Decomposition of aggregate labour productivity growth into industry contributions – Methodology applied in the OECD Compendium of Productivity Indicators 2023

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Introduction

This note presents a decomposition of aggregate labour productivity growth, defined as a ratio of value added and hours worked, into the effect of labour productivity growth at industry level (*within-industry effect)*, and reallocations of hours worked across industries with different labour productivity levels (*static reallocation effect)* or growth rates (*dynamic reallocation effect)*. The sum of the static and dynamic reallocation effects is referred to as the *overall reallocation effect*.

The main difficulty in this otherwise standard shift-share decomposition is related to the way inflation is controlled for. At the aggregate level, this can be done by compiling labour productivity as a ratio between value added in volume terms and hours worked. Nevertheless, all OECD countries except Mexico currently compile *chain-linked* volumes[1] of value added, which are not additive across industries.[2]

This note builds on the fact that all OECD countries except three compile Laspeyres chain-linked volumes, in which case volume growth between consecutive years can be calculated by combining value added at current prices and value added at previous-year prices, which are both additive across industries. This idea can be easily extended to the only OECD country still compiling fixed-year volumes (Mexico). For all these countries, the proposed decomposition is exact.

The only drawback of the proposed method is that it is not exact for Canada and the United States, which compile Fisher volume indices. In this case, contributions do not sum exactly to aggregate labour productivity growth, but the residual term is usually small, especially if labour productivity growth is averaged over several years.

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[1] In the following, the terms *chain-linked volumes* and *fixed-year volumes* are used. They are synonymous to *volumes at chain-linked prices* and *volumes at fixed-year prices*, respectively.

[2] The volume growth of macroeconomic aggregates is measured more accurately when reference prices are updated every year. This is the main reason for releasing volumes at chain-linked prices rather than volumes at fixed-year prices. The main drawback of doing so is that chain-linked volumes are not additive across industries. Mexico compiles volumes at fixed-year prices, and these measures are additive across industries.

Notations

Value-added and hours worked

$Y_{tot,t}$, $Y_{i,t}$: value added in year t, at current prices, at aggregate level (tot) and in industry i

$Y_{tot,t}^{PY}$, $Y_{i,t}^{PY}$: value added in year t, at previous-year (PY) prices, i.e. prices of year (t-1), at aggregate level (tot) and in industry i

$Y_{tot,t}^{FY}$, $Y_{i,t}^{FY}$: value added in year t, at fixed-year (FY) prices, at aggregate level and in industry I

$IVOL\_Y_{tot,t}$: volume index of aggregate value added in year t

$H_{tot,t}$, $H_{i,t}$: total hours worked in year t, at aggregate level and in industry i

$S_{i,t}$: share of hours worked in industry i in total hours worked, in year t

Labour productivity

$LP_{tot,t}$, $LP_{i,t}$: labour productivity per hour worked in year t, at aggregate level and in industry i, with value added measured at current prices of year t

$LP_{tot,t}^{PY}$, $LP_{i,t}^{PY}$: labour productivity per hour worked in year t, at aggregate level and in industry i, with value added measured at prices of year (t-1)

$LP_{tot,t}^{FY}$, $LP_{i,t}^{FY}$: labour productivity per hour worked in year t, at aggregate level and in industry i, with value added measured at fixed-year prices

Decomposition of aggregate labour productivity growth for countries compiling Laspeyres chain-linked volumes

For OECD countries compiling Laspeyres chain-linked volumes,[3] the volume index of value added in year t can be calculated as the ratio of value added in year t expressed at prices of year t-1, and value added at current prices in year t-1:

$$IVOL\_Y_{tot,t} = \frac{Y^{PY}_{tot,t}}{Y_{tot,t-1}} \qquad (\mathbf{1})$$

Chain-linked volume indices over longer periods are compiled by multiplying such indices over consecutive years. The decomposition in this note takes advantage of the fact that value added at current prices and value added at previous-year prices are both additive across industries, contrary to value added at chain-linked prices.

