

SUBSTITUTION BETWEEN DIFFERENT CATEGORIES OF LABOUR, RELATIVE WAGES AND YOUTH UNEMPLOYMENT

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INTRODUCTION

The issue of youth unemployment has attracted increasing attention from economists and other students of social problems'. Various explanations of the causes of the phenomenon have been proposed. The purpose of this essay is to examine those strands of the argument relating to the demand for young workers. Particular focus is on two points that have been made during this debate: *i)* that growth in the youth labour force has been so rapid that the demand for young workers has been unable to keep up; and *ii)* that competition from adult female workers has had deleterious effects on the job opportunities of young workers. The first section of this paper presents background data depicting the situation in various countries. Since understanding the ongoing debate requires a knowledge of the underlying theory of the demand for labour, the second section of this essay outlines that theory. The last section presents the evidence on the demand for young workers and probes the ability of these demand-side arguments to account for youth unemployment. The conclusions – summarised at the end – have to be tentative due to the limitation of thorough empirical studies to North America, Australia and Great Britain. Indeed, one clear implication of this study is the need for more work on labour demand and substitution between different categories of labour using data from other developed countries.

I. BACKGROUND DATA

In order to gauge the relative importance of the various explanations for the problem, it is essential to examine the available data describing a large number of developed countries. As the data will show, the picture presented by the experience of any particular country is too idiosyncratic to provide a general insight into the problem. Taken together, though, the experiences of a large number of countries provide sufficient common strands to allow inferences to be drawn about the overall importance of alternative forces affecting the youth labour market.

The extent to which adult women could have "taken jobs away" from young workers depends in the first instance on whether the work force of women has

Table 1. Labour force share of women and young people
Percent

Country (Reporting years)	(a) Women		(b) Youths under 25 years of age	
	Early 1960s	Early 1980s	Early 1960s	Early 1980s
Canada (1961,1981)	27.4	40.7	20.7	26.2
United States (1960,1982)	32.1	42.7	17.2	22.9
Japan (1965,1981)	38.9	38.7
Australia (1966,1981)	29.6	37.6
Austria (1961,1982)	40.4	38.6
Belgium (1961,1981)	26.6	38.5
Denmark (1965,1981)	33.9	44.4	25.0	20.0
Spain (1960,1981)	18.2	29.5	23.6	22.6
Finland (1960,1981)	39.4	47.6	20.8	16.2
France (1965,1982)	34.5	39.3	20.9	16.7
Germany (1961,1982)	36.7	38.9	26.8	20.7
Italy (1961,1982)	24.9	34.1	25.4	18.9
Norway (1960,1982)	22.9	42.2	18.8	16.0
Netherlands (1960,1982)	22.3	33.4	27.9	21.5
Sweden (1965,1982)	33.6	46.3	18.7	15.7
United Kingdom (1966,1981)	37.8	39.1

Sources: ILO, *Yearbook of Labour Statistics*, various years, and *Bulletin of Labour Statistics*; OECD, *Labour Force Statistics*; statistical publications of individual countries.

grown relative to the total labour force. Section (a) in Table 1 presents the corresponding data for a large number of developed economies. For most, the share of women in the total labour force has grown rapidly since the early 1960s. Growth has been especially rapid in North America and Scandinavia, but it has characterised most of the labour markets of the OECD countries. Only in Japan, Austria, the Federal Republic of Germany and the United Kingdom has the female labour force

not grown significantly relative to the total. The data suggest that the trends in female labour-force participation have been sufficiently widespread and strong to provide scope for increased competition between women and young workers,

Section (b) in Table 1 shows the share of persons under age 25 in the labour force in selected countries. North America, which experienced a sharp increase in the birth rate in the 1950s, saw a rapid expansion of the youth labour force in the 1960s and 1970s, an experience unique to that continent. In every other country represented in the Table young workers' labour-force share declined between the early 1960s and the early 1980s. In some countries this was due as much to the effects of imposed increases in school-leaving ages as to the absence of a prior baby boom.

Table 2 presents trends in unemployment differentials between age-sex groups for many of the countries for which data were presented in Table 1. In the United States, Australia and Japan there is no evidence that youth unemployment has worsened relative to the state of the entire labour market, i.e. youth

Table 2. Unemployment rates as a fraction of the adult male unemployment rate

Country	Reporting year	Males under 20	Females under 20	Males 20-24	Females 20-24
Canada	1967	3.08	1.97	1.65	0.86
	1974	3.67	3.33	2.36	2.24
	1982	3.13	2.48	2.20	1.65
United States	1967	5.30	6.75	1.80	3.50
	1974	4.97	5.69	2.69	3.24
	1982	3.25	2.92	2.19	1.76
Japan	1969	2.40	1.80	1.70	1.70
	1974	3.42	1.75	2.00	1.83
	1982	2.98	1.97	1.64	2.00
Australia	1967	3.44	4.00	1.89	3.33
	1974	4.17	5.58	2.42	3.00
	1982	4.11	4.81	2.61	2.43
France	1967	3.08	4.50	1.75	2.67
	1973	3.64	7.36	2.36	4.09
	1977	5.12	10.37	2.58	5.04
Sweden	1967	2.65	3.94	1.59	2.06
	1974	3.71	5.79	1.93	2.79
	1981	4.10	5.25	2.40	2.30
United Kingdom	1967	1.17	0.74	1.09	0.61
	1974	1.37	0.89	1.30	0.56
	1981	2.17	1.92	1.83	1.25

Sources: Richard Layard and Christopher Pissarides (1982), "Youth wages and youth unemployment", Centre for Labour Economics, London School of Economics, Working Paper No. 393; OECD, Quarterly Labour Force Statistics; ILO, Yearbook of Labour Force Statistics; statistical publications of individual countries.

unemployment rates and total unemployment rates have risen roughly proportionately. It is interesting to note that the absence of a trend in relative youth unemployment rates was accompanied by a rapid rise in the female labour force in the United States. In Canada there has been a rise in youths' relative unemployment rates except among male teenagers, with the increase especially pronounced among 20-24-year-olds. In the three European countries there have been clear trends in most categories of the youth labour force toward relatively higher unemployment rates. The trends have been more pronounced among teenagers than among young adults. Interestingly, the same trends are observed in the United Kingdom, which experienced no increase in adult females' share in the work force, and in Sweden, which had one of the sharpest increases.

The other labour-market outcome of interest is the wage rate obtained by workers in various age-sex groups. Table 3 contains the relative wage rates of full-time young workers and women compared to those received by adult men. These data are less reliable than those in the previous tables. Changes in the skill mix of workers within each demographic aggregate and the focus on wage rates rather than on average labour costs per worker are only two of the data problems. For

Table 3. Wage rates as a percent of the adult male wage

Country	Reporting year	Age-sex group				
		Teen males	Young adult males	Teen females	Young adult females	Adult women
Canada	1967	45	69	33	51	59
	1979	49	68	41	51	63
United States	1967	74	74	57	57	60
	1982	60	60	50	50	64
Japan	1969	48	73	40	52	54
	1978	47	70	43	54	53
Australia	1967	48	49	38	38	58
	1979	53	53	49	49	74
France	1967	34	68	33	54	66
	1977	35	63	37	54	69
Sweden	1967	33	62	32	46	61
	1978	47	62	47	55	73
United Kingdom	1967	51	51	38	38	57
	1981	60	60	40	40	73

Sources: Richard Layard and Christopher Pissarides (1982), "Youth wages and youth unemployment", Centre for Labour Economics, London School of Economics, Working Paper No. 393; for 1981, United Kingdom, annual statistical abstract; for 1982, United States, *Employment and Earnings*.

For the United States and Japan the data represent usual or contractual weekly or monthly earnings; for the United Kingdom and France they are hourly or annual earnings; for the other countries they are full-time workers' earnings per time period.

these reasons any inferences drawn from them must necessarily be quite tentative.

Relative wages in the seven countries show no uniform trend. In the United States there has been a very sharp decline in relative youth wage rates, while they have risen in Australia, Sweden and the United Kingdom. Relative wages for adult women have risen in all the countries shown except Japan. This rise has occurred in Canada, the United States, Australia, France and Sweden, where the adult female labour force has grown rapidly, and in the United Kingdom, where it has not. Combining the patterns for youths and women, one can infer that young males' wages have fallen relative to those of adult women in the United States and the United Kingdom, and stayed roughly constant elsewhere.

