Productivity Measures”, *Monthly Labor Review*, February.

Eurostat (2001), *Handbook on Price and Volume Measures in National Accounts*, forthcoming.

Ezaki, Mitsuo and Dale W. Jorgenson (1995), “Measurement of Macroeconomic Performance in Japan, 1951-1968”, in Dale W. Jorgenson (ed.), *Productivity Volume II*, MIT Press.

Farrell, M.J. (1957), “The Measurement of Productive Efficiency”, *Journal of the Royal Statistical Society*, pp. 253-281.

Fisher, Irving (1922), *The Making of Index Numbers*, Boston.

Fisher, M. Franklin and Karl Shell (1998), *Economic Analysis of Production Price Indexes*, Cambridge University Press.

Fosgerau, Mogens, Svend E. Hovgaard Jensen and Anders Sørensen (2000), “The Impact of Education on Labour Quality and Economic Growth”, CEBR Discussion Paper, www.cebr.dk (forthcoming).

Foss, M.F. (1963), “The Utilisation of Capital Equipment: Post-war Compared with Pre-war”, *Survey of Current Business* 43.

Gersbach, Hans and Bart van Ark (1994), *Microfoundations of International Productivity Comparisons*, Research Memorandum, Institute of Economic Research, University of Groningen.

Goldman, S.M. and H. Uzawa (1964), “A Note on Separability in Demand Analysis”, *Econometrica*, Vol. 32, No. 3.

Gollop, Frank (1979), “Accounting for Intermediate Input: The Link between Sectoral and Aggregate Measures of Productivity”, in *Measurement and Interpretation of Productivity*, National Academy of Sciences, Washington.

Griliches, Zvi (1987), “Productivity: Measurement Problems”, in J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: A Dictionary of Economics*.

Griliches, Zvi (1990), “Hedonic Price Indexes and the Measurement of Capital and Productivity: Some Historical Reflections”, in Ernst R. Berndt and Jack E. Triplett (eds.), *Fifty Years of Economic Measurement*, Studies in Income and Wealth 54, National Bureau of Economic Research.

Griliches, Zvi (1996a), “The Discovery of the Residual: A Historical Note”, *Journal of Economic Literature*, Vol. XXXIV.

Griliches, Zvi (1996b), “R&D and Productivity: Econometric Results and Measurement Issues”, in Stoneman (ed.) (1996).

Griliches, Zvi (1997), “R&D and the Productivity Slowdown: Is Recovery Around the Corner?” unpublished mimeo.

Guellec, Dominique and Bruno van Pottelsberghe de la Potterie (2000), “R&D and Productivity Growth: Panel Data Analysis of 16 OECD Countries”, *STI Working Paper* 2001/3, OECD, Paris.

Gullickson, William and Michael J. Harper (1999a), “Possible Measurement Bias in Aggregate Productivity Growth”, *Monthly Labor Review*, February.

Gullickson, William and Michael J. Harper (1999b), “Production Functions, Input-Output Tables, and the Relationship Between Industry and Aggregate Productivity Measures”, unpublished.

Hall, Robert E. and Dale W. Jorgenson (1967), "Tax Policy and Investment Behaviour", *American Economic Review*, Vol. 57.

Haltiwanger, John (2000), "Aggregate Growth: What have we learned from Microeconomic Evidence?", *OECD Economics Department Working Papers No. 276*, OECD, Paris.

Harberger, Arnold C. (1998), "A Vision of the Growth Process", *American Economic Review*, March.

Harper, Michael J., Ernst R. Berndt, and David O. Wood (1990), "Rates of Return and Capital Aggregation Using Alternative Rental Prices", in Dale W. Jorgenson and Ralph Landau (eds.), *Technology and Capital Formation*, MIT Press.

Hill, Peter (1971), *The Measurement of Real Product: A Theoretical and Empirical Analysis of the Growth Rates for Different Industries and Countries*, OECD, Paris.

Hill, Peter (1988), "Recent Developments in Index Number Theory and Practice", *OECD Economic Studies No. 10*, OECD, Paris.

Hill, Peter (1999a), "The Productive Capital Stock and the Quantity Index for Flows of Capital Services", paper presented at the third meeting of the Canberra Group on Capital Stock Statistics, Washington, DC.