Aggregate labour productivity at current prices can be expressed as the sum of labour productivity in each industry, weighted by the industry's share in total hours worked. The same holds for aggregate labour productivity at previous-year prices:

$$LP^{PY}_{tot,t} \equiv \frac{Y^{PY}_{tot,t}}{H_{tot,t}} = \frac{\sum_i Y^{PY}_{i,t}}{\sum_i H_{i,t}} = \sum_i \underbrace{\left(\frac{H_{i,t}}{H_{tot,t}}\right)}_{\equiv S_{i,t}} \underbrace{\left(\frac{Y^{PY}_{i,t}}{H_{i,t}}\right)}_{\equiv LP^{PY}_{i,t}} = \sum_i S_{i,t} LP^{PY}_{i,t} \qquad (\mathbf{2})$$

(2) can be used to decompose the difference between aggregate labour productivity in year t expressed at previous-year prices, and aggregate labour productivity in year t-1 expressed at current prices:

$$\begin{aligned} LP^{PY}_{tot,t} - LP_{tot,t-1} &= \sum_i \left[S_{i,t} LP^{PY}_{i,t} - S_{i,t-1} LP_{i,t-1}\right] \\ &= \sum_i \left[S_{i,t-1}\left(LP^{PY}_{i,t} - LP_{i,t-1}\right) + \left(S_{i,t} - S_{i,t-1}\right) LP^{PY}_{i,t}\right] \\ &= \sum_i \left[S_{i,t-1}\left(LP^{PY}_{i,t} - LP_{i,t-1}\right) + \left(S_{i,t} - S_{i,t-1}\right) LP_{i,t-1} + \left(S_{i,t} - S_{i,t-1}\right)\left(LP^{PY}_{i,t} - LP_{i,t-1}\right)\right] \\ &= \sum_i \left[S_{i,t-1}\left(LP^{PY}_{i,t} - LP_{i,t-1}\right) + \left(S_{i,t} - S_{i,t-1}\right)\left(LP_{i,t-1} - LP_{tot,t-1}\right)\right. \\ &\qquad \left. + \left(S_{i,t} - S_{i,t-1}\right)\left(LP^{PY}_{i,t} - LP_{i,t-1}\right)\right] \qquad (\mathbf{3}) \end{aligned}$$

[3] All OECD countries except Canada and the United States (compiling Fisher chain-linked volumes) and Mexico (compiling fixed-year volumes)

(3) can then be used to decompose aggregate labour productivity growth in volume terms between years t-1 and t:

$$\frac{LP^{PY}_{tot,t} - LP_{tot,t-1}}{LP_{tot,t-1}} = \sum_i \left[S_{i,t-1} \frac{LP_{i,t-1}}{LP_{tot,t-1}} \left(\frac{LP^{PY}_{i,t} - LP_{i,t-1}}{LP_{i,t-1}} \right) + \left(S_{i,t} - S_{i,t-1} \right) \left(\frac{LP_{i,t-1}}{LP_{tot,t-1}} - 1 \right) + \left(S_{i,t} - S_{i,t-1} \right) \frac{LP_{i,t-1}}{LP_{tot,t-1}} \left(\frac{LP^{PY}_{i,t} - LP_{i,t-1}}{LP_{i,t-1}} \right) \right] \qquad (\mathbf{4})$$

Moreover:

$$\frac{Y_{i,t-1}}{Y_{tot,t-1}} = S_{i,t-1} \frac{LP_{i,t-1}}{LP_{tot,t-1}} = \frac{H_{i,t-1}}{H_{tot,t-1}} \frac{Y_{i,t-1}}{H_{i,t-1}} \frac{H_{tot,t-1}}{Y_{tot,t-1}} \qquad (\mathbf{5})$$

and

$$\frac{S_{i,t}}{S_{i,t-1}} \frac{Y_{i,t-1}}{Y_{tot,t-1}} = S_{i,t} \frac{LP_{i,t-1}}{LP_{tot,t-1}} = \frac{H_{i,t}}{H_{tot,t}} \frac{Y_{i,t-1}}{H_{i,t-1}} \frac{H_{tot,t-1}}{Y_{tot,t-1}} = \frac{H_{i,t}/H_{i,t-1}}{H_{tot,t}/H_{tot,t-1}} \frac{Y_{i,t-1}}{Y_{tot,t-1}}$$

$$\Rightarrow \left(S_{i,t} - S_{i,t-1} \right) \cdot \frac{LP_{i,t-1}}{LP_{tot,t-1}} = \left(\frac{S_{i,t}}{S_{i,t-1}} - 1 \right) \cdot \frac{Y_{i,t-1}}{Y_{tot,t-1}} \qquad (\mathbf{6})$$