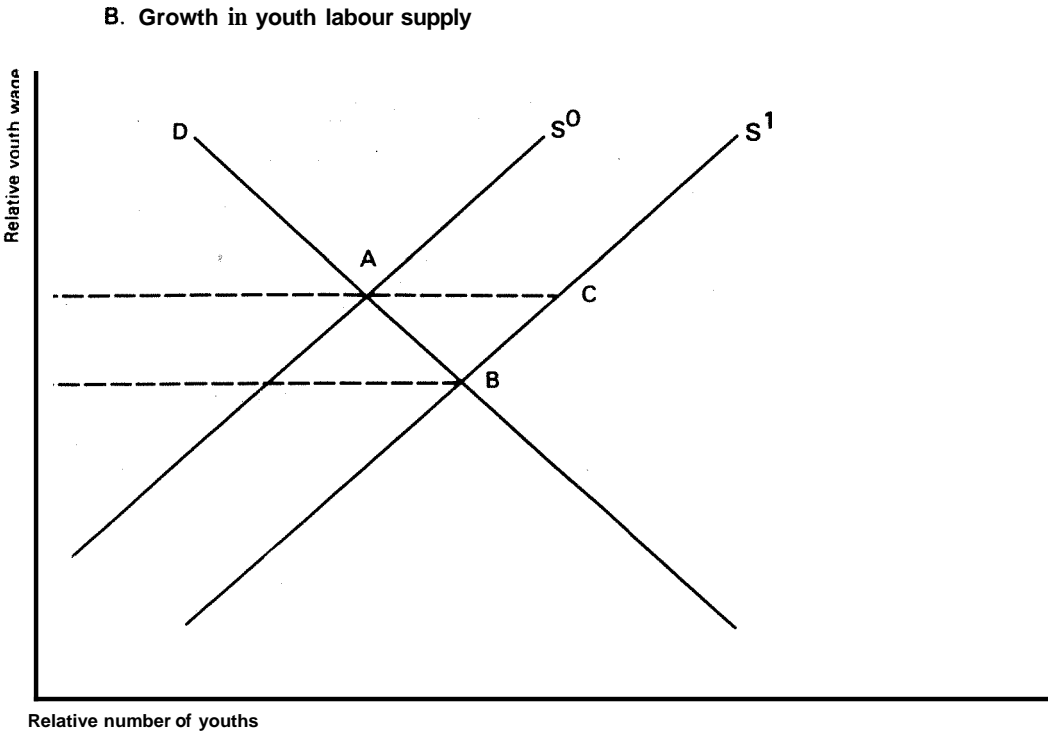
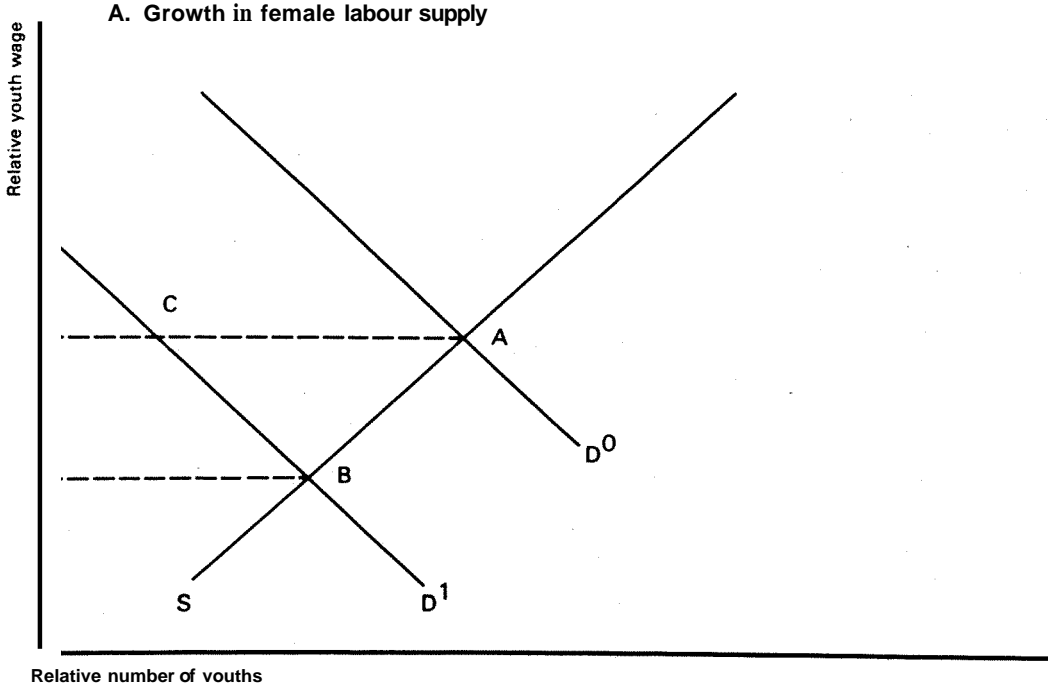
II. THEORETICAL CONSIDERATIONS

Growth in the adult female labour force will affect the youth labour market, but need not increase the unemployment rate of young workers. Consider the hypothetical youth labour market depicted in Chart 1A, where the share of young workers in the labour market is shown on the horizontal axis, and their real wage rate relative to the economy's average level of real wages is shown on the vertical axis. Throughout this part of the discussion the average level of real wages in the economy is assumed to remain constant. The supply of young workers responds positively to their real rewards, as shown by the upward-sloping curve S ; employers' demands for young workers decrease as the cost of employing them rises, as shown by the demand curve D^0 . The true shape of these curves – how steeply sloped they are – is an empirical issue discussed below.

Assume that the adult female labour force has grown relative to the entire civilian labour force and that this change reduces the number of young workers employers wish to hire at any given wage rate of youths relative to all workers. This assumption means that the demand for young workers shifts inward, from D^0 to D^1 . Whether the shift is really in this direction is also an empirical question. However, if it did not change this way, the growth in the number of adult female workers could not have reduced job opportunities for youths. The shift in demand can produce two effects:

- i)* If the wage of youths relative to other workers is free to drop, the pressure of the enlarged adult female labour force will cause such a decline. The outcome, denoted by point B in Chart 1A, will be both lower relative wages among youths and reduced employment of young workers compared to what would have been observed had there been no change in the composition of the labour force.

Chart 1
YOUTH LABOUR MARKET



- ii) Alternatively, if the wage rate of youths cannot drop relative to that of all other workers, employers will be unwilling to absorb all the young workers who seek jobs. They will hire adult women instead. But *unlike* the first case, the continued high relative wage of youths will induce young workers to stay in the labour market in the hope of finding employment. Unemployment indicated by the horizontal distance CA will be observed in the youth labour market.

If the relative wage rate of young workers adjusts sufficiently, there will be no special youth unemployment problem regardless of how much youths and adult women compete for the same jobs. The only effect of such competition after an influx of adult women into the labour force will be a decline in the real wages of youths. If, however, real wages are rigid downward, and thus youths' relative labour cost cannot fall, such an influx will induce higher unemployment among youths, *if* the two groups compete for the same jobs. Thus, if such competition does exist, there will be a youth unemployment problem in those economies in which the relative wage rates of young workers have not fallen in response to the increased labour-force entry of women.

Youth unemployment can also result from an influx of young workers into the labour force. Consider the youth labour market shown in Chart 1B. Again, the demand for young workers is shown by the downward-sloping curve **D**, while the initial supply of young workers at each real wage rate is shown by S^0 . Suppose the youth labour force grows suddenly, perhaps because of pure demographic effects (an unusually high birth rate fifteen to twenty-five years earlier), or perhaps because more young people choose to work rather than attend school. This shock will shift the relative supply of young workers outward at each wage, from S^0 to S^1 . Again, two cases are possible:

- i) If relative wages can adjust, the influx of young workers will be absorbed into jobs. The youths, competing for the same jobs, will bid down the wages offered on those jobs, inducing employers to hire them. The youth labour market will then be characterised by real wage and employment outcomes at point B in Chart 1B. Wages will be lower in real terms and relative to the wages of other workers; more youths will be employed; and there will be no special youth unemployment problem.
- ii) If real wages do not adjust, some of the young labour-force entrants will not be hired. Unemployment of youths, in the amount AC, will be observed.

