The Measurement of Business Capital, Income and Performance

by

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1.0   Introduction

"In economics it is difficult to prove originality; for the germ of every new idea will surely be found over and over again in earlier writers".

Irving Fisher[1930; ix]

"Capital (I am not the first to discover) is a very large subject, with many aspects; wherever one starts, it is hard to bring more than a few of them into view. It is just as if one were making pictures of a building; though it is the same building, it looks quite different from different angles".   *John Hicks*[1973; v]

The main topic to be addressed in this paper is how to measure the contribution of capital to periodic business income.

As many economists and accountants have observed over the past century, economic measurement cannot be divorced from the purpose at hand.[1] The main purpose of this paper is to answer the following question: how should capital input be measured in the context of evaluating business performance over a number of accounting periods?

The fundamental problem associated with measuring the contribution of a capital input to the period by period economic performance of a business unit is the *durability* of capital: a capital input is purchased in an initial accounting period but its contribution to the production of outputs persists over several subsequent periods. Thus the initial purchase cost of the capital input cannot be *entirely* allocated to the period of purchase but it is difficult to know precisely how the initial cost should be allocated over subsequent periods. This problem of determining the period by period contributions to production and the associated costs is perhaps the fundamental problem in accounting theory.[2] The difficulties associated with this fundamental allocation problem are greatly magnified if the price level is not stable. In this paper, we will *not* assume stability of prices.

Once the initial purchase cost of a durable capital input has been allocated across accounting periods, period costs can be subtracted from period revenues and accounting period income or profits can be calculated. Thus the measurement of capital goes hand in hand with the measurement of business income: different measures for the period by period cost of capital will give rise to different income measures.[3]

However, there are still some additional problems to be addressed, associated with timing problems; i.e., exactly when should a revenue or cost item be recognized by the business unit? The following quotation from the accounting literature summarizes these timing and recognition problems:

"In the earlier, simpler days of accounting, the convention called for realization in actual cash, which was later modified by the assumption that legal transfer of title constituted realization, the resulting account receivable merely representing a deferred change to cash, while still later the mere act of shipment and billing was assumed to be equivalent to the legal transfer of title".

Stephen Gilman[1939; 212]

The questions raised by the above quotation cannot be answered unless we have a framework or model in mind in order to classify and evaluate these problems.[4] The framework that we will use to help us answer our measurement questions is that of production theory: we view the business unit as a black box that transforms inputs used during an accounting period into outputs that were produced during the period. This is the point of view that is adopted in Social or National Income Accounting.[5] However, it must be kept in mind that most accountants think that sale of commodities is more important than production and that realization of revenue (through sale) is more fundamental than any imputed accrual of revenues that might occur through production without sale.[6] We will develop our production theory framework in more detail in section 5 below but for now, we note that the main ideas are due to the economist Hicks [1946; 191-201 and 325-326] and the accountants Edwards and Bell [1961; 71-72]. Hicks[7] realized the usefulness of his intertemporal production theory framework in being able to cast some light on a wide variety of economic phenomena; we shall make use of his intertemporal framework in sections 5, 10 and 11 below.

Once the revenues and costs which pertain to the outputs produced and the inputs utilized by a production unit during an accounting period have been determined, we can address the problem of evaluating the performance or efficiency of the unit. This can be done in at least three ways: (i) the net income or profits (the difference between revenues and costs) of the unit in one period can be compared to the profits of other units or the profits of the same unit in a previous period; (ii) the rate of return on assets employed (equal to net income divided by the value of assets employed by the unit at the beginning of the accounting period) can be compared to rates of return on similar business units or the same unit in a prior period, and (iii) the productivity change of the business unit going from one period to another can be computed.

If there were only one output produced and one input utilized by a production unit, the ex post evaluation of the performance of one period (say period 1) with another period (say period 0) would be very straightforward from the viewpoint of the productivity criterion. Let y^0 and y^1 be the physical amounts of output produced during periods 0 and 1 and let x^0 and x^1 denote the physical amounts of inputs used during the two periods by the production unit. Then the productivity change going from period 0 to 1 can be defined as the rate of growth of output y^1/y^0 divided by the rate of growth of input x^1/x^0,

or alternatively, as the ratio of the period 1 output-input coefficient y^1/x^1 to the period 0 output-input coefficient y^0/x^0; i.e., we have

(1)   Productivity $\equiv [y^1/y^0]/[x^1/x^0] = [y^1/x^1]/[y^0/x^0]$.

If the productivity number defined by (1) is greater than unity, then we say that the production unit exhibited a productivity improvement or that the production unit was more efficient in period 1 than in period 0.

In the case of many inputs and many outputs, the definition of productivity change or efficiency improvement is much more difficult and controversial. One can approach the problem of measuring productivity in the multiple input, multiple output context in at least three ways: (i) use output and input price information to weight the individual output and input growth rates (i.e., use index number techniques); (ii) use econometric techniques to estimate production or cost functions or (iii) use information on output and input quantities to construct nonparametric estimates of production unit efficiency along the lines pioneered by Farrell [1957] and Afriat [1972]. We will not utilize approaches (ii) and (iii) to efficiency measurement in the present paper[8] but the index number approach (i) will be discussed briefly in section 12 below and used in section 13.

We summarize the above discussion as follows: we want to measure the cost of capital inputs in an inflationary environment in the context of determining the period by period costs and revenues that pertain to a business unit. Once these periodic costs and revenues have been determined, periodic business income (or profits) can be calculated along with other measures of business performance.

Once we have considered the problems involved in defining periodic business income, it is natural to consider the implications of taxing alternative definitions of business income. Thus in section 11 below, we shall devote some attention to the problem of determining an appropriate base for taxing business income.

Edwards and Bell [1961; 271] noted that accounting measurement has three main functions: (i) to prevent fraud and theft; (ii) to evaluate business performance[9] and (iii) to provide a sound and equitable basis for taxation. Carsberg [1982; 64] added a few more functions: (iv) the measurement of current income may help investors estimate future cash flows from the enterprise and (v) accounting income is an indicator of the amount that can be distributed to owners without impairing the long run viability of the enterprise.[10] It can be seen that our examination of the problems involved in the measurement of capital has led us to consider some of the same measurement problems that have been studied in the accounting literature–namely items (ii) and (iii) in the above list.[11]

We turn now to a brief description of the contents of each section in the present paper. In section 2, we develop the idea of the ex post cost of a durable input. This is the fundamental concept which allows us to allocate the cost of a durable input across

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the accounting periods during which it is used in production. An interest rate plays a key role in the definition of user cost. Some accountants maintain that interest (in particular imputed interest on equity capital) is not a cost of production, so in section 3, we consider whether interest is a "valid" cost or not. Depreciation rates also play a role in user cost formulae. Hence in section 4, we consider the problems involved in empirically determining these depreciation rates.

In section 5 below, we examine more closely the production function model that we are using to frame our analysis of capital measurement problems. We shall distinguish the Hicksian multiperiod intertemporal production function, the single period Austrian production function, the single period reduced form "traditional" production function and the interrelationships among these functions. We will also discuss the fundamental question of what exactly are admissible productive activities and the implications of answers to this question for capital measurement.

Sections 6 and 7 deal with the core controversies in the capital measurement literature. Section 6 looks at alternative methods of asset valuation; i.e., alternative methods for determining the period by period value of a durable capital input that is held by a business enterprise over multiple accounting periods. Section 7 then looks at the implications of alternative asset valuation methods for the determination of the business unit's period by period income.

Section 8 deals with a question which is rarely discussed in the measurement literature: what is the ideal length of the accounting period? It turns out that the amount of inflation that the economy is experiencing impacts on the answer to this question. In order to calculate productivity measures for a business unit, it is necessary to decompose periodic value flows pertaining to the same commodity (i.e., sales or purchases of a homogeneous commodity made over the accounting period) into price and quantity components. Exactly how this decomposition should be accomplished is also discussed in section 8.

Section 9 focuses on the measurement problems that arise when we attempt to adapt our basic user cost framework to the treatment of inventories. The analysis here draws heavily on Diewert and Smith [1994].

Section 10 attempts to demonstrate the following proposition: with a proper treatment of interest, *any* method of depreciation (including immediately expensing the purchase of a durable capital input) will lead to the same discounted stream of period by period profits for a business unit. However, this proposition does not mean that the method of depreciation is unimportant: *different* depreciation methods will lead to *different* estimates of short run periodic profits and hence to *different* estimates of the short run efficiency of the business enterprise.

In section 11, we consider some of the problems posed by the existence of business income taxes. Are these taxes costs or distributions of financial capital to the government?

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If they are costs, can they be allocated to specific commodities? We also draw on the results of section 10 to analyze the economy wide efficiency and neutrality aspects of alternative business income concepts as bases for income taxation. The distortions generated by basing business income taxes on a historical cost income concept in a period of high inflation have been noted by accountants for at least 75 years.[12]

Section 12 discusses various approaches to the measurement of business efficiency over two or more accounting periods (or comparable business units). In particular, index number methods for making productivity comparisons are discussed.

Section 13 draws on sections 5 and 12 in order to discuss alternative index number methods for aggregating capital. In particular, the merits of gross versus net capital stock concepts will be discussed.

Traditional bilateral index number theory is based on the implicit assumption that the set of commodities to be aggregated is the *same* in the two periods under consideration. The existence of new capital goods creates problems for this traditional paradigm; we address these problems in section 14.

Up to this point, we have been concerned only with the problems involved in accounting for durable capital inputs. In section 15, we present a brief discussion on how our user cost framework can be extended to deal with the problems involved in accounting for financial assets.

The problems involved in accounting for capital in a regulatory context are more severe than in the unregulated context because we cannot rely on the assumption that the "market" will be able to see through any distortions of the firm's financial position that might be generated by the conventions of historical cost accounting. In the context of traditional regulation, the regulator forces the revenues of the regulated business unit to equal its costs. If historical cost accounting is used in an inflationary environment, costs will be understated and the long run viability of the regulated firm will be impaired. Section 16 discusses some of these problems of accounting in a regulatory environment.

As the reader progresses through the various sections of this paper, it will become apparent that historical cost accounting is a very flawed instrument in the context of evaluating firm performance and of providing an equitable base for income taxation, particularly in an inflationary environment. Even though accountants have known about these problems associated with the use of historical cost for a long time, historical cost accounting persists in virtually all countries to the present day. In section 17, we attempt to answer the question: why does historical cost accounting persist?

Section 18 concludes.

I conclude this section with some personal observations. As will become apparent in reading this paper, virtually all of the ideas presented here have their roots in rather ancient academic literature. However, I hope that the present paper may be useful to accountants,

economists, engineers and management scientists who may not be aware of contributions to the capital measurement literature which have been made by other disciplines. Also I hope that the organization of the paper and the framing of the issues will help the reader absorb the main points of a rather vast literature in a relatively painless manner. In particular, section 7 may help the reader grasp the essence of alternative income concepts as special cases of a general valuation framework. The reader will also note that I have attempted to trace the origins of ideas as far back as limited time resources would allow. Undoubtedly, many ideas could be traced back even further but I hope that the detailed references to the literature will be helpful to readers who want more background information or alternative treatments of the topic under consideration. Finally, this paper can be viewed as a sequel to my first survey of capital measurement problems, Diewert [1980b]. There is nothing terribly wrong about this earlier paper (as far as I can see), but I now realize (as Hicks [1973; v] did later in life) that there are many aspects to capital measurement and my earlier survey did not address many of the topics considered in the present paper.

2.0   The Ex post User Cost of a Durable Input

"The machine should be sold if its expected quasi-rent for the coming period is less than the interest on disposal value now plus the expected decline in disposal value over the period".   *Edgar O. Edwards and Philip W. Bell*[1961; 172]

"In a perfectly competitive world the annual rent of a machine would equal the marginal product of its services. The rent itself would be determined by the interest costs on the investment, the deterioration in the future productivity of the machine due to current use, and the expected change in the price of the machine (obsolescence)".   *Zvi Griliches*[1963; 120]

In order to measure the contribution of a capital stock component to the production of current period outputs, it is necessary to recognize that a capital input is not like other nondurable inputs for at least two reasons: (i) a capital input is *durable* and hence by definition, it is not completely used up in the period of its purchase and hence (ii) the purchase price of a capital input should not be entirely allocated to current period costs but instead it should be allocated over all periods from the period of purchase to the period when the capital good is scrapped or sold. How should this cost allocation over the useful life of the capital input be accomplished? We shall give a preliminary answer to this question in this section but we shall revisit this problem of intertemporal cost allocation in subsequent sections.

To provide a preliminary answer to the cost allocation problem, we specialize the dynamic production theory framework of Hicks [1946; 193-4] to work out the *ex post user cost* for a durable input. Consider a durable input that can be purchased at the beginning of period 0 at the price P^0. After using the input during period 0 the producer could sell the depreciated or "used" capital input at the beginning of period 1 price P_u^1. Assume that the producer's opportunity cost of financial capital at the beginning of period 0 is r^0. Then the present value of the net cost of buying one unit of the durable input, using it for one accounting period and selling it at the end of period 0 is the following *ex post user cost*:[13]

$$w^0 \equiv P^0 - P_u^1/(1+r^0)$$
$$(2) \qquad = [r^0 P^0 + (P^0 - P_u^1)]/(1+r^0).$$

The first term on the right hand side of (2) is an interest cost term while the second term combines the effects of depreciation and capital gains (or losses) on the sale of the asset. These effects can be separated as follows. Let P^1 be the beginning of period 1 price for one unit of a capital input which is physically equivalent to the capital input which was purchased at the beginning of period 0. We can now use the end of period 0 "new" good price P^1 and the end of period 0 "used" good price P_u^1 to define the end of period 0 *economic depreciation rate* δ^0 as follows:[14]

$$(3) \quad 1 - \delta^0 \equiv (P_u^1/P^1).$$

The logic behind definition (3) is that P^1 represents the price of a "new" capital input at the end of period 0 while P_u^1 represents the end of period 0 price of the same capital input that has been used for one additional accounting period. Thus $(1 - \delta^0) = P_u^1/P^1$ is the ratio of "used" to "new" value evaluated at the end of period 0. Replacing P_u^1 in (2) by $(1 - \delta^0)P^1$ leads to the following user cost formula:

$$(4) \quad w^0 = [r^0 P^0 + \delta^0 P^1 - (P^1 - P^0)]/(1+r^0).$$

The three terms in the numerator of (4) are the usual components that appear in user cost formulae: the first term $r^0 P^0$ reflects the interest opportunity cost of using financial capital to purchase and hold the input over the accounting period; the second term $\delta^0 P^1$ is a depreciation term that reflects the fact that the input has been utilized during period 0 and hence its useful life has been reduced by one period and the final term, $-(P^1 - P^0)$, is the negative of the capital gains made by a new unit of the asset over period 0.

The ex post user cost formula (4) can be further simplified if we define the "new" asset inflation rate i^0 over period 0 by

$$(5) \quad 1 + i^0 \equiv P^1/P^0$$

and substitute (5) into (4) in order to obtain the following ex post user cost formula:

$$(6) \quad w^0 = [r^0 - i^0 + \delta^0(1 + i^0)]P^0/(1 + r^0).$$

Note that $r^0 - i^0$ is an asset specific real interest rate.

What are the informational requirements for the calculation of the ex post user cost defined by (6)? We require time series for: (i) new asset prices P^t; (ii) the opportunity cost of capital r^t and (iii) economic depreciation rates δ^t.

The user cost formulae (2), (4) and (6) discounted the end of period 0 price of the asset back to the beginning of period 0. Thus these user costs can be interpreted as beginning of the accounting period prices. For accounting purposes, it is often more convenient to work with end of the period prices.[15] To obtain end of period user costs, we need only multiply the w^0 defined by (2), (4) or (6) by $(1 + r^0)$. Thus the *end of period 0 ex post user cost* counterparts to (2), (4) or (6) are:

$$(7) \quad w_e^0 \equiv r^0 P^0 + P^0 - P_u^1$$
$$(8) \quad = r^0 P^0 + \delta^0 P^1 - (P^1 - P^0)$$
$$(9) \quad = [r^0 - i^0 + \delta^0(1 + i^0)]P^0.$$

User cost formulae of the type (9) (with i^0 set equal to zero) date back to Walras [1874; translated 1954; 269]. Other contributors to the user cost literature who derived more complex user cost formulae with income tax complications include Jorgenson [1963] and Hall and Jorgenson [1967] (in a continuous time framework) and Christensen and Jorgenson [1969] (in a discrete time framework).[16]

To see how (7) could be justified directly from an accounting perspective, suppose that the production unit purchases one unit of the asset at the beginning of period 0 at the price P^0 and at the end of period 0 (or equivalently, at the beginning of period 1) the asset is worth[17] P_u^1. Thus the net cost of using the asset over the period would appear to be $P^0 - P_u^1$. However, this net cost neglects interest cost considerations. Suppose that the production unit can issue debt at the beginning of period 0 at the interest rate r_d^0 and can raise equity capital at the beginning of period 0 cost of capital r_e^0. Suppose further that the production unit finances the fraction f^0 of its beginning of period 0 purchase of the asset by issuing debt and the fraction $1 - f^0$ by issuing equity. Then its average cost of capital (including the opportunity cost of equity capital) will be:

$$(10) \quad r^0 \equiv f^0 r_d^0 + (1 - f^0) r_e^0.$$

Thus the net cost of using the asset over period 0, including interest and the opportunity cost of equity capital, is $P^0 - P_u^1 + r^0 P^0$, which is (7).

Should we use the beginning of the period user cost formulae (2), (4) or (6) or the end of period formulae (7) - (9) in order to determine the period 0 cost of a durable capital input which is available for use at the beginning of the period? Now period 0 capital costs are going to be determined as the product of user costs times the number of units of each capital stock component which is available at the beginning of the period. These period 0 capital costs will be added to other period 0 flow costs (such as labour, fuel and telecommunications costs) in order to determine total period 0 costs. Thus if we use the user cost formulae (2), (4) or (6), we are implicitly assuming that all period 0 flow costs are equivalent to beginning of the period values or equivalently, that all of these flow costs are "realized" at the beginning of the period. On the other hand, if we use formulae (7) - (9), we are implicitly assuming that all period 0 flow costs are "realized" at the end of period 0 (which seems more sensible). In fact, it may be more accurate to assume that flow costs are "realized" on average at the midpoint of the accounting period and to provide an interest rate adjustment to these flow costs to make them comparable to either the beginning or end of the period user costs. However, for most practical purposes, it will not be worth the trouble of carrying out this extra discounting adjustment. In view of the fact that the treatment of interest as a cost is much more straightforward when we use the user cost formula (7), we recommend that the user cost formulae (7) - (9) be used together with the usual period 0 costs for nondurable inputs.[18]

In the following section, we briefly consider the question of whether interest really is a cost of production.

3.0   Interest as a Cost of Production

"The stock which is lent at interest is always considered as a capital by the lender. He expects that in due time it is to be restored to him, and that in the mean time the borrower is to pay him a certain annual rent for the use of it". *Adam Smith*[1776; reprinted 1963; 271]

"And human nature being what it is, we are justified in speaking of the interest on capital as the reward of the sacrifice involved in the waiting for the enjoyment of material resources, because few people would save much without reward; just as we speak of wages as the reward of labour, because few people would work hard without reward". *Alfred Marshall*[1920; 232]

It is clear that economists from Adam Smith through Alfred Marshall regarded interest as a reward (to the lender of financial capital for abstaining from or deferring consumption and as a cost of production for producers who are recipients of the financial capital.[19]

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However, the problem of explaining the factors that determined the interest rate was a much more difficult task. Böhm-Bawerk [1888; translated 1891; 24-72] summarized the literature on this topic up to his time and provided a verbal description of a modern theory of interest[20] while Fisher [1930] presented a very convincing algebraic and geometric description of the same theory.[21] Böhm-Bawerk [1891; 285-6], Fisher [1897; 522] and Hicks [1946; 141-2] explained how the present price of a good purchased now for delivery next period is equal to the (spot) price of the good next period divided by one plus the current period interest rate. Hicks [1946; 136] generalized the simple one (spot) commodity and multiple time period models of Fisher [1930] into a general model with many commodities and many time periods (his "Futures Economy" where all commodities can be bought and sold on forward markets) and Debreu [1959] provided a rigorous proof of the existence of equilibrium in such a model. But Hicks [1946; 119-127] also developed another model of intertemporal equilibrium that had a theory of the interest rate built into it– the temporary equilibrium model.[22] This second Hicksian model used the same building blocks as the futures economy model, except that instead of assuming the existence of futures markets, Hicks assumed the existence of current period (spot) markets for commodities and financial capital and the existence of definite expectations about future period spot prices (which could depend on current period prices) for all consumers and producers in the economy. In this model, these expected future period spot prices were used by producers and consumers in their intertemporal profit maximization and utility maximization plans.

The above sketch of the role of interest in modern economic theory would seem to indicate that economists generally accept interest as a valid cost of production, and indeed, interest plays a vital role in the intertemporal allocation of resources. However, many accountants and some economists object to interest as a cost of production; in particular, they object to interest that is imputed to the equity capital employed by a business unit.

A few accountants objected to associating an interest cost with the use of a durable input over an accounting period (in addition to a depreciation cost) on the grounds that such interest rate adjustments are likely to be minor in view of the errors involved in estimating depreciation.[23] However, if interest rates are high and the durable input is long lived, other accountants have pointed out that the neglect of interest can lead to substantial underestimation of costs.[24] On the other hand, the main objection of accountants[25] and some economists[26] to the inclusion of interest on equity capital as a cost is that it is an imputation or estimated value and accountants should stick to recording values rather than creating them. These objectors have a valid point: it is not a trivial matter to determine precisely what are the relevant debt and equity interest rates, r_d^0 and r_e^0, which appeared in (10) above.

Let us consider the problem of estimating the period 0 debt interest rate r_d^0 first. Suppose that we can determine the beginning of period 0 market values for all of the debt obligations of the enterprise under consideration, say B_j^0 for $j = 1, \ldots, J$ where B_j^0 is the beginning of period 0 market value of the jth bond. Suppose also that we can determine the interest payments b_j^0 that can be associated with the jth bond for period 0 for $j = 1, \ldots, J$. The effective period 0 interest rate r_j^0 for the jth debt instrument is defined as:

(11) $\quad r_j^0 \equiv b_j^0 / B_j^0, j = 1, \ldots J.$

Define the total market value of the beginning of period 0 debt as[27]

(12) $\quad B^0 \equiv \Sigma_{j=1}^J B_j^0.$

The overall ex ante period 0 debt interest rate could now be defined as

(13) $\quad r_d^0 \equiv \Sigma_{j=1}^J B_j^0 r_j^0 / B^0 = \Sigma_{j=1}^J b_j^0 / B^0.$

The first equation in (13) indicates that r_d^0 can be regarded as a weighted average of the individual bond effective interest rates r_j^0 while the second equation indicates that the overall debt interest rate is equal to total period 0 interest payments $\Sigma_{j=1}^J b_j^0$ divided by the initial market value of the debt B^0.

If all of the debt instruments were one period bonds, (13) would be the end of the story. However, if some of the bonds had a period to maturity greater than the length of period 0 (the usual situation!), then the above analysis is unsatisfactory since it ignores possible changes in the market value of the bonds as we go from the beginning of period 0 to its end. To deal with this complication, define B_j^1 as the market value of the jth bond at the end of period 0.[28] Any appreciation in value of the jth bond over period 0, $B_j^1 - B_j^0$, should be added to the end of period 0 interest payments b_j^0 to get the total cost that can be imputed to the jth debt instrument over period 0. Thus the end of period 0 effective interest rate for the jth bond (including accrued capital costs) is

(14) $\quad \bar{r}_j^0 \equiv [b_j^0 + (B_j^1 - B_j^0)] / B_j^0; j = 1, \ldots, J.$

If market interest rates increase over period 0 (due to say unanticipated general inflation), then B_j^1 will be less than B_j^0 and the terms $B_j^1 - B_j^0$ in (14) will be negative and hence will act to reduce effective interest rates. Now define the overall ex post[29] debt interest rate as the following weighted average of the individual $\bar{r}_j^0$ defined by (14):

(15) $\quad \bar{r}_d^0 \equiv \Sigma_{j=1}^J B_j^0 \bar{r}_j^0 / B^0$

(16) $\quad\quad = \Sigma_{j=1}^J [b_j^0 + (B_j^1 - B_j^0)] / B^0$

where (16) follows from (15) and (14). The $\bar{r}_d^0$ defined by (15) could be used as the debt interest rate r_d^0 which appears in (10) above.[30] Thus if beginning and end of period market values for the firm's debt instruments are available, the problem of defining an overall effective ex post debt interest rate seems straightforward.

However, the problems involved in estimating the firm's period 0 opportunity cost of equity capital, the r_e^0 which appears in (10), are far from straightforward. To conclude this section, we shall outline six possible approaches to the determination of r_e^0.

Approach 1:   Discounted Cash Flow

Suppose that a company's current period 0 dividends D^0 are expected to grow at the constant real rate $\tilde{g}$ for the indefinite future and that the expected inflation rate for the indefinite future is $\tilde{i}$. The company's current share price S^0 should equal the discounted future expected dividends. The discount rate should be the long run cost of equity capital r_e^0 minus the anticipated inflation rate $\tilde{i}$. Under these assumptions, we should have the following relationship between the company's current share price S^0 and current dividend rate D^0:

$$(17) \quad S^0 = D^0/[\tilde{r}_e^0 - \tilde{i} - \tilde{g}].$$

Formula (17) can be rearranged to give the following formula for the cost of capital:

$$(18) \quad r_e^0 = [D^0/S^0] + \tilde{i} + \tilde{g}.$$

This method for determining the opportunity cost of equity capital is due to Williams [1938] and Gordon and Shapiro [1956]. According to Myers [1992; 489], this method is widely used to determine allowed equity rates of return for regulated utilities in the United States.

There are many problems with this method. The determination of the anticipated future inflation rate $\tilde{i}$ will be problematical given that past inflation rates have been very variable during the past two decades. Dividend growth rates are also variable over the business cycle. Finally, dividend price ratios of the form D^0/S^0 are also tremendously variable and moreover, this method is not suitable for the determination of an economy wide equity cost of capital since many businesses are not incorporated and many incorporated businesses do not have publicly traded shares.

Approach 2:   The Capital Asset Pricing Model

Under certain assumptions, the capital asset pricing model of Sharpe [1964], Lintner [1965] and Mossin [1966] yields the following relationship between the expected cost of capital for a company $\tilde{r}_e^0$, a safe or riskless interest rate r_s^0 and the expected return on a market portfolio of assets $\tilde{r}_m^0$:

(19) $\tilde{r}_e^0 = r_s^0 + \beta[\tilde{r}_m^0 - r_s^0]$

where β is the covariance between the company's equity rate of return and the market portfolio rate of return divided by the variance of the market portfolio rate of return. Given a time series of ex post company rates of return r_e^t, market rates of return r_m^t and the safe rate of return r_s^t, ex post returns can be substituted into (19) in place of anticipated returns and β can be estimated in a regression model.[31] Alternatively, an estimate for β can be constructed by taking an average of past covariances $Cov(r_e^t, r_m^t)$ divided by $Var(r_m^t)$. Given this estimator for β, an ex ante $\tilde{r}_e^0$ can be calculated as the right hand side of (19) where $\tilde{r}_m^0$ is a forecast for the period 0 market rate of return.

Some of the assumptions that are required to derive (19) are: (i) each investor is a von Neuman and Morgenstern expected utility maximizer with the same preferences over current period consumption and end of the period wealth; (ii) a riskless one period asset actually exists; (iii) all investors have preferences over the same set of risky assets and the common riskless asset; (iv) all investors have the same expectations about the returns, variances and covariances of the risky assets and (v) there are no transactions costs. All of these assumptions are somewhat suspect from the empirical point of view. Machina [1992; 860-862] documents some of the empirical evidence which contradicts the expected utility model. In particular, Mehra and Prescott [1985] show that the equity premium over the safe asset seems to be too large for generally agreed upon values of relative risk aversion. Epstein and Zinn [1990] explain this equity premium by a generalization of the usual expected utility model that allows for first order risk aversion.[32] Assumption (ii), the assumption that a perfectly safe one period asset exists, is also problematical: nominal government bonds are not risk free due to inflation risk.[33] Assumption (iii) is also dubious: what is the relevant set of risky assets facing any investor? Should we include housing or foreign stock markets? Our β estimates will generally change as we change our definition of the "market" for risky assets. Assumption (iv) is also problematical: what will happen to our estimate for β as we include or exclude data for 1987–the year of the great worldwide stock market crash? Finally, assumption (v) is also far from being satisfied.

Although the capital asset pricing model could be used to estimate the cost of equity capital for some companies whose shares are traded in a stock market, it cannot be used

to estimate the cost of equity capital for many companies and for the economy as a whole since a large proportion of private sector companies are not listed on any stock exchange.

Approach 3:   The Ex Post Return Method

In this approach, the company's ex post operating income for the period under consideration is equated to the sum of ex post user costs of the type defined by (9) where each user cost is weighted by the quantity of capital used. The resulting equation is solved for r_e^0, the ex post return to equity capital. Suppose that there are J types of capital in use during period 0. Denote the beginning of period 0 price of the jth type of capital by P^{0j}, denote the jth depreciation rate by δ^{0j} and denote the jth asset appreciation rate in period 0 by i^{0j} for $j = 1, \ldots, J$. Finally denote the quantity of the jth type capital used during period 0 by k^{0j} for $j = 1, \ldots, J$. The ex post rate of return on equity capital r_e^0 can then be determined as the solution to the following linear equation where OI^0 represents period 0 operating income:

(20) $\quad OI^0 = \Sigma_{j=1}^{J}[f^0 r_d^0 + (1 - f^0)r_e^0 - i^{0j} + \delta^{0j}(1 + i^{0j})]P^{0j}k^{0j}.$

Formula (20) can be simplified if we replace the weighted sum of debt and the equity cost of capital, $f^0 r_d^0 + (1 - f^0)r_e^0$, by a single overall ex post cost of capital r^0. Thus r^0 can be determined by solving the following equation:

(21) $\quad OI^0 = \Sigma_{j=1}^{J}[r^0 - i^{0j} + \delta^{0j}(1 + i^{0j})]P^{0j}k^{0j}.$

The ex post cost of capital method for determining the opportunity cost of capital that is based on solving equation (21) for r^0 is due to Christensen and Jorgenson [1969] and has recently been advocated in the regulatory context by Christensen, Schoech and Meitzen [1995]. In addition to the simplicity of this method, Christensen, Schoech and Meitzen [1995; 10] note that this method can be applied in a symmetric manner to both a single enterprise as well as to the economy as a whole.

The problem with the ex post return method for estimating r^0 is that it does *not* correspond to a true opportunity cost of capital for the business unit; instead, it corresponds to an ex post measure of period 0 *performance* for the business unit.

Approach 4:   The Weighted Average of Past Ex Post Returns

Instead of using the ex post returns to capital method outlined in Approach 3 above, we could switch to a forecasting framework, using a weighted average of past ex post returns to capital to forecast a current period opportunity cost of capital.

The problem with methods of this type is their arbitrariness: which ex post approach to the determination of the opportunity cost of capital should be used? Which forecasting method should be used? How far back in time should we go? It will be difficult to reach agreement on what is the most reasonable specific method in this general class of methods. Moreover, as we argued at the end of Approach 3 above, ex post returns incorporate pure profits (or losses) and hence are not true opportunity costs for equity capital.

Approach 5:   The Use of An Exogenous Market Interest Rate

In this method, a relevant market interest rate is used as a proxy for the equity opportunity cost of capital. This market interest rate could be: (i) the prime lending rate that banks or other financial institutions charge borrowers in "similar" lines of business; (ii) the business unit's period 0 ex ante average interest rate defined by (13) or (iii) an index of ex ante interest rates of the form (13) where the B_j^0 now refer to market capitalizations of beginning of period 0 debt instruments for business units j that are "similar" to the business unit being studied.[34] As an example of (iii), Christensen, Schoech and Meitzen [1994] used the Moody's public utility bond as a proxy for the cost of capital for a regulated utility. These authors noted that this method has the advantages that the Moody bond yield is publicly available and is updated annually.[35]

Approach 6:   The Use of An Official Rate of Return

In this approach, a government or regulatory agency would set an "official" interest rate that could be used to approximate a business unit's cost of equity capital. For example, the "official" rate might be: (i) the interest rate that is used by the taxation authorities to assess late payment of income taxes; (ii) an equity interest rate that is recommended by the country's accounting standards board or (iii) the midpoint of a regulator's range of acceptable returns to equity capital for a regulated firm.

A problem with this method is that there is no guarantee that the "official" rate set by a taxation authority, accounting standards board or regulator will be "reasonable"; i.e., this method gives no guidance on how the authority will in fact determine the "official" rate. In practice, official rates determined by the tax authorities are probably based on Approach 5(iii) outlined above.

Of the 6 broad approaches to the empirical determination of an equity opportunity cost of capital, I feel that Approaches 5 and 6 are fairly satisfactory.

Why is it important to recognize imputed interest on equity capital as a cost of production? The following quotations answer this question:

> "Once again, the basic reason why interest on the use of total capital should be recorded as a cost is that interest *is* a cost .... A company has not performed satisfactorily, either for its shareholders or for society, if it has not generated enough revenue to cover all its costs, including the cost of using capital. The current income statement does not show whether or not the company has met this fundamental test. Its implication is that any earnings above the cost of debt interest are a 'plus' ".   *Robert N. Anthony*[1973; 96]

> "The argument in favor of including interest as an element of cost is twofold. From the viewpoint of the business as a whole, it helps to point out an important fact to the managers of any enterprise which persistently fails to return a normal current rate of interest on the investment. From the more detailed cost accounting viewpoint, it is said to make an important cost distinction between those manufacturing departments using costly machinery and those using inexpensive machinery or none at all".   *Stephen Gilman*[1939; 322]

We turn now to the problems involved in determining the depreciation rates which appear in user cost formulae.