Hill, Peter (1999b), "Capital Stocks, Capital Services and Depreciation", paper presented at the third meeting of the Canberra Group on Capital Stock Statistics, Washington, DC.

Hill, Peter (2000), "Economic Depreciation and the SNA", paper presented at the 26th Conference of the International Association for Research on Income and Wealth, Cracow, Poland.

Hotelling, Harold (1925), "A General Mathematical Theory of Depreciation", *Journal of the American Statistical Association*, September, pp. 340-353.

Hulten, Charles R. (1973), "Divisia Index Numbers", *Econometrica* 41.

Hulten, Charles R. (1978), "Growth Accounting with Intermediate Inputs", *Review of Economic Studies* 45.

Hulten, Charles R. (1986), "Productivity Change, Capacity Utilization, and the Sources of Efficiency Growth", *Journal of Econometrics* 33.

Hulten, Charles R. (1990), "The Measurement of Capital", in Ernst R. Berndt and Jack Triplett (eds.) *Fifty Years of Economic Measurement*, NBER.

Hulten, Charles R. (1996), "Issues in the Measurement of Depreciation: Introductory Remarks", *Economic Inquiry*, Vol. 34, pp. 10-23.

Hulten, Charles R. (2001), "Total Factor Productivity: A Short Biography", in Hulten, Dean and Harper (eds.).

Hulten, Charles R., Edwin R. Dean and Michael J. Harper (eds.) (2001), *New Developments in Productivity Analysis*, University of Chicago Press for the National Bureau of Economic Research.

Hulten, Charles R. and Frank C. Wykoff (1980), "Economic Depreciation and the Taxation of Structures in United States Manufacturing Industries: An Empirical Analysis", in Dan Usher (ed.), *The Measurement of Capital*, University of Chicago Press.

Jaffé, William (1954), Translation of Léon Walras *Elements of Pure Economics*, Homewood, Illinois, RD.

Jorgenson, Dale (1963), "Capital Theory and Investment Behaviour", *American Economic Review*, Vol. 53, pp. 247-259.

Jorgenson, Dale (1989), "Productivity and Economic Growth", in Ernst R. Berndt and Jack E. Triplett (eds.), *Fifty Years of Economic Measurement*, University of Chicago Press.

Jorgenson, Dale (1995a), *Productivity Volume 1: Postwar U.S. Economic Growth*, MIT Press.

Jorgenson, Dale (1995b), *Productivity Volume 2: International Comparisons of Economic Growth*, MIT Press.

Jorgenson, Dale (1996), "Empirical Studies of Depreciation", *Economic Inquiry*, Vol. 34, pp. 24-42.

Jorgenson, Dale and Zvi Griliches (1967), "The Explanation of Productivity Change", *Review of* Economic *Studies* 34.

Jorgenson, D., F. Gollop and B. Fraumeni (1987), *Productivity and U.S. Economic Growth*, Harvard University Press, Ma.

Lau, Lawrence J. (1976), "A Characterisation of the Normalized Restricted Profit Function", *Journal of Economic Theory* 12, pp. 131-163.

Lavoie, Marie and Richard Roy (1998), "Employment in the Knowledge-Based Economy: A Growth Accounting Exercise for Canada", *Human Resources Development Canada Research Paper R-98-8E.*

Lowe, Robin (1996), "Handling Quality Changes in the Canadian National Accounts Price Deflators", in OECD(1996a).

Mairesse, Jacques (1972), "L'évaluation du capital fixe productif – méthodes et résultats", *Les collections de l'INSEE* série C No 18-19.

Malmquist, Sten (1953), "Index Numbers and Indifference Surfaces", *Trabajos de Estatistica* 4, pp. 209-242.

Metcalfe, Stan (1996), "The Economic Foundations of Technology Policy: Equilibrium and Evolutionary Perspectives", in Stoneman (ed.) (1996).

Milana, Carlo and Kiyoshi Fujikawa (1996), "Bilateral and Multilateral Comparisons of Productivity in Input-Output Analysis Using Alternative Index Numbers", in OECD (1996a).

Morrison, Catherine J. (1986), "Productivity Measurement with Non-Static Expectations and Varying Capacity Utilization: An Integrated Approach", *Journal of Econometrics* 33, pp. 51-74.