The amount of youth unemployment, if real wages cannot fall far enough (or the size of the reduction in real wages received by youths, if they can) depends partly on the responsiveness of youths' labour supply to the returns to work. Though the evidence on this will not be examined in great detail, in Section III.A, those aspects of this issue that affect the discussion of labour demand will be analysed. The

outcomes also depend on the nature of demand for young workers. The responsiveness of that demand to changes in the number of, and wages received by, other workers will affect the employment, unemployment and wage rates of young workers.

Unfortunately, there are no empirical estimates of labour-demand relationships for youth labour markets (or any other) under generalised assumptions about the nature of labour supply. The analysis of labour demand has been carried out under either of two extreme assumptions about the nature of the supply of particular subgroups of workers. Under the first of these approaches it is assumed that labour supply is infinitely elastic. Firms take the wage as fixed and hire as many workers at that wage as they wish given their goal of maximising profits. In this case labour demand determines the amount of employment, but the wage rates of each group of workers are assumed to be determined exogenously. On the other hand, we can assume that there is a fixed supply of workers of a certain type, independent of the group's relative wage. In this case employers are assumed to compete for the fixed stock of workers of each type, bidding the relative wage up or down as the stock falls or rises.

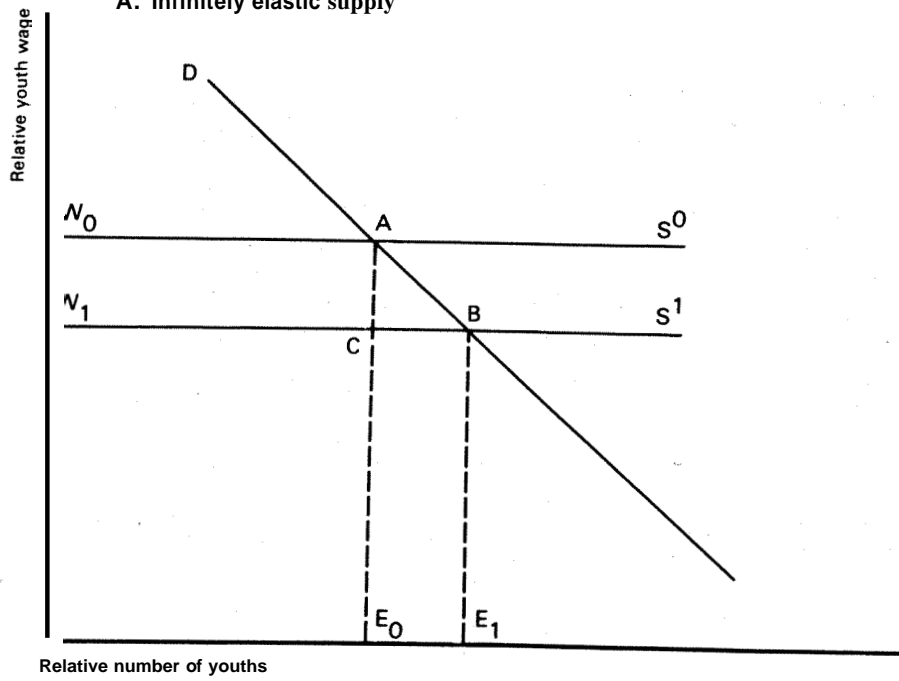
Consider the first case, shown in Chart 2A. By assumption the relative supply of young workers is infinitely elastic at the going wage rate, W_0 . Assume that the relative supply curve shifts downward. This could be due to a decline in the average human capital of young labour-force participants, a subsidy that induces them to enter the labour force, or another such change. Employers, who take the relative wage as given at the lower level, W_1 , are willing to hire more young workers, since they are now relatively less expensive than before. They expand the relative employment of young workers from E_0 to E_1 . The percentage change in the relative number of young workers employed in response to a 1 per cent fall in their relative wage rate is the elasticity of *factor* demand for young workers. This concept and the other parameters discussed in this section are defined formally in the Annex.

The demand for young workers might also vary with the wage rates of other types of workers. For example, an increase in the relative wage rate of adult males might make young workers more attractive new hires. In terms of Chart 2A, the demand curve would shift out. In such a case young workers and adult males are *p-substitutes*: a rise in the relative wage of one group leads employers to hire more members of the other group. If, on the other hand, the higher relative wage of adult males leads employers to hire fewer youths (as well as a reduced number of adult males), the two groups are *p-complements*. In this case the relative demand curve in Chart 2A would shift leftward.

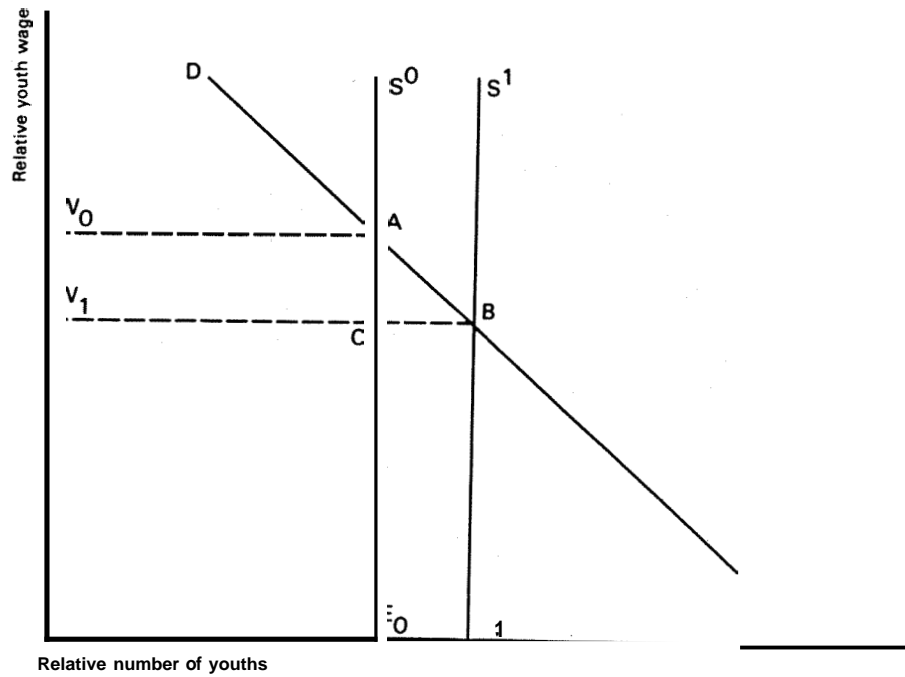
Note that if two types of workers are *p-substitutes*, a rise in the relative wage of one group induces an increase in the relative employment of the other group. The ease of substituting workers in different groups is measured by the partial elasticity of substitution, the percentage effect on their relative employment of a 1 per cent

ALTERNATIVE LABOUR SUPPLY ASSUMPTIONS

A. Infinitely elastic supply



B. Inelastic supply



change in their relative wages. If the two types of labour are p-substitutes, this elasticity is positive; if they are p-complements, it is negative. In reviewing the evidence on the extent of labour-market competition between young workers and other groups of workers, the partial elasticity of substitution is the parameter used to summarise the relevant information².

Thus far we have assumed that total output is unchanged. If, though, the drop in the relative wage of youths means that average labour costs are lower, employers will be making higher profits. If markets are competitive, new firms will enter and add to production³. The expansion in output will increase employers' demand for all types of labour. This **output effect** means that the elasticity of factor demand understates the impact of a fall in the real wage rate on the employment of young workers.

Examining labour demand using empirically-determined elasticities of factor demand and partial elasticities of substitution requires the assumption that the supply of each type of labour is horizontal at the going market wage rate. Broad shifts in the supply of labour of different age-sex groups have in the post-war period often resulted from pure demographic changes – the effects of prior changes in birth rates or reductions in mortality rates. In the case of adult women, they have occurred mainly because of changes that are not apparently closely linked to the relative financial rewards for entering the labour market. For that reason this approach has in recent years been used less frequently to study the demand for workers classified by demographic group.

An alternative approach assumes that the relative supply of workers of each particular type does not change as the relative wage they receive varies. Under such conditions the effect of a change in the number of workers of one type on the relative wage is illustrated in Chart 2B. The relative supply of young workers is assumed to be given at S^0 . This fixed supply induces employers, whose demand is characterised by the line D, to compete for the available workers by offering a wage rate of W_0 . If the relative supply of young workers shifts from S^0 to S^1 , employers will only be willing to hire the additional young workers if their relative wage can be bid down from W_0 to W_1 .

