

# **Estimating Capital Inputs for Productivity Measurement: An Overview of Concepts and Methods**

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# Estimating Capital Inputs for Productivity Measurement: An Overview of Concepts and Methods

by Michael J. Harper

## Introduction

Productivity measurement involves comparing trends in output with trends in inputs. The microeconomic theory of the firm uses a "*production function*" to formally describe the relationship between inputs and output. In its simplest form<sup>1</sup>, a production function treats inputs as if they are consumed in the production of outputs. Capital is, of course, one type of input. However, capital goods do not neatly conform with the simple production model. Among other things, they are not consumed in production.<sup>2</sup> Nonetheless, capital goods must specifically be deployed in production for a period of time in order to *render services*. A measure of capital input which would be consistent with theory is therefore the quantity of the *flow of services* provided by capital goods.

The capital service flow is a rather abstract notion and it is rarely possible to measure it directly.<sup>3</sup> Instead, estimation of the service flow depends on applying theory to related information which is more readily obtained such as data on investment. This estimation process depends on a careful analysis of the relationship between capital services and the goods which produce them. The approach used by the U.S. Bureau of Labor Statistics (BLS) to measure multifactor productivity<sup>4</sup> (MFP) is in close conformance with the literature on productivity measurement based on the *neoclassical* production model.<sup>5</sup>

Neoclassical theory deals with the difficult problem of how to treat capital services in a production function. This theory involves a set of assumptions. First, the quantity of capital services is defined in terms of investment goods. Second, investors are assumed to have perfect foresight regarding the future results of their investments. Finally, investment behaviour is assumed to be

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<sup>1</sup> One simple production function,  $f$ , expresses the relationship as  $Y = f(x^1, x^2, \dots, x^n, t)$  where  $Y$  is the quantity of output,  $x^i$  are quantities of inputs, and  $t$  is a time index. Productivity growth occurs as  $f$  shifts outward over time.

<sup>2</sup> This is a property capital shares with labor!

<sup>3</sup> The owner and user of a capital asset are often the same firm. When this is the case, we do not observe a transaction for capital services.

<sup>4</sup> This is the approach BLS uses to measure "multifactor productivity" (MFP). MFP is often referred to as "total factor productivity". MFP involves comparing output with several inputs rather than just labor input. For the US private nonfarm business sector, BLS publishes measures of real final output per combined unit of capital and labor input. For more specific industries including total manufacturing, BLS publishes real "sectoral" output per combined unit of capital, labor, and purchased intermediate inputs of energy, materials, and services. The most recent report which summarizes and presents these measures is BLS [1996].

<sup>5</sup> In this paper, we will speak of the neoclassical "model" or "procedures" for measuring capital or productivity. We will be referring to a somewhat loosely defined scholarly line, with origins in works by Paul Samuelson [1947] and Robert Solow [1957], and culminating in a book by Dale Jorgenson, Frank Gellop and Barbara Fraumeni [1987]. While there are differences between the BLS procedures and those of Gellop, Jorgenson, and Fraumeni, many of the most important concepts are similar.

consistent with long run competitive behaviour: the price of an asset will equal the discounted value of the services it will provide in the future.<sup>6</sup>

In the next section we present an overview<sup>7</sup> of the concepts used by BLS to measure capital. Following this we discuss how these concepts help determine the choices we make on many practical issues of empirical implementation. This discussion also serves as a brief summary<sup>8</sup> of the procedures used at BLS.

## Concepts

Often, the goal of MFP measurement is to isolate the degree to which efficiency and technology influence economic growth.<sup>9</sup> In the complex process of technological advance, capital is a link between the past, the present and the future. Investment decisions made at each point in time influence subsequent production possibilities. By "investing", economic agents try to divert some of the output from one time period's production into making even more output in subsequent time periods. Because of this dynamic, estimation of capital inputs requires a set of concepts which carefully distinguish the quantities and prices influencing these time-related decisions. Neoclassical capital theory provides such concepts.

To understand neoclassical theory, it is useful to think of two separate types of decisions involving capital which are made by producers. The first type is investment decisions, while the second type is production decisions. Investment decisions involve the creation of capital assets for future use, while production decisions involve the efficient deployment of existing assets in the current period.

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6 These service values are realized either explicitly as rents or implicitly as the difference between the value of output and the cost of "variable" inputs.

7 Mathematical derivations of capital services from capital stocks are provided by Robert Hall [1968] (in continuous functions) and by Dale Jorgenson [1973] (in terms of discrete annual data). In this paper, we provide a simplified (but hopefully intuitive) discussion of the issues these derivations address.