Aggregate labour productivity growth can finally be decomposed as follows:

$$\underbrace{\frac{LP^{PY}_{tot,t} - LP_{tot,t-1}}{LP_{tot,t-1}}}_{\textbf{Aggregate LP growth}} = \sum_i \left[\underbrace{\frac{Y_{i,t-1}}{Y_{tot,t-1}} \left(\frac{LP^{PY}_{i,t} - LP_{i,t-1}}{LP_{i,t-1}} \right)}_{\textbf{Within-industry effect}} + \underbrace{\left(S_{i,t} - S_{i,t-1} \right) \left(\frac{LP_{i,t-1}}{LP_{tot,t-1}} - 1 \right)}_{\substack{\text{Reallocation across industries with different productivity levels} \\ \rightarrow \textbf{Static reallocation effect}}} + \underbrace{\left(\frac{S_{i,t}}{S_{i,t-1}} - 1 \right) \left(\frac{Y_{i,t-1}}{Y_{tot,t-1}} \right) \left(\frac{LP^{PY}_{i,t} - LP_{i,t-1}}{LP_{i,t-1}} \right)}_{\substack{\text{Reallocation across industries with different productivity growth rates} \\ \rightarrow \textbf{Dynamic reallocation effect}}} \right] \qquad (\mathbf{7})$$

This decomposition includes three main terms, each of them corresponding to a sum of industry contributions:

- *A within-industry effect*, where labour productivity growth in each industry is weighted by the industry share in total value added at current prices in year t-1.
- A *static reallocation effect*, accounting for changes between t-1 and t in the total hours worked share of industries with different productivity levels. Note that industries with an increasing share in total hours worked contribute positively to aggregate labour productivity growth only if they have an above-average labour productivity level (i.e. if $\frac{LP_{i,t-1}}{LP_{tot,t-1}} > 1$).
- A *dynamic reallocation effect*, accounting for changes between t-1 and t in the total hours worked share of industries with different productivity growth rates. An increase in the total hours worked share of industries with positive productivity growth has a positive effect on aggregate labour productivity growth. This effect is all the more significant that the industry value added is high.

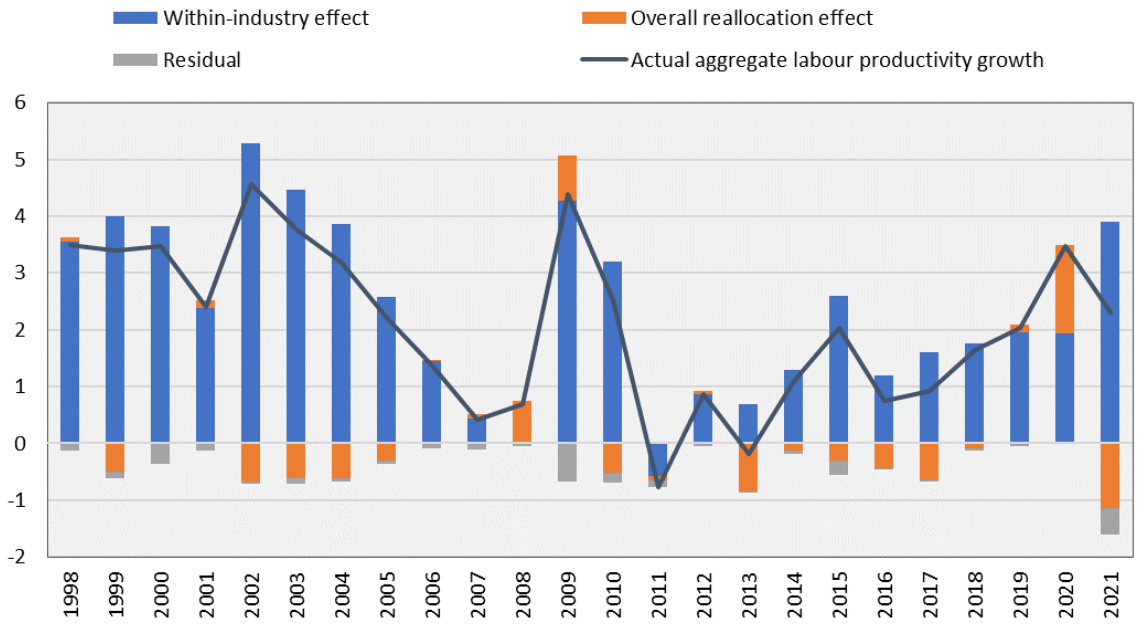
In most cases, the static reallocation effect largely dominates the dynamic reallocation effect. In this note and the in the OECD Compendium of Productivity Indicators, the sum of these two effects is referred to as *overall reallocation effect*.