The percentage change in the relative wage of young workers in response to a 1 per cent increase in their relative supply is the **elasticity of factor price** of young workers. It measures how easily additional young workers will be absorbed when their relative supply increases. If they are easily absorbed, a small drop in their wages relative to a fixed average of wages and other input costs will suffice to induce employers to hire the new labour-force entrants. In that case the elasticity of factor price will be small (but always negative).

If the number of workers in other groups changes, employers' demand for young workers will be changed too. The increased supply of adult men might reduce the demand for young workers, if the two groups are employed in similar jobs. In that case the relative wages of young workers would fall, and the two groups are

q-substitutes. Alternatively, if a relative increase in the number of adult males in the labour market induces a rise in the relative wage of young workers, adult males and young workers are **q-complements.**

Whether groups of workers are q-complements or q-substitutes is measured by the **partial elasticity of complementarity**, the percentage change in their relative wages in response to a 1 per cent increase in their relative supply. If it is positive, the groups are q-complements; if it is negative, they are q-substitutes. The elasticity of complementarity thus measures the extent to which two groups of workers whose supply varies independently of their wage rates compete with each other in the labour market. The lower it is, the greater the negative effect of an increase in the number of workers in one group will be on the relative wage of the other group; or, if the real wage rate of the group cannot fall while average factor cost remains constant, the greater will be the increase in group-specific unemployment that results⁴.

This discussion is based on the assumption that the cost of producing a unit of output is unchanged when all the wage rates change in response to an increase in the supply of workers in a particular group in the labour force. (This assumption is analogous to the assumption of fixed output made above in the discussion of labour demand under infinitely elastic labour supply.) However, the growth in employment in this group means additional output is produced. In order to sell this output employers must cut prices. To maintain their profits with the lower price of output, labour costs per unit must fall. This **cost effect** of an increase in the supply of one group of workers means that the elasticity of factor prices understates the decline in the real wage of a group of workers whose supply increases. The total effect will be more negative.

On first consideration it seems that the two ways of viewing labour demand are merely mirror images, and that one can infer the elasticities of factor demand from elasticities of factor prices, and **vice versa.** (After **all**, it seems to be just a matter of looking at the relative demand curve D along the employment or along the wage-rate axis in Chart 2.) More specifically, it might seem that two groups must be p-substitutes and q-complements, or p-complements and q-substitutes. That is true if there are only two inputs, for example, capital services and some homogeneous input called labour. If there are several inputs, though, the simple correspondence between the two ways of viewing labour demand disappears, as shown in the Annex. Two types of labour can be p-substitutes (a rise in the relative wage of one induces employers to increase the relative use of the other) and be q-substitutes at the same time (so that a rise in the relative supply of one induces a drop in the relative wage of the other). Intuitively, the reason for this potential lack of correspondence is that the p-approach assumes that output is unchanged and allows relative quantities used to vary, while the q-approach assumes that unit cost does not change and allows relative wages to vary. It is difficult to think of examples where this might occur. It may be, though, that employers would react to an

imposed increase in the relative wage of young workers (perhaps the abolition of special youth wage rates) by substituting adult women for youths. If so, adult women and youths would be p-substitutes. At the same time, if youths' relative wages were driven down when more adult women entered the labour force, the two groups would be q-substitutes as well.

III. EMPIRICAL EVIDENCE

A. Labour supply behaviour

The preceding discussion makes it clear that the analysis of the effects of labour demand on the (un-)employment incidence among young workers cannot proceed without some discussion of the supply behaviour characterising this and related markets. No complete survey of the empirical results on labour supply behaviour is presented here. [For that, see, e.g. Borjas-Heckman (1978) and Killingsworth (1981).] Rather, a summary of results essential to shed light on the appropriateness of the two approaches to studying the demand for labour will be given.

In the case of the supply of young workers it is important to distinguish the effects of wage rates from those of job opportunities. Thus a higher wage rate can be expected to elicit an increased supply from a fixed population of young workers; but if that wage increase reduces the amount of employment offered, and reduces new workers' chances of finding a job, it may deter some new workers from entering the labour force.

A few studies have examined the sensitivity of the supply of young workers to changes in the real wage rate they can obtain at a constant rate of unemployment. An early study, Hall (1973), finds little indication that the supply of young workers is increased by higher real wages. However, Gustman and Steinmeier (1981), using more sophisticated techniques and a much larger sample, show that higher wage rates induce a greater supply of young workers. Even in their study, though, the effect of doubling the real wage is only a small increase in hours supplied and presumably in participation rates, too. Evidence of a much higher wage elasticity of supply (based only on the participation of young white married women) is found by Schultz (1980) for the United States, but for blacks little response is found.

There is only weak evidence of a large positive response of the supply of youth labour to higher real wages *at a fixed rate of unemployment*. A large number of studies have, though, used both time-series and cross-section data to demonstrate that the supply of young workers is sensitive to variations in their unemployment rate. Thus Mincer (1976) for the United States and Swidinsky (1980) for Canada

find that the imposition of a higher effective minimum wage rate reduces the labour-force participation rate of young workers. Presumably the reduction in job opportunities induced by a higher minimum wage more than offsets the inducement to seek work due to the increased minimum wage. Higher unemployment rates over the business cycle sharply reduce the labour-force participation rates of youths in the United States; and those rates are lower in cities in which the unemployment rate is higher. [See Bowen-Finegan (1969) and Gustman-Steinmeier (1981), among others.] These effects are most pronounced among the youngest workers.

The evidence seems fairly clear that supply curves for young workers, like S in Chart 1A, are quite steeply sloped. While such workers do respond strongly to changes in job opportunities, the available evidence (which is admittedly quite sparse) suggests their response to changes in real wages is not very large. This suggests that the better evidence on the nature of the demand for young workers will be derived from studies that assume the supply of youths is invariant with respect to the real wage. This conclusion is reinforced by the observation that the major fluctuations in the labour supply of young workers in North America (the source of most of the studies of labour demand) have been due to changes in the size of age cohorts.

The labour market for adult males is characterised by a response of hours supplied and participation rates that is remarkably insensitive to changes in job opportunities and real wage rates. Bowen-Finegan (1969) and others have shown that adult males' participation is less sensitive to the business cycle than that of any other group. Borjas-Heckman (1978), summarising nine studies of the response of prime-age males' labour supply to changes in real wage rates, conclude that it is essentially zero, if not negative. More complex methods, designed to measure very long-run effects of wages on the supply of adult men, also find only very small responses. It seems quite safe to conclude that any changes in the market for adult males that impinge upon the youth labour market must be the result of shifts in what is essentially a vertical supply curve of adult male labour. Thus the demand for young workers should also be analysed as if the supply of workers of different types is not subject to changes induced by variations in real wage rates.

The overwhelming majority of studies of labour supply have dealt with adult women, particularly married women. The range of estimates is truly remarkable, as early and more recent surveys [Cain-Watts (1973); Killingsworth (1981)] show. That range is centred roughly around a wage elasticity of supply of between 0.5 and 1.0. This suggests that the labour supply of this group cannot be treated as independent of wages in studies of the demand for labour. Since that of other groups, as shown above, can be treated this way, the question is: How much error is introduced into estimates of labour-demand characteristics by assuming that the labour supply of *all* groups is independent of wages? The argument that such errors are small rests on the assumption that fluctuations in the supply of adult women have mainly resulted from changes in the value of their time at home and in their

social characteristics. At least for the United States this assumption seems reasonable⁵. Thus we may conclude that the best available evidence on the demand for labour classified by demographic group comes from studies that treat the supply of labour as if it were unaffected by changes in real wage rates⁶.

Given that it is preferable to treat the youth labour force as if it is chiefly affected by changes that shift the (vertical) supply curve rather than by changes in real wages, it is worth remembering that Table 1(b) showed that only in North America has the youth labour force increased in relative size. In terms of the view of labour demand we have expounded, this suggests that in other countries the relative drop in the supply of youths should have either caused their relative wages to increase, reduced their unemployment rates, or both.