8 In this summary we refer to other documents which provide more details on various particulars. Perhaps the most complete of these is Appendix C of a BLS [1983] Bulletin. We do not, at present have a single document which provides both an up-to-date and thorough summary of our procedures- The U.S. Bureau of Economic Analysis is planning soon to publish new detailed investment data and new information on economic depreciation our present plan is to review this work, incorporate it into our procedures, and then to complete an up-to-date and thorough summary of our procedures.

9 As a practical matter, this goal is never fully realized. MFP, the ratio of output to inputs, reflects many influences in addition to efficiency and technological change. Among them are the effects of returns to scale, unmeasured inputs, and measurement errors for inputs or outputs. However, MFP measures are a step closer to the goal than are measures of output per hour in that MFP "adjusts out" the effects of increased capital intensity.

It is also useful to think of these two types of decisions as being made by two distinct types of economic agents: *investors* and *production managers*. The investor is responsible for the long run and the production manager is responsible for the short run. Imagine that capital goods cannot be created in the short run, and therefore the available supply of capital goods is fixed in the short run. Imagine transactions in discrete time periods between investors who own all of the assets and production managers who need them.

In the long run, the investor decides to order a certain *quantity of capital goods* when she perceives an opportunity to earn sufficient *returns* in future time periods to justify the expenditure of funds. She subsequently earns these returns by charging production managers (explicitly or implicitly) for the use of the capital goods. We assume these *rental* transactions occur in fully competitive markets. We also assume that capital goods cannot be created in the short run the supply of capital goods clearing through the rental market is fixed by past decisions.

Consistent with all of this, we further assume that the production manager's goal is simply to **minimize the cost** of each unit of output. We thus shift the analytically more difficult objective of profit maximization to the investor. In fact, since the investor's perspective is long run, she has the even more complex<sup>10</sup> goal of maximizing the value of her portfolio of assets. By shifting these complications to the investor, the production manager's problem can be described as the minimization of costs subject to a simple production function and price-taking behaviour.

The production manager hires laborers, buys materials, and contracts (explicitly or implicitly) with investors for the use of capital goods. If we could measure the quantities and prices of capital services, we could ignore the investor and measure MFP in terms of the production manager's simple production function. However, since it is usually impossible to measure capital services, economists turn to capital goods. It is clear that there is a direct relationship between the services and the goods: *capital services are obtained by renting a specified quantity of capital goods for a specified (short run) period at a competitive rental price*. It is relatively easy to measure capital goods due to their *tangibility*. A crucial element of the neoclassical approach is to define the *quantity of capital services* in terms of *the quantity of capital goods*.<sup>11</sup>

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<sup>10</sup> Analysis of investment decisions gets into issues which are beyond the scope of most productivity analysis- The investor faces uncertainty about the future returns she will earn and she faces constraints, such as limits in her ability to gain control of funds with which to acquire assets.

<sup>11</sup> Jorgenson and Griliches [1967] developed the idea of adjusting the quantity of capital goods with an indicator of "capacity utilization" to get at the idea of variations in the service flow coming from the capital goods. However, this creates a new measurement problem: the appropriate measurement of capacity utilization. It was subsequently demonstrated by Hulten [1986] and by Berndt and Fuss [1986] that the adjustment is unnecessary in the context of neoclassical theory. Explicit adjustments for capacity utilization are no longer part of standard practise. More on this later.

Now if there were only a single type of capital good, the distinction between the quantities of services and goods would be almost trivial. We could define the quantity of services to be the quantity of goods used for one period. The rental price then would be the cost of renting one unit of the good for one period. However, in the real world there are many types of capital and therefore we face *aggregation problems*. Neoclassical capital theory essentially treats each different type of capital as a separate input and then describes conditions and procedures for consistent aggregation of these inputs.<sup>12</sup>

Neoclassical theory essentially uses analysis of our investor to determine appropriate weights for capital aggregation. The crucial assumption is that *the price of an investment good will equal the discounted value of the services it will provide in the future*. This assumption follows from long run competitive equilibrium in markets for investment goods. It has implications for weights in aggregation which we will discuss shortly.

## Issues of empirical Implementation

### *Which asset types to include*

In its broadest sense, capital can include anything which involves foregoing something in the present to earn returns in the future. It can include reproducible equipment and structures, land, inventories, financial assets such as stocks and bonds, "human capital" acquired through education, training or experience, and finally intangibles such as software development costs, advertising expenses, or organizational efforts.

However, in the productivity literature this list usually is limited to equipment, structures, land, and inventories.<sup>13</sup> Why are these types of capital included while others are left out? There is no apparent conceptual reason for the exclusions. However, it is difficult to measure these items *in a manner consistent with the production model*. Thus measurement difficulties seem to be the root cause of the exclusions.