4.0   The Determination of Depreciation Rates

> "Depreciation is defined simply as rate of decrease of value".
>   *Harold Hotelling*[1925; 341]

> "The net stock concept is motivated by the observed fact that the value of a capital good declines with age (and/or use). This decline is due to several factors, the main ones being the decline in the life expectancy of the asset (it has fewer work years left), the declines in the physical productivity of the asset

(it has poorer work years left), and the decline in the relative market return for the productivity of this asset due to the availability of better machines and other relative price changes (its remaining work years are worth less). One may label these three major forces as exhaustion, deterioration, and obsolescence".

Zvi Griliches[1963; 119]

Definition (3) in section 2 above defined the depreciation rate of a durable input in terms of the decline in value of a "new" machine or other durable input compared to a "used" machine that had been used one additional accounting period. The two values that were compared were market values that pertained to the end of the accounting period. If the price of "new" machines were the same at the beginning and the end of the accounting period (so that $P^0 = P^1$), then the decline in the value at the end of the period, P_u^1/P^1, would be the same as the decline in value over the period, P_u^1/P^0. Many of the early treatments of depreciation implicitly assumed price stability (i.e., $P^0 = P^1$), and hence depreciation was identified with the decline in value of the durable input over the accounting period.[36] As the above quotation by Griliches [1963; 119] indicates, economists tried to analyze the factors that determine depreciation rates.[37]

Accountants, engineers, statisticians and economists have all made contributions to the literature on depreciation. We shall review many of their approaches in sections 4.1 to 4.6 below. Sections 4.1 and 4.2 summarize some of the early accounting approaches to the treatment of depreciation. These early approaches do not lead to empirical estimates of depreciation rates as we have defined them (recall (3) above); however, the approaches outlined in sections 4.3 to 4.6 do lead to estimates of depreciation rates. Section 4.3 reviews accounting approaches while sections 4.4 – 4.6 review some economic approaches.

4.1   Early Accounting Approaches to the Measurement of Depreciation

"[There are] various methods of estimating the Depreciation of a Factory, and of recording alteration in value, but it may be said in regard to any of them that the object in view is, so to treat the nominal capital in the books of account that it shall always represent as nearly as possible the real value. Theoretically, the most effectual method of securing this would be, if it were feasible, to Revalue everything at stated intervals, and to write off whatever loss such valuations might reveal without regard to any prescribed rate …. The plan of valuing every year instead of adopting a depreciation rate, though it might appear the more perfect, is too tedious and expensive to be adopted …. the next best plan, which is that generally followed … is to establish average rates

which can without much trouble be written off every year, to check the result by complete or partial valuation at longer intervals, and to adjust the depreciation rate if required".   *Ewing Matheson*[1884; 35]

"One of the first clear references to depreciation accounting was in the annual report of the Baltimore and Ohio Railroad for the year ended September 30, 1835. That report explained that income for the year was determined 'after carrying \$75,000 to the debit of profit and loss to make good deterioration of the railway and machinery ...'. During the years following 1835, there was no consistent policy followed by any group of companies or even by any one company. Apparently, some companies made a separate provision for depreciation as did the Baltimore and Ohio Railroad, while other companies charged replacement costs to expense in lieu of depreciation".   *P.D. Woodward*[1956; 71]

The very earliest treatments of the depreciation problem seem to have been on the basis of periodic appraisals of the value of fixed assets. Thus our first early approach to the determination of depreciation is (i) the appraisal approach: changes in appraised values, if negative, were regarded as costs to be charged to the accounting period between appraisals.[38] However, as the quotation by Woodward above indicates, there were two additional early treatments of depreciation: (ii) engineers made estimates of the value of the physical deterioration and loss of productive life that equipment and machinery might have experienced during an accounting period and (iii) new purchases of durable inputs were simply expensed in the period of purchase.

Obviously, the third approach (which is consistent with cash flow accounting) is not helpful in the determination of periodic income, which is the focus of the present paper. The other two approaches are reasonable but not helpful in the context of the explicit determination of depreciation rates: approach (i) mixes up capital gains with the determination of depreciation rates while approach (ii) gives no indication as to how depreciation rates would be determined.

4.2 The Provision for Future Replacement or Sinking Fund Approach

"The depreciation problem, in general terms, is the problem of writing off from fiscal period to fiscal period sums sufficient to return the capital invested in a property when that property has outlived its usefulness".

J.S. Taylor[1923; 1010]

"After the straight line formula the one perhaps most widely used is the sinking fund formula and the modifications of this method".

John B. Canning[1929; 273]

As the above quotations indicate, some early statisticians and accountants viewed the depreciation problem as a method for funding the future replacement of a durable input. In this sinking fund method, the focus shifted from changes in asset values to the problem of setting aside period by period accounting charges into a fund which will cumulate over the useful life of a fixed asset into an amount which will be sufficient to replace the asset on its retirement date. This treatment of depreciation has some elements in common with the maintenance of physical capital approach to income determination to be discussed in section 7.4 below, but the two methods are distinct.

In the 1930's, the sinking fund approach to depreciation was successfully attacked by a number of accountants on the grounds that depreciation accounting should be viewed as a method of spreading the initial cost of a durable input over its useful life rather than as a solution to the logically distinct problem of deciding whether the asset should be replaced at the end of its life:

"There is a much larger number who in some way try to identify the deposits to a sinking fund or the deposits and earnings thereon with 'depreciation expense'. The only source of an 'expense' of depreciation is the outlay or outlays made or agreed to be made for the asset in order to have the enjoyment of the service". *John B. Canning*[1929; 274]

"Depreciation exists whether the property being used is to be replaced or not. In no sense does the depreciation allowance ('reserve') account represent the accumulation for the purpose of acquiring future assets".

M.B. Daniels[1933; 306]

"Generally it is conceded that it is the purpose of recording depreciation to recover the original expenditure, the purchase of a new truck being a separate and distinct transaction having no possible connection with, or relation to, the

recovery of the original investment. It is simple to point out that the truck at the end of six years may be replaced at a higher price or a lower price, or at the same price, or by teams and wagons, or that no replacement will be made if the need for a truck has disappeared. The above arguments seem sufficiently convincing to discredit this replacement theory in so far as it is related to fixed assets ".   *Stephen Gilman*[1939; 349]

"The availability of money for replacement may offer serious financial problems. The problem of financing replacements may be sufficiently difficult to tax the resourcefulness and foresight of business men but it is in no sense whatever an accounting problem. The originally acquired asset was a deferred charge and its cost is recovered by the depreciation program. The replacement whether it be an identical item or not, is a fresh transaction resulting in the creation of a new deferred charge the cost of which in turn must, from the accounting viewpoint, be recovered over the years which follow its acquisition".

Stephen Gilman[1939; 494]

We turn now to accounting treatments of depreciation that take the intertemporal cost allocation viewpoint.

4.3   Straight line Depreciation and Other Accounting Allocation Methods

"A plough, for instance, which lasts twenty years, will contribute a twentieth part of its life-work and use to the ingathering of twenty different harvests".

Eugen von Böhm-Bawerk[1891; 305]

"*Straight Line Formula* ... In general, only two primary estimates require to be made, viz., scrap value at the end of n periods and the numerical value of n .... Obviously the number of periods of contemplated use of an asset can seldom be intelligently estimated without reference to the anticipated conditions of use. If the formula is to be respectable at all, the value of n must be the most probable number of periods that will yield the most economical use".

John B. Canning[1929; 265-6]

The first method that comes to mind in attempting to determine a sequence of depreciation rates of the form (3) for a durable capital input as it ages is the one suggested by Böhm-Bawerk above: estimate the expected number of accounting periods n that the input is likely to be used in production and assume that the single period depreciation

rate is $\delta = 1/n$. This straight line method of depreciation can be used to allocate the initial purchase cost of the asset, say P^0, across the n periods of its life; these historical cost allocations under straight line depreciation would be $(1/n)P^0, (1/n)P^0, \ldots, (1/n)P^0$, a sequence of n equal allocations. The straight line depreciation method can also be used in conjunction with current values of new units of the asset, yielding the following sequence of current value depreciation charges: $(1/n)P^1, (1/n)P^2, \ldots, (1/n)P^n$, where P^t is the price of a new unit of the asset at the beginning of period t, for $t = 1, 2, \ldots, n$.

Another commonly used method for the determination of depreciation rates rests on the assumption that depreciation occurs on the undepreciated value of the asset at a constant geometric rate δ where $0 < \delta < 1$. The sequence of historical cost allocations of original cost P^0 that this method generates is $\delta P^0, \delta(1-\delta)P^0, \delta(1-\delta)^2 P^0, \delta(1-\delta)^3 P^0, \ldots$ while the corresponding stream of periodic current cost accounting charges is $\delta P^1, \delta(1-\delta)P^2, \delta(1-\delta)^2 P^3, \delta(1-\delta)^3 P^4, \ldots$. This method of depreciation is sometimes called the reducing balance method[39] or the declining balance method.[40] As we shall see in section 13 below, this method of accounting for depreciation (applied to current values) is very convenient when it is necessary to construct capital aggregates for productivity measurement purposes.[41] Empirical estimates for the declining balance depreciation parameter δ generally come from: (i) "official" estimates by broad asset class made by the national tax or regulatory authorities[42]; (ii) estimates made by the engineers or managers of the business unit[43], or (iii) statistical studies such as those to be discussed in sections 4.4 and 4.5 below.

Saliers [1922] and Canning [1929; 260-309] list many other rather arbitrary methods that accountants have used to estimate depreciation rates. The arbitrariness of these accounting depreciation methods and the fact that the estimates are generally based on a prior reasoning rather than on empirically observable declines in value[44] has of course attracted comment from many accountants and economists over the years:

> "Accountants immediately discard their own figures and demand an appraisal of the plant and other fixed assets, whenever they are called upon to compute capital value for the purpose of sale, reorganization etc. Apart from such occasions they adhere to their depreciation methods with the proviso that the method itself matters less than consistent adherence to it, once it has been adopted. These methods generally limit guessing to a minimum considered unavoidable in the circumstances". *Gabriel A.D. Preinreich*[1938; 240]

> "For the past hundred years accountants have been searching for the 'true' depreciation method which would allocate the cost of the machine over its lifetime in accordance with the rate at which it is actually being 'used' up. They have reluctantly concluded that there is no 'true' depreciation method, and that all

the methods used or proposed are mere conventions, the choice between which is a matter of convenience". *F. Lutz and V. Lutz*[1951; 7]

However, historical cost accountants such as Daniels and Ijiri have defended the arbitrariness of accounting cost allocations as follows:

"The function of depreciation is recognized by most accountants as the provision of a means for spreading equitably the cost of comparatively long lived assets. Thus, if a building will be of use during twenty years of operations, its cost should be recognized as operating expense, not of the first year, nor the last, but of all twenty years. Various methods may be proper in so allocating cost. The method used, however, is unimportant in this connection. The important matter is that at the time of abandonment the cost of the asset shall as nearly as possible have been charged off as expense, under some systematic method". *M.B. Daniels*[1933; 303]

"However, there is a diametrically opposite problem in historical cost accounting ... the problem is one of disaggregation or allocation. Suppose that resources A and B are purchased together for \$20, but at the end of the year the firm had only Resource A. How much of the \$20 should be assigned to Resource A? Depreciation is a typical problem of this kind. However, accountants have devised many methods, however arbitrary they may be, by which such allocations are carried out objectively". *Yuji Ijiri*[1979; 67]

Both of the above authors recognize the arbitrariness of historical cost accounting allocations of asset cost; the best that can be said of these methods is that they are "systematic". If the tax authorities specify that one or more depreciation formulae must be used for tax purposes, then the use of the resulting historical cost allocations might also be characterized as "objective".

Since historical cost accountants regularly criticize current value accountants for their use of imputed or estimated values, it is important to recognize that historical cost accounting is subject to precisely the same criticism: historical cost accounting, by accepting an arbitrary a priori pattern of depreciation rates *imputes period by period depreciation costs.* If we attempt to estimate the period by period durable input costs accurring to a business unit, then *any* method of accounting will have to resort to imputed or estimated values.[45]

We conclude this section with two additional criticisms of historical cost depreciation allocations.

The first criticism is due to Canning[46] who asked that criteria be developed to choose among the many depreciation methods that were used by historical cost accountants.

This request for a rational criterion for choosing a depreciation formula has not been answered because the answer cannot be given on the basis of a priori reasoning: period by period empirical evaluation of the physical condition and market value of the assets to be depreciated is required.

The second criticism is due to Edwards and Bell[47] who noted that the historical cost accountant would need to be clairvoyant in order to determine the useful life of an asset; i.e., identical new assets are not all retired at the same time.[48] On the other hand, current value accounting techniques, by estimating period by period used asset values (recall formula (7) above), avoid in principle the difficult problems that arise when durable inputs are used at different intensities. We can paraphrase this second criticism of historical cost accounting techniques as follows: different historical cost accountants will estimate different lengths of life (and scrap values) for the same asset, leading to variable or "nonobjective" period by period depreciation estimates. Parker[49] in an ingenious empirical study showed that current value estimation of depreciation for a certain calculating machine gave rise to less dispersion in the estimates for depreciation expense than historical cost estimation. Thus current value accounting may be more "objective" than historical cost accounting.

We now turn to an examination of economic methods for estimating depreciation rates. These methods are based on the observation of current market values.

4.4   Economic Approaches Based on Used Asset Prices

"If there is a perfect second hand market for the goods in question, so that a market value can be assessed for them with precision, corresponding to each particular degree of wear, then the value-loss due to consumption can be exactly measured ...".   *John R.Hicks*[1939; 2nd edition 1946; 176]

"Some depreciation patterns have very little economic justification (except accounting convenience), but most of them at least purport to approximate the decline in the economic value of the remaining services (i.e., market value). Of the various possible depreciation schemes (net stock measures), two measures seem to be of the most interest: (a) a net stock concept based on a purely physical deterioration depreciation scheme, and (b) the market value of the existing stock of capital. The latter figure can be approximated by the use of depreciation rates derived from studies of used machinery prices".   *Zvi Griliches*[1963; 120]

Economists (like Hicks and Griliches) and accountants (like Bell and Edwards[50]) have long realized that a possible method of estimating the decline in value of a durable input

due to its use over an accounting period is to use information on the market prices of used assets at a point in time and to compare differences in price as a function of the age of the input.

Let t index the beginning of a time period and suppose that we know the purchase price of a new durable input at the beginning of period t and we have collected information on the average market price at time t for the same input that has been used for n periods, P^{tn}, for $n = 1, 2, \ldots, N$.[51] Then the one period depreciation rate for a new asset could be defined by δ_0 in the following equation (provided that depreciation is independent of the time period t):[52]

$$(22) \quad P^{t1}/P^{t0} \equiv (1 - \delta_0).$$

Given that δ_0 has been determined, then the one period depreciation rate for a one period old asset can be defined by δ_1 in the following equation:

$$(23) \quad P^{t2}/P^{t0} = (1 - \delta_0)(1 - \delta_1).$$

In general, given that the one period depreciation rates $\delta_0, \delta_1, \ldots, \delta_{n-1}$ have been determined, then the one period depreciation rate δ_n for an asset that is n periods old at the start of an accounting period can be defined by δ_n in the following equation:

$$(24) \quad P^{tn+1}/P^{t0} = (1 - \delta_0)(1 - \delta_1) \ldots (1 - \delta_n), \quad n = 0, 1, \ldots, N - 1.$$

Thus given market data on the prices of the used asset, period by period depreciation rates δ_n can be obtained by solving equations (24).[53] If we have information on used asset prices for many different time periods t and we are willing to make the assumption that depreciation rates are stable over time, then a stochastic specification of a variant of (23) can be made and econometric techniques can be used to estimate the sequence of one period depreciation rates. If the market data on used asset prices is sparse, then instead of estimating a completely general pattern of period to period depreciation rates $\delta_0, \delta_1, \ldots, \delta_{N-1}$, various restrictions on these parameters can be imposed.[54] The simplest such restriction is that δ_n be constant from period to period; i.e.,

$$(25) \quad \delta_n = \delta_0 \text{ for } n = 1, 2, \ldots, N - 1$$

which is the geometric or declining balance depreciation model mentioned in section 4.3 above.

Empirical studies of depreciation rates using second hand asset prices have been made by the accountant Beidleman [1973] [1976][55] and the economists Hall [1971], Hulten and Wykoff [1981a] [1981b] and Oliner [1996]. The literature on this used asset approach is ably reviewed by Hulten and Wykoff [1996][56] and Jorgenson [1996c].

Many economists and accountants have objected to the use of second hand data to estimate depreciation rates for a variety of reasons:

"We readily agree that where a market is sufficiently large, generally accessible, and continuous over time, it serves to coordinate a large number of subjective estimates and thus may impart a moment of (social) objectivity to value relations based on prices formed on it. But it can hardly be said that the second-hand market for industrial equipment, which would be the proper place for the determination of the value of capital goods which have been in use, satisfies these requirements, and that its valuations are superior to intra-enterprise valuation". *L.M. Lachmann* [1941; 376-377]

"But why, if market values are the key to asset values, does not the accountant find depreciation by direct reference to market quotations for assets of different ages, and abandon his formulae? Various answers suggest themselves .... Second-hand markets tend to be small and scrubby, so that quotations may be hard to find and harder to trust. Many assets are built specifically for the one firm, and therefore worn replicas do not exist. An owner usually regards his own worn assets as different from, and better than, replicas in the market, because he knows their history, condition and foibles". *William T. Baxter* [1971; 31]

"One argument, drawing on the Akerlof Lemons Model, is that assets resold in second hand markets are not representative of the underlying population of assets, because only poorer quality units are sold when used. Others express concerns about the thinness of resale markets, believing that it is sporadic in nature and is dominated by dealers who under-bid".

Charles R. Hulten and Frank C. Wykoff [1996; 17-18]

Inspite of the above objections to the use of the second hand market method for estimating depreciation rates, this method seems more "objective" than simply guessing at the appropriate rates.

A more serious objection to the above model of depreciation rate determination is that the method includes only length of asset service or time in use as an explanatory variable and thus the method neglects variations in the intensity of use of the durable input.[57] There are at least two ways of meeting this criticism: (i) we can follow the advice of Edwards and Bell[58] and estimate separate sequences of depreciation rates that pertain to assets that are used with approximately the same intensity and have similar maintenance policies or (ii) we can incorporate utilization and maintenance variables as explanatory variables in stochastic versions of (24).[59] Jorgenson [1996c, 27-29] reviews the literature on these extensions of the basic used asset model of depreciation rate determination.

We turn now to another economic method for the determination of depreciation rates.

4.5 Economic Approaches Based on Production Function Estimation

Suppose a durable input is used by a business unit and it purchases I^t units of it at the beginning of period t in order to produce y^t units of output using the vector x^t of nondurable inputs[60] during period t as well as the services of past purchases of the durable input. Suppose further that the durable input lasts N periods and that after adjusting for physical loss of efficiency, all unretired units of the durable input are perfect substitutes in production. The production function which relates output flow to inputs used during a period is F and if there is no technological progress, we have the following relationship between output produced and inputs used during period t:

$$(26) \quad y^t = F(x^t, I^t + (1 - \delta_0)I^{t-1} + (1 - \delta_0)(1 - \delta_1)I^{t-2} + ... + (1 - \delta_0)...(1 - \delta_{N-1})I^{t-N}).$$

The production function method for determining depreciation rates works as follows: (i) collect data on output produced during period t, nondurable inputs used x^t and durable input purchases I^t for a number of periods t; (ii) assume a functional form for the production function F; (iii) add a stochastic specification to equations (26) for $t = 0, 1, ..., T$ and (iv) use econometric techniques to simultaneously estimate the unknown parameters which appear in the production function F as well as the depreciation rates $\delta_0, \delta_1, ..., \delta_{N-1}$. Variants of this basic method include: (i) restricting the depreciation parameters $\delta_0, \delta_1, ..., \delta_{N-1}$ in some a priori fashion (e.g., recall (25) above); (ii) using the assumption of short run profit maximizing or cost minimizing behavior on the part of the business unit in order to add extra estimating equations involving period t prices to the single estimating equation (26) or (iii) instead of estimating the production function F, estimating the unknown parameters in the dual cost or profit function.[61]

Empirical studies using this approach to the estimation of depreciation rates include Epstein and Denny [1980], Pakes and Griliches [1984], Nadiri and Prucha [1996] and Doms [1996].

It should be noted that the depreciation rates which are estimated using this production function approach will, in general, be different from the estimates that result from the used asset approach studied in section 4.4 above. The latter approach incorporates the effects of both exhaustion and deterioration (to use Griliches' terminology), while the production function approach incorporates only the effects of physical deterioration.[62]

There are two major problems with the production function approach: (i) the approach will only work in a highly aggregated model with a small number of outputs, nondurable inputs and durable inputs due to the difficulties involved in estimating the parameters pertaining to a general production function when there are numerous inputs and outputs[63] and (ii) the assumption that the different vintages of capital can be combined together in the additive capital aggregate that appears as the last term on the right hand side of (26) is restrictive; i.e., it is assumed that the deterioration adjusted different vintages of capital are perfect substitutes in production, an assumption which may or may not be true.

Thus as a good general method for the empirical determination of depreciation rates, the production function method is unsatisfactory.

We turn now to our final class of economic methods for the determination of depreciation rates.

4.6   Economic Approaches Based on Rental Prices

Suppose that a rental market for a durable input exists so that we can observe in period 0 the rental price w^0 which appears in the user cost formulae (2) or (4). Then given information on the firm's cost of capital r^0 and the prices P^0 and P^1 of a "new" unit of the durable input being used by the firm at the beginning and end of period 0, we can use equation (4) to solve for the period 0 depreciation rate δ^0. Thus given that rental markets exist for durable inputs being used by a business unit, these rental prices can be equated to the corresponding user costs and depreciation rates can be derived from the resulting equations. Of course, if rental markets do not exist, then this method will not work, an obvious point made by Hulten and Wykoff [1996;15]. Jorgenson [1996c;32] reviewed the few studies that have used this method.

Even when rental markets exist, this method is unlikely to generate very accurate depreciation rates, particularly when prices are changing rapidly over the course of the accounting period. The problem is that rental prices are generally determined at the beginning of the period *before* the price of the used durable at the end of the period is known. Hence an observed rental price is actually an ex ante price of the form

$$\tilde{w}^0 \equiv \quad P^0 - \tilde{P}_u^1/(1+r^0)$$
$$= [r^0 - \tilde{i}^0 + \delta^0(1+\tilde{i}^0)]P^0/(1+r^0)$$

where $\tilde{P}_u^1$ is the (beginning of period 0) *anticipated* price of the used durable input (at the end of period 0) and $\tilde{i}^0$ is the *anticipated* period 0 inflation rate for "new" units of the durable input. Thus equating the right hand side of (28) to an observed rental price w^0 and solving for δ^0 is impossible unless we *know* the anticipated inflation rate $\tilde{i}^0$ at the beginning of period 0, an unlikely possibility.[64]

Even though the rental price method is unlikely to be a useful method for the empirical determination of depreciation rates, rental prices are useful when they exist–they can be used as period 0 (opportunity) costs for the use of the corresponding durable inputs during period 0.[65]

Our overall conclusion in evaluating different methods for the empirical determination of depreciation rates is that the used asset price method seems best when the relevant second hand markets actually exist. However, for firm specific assets that are not traded in second hand markets, it appears that depreciation will have to be determined on the basis of engineering estimates or appraisals.

It is evident from our discussion of depreciation that different authors have different concepts of depreciation. Goldberg summed up some of the alternative meanings as follows:

> "If, by 'charging depreciation' we mean an allocation of historical cost, I suggest that we use words (such as 'cost allocation' and 'proportion' of cost allocated against past revenue') which will convey this meaning. If we mean attempting to provide resources for future replacement of assets, why not use words (such as 'provision for future replacement') which will bring this meaning out? If we mean adjustment to present market costs, why not use words which say so; if we mean estimate of wearing out, let us indicate this clearly and unequivocally. To use a word like 'depreciation' or a phrase like 'provision for depreciation' which is now so confused is not quite fair to ourselves or to the readers of our statements and reports". *Louis Goldberg*[1955; 484]

We interpret "depreciation" to encompass exhaustion and deterioration over an accounting period but to exclude capital gains or losses due to changes in the prices of assets over the accounting period; i.e., depreciation is the hypothetical loss of value of an asset that would take place over an accounting period under "normal" use, holding the structure of new and used asset prices fixed.[66]

We turn now to a closer examination of the production function concept and the meaning of "production."

5.0   The Production Function Framework

"Thus far, however, we have left out of consideration the fact that commodities are *products* which result from the combination of productive factors such as land, men and capital goods".   *Léon Walras*[1954; 211]

"Almost all of our theorizing about investment and the desired stock of capital rests implicitly on some technological considerations and is derived from some kind of general production function. As long as we stick to the production function framework, it is clear that *quantity* rather than *value* is the relevant dimension, since the production function is defined as a relationship between the quantity of output and the quantity of various inputs".

Zvi Griliches[1963; 118]

In order to measure the contribution of capital to the production of outputs, it is useful to have an idealized model of how capital inputs interact with other flow inputs to produce outputs. The idealized models that economists utilize are based on *production functions*, or more specifically, on *production possibilities sets* which are technologically feasible sets of inputs and outputs that can be produced by a specified business unit in a specified time period. There are a number of different production function concepts that can be distinguished. Thus in section 5.1, we discuss the short run production function which distinguishes capital as an input at the beginning of the accounting period and (depreciated) capital as an output at the end of an accounting period. In section 5.2, we consider an intertemporal production function which relates inputs to outputs over many accounting periods. In this production function concept, the capital stocks the firm has available at the start of the first accounting period are distinguished as inputs and the (depreciated) capital stocks at the end of the last accounting period (when the assets of the firm are sold) are distinguished as outputs, but there is no apparent necessity to keep track of used capital inputs in intermediate accounting periods in this framework (unless they are sold before the final period). Purchases of new capital inputs over intermediate periods are distinguished in this framework. In section 5.2, we also attempt to reconcile this intertemporal production function concept with the one period "Austrian" production function concept in section 5.1. In section 5.3, we indicate how the usual one period production function that treats capital just as an input in each accounting period can be extracted from the Austrian production function framework.

In section 5.4, we address a very fundamental question in accounting theory; namely, is speculative activity productive? The answer to this question will determine the validity of including asset inflation rates in our user cost formulae.

Finally, in section 5.5, we discuss briefly what are the factors determining the production function for a firm or business unit.

5.1 The Austrian Production Function

"We must look at the production process during a *period* of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output".   *John R. Hicks*[1961; 23]

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity".

Edgar O. Edwards and Philip W. Bell[1961; 71-72]

Hicks [1961; 23] and Edwards and Bell [1961; 71-72] obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period). We call this an Austrian model of production in honour of the Austrian economist Böhm-Bawerk [1891] who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.[67]

It should be noted that the neo-Austrian model of Hicks [1973] is different from the model that we are describing in this section. Hicks [1973; 7] interpreted Böhm-Bawerk's production model as follows:

> "Like Böhm-Bawerk (or Hayek) I think of the general productive process as being composed of a number (presumably a large number) of *separable elementary processes ....* The elementary process, of the older Austrian theory, was of a simple, too simple, type. There was associated with a unit of output, forthcoming at a particular date, a sequence of units of input at particular previous dates. The sequence of inputs, and the single output, constituted the process".

Hicks [1973; 8] then asserted that Böhm-Bawerk's production model was inconsistent with the existence of fixed capital inputs:

> "For the only kind of capital-using production which will fit into the old Austrian scheme is production without fixed capital, production that uses working capital (or circulating capital) only. Fixed capital (plant and machinery) will not fit in. For fixed capital goods are 'durable-use goods'; their essential characteristic is that they contribute, not just to one unit of output, at one date, but to a sequence of units of output, at a sequence of dates".

However, Böhm-Bawerk [1891; 299-300] certainly mentioned various durable capital inputs such as "tools", "machines" and "agricultural implements" as inputs in his model of production; what he did not explain explicitly is how time and use (i.e., depreciation) would transform these inputs into less valuable outputs at the end of a production period. What Böhm-Bawerk emphasized was the transformation of partly finished goods into more valuable partly finished goods and final products.[68]

It will be useful in subsequent sections to develop some notation to describe the one period Austrian model of production of this section. We suppose that there are M durable inputs that the business unit is using at the beginning of period 0. These durable inputs include machines, transportation equipment, other equipment, computers, plant structures, office buildings, tools, office furnishings and furniture, etc. These fixed capital stock components are classified into discrete categories according to their age and other relevant physical characteristics. The list of durable inputs also includes circulating capital stock components: inventories of raw materials, finished goods and partly finished goods (goods in process). Finally, we include in the business unit's list of initial capital stock components any patents or other marketable knowledge products as well as any holdings of land or other natural resources that it might possess. We denote the business unit's beginning of period 0 holdings of durable capital inputs by the nonnegative vector $\bar{k}^0 \equiv [\bar{k}_1^0, \bar{k}_2^0, \ldots, \bar{k}_M^0]$ where $\bar{k}_m^0 \geq 0$ denotes the initial stock of durable input m for $m =$

$1, \ldots, M$. We also suppose that $P_m^0 \geq 0$ is the beginning of period 0 market opportunity cost[69] for a unit of durable input m for $m = 1, \ldots, M$ and the vector of these initial market values is $P^0 \equiv [P_1^0, P_2^0, \ldots, P_M^0]$.

Next, we suppose that there are N outputs or inputs that the business unit can sell or purchase in the marketplace during period 0. The vector of average market prices that the business unit faces for the N commodities in period 0 is $p^0 \equiv [p_1^0, p_2^0, \ldots, p_N^0]$ where $p_n^0 \geq 0$ is the average market price for commodity n in period $0, n = 1, \ldots, N$. The vector of *net outputs* that the business unit produces during period 0 is denoted by $y^0 \equiv [y_1^0, y_2^0, \ldots, y_N^0]$; if $y_n^0 > 0$, then y_n^0 units of commodity n are being produced by the business unit during period 0 while if $y_n^0 < 0$, then $-y_n^0 > 0$ units of commodity n are being used as inputs during period 0.[70] The list of period 0 "flow" inputs includes various types of labour services, including both establishment employees and contracted professional services, purchases of electricity, heating fuels and telecommunications services. In principle, the entire list of durable capital stock components could be included in the list of flow inputs, since the business unit could purchase additional units of capital during period 0 to add to its initial stocks. The list of "flow" outputs will include the usual outputs that the business unit produces, classified as finely as seems necessary for the purpose at hand.[70] In principle, all of the initial capital stock components held by the firm at the start of period 0 should appear in the list of flow outputs, since these stocks could be sold in the marketplace during period 0. Note that we are distinguishing as separate flow commodities *sales* of initial capital stock components from additional *purchases* of capital stock components during period 0 for two reasons: (i) the selling price of an asset will usually differ from the purchase price of a similar unit of the asset due to transactions costs and (ii) the technological impact of the sale of a fixed capital stock component is often quite different from the purchase of an additional unit due to internal transactions costs (such as installation and training costs for purchases and dismantling and renovation costs for sales). Thus the dimensionality of the "flow" commodity space N will generally be much greater than the dimensionality of the initial durable input "stock" space M.

Finally, at the end of period 0, the business unit will have at its disposal a vector $k^1 \equiv [k_1^1, k_2^1, \ldots, \ldots, k_M^1]$ of durable inputs which can be valued at the end of period 0 (or beginning of period 1) nonnegative market opportunity costs vector $P^1 \equiv [P_1^1, P_2^1, \ldots, P_M^1]$. The components of k^1 consist of depreciated units of the business unit's beginning of period 0 vector of capital stocks k^0 that were not sold during period 0 plus any additional units of capital that might have been purchased during period 0 .[71]

We now turn our attention to the definition of the period 0 Austrian production function or more generally, the period 0 Austrian production possibilities set for the business unit under consideration. Several definitions are possible.

The broadest definition for the period 0 production possibilities set S^0 is to define S^0 as the set of all technologically feasible vectors of the form (k^0, y^0, k^1) where $k^0 \geq 0_M^{72}$ is a nonnegative beginning of period 0 vector of capital inputs, y^0 is an N dimensional vector of period 0 net outputs that can be produced given k^0 and $k^1 \geq 0_M$ is a nonnegative vector of capital stocks that are left over at the end of period 0. In this broadest definition of the period 0 production possibilities set, we allow the business unit to choose its initial vector of capital stocks k^0. In our next definition of the period 0 production possibilities set, we restrict the business unit's choices for the initial capital stock vector k^0 to be an observed vector of capital stocks $\bar{k}^0$ for the business unit under consideration. In this case, the period 0 production possibilities set for the business unit can be described as a feasible set of net outputs and end of the period capital stocks $\{y^0, k^1\}$ that could be produced using the observed initial vector of capital stocks $\bar{k}^0$; i.e., the technology set now has the form[73]

(29) $\{(y^0, k^1) : (\bar{k}^0, y^0, k^1) \in S^0\}$.

The production possibilities set defined by (29) is our most narrow definition for the period 0 feasible set of inputs and outputs for the business unit under consideration. A third possible definition for the period 0 production possibilities set might be the following one:

(30) $\{(k^0, y^0, k^1) : (k^0, y^0, k^1) \in S^0, k^0 \leq \bar{k}^0\}$;

i.e., in definition (30), we allow the business unit to dispose of units of its initial capital vector $\bar{k}^0$, so that the capital stock vector k^0 actually used in the period 0 production process is equal to or less than $\bar{k}^0$.