B. Demand for young workers

One way of examining the demand for young workers is to consider the effect of minimum wages on their employment. Since minimum wage laws, if they are well enforced, make the wage exogenous to the labour market, their effects on youth employment can provide evidence on the elasticity of demand for young workers. A huge literature on the effects of minimum wage rates on the employment of young workers has been produced over the past twenty years. [For recent surveys of this area of research see Brown et al. (1982) and Martin (1983).] In the data for the United States there is a surprising consistency of results on the employment effects of higher minimum wage rates: each 1 per cent increase in the minimum relative to other wages has reduced teenage employment by 0.1 per cent.

By itself this elasticity tells us little about the responsiveness of the demand for teenage labour to changes in its cost, for we do not know how many young workers' jobs are affected by the minimum wage⁷. Available data do not allow a precise measure of the fraction of teenage workers whose wages are directly affected by the minimum; but Meyer-Wise (1983) have produced estimates for the United States for a number of years under specific assumptions about the nature of the underlying distribution of wage rates. These distributions suggest that no more than one-third of teenage workers could be directly affected by a marginal increase in the minimum wage in the United States. Coupled with the result that the employment-minimum wage elasticity is -0.1 , this implies that the elasticity of demand for young workers corresponding to the effects of the minimum wage is at least -0.3 .

Other sources of evidence are studies of the effect of cohort size on earnings. Using data from the United States Current Population Survey, Welch (1979) found that each 1 per cent change in relative cohort size reduced the relative earnings of its members by between 0.1 and 0.2 percentage points. Similar effects have been found by Freeman (1979) and Berger (1983); only Stapleton-Young (1984) fail to find similar evidence for the United States. These effects, which can be viewed as

analogous to elasticities of factor prices, suggest that the labour market, at least in the United States, is characterised by substantial flexibility: small changes in relative wages are sufficient to accommodate fairly large changes in the relative supply of workers of different age groups. This evidence is too sparse to allow any exact inferences to be made about the size of the underlying factor-price elasticities, however.

A wide variety of methods has been used to produce direct estimates of elasticities of factor demand or factor prices for young workers. Outlining these methods and the extent to which they are likely to provide good estimates of the parameters under investigation is beyond the scope of this essay [but see Hamermesh (1984)]. Nonetheless, in examining results among the studies summarised here, it is possible to comment on their comparative reliability and thus to draw conclusions about the likely magnitude of the demand effects.

Pervading all the estimates is a host of assumptions that researchers have made about such issues as the particular disaggregation of the labour force, how to treat full- and part-time workers, and how to treat technical change. Perhaps the most difficult of these is that of disaggregation. The choices made have usually been dictated by the availability of data or by a specific policy issue on which the researcher wished to focus. This means that econometric issues of biases due to inappropriate aggregation have frequently been ignored. Most important, the implicit assumption that all workers within a particular subgroup of the labour force are perfect substitutes may be seriously in error and could lead to very misleading estimates. Other than studying whether labour as a whole can be aggregated [see Berndt-Christensen (1974), Denny-Fuss (1977), and Grant-Hamermesh (1981)], nothing has been done to examine this issue.

Most of the studies use hours of labour input (or full-time equivalent workers) rather than number of workers. Thus the complications arising from part-time work are circumvented, if hours of part-time workers can be aggregated with those of full-time workers. Since there is no evidence on this issue, we cannot infer the extent of biases that may be caused by this aggregation. The issue of biases due to Hicks-neutral technical change is more complex. Clearly, it does not induce biases among those studies based on cross-section data. Among those using time-series data, all assume neutral technical change, but none accounts for the possibility of biases arising from the existence of non-neutral technical change. These biases may be important; the only consolation is that the one study [Berndt-Khaled (1979)] that examined the issue (unfortunately for our purposes, using a set of factor inputs that did not disaggregate the work force by demographic characteristic) found little change in factor-demand elasticities when non-neutrality was assumed.

Table 4 summarises the studies of labour demand and substitution based on **disaggregations** of the work force along the lines of age, sex or race. As inspection of the table makes clear, most of the work on this issue has been conducted using data from the United States, though studies of the United Kingdom, Canada and Australia

have also been made. The column labelled "data and method" lists the country, date and sector coverage. Also listed is an indication of the method the researcher used to estimate the parameters describing labour demand. The notations "translog" or "generalised Leontief" refer to what one might call "state-of-the-art" methods of producing estimates. The CES method is a restrictive technique that does not allow for the full scope of substitution possibilities among all the types of labour being studied; and factor-demand equations, while allowing for any possible degree of substitution among groups of workers, are not usually used in a way that allows the researcher to impose all the requirements of the theory of labour demand (such as that D be downward-sloping)*.

The third column in Table 4 shows the particular groups of labour being examined. Fortunately for our purposes most of the studies have disaggregated the work force by age and sex rather than by race and sex. The penultimate column summarises information on substitution and complementarity elasticities among groups, which will be discussed below: the last column presents estimates of factor-demand and wage elasticities. The first part of the table shows estimates from studies that take the first approach to labour demand: the second part lists those that use the second approach, which as argued above is considered more appropriate.

Within each of the two parts, the studies are classified by whether a measure of the quantity or price of capital services is included. While data on these variables are often unreliable, those studies that neglect them may produce biased estimates of measures of substitution. The three studies that have considered the issue – Berndt-Christensen (1974), Denny-Fuss (1977) and Grant-Hamer mesh (1981) – all find that estimates characterising labour demand are incorrect if one assumes they are unaffected by the price or quantity of capital services. Thus those studies that ignore capital will in general provide worse measures of the true extent of substitutability and the true responsiveness of labour demand to changes in wages or supply than those that include it.

Before examining the results of the studies in detail, two robust empirical conclusions stemming from the broader literature on substitution and labour demand should be stressed [see Hamer mesh (1984)]. First, skilled labour and physical capital are p-complements, while capital and unskilled labour are p-substitutes. This **capital-skill complementarity hypothesis** [originally in Griliches (1969)] has been demonstrated many times. In its application to the labour market for youths, it implies that tax credits for investment in new capital equipment (which lower the cost of capital) will produce decreases in the relative employment of young (presumably unskilled) workers, though their absolute employment may rise if output effects are sufficiently large. Second, the demand curve for a particular group of workers is steeper (factor-demand elasticity is lower, and factor-price elasticity is higher), the more skill (education and on-the-job training) is embodied in that group of workers. This suggests that the demand curve for young workers is flatter than

Table 4. Studies of substitution among age and sex groups

Category	Study	Data and method	Types of labour	Elasticities of substitution or complementarity	Elasticities of demand or wages
1. SUBSTITUTION AND DEMAND ELASTICITIES					
A. Capital excluded					
	Welch-Cunningham (1978)	States, Census of Population, 1970;CES	14-15 years 16-17, 18-19 years Teenage labour	All are positive	-1.34
	Government of Australia (1983)	17 Australian industries, 1976-81; factor-demand equations	Males under 21 Females under 21 Males over 21 Females over 21	All are positive	-1.80 -4.58 -0.59 -2.25
	Wells (1983)	Britain, 1969-81; factor-demand equations	Males under 18 Males over 18 Females	All are positive	-5.85
	Layard (1982)	British manufacturing, 1949-69;translog	Males under 21 Females under 18 Males over 21 Females over 18	All are positive except females under 18 vs. females over 18	-1.25 -0.31 -0.35 -1.59
	Layard-Pissarides (1982)	Britain 1959-79;translog	Young males Women Men	All are positive except women vs. young men	-0.25 -0.08 -0.18
B. Capital included					
	Merrilees (1982)	Canada, entire economy 1957-78;factor-demand equations	Young males Young females Adult males Adult females	Mixed, but all involving adult male wages are negative	0.56 -0.44 -0.07 0.11
	Hamermesh (1982)	U.S. economy, 1955-75; translog, mean elasticities	14-24 years Over 25	Positive	-0.59 -0.01
	Grant (1979)	SMSAs, Census of Population, 1970;	14-24 years 25-44 years	All are positive	-9.68 -2.72