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<sup>12</sup> What we now think of as the neoclassical approach gradually emerged in the 1950s and 1960s after a debate in the literature which was sometimes referred to as "the controversy of the two Cambridges." Samuelson and Solow pushed the idea of a production function while Sir John Hicks and Joan Robinson pressed them with necessary qualifications to the underlying logic.

<sup>13</sup> The BLS follows this convention. The BLS also estimates the effects of "human capital" on productivity, reflecting changes in the education and experience of the work force. The returns to human capital are presumed to be earned by the individuals involved in the form of increased wages- They are therefore treated as a factor augmenting raw hours worked by labor rather than as part of the BLS capital measures.

In the case of financial assets, many relevant data are often available. Furthermore, management of financial assets is essential to a firm's existence. However, it is difficult to identify a systematic relationship between decisions about financial assets and production decisions<sup>14</sup>. By excluding financial assets, productivity economists have effectively classified portfolio management decisions as "investor" issues. These issues presumably are not of concern to our stylized "production manager".<sup>15</sup>

Intangible capital assets are excluded for slightly different reasons. While intangible capital may play a more direct role in production than financial assets, they are excluded because there is no basis for measuring their services. Since these assets are intangible, we can't rely on tying the quantity of services to the quantity of goods. Computer software can be used to illustrate the problem. It is not only difficult to identify the capital services --- it is equally difficult to identify the capital good. From the economy's point of view, the true investment is in development of software programs. Once developed, software can be reproduced at negligible costs. It is not clear whether the "quantity" of software investment is best defined in terms of characteristics of the software, the number of copies produced, or both.<sup>16</sup> A successful application of neoclassical theory to measurement of an "intangible asset" would effectively involve solving all the measurement issues so as to treat the asset as tangible.

Why are equipment, structures, inventories and land included? Equipment and structures are capital goods which are reproducible (unlike land) and depreciable (they lose value as they age), we can readily determine the cost of producing each unit of them and we can identify "real quantities" by a) counting them, b) observing how they deteriorate over time, and c) adjusting for quality improvements in newer goods by using hedonic models or linking procedures.

Land, like equipment and structures, is a fixed asset which can be readily measured. It differs in that it is generally assumed not to depreciate. If anything this makes it simpler to account for. Classical work by David Ricardo described the rental market for a fixed supply of land. While the total quantity of land may be fixed, land can be traded among economic sectors. Also the characteristics of a plot of land from the standpoint of production can change, even if the land itself is intrinsically the same. For example, development can occur nearby which may enhance the usefulness of the plot. Acknowledging that the quantity and price of land can change, neoclassical theory treats land symmetrically with equipment and structures.

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<sup>14</sup> For banks, insurance companies, and investment firms, financial transactions are at the heart of their service output. Development of a reasonable production function model is quite difficult. A model for banks was proposed Fixler and Zieschang[1992]. An approach to measuring insurance company output was proposed by Sherwood[1996].

<sup>15</sup> Miller and Modigliani [1966] discussed the conditions under which production decisions could be presumed separable from decisions affecting the firms financial portfolio. It is clear that most of the productivity literature has assumed these decisions are separable.

<sup>16</sup> Other examples of intangible assets are "name recognition, acquired by advertising and "good will" acquired by careful quality control. These assets involve additional measurement problems to the extent they contradict the assumption of competitive markets made in neoclassical theory.

Inventories are goods and are therefore relatively easy to measure. However, the rationale for including inventories with capital is a bit less obvious. One obscuring factor is that goods held as inventories are often outputs (finished inventories) or inputs (purchased raw materials) of the firm- However, these same goods play a second role --- that of capital --- to the extent that firms deliberately maintain "buffer stocks" of these goods to facilitate production- For example raw materials are typically received in discrete batches and significant amounts are stockpiled to protect against uncertainties about deliveries. Similarly, a firm stockpiles finished goods to ensure it can promptly fill new orders without excessive labor costs.<sup>17</sup>

*What data do we have that are directly related to the capital service flows?*

We usually cannot obtain direct measures of the prices or quantities of capital service flows. However, one important type of data, which relates directly to this "shadow" market, is readily available: *property income*. Property income is defined as nominal revenues minus expenses for variable inputs (labor and purchased materials and services).

In the productivity literature, property income is often identified with the total income from capital, It represents the nominal cost paid by the production manager and it corresponds to the return received by the investor. Capital's cost share is the ratio of property income to the nominal value of output. Capital's cost share is the weight applied to capital input in constructing an index of total input for MFP measurement.