Note that the "narrower" or more restricted production possibilities sets defined by (29) and (30) are defined in terms of the "broader" or least restricted production possibilities set S^0 and the business unit's initial observed capital stock vector $\bar{k}^0$.

The business unit's competitive profit maximization problem that corresponds to the broadest definition of the period 0 production possibilities set S^0 can be formalized as:

(31) $max_{k^0, y^0, k^1}\{-P^0 \cdot k^0 + (1 + r^0)^{-1}p^0 \cdot y^0 + (1 + r^0)^{-1}P^1 \cdot k^1; (k^0, y^0, k^1) \in S^0\}$

where $P^t \cdot k^t \equiv \Sigma_{m=1}^{M} P_m^t k_m^t$ for $t = 0, 1, p^0 \cdot y^0 \equiv \Sigma_{n=1}^{N} p_n^0 y_n^0, r^0$ is the period 0 interest rate or opportunity cost of capital and S^0 is the period 0 production possibilities set for the business unit. Note that we have divided the net "flow" revenues for period 0, $p^0 \cdot y^0$, and the market value of the business unit's end of the period holdings of capital stocks, $P^1 \cdot k^1$, by one plus the interest rate, $(1 + r^0)$. Thus we are assuming that period 0 "flow" revenues and costs $p^0 \cdot y^0$ are "realized" at the end of period 0 along with the end of period

0 value of the business unit's capital stocks, $P^1 \cdot k^1$. These end of period 0 capital stocks are discounted to make them equivalent to beginning of period 0 values, as is traditional in economics. However, from the perspective of accounting theory, it is more natural to express all values in terms of end of the period values and thus from this perspective, the business unit's period 0 profit maximization problem becomes:

$$(32) \quad max_{k^0, y^0, k^1}\{-(1+r^0)P^0 \cdot k^0 + p^0 \cdot y^0 + P^1 \cdot k^1 : (k^0, y^0, k^1) \in S^0\}.$$

Now let us shift our focus to the business unit's profit maximization problem in period 1. Let $S^1 \equiv \{(k^1, y^1, k^2)\}$ denote the business unit's unrestricted period 1 production possibilities set, which consists of feasible vectors of starting capital stocks k^1, period 1 "flow" inputs and outputs y^1 and end of period 1 finishing capital stock vectors k^2. If there is no technological progress or managerial improvement in the organization of production, S^1 will equal S^0; i.e., the period 1 and period 0 production possibilities sets will be the same.

The period 1 counterpart to the period 0 unrestricted profit maximization problem (32) is:

$$(33) \quad max_{k^1, y^1, k^2}\{-(1+r^1)P^1 \cdot k^1 + p^1 \cdot y^1 + P^2 \cdot k^2 : (k^1, y^1, k^2) \in S^1\}$$

where r^1 is the beginning of period 1 opportunity cost of capital; $P^1 \equiv (P_1^1, \ldots, P_M^1) \geq 0_M$ is the vector of beginning of period 1 opportunity costs for capital stock components; $P^2 \equiv (P_1^2, \ldots, P_M^2) \geq 0_M$ is the vector of end of period 1 opportunity costs for capital stock components; $p^1 \equiv (p_1^1, \ldots, p_N^1) \geq 0_N$ is the vector of period 1 average prices for units of outputs and inputs and $y^1 \equiv (y_1^1, \ldots, y_N^1)$ is a period 1 net output vector (positive components of y^1 denote outputs, negative components denote inputs).

Obviously, one period profit maximization problems that are analogous to (32) and (33) can be defined for each accounting period t that the business unit is in operation.

We can also define a one period profit maximization problem that has the same structure as (32) except that the restricted production possibilities set defined by (29) is used in place of S^0. This restricted period 0 profit maximization problem (with k^0 restricted to equal the fixed initial capital stock vector $\bar{k}^0$) is:

$$(34) \quad max_{y^0, k^1}\{-(1+r^0)P^0 \cdot \bar{k}^0 + p^0 \cdot y^0 + P^1 \cdot k^1 : (\bar{k}^0, y^0, k^1) \in S^0\}.$$

Suppose $\bar{y}^0$ and $\bar{k}^1$ solves (34). Then the end of period 0 capital stock vector $\bar{k}^1$ can serve as a vector of fixed starting capital stocks for the business unit's period 1 restricted profit maximization problem which is analogous to (33) except that k^1 is fixed at $\bar{k}^1$:

$$(35) \quad max_{y^1, k^2}\{-(1+r^1)P^1 \cdot \bar{k}^1 + p^1 \cdot y^1 + P^2 \cdot k^2 : (\bar{k}^1, y^1, k^2) \in S^1\}.$$

Finally, we can define a one period profit maximization problem that has the same structure as (32) except that the production possibilities set defined by (30) is used in place of S^0:

(36) $\quad max_{k^0, y^0, k^1}\{-(1+r^0)P^0 \cdot k^0 + p^0 \cdot y^0 + P^1 \cdot k^1 : (k^0, y^0, k^1) \in S^0; k^0 \leq \bar{k}^0\}.$

The differences between the three period 0 profit maximization problems (32), (34) and (36) can be explained as follows: in (32), the business unit is allowed to sell its initial holdings of capital (the components of the vector $\bar{k}^0$) or buy additional units of capital at the beginning of period 0 at the prices P^0; in (36), the business unit can only sell its initial holdings of capital $\bar{k}^0$ at the prices P^0; in (34), the business unit is stuck with its initial holdings of capital $\bar{k}^0$ at the beginning of period 0 and values these initial holdings at the prices P^0. Thus the different period 0 profit maximization problems reflect different assumptions about what options are open to the business unit at the beginning of period 0. However, for each of the three problems, the Austrian production possibilities set S^0 plays a crucial role.

In the following section, we no longer assume that the business unit's decision horizon is limited to a sequence of single periods; we will allow the business unit to make production plans that extend over a number of periods.

5.2 The Intertemporal Production Function

"An option is any possible income stream open to an individual by utilizing his resources, capital, labor, land, money, to produce or secure said income stream. An investment opportunity is the opportunity to shift from one such option, or optional income stream, to another .... Some of the optional income streams, however, would never be chosen, because none of their respective present values could possibly be the maximum."   *Irving Fisher*[1930; 151]

"The problem of the firm, dynamically considered, is to find that stream of outputs, capable of being produced from the initial equipment, which shall have the maximum capital value ... . If we write $x_{r0}, x_{r1}, x_{r2}, \ldots, x_{rv}$ for the [net] outputs of x_r planned to be sold in successive 'weeks' from the present, then the production function takes the form $f(x_{10}, x_{20}, \ldots, x_{n0}; x_{11}, x_{21}, \ldots, x_{n1};$ $x_{12}, x_{22}, \ldots, x_{n2}; \ldots; x_{1v}, x_{2v}, \ldots, x_{nv}) = 0$ assuming that the plan extends forward for v weeks. The capitalized value of the plan is $C = \Sigma_{r=1}^{n} \Sigma_{t=0}^{v} (\beta_t p_{rt} x_{rt})$ where $\beta_t = 1/(1+i_t)$, and i_t is the rate of interest per week for loans of t weeks;

In this section, we will utilize the intertemporal production function concepts developed by Fisher [1930; 151] and Hicks [1946; 136]. As in section 5.2, we assume that there are M types of durable capital equipment and that the business unit's initial holdings of capital stock components at the beginning of period 0 is $\bar{k}^0 \equiv [\bar{k}_1^0, \dots, \bar{k}_M^0] \geq 0_M$. We now assume that the business unit's time horizon extends over T periods. Denote a vector of planned net outputs for period t by $y^t \equiv [y_1^t, \dots, y_N^t]$ for $t = 0, 1, 2, \dots, T-1$ and denote the corresponding vector of anticipated average prices for period t by $p^t \equiv [p_1^t, \dots, p_N^t] \geq 0$ for $t = 0, 1, 2, \dots, T-1$. At the end of period $T-1$ (or equivalently, at the beginning of period T), we assume that the business unit is sold[74]; i.e., the components of its beginning of period T capital stock vector $k^t \equiv [k_1^T, \dots, k_M^T] \geq 0$ are sold at the prices $P^T \equiv [P_1^T, \dots, P_M^T] \geq O_M$. The *intertemporal production possibilities set* $S \equiv \{(k^0, y^0, y^1, \dots, y^{T-1}, k^T)\}$ is the feasible set of net output vectors $y^0, y^1, \dots, y^{T-1}$ for periods $0, 1, \dots, T-1$ and beginning of period T (depreciated) capital stock components k^T that can be produced by existing technology (and technology that can be anticipated to exist over the time horizon of the business unit) and an initial vector of capital stock inputs, k^0.

Let r^t be the interest rate or opportunity cost of financial capital that is relevant to the business unit at the beginning of period t for $t = 0, 1, \dots, T-1$ and let $P^0 \equiv [P_1^0, \dots, P_M^0] \geq 0_M$ be the vector of opportunity costs for capital stock components at the start of period 0. Then assuming that period t "flow" net revenues, $p^t \cdot y^t \equiv \Sigma_{n=1}^N p_n^t y_n^t$, are "realized" at the end of period t, the business unit's intertemporal planned profit maximization problem can be written as follows:

$$
(37) \quad
\begin{aligned}
max_{k^0, y^0, y^1, \dots, y^{T-1}, k^T} \{ &-P^0 \cdot k^0 + (1+r^0)^{-1} p^0 \cdot y^0 + (1+r^0)^{-1}(1+r^1)^{-1} p^1 \cdot y^1 \\
&+ \dots + (1+r^0)^{-1} \dots (1+r^{T-1})^{-1}(p^{T-1} \cdot y^{T-1} + P^T \cdot k^T) : \\
&(k^0, y^0, y^1, \dots, y^{T-1}, k^T) \in S \}.
\end{aligned}
$$

Note that all values in the objective function of (37) that are realized after the beginning of period 0 are discounted by interest rate terms $(1+r^t)$. Thus all values are expressed in beginning of period 0 equivalent values. Note also that the intertemporal profit maximization problem (37) reduces to the single period Austrian profit maximization problem (31) if the business unit's time horizon is only one period; i.e., if $T = 1$. Note also that in both (31) and (37), we allowed the initial vector of beginning of period 0 capital stocks k^0 to be variable. A counterpart to (37) which freezes k^0 to equal the business unit's historically determined capital stocks $\bar{k}^0$ is[75]:

$$(38) \quad max_{y^0, y^1, \ldots, y^{T-1}, k^T} \{ -P^0 \cdot \bar{k}^0 + (1 + r^0)^{-1} p^0 \cdot y^0 + (1 + r^0)^{-1}(1 + r^1)^{-1} p^1 \cdot y^1$$
$$+ \ldots + (1 + r^0)^{-1} \ldots (1 + r^{T-1})^{-1} (p^{T-1} \cdot y^{T-1} + P^T \cdot k^T) :$$
$$(\bar{k}^0, y^0, y^1, \ldots, y^{T-1}, k^T) \in S \}.$$

If we divide the objective function in (34) by $(1 + r^0)$, it can be seen that the resulting version of (34) is the same problem as (38) if $T = 1$; i.e., the intertemporal profit maximization problem (38) is equivalent to our restricted one period Austrian profit maximization problem (34) when the business unit's time horizon is only a single period.

In the case where the business unit has a multiperiod planning horizon (i.e., the case where $T > 1$), it is possible to relate the one period technology sets of section 5.2 to the intertemporal technology set S of the present section. Suppose that the one period Austrian technology sets $S^0, S^1, \ldots, S^{T-1}$ are given for periods $0, 1, \ldots, T - 1$. Then these Austrian technology sets can be used to define a Hicksian intertemporal technology set S as follows:

$$(39) \quad S \equiv \{ (k^0, y^0, y^1, \ldots, y^{T-1}, k^T) : (k^0, y^0, k^1) \in S^0, (k^1, y^1, k^2) \in S^1, \ldots,$$
$$(k^{T-1}, y^{T-1}, k^T) \in S^{T-1} \};$$

i.e., in defining (39), we simply force the end of period t capital stocks k^{t+1} to be equal to the starting capital stocks for period $t + 1$ for $t = 0, 1, \ldots, T - 1$. Thus we do not allow the business unit to sell or purchase any units of capital at the very end of each period t in definition (39) for $t = 0, 1, \ldots, T - 2$.[76]

We conclude section 5.3 by indicating that under certain conditions, solutions to the Hicksian intertemporal profit maximization problem (38) are also solutions to a sequence of Austrian single period profit maximization problems, provided that the period by period capital stock valuation vectors $P^1, P^2, \ldots, P^{T-1}$ that appear in the Austrian problems (but do not appear in (38)) are chosen appropriately. In order to minimize notational complexity, we will demonstrate the above assertion for the case of a two period intertemporal technology; i.e., we will assume $T = 2$ in (38).[77]

Suppose that y^{0*}, y^{1*}, k^{2*} solves (38) when $T = 2$. Under certain conditions, we can define end of period 0 or beginning of period 1 capital stock price and quantity vectors P^{1*} and k^{1*} such that: (i) y^{0*} and k^{1*} solve the period 0 profit maximization problem (34) provided that $P^1 = P^{1*}$ and (ii) (k^{1*}, y^{1*}, k^{2*}) solves the period 1 profit maximization problem (33) provided that the P^1 which appears in (33) is equal to P^{1*}. The translation of the last rather technical sentence is this: period by period "Austrian" profit maximization can be consistent with the intertemporal Hicksian profit maximization model (38) provided that the correct "economic" capital stock prices P^{t*} are used in the single period profit maximization problems.

In order to establish the above assertion, it is necessary to introduce the period t variable profit function π^t that is dual to the Austrian technology set S^t for $t = 0, 1$:[78]

(40)　$\pi^t(p^t, k^t, k^{t+1}) \equiv max_{y^t}\{p^t \cdot y^t : (k^t, y^t, k^{t+1}) \in S^t\}, t = 0, 1.$

Using definition (40), the single period constrained profit maximization problem (34) can be rewritten as the following unconstrained profit maximization problem involving only the components of k^1:

(41)　$max_{k^1}\{-(1 + r^0)P^0 \cdot \bar{k}^0 + \pi^0(p^0, \bar{k}^0, k^1) + P^1 \cdot k^1\}.$

Suppose that $\hat{k}^1$ is a solution to (41) and each component of $\hat{k}^1$ is positive; i.e., $\hat{k}^1 \gg 0_M$. If $\pi^0(p^0, \bar{k}^0, k^1)$ is differentiable with respect to the components of k^1 at $k^1 = \hat{k}^1$, then the vector of first order partial derivatives of π^0 with respect to the components of k^1, $\nabla_{k^1}\pi^0(p^0, \bar{k}^0, \hat{k}^1) \equiv [\partial\pi^0(p^0, \bar{k}^0, \hat{k}^1)/\partial k_1^1, \ldots, \partial\pi^0(p^0, \bar{k}^0, \hat{k}^1)/\partial k_M^1]$, will satisfy the following first order necessary conditions to solve (41):[79]

(42)　$\nabla_{k^1}\pi^0(p^0, \bar{k}^0, \hat{k}^1) + P^1 = 0_M.$

Now use definition (40) for $t = 1$ and rewrite the period 1 constrained profit maximization problem (33) as the following unconstrained profit maximization problem involving the vector of beginning of period 1 capital stocks k^1 and the vector of end of period 1 capital stocks k^2:

(43)　$max_{k^1, k^2}\{-(1 + r^1)P^1 \cdot k^1 + \pi^1(p^1, k^1, k^2) + P^2 \cdot k^2\}.$

Suppose that $\tilde{k}^1 \gg 0_M$ and $\tilde{k}^2 \gg 0_M$ are solution vectors for (43) and that $\pi^1(p^1, k^1, k^2)$ is differentiable with respect to the components of k^1 and k^2 at $(k^1, k^2) = (\tilde{k}^1, \tilde{k}^2)$. Then the vector of first order partial derivatives of π^1 with respect to the components of k^1, $\nabla_{k^1}\pi^1(p^1, \tilde{k}^1, \tilde{k}^2)$, and the vector of first order partial derivatives of π^1 with respect to the components of k^2, $\nabla_{k^2}\pi^1(p^1, \tilde{k}^1, \tilde{k}^2)$, will satisfy the following first order necessary conditions to solve (43):[80]

(44)　$\nabla_{k^2}\pi^1(p^1, \tilde{k}^1, \tilde{k}^2) + P^2 = 0_M;$

(45)　$\nabla_{k^1}\pi^1(p^1, \tilde{k}^1, \tilde{k}^2) - (1 + r^1)P^1 = 0_M.$

Now assume that the intertemporal production possibilities set S is constructed using the one period technology sets S^0 and S^1 and definition (39) when $T = 2$. Using definitions (40), we can rewrite the constrained intertemporal profit maximization problem (38) (when $T = 2$) as the following unconstrained profit maximization problem involving the beginning and end of period 1 capital stock vectors k^1 and k^2 as decision variables:[81]

$$(46) \quad max_{k^1,k^2}\{-P^0 \cdot \bar{k}^0 + (1+r^0)^{-1}\pi^0(p^0, \bar{k}^0, k^1) + (1+r^0)^{-1}(1+r^1)^{-1}\pi^1(p^1, k^1, k^2)$$
$$+ (1+r^0)^{-1}(1+r^1)^{-1}P^2 \cdot k^2\}.$$

Assume that $k^{1*} >> 0_M$ and $k^{2*} >> 0_M$ solves (46) and that π^0 and π^1 are differentiable with respect to the components of k^1 and k^2 when $(k^1, k^2) = (k^{1*}, k^{2*})$. Then k^{1*} and k^{2*} will satisfy the following first order necessary conditions for solving (46):

$$(47) \quad (1+r^0)^{-1} \bigtriangledown_{k^1} \pi^0(p^0, \bar{k}^0, k^{1*}) + (1+r^0)^{-1}(1+r^1)^{-1} \bigtriangledown_{k^1} \pi^1(p^1, k^{1*}, k^{2*}) = 0_M;$$
$$(48) \quad (1+r^0)^{-1}(1+r^1)^{-1} \bigtriangledown_{k^2} \pi^1(p^1, k^{1*}, k^{2*}) + (1+r^0)^{-1}(1+r^1)^{-1}P^2 = 0_M.$$

We use the vector of partial derivatives $\bigtriangledown_{k^1}\pi^0(p^0, \bar{k}^0, k^{1*})$ in order to define a vector of end of period 0 "economic values" or shadow prices of capital P^{1*} :[82]

$$(49) \quad P^{1*} \equiv - \bigtriangledown_{k^1} \pi^0(p^0, \bar{k}^0, k^{1*}).$$

Now suppose that the P^1 in (41) and (42) is equal to P^{1*}. Since k^{1*} satisfies (49), then k^{1*} will also satisfy (42) if the P^1 in (42) is replaced by P^{1*} (and $\hat{k}^1$ is replaced by k^{1*}). Thus k^{1*} will solve the period 0 Austrian profit maximization problem (41) if the vector of shadow prices P^{1*} defined by (49) is used as the end of period 0 capital stock price vector P^1 in (41). In this case, the one period maximization of profits coincides with the business unit's long run intertemporal maximization of profits.

Now substitute the definition (49) of the shadow prices P^{1*} into the first order conditions (47). The resulting equation is (after simplification):

$$(50) \quad -(1+r^1)P^{1*} + \bigtriangledown_{k^1}\pi^1(p^1, k^{1*}, k^{2*}) = 0_M,$$

which is (45) if we replace P^1 by P^{1*} (and $\tilde{k}^1, \tilde{k}^2$ are replaced by k^{1*}, k^{2*}). Finally, after simplification, the first order conditions (48) are equivalent to the first order conditions (44) (where $\tilde{k}^1, \tilde{k}^2$ are replaced by k^{1*}, k^{2*}). Thus k^{1*}, k^{2*} will solve the period 1 Austrian profit maximization problem (43), provided that the beginning of period 1 price vector for capital stock components P^1 is replaced by the vector of shadow prices P^{1*} defined by (49).

The thrust of the above algebra is this: under some regularity conditions,[83] single period Austrian profit maximization is perfectly consistent with the long run intertemporal maximization of profits, provided that the business unit uses "economic" prices to value its end of period capital stock components. The problem with this result is that it is usually extremely difficult or impossible to determine these economic prices as outside observers of the business unit (or even as insiders); i.e., at the end of period 0, how can we determine P^{1*} defined by (49)? The difficulties involved in the practical determination of "economic" prices explain why most accountants dismiss the use of "economic values" as a practical

alternative for the valuation of a business unit's end of the period capital stocks. We will discuss alternative methods of valuation in more detail in section 6 below.

Traditional production function models do not distinguish capital as an input at the beginning of a period and capital as an output at the end of the same period as was done in the Austrian production function. In the following section, we indicate how a traditional production function can be derived from an Austrian production function.

5.3 The Traditional Production Function

"I belong to the party which is still looking to find, at the end of its journey, a rehabilitation of the so-called 'Production Function' $P = f(L, C)$ [where P is product, L is labour input and C is capital input] in some form or other; what I am looking for is a concept of capital which will ultimately allow us to think, more or less, in those terms". *John R. Hicks*[1961; 18]

"In the context of the Hicksian model, it is clear that we can construct several capital aggregates that must be carefully distinguished: (a) a current period capital stock aggregate (an input from the viewpoint of the current period) using current period capital stock prices as weights in the aggregation procedure; (b) a (depreciated) following period capital stock aggregate (an output from the viewpoint of the current period) using discounted expected following period capital stock prices as weights; (c) a current period investment aggregate (an output) using current period investment goods prices as weights in the aggregation procedure; and (d) a capital aggregate that is an aggregate of (a) and (b) where capital as an input and capital as an output are oppositely signed in the index number formula that is used". *W. Erwin Diewert*[1980b; 474-475]

We return to the Austrian model of section 5.1 and note that there is an easy way of simplifying the model so that we do not have to distinguish each durable commodity as both an input and an output: we need only use Leontief's [1936; 54-57] Aggregation Theorem. This result says that if two commodities are always used or produced in fixed proportions by a production unit in each period t, then the two commodities can be aggregated into a single composite commodity. More specifically, let x_1^t and x_2^t denote the quantities of say two inputs used during period t and let $p_1^t \geq 0$ and $p_2^t \geq 0$ denote the period t average price for each commodity. If $x_1^t = \alpha x_2^t$ for all periods t under consideration, then the two commodities can be aggregated into a composite commodity with period t

aggregate input X^t equal to the quantity of input 1 during period t, x_1^t and with period t composite price P^t equal to the period t value of the two commodities divided by x_1^t; i.e.,

$$(51) \quad X^t \equiv x_1^t; \ P^t \equiv [p_1^t x_1^t + p_2^t x_2^t]/x_1^t = p_1^t + \alpha p_2^t.$$

Definitions (51) can still be used to aggregate the two commodities even if say commodity 1 is an input and commodity 2 is an output; in this case, x_1^t and x_2^t have opposite signs and α, the factor of proportionality, is negative.

Now consider the case of a single durable input that lasts 2 or more periods and whose productivity declines only with age (and not use). Suppose that $k^0 > 0$ units of the (new) durable input were purchased at the start of period 0 at price $P^0 > 0$, and suppose that the end of period 0 price for depreciated units is $P_u^1 >> 0$. Then from the perspective of the end of period 0, the net cost of using k^0 units of the durable input during period 0 is

$$(52) \quad (1 + r^0)P^0 k^0 - P_u^1 k^0 = w_e^0 k^0$$

where w_e^0 is the end of period 0 user cost defined by (7) above. Now define $x_1^t \equiv k^t, x_2^t \equiv -k^t, p_1^t \equiv (1 + r^t)P^t$ and $p_2^t \equiv P_u^t$ for $t = 0, 1, \ldots$ and apply Leontief's Aggregation Theorem. It can be seen that $X^t = k^t$ and $P^t = w_e^t$ for $t = 0, 1, \ldots$; i.e., the ex post end of period 0 user cost w_e^0 can be viewed as the period 0 price for the use of one unit of an aggregate of capital where the two capitals are capital input at the beginning of period 0 and capital output at the end of period 0. The resulting aggregate capital can be viewed as the capital input which appears in a "traditional" production function and a user cost is the price which is associated with the capital aggregate.

Obviously, the above aggregation technique can be applied to all vintages of a capital input provided that declines in value over the period are independent of use. If declines in value are not independent of use, then we need to distinguish different end of period prices that depend on the intensity of use of the durable input over the accounting period. For example, suppose that k_L^0 units of our new durable good in the previous paragraph were used "lightly" during period 0 and that k_H^0 units were used "heavily" (or more intensively), where $k^0 = k_L^0 + k_H^0$. Suppose that the end of period 0 price for a unit that is used lightly is P_{uL}^1 and for a unit that is used heavily is P_{uH}^1. We can define period 0 Leontief aggregate prices for new units of capital that are lightly and heavily used, w_{eL}^0 and w_{eH}^0 respectively, as follows:

$$(53) \quad w_{eL}^0 \equiv (1 + r^0)P^0 - P_{uL}^1; \ w_{eH}^0 \equiv (1 + r^0)P^0 - P_{uH}^1.$$

The Leontief aggregate quantities that are matched up with the above user costs are k_L^0 and k_H^0 respectively. Of course, the disaggregation of each type of beginning of the period capital input into separate categories depending on period 0 use can be carried out as finely

as seems empirically necessary.[84] Thus Leontief's Aggregation Theorem can be applied to aggregate capital inputs in an Austrian production function even if the value of the assets declines with use as well as with age.

The above aggregation technique will not work for assets that lose their identity during the period 0 production process; e.g., a computer chip on hand at the beginning of the period emerges as part of a computer at the end of the period[85] or a concrete foundation at the beginning of the period becomes part of a building at the end of the period, etc.

We turn now to a discussion of what seems to be a rather philosophic question: what exactly is productive activity? It turns out that answers to this question are crucial in determining how to define business income.

5.4 The Three Basic Forms of Productive Activity

"Production, in the narrow sense, changes the form and nature of products. Trade and transport change their external relations".

Alfred Marshall[1920; 64]

"If the difference of the *place* at which goods are available is a sound economic reason for exchanging fungible goods that are in other respects entirely similar, and if the advantage and convenience of the present *place* may justify the claim and allowance of a premium, just as much may the difference of the *time* at which similar goods are available be a sound reason for their exchange, and a guarantee that there will be premium on the – more valuable – present goods. This premium, and nothing else, is Interest".

Eugen von Böhm-Bawerk[1891; 295]

Over the years, three general forms of productive activity have been identified: (i) the *transformation* of less valuable commodities into more valuable commodities; (ii) the *transportation* of commodities from one location to a more valuable location, and (iii) the *storage* or *holding* of goods from one time period to a future time period when they will be more valuable.

The first two types of productive activity are not controversial and were identified long ago by Marshall [1920; 64] in 1890; the transformation activity is regarded as being the most fundamental type of productive activity by economists and accountants.[86] Böhm-Bawerk [1891; 295] noticed the analogy between transportation and storage activities and introduced the third type of productive activity into the literature. Over the years, many accountants (such as Schmidt,[87] Edwards and Bell[88] and Chambers[89]) and economists

(such as Lerner[90] and Debreu[91]) have argued that the third type of productive activity is just as valuable as the other two types. However, when the third type of activity is labelled as "speculative activity" instead of "storage activity", many economists and accountants have objected to treating the third type of productive activity in exactly the same manner as the first two types. Specifically, these economists and accountants argue against the inclusion of capital gains on assets held by a business unit over an accounting period. These objections will show up in section 6 below when we consider various approaches to the valuation of end of the period capital stocks. However, in the present section, we shall present some of the arguments that have been advanced by economists and accountants to *deny* treating speculative gains or capital gains on assets held through an accounting period as net revenues or components of business income in a manner that is symmetric to the treatment of transformation and transportation activities. We shall consider four types of objection.

The first objection states that capital gains on assets held through the accounting period are capital losses to someone else and hence there is no net gain to the community. Consider the following quotations by the accountants Schmidt and Crandell:

> "Only in one case can appreciation be real profit to the business man, viz., when he uses money credit to buy goods for speculation outside of his regular business needs. If his selling prices thereafter are higher than the money lent plus interest and costs after selling the goods, the difference will be his realized speculative gain. This kind of profit is especially high in times of rising general price levels. But this kind of private profit is no profit to the community, because the lender of money loses the same buying power on his money that the borrower gains".
> *Fritz Schmidt*[1931; 291]

> "What treatment should be accorded the speculative gains and losses realized from trading among individuals in securities? It is obvious that these sorts of transactions cannot increase the national wealth, hence the national income cannot be affected thereby. Whatever one gains the other loses".
> *William T. Crandell*[1935; 399]

The argument that the capital gains made by one business unit must be offset by capital losses made by some other consumer or business unit does not seem to be correct. Consider the case of a one person economy that controls a single business unit. Any capital gains made by the business unit that result from an optimal intertemporal allocation of resources are not offset by capital losses.[92]

The second objection to the inclusion of capital gains in income is more subtle: speculative holding activities do not enhance the productive powers of the economy and hence

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any increase in revenues resulting from these activities should not be recognized as a benefit to the economy; in fact, focusing on speculative gains may be bad for the economy because it will cause managers to not focus on the first two types of productive activities[93] (transformation and transportation). Consider the following quotations which are representative of this point of view:

"Some theoretical explanation of the reasons why appreciation cannot be profit is needed at this point. For this purpose we must consider the enterprise as a part of the national production machine. It will then be clear that a maintenance of total productivity as of a certain moment will only be possible, if the productive instrumentalities of all individual enterprises concerned are preserved intact. The maintenance of productive power as a whole is not possible if accounting is based on an original value basis. The reason is that pure appreciation would then appear as profit whenever a change of value has taken place between the purchase and selling dates for the materials and wages that compose a product".   *Fritz Schmidt*[1931; 289]

"The appreciation in value of capital assets and land must not be treated as an element in national income. Depreciation due to physical wear and tear and obsolescence must be treated as a charge against current income, but not the depreciation of the money value of an asset which has remained physically unchanged. Appreciation and depreciation of capital were included in the American statistics of national income prior to 1929, but now virtually the same convention has been adopted in all countries".   *Colin Clark*[1940; 31]

"Enhancement of asset values as a result of increased market prices does not, without realization of such appreciation through sale, constitute a basis for recognition of revenue to the business enterprise. However, the realization of gain on the sale of a capital asset does not necessarily imply any contribution by the seller to the social product during the period of realization. Because such gains are irrelevant to production of the period, capital gains (and losses) are excluded from calculations of national income and product. It is seen then that, whereas standards of accounting for revenue provide for recognition of capital gains once they have been realized, such gains find no place at all in the accounting for the economy".   *Gilbert P. Maynard*[1952; 190]

"The essence of the difference between financial capital maintenance and all concepts of physical capital maintenance is in the treatment of the effects of price changes while assets are held. Under financial capital maintenance, all such effects are included in income .... Under physical capital maintenance,

the effects of price changes are excluded from income on the grounds that, if positive, they do not enable an enterprise to increase its operating capability or, if negative, they do not force a reduction".   *Bryan Carsberg*[1982; 62]

The above authors have implicitly ruled out storage and holding activities as being productive like transformation and transportation activities. However, we can follow Böhm-Bawerk and argue that holding activities are completely analogous to transportation activities. Since transportation activities are regarded as being productive, so should holding activities.

The third objection to the inclusion of capital gains in the period by period income statements of a business unit has been made by accountants and it is an objection only to the inclusion of unrealized capital gains (i.e., no *sale* of the asset which has experienced a capital gain over the accounting period has been made) in income, not to the inclusion of realized capital gains (i.e., the appreciating asset has been sold during the accounting period). The objection is that unrealized capital gains should not be included in the period's income due to their *hypothetical* and *unverifiable* nature. Consider the following quotations:

"*Appreciation, Capital Gains and Losses.* A part of the ultimate net income of an enterprise can be assigned in some cases to natural growth and other increases in value. In the case of timber tracts, orchards and similar properties, natural increase, commonly called *accretion*, is an important factor in financial history. In other cases enhancement of property values due to changing business and general economic conditions, a general rise in the price level, or other factors which result in an increase in effective value over actual cost, usually referred to as *appreciation*, are of marked significance. At what point, in the succession of events that lead to final fruition of these gains in cash, should the accountant recognize the change? Eventually, if no cognizance is taken of it before, the gain will be realized in cash when the property itself, or the product resulting from its use, is sold. Until such time as the gain is validated by sale, the increased value is commonly characterized as unrealized and the gain as 'unearned' or 'unrealized income'".   *William T. Crandell*[1935; 389]

"The various codifications of accounting doctrine during the past two decades have been in general agreement that revenue should be recognized in the accounts only when certain tests of realization have been met. In the vast majority of cases, revenue realization is marked by a discrete event, that of sale and delivery of goods or services. Thus accountants draw a distinction between the earning

or accrual of revenue throughout the productive processes and the realization of revenue, giving recognition in the accounts only to the latter….