	1972 elasticities	Over 45		-3.45 -3.99
II. ELASTICITIES OF COMPLEMENTARITY AND WAGES				
A. Capital excluded				
Borjas (1983b)	Entire U.S. economy, microdata 1975; generalised I-eontief	Blacks Hispanics Whites	All are positive	-0.07 ^a -0.64 ^a -0.001 ^a
B. Capital included				
Borjas (1983a)	Census of Population, 1970; generalised Leontief	Black males Females Hispanic non-migrants Hispanic migrants White non-migrants White migrants	All positive except those involving females, and some involving Hispanics	1.02 ^a 2.90 ^a -2.66 ^a -1.98 -0.03 1.02 ^a
Grant-Hamermesh (1981)	SMSAs, Census of Population 1970; translog	Youths Blacks over 25 White Males over 25 White Females over 25	All positive except youths vs. females over 25	-0.03 -0.43 -0.13 -0.19
Berger (1983)	States, U.S., 1967-74; translog, mean elasticities	Males, 0-15 years of schooling, 0-14 years' experience Males over 16 years of schooling, 0-14 years' experience Males 0-15 years of schooling, over 15 years' experience Males over 16 years of schooling, over 15 years' experience	All are negative except young vs. old college graduates	-0.51 ^a -3.45^a -0.80 ^a -1.48^a
Freeman (1979)	Entire U.S. economy, 1950-74; translog, mean elasticities	Males 20-34 years Males 35-64 years Females	Only males 20-34 years vs. females is positive	-0.29' -0.38 -0.49 -0.71

a) Own-quantity elasticities of complementarity.

that for adult males. Taking the second approach to labour demand, this means that their relative wages will not rise as much in response to a given decrease in the youth labour force as would the wages of other groups in response to a similar change. Taking the first approach, this means that offering employers a subsidy of a fixed percent of wages will produce greater percentage increases in the relative employment of youths than of other groups.

The range of estimates of the elasticity of factor demand for young workers implied by the last column of Part I of Table 4 is very large. Some, however, are less reliable than others, and this permits us to narrow the probable range substantially. Thus the studies by Grant (1979) and Anderson (1977) measure these elasticities using a method that can easily induce severe errors^S. Merrilees' (1982) study for Canada does not have any inherent methodological problems. However, the estimates are inconsistent with the most basic premise of the theory of the demand for labour, namely that demand curves slope down. (Note that two of the four factor-demand elasticities are positive.) The causes of this result are not clear. Because of this difficulty this study too does not seem to be a very reliable guide to finding the "true" elasticity of demand for young workers.

Among the other five studies in Part I of Table 4, the range of estimated factor-demand elasticities is still quite wide, from **-0.25** for young males in Layard-Pissarides (1982), to **-5.85** for males under age **18** in Wells (1983). Except for this estimate and Government of Australia (1983) for young females, though, none of the estimated elasticities exceeds two. Moreover, the only estimates for the aggregate of young workers, in Welch-Cunningham (1978) and Hamermesh (1982), lie above, but not too far from one. That the estimates for subgroups in the youth labour force vary more widely is to be expected, for the degree of precision of these estimates seems generally to drop, the finer the degree of disaggregation that is used. Also, there is no reason to expect estimates for different subgroups' factor-demand elasticities to be identical. The general result that they are lower among groups with greater human capital suggests that the elasticity may be far above one for teenagers aged **14-17**, and somewhat lower for youths aged **18** and over.

The data do not permit to draw firm conclusions about the nature of the factor-demand elasticity for young workers. Nonetheless, they seem most consistent with an estimate larger than one in absolute value, but probably below two¹⁰. This inference suggests that employers do respond to a change in the relative wage of young workers by changing the number of young workers they seek to employ.

All five studies that have examined the demand for workers disaggregated by age, sex and/or race using the second approach have been based on United States data. This is unfortunate, because this approach should provide a better indication of how the demand for labour influences outcomes in the youth labour market. Among the studies listed in Part II of Table 4, neither of those by Borjas presents estimates

of the elasticity of factor price for youths; they are included only for comparison and reference.

Freeman's (1979) study does not present estimates for a group that is explicitly composed only of young workers. If, though, we include among the "young" males aged 20-34, we may infer that the demand for young workers is quite responsive to market changes: Each 1 per cent increase in the relative supply of young males is absorbed by employers with only a 0.38 per cent drop in the relative wage of this group. Berger's (1983) study also does not explicitly examine young workers, but one can treat workers with 0-14 years of experience as young for the purpose of discussion. Berger does not present estimates of factor-price elasticities. Instead, what are listed in the final column of Table 4 for this study are partial elasticities of complementarity – factor-price elasticities divided by the fraction of total costs accounted for by the particular input (see the Annex). However, consideration of the magnitudes involved suggests that the elasticity of factor price for youths with 0-15 years of schooling is no greater than -0.1 , while that for youths with at least sixteen years of school does not exceed -0.5 ¹¹.

Grant-Hamermesh (1981) is the only study that explicitly examines the demand for youths (defined as persons aged 14-24) using the second approach. They find that the factor-price elasticity for young workers is nearly zero. Interestingly, both their estimates and those of Freeman are consistent with the notion that the demand for more skilled labour is less sensitive to external shocks: in both studies the factor-price elasticities for other groups are larger, indicating a greater difficulty in absorbing an increase in relative supply.

Grant-Hamermesh may underestimate the size of the factor-price elasticity for young workers: but, together with Freeman's and Berger's results, their estimate implies that employers are quite responsive to changes in the relative supply of young workers. This indicates that, *if relative wages are free to adjust*, the market can absorb an increase in the size of the youth labour force with only a small drop in the relative wage rate paid to young workers. Whether the factor-price elasticity for youths is closer to -0.1 or -0.5 is not clear at this point; but the evidence suggests quite strongly that it is below one in absolute value. The true relative demand curve for young workers (of the kind drawn in Charts 1 and 2) seems to be far from vertical.

The penultimate column of Table 4 provides a summary of available research on the degree of substitutability among workers classified by demographic group. While specific estimates of elasticities of substitution and complementarity are not presented, the signs (positive or negative) are listed. Part I of the table thus indicates whether pairs of labour inputs are p-substitutes (a positive elasticity of substitution) or p-complements (a negative one); in Part II a positive sign denotes the inputs are q-complements, and a negative sign indicates they are q-substitutes.

With few exceptions there is substantial agreement among the studies listed in Part I of Table 4 on the p-substitutability of pairs of types of labour disaggregated by

age or sex. Only Layard-Pissarides (1982) for the pair, adult women and young men; Layard (1982) for the pair, adult women and young women; and Merrilees (1982) for the pairs, adult men and young men, and adult men and young women, find any evidence of p-complementarity between young workers and others. As noted in the previous section, Merrilees' results are inconsistent with the basic tenets of the theory of the demand for labour.

It is more difficult to explain the discrepancy between the results of Layard, and Layard-Pissarides, and those of other work. One possibility is that the nature of production, or the characteristics of the demographic groups being studied, are sufficiently different from those in the other countries that the results for the United Kingdom shown in the table are correct. A second possibility is that they result from the exclusion of measures of the price of capital services. As discussed above, the failure to include such measures can lead to biased estimates of the extent of substitutability among pairs of labour inputs. It is not clear from the few studies that have examined this issue whether the extent of these biases is sufficient to explain the differences in results presented here.

The other studies – Government of Australia (1983), Hamermesh (1982), Grant (1979) and Anderson (1977) – all find that young workers are p-substitutes for older workers. The current state of knowledge is insufficient to allow drawing inferences about the likely magnitude of the partial elasticity of substitution between young and older workers. All that can be inferred at this point is that a decrease in the relative wage of adult workers will lead employers to substitute away from young workers. If the real wage rate of younger workers cannot fall, this will in turn lead to increased unemployment among youths.

Casual observation of the labour market would suggest that the ease of substitution between adults and young men, and adults and young women, would differ. Unfortunately, there just is not enough evidence on this issue to draw any conclusions on whether, and to what extent, the degree of p- or q-substitutability with adults differs by sex within the youth labour force. The only study that looks at this issue [Wells (1983)] finds weak evidence that teenage males are more p-substitutable for adults of both sexes than are teenage females. The results are, however, highly sensitive to the definition of the wage measures used; thus this is clearly an issue that requires additional research.

Part II of Table 4 presents estimated partial elasticities of complementarity for demographic groups in the United States. One result stands out: the apparent q-substitutability between adult females and young workers. This finding is clear in Grant-Hamermesh (1981), but it is also implied by most other work as well. Treating men from 20 to 34 years of age in Freeman's study as young workers, they are weak q-complements for adult women. However, women are q-substitutes for all types of male labour, but especially strongly so for young high-school graduates, in Berger (1983). Borjas (1983a), in additional estimates not summarised in the Table,

finds that q-substitution between females and black males is particularly strong between females and young black males.