Nadiri, Ishaq (1998), “Production: Neoclassical Theories”, in Eatwell, John, Murray Milgate and Peter Newman (eds.), *The New Palgrave Dictionary of Economics*, Stockton Press Ltd.

Nadiri, Ishaq and Ingmar Prucha (2001), “Dynamic Factor Demand Models and Productivity Analysis”, in Hulten, Dean and Harper (eds.).

Nelson, Richard (1981), “Research on Productivity Growth and Productivity Differences: Dead Ends and New Departures”, *Journal of Economic Literature*.

Nelson, Richard and Sidney Winter (1982), *An Evolutionary Theory of Economic Change*, Harvard University, Cambridge, Mass.

Nickel, Stephen J. (1978), *The Investment Decisions of Firms*, Oxford.

Nicoletti, Guiseppe and Lucrezia Reichlin (1993), “Trends and Cycles in Labour Productivity in the Major OECD Countries”, *Centre for Economic Policy Research Discussion Paper No. 808*.

OECD (1996a), *Industry Productivity: International Comparison and Measurement Issues*, OECD, Paris.

OECD (1996b), *Services: Measuring Real Value Added*, OECD, Paris.

OECD (1998a), *Technology, Productivity and Job Creation*, OECD, Paris.

OECD (1998b), “Employment Measures in the OECD National Accounts: Comparison between National Accounts and Labour Force Statistics”, OECD Meeting of National Accounts Experts, OECD, Paris.

OECD (2000), *A New Economy? The Changing Role of Innovation and Information Technology in Growth*, OECD, Paris.

OECD (forthcoming), *OECD Handbook on the Quality Adjustment of Price Indices for ICT Products*, OECD, Paris.

Rymes, Thomas K. (1971), *On Concepts of Capital and Technological Change*, Cambridge.

Sato, Kazuo (1976), “The Meaning and Measurement of the Real Value Added Index”, *Review of Economics and Statistics* 57.

Seiford, Lawrence M. and Robert M. Thrall (1990), “Recent Developments in DEA”, *Journal of Econometrics* 46.

Shepard, R.W. (1953), *Cost and Production Functions*, Princeton University Press.

Solow, Robert (1957), “Technical Change and the Aggregate Production Function”, *Review of Economics and Statistics*, Vol. 39, pp. 312-320.

Stoneman, Paul (ed.) (1996), *Handbook of the Economics of Innovation and Technological Change*, Blackwell Handbooks in Economics.

Szulc, Bohdan J. (1983), “Linking Price Index Numbers”, in Erwin W. Diewert and Claude Montmarquette (eds.), *Price Level Measurement*, Statistics Canada.

Tinbergen, Jan (1942), “Zur Theorie der langfristigen Wirtschaftsentwicklung”, *Weltwirtschaftliches Archiv*, Band 55:1.

Triplett, Jack (1989), “Price and Technological Change in a Capital Good: A Survey of Research on Computers”, in Dale W. Jorgenson and Ralph Landau (eds.) (1989), *Technology and Capital Formation*, MIT Press.

Triplett, Jack (1996a), “High-tech Industry Productivity and Hedonic Price Indices”, in OECD (1996a).

Triplett, Jack (1996b), “Depreciation in Production Analysis and in Income and Wealth Accounts: Resolution of an Old Debate”, *Economic Inquiry*, Vol. 34, pp. 93-115.

Triplett, Jack (1998), *A Dictionary of Usage for Capital Measurement Issues*, presented at the Second Meeting of the Canberra Group on Capital Stock Statistics.

United Nations Organisation (1990), *International Standard Industrial Classification*, Statistical Papers, Series M, No. 4, Rev. 3, New York.

Varian, Hal R. (1984), *Microeconomic Analysis*, Norton and Company.

Wykoff, Frank C. (1989), “Economic Depreciation and Business-Leased Automobiles”, in Dale W. Jorgenson and Ralph Landau (eds.) (1989), *Technology and Capital Formation*, MIT Press.

OECD PUBLICATIONS, 2, rue André-Pascal, 75775 PARIS CEDEX 16
PRINTED IN FRANCE
(92 2001 12 1 P) ISBN 92-64-18737-5 – No. 51987 2001