The national income accountant is concerned with the *creation* of product, not alone with its subsequent sale".   *Gilbert P. Maynard*[1952;189][94]

"There is another important respect in which business and social accounting differ which is worthy of comment here. Although business accountants are fully aware of the tentative nature of their measurements of income, they place great emphasis upon the objectivity and verifiability of the business data to which they grant recognition in the accounts".   *Gilbert P. Maynard*[1952; 193]

The accountant's objection to the inclusion of *unrealized* capital gains as a contribution to the income of an accounting period due to their hypothetical nature is a valid one. However, the traditional accounting solution to the unrealized capital gains problem is to assume that no capital gains occur in any accounting period unless a realization occurs in some period (i.e., the asset is sold) in which case, all of the capital gains that accrued over the many accounting periods that the asset was held are imputed to the period of sale. This historical cost treatment of capital gains can create tremendous distortions (particularly in inflationary environments) to both the periodic income statements and balance sheets of the business unit.[95] Thus the accountant's treatment of unrealized capital gains (i.e., to exclude them from the income statement) is just as hypothetical (and more misleading in an inflationary environment) as including them in periodic income.[96] However, the historical cost accountant's objection to the hypothetical nature of period by period valuations of the capital stock components held by the business unit could be used to justify a separate treatment of unrealized capital gains on income statements rather than simply lumping them in with the more objective (transformation and transportation) sources of income.[97]

A fourth argument against the inclusion of capital gains in income statements runs as follows: for most businesses, capital gains or losses are an unintended consequence of their normal productive activities and moreover, in the long run, these gains and losses will tend to cancel. Hence it is not worth the bother of including these gains and losses as income, particularly when income may be taxed and hence a large unrealized capital gain may lead to a large tax bill which in turn may lead to a curtailment of the firm's normal productive activities. However, this line of thought led to a difficulty: what if the normal activity of a business unit was speculative (e.g., a commodities trader or a land speculator)? These business units would seem to be excluded from paying any income taxes on their earnings from speculative activities. To get around this difficulty, Plehn and other economists introduced the concept of *recurrence* of income:

"Income is essentially wealth available for recurrent consumption recurrently (or periodically) received. Its three essential characteristics are: *receipt, recurrence, and expendability*".   *Carl C. Plehn*[1924; 5]

"It will, I think be readily admitted that those particular gains and profits which are recurrent, expendable receipts are the ones about whose income character there is seldom any doubt. Thus the gains and profits of a merchant are his income. The possible or even probable irregularity or uncertainty which distinguishes them from some other incomes does not seem to militate against their inclusion in income, provided they are *expected* to be recurrent. The same is true of the gains and profits of dealers in capital assets, for the lands, stocks and bonds, houses and the like are their stock in trade .... But it is when gains and profits lack one or two of the three characteristics of income, or have them in less than complete form, that a question arises. The one that is most often lacking is recurrence. Thus gains and profits from transactions outside of one's regular vocation or line of business, like the profit from the sale of a home, are of doubtful income character".   *Carl C. Plehn*[1924; 10]

"The British income tax places very heavy stress upon the annual character of income. For an explanation of this conception, which results in the exclusion from taxable income of gains of an irregular nature, one must go back as far as the fifteenth century, when, with an agricultural society where few fortuitous gains developed, the idea of receipts as being annual in character became deeply impressed upon the minds of the people. It became the habit to think of one's regular receipts as his income, and to consider irregular receipts as additions to capital".   *Robert Murray Haig*[1921; reprinted 1959; 69]

Thus if a business unit *regularly* makes profits on its speculative activities, the resulting profits are regarded as income but any capital gains on *occasional* speculative activities are not regarded as income according to the recurrence criterion for income. Of course, the problem with this concept is that it is difficult to draw the *boundaries* of recurrence:

"When is income recurrent? Professor Plehn expressly says it need not be perfectly regular. But how irregular can it be and still be 'recurrent'? The big profit on the sale of an old homestead may well occur twice in a lifetime. Does it not then 'recur'? If we extend the picture through two or more lifetimes 'recurrence' becomes altogether likely. In the case of corporations whose life goes on indefinitely every windfall, or extraordinary profit, may some day be duplicated. Evidently the 'recurrency' concept turns out to be too elusive to pass muster as a basis for analysis".   *Irving Fisher*[1924; 666]

More fundamentally, if capital gains are regarded as being valid additions to income in some contexts, then why should they be excluded in other contexts?[98] When we view speculative activity as being analogous to transportation activities, it is obvious that a strong case can be made for including it as a valid form of productive activity.

We turn now to our final topic in the general area of production theory.

5.5   Where does the Production Function Come From?

"The theory of the firm subsumed in general equilibrium theory is incredibly lean. It is summarized in the representation of a firm as a subset of the commodity space (preferably convex, closed, etc.). The simplicity of this theory derives from the activity analysis model of production which describes a firm as a given technology–a book of proprietary information about feasible transformations of inputs into outputs. A technology can evolve over time but it is not a matter of choice; indeed, a basic premise of general equilibrium theory is that information is not a decision variable".   *Robert Wilson*[1975; 184]

"An invention is viewed as a new process of production, or as a new vector of input-output coefficients".   *William D. Nordhaus*[1969; 19]

Up to now, we have assumed that the single period production functions of section 5.1 or the intertemporal production function of section 5.2 are exogenously determined and known to the business unit. We think of the corresponding production possibilities sets as a given set of plans or operating procedures that are known to the management of the production unit. But where does this knowledge of the production possibilities set come from? How does this knowledge expand over time; i.e., how does innovation occur? These are the questions that we wish to address in this section.[99]

Knowledge of the set of feasible input and output combinations that a business unit in a specific geographic location[100] could use and produce during an accounting period comes from at least three sources: (i) operating manuals or other written (or computer accessible) materials that are available in the establishment; (ii) knowledge of production techniques that is embodied in employees and managers who work in the establishment and (iii) knowledge that is embedded in establishment machines. This provides a brief answer to the first question above. Our answer to the second question will not be so brief.

How does knowledge of new techniques of production (process innovations) and of new products (product innovations) get created? Traditional production theory as is embedded in general equilibrium theory[101] is silent on this point (even though many economists have

noted that knowledge creation cannot be regarded as exogenous[102] and critics[103] have noted this deficiency of traditional production theory).

Obviously, specialized schools, universities and publicly supported research labs are a primary source of the creation of new knowledge but a considerable amount of innovative activity is undertaken by individual inventors and the research departments of private firms.

Arrow[104] and others[105] have attributed increases in productivity (more output for the same amount of input) to experience or the incidental effect of new investments. Arrow [1962; 155-157] explains his theory of innovation as follows:

> "I would like to suggest here an endogenous theory of the changes in knowledge which underlie intertemporal and international shifts in production functions. The acquisition of knowledge is what is usually termed 'learning' and we might perhaps pick up some clues from the many psychologists who have studied this phenomenon .... I advance the hypothesis here that technical change in general can be ascribed to experience, that it is the very activity of production which gives rise to problems for which favorable responses are selected over time .... The first question is that of choosing the economic variable which represents 'experience' ... I therefore take instead cumulative gross investment (cumulative production of capital goods) as a index of experience".

A somewhat similar theory of innovation was advanced by Allen [1983] which he called collective invention.[106] Allen explained his theory as follows:

> "Thus, if a firm constructed a new plant of novel design and that plant proved to have lower costs than other plants, these facts were made available to other firms in the industry and to potential entrants. The next firm constructing a new plant could build on the experience of the first by introducing and extending the design change that had proved profitable .... Collective invention was thus like modern research and development in that firms (and not individual inventors) generated the new technical knowledge. However, collective invention differs from R & D since the firms did not allocate resources to invention–the new technical knowledge was a by-product of normal business operation–and the technical information produced was exploited by agents other than the firms that discovered it".   *Robert C. Allen*[1983; 2]

> "As long as the rate of investment was high, the rate of experimentation and the discovery of new technical knowledge was also high. On the other hand, if the rate of investment fell for any reason, the rates of experimentation and invention fell with it".   *Robert C. Allen*[1983; 3]

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Allen illustrated his theory using data on changes in the height and operating temperatures of blast furnaces in England between 1850 and 1875 and he summarized his results as follows:

> "Increasing furnace height and blast temperature led to lower fuel consumption and costs. The first firms to build tall furnaces might have treated this knowledge as a trade secret, but they did not. This information was made available to other parties through two channels–informal disclosure and publication in the engineering literature".   *Robert C. Allen*[1983; 6-7]

Thus Allen modelled innovation as follows: as firms invested in new facilities, bolder firms undertook marginal changes in the design of their facilities or machines; successful design changes were then communicated to the industry as a whole through trade associations or formal publication in journals or magazines. It is interesting to note that Marshall advanced similar ideas many years ago.[107]

The next batch of theories of innovation date back to the origins of economics.

Adam Smith [1963; 8] observed that many inventions or innovations are made by workers who simply figure out better ways of accomplishing a task that they are presently engaged in:

> "I shall only observe, therefore, that the invention of all those machines by which labour is so much facilitated and abridged, seems to have been originally owing to the division of labour. Men are much more likely to discover easier and readier methods of attaining any object when the whole attention of their minds is directed towards that single object, than when it is dissipated among a great variety of things. But in consequence of the division of labour, the whole of every man's attention comes naturally to be directed towards some one very simple object. It is naturally to be expected, therefore, that some one or other of those who are employed in each particular branch of labour should soon find out easier and readier methods of performing their own particular work, whenever the nature of it admits of such improvement".[108]

Smith also observed that many improvements in productivity result from the specialization of labour: a worker who is able to concentrate or specialize on one task will become more proficient at that single task due to: (i) improvements in dexterity or physical skill and (ii) the elimination of the fixed costs in going from one type of task to another:

> "This great increase of the quantity of work, which, in consequence of the division of labour, the same number of people are capable of performing, is owing to three different circumstances; first, to the increase of dexterity in every

particular workman; secondly, to the saving of the time which is commonly lost in passing from one species of work to another; and lastly, to the invention of a great number of machines which facilitate and abridge labour, and enable one man to do the work of many". *Adam Smith*[1963; 7]

Note that Smith suggested a third productivity benefit due to the increased specialization of labour: specialized routine operations by workers lend themselves to replacement by more efficient machines. Marshall[109] and Young[110] made similar observations. These observations are still valid today; e.g., many clerical and lower level managerial jobs are being replaced by computers.[111]

Smith [1963; 14] also pointed out that the division of labour was limited by the extent of the market; i.e., as the scale of the establishment grows due to the growth of markets for its outputs, the possibility of using specialized labour (and capital!) inputs also grows. As a corollary to his general principle, Smith pointed out that cities had larger markets than small towns and hence would support a higher degree of specialization in labour markets:

"There are some sorts of industry, even of the lowest kind, which can be carried on no where but in a great town. A porter, for example, can find employment and subsistence in no other place. A village is by much too narrow a sphere for him; even an ordinary market town is scarce large enough to afford him constant occupation". *Adam Smith*[1963; 14]

Alfred Marshall further refined Adam Smith's idea that increases in the scale of an enterprise would generally lead to more efficient production by introducing the ideas of internal and external economies of scale:

"We may divide the economies arising from an increase in the scale of production of any kind of goods, into two classes–firstly, those dependent on the general development of the industry; and, secondly, those dependent on the resources of the individual houses of business engaged in it, on their organization and the efficiency of their management. We may call the former *external economies*, and the latter *internal economies*". *Alfred Marshall*[1920; 266]

Internal economies of scale occur if output expansion leads to a less than proportional increase in the use of inputs; i.e., internal economies are equivalent to increasing returns to scale in more modern language. The increasing returns to scale phenomenon could be regarded as meaning that the production possibilities set of an establishment has a particular shape and hence it might appear that the increasing returns to scale phenomenon can be accommodated by traditional production theory. This is true once a business unit has actually run an establishment at a higher scale and has demonstrated that the

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technology works at the higher output levels, but the *first* successful demonstration of operating a technology at a higher scale has much the same character as establishing the feasibility of an innovation.[112]

There appear to be four main sources of internal economies of scale:

(i)   *Indivisibilities*; i.e., most labour and capital inputs cannot be purchased in fractional amounts and all capital inputs have upper and lower limits on their capacities.[113]

(ii)   *The Laws of Physics*; i.e., Kaldor[114] (and Marshall[115]) noted that the three dimensional nature of space leads to certain economies of scale.

(iii)   *The Existence of Fixed Costs*; i.e., these are the efficiencies which result from averaging or amortising fixed costs over higher output levels. Before a machine yields a benefit from its operation, it may require the services of an operator who may have to be transported from one location to another[116] and the machine may require a warming up period before production can begin. These are examples of fixed costs whose effect becomes relatively smaller the greater the length of time that the machine is continuously operated.

(iv)   *The Law of Large Numbers*; i.e., these are efficiencies that result from the laws of probability theory. For example, consider a power plant that uses a number of identical engines. If the probabilities of engine failure are independently distributed, then having one set of spare parts on hand will generally be sufficient whether the plant has one engine or ten engines. Similarly, a large bank will not require as high a proportion of cash reserves to meet random demands as a small bank.[117] In a similar vein, a large property insurance company whose risks are geographically diversified faces a smaller probability of bankruptcy than a small insurance company, etc.

We note that operations research and management science have developed mathematical techniques which enable the business unit to achieve internal economies with respect to the factors listed in (i), (iii) and (iv) above.

Two examples of Marshall's external economies of scale are: (i) reduced prices for inputs due to bulk purchasing[118] and (ii) the large scale of a business unit translates into a large demand for inputs and this in turn can encourage specialized suppliers to come into existence.[119] Thus external economies of scale reflect favorable changes in the environment facing the expanding business unit (lower input prices and new intermediate input suppliers).

What is the underlying cause of both internal and external economies? It seems that Adam Smith [1963; 14] had the answer to this question: growth of the market.

Some of the obvious factors that facilitate growth of the market are: (i) transportation improvements[120]; (ii) population growth[121]; (iii) reduction in trade barriers[122]; (iv)

improvements in advertising[123]; (v) improvements in communications[124]; and (vi) growth of physical and human capital which leads to income growth which in turn leads to a growth in effective demand.

Perhaps the most interesting aspect of the creation of new knowledge about new production techniques is not the initial creation of the new knowledge but its diffusion to the local establishment level. The fact that a new product or production process has been developed somewhere in the world is of little significance to a local establishment that could use the innovation if the original knowledge is not transmitted or diffused to the establishment. Some of the factors that facilitate the rapid diffusion of new (and old) knowledge into a local market area are: (i) access to public libraries and university libraries[125]; (ii) access to newspapers, periodicals, journals, magazines, how to do it books, etc.[126]; (iii) memberships in trade associations, industry associations, professional societies, etc.;[127] (iv) access to international meetings and trade fairs where knowledge can be transmitted on a face to face basis[128] (adequate local transportation infrastructure will facilitate this access[129]); (v) access to good schooling and specialized training programs[130]; (vi) access to specialized consulting services and product information and (vii) access to telecommunications services[131] (i.e., having good local telecommunications infrastructure). The point that we are trying to make here is that a local market area does not necessarily have to devote a high percentage of its resources to primary research and development (i.e., to the creation of new products and processes): it need only have easy access to the sources of new knowledge.

We can summarize the results of this section as follows: (i) Traditional production theory that regards the feasible set of inputs and outputs for a business unit as an exogenous set is a serious oversimplification of reality. (ii) Current operating techniques can be regarded as exogenous, but the search for new products and new processes is an endogenous activity of the business unit that leads to an expansion of its single period production possibilities set. (iii) Exactly how this endogenous expansion takes place has not yet been adequately modeled, but it seems that the process of creating an innovation involves fixed costs and uncertain benefits.[132] (iv) It is difficult to separate a shift in the establishment production function (due to innovative activity) from a movement along a production function (due to a change in scale or to a change in input prices[133]); any new choice of inputs and outputs by a firm during an accounting period may involve innovative activity or it may simply involve a routine response to a change in the market environment.

We turn now to the discussion of a much narrower topic that will help us to evaluate the period by period performance of a business unit.

6.0   Alternative Approaches to Asset Valuation

"There are four possible bases which might be adopted: (1) liquidation value, or that value which is likely to be realized if the assets were thrown onto the market in the process of an orderly or forced liquidation; (2) original cost with proper allowance for decline in value of current assets and allowances for depreciation and depletion of fixed assets; (3) capitalized income producing value; (4) present market price of replacing or reproducing a similar asset in its present state of condition".   *H.C. Daines*[1929; 98]

"Regarded from the more technical viewpoint of accounting, the problems faced and solved by German accountants during the period of absurd price fluctuations are quite worthy of study by accountants of all other countries. For the problems associated with keeping and interpreting financial records that must be expressed in a monetary unit oscillating even hourly are problems that the rest of the world must face to a less extreme degree".

Henry W. Sweeney[1927; 180-181]

In this section, we return to the fundamental problem of accounting: how to determine end of the accounting period prices for durable assets that are held by the business unit for multiple accounting periods. As can be seen from the above quotation by the accountant Daines, there are many possible methods for asset valuation that could be used. We shall consider six methods: (i) historical cost valuations; (ii) general purchasing power adjusted historical costs; (iii) net realizable values or appraisal values; (iv) replacement costs; (v) future discounted cash flows and (vi) asset specific index number adjusted historical cost. The method of valuation that is in general use today by accountants is the first method: historical cost accounting. However, the quotation by Sweeney should alert us to the problems associated with this method when there is general inflation in the economy. The other five methods attempt to deal with the valuation problem when there is general price instability.

6.1   Historical Cost Valuation

"Its greatest advantage is the fact that an original cost method is most easily subject to objective verification; it is the easiest to use in practice".

H.C. Daines[1929; 98]

"Today's dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of

different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber tape-line".

<div align="right">*Livingston Middleditch*[1918; 114-115]</div>

We have already discussed how historical cost depreciation (i.e., decline in asset value over an accounting period) is determined in section 4.3 above: once a useful life for an asset has been estimated and a corresponding depreciation schedule has been determined, the initial purchase cost of the asset is allocated across accounting periods as a sum of periodic depreciation allowances. The corresponding historical cost value of the asset at the end of an intermediate accounting period is simply the initial purchase cost less the accumulated depreciation allowances over prior periods.

The main problem with historical cost valuation of assets shows up if there is a large change in the price of the asset (due to general inflation for example) from the time of its purchase to the end of the current accounting period: the historical cost valuation may bear no resemblance at all to a current market valuation for the asset. Thus in an inflationary situation, historical cost depreciation allowances will be understated, income will be over-stated and income taxes may become capital taxes.[134] The problem is that historical cost accounting implicitly assumes that monetary values at the end of an accounting period are comparable to monetary values at the beginning of the accounting period; i.e., there is an implicit assumption of price level stability. The accountant Middleditch [1918] challenged this implicit assumption, having observed the tremendous inflation that occurred during World War I.[135]

There are two main virtues that are claimed for historical cost accounting: (i) it is *objective* and (ii) it is *conservative*. Both of these virtues are subject to criticism. We have already seen in section 4.3 that the determination of the length of life of an asset and the determination of the associated depreciation rates are far from being objective. Moreover, even if "economic" depreciation rates were used in order to allocate the initial purchase cost of an asset over its useful life, the resulting historical cost end of period values will be completely meaningless in a high inflation environment; i.e., they will not reflect current opportunity costs or market values. Thus historical cost accounting values might be *objective* but *irrelevant*.[136] *Conservatism*, on the other hand, conflicts with *accuracy*; i.e., if we wanted to be super conservative, why not assume all intermediate asset values are zero? The absurdity of this statement should make us realize that accuracy is a much more important virtue than conservatism.[137]

It is perhaps useful to elaborate a bit more on the meaning of "accuracy" in the context of determining period by period values for the assets of a business unit. It seems clear that there cannot be an answer to the problem of constructing intermediate values that is as unambiguous as the actual selling price of an asset; i.e., we can only make estimates of these intermediate values. Thus it seems reasonable to follow the example

<div align="center">55</div>

of Morgenstern [1963; 77] and regard these estimated intermediate values as probability distributions. "Accuracy" in this context could be defined as providing a suitable measure of central tendency (e.g., a mean valuation) along with a measure of dispersion (e.g., a variance). Unfortunately, accounting theory (and practice) has not proceeded along these lines[138], although occasionally, accountants recognize that introducing statistical concepts into accounting would be useful.[139]

We turn now to a discussion of other methods for valuing assets on a periodic basis–methods that will more closely approximate current market values or opportunity costs.

6.2   Purchasing Power Adjusted Historical Cost

"It is obvious, therefore, that if quantities, whether measured in pounds or bushels or dollars, are to be correctly combined or compared, the unit of measurement must be homogeneous . . .. Yet many men who are not measuring their heights with fluctuating rulers, and who would throw verbal stones at such a silly doing, are complacently living in a similar kind of glass house, a business structure where in the substance of value continues to be measured by a dollar of seriously fluctuating size".   *Henry W. Sweeney*[1936; reissued 1964; 11]

"Professor Baxter [1976] has characterized the development of Latin American inflation accounting systems as having two stages: firstly, fixed assets and depreciation are adjusted by reference to a general index, and, secondly, at a later stage, the 'time-log' error on stocks [inventories] and monetary working capital is corrected by the application of an index".

David Tweedie and Geoffrey Whittington[1984; 243]

This method of constructing a current value at the end of an accounting period originates with Middleditch [1918][140] and works as follows. Suppose an asset was purchased at the beginning of accounting period 0 at the price P^0, the period 0 depreciation rate is δ^0 and a general rate of price inflation over period 0 is ρ^0 (i.e., the general price level at the end of the period divided by the general price level at the beginning of the period is $1 + \rho^0$). Then the historical cost accounting value of the asset at the end of the period is $(1 - \delta^0)P^0$ but the *GPLA (General Price Level Adjusted)* value is

(54) $(1 - \delta^0)(1 + \rho^0)P^0$.

The advantage of this method for constructing current asset values on a period by period basis is its relative *simplicity* (adjusted historical cost values at the beginning of the period

need only be inflated by the common indexation factor $1 + \rho^0$) and its *objectivity* (once the appropriate indexation factor $1 + \rho^0$ has been chosen).[141]

In response to rapid inflation or a hyperinflation, GPLA accounting is the main form of current value accounting that has been used historically.[142]

Note the difference between ρ^0, a general inflation rate, and the asset specific inflation rate i^0 defined earlier by (5). In general, ρ^0 will not equal i^0 and hence the GPLA value for the asset will not equal its end of period selling price replacement value. This is the weakness of General Price Level Adjusted accounting. However, its strength is that it will adjust for the effects of general inflation.

The remaining topic to be discussed is how to choose the general inflation rate ρ^0.[143]

One of the simplest choices is to use the inflation rate for a widely traded commodity (such as gold[144]) as the index of general inflation. Another alternative is to use the rate of increase in the exchange rate of the country where the business unit is (primarily) located against a stable currency.[145] Instead of using the price of gold or any single commodity as the indicator of inflation, the general inflation between the beginning and the end of the accounting period might be better captured by looking at the price change of a "representative" basket of goods. As a further refinement, we could replace a fixed basket price index by a more general price index such as the Fisher [1922] ideal price index, which allows for substitution in response to price changes.[146]

Accountants and economists have struggled with the problem of choosing an appropriate price index to represent inflation for approximately a century. Many of the problems have still not been resolved[147]: (i) Which commodities should be included in the index? (ii) How should the individual price ratios be weighted[148]; i.e., what is the theoretically correct functional form for the price index? (iii) If the accounting period is shorter than a year, how can we deal with seasonal commodities that might be present in the index?[149] (iv) If we decide to use a consumer price index as a measure of general inflation, what is the appropriate class of consumers whose preferences should be reflected in the price index; i.e., should we construct a consumer price index that pertains only to investors in the business unit (some of whom may reside in foreign countries[150]) or should we broaden the class of consumers to be included in the index to include employees, creditors and potential investors?[151]

Even though the above questions are difficult to answer, we agree with Staubus that adjusting historical costs for general inflation by an imperfect index will generally be an improvement over historical cost accounting:

> "The argument that the corporate accountant cannot use the different purchasing power indexes of each individual shareholder must be read as either a weak excuse for inaction or an insistence on a degree of perfection that accountants have not reached in the past and are not likely to reach in the future.

Surely a broadly based price index provides a better measure of the change in the measuring unit than the assumption that there is no change at all, as the millions of people who base contracts on such indexes recognize".

<div align="right">George J. Staubus[1975; 44-45]</div>

It is sometimes asserted that General Price Level Accounting adds no additional information over that which is available from reading historical cost accounting balance sheets[152]; i.e., if investors know historical cost values and they can look up the relevant general inflation index, then they can readily calculate the adjusted asset values defined by (54). This would be true if the business unit made the following information available to investors in each accounting period: (i) the value of new investments made in each period and (ii) the historical cost residual value of all assets that are sold or retired during the accounting period. In general, this information is not provided in balance sheets; hence providing investors with an aggregate GPLA asset value will provide new information that could not be calculated by individual investors.

6.3   Net Realizable Values (Exit Values)

"Some economists, notably Professor Jacob Viner of the University of Chicago, hold the belief that the value which the assets would bring in the market is the only proper basis of value for use in accounting".   H.C. Daines[1929; 98]

"These markets [for assets] can be divided into two kinds, the markets in which the firm could buy the asset in its specified form and at the specified time and the markets in which the firm could sell the asset in its specified form and at the specified time. The prices obtained in markets of the first group we shall call entry prices; the prices obtained in markets in the second group we shall call exit prices".   Edgar O. Edwards and Philip W. Bell[1961; 75]

In our earlier discussion of depreciation, we saw that a century ago, it was not unusual for accountants to value the fixed assets of a business unit at the end of an accounting period by appraised values; i.e., estimates of the net realizable values that the assets would bring in the market at the moment in time. However, during the first 35 years of the current century, many business firms arbitrarily revalued their fixed assets to suit their immediate purposes.[153] By the 1930's, the accounting profession reacted against these abuses by adopting the historical cost accounting methodology for valuing assets, and the accounting profession as a whole has stuck to this position since that time (except when an economy

experienced very rapid inflation in which case General Price Level Adjusted accounting has been temporarily adopted). However, most economists and some accountants, such as Sweeney [1936; 44-53], Staubus[154], Edwards and Bell [1961], Chambers[155] and Sterling[156], have advocated the use of current values to value assets at the end of each accounting period.

The basic problem with the use of current values is that it is difficult to determine exactly what is the "correct" concept for a current value. Edwards and Bell [1961; 75] distinguish between an *entry value* (the minimum cost of purchasing a replacement for a currently held asset) and an *exit value* (the maximum price a currently held asset could be sold for in the market less the transactions costs of the sale; i.e., the net realizable value for the asset).[157] In this section, we will focus on the problems associated with the use of exit values and we will deal with entry values in the next section.

Historical cost accountants have two principle objections to the use of (imputed) net realizable values to value assets held by a business unit at the end of an accounting period: (i) they are not *objective* and (ii) they are not *additive*.

On the lack of objectivity of net realizable values, consider the following quotations:

> "Which alternative should be used as a basis? The highest, or the lowest, or an average? How should the search area, to get offers or find prices, be determined?". *Yuji Ijiri*[1979; 66]

> " 'Forced liquidation value' is also ill defined, but it sometimes seems to mean the price that could be obtained by selling to the first man on the street that one happened to meet. If this is the meaning, then we agree that it would be absurd to report such values. A less radical notion of immediate exit price is obviously called for". *Robert R. Sterling*[1970; 328]

Thus to find an estimated net realizable value for an asset, it is necessary to determine what is the appropriate set of potential buyers and how their price bids could be elicited. If instead of seeking prices from potential buyers of the asset, we resort to appraisal values for the asset, we again encounter a certain lack of determinancy: how many appraisals should be made; what are the credentials of the appraisers; what criteria do the appraisers use[158]; etc.

Rather than saying that hypothetical net realizable values or appraised values are not objective[159], it might be more accurate to say that they do not pass the *reproducibility test*; i.e., two accountants attempting to construct net realizable values for a firm's assets would not generally come up with the same values. This is the major advantage of historical cost accounting and general price level adjusted accounting; aside from the major problems

involved in defining asset lives and depreciation rates[160], these two methods of accounting can claim that they pass the reproducibility test.

Turning now to the lack of additivity of net realizable values, consider the following quotation:

> "The second factor which makes current cost income more disputable than historic cost income is the non-additivity of current costs. The historical cost of Resource A and Resource B is by definition the sum of the historical cost of Resource A and Resource B …. This addivity does not exist in current cost valuation, insofar as the price of a resource is not necessarily equal to the sum of the prices of its components. If the current cost of Resource A is $20 and Resource B is $30 but that of A and B together is $60, should we use $50 or $60 as the current cost of Resource A and Resource B?".   *Yuji Ijiri*[1979; 67]

Thus if we have two assets that can be combined together to produce an extraordinary revenue stream (e.g., a machine and a building to house the machine that together produce a new product with a high profit margin), then the joint asset may have a net realizable value that is much greater than the sum of the separated net realizable values; i.e., net realizable values for assets are not necessarily additive.[161]

In order to overcome the lack of additivity of net realizable values, it will be necessary to make some rather arbitrary judgments. For example, current values could be obtained for each asset that was purchased separately (or for each group of assets that was purchased jointly) on a stand alone basis; e.g., if a tractor were purchased with several supplementary attachments, then we could attempt to find a net realizable value for the entire asset package. Thus the additivity problem is "solved" by restricting the collection of net realizable values to the asset combinations that were actually purchased by the business unit.[162]

To overcome the lack of reproducibility objection to the use of net realizable values is a bigger task and might involve considerable costs.[163] Accounting standards organizations or the government (in its role as a collector of business income taxes) would have to specify acceptable methods for constructing net realizable values. One possible (partial) solution might be to utilize appraised values for property insurance purposes. Insurance companies have an incentive to insure property up to its maximum value to the business unit (if premium revenue is proportional to insured value) but they also have an interest in not allowing overinsurance (in order to minimize carelessness and fraud on the part of the insured business unit). Another possible solution to the lack of reproducibility problem would be for the Accounting Standards Board or the Government to develop appraisal criteria and to train and license appraisers.

We leave the final words on possible methods for the objective or reproducible determination of net realizable values to Chambers:

"We will take a more or less common sense view–namely that a statement of financial position as at a date will include singular statements, in respect of plant assets, which are indicative of one or more of the following: the cost at that date of acquiring plant in the condition in which it then stands, the valuation which a lender might place on it as a security for a loan, the valuation which the owner might place on it for insurance purposes, or the price which might be obtained for it if it were decided to change the character of the company's investments. Anyone is at liberty to contend that these would all be different; but they have one thing in common, they are all estimates made in the context of conditions operating about the time at which the financial statements are prepared. They are approximations to contemporary value in the market".

Raymond J. Chambers[1964; 270]

We turn now to a discussion of entry values.

6.4   Replacement Costs (Entry Values)

"The replacement cost is the sum of money which would have to be expended at the present time to reproduce a physical property identical with that in existence at the present time and used for the benefit of the public".

Hammond V. Hayes[1913; 618]

"The values which the accountant uses in closing the books and preparing statements ideally should be based upon economic conditions at the moment of closing. If plant and equipment assets were valued at the close of each period on the basis of costs of replacement–effective current costs–depreciation changes would be increased in a period of rising prices and the other concomitant effects would be registered in the accounts in a rational manner".

William A. Paton[1920; 6-7]

The description of an entry price or replacement value of an asset has already been provided in section 6.3 above: it is the current market cost of purchasing a physically identical replacement for an asset currently being held by a business unit. As can be seen from the above quotations, the concept of a replacement value dates back at least 80 years.

Replacement cost as a basis for asset valuation grew in popularity during the 1920's due to the inflationary upheavals that took place at that time and in the prior decade:

"In Germany, during the severe inflation period, the orthodox practice of calculating depreciation on the basis of original book costs was eventually swept aside because accountants and business men came to perceive that, in maintaining the substance of capital, it was no longer useful. At first various supplementary measures were adopted, such as charging all new fixed asset costs to expense and creating a special reserve to provide for maintenance of plant value and business efficiency (e.g., the prevalent Werkerhaltungskonto). Later, computation of depreciation on the basis of reproductive cost grew in popularity, which, indeed, is still evident from a survey of contemporary German depreciation theory".   *Henry W. Sweeney*[1931; 166]

"Prices go up and prices go down, and with each change in the price level the discussion of replacement cost usage recurs. It appears that businessmen and accountants were willing to experiment with the use of replacement cost in the 1920's and early 1930's. But this receptivity to its use has declined steadily since then: in the 1940's practicing accountants were opposed to its use; . . .. Thus if past experience holds true for the future, replacement cost will still receive its share of attention from theoreticians while practicing accountants largely ignore it".   *Germain Boer*[1966; 97]

Even though replacement cost accounting is no longer used by business accountants, it should be noted that it is still used today by national income accountants (or social accountants as they are sometimes called) as the basis for computing depreciation on a current cost basis.

The net realizable value and replacement cost of an asset can be regarded as the selling and buying prices for the asset in the relevant second hand market. Replacement cost will generally exceed the corresponding net realizable value due to the existence of transactions costs.

There is a variant of replacement cost accounting that at first sight seems to eliminate the need to consider second hand markets: find a current purchase price for a new asset that corresponds to the used asset on hand, apply the same method of depreciation to this new asset price (instead of the original historical cost price for the asset) and the resulting depreciated current price is an estimate for replacement value. However, this method of constructing replacement values implicitly assumes that the business unit is using the "correct" depreciation rates (the "correct" rates are reflected in current used asset markets).

Replacment cost can exceed the corresponding net realizable value for reasons other than transactions costs. Consider the following example due to Paton:

> "One example will be sufficient to show the ruinous error which may flow from a slavish adherence to the cost-of-replacement theory in appriasals. In 1924, a valuation was made of the properties of the Kansas City Railways by two independent engineers. One of the items to be appraised was three old engines in the power house. These were of the massive type, with enormous flywheels, and were standard equipment twenty or twenty-five years ago, or more. This equipment was in excellent physical shape, but was utterly obsolete, and a couple of the engines were no longer even connected. The company's power at the time of appraisal was entirely supplied by other and more modern equipment, although the old units were capable of giving service if required. One of the engineers went to the Westinghouse Company, with complete specifications, and secured an estimate of what it would actually cost, as of the date of the appraisal, to construct these engineers, on special order. He then made an estimate of the cost of shipment, installation, etc. The result was a cost of replacement figure considerably over a million dollars. The other engineer treated the units as scrap and gave them a net value of $20,000". *William A. Paton*[1931; 95]

What happend in the above example is that technical progress occurred which caused the net realiable value for the used asset to plummet, but the replacement value for the asset was high, since the old asset was no longer being produced.