The evidence is not as yet as complete as one would desire to draw conclusions for policy purposes. Nonetheless, the available evidence strongly suggests that women and youths are q-substitutes in the labour market. While it is impossible, given the paucity of research on the issue, even to attempt to pick a "best-guess" estimate of the size of the partial elasticity of **complementarity** between young men and adult women, the evidence that the parameter is negative is strong.

The research indicates that, when the relative supply of women to the labour market increases, as it did in most developed countries in the 1960s and 1970s, the wage rate of young workers must fall relative to other wages if current young employees are to keep their jobs. At this point the research results do not permit a reliable estimate of the size of the decline in relative wages that is required. All that can be inferred from the finding of q-substitutability between women and youths is that rigidity of the relative wage rate of youths in the presence of an expanding labour force of adult women will result in an increase in the unemployment rate of young workers.

CONCLUSIONS

In nearly all developed countries, the last twenty years have seen a rapid rise in the share of women in the labour force. At the same time, the share of young workers has risen in a few countries and fallen sharply in most. How these two crucial changes in the labour markets of developed countries could have interacted to affect the extent of unemployment among youths depends on two factors:

- i)* The extent to which adult females and young workers compete for the same job opportunities; and
- ii)* The extent to which the relative wages of young workers respond to forces tending to depress them and lead employers to hire more young workers.

The evidence seems quite suggestive, though by no means conclusive, that adult women and youths are substitutes in the sense that an increased supply of one puts downward pressure on the relative wage of the other. That is, a relative influx of adult women into the labour force creates pressure for a fall in the relative wage of young workers; or, if the relative wage cannot fall, for a rise in the unemployment rate of young workers. There is some inferential evidence that these mechanisms have been important in some countries. In the United States, where the biggest growth in the youth labour force has combined with rapid growth of the adult female labour force, relative wages of youths have fallen markedly. This decline appears to

have been sufficient to prevent any rise in the unemployment rate of youths relative to other groups. In Canada, the only other developed country to experience growth in the labour-force shares of both groups, relative wages of youths have not fallen. This survey suggests that this relative wage rigidity, coupled with the pressures for substitution induced by the simultaneous increases in both groups in the labour force, may have contributed to the relative rise in the unemployment rates of youths that has occurred in Canada.

In none of the other countries examined has the share of both youths and adult women in the labour force increased. Thus, even though the entry of adult women would have put downward pressure on the relative wages of young workers, the relative decline in the labour-force share of youths would have reduced this pressure. Youth unemployment rates have increased relative to others in France, Sweden and the United Kingdom (among the seven countries examined in Table 2); this could be attributed to competition between youths and women only if the influx of adult females into the labour force has more than offset the relative wage repercussions of the decrease in relative youth labour supply.

NOTES

1. See, for example, Freeman and Wise (1982) and OECD (1980).
2. If there are only two inputs, they must be p-substitutes. With three or more inputs, though, it is possible for a particular input to be a p-complement to other inputs, but it still must be a p-substitute for at least one other input.
3. Even if markets are controlled by a monopolist, profit-maximising behaviour will lead that monopolist to lower the price of the product, and thus expand output.
4. With only two inputs into production, they must be q-complements. With three or more inputs, though, any input can be q-substitutable for others; but it must be a q-complement for at least one other input.
5. Shapiro-Shaw (1983) demonstrate for the United States that only half the increase in female labour supply in the late 1960s and 1970s could be accounted for by changes in all the usual variables that are assumed to affect supply, many of which are themselves more demographic than narrowly economic. The other half presumably represents the effects of a shift in tastes or in unmeasurable components of the value of time in the household.
6. Clearly, though, the best estimates would be those based upon a carefully specified system of demand and supply relations for workers in different demographic groups. Unfortunately, this is a very difficult task, and no such estimates are currently available. As noted in Section III, that is why one needs to make the polar assumptions that generate the dichotomy between the q- and p-approaches.
7. If only 10 per cent of all teenage workers were in jobs affected by the minimum wage, then a 1 per cent increase in the minimum that reduced *total teen* employment by 0.1 per cent would imply an elasticity of labour demand of -1.0 , which is a rather substantial response.
8. Details on how these methods are used and the underlying production and cost functions are discussed in Hamermesh (1984).
9. Their method requires that a matrix of estimates of a large number of parameters be inverted to obtain each elasticity. (Rather than using a cost function, which, as the Annex shows, makes the s_{ij} dependent only on one vector of parameter estimates from a translog tableau, they use a production function, which requires the use of all the estimated translog parameters.) Since that inversion procedure means that each elasticity is based on **all** the parameters that are estimated, errors in any one of these will affect the estimated elasticity of factor demand.
10. If the economy is fully employed, this is the correct estimate to use to discover the effect on youth employment of a change in the real wage of young workers. If, however, there are unemployed resources, the output effect must be added to this figure. Thus a wage subsidy of, for example, 10 per cent applied to young workers in times of high unemployment will have an impact on employers' demands for them of more than 10 per cent. Assuming competitive product markets, the output effect will increase the demand for young workers by an additional 10 per cent of the fraction of total costs accounted for by the wages paid to young workers. Elsewhere [Hamermesh (1982)] it is shown that for the United States this fraction is about 3 per cent. If this is true in other economies, as consideration of the relative wage rates and employment shares of youths suggests it is, the output effect is probably less important in predicting the effects of subsidies targeted to specific groups than are the substitution effects the subsidies engender.
11. Since the share of capital is found to be at least 0.4 in studies using cross-section data, and since wages are higher among more experienced workers, it is unlikely that the share of total costs accounted for by either of the two groups of less-experienced workers exceeds 0.15.

ANNEX

While all the definitions in the text can be derived generally, their meanings can be seen more clearly if a specific functional form is used. Accordingly, using a three-input translog formulation, the production function is defined as:

$$\ln(Q) = \ln(a_0) + \sum_{i=1,3} a_i \ln(X_i) + 1/2 \sum_{i=1,3} \sum_{j=1,3} e_{ij} \ln(X_i) \ln(X_j) \quad [1]$$

where $e_{ij} = e_{ji}$ is usually assumed, technology parameters are a_0, a_i , and e_{ij} , Q is output, and the X_i are inputs. Similarly, the constant returns to scale cost function, $C = Qg(P_1, P_2, P_3)$, has the translog form:

$$\ln(C) = \ln(Q) + \ln(b_0) + \sum_{i=1,3} b_i \ln(P_i) + 1/2 \sum_{i=1,3} \sum_{j=1,3} d_{ij} \ln(P_i) \ln(P_j) \quad [2]$$

where $d_{ij} = d_{ji}$, technology parameters are b_0, b_i, d_{ij} , and C and P_i represent the total cost and input prices, respectively.

Assuming input markets are competitive, cost minimisation implies that $\delta Q / \delta X_i = P_i, i = 1, 2, 3$. Shephard's Lemma demonstrates that $\delta C / \delta P_i = X_i$. These imply for [1] and [2], respectively:

$$\delta \ln(Q) / \delta \ln(X_i) = P_i X_i / Q = S_i \quad [3a]$$

$$\delta \ln(C) / \delta \ln(P_i) = X_i P_i / C = S_i \quad [3b]$$

where S_i is the cost share of input X_i in the total cost of producing Q and the output price (P) is unity. Taking partial logarithmic derivatives and equating them with the cost shares for the production function, gives:

$$S_1 = a_1 + e_{11} \ln(X_1) + e_{12} \ln(X_2) + e_{13} \ln(X_3) \quad [4]$$

$$S_2 = a_2 + e_{12} \ln(X_1) + e_{22} \ln(X_2) + e_{23} \ln(X_3)$$

$$S_3 = a_3 + e_{13} \ln(X_1) + e_{23} \ln(X_2) + e_{33} \ln(X_3)$$

For the cost functions, one obtains:

$$S_1 = b_1 + d_{11} \ln(P_1) + d_{12} \ln(P_2) + d_{13} \ln(P_3) \quad [5]$$

$$S_2 = b_2 + d_{12} \ln(P_1) + d_{22} \ln(P_2) + d_{23} \ln(P_3)$$

$$S_3 = b_3 + d_{13} \ln(P_1) + d_{23} \ln(P_2) + d_{33} \ln(P_3)$$

Consider Allen's (1938) general definition of s_{ij} , the partial elasticity of substitution, as :

$$s_{ij} = (Q / X_i X_j) (|F_{ij}| / |F|) \quad [6]$$

where $|F|$ is the determinant of the bordered Hessian matrix F , corresponding to the implicit production function $f(\cdot)$:

$$F = \begin{bmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & & & \\ f_2 & & f_{ij} & \\ f_3 & & & \end{bmatrix} \quad [7]$$

and $|F_{ij}|$ is the determinant of the cofactor f_{ij} .