Decomposition of aggregate labour productivity growth for countries compiling Fisher chain-linked volumes (Canada and the United States)

Canada and the United States are the only two OECD countries relying on Fisher chain-linked volume indices in their national accounts (Landefeld et al., 2003). Consequently, value added growth in volume terms cannot be expressed any more as a ratio of value added at current and previous year prices, nor as a ratio of measures that are additive across industries. In practice however, Fisher volume indices between consecutive years can be quite well approximated by Laspeyres volume indices. Therefore, the previous decomposition can be applied to countries compiling Fisher chain-linked volumes of value added, but it gives rise to a residual term. As shown in Figure 1 below, this residual is usually small, especially if labour productivity growth is averaged over several years.[4]

Figure 1. Decomposition of aggregate labour productivity growth for the United States

Percentage points



Source: OECD calculations based on data sourced from the US Bureau of Economic Analysis and the US Bureau of Labor Statistics, December 2022.

[4] It amounts to -0.1 percentage point on average between 1998 and 2021.

Decomposition of aggregate labour productivity growth for countries compiling fixed-year volumes (Mexico)

Mexico is the only OECD country compiling fixed-year, i.e. not chain-linked, volumes of value added in its national accounts. In this case, the growth rate of value added in volume terms between years t-k and t ($k \geq 1$) can be compiled as a ratio of fixed-year volumes in years t and t-k. Moreover, fixed-year volumes are additive across industries. Therefore, the above decomposition can be applied by simply replacing labour productivity at current and previous-year prices with labour productivity at fixed-year prices, and it is exact (no residual term).

$$\underbrace{\frac{LP^{FY}_{tot,t} - LP^{FY}_{tot,t-1}}{LP^{FY}_{tot,t-1}}}_{\textbf{Aggregate LP growth}} = \sum_i \left[\underbrace{\frac{Y^{FY}_{i,t-1}}{Y^{FY}_{tot,t-1}} \left(\frac{LP^{FY}_{i,t} - LP^{FY}_{i,t-1}}{LP^{FY}_{i,t-1}} \right)}_{\textbf{Within-industry effect}} + \underbrace{\left(S_{i,t} - S_{i,t-1}\right) \left(\frac{LP^{FY}_{i,t-1}}{LP^{FY}_{tot,t-1}} - 1 \right)}_{\substack{\text{Reallocation across industries with different productivity levels} \\ \rightarrow \textbf{Static reallocation effect}}} \right.$$

$$\left. + \underbrace{\left(\frac{S_{i,t}}{S_{i,t-1}} - 1 \right) \left(\frac{Y^{FY}_{i,t-1}}{Y^{FY}_{tot,t-1}} \right) \left(\frac{LP^{FY}_{i,t} - LP^{FY}_{i,t-1}}{LP^{FY}_{i,t-1}} \right)}_{\substack{\text{Reallocation across industries with different productivity growth rates} \\ \rightarrow \textbf{Dynamic reallocation effect}}} \right] \qquad (8)$$

Relation to other decompositions proposed in the literature

Similarly to the decomposition proposed in this note, other decompositions in the literature break down aggregate labour productivity growth into within-industry and reallocation effects. The main differences between these decompositions relate to how inflation is controlled for.

- The decomposition that was recently applied to EU countries by the European Central Bank (ECB, 2021) relies on chain-linked volumes of value added and disregards the fact that these measures are non-additive across industries. This leads to a residual term in the decomposition. Moreover, all industries with an increasing share in total hours worked contribute positively to aggregate labour productivity growth in the ECB decomposition. By contrast, the decomposition presented in this note ensures that only industries with an above-average productivity level contribute positively to aggregate labour productivity growth when their share in total hours worked increases.
- The decomposition proposed by Tang and Wang (2004) deflates value added at both aggregate and industry levels using the aggregate value-added price deflator. This method ensures that no distinction needs to be done between countries compiling fixed-

year volumes, and Laspeyres or Fisher chain-linked volumes. Its main drawback is that the industry weights used to compute the reallocation effects depend on changes in both industry shares in total hours worked and relative industry prices (i.e. industry value added prices compared to aggregate value-added prices). This makes the interpretation of the reallocation effects more difficult than in the decomposition presented in this note.

- The decomposition proposed by Diewert (2015) builds on the one proposed by Tang and Wang (2004) and separates reallocation effects between those related to changes in industry shares in total hours worked and those related to changes in relative industry prices. Its main drawback is that the reallocation effects related to relative industry prices are difficult to interpret.

References

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