There is a logical difficulty associated with the use of replacement cost values for unique assets such as a specially constructed machine or an engineering structure that is specific to the business unit: no replacement cost values are available in the marketplace for unique assets.[164] A solution to this difficulty is inherent in the approach of the first engineer in the above example: simply calculate the estimated cost of building the specific asset using the technology and input prices that pertain to the end of the accounting period.[165]

Replacement cost values are subject to the same two difficulties that were associated with the use of net realizable values: replacement costs are not generally *reproducible* (different accountants will generally obtain different estimates of replacement cost) and replacement costs are not generally *additive* (if a group of assets is replaced, the aggregate replacement cost may be less than the sum of the individual replacement costs). The lack of additivity is not a serious problem: we can again impose additivity by seeking replacement costs for assets according to how they were originally purchased; i.e., if a group of assets were jointly purchased, then we attempt to find a joint replacment cost for

the same group of assets. However, the lack of reproducibility is a serious limitation on the use of replacement values.[166]

In this section, we considered the use of replacement costs and in section 6.3, we considered the use of net realizable values as a basis for valuing the assets held by a firm at the end of an accounting period. Is there a rational basis for choosing between these alternative valuation methods? One way of answering this question is to consider whether the business unit is likely to *buy* additional units of the asset in the near future (in which case an appropriate opportunity cost would appear to be *replacement cost*) or whether the business unit is likely to *sell* the asset in question (in which case the relevant opportunity cost would appear to be *net realizable value*). Thus several accountants[167] have argued for the use of replacement values for raw material inventories and for net realizable values for inventories of finished products. Following this same logic, an expanding firm might value its fixed capital stock components at replacement values while a contracting firm might use net realizable values. While this line of reasoning does not provide a complete answer to the question of which valuation base to use, it does seem helpful.

We turn now to a brief discussion of yet another basis for interim asset valuations.

6.5   Future Discounted Cash Flows

"The flow of services issuing from an article of capital may have any duration and any distribution of rate. In every case the capital value of the article is the discounted value of its anticipated services".   *Irving Fisher*[1897; 527]

"If one could approximate the whole future series of money outgoes and of money receipts of an enterprise, one could find, given a rate of discount, a direct capital value of that enterprise".   *John B. Canning*[1929; 207]

The view that the appropriate value for an asset is the discounted stream of the future net revenues that can be attributed to it was actively advocated by Irving Fisher [1897] [1930]. In the accounting literature, estimating a current asset value as the discounted stream of its future expected returns is known as the *economic approach* to asset valuation. Of course, a current purchase price for an asset can be thought of as representing a lower bound to the asset's economic value to the purchaser, but in this section, we will define an asset's economic value as an estimated discounted stream of net returns that can be attributed to the asset.

Accountants pointed out that the economic approach to asset valuation suffers from two fatal flaws: (i) future discounted net returns are generally not known with any degree

of certainty and hence the resulting estimates will not be *reliable*[168] and (ii) even if we did know future revenue flows with certainty, revenue flows are produced by the joint efforts of all assets and it is generally impossible to allocate the resulting joint net revenue flows to individual assets.[169] Another way of phrasing the first objection is to say that economic values will not generally pass the *reproducibility test*; i.e., different accountants will generally obtain different estimates for economic values. In principle, the second objection to the economic approach can be overcome–the economic values or shadow prices defined by (49) in section 5.3 above provide the "correct" allocations of future discounted expected profits to the end of period 0 capital stock components held by the business unit. However, it is not a straight-forward matter to compute these shadow prices–an econometric estimate for the firm's Austrian variable profit function would have to be obtained and the resulting shadow prices would still depend on somewhat arbitrary assumptions about future technical progress and about future expected input and output prices that the firm will face.

Inspite of the above rather negative evaluation of the Fisherian economic approach to asset valuation, accountants[170] have recognized that for certain *unique* assets held by a business unit, the economic approach may be the only relevant approach for obtaining current asset values. For example, a reasonable estimate for the value of a unqiue oil field held by an exploration company might be the estimated discounted net revenues generated by the crude oil pumped out of the field over the life of the field. In order to obtain these estimates, it will be necessary to: (i) estimate how much crude will be extracted in each future period; (ii) estimate future spot prices per barrel of crude (less applicable taxes); (iii) estimate future extraction costs and (iv) provide an appropriate discount rate. In fact, there are engineering firms that will provide such estimates and accountants accept their valuations in order to put an estimated value on oil reserves. As another example, suppose a business unit holds the rights to a movie or a patent; (both are unique assets). Then a reasonable current asset value for the movie might be the discounted value of future expected rental income and for the patent might be the discounted value of future anticipated royalty payments.

In order for economic valuations to pass the objectivity or reproducibility test, it seems necessary that these valuations be done by specialized valuation firms, which could be accredited by the relevant accounting standards board or by the relevant governmental authority.

We turn now to our final class of methods for valuing assets.

6.6 Specific Price Level Adjusted Historical Cost

"On account of the expense involved, to argue for yearly appraisals of fixed assets, would sound impractical. When price levels remain fairly constant they would prove to be unnecessary. During periods of price fluctuation an adjustment could be made in previous appraisals to reveal this condition or an entirely new appraisal resorted to. In this connection, price indexes may prove very helpful in the future to both the accountant and the appraisal engineer".

H.C. Daines[1929; 101]

"Knowing the exact composition of the client's property as at the date for which the new appraisal is to be made, the appraisal company then values such property at the prices prevailing on that date .... A method that may very conveniently and profitably be used as a quick and cheap substitute under certain conditions is the index-number method. This method is a phase of 'stabilized accounting', which is concerned with the use of index numbers to restate accounting figures in a uniform price level before combining or comparing them".

Henry W. Sweeney[1934; 110]

The specific price level method for constructing current values for an asset held by a business unit through successive accounting periods was suggested by Daines [1929; 101], Sweeney [1934; 110] and many other accountants.[171] The method works as follows. First, assets held by the business unit at the beginning of period 0 are classified into a finite number of distinct asset classes. Secondly, it is supposed that index numbers that pertain to each asset class are available at the beginning and end of each accounting period. Finally, suppose that an asset was purchased at the beginning of accounting period 0 at the price P^0 [172], the period 0 depreciation rate for the asset is δ^0 and the asset inflation rate for the relevant asset class over period 0 is i^0 (i.e., the specific asset index number at the end of the period divided by the specific asset index number at the beginning of the period is $1 + i^0$). Then the *Specific Price Level Adjusted* (SPLA) historical cost of the asset at the end of period 0 is defined as

$$(55) \quad (1 - \delta^0)(1 + i^0)P^0.$$

Comparing (55) with (54), we see that the present specific price index number method for constructing an end of period estimated asset value is very similar to the General Price Level Adjusted asset value defined earlier by (54); the only difference is that now a presumably more relevant specific price index is used for revaluation purposes rather than an index of general inflation.

If the same set of asset specific price indexes is given to all accountants, then Specific Price Level Adjusted values will satisfy the reproducibility test. The SPLA asset value should also be closer to its end of period market value (i.e., an end of period purchase cost or net realizable value) since presumably, the index numbers reflect a sample of market transaction prices for new units of the asset (or similar assets) during a time period that includes the end of period 0. Thus SPLA values will tend to be *reproducible* and *relevant*. We also note that Specific Price Level Adjusted accounting is not completely impractical since it has occasionally been used historically.[173]

There are some problems associated with the use of Specific Price Level Adjusted values: (i) None of the available specific price indexes may be relevant for the particular asset on hand.[174] A related problem is that different accountants may classify the same asset into different asset classes thus destroying the reproducibility property for the method.[175] (ii) The asset specific index numbers will generally pertain to a discrete interval of time instead of the precise date at which the accounting period ends. Under these conditions, the exact adjustments (if any) that the accountant should make to the specific indexes is ambigous.[176] (iii) The related issue of the timeliness of the specific indexes should also be raised: annual specific price indexes for capital stock components that appear with a half year time lag will be useless in the context of quarterly accounting.[177] (iv) The construction of SPLA values is mainly suitable for the valuation of fixed capital stock components and not circulating capital stock components.[178] How then should end of period prices for inventory stocks be constructed? We shall address this problem in section 9 below. The problems involved in constructing current prices for inventory items are generally not as severe because relevant market prices for inventory components held by the business unit are often available in the records of the business unit: market prices for used fixed assets are more difficult to obtain. (v) The SPLA values for assets at the end of the accounting period are still dependent on the rather arbitrary depreciation rates (recall δ^0 in (55)) that are associated with historical cost accounting.[179] To cure this lack of reproducibility in the method, the Agency that provides the asset specific index numbers should also provide "standard" depreciation rates for assets in each class (or alternatively, provide index numbers for not only new assets but also used assets[180]). The adoption of this last suggestion will not only lead to reproducible SPLA values, but it will also lead to reproducible estimates of depreciation.[181]

Which Agency should provide the relevant index numbers and depreciation rates? Three possible choices are: (i) the relevant National Statistical Agency; (ii) the relevant Accounting Standards Board or (iii) an Agency or Department of the relevant National Government (e.g., the income taxation authority).

We note that historical cost valuations for fixed assets have proved to be very resilient from a historical perspective, being temporarily abandoned only in the face of

dramatic inflationary shocks when the method clearly became absurd.[182] It seems likely that the alternative valuation methods described above in sections 6.2-6.6 have failed to be adopted permanently for a number of reasons: (i) the alternative method was thought to be too *inaccurate* (General Price Level Adjusted valuations); (ii) the alternative method was thought to be too nonobjective or not *reproducible* (all other methods) or (iii) the alternative method was thought to be too *expensive* or too *complex*. However, it seems possible that all of these objections could now be overcome with the use of Specific Price Level Adjusted values, provided that a National Authority could provide the accounting profession with the relevant asset specific index numbers and standard depreciation rates.

We leave our last words on the subject of asset valuation to one of the pioneers of current value accounting:

> "Even crude attempts should result in an improvement over present depreciation practices. During periods of rapidly changing prices crude measurements of a relevant item are likely to be much more meaningful than accurate measurements of an irrelevant one (in this case, historic cost)".
>
> *Edgar O. Edwards*[1954; 268]

7.0   Alternative Income and Cost of Capital Concepts

"Economists and accountants, at least by implication, seem initially to agree that income is something related to capital so closely that the determination of one involves that of the other".   *Frank A. Fetter*[1937; 9]

"The literature that may be designated as accounting theory deals with (1) the choice among alternative definitions of income, and (2) the measurement rules that follow from and implement or that supplement a definition".
Myron J. Gordon[1960; 606]

"What is 'business income'? How should it be measured? These problems have been constantly discussed by both accountants and economists. Yet no close agreement has ever been reached. Accountants complain that economists are too idealistic and their concepts are impractical; economists charge that accountants are too mechanical and their procedures are not based on sound principle".
Emily Chen Chang[1962; 636]

"We are touching, of course, on the definition of income. But must we settle on one?".   *Edgar O. Edwards*[1975; 238]

In this section, we shall consider alternative definitions for the concept of business income for an accounting period. Synonyms for the "business income" concept are "net earnings" and "profits".

From the 1880's to the 1930's, three broad approaches to the measurement of business income can be distinguished in the accounting and economics literature.

In the first approach, business income was simply defined as the change in asset value over the accounting period; i.e., the emphasis was on change in balance sheet values for the business unit over an accounting period. We discussed this approach earlier in section 4.1 above. This approach to the measurement of business income was a natural outgrowth of the very early "completed venture" approach to accounting.[183]

In the second broad approach, the business income of a business unit over an accounting period was defined as operating profits (current period revenues less current period variable costs) plus the net change in the value of assets held through the accounting period.[184] This second approach to income is not really that different from the first approach since any operating profits made by the business unit in the second approach would show up as increased asset holdings in the first approach. However, in the second approach, the focus shifts from the backward looking changes in asset values approach to the forward looking projection of current operating profits approach; i.e., the focus shifts from

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the balance sheet (which lists asset values) to the income statement (which lists current operating profits). This second approach to the measurement of business income is due to Marshall, Schanz and Haig:

"When a man is engaged in business, his profits for the year are the excess of his receipts from his business during the year over his outlay for his business. The difference between the value of his stock of plant, material, etc. at the end and at the beginning of the year is taken as part of his receipts or as part of his outlay, according as there has been an increase or decrease of value. What remains of his profits after deducting interest on his capital at the current rate (allowing, where necessary, for insurance) is generally called his *earnings of undertaking or management.* The ratio in which his profits for the year stand to his capital is spoken of as his *rate of profits.* But this phrase, like the corresponding phrase with regard to interest, assumes that the money value of the things which constitute his capital has been estimated: and such an estimate is often found to involve great difficulties".   *Alfred Marshall*[1920;74][185]

"If income is defined as the total accretion in one's economic strength between two points of time, as valued in terms of money, it is clear that his income will reflect every change in the value of money between those two points of time in so far as the items entered on the balance sheets at those times affect the computation. If the level of prices goes up ten per cent the money value of my assets will ordinarily follow at a like rate. That particular increase in value does not really indicate an increase in my economic strength. My power to command economic goods and services has likewise increased .... If it were possible to modify the concept of taxable income so as to eliminate this variation it would certainly be desirable to do so. The prospect for a complete solution of the difficulty pointed out, however, is identical with the prospect for a perfect monetary standard. But an approximate solution might be realized if we were able to evolve a satisfactory index of the level of prices. If it was accurately known what the change in price level in a given year had been, it might be possible to qualify the results shown by a comparison of the balance sheets for the beginning and the end of the period in such a way as to eliminate the influence of the changing standard. But even this refinement is not likely to be introduced soon. Indeed, the desirability and urgency of its introduction is dependent largely upon the complete solution of the accounting problem, which solution is certainly not imminent".   *Robert Murray Haig*[1921; reprinted 1959; 67-68]

"The broadest definition of income for taxation purposes was formulated by Georg Schanz in 1894 and was independently advanced over two decades later

by R.M. Haig, who defines income as 'the money value of the net accretion to one's economic power between two points in time'. This view is in substantial agreement with the accountant's practise of ascertaining the annual profit of an enterprise by comparing the balance sheet at the opening and the close of the year .... The widening of the income concept, however, brought forth a number of difficult problems. The first problem is that of appreciation in value. Is this income or only addition to capital? In the light of the Schanz concept it undoubtedly is income. Theoretically this view seems sound .... There are, however, some practical difficulties in the way of accepting this conclusion. In the first place, an income tax is supposed to be paid out of income. But if the taxpayer's entire wealth consists in the house that has appreciated in value, he has no funds with which to pay the tax .... A further objection is to the effect that in case of a change in the general level of prices the money worth of a commodity at the end of the period may be greater than at the beginning. The owner may then be assessed for what is nothing but a fall in the value of gold. This objection might, however, be removed by reference to an index of prices, as is done in some of the foreign unearned increment taxes".

Edwin R.A. Seligman [1932; 628-629]

The third broad approach to the period by period measurement of business income is the same as the second except that the third approach attempts to make an adjustment to the net change in the value of assets that will take into account either general price change or specific price change (or both) in the prices of assets held from the beginning to the end of the accounting period. This concept of income leads us directly into the fundamental problem of accounting: how are we to value assets which do not trade in every period? Thus the different methods for valuing assets which were discussed in section 6 above can be used in the present section in order to define alternative income concepts.[186] But, as can be seen from the quotations by Haig and Seligman above, income determination is more complicated than merely choosing the appropriate method for valuing assets; according to these authors, some value changes (due to asset price changes over the accounting period) are to be excluded from the business income concept.[187]

In sections 7.1-7.7 below, we shall outline seven alternative income concepts that have been suggested over the years, beginning with historical cost income (section 7.1) and ending with cash flow income (section 7.7). In order to focus on the essential issues, we shall assume that a business unit produces current outputs, uses current inputs but holds only a single depreciable fixed asset over the accounting period. In this highly simplified framework, we shall see that alternative business income concepts give rise to alternative user cost of capital formulae; in particular, the "economic" income concept

defined in section 7.6 gives rise to the ex post user cost of capital, formula (9) defined in section 2 above.

We conclude this introductory section with some observations made by accountants over the years on the changing nature of accounting and on the importance of defining the "correct" income concept.

Our first general observation on the importance of the income concept was made by Seligman and many others:[188]

> "An essential attribute of income therefore is that it must be so defined as to exclude the impairment of the capital. Otherwise the so-called income would include a periodic fraction of the capital, with the result that in the end there would be neither income or capital". *Edwin R.A. Seligman*[1932; 631]

A second general observation on income measures is that the income or profits measure that is chosen should not *overstate* the "true" income of the business unit, because income overstatement may lead to excessive: (i) dividend payments; (ii) wage settlements and (iii) income tax payments, all of which may deplete the capital of the business unit and hence put its long term survival in doubt:

> "The first problem created by changes in the price level is to determine how much of money profit represents an increase in the current purchasing power of the firm. Its solution has significance for tax, dividend, and wage policies particularly. The levying of taxes on money profits when prices are rising means that a part of the firm's original real investment is being taxed; a firm may be paying taxes even though its real investment has decreased over the period. If management is misled by money profits, it may bring about the same result by paying dividends that deplete the firm's real investment. Or management, on the basis of high money profits, may contract to pay higher wages even though the firm's real profit is small or negative".
> *Edgar O. Edwards and Philip W. Bell*[1961; 123]

A third general observation is that the chosen income measure should not *understate* the "true" income of the business unit because income understatement may lead to: (i) insiders making unfair profits on their knowledge of the business unit's "true" situation[189] and (ii) underpayment of income taxes. The understatement problem can occur if the historical cost income concept is chosen and there is asset inflation. This will lead to understated historical cost income for accounting periods during which the assets were held (and overstated income for the accounting period when the assets were sold). Thus many early accountants advocated the reporting of capital gains in the periods when they

occurred (whether realized or not) but they also advocated that unrealized capital gains should be reported separately on the income statement:

> "Income is the economic benefit coming in during a period of time. It consists of current income and of capital gains and losses. The general accounting habit of regarding income as though it were synonymous with realized income, although of some usefulness in practical affairs because it tends to substitute fact for fancy, should be more clearly recognized as both illogical and generally causative of the compilation of false information. *All* the income–realized, unrealized and the total thereof–coming into existence during a period should be credited to the period, but the realized should continue to be separated from the unrealized".
> *Henry W. Sweeney*[1933; 335]

Finally, the economist Haig[190] observed that income concepts (e.g., historical cost income) that were suited to the early days of business accounting (before stock markets became widespread and before the invention of the business income tax) were no longer suited to the current accounting environment:

> "While the accounting ideal as stated by the leading theorists in the accounting field is in entire harmony with the economic analysis, it should be pointed out that many so-called accounting principles which are generally accepted are little more than rules of action formulated during an obsolete period when the use of accounts for tax purposes did not exist. So long as the chief purposes of the accounts were to provide a basis for applications for credit, and for the distribution of dividends, rules which tended toward a conservative statement of profits were certainly full of virtue. The increase in the tax burden has added a new primary use for the accounts, a use which demands certain qualities which are not important in the other cases. To form an entirely satisfactory basis for the imposition of income taxes the accounts must reflect the full, true, economic position of the taxpayer; and in so far as arbitrary rules of inventory valuations operate to build up hidden reserves, or other accounting practices tend to befog the picture, they must ultimately be eliminated and they have no place in truly scientific accounting".
> *Robert Murray Haig*[1921; reprinted 1959; 68]

7.1 Historical Cost Income

> "The actual taking of a price by the seller and its payment by a buyer, each acting irrevocably in what he conceives to be his own interest and each

presumably having some skill in appraising *the practicable options available to him*, provides a working valuation for the land. The accountant merely adopts this 'prudent investor' figure until he obtains equally reliable evidence to support a different valuation".   *John B. Canning*[1929; 197-198]

"Insofar as objectivity is regarded as an indispensible quality of an income concept which is to have any claim to being practical, accounting income is practical enough. But this is of little moment if it does not measure what we want to measure. Objectivity without relevance is not much of a virtue".

                                                      David Solomons[1961; 378]

We shall use the notation that was introduced in section 5.1 above: the vector of average market prices that the business unit faces for the N variable inputs and outputs used and produced during period 0 is $p^0 \equiv [p_1^0, \ldots, p_N^0]$, the quantity vector of net outputs that the business unit produces during period 0 is $y^0 \equiv [y_1^0, \ldots, y_N^0]$ (if $y_n^0 > 0$, then commodity n is produced during period 0 while if $y_n^0 < 0$, then commodity n is used as an input) and period 0 operating profits are $p^0 \cdot y^0 \equiv \Sigma_{n=1}^N p_n^0 y_n^0$. We shall simplify the notational complexity of section 5.1 by assuming throughout section 7 that the business unit uses only $k^0 > 0$ units of a single durable input during period 0. Again for simplicity, we assume that the units of the durable input were purchased at the beginning of period 0 at the per unit price $P^0 > 0$ and that no other durable inputs were purchased during period 0. We can now repeat the same assumptions about depreciation, asset inflation and interest rates that were made in section 2 above; i.e., define: (i) the one period depreciation rate δ^0 by (3); (ii) the period 0 inflation rate for new assets i^0 by (5); (iii) the period 0 costs of debt and equity capital by r_d^0 and r_e^0 respectively; (iv) the debt and equity financial capital at the beginning of period 0 by $D^0 \equiv f^0 P^0 k^0$ and $E^0 \equiv (1 - f^0)P^0 k^0$ respectively and (v) the overall period 0 cost of capital (or interest rate) r^0 by (10). We shall maintain these definitions and assumptions throughout section 7.

In general, following Marshall, a period 0 income definition will have the following structure: income in period 0 equals operating profits in period 0 less interest paid in period 0 plus the change in net asset value over period 0; i.e.,

(56) $I^0 \equiv p^0 \cdot y^0 - \text{interest} + A^1 - A^0$

where $p^0 \cdot y^0$ equals operating profits (value of current outputs minus value of current inputs), A^0 is the beginning of period 0 value of the business unit's fixed assets less the debt D^0 and A^1 is the end of period 0 value of these same assets.[191]

Historical cost income I_{HC}^0 is defined along the lines of (56) using the historical cost valuation principle discussed in section 6.1 above:

(57) $\quad I_{HC}^0 \equiv p^0 \cdot y^0 - r_d^0 f^0 P^0 k^0 + [(1 - \delta^0) P^0 k^0 - D^0] - [P^0 k^0 - D^0]$

(58) $\qquad = p^0 \cdot y^0 - [(r_d^0 f^0 + \delta^0) P^0] k^0.$

Note that $P^0 k^0 - D^0 \equiv A^0$ is the beginning of period 0 net asset value for our business unit, $(1 - \delta^0) P^0 k^0 - D^0$ is the end of period 0 historical cost net asset value and interest paid on asset debt is $r_d^0 f^0 P^0 k^0$. Further note that there are three terms on the right hand side of (57) involving the beginning of period 0 capital stock k^0. These three terms have been consolidated in (58) and the term in square brackets, $[(r_d^0 f^0 + \delta^0) P^0] \equiv w_{HC}^0$, can be regarded as the *user cost of capital* that corresponds to *historical cost accounting.*

   If $f^0 = 0$ (so that the initial physical capital stock of the business unit is financed entirely by equity financial capital), then the above historical cost accounting user cost reduces to

(59) $\quad \delta^0 P^0$

while if $f^0 = 1$ (so that the initial physical capital stock of the business unit is financed entirely by debt), then the above historical cost accounting user cost reduces to

(60) $\quad (r_d^0 + \delta^0) P^0.$

The historical cost accounting user costs defined by (59) and (60) can be compared to our earlier end of period 0 ex post user cost defined by (9), $[r^0 - i^0 + \delta^0 (1 + i^0)] P^0$. Note that (59) and (60) will generally not equal (9).

   The historical cost income defined by (57) was derived under the assumption that the business unit did not sell its initial capital stock during period 0. Suppose now that the business unit sold its initial capital stock at the end of period 0 at the market price $(1 - \delta^0) P^1$. Under these conditions, historical cost income is no longer defined by (57); it is now defined as follows (*realized historical cost income*):

(61) $\quad I_{HCR}^0 \equiv p^0 \cdot y^0 - r_d^0 f^0 P^0 k^0 + [(1 - \delta^0) P^1 k^0 - D^0] - [P^0 k^0 - D^0]$

(62) $\qquad = p^0 \cdot y^0 - [\{f^0 r_d^0 - i^0 + \delta^0 (1 + i^0)\} P^0] k^0.$

The three terms involving the initial capital stock k^0 in (61) have been collected in (62); the term in square brackets in (62) is the *the user cost of capital* w_{HCR}^0 that corresponds to *realized historical cost income.* Note that if $f^0 = 1$ (so that the initial capital stock is financed entirely by debt), then the realized historical cost accounting user cost of capital is equal to our earlier ex post user cost of capital defined by (9) above. However, if $f^0 < 1$, then in general, neither of the historical cost accounting user costs defined in (58) and (62) will coincide with (9).

In section 6.1 above, we discussed the benefits associated with the use of historical cost accounting valuations for capital stock components. The same benefits carry over to the current situation. The main benefits that can be attributed to the use of historical cost incomes defined by (57) and (61) are their *low costs* and *objectivity* (in the sense of being *reproducible* by any accountant with the same primary data).[192]

On the negative side of the ledger, accountants have identified five problem areas associated with the use of historical cost accounting incomes:

(i)   Historical cost incomes and balance sheet values are often *irrelevant*; i.e., historical cost income is often not a surplus that can be distributed to stakeholders in a costless manner without impairing the long run viability of the business unit and historical cost accounting asset values are often far from current asset values.[193]

(ii)   The realization convention of historical cost accounting *distorts* period by period income: under conditions of high asset inflation, historical cost income will tend to be too high in periods when assets are held[194] and will be definitely too high in the periods when assets are eventually sold. In the realized case, the problem with historical cost accounting income is that cumulative capital gains made by the asset are recognized *only* in the period of the asset sale rather than being distributed over all prior accounting periods during which the asset was held.[195]

(iii)   Attempts by historical cost accountants to improve the measurement of income by using more current values for end of the period inventory stocks (e.g., the use of Last In, First Out (LIFO) inventory valuations) generally lead to grossly *understated* values for these inventory stocks on balance sheets.[196] We shall consider the problems of inventory valuation in more detail in section 9 below.

(iv)   Period 0 income should depend only on opportunity costs for assets at the beginning and end of period 0 and on period 0 prices and quantities for variable inputs and outputs. An income concept that satisfies this desirable property could be said to satisfy the *relevance property* or to satisfy *current period prices and quantities test*. Historical cost accounting income does not satisfy this property (except if assets are bought and sold every period); historical cost asset values are generally based on prices that pertain to periods in the distant past (in the case of long lived assets). The thrust of this criticism of historical cost accounting is contained in the following two quotations:

> "As applied at present, accounting techniques tend to result in an undesirable state of affairs in which firms producing the same goods, using identical plant and with no market difference in efficiency are subject to a considerable variation in costs and profit margins simply because of differing times of acquisition of plant and equipment".   *J.C. Latham and others*[1952; 95]

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"Traditional accounting operating profit cannot be expected to yield identical data for two different periods whose current events are identical. Current operating profit does fulfill this criterion; it depends for its determination on the events which are current in the period under consideration. The figures derived when historically oriented profit concepts are used depend only partly on current events; their measurement is influenced heavily by events of preceding periods, namely, historic costs. Yet it seems reasonable to expect that if a firm operates during two periods with the same endowment of assets, contributes the same factors to the same production and sales processes, finds the prices of its factors to be the same, and sells the same output at the same prices, the firm should report the same profit. The fact that present accounting procedures will not meet this criterion we regard as a strong argument in favor of the kind of accounting modification we propose".   *Edgar O. Edwards and Philip W. Bell*[1961; 226]

(v)   Historical cost accounting does not recognize (imputed) *interest on equity capital* as a valid cost of production.[197] We shall pursue this point in more detail in section 7.6 below.

In view of the above criticisms of historical cost accounting income, we now turn our attention to alternative income concepts.

7.2   Restated Historical Cost Income (CPP Income or GPLA Income)

"In his quest for certainty, the accountant has come to assume certain values as constant which are in fact variable. In particular, the value of the monetary unit has been regarded as constant and the measure of all other values. No increase in value of any other asset has, in general, been recognized until its credentials have been validated by 'realization', i.e., exchange for money. Such unquestioning faith in the monetary unit must indeed have been shaken by the great rise of prices in almost all countries during and after World War II. In some countries, where at various times the currencies have declined catastrophically in value, a rejection of the local monetary unit has been common and accounts have been kept in gold or in foreign monetary units".

Sidney S. Alexander[1950; revised 1962; 132]

"It was pointed out that even an imperfect index would give more significant results than ignoring changes in the value of the dollar altogether".

George O. May and others[1952; 54]

Restated historical cost income is defined in much the same way as historical cost income was defined in the previous section *except* that assets held at the beginning and end of the accounting period are revalued using a general index, $1+\rho^0$, of purchasing power change over the accounting period. Thus end of period values of assets are equal to the corresponding historical cost accounting values times the general inflation factor $(1 + \rho^0)$. These are GPLA (General Price Level Adjusted) values which were discussed in section 6.2 above; see (54) for a definition of the end of period 0 restated historical cost value for our representative asset which cost P^0k^0 at the beginning of period 0. The justification for this method of end of period asset valuation is that it is a *low cost* and *reproducible* method for obtaining approximate current values for *assets*. However, when calculating restated historical cost *income*, we also multiply the beginning of the accounting period asset values by the accounting period inflation factor, $1 + \rho^0$. This beginning of period 0 asset revaluation is justified on the grounds that *beginning* of the period monetary asset values should be converted into units of purchasing power that are equivalent to the *end* of period 0 value of money; thus if there were no general inflation over period 0 (so that $\rho^0 = 0$), then this conversion to units of stable purchasing power would not be necessary.

Restated historical cost income is sometimes called CPP (Constant Purchasing Power) income[198] or GPLA (General Price Level Adjusted) income.[199] This income concept is due to the accountant Middleditch [1918].[200]

The above discussion of CPP accounting has implicitly assumed that the business unit has no debt. If the business unit does have debt, then the details of CPP accounting are more complex. The beginning of the period net asset value A^0 (which is equal to the value of the physical capital stock P^0k^0 less debt $D^0 = f^0P^0k^0$ which in turn is equal to equity $E^0 = (1 - f^0)P^0k^0$) is multiplied by the period 0 general inflation factor $1 + \rho^0$ to convert the beginning of period 0 values into their purchasing power equivalents at the end of period 0. If there is no sale of the initial capital stock, the end of the period value of the (depreciated) capital stock is defined to be the (depreciated) historical cost asset value $(1 - \delta^0)P^0k^0$ times the general inflation factor $(1 + \rho^0)$ less the value of the initial debt D^0; i.e., we do *not* escalate D^0 by $1 + \rho^0$ because the debt is already expressed in terms of the end of period 0 value of money.[201] Thus in the case where the initial capital stock is not sold during period 0, *constant purchasing power income* is defined as follows:

$$
\begin{aligned}
I^0_{CPP} &\equiv p^0 \cdot y^0 - \text{interest paid} + A^1_{CPP} - A^0 \\
&= p^0 \cdot y^0 - r^0_d f^0 P^0 k^0 + [(1 + \rho^0)(1 - \delta^0)P^0 k^0 - D^0] - (1 + \rho^0)[P^0 k^0 - D^0] \\
&= p^0 \cdot y^0 - r^0_d f^0 P^0 k^0 - (1 + \rho^0)\delta^0 P^0 k^0 + \rho^0 f^0 P^0 k^0 \\
&= p^0 \cdot y^0 - [\{(r^0_d - \rho^0)f^0 + \delta^0(1 + \rho^0)\}P^0]k^0.
\end{aligned}
$$
(63)

The term in square brackets in (63) is the *user cost of capital* w^0_{CPP} that is implied by constant purchasing power accounting in the case where the asset is held during period 0; i.e., we have

$$(64) \quad w^0_{CPP} \equiv [(r^0_d - \rho^0)f^0 + \delta^0(1 + \rho^0)]P^0$$

$$(65) \qquad\qquad = [r^0_d f^0 + \rho^0(1 - f^0) - \rho^0 + \delta^0(1 + \rho^0)]P^0$$

where (65) follows from (64) by adding and subtracting the term $\rho^0(1 - f^0)P^0$.