Similarly, c_{ij} , the partial elasticity of complementary, is defined in terms of the implicit cost function as:

$$c_{ij} = (C/P_i P_j) (|G_{ij}|/|G|)$$

where $|G|$ is the determinant of the bordered Hessian matrix G derived from cost function $g(\cdot)$

The corresponding dual relationships are:

$$s_{ij} = gg_{ij}/g_i g_j, \quad i \neq j$$

$$c_{ij} = ff_{ij}/f_i f_j, \quad i \neq j$$

Because the c_{ij} are defined with respect to changes in factor quantities, and the s_{ij} are defined with respect to changes in factor prices, X_i and X_j are termed:

q-complements (substitutes) as $c_{ij} > 0 (< 0)$

p-complements (substitutes) as $s_{ij} > 0 (< 0)$

If $i > 2$, nothing requires that c_{ij} and s_{ij} have the same sign. For the translog production function $h(\cdot)$ particularly:

$$s_{ij} = |H_{ij}|/|H|$$

where $|H|$ is the determinant of the symmetric matrix:

$$H = \begin{bmatrix} 0 & h_1 & h_2 & h_3 \\ h_1 & & & \\ h_2 & & h_{ij} & \\ h_3 & & & \end{bmatrix}$$

$|H_{ij}|$ is the determinant of the cofactor of h_{ij} , and:

$$h_i = S_i$$

$$h_{ii} = e_{ii} + S_j^2 - S_i$$

$$h_{ij} = e_{ij} + S_i S_j$$

Also

$$c_{ij} = (e_{ij} + S_i S_j)/S_i S_j, \quad i \neq j$$

For the dual translog cost specification:

$$s_{ij} = (d_{ij} + S_i S_j)/S_i S_j, \quad \text{if } J$$

Also, $c_{ij} = |K_{ij}|/|K|$, where K is a symmetric matrix defined in a manner similar to H

The factor price elasticity is (Sato-Koizumi, 1973, pp. 4849):

$$\delta \ln(P_i) / \delta \ln(X_i) = S_j c_{ij}$$

The constant-output factor demand elasticity is (Allen, 1938, pp. 503-509):

$$\delta \ln(X_i) / \delta \ln(P_i) = S_j s_{ij}$$

BIBLIOGRAPHY

- Allen, R.G.D. (1938), *Mathematical Analysis for Economists*, Macmillan.
- Anderson, J. (1977), "Labor force age structure changes and relative wages", Unpublished paper, Harvard University.
- Berger, M. (1983), "Changes in labor force composition and male earnings: a production approach", *Journal of Human Resources* (Spring), pp. 177-196.
- Berndt, E. and Christensen, L. (1974), "Testing for the existence of a consistent aggregate index of labor inputs", *American Economic Review* (June), pp. 391-404.
- Berndt, E. and Khaled, M. (1979), "Parametric productivity measurement and choice among flexible functional forms", *Journal of Political Economy* (December), pp. 1220-1245.
- Borjas, G. (1983a), "The demographic determinants of the demand for black labor", Unpublished paper, University of California – Santa Barbara.
- Borjas, G. (1983b), "The substitutability of black, Hispanic and white labor", *Economic Inquiry* (January), pp. 93-106.
- Borjas, G. and Heckman, J. (1978), "Labor supply estimates for public policy evaluation", *Proceedings of the Industrial Relations Research Association*, pp. 320-331.
- Bowen, W. and Finegan, T.A. (1969), *The Economics of Labor Force Participation*, Princeton University Press.
- Brown, C., Gilroy, C. and Kohen, A. (1982), "The effect of the minimum wage on employment and unemployment", *Journal of Economic Literature* (June), pp. 487-528.
- Cain, G. and Watts, H. (1973), *Income Maintenance and Labor Supply*, Rand McNally.
- Denny, M. and Fuss, M. (1977), "The use of approximation analysis to test for separability and the existence of consistent aggregates", *American Economic Review* (June), pp. 404-18.
- Freeman, R. (1979). "The effect of demographic factors on age-earnings profiles", *Journal of Human Resources* (Summer), pp. 289-318.
- Freeman, R. and Wise, D., eds. (1982), *The Youth Labor Market Problem*, University of Chicago Press.
- Government of Australia, Bureau of Labour Market Research (1983), *Youth Wages, Employment and the Labor Force*, Australia Government Publishing Service.
- Grant, J. (1979), "Labor substitution in U.S. manufacturing", PhD. dissertation, Michigan State University.
- Grant, J. and Hamermesh, D. (1981), "Labor-market competition among youths, white women and others", *Review of Economics and Statistics* (August), pp. 354-360.
- Griliches, Z. (1969), "Capital-skill complementarity", *Review of Economics and Statistics* (November), pp. 465-68.
- Gustman, A. and Steinmeier, T. (1981), "The impact of wages and unemployment on youth enrolment and labor supply", *Review of Economics and Statistics* (November), pp. 553-560.
- Hall, R. (1973), "Wages, income and hours of work in the U.S. labor force", in Cain, G. and Watts, H., eds. *Income Maintenance and Labor Supply*, Rand McNally.
- Hamermesh, D. (1984), "The demand for labor in the long run", Working Paper No. 1297, National Bureau of Economic Research.

- Hamermesh, D. (1982), "Minimum wages and the demand for labor", *Economic Inquiry* (July), pp. 365-380.
- Killingsworth, M. (1981), "A survey of labor supply models: theoretical analysis and first-generation empirical results", *Research in Labor Economics*, pp. 1-52.
- Layard, R. (1982), "Youth unemployment in Britain and the United States compared", in Freeman R. and Wise, D., eds. *The Youth Labor Market Problem: Its Nature, Causes and Consequences*, University of Chicago Press.
- Layard, R. and Pissarides, C. (1982), "Youth wages and youth unemployment", Working Paper No. 393, Centre for Labour Economics, London School of Economics.
- Martin, J. (1983), 'Effects of the minimum wage on the youth labour market in North America and France', OECD.
- Merrilees, W. (1982), "Labour market segmentation in Canada: an econometric approach", *Canadian Journal of Economics* (August), pp. 458-473.
- Meyer, R. and Wise, D. (1983), "The effects of the minimum wage on the employment and earnings of youth", *Journal of Labor Economics* (January), pp. 6-100.
- Mincer, J. (1976), "Unemployment effects of minimum wages", *Journal of Political Economy* (August), pp. S87-S104.
- OECD (1980), *Youth Unemployment: The Causes and Consequences*, OECD.
- Sato, R. and Koizumi, T. (1973), "On the elasticities of substitution and complementarity", *Oxford Economic Papers* (March), pp. 44-56.
- Schultz, T.P. (1980), 'Estimating labor supply functions for married women', in Smith J. ed. *Female Labor Supply*, Princeton University Press.
- Shapiro, D. and Shaw, L. (1983), "Growth in the labor force attachment of married women: accounting for changes in the 1970s", *Southern Economic Journal* (October), pp. 461-473.
- Stapleton, D. and Young, D. (1984), "The effects of demographic change on the distribution of wages, 1967-1990", *Journal of Human Resources* (Spring), pp. 175-201.
- Swidinsky, R. (1980), "Minimum wages and teenage unemployment", *Canadian Journal of Economics* (February), pp. 158-171.
- Welch, F. (1979), "Effect of cohort size on earnings: the baby boom babies' financial bust", *Journal of Political Economy* (October), pp. 565-598.
- Welch, F. and Cunningham, J. (1978), "Effects of minimum wages on the level and age composition of youth employment", *Review of Economics and Statistics* (February), pp. 140-45.