The CPP user cost defined by (64) or (65) can be compared to our earlier ex post user cost of capital w^0_e defined by (9) and the historical accounting user cost w^0_{HC} defined by the expression in square brackets in (58) above. Recall that $r^0 \equiv r^0_d f^0 + r^0_e(1 - f^0)$ where r^0_d and r^0_e are the period 0 interest rates for debt and equity respectively. Comparing $w^0_e \equiv [r^0 - i^0 + \delta^0(1 + i^0)]P^0$ and w^0_{CPP} defined by (65), it can be seen that

$$(66) \quad w^0_{CPP} = w^0_e \text{ if } \rho^0 = r^0_e \text{ and } \rho^0 = i^0;$$

i.e., constant purchasing power user cost w^0_{CPP} equals economic user cost w^0_e provided that the period 0 inflation rate ρ^0 equals *both* the period 0 equity opportunity cost of capital r^0_e as well as the period 0 asset inflation rate i^0. In the unrestricted general case, we have

$$(67) \quad w^0_e - w^0_{CPP} = [(r^0_e - \rho^0)(1 - f^0) - (i^0 - \rho^0)(1 - \delta^0)(1 - \delta^0)]P^0.$$

The above equation can be used to analyze why the two user costs might be different. Note in particular that the difference between the two user costs depends only on the initial price of capital P^0, the depreciation rate δ^0, the fraction of asset purchases financed by debt f^0 and the two *real interest rates*, $r^0_e - \rho^0$ and $i^0 - \rho^0$.[202]

Taking the difference between historical cost income defined by (58) and constant purchasing power income defined by (63) yields the following equation:

$$(68) \quad I^0_{HC} - I^0_{CPP} = (\delta^0 - f^0)\rho^0 P^0 k^0$$

$$(69) \qquad\qquad\qquad > 0 \text{ if } \delta^0 > f^0 \text{ and } \rho^0 > 0.$$

Let us assume that $\rho^0 > 0$ so that there is general inflation in period 0. Then period 0 HC income will *exceed* CPP income due to the term $\delta^0 \rho^0 P^0 k^0$ (this term indexes depreciation for inflation) and HC income will be *less* than CPP income due to the term $-f^0 \rho^0 P^0 k^0$ (this term reflects the decline in the real value of debt due to period 0 inflation). Hence if the firm's debt is zero or small (more precisely, if the debt fraction f^0 is less than the depreciation rate δ^0), then historical cost income will *exceed* CPP income, as we would expect. However, if the firm is highly indebted (more precisely, if the debt fraction f^0 is greater than the depreciation rate δ^0), then HC income will be *less* than CPP income. This last result is perhaps contrary to our expectations since we would expect the indexation

of assets for inflation to lead to a lower income than historical cost income which does not index assets for inflation. However, as long as the initial capital stock is not *entirely* financial by debt, CPP income will *eventually* fall far below the corresponding historical cost income provided that the inflation persists over the life of the asset. We will prove this assertion for the case of an asset that lasts 3 periods[203] under the following simplifying assumptions:

$$(70) \quad \rho^0 = r_d^0 > 0; \rho^1 = r_d^1 > 0; \rho^2 = r_d^2 > 0;$$

$$(71) \quad D^0 = f^0 P^0 k^0; D^1 = f^0 (1 - \delta^0) P^0 k^0; D^2 = f^0 (1 - \delta^0)(1 - \delta^1) P^0 k^0;$$

$$(72) \quad 0 \le f^0 < 1;$$

i.e., (70) means that the period i inflation rate ρ^i equals the period i debt interest rate r_d^i for each period i; (71) means that a constant fraction of the (depreciated) historical cost asset value is financed by debt in each period, and (72) means that the business unit has some equity so that not all of the initial asset value $P^0 k^0$ is financed by debt. Note that the historical cost depreciation allowances in periods 0, 1 and 2 are $\delta_0 P^0 k^0, \delta_1 (1 - \delta_0) P^0 k^0$ and $\delta_2 (1 - \delta_0)(1 - \delta_1) P^0 k^0$ respectively and we assume that $\delta_2 = 1$ so that the asset is fully depreciated at the end of period 2.

We now define CPP incomes for periods 0, 1 and 2:

$$(73) \quad I_{CPP}^0 \equiv p^0 \cdot y^0 + [(1 + \rho^0)(1 - \delta^0) P^0 k^0 - (1 + r_d^0) D^0] - (1 + \rho^0)[P^0 k^0 - D^0]$$
$$= p^0 \cdot y^0 + (1 + \rho^0)(1 - \delta^0) P^0 k^0 - (1 + \rho^0) P^0 k^0 \quad \text{using (70);}$$

$$I_{CPP}^1 \equiv p^1 \cdot y^1 + [(1 + \rho^1)(1 + \rho^0)(1 - \delta^1)(1 - \delta^0) P^0 k^0$$
$$- (1 + r_d^1) D^1] - (1 + \rho^1)[(1 + \rho^0)(1 - \delta^0) P^0 k^0 - D^1]$$
$$(74) \qquad = p^1 \cdot y^1 + (1 + \rho^0)(1 + \rho^1)(1 - \delta^0)(1 - \delta^1) P^0 k^0 - (1 + \rho^0)(1 + \rho^1)(1 - \delta^0) P^0 k^0$$
$$\text{using (70);}$$

$$I_{CPP}^2 \equiv p^2 \cdot y^2 + [(1 + \rho^2)(1 - \delta^2)(1 + \rho^0)(1 + \rho^1)(1 - \delta^0)(1 - \delta^1) P^0 k^0 - (1 + r_d^2) D^2]$$
$$- (1 + \rho^2)[(1 + \rho^0)(1 + \rho^1)(1 - \delta^0)(1 - \delta^1) P^0 k^0 - D^2]$$
$$(75) \qquad = p^2 \cdot y^2 - (1 + \rho^0)(1 + \rho^1)(1 + \rho^2)(1 - \delta^0)(1 - \delta^1) P^0 k^0 \quad \text{using } \delta^2 = 1 \text{ and (70).}$$

Now calculate the discounted stream of CPP incomes over the useful life of the asset using the inflation factors $(1 + \rho^i)$ as the discount rates. Using (73) - (75), we find that

$$(76) \quad (1 + \rho^0)^{-1} I_{CPP}^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} I_{CPP}^1 + (1 + \rho^0)^{-1}(1 + \rho^0)^{-1}(1 + \rho^2)^{-1} I_{CPP}^2$$
$$= (1 + \rho^0)^{-1} p^0 \cdot y^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} p^1 \cdot y^1 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}$$
$$(1 + \rho^2)^{-1} p^2 \cdot y^2 - P^0 k^0.$$

Equation (76) shows that the discounted (to the beginning of period 0) stream of CPP incomes over the useful life of the asset is equal to the discounted sum of gross operating profits $p^i \cdot y^i$ less the initial purchase price of the assets $P^0 k^0$.

We now define the corresponding period 0, 1 and 2 historical cost incomes:

$$I_{HC}^0 \equiv p^0 \cdot y^0 - r_d^0 D^0 + [(1 - \delta^0)P^0 k^0 - D^0] - [P^0 k^0 - D^0]$$

(77)
$$= p^0 \cdot y^0 - \rho^0 f^0 P^0 k^0 + (1 - \delta^0)P^0 k^0 - P^0 k^0 \qquad \text{using (70) and (71)};$$

$$I_{HC}^1 \equiv p^1 \cdot y^1 - r_d^1 D^1 + [(1 - \delta^1)(1 - \delta^0)P^0 k^0 - D^1] - [(1 - \delta^0)P^0 k^0 - D^1]$$

(78)
$$= p^1 \cdot y^1 - \rho^1 f^0 (1 - \delta^0)P^0 k^0 + (1 - \delta^1)(1 - \delta^0)P^0 k^0 - (1 - \delta^0)P^0 k^0$$

$$\text{using (78) and (71)};$$

$$I_{HC}^2 \equiv p^2 \cdot y^2 - r_d^2 D^2 + [(1 - \delta^2)(1 - \delta^1)(1 - \delta^0)P^0 k^0 - D^2]$$
$$\qquad - [(1 - \delta^1)(1 - \delta^0)P^0 k^0 - D^3]$$

(79)
$$= p^2 \cdot y^2 - \rho^2 f^0 (1 - \delta^0)(1 - \delta^1)P^0 k^0 - (1 - \delta^1)(1 - \delta^0)P^0 k^0$$

where (79) follows from the line above using (70), (71) and $\delta^2 = 1$. Now calculate the discounted stream of HC incomes over the useful life of the asset using the inflation factors $(1 + \rho^i)$ as discount rates:

$$(1 + \rho^0)^{-1} I_{HC}^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} I_{HC}^1 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1} I_{HC}^2$$
$$= (1 + \rho^0)^{-1} p^0 \cdot y^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} p^2 \cdot y^2 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1} p^3 \cdot y^3$$
$$\qquad - f^0 P^0 k^0 [(1 + \rho^0)^{-1} \rho^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} \rho^1 (1 - \delta^0)$$
$$\qquad \qquad + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1} \rho^2 (1 - \delta^0)(1 - \delta^1)]$$
$$\qquad - P^0 k^0 [1 - (1 + \rho^0)^{-1} \rho^0 - (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 - \delta^0)\rho^1$$
$$\qquad \qquad - (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1}(1 - \delta^1)(1 - \delta^0)\rho^2] \quad \text{using (77) - (79)}$$

(80)
$$= (1 + \rho^0)^{-1} p^0 \cdot y^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} p^1 \cdot y^1$$
$$\qquad + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1} p^3 \cdot y^3 - P^0 k^0$$
$$\qquad + (1 - f^0)[(1 + \rho^0)^{-1} \rho^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 - \delta^0)\rho^1$$
$$\qquad + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1}(1 - \delta^0)(1 - \delta^1)\rho^2] P^0 k^0$$

(81)
$$= (1 + \rho^0)^{-1} I_{CPP}^0 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1} I_{CPP}^1 + (1 + \rho^0)^{-1}(1 + \rho^1)^{-1}(1 + \rho^2)^{-1} I_{CPP}^2$$
$$\qquad + (1 - f^0)[\alpha] P^0 k^0$$

where (81) follows from (80) using (76) and the distortion factor α is defined as the term in square brackets in (80). Note that assumptions (70) imply that α is greater than 0. Thus the discounted sum of historical cost incomes *exceeds* the corresponding sum of constant purchasing power incomes by the amount $(1 - f^0)\alpha P^0 k^0$, which is positive using (72).

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The above proof shows that in the high debt, low depreciation rate case (i.e., $f^0 - \delta^0 > 0$), historical cost income will initially be lower than the corresponding CPP income but eventually this initial inequality will be reversed and the discounted sum of historical cost incomes will always exceed the corresponding discounted sum of CPP incomes under assumptions (70) and (71) by the positive term $(1 - f^0)\alpha P^0 k^0$.

We now turn our attention to the case where the business unit sells its initial capital stock at the end of period 0 at the market price $(1 - \delta^0)P^1 = (1 - \delta^0)(1 + i^0)P^0$. Under these conditions, period 0 CPP income is no longer defined by (63); it is now defined as follows (*realized constant purchasing power income*):

$$(82) \quad I^0_{CPPR} \equiv p^0 \cdot y^0 - \text{ interest paid } + A^1_{HCR} - (1 + \rho^0)A^0$$
$$= p^0 \cdot y^0 - r^0_d f^0 P^0 k^0 + [(1 + i^0)(1 - \delta^0)P^0 k^0 - D^0] - (1 + \rho^0)[P^0 k^0 - D^0]$$
$$(83) \qquad = p^0 \cdot y^0 - [\{(r^0_d - \rho^0)f^0 - (i^0 - \rho^0) + \delta^0(1 + i^0)\}P^0]k^0 \quad \text{using} \quad D^0 = f^0 P^0 k^0.$$

In definition (82), the realized end of period 0 CPP net value of assets A^1_{CCPR} equals the corresponding realized historical cost net value of assets

$$(84) \quad A^1_{HCR} \equiv (1 - \delta^0)P^1 k^0 - D^0 = (1 - \delta^0)(1 + i^0)P^0 k^0 - D^0 \equiv A^1_{CCPR}.$$

Note also that in definition (82), the beginning of period 0 net value of assets A^0 defined as

$$(85) \quad A^0 \equiv P^0 k^0 - D^0 = P^0 k^0 - f^0 P^0 k^0 = (1 - f^0)P^0 k^0$$

is escalated by the inflation factor $1 + \rho^0$.

The term in square brackets in (83) is the *user cost of capital* w^0_{CPPR} that is implied by constant purchasing power accounting in the case where the asset is sold at the end of period 0; i.e., we have

$$(86) \quad w^0_{CPPR} \equiv [(r^0_d - \rho^0)f^0 - (i^0 - \rho^0) + \delta^0(1 + i^0)]P^0$$
$$(87) \qquad = [r^0_d f^0 + \rho^0(1 - f^0) - i^0 + \delta^0(1 + i^0)]P^0$$

where (87) follows from (86) by rearranging terms.

Comparing the realized CPP user cost of capital w^0_{CPPR} defined by (86) with our economic user cost of capital w^0_e defined by (9), we see that

$$(88) \quad w^0_e - w^0_{CPPR} = (r^0_e - \rho^0)(1 - f^0)P^0.$$

Usually, the period 0 equity opportunity cost of capital r^0_e will exceed the period 0 inflation rate and so if $f^0 < 1$, w^0_e will usually exceed w^0_{CPPR}.

Finally, we compare realized CPP income defined by (82) or (83) to realized historical cost income I_{HCR}^0 defined by (61) or (62). Comparing (61) and (82), it can be seen that I_{HCR}^0 differs from I_{CPPR}^0 by the inflation indexation term $\rho^0 A^0 = \rho^0[P^0 k^0 - D^0] = \rho^0(1 - f^0)P^0 k^0$; i.e., we have

(89) $I_{HCR}^0 = I_{CPPR}^0 + \rho^0(1 - f^0)P^0 k^0$
$> I_{CPPR}^0$ if $\rho^0 > 0$ and $f^0 < 1$.

Thus in the case of an asset sale, realized historical cost income will exceed realized CPP income, provided that inflation is positive ($\rho^0 > 0$) and the firm has some equity capital ($f^0 < 1$).

Our overall evaluation of CPP accounting is that it is a considerable improvement over historical cost accounting as long as there is general inflation (or deflation) in the economy. *CPP incomes will more closely approximate economic incomes* that are based on current opportunity costs than the corresponding historical cost incomes, provided that there is general price change over the accounting period. Moreover, CPP accounting is a *low cost* alternative to HC accounting; with a few multiplications by the general inflation factor $1 + \rho^0$, period 0 historical accounts can readily be converted into CPP accounts. Finally, CPP accounting is *reproducible* if the period by period general inflation factors $1 + \rho^0, 1 + \rho^1, 1 + \rho^2$, etc. are given to accountants by a suitable authority.

The weakness of CPP accounting is that it assigns too many roles to the general inflation rate ρ^0; i.e., in comparing (87) to (9) and (65) to (9), we see that CPP accounting implicitly assumes that the opportunity cost of equity r_e^0 equals ρ^0 and (in the unrealized case) that the asset inflation rate i^0 equals ρ^0 as well. Perhaps this lack or accuracy of CPP accounting has offended the sensibilities of many precision oriented historical cost accountants.

We conclude this section by discussing some of the objections made by accountants over the years to CPP accounting.

The first objection we list is one that we already noted in section 6.2; i.e., that the general price index $1 + \rho^0$ is faulty:

> "Furthermore, the use of a general price index for the purpose of modifying dollar values and dollar results assumes that all investors are alike, having the same purchasing habits".   *Stephen Gilman*[1939; 6]

The answer to this objection is that if general price change is significantly different from 0 over the accounting period, it is better to make some adjustment for the change in the purchasing power of money than make no adjustment at all.

A second objection is also due to Gilman:

"[Sweeney's] proposal has been given serious consideration by accounting thinkers. During severe inflationary periods his plan, or a variation of it, may be almost a necessity. It is, however, rather unlikely that stabilized accounting will win much acceptance until an actual inflation sets in, since there is no powerful group behind it and no tax-saving advantage inherent in it".

Stephen Gilman[1939; 6]

However Gilman's assertion that switching to CPP accounting when there is general inflation would not result in a reduced stream of income and hence lower income taxes seems to be incorrect; recall (81) and (89) above. Thus with even modest inflation, businesses would pay lower income taxes under CPP accounting than under HC account.[204]

A third set of objections is due to Dein:

"Current costs (interpreted as opportunity costs, a proper basis for an inquiry of this sort) are spuriously equated to historical investment costs restated in terms of current dollars. This equating is appropriate only if obsolescence, physical or functional, has not occurred or is not in prospect. Since the fact of obsolescence typifies American industry generally, the adjustment procedure results in adjusted changes against revenue, which exceed the properly determined current costs. The amount of this overstatement of costs may vary over a wide range, making it hazardous to use these adjusted statements".

Raymond C. Dein[1955; 15]

Dein's first objection that restated historical costs are not equal to the actual value of the asset is valid; in our notation, he is saying that ρ^0 (the general inflation rate) will not equal i^0 (the asset specific inflation rate). Our response to this objection is that some adjustment for inflation will usually be better than no adjustment. Dein's second objection is that CPP accounting does not take into account obsolescence. Thus is true but it is a charge that is equally applicable to historical cost accounting. For either system of accounting, the solution to the obsolescence problem is the same: an extra depreciation charge (or price level charge) must be made against the income of the period over which the obsolescence took place.

A final objection to CPP accounting was made by Smith:

"Books must be kept in the established monetary unit. Any attempt to adjust the books to conform to the value of the monetary unit will likely result in a hodgepodge of practices that will not be beneficial either to our business institutions or to our national economy, in my opinion".

Charles W. Smith in G.O. May and others[1952; 125]

Smith's objection to CPP accounting seem to be that: (i) comparability across firms will be lost if some firms use CPP accounting and some use HC accounting[205] and (ii) even if all firms switch to CPP accounting, there is no guarantee that the reproducibility test would be satisfied; i.e., different accountants might come up with different CPP incomes. The force of Smith's objections vanishes if a national tax authority or Accounting Standards Board were to force CPP accounting on all firms and to publish suitable inflation factors $1 + \rho^0, 1 + \rho^1$, etc. that would be used universally. Moreover, it seems that Smith's first objection to CPP accounting could be more properly be directed towards HC accounting; recall our discussion of the current period prices and quantities test in section 7.1 above and the quotations by Latham [1952; 95] and Edwards and Bell [1961; 226].

Inspite of our (academic) approval of CPP accounting as being an improvement over HC accounting, it must be recognized that accountants around the world, for whatever reasons, remain tremendously attached to historical cost accounting:

> "Presumably in response to the high rates of inflation experienced, legal requirements have been introduced in Brazil and Chile for the adjustment of historical cost statements to reflect general price-level changes. In Argentina, historical cost statements are accompanied by statements similarly adjusted. However, the replacement of historical cost statements with statements adjusted for the effects of *general price-level* changes is prohibited by law or by the accountancy profession in forty-three countries. The replacement of historical cost statements with statements adjusted for the effect of *specific price* changes is similarly prohibited in forty-six countries. It is required in none and minority practice in only one country–the Netherlands".
>
> *R.D. Fitzgerald, A.D. Stickler and T.R. Watts*[1979; 12]

We conclude our discussion of CPP accounting by quoting the economist Alexander and the accountant May[206] on the problems associated with historical cost accounting when general inflation is significant:

> "Occasionally tremendous changes in the general price level demonstrate the instability of the monetary measure on which business accounts are based. In extreme cases of inflation the maintenance of capital intact in money value becomes a ludicrous preoccupation. If income is to be defined as the amount a man can dispose of and yet be as well off at the end of the period as at the beginning, it is clearly inappropriate to use money value as a measure of well being in a period during which money's command over goods and services is shrinking rapidly".   *Sideny S. Alexander*[1950; revised 1962; 188]

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"Business accounts play a part also in the division of the income produced in an industry among suppliers of capital, management, labor, and government. In such instances business income taxes will tend to be higher on the monetary than on the purchasing power postulate. It is likely also that wages will tend to be higher: textbooks on collective bargaining include 'ability to pay' among the factors determining wages. And salaries, pension contributions, and bonuses to management may also tend to be higher".

<div align="right">George O. May and others[1952; 78]</div>

7.3 Current Cost Accounting Income (Maintenance of Financial Capital)

"The gross revenue of all the inhabitants of a great county, comprehends the whole annual produce of their land and labour; the neat [net] revenue, what remains free to them after deducting the expence of maintaining; first, their fixed; and secondly, the circulating capital; or what, without encroaching upon their capital, they can place in their stock reserved for immediate consumption, or spend upon their subsistence, conveniences, and amusements".

<div align="right">Adam Smith[1776; reprinted 1963; 218-219]</div>

"Income No. 1 is thus the maximum amount which can be spent during a period if there is to be an expectation of maintaining intact the capital value of prospective receipts (in money terms). This is probably the definition which most people do implicitly use in their private affairs; but it is far from being in all circumstances a good approximation to the central concept".

<div align="right">John R. Hicks[1946; 173]</div>

The income concept in this section can be described as a maintenance of financial capital concept; it is basically the Schanz, Marshall [1920; 74] and Haig [1959; 67-68] income concept that was described in section 7.0 above. As the quotations leading off this section indicate, this income concept can be traced back to Adam Smith and it also corresponds to Hicks' Income No. 1. In the accounting literature, this income concept is known as Current Cost Accounting (CCA) income.

CCA income is very similar to historical cost income except that CCA income always attempts to value capital stock components at their current market value. Making the same notational assumptions for our highly simplified model as were made in sections 7.1 and 7.2 above, we define period 0 *current cost accounting income* I_{CCA}^0 as follows:

(90) $\quad I^0_{CCA} \equiv p^0 \cdot y^0 - \text{ interest paid } + A^1_{CCA} - A^0$

$\qquad\qquad = p^0 \cdot y^0 - r^0_d f^0 P^0 k^0 + [(1 + i^0)(1 - \delta^0)P^0 k^0 - D^0] - [P^0 k^0 - D^0]$

(91) $\qquad\qquad = p^0 \cdot y^0 - [\{r^0_d f^0 - i^0 + \delta^0(1 + i^0)\}P^0]k^0.$

In definition (90), the end of period 0 CCA net value of assets A^1_{CCA} is an estimate of the end of period 0 current net value of assets; i.e.,

(92) $\quad A^1_{CCA} \equiv (1 - \delta^0)P^1 k^0 - D^0 = (1 - \delta^0)(1 + i^0)P^0 k^0 - D^0$

where $D^0 \equiv f^0 P^0 k^0$ is the beginning of period 0 value of debt and $(1 - \delta^0)P^1 k^0$ is an estimate of the end of period 0 current value of the depreciated assets; (recall sections 6.3 to 6.6 above for alternative methods for estimating current values). In the case where the asset is sold at the end of period O, A^1_{CCA} is the same as the realized historical cost net asset value A^1_{HCR} defined earlier by (84).

Note that the present CCA income concept does not index the beginning of period 0 net value of assets $A^0 \equiv P^0 k^0 - D^0$ for any inflation or general price change that might have occurred between the beginning and end of period 0. This is a fatal flaw that is associated with this income concept since this lack of indexation will lead to a hugh overstatement of income in the case of high general inflation.

The term in square brackets in (91) is the period 0 CCA *user cost of capital* w^0_{CCA}; i.e., we have

(93) $\quad w^0_{CCA} \equiv [r^0_d f^0 - i^0 + \delta^0(1 + i^0)]P^0.$

Comparing the CCA user cost of capital defined by (93) with the economic user cost of capital w^0_e defined by (9) leads to the following equation:

(94) $\quad w^0_e - w^0_{CCA} = r^0_d(1 - f^0)P^0$

$\qquad\qquad\qquad > 0 \qquad \text{if } r^0_d > 0 \quad \text{and} \quad (1 - f^0) > 0;$

i.e., the CCA user cost of capital will be *too low* compared to the economic user cost of capital (provided that the business unit has some equity so that $1 - f^0 > 0$) and CCA income defined by (90) will be *too high* compared to economic income.

Comparing CCA income defined by (91) to CPP income defined by (63) and to realized CPP income defined by (83) leads to the following relationships:

(95) $\quad I^0_{CCA} - I^0_{CPP} = [\rho^0(1 - f^0) + (i^0 - \rho^0)(1 - \delta^0)]P^0 k^0$

$\qquad\qquad\qquad > 0 \qquad \text{if } 1 - f^0 > 0 \quad \text{and} \quad i^0 \geq \rho^0 > 0;$

(96) $\quad I^0_{CCA} - I^0_{CPPR} = \rho^0(1 - f^0)P^0 k^0$

$\qquad\qquad\qquad > 0 \qquad \text{if } 1 - f^0 > 0 \quad \text{and} \quad \rho^0 > 0;$

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Thus under normal circumstances where $\rho^0 > 0$ (positive general inflation), $1 - f^0 > 0$ (positive initial equity capital) and $i^0 \geq \rho^0$ (specific asset inflation is equal to or greater than general inflation), CCA income will exceed CPP income.

The CCA income concept rests on the assumptions that equity dollars at the beginning of the accounting period are equivalent to equity asset dollars at the end of the accounting period and so the difference $A^1_{CCA} - A^0$ can be regarded as income; i.e., CCA income is implicitly a maintenance of financial capital concept (unadjusted for general price level change). In periods of high inflation, it is generally recognized that maintenance of financial capital leads to a rather silly income concept.[209] Thus we turn our attention to a more sensible income concept that rests on a maintenance of physical capital assumption.[210]

7.4 Replacement Cost Accounting Income (Maintenance of Physical Capital)

Footnotes

1.   Consider the following quotations:

> "The answer to the question what is the *mean* of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are: as many kinds of average as there are purposes; and we may almost say in the matter of prices as many purposes as writers. Hence much vain controversy between persons who are literally at cross purposes".   *F.Y. Edgeworth*[1888a; 347]

> "There may be more than one *Money Value of the Social Income*, each corresponding to a different purpose of calculation".   *John R. Hicks*[1940; 107]

> "The concept of business income is dependent on the ends served by the use of the income measurement. There is no one correct method of computing net profit. The question of which procedure to use can be decided only on the basis of the purpose or purposes for which the figures are to be used, not on an abstract basis. Hence, the result will vary according to the purposes for which the calculations are being made".   *Albert L. Bell*[1953; 44]

> "Conceptual issues must be settled before measurement issues; we need to know *what* to measure before we can decide *how* to measure. On the other hand, serious measurement problems can severely diminish the usefulness of even the most superior conceptual approach".
>   *John Leslie Livingstone and Roman L. Weil*[1982; 251]

> "In this article, I propose a resolution for the controversy over depreciation …. the essence of the controversy arose from two different views of what it means to maintain capital 'intact'. … There are two objectives, not one – income and wealth accounting, and productivity measurement. Part of the depreciation controversy arose from the presumption, usually implicit, that identical capital concepts could be used for different purposes".   *Jack E. Triplett*[1996; 94]

2.   The following quotations illustrate this point and also indicate that the problems of allocating costs across periods become more severe as the length of the accounting period is shortened:

"Early enterprises and partners working in the main in isolated trading ventures, needed only an irregular determination of profit. But before the business corporation had been very long in operation it was evident that it needed to be treated as a continuing enterprise. For example, calculating dividends by separate voyages was found impractical in the East India Company by 1660. Profit calculation therefore became a matter of periodic estimates in place of the known results of completed ventures".   *A.C. Littleton*[1933; 270]

"The difficulty of inputing expenses to individual sales or even to the gross earnings of the accounting period, the month or year, is an ever present problem for the accountant in the periodic determination of enterprise income. The longer the period for which the income is to be determined, the smaller the relative amount of error. Absolute accuracy can be attained only when the venture is completed and the enterprise terminated".

William T. Crandell[1935; 388-389]

"The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple matter of recording completed and fully realized transactions: an act of primitive simplicity".

Stephen Gilman[1939; 26]

"*All* the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter".   *David Solomons*[1961; 378]

3.   "The definition of the profit of an enterprise depends upon which concept of capital we choose as our starting point".   *Fritz Schmidt*[1930; 235]

4.   "Every science, methodology, or other body of knowledge is oriented to some conceptual structure–a pattern of ideas brought together to form a consistent whole or a frame of reference to which is related the operational content of that field. Without some integrating structure, procedures are but senseless rituals without reason or substance . . .".   *William J. Vatter*[1947; 1]

5.   "A productive system may be regarded as a system of transformation processes in which natural resources, other factors of production and products are combined to produce other products. These transformation processes take place in establishments and may conveniently be grouped together in industries".

<div align="right">*Richard Stone*[1956; 27]</div>

"The activity of production is fundamental. In the System, production is understood to be a physical process, carried out under the responsibility, control and management of an institutional unit, in which labour and assets are used to transform inputs of goods and services into outputs of other goods and services".

<div align="right">*United Nations*[1993; 4]</div>

6.   However, note that "sale" still differs from "revenue collected" and hence the traditional accounting concept of revenue is still somewhat hypothetical and differs from cash flow accounting. Some representative quotations which illustrate some of the differences between accounting and economic revenues follow.

"Some accountants and economists express the opinion that income is the result of production rather than of sale, and advance certain arguments accordingly".

<div align="right">*Stephen Gilman*[1939; 121]</div>

"Why do economists insist on accrual and accountants on realization? Once more we may ask if the difference is one of principle or practicality. Economists' insistence on accrual stems from the fundamental idea of income as a difference between wealth at two difference points of time. If a corporation has more valuable assets at the end of the period than at the beginning this is an increase in wealth, whether a sale has taken place or not. The accountant may reply that he cannot be sure the gain has been made until the sale has taken place".

<div align="right">*Sidney S. Alexander*[1950; republished in 1962; 171-172]</div>

"In accounting, business income is generally conceived as the residual from matching revenue realized against costs consumed …. The main task of income accounting is to match revenue realized against costs consumed in generating the revenue".

<div align="right">*Emily Chen Chang*[1962; 636]</div>

7.   "I believe I have had the fortune to come upon a method of analysis which is applicable to a wide variety of economic problems".   *John R. Hicks*[1946; 1]

8.   All three approaches are briefly surveyed in Diewert [1980a] while the first two approaches are more intensively surveyed in Diewert [1992]. For further information and references to the nonparametric approach to efficiency measurement, see Diewert and Parkan [1983], Varian [1984] and Charnes and Cooper [1985]. In the operations research and management science literature, approach (iii) is known as Data Envelopment Analysis.

9.   Gilman [1939; 378] noted the early attempts of cost accountants to improve operating efficiency: "Giving but secondary consideration to the balance sheet viewpoint or to general problems of financial accounting, cost accountants, engineers, and efficiency men approached accounting problems from the engineering side with the result that they looked upon accounting data as statistical measures of operating efficiency".

10.   Schmalenbach [1919; translated 1959; 17] traced this function of accounting measurement to early German legislation to prevent fraud: "The main accounting aspects of the 1884 legislation were: Companies must declare their true profits to shareholders, and those who intend to become shareholders, and may not pay out dividends which are distributions of capital and not of profits".

11.   We will discuss some of the other items on the accountant's list as well.

12.   "The apparent economic gain measured in dollars is often largely or entirely nominal because of the lessening value of the dollar. Many a taxpayer has felt the injustice of being obliged to pay large sums in income and profits taxes from net earnings determined by orthodox accounting methods which were fictitious as a measure of true improvement in economic condition".

William A. Paton[1920; 3]

13.   We call w^0 an ex post user cost because it can be calculated only when end of period 0 information on the price of the used asset, P_u^1, becomes available. An ex ante user cost would have the same form as (2) except that the actual end of period value for the asset, P_u^1, would be replaced by a (beginning of period 0) *forecast* for the used asset price, say $\tilde{P}_u^1$. From the viewpoint of economic decision making, the ex ante concept is more relevant but there is the practical difficulty of determining how the firm made its forecasts. The ex post concept is more useful from the point of view of evaluating the actual performance of the firm over the accounting period and so we will use the ex post concept in the remainder of the paper. For discussions of the usefulness of ex ante and ex post concepts, see Hicks [1946; 178-179] and Sterling [1970; 8-9].

14.   We discuss alternative methods for estimating economic depreciation rates in section 4 below.

15.   "It is most convenient to express any real measure of income in items of dollars of purchasing power as of a specified time ... the real income of any year is expressed in dollars of purchasing power as of the end of the year."

Sidney S. Alexander[1950; republished 1962; 189]

16.   In fact, Christensen and Jorgenson [1969; 302] derived precisely formula (9) but under the hypothesis of geometric or declining balance depreciation rates. Edwards and Bell [1961; 172] provided a verbal description of the user cost formula (7) while Griliches [1963; 120] provided a verbal description of the user cost formula (8). The technique of specializing Hicks' [1946; 193-194] general intertemporal framework in order to derive user cost formula of the form (2), (4) and (6) was repeatedly used by Diewert [1974; 503-504], [1980b; 470-475] [1983a; 211-216][1992a; 191-195]. However, the essence of this technique can be traced back to Böhm-Bawerk [1891; 344].

17.   In sections 4 and 6 below, we consider alternative methods for the determination of used asset prices.

18.   We will examine some of these timing problems in section 8 below.

19.   Böhm-Bawerk [1888; translated 1891; 301-303 and 365-371] and Fisher [1930; 48-52] patiently refuted the Marxian exploitation theory which denied that interest was a valid cost of production.

20.   "In our science there are three views in circulation as to the formation of capital. One finds its origin in Saving, a second in Production, and a third in both together. Of these, the third enjoys the widest acceptance, and it is also the correct one".   *Eugene von Böhm-Bawerk*[1888; translated 1891; 100]

21.   For the case of one (spot) commodity and two periods, his geometric analysis was particularly compelling. Fisher [1930; 246] introduced a family of "willingness lines" (i.e., indifference curves over combinations of consumption today and consumption tomorrow) and in Fisher [1930; 265], he introduced the "opportunity line" (i.e., the combinations of consumption today and consumption tomorrow that the economy could produce–the intertemporal production possibilities set). Unfortunately, Fisher [1930; 57] viewed interest as a capital gain and hence interest payments were not negative "income" for a producer and hence could not be a "cost" in his view. Fisher's "income" was what everybody else called "consumption" and so his terminology caused great confusion in the economics literature of his time.

22.   The Hicksian model of temporary equilibrium can be viewed as a generalization of a model of capital formation due to Walras [1874; translated 1954; 267-306]. In order to justify a demand to save or defer present consumption, Walras [1954; 274] assumed the existence of a single future period commodity that would be consumed in fixed proportions indefinitely and hence had a price which was proportional to the reciprocal of the one period interest rate, which was assumed to be constant over time. In deriving the price of this future composite commodity, Walras assumed that the spot price of the composite commodity would remain constant over future periods; i.e., he made a static expectations assumption. Hicks [1946; 227-232] was able to dispense with these restrictive assumptions by assuming general intertemporal preferences for consumers and definite expectations about future prices. Walras' [1954; 239-240] theory of production was also somewhat primitive. He essentially assumed: no joint production, constant returns to scale and fixed coefficient or no substitution production functions (which have come to be known as Leontief [1941] production functions). On the other hand, Hicks [1946; 325-6] had a perfectly general intertemporal production function. For a modern interpretation of Walras' theory of capital formation and interest rate determination, see Diewert [1977].

23.   "In the muddy pool of controversy over the question of 'interest on the investment' , one finds all kinds of slippery arguments about what rate is to be employed. Most who discuss the question seem to think that, given a correct rate, there is little to object to except the arbitrariness of the rate chosen and of the valuations that result, and the clerical labor involved in the calculating and recording .... But the present writer wishes to urge a somewhat different objection. If interest is to be charged at some agreed rate, into what quantity shall that rate be multiplied? Into book value of the assets? But look at these book values. Consider how they are themselves determined and consider how far they are likely, under any valuation rule yet discovered, to miss the ideally useful and convenient figure. To go through the book valuations found by the simple [accounting depreciation] formulas in Appendix A, where exact prevision of events is supposed, and begin charging interest upon them, is as absurd as trying to correct for the earth's rotation in a snowball fight. To attempt a 5% alternation of an amount which may be 50% in error in a direction unknown to us, is what the kindergarten teachers call 'busy work"'.

John B. Canning[1929; 297]

24.   "Cost-of-capital is seldom recognized when practical men calculate depreciation. This is not unreasonable if the rate is low or the life is short. But cost-of-capital alters the figures materially if the rate is high or the life is long

(as they often are): its recognition then gives useful extra information about asset values, income and costs". *William T. Baxter*[1975; 159]

25.   "The whole question of whether actual historical costs or imputed costs (this includes all costs that are not actual, from the point of view of explicit transactions entered into between the particular enterprise and other parties) should constitute accounting data is that of whether the accountant can or should attempt to account for enterprise income on the basis of the same set of concepts that the economist uses. The inclusion of imputed costs of any variety, in the accounts, invariably results in a confusion of expenses with aspects of business income or its distribution". *William T. Crandell*[1935; 387]

"Although few would assert that equity interest is not a cost, many persons argue that it is an imputed cost and that accounting does not record imputed costs". *Robert N. Anthony*[1973; 90]

26.   Economists who are social or national income accountants tend to resist inputing an interest cost for the use of equity capital as the following quotations indicate:

"The amounts of rents and interest actually payable on rented land and borrowed funds are recorded in the allocation of primary income account, and the entrepreneurial income account, but the implicit rents on land owned by the enterprise and the implicit interest chargeable on the use of the enterprise's own funds are not recorded in the accounts of the System". *United Nations*[1993; 175]

"The volumes of intermediate consumption, consumption of fixed capital and any taxes on production measured at the prices or rates of the previous year or the fixed base year should be added to obtain a comprehensive volume measure covering all inputs". *United Nations*[1993; 402-3]

The last quotation indicates that the current U.N. System of National Accounts does not recognize interest, either imputed or paid, as a period cost that should be associated with the use of capital inputs–only depreciation and tax costs are recognized by the System.

27.   If the enterprise is also a lender of funds as well as a borrower, then we let the corresponding B_j^0 and b_j^0 be negative (so that r_j^0 defined by (11) is still positive). We assume that all $B_j^0 \neq 0$ and $B^0 > 0$, so that the enterprise is a net borrower in period 0.

28.   We include any repayments of principal in B_j^1 that may have occurred for the jth debt instruments over period 0.

29.   Overall ex ante debt interest rates $\tilde{r}_j^0$ and $\tilde{r}_d^0$ could be defined in a manner analogous to (14) and (15) except that the actual market values of the firm's bonds at the end of period 0, the B_j^1, would be replaced by anticipated or forecast values for these market values, say $\tilde{B}_j^1$ for $j = 1, \ldots, J$. However, in keeping with our ex post evaluation framework explained in footnote 13 above, we will use ex post actual effective interest rates rather than ex ante forecasted effective interest rates.

30.   We will consider an alternative treatment of debt instruments in section 15 below.

31.   See Nagorniak [1972; 351] for an example of this approach.

32.   For more recent discussions of the equity premium puzzle, see Benninga and Protopapadakis [1992; 770] and Burnside and McCurdy [1992].

33.   New financial instruments have recently come onto the market place that create the equivalent of indexed (for inflation) bonds. However, there will still be some small risk elements associated with the use of these instruments since the indexation may not be perfect.

34.   I believe that ex ante interest rates of the form defined by (13) are more relevant than ex post interest rates of the form defined by (15) in providing an approximation to the business unit's beginning of period 0 opportunity cost of equity capital, which seems to me to be an inherently ex ante concept.

35.   Moreover, these authors pointed out that the Moody's bond yield for the entire private sector economy could be used as the cost of capital in an economy wide total factor productivity study so that the cost of capital for a regulated utility and the entire private sector economy could be treated in a symmetric manner using this exogenous bond rate approach.

36.   Recall the above quotation by Hotelling. It should be noted that the statistician Hotelling [1925; 345] deduced a continuous time counterpart to the user cost formula (7).

37.   Of the three factors identified by Griliches, only the first two are factors in determining depreciation rates as we have defined them; the third factor, obsolescence, would tend to decrease both P_u^1 and P^1, leaving the depreciation rate δ^0 defined by (3) approximately unchanged. It is interesting to note that Pigou identified more or less the same three factors affecting depreciation as Griliches:

"Allowance must be made for such part of capital depletion as may fairly
be called 'normal'; and the practical test of normality is that the depletion is

sufficiently regular to be foreseen, if not in detail, at least in the large. This test brings under the head of depreciation all ordinary forms of wear and tear, whether due to the actual working of machines or to mere passage of time – rust, rodents and so on – and all ordinary obsolescence, whether due to technical advance or to changes of taste. It brings in too the consequences of all *ordinary* accidents, such as shipwreck and fire, in short of all accidents against which it is customary to insure. But it leaves out capital depletion that springs from the act of God or the King's enemies, or from such a miracle as a decision tomorrow on the part of this country to forbid the manufacture of whisky or beer. These sorts of capital depletion constitute, not depreciation to be made good before current net income is reckoned, but capital losses that are irrelevant to current net income". *A.C.Pigou*[1935; 240-241]

38.   "*Bookkeeping Modernized* by John Mair (2nd edition, 1768). The same general procedure is described as in *Bookkeeping Methodiz'd* (17) and there are no illustrations of depreciation but the inference might be made that if the 'value' of the property were less than cost, this decrease in value would be included in the change of Profit and Loss". *Perry Mason*[1933; 210-211]

      "There is little reason to doubt that depreciation was originally calculated on the basis of appraisals. The appraisal, it may be conjectured, was originally on a market price basis in order to obtain a figure roughly equivalent to what would have been realized at the date of the appraisal had the asset actually been sold …. After general adoption of the accounting period convention, such appraisals were probably made at the end of each accounting period. It must, however, soon have been obvious that such periodic appraisals gave erratic results depending, of course, upon who made them, how they were made and the general state of business at the time they were made". *Stephen Gilman*[1939; 488]

39.   See Hotelling [1925; 350].

40.   See Canning [1929; 276-7].

41.   See Christensen and Jorgenson [1969] and Jorgenson [1996a][1996b].

42.   'It is undoubtedly true that the methods now in use for the systematic accounting for depreciation and many of the now generally accepted concepts of depreciation have a comparatively recent origin, and that much of the development of the subject has taken place since and as a result of the establishment

of governmental regulation of public utilities and the enactment of income tax legislation".   *Perry Mason*[1933; 209]

"Under the impetus of increased income tax rates due to World War I, the interest in depreciation calculation increased. The first edition of Bulletin *F* issued by the Bureau of Internal Revenue in August 31, 1920, advocated the straight-line age-life method".   *P.D. Woodward*[1956; 74]

"Rates of depreciation must of necessity conform to Internal Revenue edicts– on documents submitted to the Internal Revenue Service. Unfortunately, the same rates are often used in accounts which do not have to be submitted to the IRS, with attendant undesirable influences on business decisions .... The writer advises engineering students as follows: 1. Disregard official rates of depreciation. 2. Estimate economical length of life for the type of service originally intended for the asset ...".   *William F. Berkow*[1964; 380]

43.   "Finally, a word should be said about the professional responsibility for valuation. Many accountants assert that valuation of fixed tangible assets is a job for appraisal engineers. Others say that it is the job of the management themselves and that accountants have discharged their whole duty when they avoid certifying statements in which the assets have been negligently or fraudulently valued. The engineers are not too happy with the burden thrust upon them. They say, at least many of them do, that it is impossible to make valuations unless the operating policy, particularly that of maintenance, upkeep and repairs, is foreknown".   *John B. Canning*[1929; 307]

44.   Wright contrasted the accountant's allocation approach with the economist's change in value approach as follows:

"There have been two distinct approaches to the solution of the depreciation problem which might be designated the 'accounting approach' and the 'economic approach', respectively. The accounting approach requires the cost of an asset less salvage, if any, to be distributed over the life of the unit 'in a systematic and rational manner'. The economic approach, on the other hand, ignores cost as an irrelevant datum: the value of an asset at any point of time is simply the sum of its discounted future services (including salvage if any).
    It seems clear that the accounting approach does not really represent an attempt at valuation: indeed, it has been officially described as 'a process of allocation, not of valuation' ".   *F.K. Wright*[1964; 81]

45.   "For a specific asset, objective verifiable values based upon external transactions are available at only two points of time: at the moment of acquisition, and at the moment of disposal. If these two events occur within the same accounting period, no depreciation problem arises. But when the events are widely separated in time (as is usually the case with fixed assets), determination of periodic income is impossible without establishing a value for the asset at the end of each intervening period. The problem of depreciation accounting is the problem of establishing these needed values without the objective verifiable basis which only external transactions can provide".   *F.K. Wright*[1964; 81]

46.   "The interminable argument that has been carried on by the text writers and others about the relative merits of the many formulas for measuring depreciation has failed, not only to produce the real merits of the several methods, but, more significantly, it has failed to produce a rational set of criteria of excellence whereby to test the aptness of any formula for any sub-class of fixed assets".
John B. Canning[1929; 204]

47.   "If historic cost is to be allocated among the asset's services as time passes, it is necessary to know in advance the total stock of these services. Otherwise there can be no basis for apportionment. Current cost depreciation, on the other hand, requires in theory no such clairvoyance. We need only know the services used or foregone this period and the price this period of those services".
Edgar O. Edwards and Philip W. Bell[1961; 175]

48.   'A thousand new Ford cars, regardless of prior statistics, may last five years, eight years, or ten years, a fact which no one can determine from examining statistics of old Ford cars".   *Stephen Gilman*[1939; 513]

49.   "The research reported upon in this article involved comparing measures of asset book value under conventional accounting practice with a specific type of market value, namely, exit value. The comparison has been with respect to the criteria of comparability and objectivity, operationally defined in such a way as to be fully consistent with a widely held public view that like assets should be shown at like amounts in financial statements ... the findings are unequivocal: (1) Exit values exhibited greater comparability than did book values. (2) Exit values were more objective than book values. (3) The major cause for the lack of objectivity in book values was dispersion in accounting estimates [i.e., in estimates of asset life and salvage value] – not accounting methods".
James E. Parker[1975; 523]

50.   'But as a practical matter the quantification and valuation of asset services used is not a simple matter and we must fall back on estimated patterns as a basis for current cost as well as historic cost depreciation. For those fixed assets which have active second hand markets the problem is not overly difficult. A pattern of service values can be obtained at any time by comparing the market values of assets of different ages or degrees of use. The differences so obtained, when related to the value of a new asset, yield the proportions of asset value which are normally used up or foregone in the various stages of asset life".

Edgar O. Edwards and Philip W. Bell[1961; 175]

51.   We suppose that all units of the durable input have been retired by the time they reach N periods of use.

52. Definition (22) is the same as definition (3): our old P^1 and P_u^1 are now P^{t0} and P^{t1} respectively for $t = 1$. We also redefined our old δ^0 as δ_0.

53.   Beidleman [1973] [1976] and Hulten and Wykoff [1981a][1996; 22] showed that equations (24) must be adjusted to correct for early retirement of assets; i.e., equations (24) assume that all units of the asset are retired at the end of N periods of use. Schmalenbach [1959; 91] noted that neglect of the survival problem leads to serious errors in the estimation of depreciation rates.

54.   See Jorgenson [1996c; 27-28] for a nice summary of the methods that have been used to date.

55.   "The findings of this chapter provide extensive evidence regarding the predominant role of age in the decline in value of certain fixed assets and the relative unimportance of valuation parameters other than age. The initial rapid decline in second hand value calculated for the regression models supports the use of accelerated depreciation techniques and the approach to finite scrap value favors declining balance methods of depreciation. The range of possible asset lives endorses the need for probability life depreciation".

Carl R. Beidleman[1973; 51-52]

56.   "We have used this approach to study the depreciation patterns of a variety of fixed business assets in the United States .... The straight-line and concave patterns are strongly rejected; geometric is also rejected, but the estimated patters are extremely close to (though steeper than) the geometric form, even for structures. Although it is rejected statistically, the geometric pattern is far closer to the estimated pattern than either of the other two candidates. This leads us

to accept the geometric pattern as a reasonable approximation for broad groups
of assets . . .".   *Charles R. Hulten and Frank C. Wykoff* [1996; 16]

57.   This criticism of depreciation theory dates back to Saliers [1922; 172-174] at least.
Many other authors noted this problem:

> "The question of charging depreciation as a function of output rather than
> of time has been discussed of late. It is more natural to consider the depreciation
> of an automobile in terms of miles than of years".
>
> *Harold Hotelling* [1925; 352]

> "While there is much to be said in favor of depreciating automobiles and
> trucks on a mileage basis, rather than by the number of years of use, and while a
> similar expedient may well be adopted in distributing the depreciation of other
> machinery and equipment, it must be observed that depreciation is seldom a sole
> function of use or time. Generally it is a combination of the two, and it is often
> desirable to check one method by applying the other".
>
> *Stephen Gilman* [1939; 345-346]

> "The two main defects of depreciation data are found to be that they ignore
> variations in the degree of utilisation, and that they are largely based on original
> rather than reproduction cost".   *L.M. Lachmann* [1941; 375]

58.   "A truck used to haul logs in timber country is not likely to yield the same
pattern of services as one used to haul produce over superhighways. Physically
identical assets having sharply different uses should be placed in separate cate-
gories and treated as different assets, for example, logging trucks and produce
trucks. How fine a distinction should be drawn is a matter of practicality".
Edgar O. Edwards and Philip W. Bell [1961; 174]

59.   The first approach could be viewed as a special case of the second if we allowed
discrete classification variables in place of continuous ones. In his study of used automobile
prices, Hall [1971] used the first approach, which Jorgenson [1996c; 27] termed the "analysis
of variance approach". Beidleman [1973][1976] used the second approach, which Jorgenson
[1996c; 28] termed the "hedonic approach".

60.   If the business unit produces more than one output, the additional outputs can be
absorbed into the x^t vector as negative inputs.

61.   For a review of duality theory and the associated functional form problems, see
Diewert [1993b].

62.   Pigou [1935; 238] clearly distinguished exhaustion (or decline in useful life) from physical deterioration:

> "A distinction should be drawn between physical changes which, while leaving the element as productive as ever, bring nearer the day of sudden and final breakdown, and physical changes which reduce its current productivity and so rentable value."

63.   If we assume a flexible functional form for the production function F, the number of parameters to be estimated will grow approximately as the square of the number of inputs and outputs that are distinguished in the model. If we do not assume a flexible functional form for F, then our a priori restrictive assumptions on the substitution possibilities for the technology will generally lead to biased estimates for the depreciation rates. These difficulties with the production function approach are discussed in more detail by Diewert [1992a; 177].

64.   Diewert [1983a; 212] [1983b; 1100-1102] discussed these expectational difficulties with ex ante user cost formulae and noted that equation (27) could be used to estimate the producer's anticipated price $\tilde{P}_u^1$. Similarly, given r^0, P^0, an observed rental price $\tilde{w}^0$ and an estimate for the depreciation rate δ^0, (28) could be solved for $\tilde{i}^0$, the anticipated inflation rate.

65.   Observing the rental price $\tilde{w}^0$, the price of a "new" asset at the beginning of the period P^0 and the opportunity cost of capital r^0 allows us to form an estimate of the end of period value of the used input, $\tilde{P}_u^1$ by solving (27). This estimated value $\tilde{P}_u^1$ incorporates depreciation and expected capital gains which is all we need for accounting purposes; i.e., it is not necessary to separately estimate δ^0 and $\tilde{i}^0$.

66.   This is more or less a verbal description of the used asset prices approach outlined in section 4.4 above.

67.   "They (entrepreneurs) buy goods of remoter rank, such as raw materials, tools, machines, the use of land, and, above all, labour, and, by the various processes of production, transform them into goods of first rank, finished products ready for consumption. ... Goods of remoter rank ... are incapable of satisfying human want; they require first to be changed into consumption goods; and since this process, naturally, takes time, they can only render their services to the wants of a future period–at the earliest, that period distant by the time which the productive process necessarily takes to change them into consumption goods".   *Eugen von Böhm-Bawerk*[1891; 299-300]

68.   Hicks [1973; 5-6] [1965; 238-250] attributed the Austrian model of this section (i.e., the model of Hicks [1961] and Edwards and Bell [1961]) to von Neumann [1937] and Malinvaud [1953], but von Neumann's model is at best only a special case of our Austrian model. Diewert [1977; 108-111] [1980b; 472-475] made extensive use of the Austrian model of production but he regarded it as a special case of the intertemporal production model of Hicks [1946; 193-194] to be studied in the following section.

69.   In section 6 below, we will consider various alternative market opportunity costs concepts that might be used. However for now, we think of P_m^0 as being the net realizable value at the beginning of period 0 for a unit of durable input m if no additional units of m are purchased by the business unit during period 0 (this will typically be the case for fixed capital stock components) or P_m^0 is the beginning of period 0 purchase price for a unit of durable input m if additional units of m are purchased during period 0 (this will typically be the case for circulating capital stock components).

70.   In some cases, it may be necessary to distinguish the same "physical" commodity by its time of production within the accounting period; e.g., electricity produced during the peak time of a day is more valuable than offpeak electricity; strawberries supplied during off seasons are more valuable than strawberries supplied during the local growing season, etc.

71.   These new purchases would show up as (negative) components of the vector y^0. Note that we have assumed that the number of components of k^0 is equal to the number of components of k^1, M. Since we allow components of k^0 and k^1 to be zero, this restriction involves no real loss of generality.

72.   Notation: $k^0 \geq 0_M$ means that each component of the M dimensional vector k^0 is nonnegative.

73.   Notation: $x \in S^0$ means "x belongs to the set S^0".

74.   This assumption was used by Hicks [1946; 193-194].

75.   This is the intertemporal profit maximization problem that Hicks [1946; 326] considered.

76.   Of course units of capital can be bought or sold during period t; these purchases or sales are components of y^t.

77.   The reader who is not interested in the technical details of our demonstration can skip to the end of section 5.3.

78.   We assume that the technology sets S^t are nonempty closed convex sets subject to free disposal. This will imply that $\pi^t(p^t, k^t, k^{t+1})$ will be jointly concave in the components of k^t and k^t+1, nondecreasing in the components of k^t and nonincreasing in the components of k^{t+1}; see Diewert and Lewis [1982; 303] or Diewert [1985; 226]. For duality theorems between S^t and π^t and references to the literature, see Diewert [1973] [1993b; 165-168].

79.   The assumption that S^0 is a convex set will imply that $\pi^0(p^0, \bar{k}^0, k^1)$ is a concave function in the components of k^1; see Diewert [1973] [1985; 226]. Thus conditions (42) are also sufficient to imply that $\hat{k}^1$ solves (41).

80.   The convexity of S^1 implies that conditions (44) and (45) are sufficient for $\tilde{k}^1 >> O_M$ and $\tilde{k}^2 >> 0_M$ to solve (43).

81.   Intertemporal profit maximization problems of this type are studied in much greater detail in Diewert and Lewis [1982] and Diewert [1985; 225-228].

82.   Since $\pi^0(p^0, \bar{k}^0, k^1)$ is nonincreasing in the components of k^1, $P^{1*} \geq 0_M$.

83.   These regularity conditions are not insignificant. In particular, the following assumptions may not be satisfied: (i) convexity of the one period technology sets S^t; (ii) differentiability of the variable profit functions π^t and (iii) the assumption of interior solutions to (46); i.e., that $k^{t*} >> 0_M$ for each t.

84.   This observation is due to Edwards and Bell [1961; 174]; recall footnote 58.

85.   We will examine the treatment of inventory capital stock components more closely in section 9 below.

86.   "The economic object of industry consists in transforming commodities, machinery, and labor, which are for the time being relatively cheap, into products which, because they are scarce, will be valued relatively high. The difference between the value of the cost-goods and the value of the final products represents the value increase".   *Fritz Schmidt*[1930; 235]

87.   "Just as industry serves society by changing the material form of goods, so commerce serves by the exchange of finished products . . .. Goods are distributed quite unevenly in different national and international areas . . .. The value difference between such markets is the cause of commerce . . .. Profits are also found in differences in time-values (Zeitwerte). While the entrepreneur can measure the degree of his success in buying and selling if the sale of his goods brings him returns which would permit him to buy or produce an additional quantity of the

same goods at the moment of selling, the purpose of the speculator is differently orientated. The latter wishes to make money produce money by investing it in such goods as will show an increase in value during the investment".

<div align="right">*Fritz Schmidt*[1930; 236]</div>

88.   "A business firm can strive to earn profit by combining factors of production having one value into a product which has a greater value, and it can attempt to make gains by holding assets while their prices rise".

<div align="right">*Edgar O. Edwards and Philip W. Bell*[1961; 272]</div>

89.   "It therefore does not seem to be realistic to suppose that holding gains or cost savings are a class apart from sellers' margins. They all, if positive, make the firm better off, that is, able to command more goods and services than before".

<div align="right">*R.J. Chambers*[1965; 740]</div>

90.   "There is another kind of speculation, which we may call *simple* or *productive* speculation. A man who does not consider himself to have any influence on the market price but who believes that the price is going to rise or is going to fall quite independently of his own actions, and who buys or sells in an attempt to make a profit, is a *simple* or *productive* speculator. If he guesses right he makes a profit, if wrong he makes a loss .... He takes a single product available at another time .... The same thing applies to the man who transports a good firm one *place* to another .... these are perfectly legitimate production activities ...".

<div align="right">*Abba P. Lerner*[1946; 69-70]</div>

91.   "The idea that a good or a service available at a certain date (and a certain location) is a different commodity from the same good or service available at a different date (or a different location) is old".   *Gerard Debreu*[1959; 35]

92.   More generally, the reader should refer to Lerner's [1946; ch. 8] analysis of the benefits to society from competitive speculation.

93.   "Net realizable value is considered significant because it purports to reveal an alternative available to a firm in which the firm, as a periodic (year end) opportunistic calculator, ever seeks to exploit by selling the assets. However valid this might be in the minds of many profit seekers, it tends to misdirect the energies of management from production and the creation of values to the trading function alone. Such misdirection could well result in limited production and thus a lower standard of living".

<div align="right">*Norton M. Bedford and James C. McKeown*[1972; 336]</div>

94.   This quotation illustrates one of the three main differences between business accounting and social (or national income) accounting: the former emphasizes the realization of revenues or the sale of products while the latter emphasizes the creation of production of products. The other two main differences are: (i) social accounting never recognizes capital gains (realized or unrealized) in income statements whereas business accounting recognizes realized capital gains and (ii) business accounting uses a historical cost accounting treatment of depreciation whereas social accounting uses a current price approach to depreciation – in principle, the approach outlined in section 4.4 above (see United Nationals [1993; 147-150]).

95.   "In effect, present accounting data are predicated on the assumption that holding activities do not represent a purposeful means by which management can enhance the market position of the firm. To the extent that the firm attempts to make gains in this fashion, traditional accounting data fail to inform management, owners, and outsiders as to the progress the firm has made during the current period. A second consequence of not counting gains when they arise is that when such gains are in fact realized, the gains earned over the entire time span during which the assets where held by the firm are attributed entirely to the period in which the gains are realized".

Edgar O. Edwards and Philip W. Bell[1961; 222]

"The third consequence of the failure to report capital gains and losses as they occur is the badly distorted balance sheet values which result".

Edgar O. Edwards and Philip W. Bell[1961; 223]

96.   However, historical cost accountants can claim that their procedure of excluding unrealized capital gains from periodic income is more "objective" and "verifiable" than any procedure that includes them in periodic income. This claim means that in section 6 below, we must pay careful attention to the "objectivity" and "verifiability" of alternative methods for valuing the assets of a business unit that are held at the end of an accounting period.

97.   The same logic would justify a separate listing for depreciation expense since it too is hypothetical.

98.   The problem of unrealized capital gains on an asset attracting income tax can be solved by the tax authorities allowing deferral of accrued tax until realization through sale or retirement of the asset.

99.   There is a vast literature on these questions that we can only touch on.

100.   This is sometimes called an establishment.

101.   See the quote above by Wilson [1975; 184].

102.   "Analysis of production functions over the last twelve years has suggested strongly that (a) a major proportion of the increase in per capita income cannot be explained by increases in the capital-labor ratio, and (b) production functions differ strongly among nations and indeed among regions .... An economist could just leave the analysis at that, asserting that the causes which determine the amount of technological knowledge at any one time and place lie as much outside his province as the tastes which determine consumption patters. But in fact, we know that significant quantities of resources are being expended by profit-making institutions on research and development .... Hence, it is suggested, we must regard the body of technological knowledge as the result as well as the cause of economic changes".   *Kenneth J. Arrow*[1969; 29]

103.   "... the basic assumptions of economic theory are either of a kind that are unverifiable–such as that producers 'maximise' their profits or consumers 'maximise' their utility–or of a kind that are directly contradicted by observation– for example, perfect competition, perfect divisibility, linear-homogeneous and continuously differentiable production functions, wholly impersonal market relations, exclusive role of prices in information flows and perfect knowledge of all relevant prices by all agents and perfect foresight. There is also the requirement of a constant and unchanging set of *products* (goods) and of a constant and unchanging set of *processes of production* (or production functions) over time .... The latest theoretical models, which attempt to construct an equilibrium path through time with all prices for all periods fully determined at the start under the assumption that everyone foresees future prices correctly to eternity, require far more fundamental 'relaxations' for their applicability than was thought to be involved in the original Walrasian scheme".

Nicholas Kaldor[1972; 1238-1239]

"Dynamic general equilibrium models with state contingent goods and convex production sets may be useful for some purposes, but the critics are right that there is something fundamental and important about the evolution of an economy that equilibrium models based on convex sets cannot capture".

Paul Romer[1994; 14]

104.   "Knowledge arises from deliberate seeking, but it also arises from observations incidental or other activities. Haavelmo, Kaldor and I ... have all stressed

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that the activities of production and investment may lead to increases in productivity without any identifiable allocation of resources to that end".

<div align="right">*Kenneth J. Arrow*[1969; 30]</div>

"The Horndal iron works in Sweden had no new investment (and therefore presumably no significant change in its methods of production) for a period of 15 years, yet productivity (output per manhour) rose on the average close to 2% per annum. We find again steadily increasing performance which can only be imputed to learning from experience". *Kenneth J. Arrow*[1969; 156]

105.   See Allen [1983] and the references in Arrow [1962; 156].

106.   "Who invents? Why do they invent? In attempting to answer these questions, economists have identified and studied three kinds of institutions–nonprofit institutions like universities and government agencies, firms that undertake research and development, and individual inventors. In this paper, it is proposed that a fourth inventive institution be recognized. This institution is called collective invention". *Robert C. Allen*[1983; 1]

107.   "Again, it is to his interest also that the secrecy of business is on the whole diminishing, and that the most important improvements in method seldom remain secret for long after they have passed from the experimental stage. It is to his advantage that changes in manufacture depend less on mere rules of thumb and more on broad developments of scientific principle; and that many of these are made by students in the pursuit of knowledge for its own sake, and are promptly published in the general interest". *Alfred Marshall*[1920; 285]

108.   Smith [1963; 8-9] illustrated this general statement by the following specific example:

"In the first fire-engines, a boy was constantly employed to open and shut alternately the communication between the boiler and the cylinder, according as the piston either ascended or descended. One of those boys, who loved to play with his companions, observed that, by tying a string from the handle of the valve which opened this communication to another part of the machine, the valve would open and shut without his assistance, and leave him at liberty to divert himself with his play-fellows. One of the greatest improvements that has been made upon this machine, since it was first invented, was in this manner the discovery of a boy who wanted to save his own labour".

109. "We are thus led to a general rule, the action of which is more prominent in some branches of manufacture than others, but which applies to all. It is, that any manufacturing operation that can be reduced to uniformity, so that exactly the same thing has to be done over and over again in the same way, is sure to be taken over sooner or later by machinery .... Thus the two movements of the improvement of machinery and the growing subdivision of labour have gone together and are in some measure connected".   *Alfred Marshall*[1920; 255]

110.   "It is generally agreed that Adam Smith, when he suggested that the division of labour leads to inventions because workmen engaged in specialised routine operations come to see better ways of accomplishing the same results, missed the main point. The important thing, of course, is that with the division of labour a group of complex processes is transformed into a succession of simpler processes, some of which, at least, lend themselves to the use of machinery. In the use of machinery and the adoption of indirect processes there is a further division of labour, the economies of which are again limited by the extent of the market. It would be wasteful to make a hammer to drive a single nail; it would be better to use whatever awkward implement lies conveniently at hand. It would be wasteful to furnish a factory with an elaborate equipment of specially constructed jigs, gauges, lathes, drills, presses and conveyors to build a hundred automobiles; it would be better to rely mostly upon tools and machines of standard types, so as to make a relatively larger use of directly-applied and a relatively smaller use of indirectly-applied labour. Mr. Ford's methods would be absurdly uneconomical if his output were very small, and would be unprofitable even if his output were what many other manufactures of automobiles would call large".

Allyn A. Young[1928; 530]

111.   Nakamura and Lawrence [1994; 248] have a nice analysis of some of the institutional differences between machines and workers that might cause managers to substitute machines for workers:

"The comparative advantages of using machine labour are readily apparent. Computers and computer controlled machines are consistent in their responses, time after time. Machines are vulnerable to feelings of boredom, fears that techological change may render them obsolete, or inopportune promotion aspirations. They never get pregnant, ask for maternity leaves, file discrimination or harassment suits, object if they are not given training opportunities, demand to be paid time-and-a-half for overtime work, or strikes. When parts of machines wear out, they can be replaced (or the whole machine can be replaced) without concerns

about Workers' Compensation or disability claims being filed. Machines may not always perform as desired, but this is never a consequence of hard-to-handle attitudes or substance abuse problems. Rather, straight-forward methods of scientific and engineering inquiry can usually be relied on to solve the performance difficulties of mechanical devices. And machines *never* have to be monitored to prevent them from intentionally shirking or stealing".

112.   Allen [1983; 10] pointed out that increasing the height of blast furnaces eventually ran into diminishing returns: "These tall furnaces proved to be disasters".

113.   For example, vehicles used to transport goods (trucks) cannot be constructed above and below certain capacities.

114.   "As was shown above, not all causes of increasing returns can be attributed to indivisibility of one kind or another and there is no reason to suppose that 'economies of scale' become inoperative above certain levels of production. There is first of all the steady and step-wise improvement in knowledge gained from experience–the so-called 'dynamic economies of scale' which have nothing to do with indivisibilities. But even in the field of 'static' or 'reversible' economies, there is the important group of cases which I described above as being due to the three dimensional nature of space–i.e., the fact that the capacity of, say, a pipeline can be quadrupled by doubling its diameter while the costs (in terms of labour and materials) are more nearly related to the diameter than to its capacity".   *Nicholas Kaldor*[1972; 1253]

115.   "A ship's carrying power varies as the cube of her dimensions, while the resistance offered by the water increases only a little faster than the square of her dimensions; so that a large ship requires less coal in proportion to its tonnage than a small one. It also requires less labour, especially that of navigation: while to passengers it offers greater safety and comfort, more choice of company and better professional attendance".   *Alfred Marshall*[1920; 290]

116.   This example of a fixed cost is of course due to Adam Smith [1963; 7]. A classic example of a returns to scale effect due to the existence of fixed costs is the square root inventory replenishment rule discovered by Harris [1915; 48-52], Allais [1947; 238-241], Baumol [1952], Tobin [1956] and many others; see Whitin [1952; 503] [1957; 32 and 230] and Hadley and Whitin [1963; 3-4] for additional references to the literation.

117.   This application of probability theory to the determination of adequate bank reserves dates back to Edgeworth [1888b; 122]; for additional applications and references

to the literature, see Whitin [1952; 506-511] [1957; 234-236] and Hadley and Whitin [1963; chapters 4-8]. Edgeworth [1888b; 124] also applied his statistical reasoning to the inventory stocking problem faced by a restaurant or club and noted that optional inventory stocks are proportional to the square root of anticipated demands: "Suppose now the number of members in the club to be doubled or trebled, while their habits are unaltered. At first sight it might appear that the reserve of provisions which the manager requires should increase proportionately. But the corrected theory is that the ratio of the new reserve to the old should not be two or three but the *square root* of two or three".

118.   Bulk purchasing means that the supplying firm can achieve internal economies of scale and thus can offer lower selling prices.

119.   This observation is of course due to Adam Smith as we have seen. Krugman summarizes Marshall's elaboration of Smith as follows:

> "It was Alfred Marshall who presented the basic classic economic analysis of the phenomenon. (Actually, it was the observation of industry localization that underlay Marshall's concept of external economies, which makes the modern neglect of the subject even more surprising). Marshall [1920] identified three distinct reasons for localization. First by concentrating a number of firms in an industry in the same place, an industrial center allows a pooled market for workers with specialized skills; this pooled market benefits both workers and firms .... Second, an industrial center allows provision of nontraded [i.e., non internationally traded] inputs specific to an industry in greater variety and at lower cost .... Finally, because information flows locally more easily than over great distances, an industrial center generates what we would now call technological spillovers ...".   *Paul Krugman* [1991; 36-37]

120.   Adam Smith [1963; 15] was well aware of this factor:
> "As by means of water-carriage a more extensive market is opened to every sort of industry than what land-carriage alone can afford it, so it is upon the sea-coast, and long the banks of navigable rivers, that industry of every kind naturally begins to subdivide and improve itself, and it is frequently not till a long time after that those improvements extend themselves to the inland parts of the country".

121.   "... every increase in [population] is likely for the time to be accompanied by a more than proportionate increase in their power of obtaining material goods. For it enables them to secure the many various economies of specialized skill and specialized machinery, of localized industries and production on a large scale: it

enables them to have increased facilities of communication of all kinds; while the very closeness of their neighbourhood diminishes the expense of time and effort involved in every sort of traffic between them, and gives them new opportunities of getting social enjoyments and the comforts and luxuries of culture in every form. No doubt deduction must be made for the growing difficulty of finding solitude and quiet and even fresh air: but there is in most cases some balance of good". *Alfred Marshall*[1920; 320-321]

122. As tariffs were reduced in the years following World War II, trade between countries grew faster than GDP growth. The North American Free Trade agreement led to a 75% increase in trade between Canada and the U.S. in 5 years. However, since World War II, domestic commodity and labour taxes have increased in most countries a factor which tends to limit the growth of the market.

123. Advertising makes potential purchasers aware of new products and thus stimulates market growth. A particularly effective recent innovation in this area is use of targetted mailing lists.

124. Particularly important today are the improvements in telecommunications technology (fax machines, the internet, etc.). Communications improvements were also important in Marshall's time:

"Meanwhile an increased in the aggregate scale of production of course increases those economies, which do not directly depend on the size of individual houses of business. The most important of these result from the growth of correlated branches of industry which mutually assist one another, perhaps being concentrated in the same localities, but anyhow availing themselves of the modern facilities for communication offered by steam transport, by the telegraph and by the printing-press". *Alfred Marshall*[1920; 317]

125. It is here where basic information on science and engineering can be obtained:

"Let us then look at those elements of the wealth of a nation which are commonly ignored when estimating the wealth of the individuals composing it .... Scientific knowledge indeed, wherever discovered, soon becomes the property of the whole civilized world, and may be considered as cosmopolitan rather than as specially national wealth. The same is true of mechanical inventions and of many other improvements in the arts of production ...".

Alfred Marshall[1920; 59]

126.   "For External economies are constantly growing in importance relatively to Internal in all matters of Trade-knowledge: newspapers, and trade and technical publications of all kinds are perpetually scouting for him and bringing him much of the knowledge he wants–knowledge which a little while ago would have been beyond the reach of anyone who could not afford to have well-paid agents in many distant places .... Although therefore the small manufacturer can seldom be in the front of the race of progress, he need not be far from it, if he has the time and the ability for availing himself of the modern facilities for obtaining knowledge".   *Alfred Marshall*[1920; 284-285]

127.   "But perhaps a greater though less conspicuous hindrance to the rise of the working man is the growing complexity of business. The head of a business has now to think of many things which he never used to trouble himself in earlier days; and these are just the kind of difficulties for which the training of the workshop affords the least preparation. Against this must be set the rapid improvement of the education of the working man not only at school, but what is more important, in after life by newspapers, and from the work of co-operative societies and trades-unions, and in the other ways".
                                                    Alfred Marshall[1920; 308-309]

128.   "While mass media play a major role in alerting individuals to the possibility of an innovation, it seems to be personal contact that is most relevant in leading to its adoption. Thus, the diffusion of an innovation becomes a process formally akin to the spread of an infectious disease".   *Kenneth J. Arrow*[1969; 33]

129.   Having an easily accessible local airport that has direct flights to many international destinations seems to be important in this respect.

130.   "Other things being equal, one person has more real wealth in its broadest sense than another, if the place in which the former lives has a better climate, better roads, better water, more wholesome drainage; and again better newspapers, books and places of amusement and instructions".
                                                        Alfred Marshall[1920; 58-59]

131.   It seems likely that internet services will eventually be substitutes for virtually all of the knowledge transmission activities (i) - (vi) listed above.

132.   Romer's [1994; 13] Figure 3 presents a single (but incomplete) paradigm for modeling innovation:

"Every real economy is presented with an almost incomprehensible number of new goods that can be introduced. Some of these goods are like good Z in Figure 3. They would increase utility. Many others, perhaps the great majority of all possible new goods, would not be worth introducing. The fixed costs are too high and the benefits too low. Out of the enormous set of possible new goods, a very small number are somehow selected and introduced".

Paul Romer[1994; 14]

133.   This point was made by Hicks [1973; 120] many years ago:

"I have so far been telling the story in the conventional terms, of shifts in technology and switches within the technology; but, at the point we have reached, do not the 'technology' and the 'technological frontier' themselves become suspect? They are essential tools of static analysis; but in dynamic analysis, such as this, do we need them? .... The notion of a 'technology', as a collection of techniques, laid up in a library (or museum) to be taken down from their shelves as required, has been deservedly criticized; in itself it is a caricature of the inventive process .... Why should we not say that every change in technique is an *invention*, which may be large or small? It certainly partakes, to some degree, of the character of an invention; for it requires, for its application, some new knowledge, or some new expertise. There is no firm line, on the score of novelty, between shifts that change the technology and shifts that do not".

134.   On the other hand, if the business unit actually sold the asset at the end of an intermediate accounting period in inflationary conditions, income would suddenly be much larger for that period under the realization conventions of historical cost accounting. This discrepancy in historical cost incomes depending on whether an asset is held or sold should alert us to the possibility that something is wrong with historical cost accounting. We will explore these problems in more detail in section 7 below.

135.   The accountant William A. Paton [1920; 2-3] was not far behind in making a similar observation:

"The significance of the dollar–the accountant's yardstick-is constantly changing .... One of the fundamental limitations of accounting arises here. The units of physical science are always the same; and hence direct comparisons of situations and phenomena arising at different times can be made in this field. Accountants deal with an unstable, untrustworthy index; and, accordingly, comparisons of unadjusted accounting statements prepared at different periods are always more or less unsatisfactory and are often positively misleading".

136.   Morgenstern [1963; 66] uses the term "meaningless statistics" to describe historical cost incomes during periods of rapid inflation.

137.   " 'Conservatism', especially when it merely means 'highly probable under-statement', is not meritorious' ".   *John B. Canning*[1929; 1054]

"Conservation infers understatement and understatement infers falsity. Falsity cannot be characterized as fundamental truth".

Stephen Gilman[1939; 204]

138.   " It is, of course, unlikely that balance sheets will be drawn up in the indicated manner; this is a matter for the future. But it is clear that present balance sheets already contain an element of expectation and speculation".

Oskar Morgenstern[1963; 78]

139.   "The accountant of the future will be a distinctly different type .... Accounting and statistics will be his tools; the entire scope of internal and external business problems that are reducible to mathematical measurement will be his field".   *H.C. Daines*[1929; 109]

"It is necessary for the accountant to realize that his measures of income or financial position are actually probability distributions".

Harold Bierman[1963; 504]

140.   Sweeney [1964; 8-11] reviews the early history of this method.

141.   "Current value accounting is easy to explain and meaningful, but hard to audit. It requires estimates of the current values of all assets and liabilities. More often than not, prices for 'used' assets are hard to get. Auditors would be required to make substantial judgemental decisions in implementing current value accounting. But we live in a litigious age, and auditors are reluctant to exercise judgement in such situations because, occasionally, subsequent events might not bear out these judgements, and costly and embarrassing lawsuits may result .... GPLA financial statements are easy to audit and are objective. Two auditors given the same historical records and the same data for the GNP Deflator are likely to derive the same general price level adjusted statements".

Sidney Davidson, Clyde P. Stickney and Roman L. Weil[1976; 225]

142.   At times, various short term emergency adjustments to historical cost accounting as a consequence of rapid inflation have been made:

"In Germany, during the severe inflation period, the orthodox practice of calculating depreciation on the basis of original book costs was eventually swept aside because accountants and businessmen came to perceive that, in maintaining the substance of capital, it was no longer useful. At first various supplementary measures were adopted, such as charging all new fixed asset costs to expense ...".   *Henry W. Sweeney*[1931; 166]

143.   "The only problem left is the selection of the index. In view of the motivation of the enterprise, it should be obvious that we think the Consumer Price Index is the most appropriate. It is the closest substitute for a utility measurement that is currently available .... The other indices which are often described as general, e.g., the implicit GNP deflator, include intermediate goods. Intermediate goods should be excluded from the purchasing power concept, because they are only indirectly productive of utility".   *Robert R. Sterling*[1970; 340-341]

144.   This choice was used by German accountants during the German hyperinflation of 1923; see Sweeney [1927] [1928].

145.   This alternative has also been used; see Wasserman [1931; 10].

146.   See Diewert [1976; 117]. Diewert [1992b; 214-222] also shows that the Fisher ideal index has very good properties from the viewpoint of the test or axiomatic approach to index number theory.

147.   Diewert [1996; 100-103] discusses the first three problems and gives references to the literature.

148.   "The simplest way to convert a money measure into a real measure is through an accepted index of the general price level. No perfectly satisfactory index of the general price level exists, nor can one be conceived. It is not only that price indexes are imperfect because of poor price reporting and inadequate coverage, but even in theory it is impossible to construct a perfect price index no matter how much information one has. Since all prices do not move together it is necessary to use an average of different price movements. The average must be weighted, and the appropriate weights change as between the beginning and end of the period over which price change is being measured .... But for practical purposes, the theoretical imperfection of index numbers need not worry us too much".   *Sidney S. Alexander*[1962; 188]

149.   In many cases, seasonal commodities are not available in all seasons and thus there will be no prices for these out of season commodities.

150.   The problem of making general price level adjustments when the business unit operates in several countries are complex and require more research. Maurice Moonitz [1970; 470] comments on this problem as follows:

> "How generalized is the money we are referring to? ... In virtually all the discussions with which I am familiar, the assumption is made, usually implicitly, that we are concerned solely with a domestic currency and with changes in its exchange value in the domestic economy .... Katano argued that price level accounting is not restricted to the sphere of a single currency, such as the U.S. dollar or the Japanese yen. The proper index, he asserts, should make accounting data comparable and consolidatable between different monetary spheres and over time".

151.   "The appropriate index to use in restating accounting measurements in units of purchasing power is an index of prices of goods consumed by investors".

George J. Staubus[1975;45]

152.   "When accounts expressed in 'diverse amounts of general purchasing power', as in historical dollar financial statements, are restated in terms of the dollar of a single point of time, nothing new is being said. No 'change' has occurred, except in the size of the units of measurement employed".

Maurice Moonitz[1970; 466]

153.   For a discussion, see Sweeney [1964; 45-47].

154.   "A difference between net realizable value and replacement cost, other than that related to direct costs of buying and selling such as commissions, transportation and taxes, indicates that the firm buys in a different market from that in which it sells .... Net realizable value of an asset is the preferable basis for measurement in this type of situation because it takes into consideration the destination of the asset rather than its source".   *George J. Staubus*[1961; 36-37]

155.   "We reach the conclusion that opportunity cost, and not the authors' current cost, is the appropriate asset measurement basis. *Opportunity costs (market resale prices) are relevant to the firm always*".   *R.J. Chambers*[1965; 736]

156.   "Edwards and Bell also build a case for exist prices, but then reject them in favor of entry prices. We were not convinced by their reasons for rejecting exit values, and we particularly disagree with the idea that exit values would be less useful to external users of the data".   *Robert R. Sterling*[1970; 328]

157.   The distinction between entry and exit values was recognized by the Prussian legal system in the 1880's according to Schmalenbach:

> "There is no basis whatsoever for the opinion held by the old school of tax jurists that the user-value allegedly meant is the value in the open market, i.e., the value on a sale. In Prussian land law the user-value was something quite different; it was the value of the property to the average person for use in its present state and therefore approximated in general to the price at which an equivalent property could be acquired".   *Eugen Schmalenbach*[1959; 20]

Economists have also long made the distinction between entry and exit prices:

> "There are three entirely separate concepts of the basis on which capital can be measured, namely *market value, replacement value* and *cost price*".
> *Colin Clark*[1940; 375]

Clark's market, replacement and cost values are the exit, entry and historical cost values of Edwards and Bell respectively.

158.   Essentially appraisers encounter the same sort of difficulties that were mentioned in the previous sentence.

159.   "Under ordinary circumstances the accountant can rely upon the operation of economic forces to determine market values; he is therefore not usually concerned with the discounting process. Market values when obtainable are also objective in character".   *H.C. Daines*[1929; 99]

160.   There are some additional more minor reproducibility problems with historical cost accounting: (i) if certain asset values are "known" to fall below historical cost, then the offending assets are to be valued at "market" value; (ii) there can be some ambiguity as to when exactly a sale is realized; i.e., it is sometimes difficult to allocate revenues to specific accounting periods and (iii) there can be uncertainty about what proportion of overdue payments will eventually become bad debts. Gilman [1939; 541] noted the inconsistency of historical cost accounting practices with respect to point (ii) above:

> "It would appear that those who condemn revaluations upward should, in all consistency, condemn downward revaluations. With some exceptions such consistency is not observed".

161.   This point did not originate with Ijiri as the following quotations indicate, but Ijiri phrased the point in the most elegant fashion:

"By and large, the reason why these writers [on asset valuation principles] could not arrive at a satisfactory theory was their premise, that the object of the balance sheet was the ascentainment of the status of capital. They did not realize that it is not possible to arrive at a value for a capital composed of a number of parts, merely by adding together the values of the individual parts".

Eugen Schmalenbach[1959; 20-21]

"Capital instruments used jointly with others in turning out goods for sale do not, properly speaking, have separate *capital* values at all".

John B. Canning[1929; 233]

"Although the correspondence between this definition of current cost and the data produced under the above rules of measurement is far from perfect, use of its alternative–market value–would raise far more formidable problems. First, an objective set of rules for measuring the market value of plant assets could not easily be established. Next, although the plant account could be assumed to be at market value, there would still remain the problem that with market value the sum of its parts is not equal to the whole".   *Myron J. Gordon*[1953; 376]

162.   More elaborate solutions to the additivity problem could be obtained by adapting the techniques used in the axiomatic cost allocation literature to this revenue allocation context. For references to the cost allocation literature, see Young [1985] [1994] and Moulin [1995].

163.   "I do not object to current cost accounting if one can show that its benefit to society is greater than its cost of implementation. Remember, however, the bill to society for establishing and running such a system can be enormous, considering the cost of assessment, calculation, and auditing (all of which must be done every year) as well as the cost of solving disputes if the firm or the accountants are challenged on the reliability of data or are accused of intending to mislead the public".   *Yuji Ijiri*[1979; 71]

164.   "In some cases, such as permanent investments, plant sites, construction jobs, etc., almost no reliable data may be obtained for use in market valuation".

H.C. Daines[1929; 101]

"There is no active trading market for large aggregates of fixed assets which have been put together into a specialized production design for specialized use. Any attempt to assign a market value to the aggregate of land, buildings, machinery, equipment and motive power constituting the average industrial plant is obviously impossible".   *Stephen Gilman*[1939; 80]

165.   Statistics Canada has used this methodology for years to estimate a construction price index; i.e., engineering and construction firms are asked to provide estimates for the cost of building a specific asset in the current survey period.

166.   Social accountants can use replacement costs as a basis for asset valuation because their efforts are never audited or questioned. This situation is starting to change in the European Union as Eurostat (the central Statistical Agency of the EU) tries to harmonize the statistical practices of its member countries.

167.   Robert R. Sterling [1970; ix] seems to have been the first accountant to argue along these lines:

> "It seems clear, for example, that one can postulate a continuing firm which is operating in two different markets (say, a retailer) and make a good case for valuing inventory at replacement cost. Under those circumstances the 'opportunity' of a unit is the cost of replacing it, since the firm *must* restock".

Edwards [1975; 240-241] argued for the use of entry values for those markets where a firm is usually a buyer and exit values for those markets where the firm is usually a seller; Davidson, Stickney and Weil [1976; 211] endorsed this argument.

168.   "The non-availability of the future series of data, except for certain fragmentary items attaching to the near future, not only prevents the systematic development of realized income statistics to the point of large usefulness but prevents also a full development of capital valuation. For without reliable estimates of all future series to be discounted, reliable present valuations are impossible".

John B. Canning[1929; 321]

169.   "If [the capitalization of the income producing value of the net assets] is impractical of application, since from the very nature of the case, the earnings of a business are the joint product of all the assets, conditions and services which the business possesses and uses. It is impossible, therefore, to impute on the basis of total earnings any particular value to any given asset".

H.C. Daines[1929; 98]

170.   "The familiar accounting dilemma of relevance versus reliability emerges for the question of how do you produce another unique asset, such as *Snow White* or a particular oil field".

John Leslie Livingstone and Roman L. Weil[1982; 253]

"Much of the discussion of changing prices at the conceptual level has assumed or implied a manufacturing environment. Different sorts of measurement problems arise for the natural resources, in financial companies, and service industries. These are often the most challenging measurement issues and these latter industries represent a considerable portion of our economy. Therefore we encourage theorists to focus added attention on these key areas".

John Leslie Livingstone and Roman L. Weil[1982; 255]

171.   "Inasmuch as the price level is not stable for any great length of time, and since this calculation is contemplated for each fiscal period, the only feasible procedure for a company with thousands of assets is the use of price index numbers".   *Albert L. Bell*[1953; 49]

"Where no market exists for new fixed assets of the type used by the firm, two means of measuring current costs are available: (1) appraisal, and (2) the use of price index numbers for like fixed assets to adjust the original cost base to the level which would now have to be paid to purchase the asset in question".

Edgar O. Edwards and Philip W. Bell[1961; 186]

172.   More generally, P^0 can be the estimated beginning of period 0 current value for the asset.

173.   "Fixed assets, as distinguished from current assets, had to be revalued by means of index corrections. The indices ..., which had to be used for any given fiscal year, were published in an official government publication. These index coefficients were computed on the basis of the wholesale price indices for construction materials, lumber and steel products".

H. Peter Holzer and Hans-Martin Schönfeld[1963; 383]

174.   "The fact that the purchasing power shown will be in terms of the index used, and not in terms of the actual purchasing power available to a given enterprise for making its purchases, is a decided limitation to the use of the index numbers in accounting".   *Donald K. Griffith*[1937; 126]

"Not many years ago standard telephone cables consisted of numerous wires encased in a lead sheeting. In the present microwave era it would be just as wrong to apply replacement-cost index numbers to the cost of the old cable and call the result value for the purpose of arriving at depreciation expense as it would be to apply price-index numbers to the cost of the famous twenty mule team and call

the result the cost of automotive transportation".

<div align="right">*Charles W. Smith in G.O. May and others*[1952; 126]</div>

175.   Note that GPLA accounting is not subject to this problem since there is only one asset class. Of course, the countervailing problem associated with GPLA accounting is that it is less relevant or accurate as an approximation to actual current values:

> "A simple general purchasing power index is proposed, but that has no real relevance to the value of capital goods".

<div align="right">*Solomon Barkin in G.O. May and others*[1952; 115]</div>

176.   Suitable rules of thumb would have to be developed. Gilman raises similar timing and domain of definition issues in the context of finding suitable estimates for end of period values for the inventory components of a business unit's capital stock:

> "Another cause of profit distortion is to be found in the methods used for determining selling prices as the preliminary basis for the proportional cost calculation. Should market quotations on the last day of each month be used? Should the daily quotations for the entire month be averaged? Should the averages for the past three months be used? Under mercurial market conditions these questions become important. The purpose of the popular three months' average plan is, according to McKee, 'to eliminate temporary market fluctuations, and reflect costs by market trends instead' ".   *Stephen Gilman*[1939; 333]

177.   "Accountants are fully aware of the difference between 'dollar accounting' and a conceivable 'purchasing power accounting', and would prefer just as the economists do, a purchasing power accounting. But the adjustment data can never become available at the time records are originally made, nor do they become available in time for report making. Whether or not it would pay to make such an accounting currently, is doubtful; but the cumulative effect of a depreciating currency upon valuations of long-lived assets and debts may be such as to require partial readjustments at relatively long intervals".

<div align="right">*John B. Canning*[1929; 196-197]</div>

178.   "In order to make the accounts reasonably reflect current conditions and to avoid abrupt value changes, numbers of accountants have recommended that fixed asset accounts be regularly adjusted by means of an index number. Gradual changes thus computed would be better than the irregular revaluations which have occurred in the past, but the recording of index number adjustments on the books conceals historical costs and at best constitutes only a partial solution to

the general problem of valuation. Even though fixed asset values were satisfactorily determined by index numbers, the more important problem of inventory valuation would still remain". *Ralph C. Jones*[1935; 172]

179. "For each account requiring adjustment the price index is of a homogeneous class of assets which includes those in the account. The use of a specific index for each account rather than a general index for all accounts follows from the use of current cost rather than purchasing power historical cost as the basis of valuation. The appropriateness of the index used for each account is, of course, limited by the knowledge of the assets included in the account, the index numbers available, and by the criterion of objectivity .... This [specific index number adjusted] quantity differs from market value in that (1) historical deferred cost is arrived at by means of arbitrary, generally straight-line, depreciation charges; (2) an index of the cost of new assets is used to adjust used assets; and (3) the impact of technological change on a firm's assets may differ radically from the recognition of technological change in an index number designed to cover a broader group of assets". *Myron J. Gordon*[1953; 375]

180. This alternative will probably be too expensive.

181. We have noted earlier that standard historical cost accounting procedures do not lead to reproducible estimates of depreciation.

182. "These factors may account for the present status of the index number accounting practice in Europe. It had its start in seemingly fertile soil, because the monetary system in Europe at that time was completely broken down, but the index number methodology has failed to develop and bear fruit. It seems reasonable to conclude that, since the index methodology has become dated, it failed to meet the fundamental and lasting needs of business. If it had met a fundamental need it would surely not have disappeared from business usage". *Donald K. Griffith*[1937; 131]

183. "There were, therefore, three stages in the development of fixed asset accounting, the first involving actual realization, followed by the second involving fictitious realization by valuation, and third a recovery of original cost based upon a preliminary estimate of the length of the asset's life. Unlike the first two, the third is not influenced by varying market prices and attempts merely to distribute the cost of the asset over the years which benefit from its use". *Stephen Gilman*[1939; 87]

184.   New asset purchases made during the accounting period could be included in current period costs (in which case, these new asset purchases would be included in the end of the period value of all assets) or the new asset purchases could be segregated and added to next period's starting value of assets held.

185.   Pigou [1924; 34-35] interpreted Marshall's business income or earnings concept as follows:

> "For the dividend may be conceived in two sharply contrasted ways as the flow of goods and services which is *produced* during the year, or as the flow which is *consumed* during the year. Dr. Marshall adopts the former of these alternatives .... Naturally, since in every year plant and equipment wear out and decay, what is produced must mean what is produced on the whole when allowance has been made for this process of attrition .... In concrete terms, his conception of the dividend includes an inventory of all the new things that are made, accompanied, as a negative element, by an inventory of all the decay and demolition of old things".

186.   "If the excess of assets over liabilities at the end of the period was greater than at the beginning, all of the excess represented profit capital. This method provided for the maintenance of asset capital as a quantum, a measure of wealth. This idea of profit was conceptually associated with the final liquidation of a company. There was not full agreement as to the proper value to put upon the assets at each balance sheet date. One leading accountant thought the single account system would require the valuing of all assets according to the market value of the capital shares; others advocated the use of the current market value of the assets themselves; and still others supported cost less actual deterioration to date as the proper value".   *D.A. Litherland*[1951; 476]

187.   Many economists and accountants wish to exclude holding gains or capital gains on assets held through the accounting period from their income concept; recall our discussion of this issue in section 5.4 above.

188.   "Moreover, all stakeholders in the firm must be interested in such key measures as profitability (however measured) and solvency, because the interests of all are affected by the entity's ability to survive".

> *Kenneth W. Lemke*[1982; 303]

189.   "Profit-and-loss statements should be designed to exhibit, in proper classification, *all* profits and losses incurred during a given period whether capital or

current, realized or unrealized. In no other way can stockholders and creditors be placed on an equal footing with informed insiders, and be given a truthful basis of present fact upon which they can base their own estimates of the future".   *Kenneth MacNeal*[1939; reprinted 1962; 69]

190.   The accountant MacNeal made similar observations:

"During this [early] era accounting practices were fairly well suited to the conditions then existing. The owner was not deceived because he knew his business intimately. The banker and trade creditors disregarded all balance-sheet values except those for current assets, and rested secure in the knowledge that these assets were worth at least as much as represented and perhaps a great deal more. The man who lent the owner money and took a mortgage on the fixed assets disregarded the accountant's valuations altogether and made his own appraisals. The small stockholder or bondholder who might place confidence in the entire balance-sheet and profit-and-loss statement did not exist. Each party looked after his own interest in his own way, and the system worked, after a fashion".   *Kenneth MacNeal*[1939; reprinted 1962; 64]

"Accountants now have an obligation to three parties–the small security holder, the management and the creditor; but they still continue to certify financial statements prepared in accordance with practices suited only to conditions existing before the advent of the small security holder".
                                    Kenneth MacNeal[1939; reprinted 1962; 65]

191.   Recall our discussion of the three basic forms of productive activity in section 5.4 above. The profits resulting from transformation and transportation productive activities are embedded in $p^0 \cdot y^0$ while the gross profits (before interest payments) from asset holding activities less depreciation are embedded in $A^1 - A^0$ if $A^1 - A^0$ is positive.

192.   "The first reason is that cost, if properly stated in the first instance, records fact rather than conjecture. It is ordinarily easier and much less expensive to maintain plant records based on cost than on other valuations".
                                    M.B. Daniels[1933; 311]

"The accountant had several reasons for disliking and misunderstanding the case for reform. The very ease with which historical costs can be gathered in a ledger gave him something of a vested interest in such figures so that he might well find it hard to admit their limitations. Again, he probably was confused by the intermixing of general with special price change; this clouds the problem by

raising questions of whether inflation charges are costs or appropriations".

<div align="right">William T. Baxter[1975; 62]</div>

193.   "Countless instances have occurred in which a company's land, its plant, or its security holdings have increased in value to a tremendous extent over many years, while this fact has been concealed from stockholders because the assets have been exhibited in the company's balance sheet only at their historical cost, in accordance with accepted accounting principles. As a result, insiders cognizant of the facts misrepresented upon the balance sheet have been enabled to accumulate the company's stock up to the date when such assets were sold, and an enormous dividend declared. This state of affairs constitutes a vicious, although perfectly legal, defrauding of stockholders, and is exactly the sort of thing that certified public accountants are theoretically supposed to prevent".

<div align="right">Kenneth MacNeal[1939; reprinted 1962; 66]</div>

"Each of us knows of the numerous aberations in our financial reporting which are created by and condoned in the name of the realization concept. A few examples, not necessarily related to mere price level changes, are: ... the innumerable situations in which publicly owned shares are traded appreciably below the net realizable values of the underlying properties owned by the entity, only because the shareholders did not know of such underlying values (nor could they find out from the financial statements prepared on a basis consistent with generally accepted accounting principles".   *Abraham J. Brilloff*[1961; 604]

"There can be little doubt that many, to their sorrow, have failed to understand the conventional limitations of accounting. Losses have been suffered by investors who believed that asset valuations appearing on balance sheets were realizable amounts".   *Stephen Gilman*[1939; 3]

"The same belief and fear which lie beneath the reiteration of the importance of objectivity have been responsible for the abandonment of the idea that balance sheets represent values. Although he admits that many users of accounting statements are interested in asset values, Professor Littleton thinks that these people may be dismissed; they are simply misinformed".

<div align="right">R.J. Chambers[1956; 588]</div>

"The conventional balance sheet is no more useful than last year's news with this year's dates superimposed".   *R.J. Chambers*[1964; 271]

"The mixed nature of amounts appearing in conventional revenue accounts and balance sheets makes it very difficult shortly to frame a hypothesis, other

than that they are the summaries resulting from applying certain operations to certain abstractions. It seems to be preferable to formulate a hypothesis which takes account of observable realities such as, for example, (a) the fact that many users of financial statements believe that income is a disposable surplus and that balance sheet figures represent values; (b) the fact that the relationship between income and the present value of the resources employed to earn it is an important test of accomplishment". *R.J. Chambers*[1956; 589-590]

194.   We shall show this in section 7.6 below. In the realized case, we can compare I^0_{HCR} defined by (62) with the corresponding economic income $I^0_E \equiv p^0 \cdot y^0 - w^e_e k^0$ where w^e_0 is the economic user cost defined by (9). Algebra shows that $I^0_{HCR} = I^0_E + r^0_e (1 - f^0) P^0 k^0 > I^0_E$ if r^0_e and $1 - f^0 > 0$.

195.   "Then why not recognize appreciation also, as it accrues, instead of waiting until a sale is made .... If this appreciation were not recognized in 1914 the item of appreciation would become revenue in 1915; and net revenue would not be correctly stated in either period". *William A. Paton*[1918; 43]

"The insistence of accountants upon the importance of differentiating between realized and unrealized income has probably proved a wise one. But the artificial showing that it causes should be clearly understood. For, as a consequence, a period may be credited with income that it did not earn, and be charged with a loss that it did not suffer". *Henry W. Sweeney*[1933; 334]

"Some limitations of accounting profit as a managerial tool can now be briefly indicated .... Capital gains are counted only when realized. This means that some of the events of past periods, notably price changes and the gains and losses associated with them, are treated as though they were events of the current period. If an asset has been held for five years and then sold, all of the gains and losses arising over the five year period are credited to the year of sale". *Edgar O. Edwards and Philip W. Bell*[1961; 116]

196.   "First, the last-in-first-out method of inventory valuation has been well established and accepted by the profession. It is an improvement in the measurement of economic income but at the price of leaving an unrealistic inventory figure for balance sheet purposes. The merchandise inventory at the end of any fiscal period may be stated at costs that are outdated by years .... Thus LIFO does not seem to be the correct method. The balance sheet and income statement are complementary. Anything that improves one should naturally and logically improve the other". *G. Fred Weber*[1960; 642]

197. "Virtually all economists view interest on capital, proprietary as well as borrowed, as an effective cost of production".   *William A. Paton*[1931; 94]

"If the machine is bought with the help of a loan, then the interest is part of the whole cost; and a similar adjustment for cost of capital should be made even where the machine is not financed by a loan".   *William T. Baxter*[1971; 100]

"In accounting, interest refers only to the cost of using *debt* capital; accountants do not record a change for the use of equity capital .... Here I wish to argue that accounting should adopt the economics concept of interest–that is, regular accounting procedures should take cognizance of the cost of using equity capital".   *Robert N. Anthony*[1973; 88]

198. See Tweedie and Whittingon [1984].

199. Sweeney [1936; reissued 1964; 42] referred to this method of income accounting as general purchasing power accounting or stabilized accounting. Sweeney had another version of stabilized accounting (which will be studied in section 7.5 below) which he called stabilization based on replacement cost.

200. Knowledge of this income concept diffused remarkably rapidly; recall our earlier quotation by the economist Haig [1921; reprinted 1959; 67-68] in section 7.0 above. See Sweeney [1964; 38-39] for additional early references to the use of this income concept, including the third edition of Schmalenbach [1959] (which was published in 1925).

201. Alternatively we can interpret the debt interest rate r_d^0 as the inflation factor for debt; i.e., instead of subtracting $r_d^0 f^0 P^0 k^0$ directly from $p^0 \cdot y^0$, we could subtract this term indirectly by defining the end of period 0 value of debt to include interest payments so that the end of period 0 interest inclusive value of debt could be defined as $(1+r_d^0)f^0 P^0 k^0$, which in turn would be subtracted from the gross end of period 0 value of assets to obtain the corresponding net value.

202. Thus the difference between the two user costs should be approximately invariant to (anticipated) inflation that occurs over period 0. This follows from Fisher's [1896; 69] observation that nominal rates of interest are approximately equal to real rates of interest plus the rate of inflation.

203. The restriction to 3 periods is made only to simplify notational complexity; the same proof works for an arbitrary number of periods.

204. There is at least one significant qualification to this statement. Firms that have substantial holdings of nondepreciable assets (such as land) may not benefit from a switch

to CPP accounting under conditions of general inflation. However, if such a firm were ever sold, its income tax liability would be reduced by a switch to CPP accounting.

205.   G. Edwardd Philips [1963; 704] made the same point:

> "Usual arguments against making price level adjustments in financial statements include: (a) difficulties of defining and measuring the 'price level' and of measuring asset values give any calculations too great a margin of error relative to the amount of the adjustments unless price level changes are extreme, (b) the cost of making the adjustments exceeds their significance, and (c) if, as is probably inevitable, adjustments are made by some firms and not others, the loss in comparability will outweight the improvement in the adjusted figures".

206.   The accountant Schmidt made observations similar to those of May:

> "The question, whether appreciation ... is a profit, is not merely the concern of bookkeepers .... Wage policies are also affected, and, of course, the computation of income taxes".   *Fritz Schmidt*[1931; 289]

207.

208.   See Tweedie and Whittington [1984; 7] or Whittington [1992; 400].

209.   Recall the quotation by Alexander [1962; 188] at the end of section 7.2 above.

210.   The accountant Schmidt clearly distinguished between the maintenance of physical (or real) capital and maintenance of financial (or abstract) capital concepts of in come:

> "It is important to a definition of profit to have a definite capital concept, or property concept. The capital or assets of an enterprise can be: 1. The stock of real property including money and money due, or 2. The *value* of all real property expressed in abstract money units. We call the first the real, capital of the enterprise, the second the abstract capital".   *Fritz Schmidt*[1930; 235]

211.

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