Measures of property income for firms are readily available from accounting records. Data on property income by industry commonly are compiled as part of national income accounting. In practice, property income is either dispersed as interest or taxes or otherwise is accounted for as depreciation allowances or profits. The measurement of property income is often straightforward and so can be regarded as fairly accurate. Property income is measured as a residual, but this merely reflects the fact that it accrues as a residual!

*What evidence is there on the price and quantity components of property income?*

Once we have time series data on property income, the measurement of the growth rate of capital services amounts to separating the growth rate of

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<sup>17</sup> Industrial proponents of "just in time" inventory practices tend to view inventories as undesirable and something a firm should work to minimize. I have heard the argument that this disqualifies them as inputs. However, quite to the contrary, this establishes their nature as inputs, since it is inputs which firms presumably try to minimize. It may be quite costly (overtime pay, inefficient planning) to adjust production in reaction to a specific order. There is presumably a tradeoff between these "startup costs" and the costs of stocking inventories.

property income into components of quantity and price change. In the approach used by BLS, we attack this separation problem from two sides. On the quantity side, we collect detailed historical data on investments. These become the building blocks for detailed capital stock measures, after aggregation across vintages. On the price side, we estimate "implicit rental prices" for detailed types of capital- we construct stocks and rental prices in as much asset detail as possible. By doing so, we can draw on neoclassical theory and index number theory to avoid restrictive aggregation assumptions. The rental prices are used in creating weights for aggregation of the various asset-type capital stocks. Index number theory is then used to aggregate these stocks into a capital input measure.

### *Vintage aggregation*

We will consider aggregation issues in light of a three-way classification of capital: by vintage, by types (e.g. lathes and locomotives), and by using industry.

In aggregating depreciable assets of different vintages, we use historical data on investment to create a capital stock. We will be treating the capital stock of each asset as a homogeneous input in production. In order to aggregate investments of different vintages appropriately, we must adjust them for differences in **prices** at the times they were created, for changes in the **quality** of the assets between the times they were created, and for any **deterioration** or **obsolescence** of the assets between the times the goods were created and the times for which we wish to measure their services.

Vintage aggregation is accomplished by BLS using the perpetual inventory method (PIM). In the BLS implementation, the PIM creates a capital stock,  $K_{i,t}$  for the  $i$ th type of capital asset at time  $t$  by adding weighted data on real historical investments,  $I(i,t-\tau)$ , in the  $i$ th type of asset of age  $\tau$ :

$$K_{i,t} = \sum_{\tau=0}^{\infty} s_{\tau} I(i, t-\tau) \quad (1)$$

The weighting function,  $s_{\tau}$ , is called an **age/efficiency** function. It is a function of age and in most cases is assumed fixed over time. The real investment data,  $I$ , are obtained by BLS from the U.S. Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce. BEA derives these data by dividing estimates of nominal investments by price deflators. In turn, the price deflators are based mainly on the BLS producer price indexes.<sup>18</sup>

The age/efficiency function is conventionally assumed to be an index with the value of 1.00 for a new good and which declines with age, eventually approaching or reaching zero. With this convention, the burden of keeping

<sup>18</sup> Typically the PPIs are weighted over many categories of goods. In some cases, BEA has developed price indexes independently of the PPI. Most notable are the structures prices indexes and the computer price indexes. Computer prices are estimated by BEA from an hedonic model.

track of quality change falls on the price deflator.<sup>19</sup> Thus, the real investment stream which enters equation (1) must be adjusted to reflect any quality improvements as "more investment". Furthermore, the burden of accounting for the deterioration and the services of a good as it ages and of the effects of obsolescence fall on the age/efficiency function,  $s\tau$ .<sup>20</sup>

Calculation of  $K_{i,t}$  using equation (1) requires that we maintain a *set of vintage accounts*, that is, we must keep track of how much investment occurred in each past year for each asset type.

A simpler PIM formula which is commonly used is:

$$K_{i,t} = I_t + K_{i,t-1} (1-\delta). \quad (2)$$

Equation (2) is easier to compute than equation (1) because it is recursive. While the answer depends on historical investments, the formula does not reference the entire investment stream each year.

The drawback to equation (2) is that it imposes a constant rate of deterioration on the efficiency function, ie.  $s_\tau = (1-\delta)^{\tau-1}$ . It is not possible to describe some plausible age/efficiency profiles in terms of a constant pattern of deterioration. A good illustrative example is that of a light bulb. It deteriorates very little (if any) through most of its lifetime, That is, its services (converting electricity into light) remain nearly constant. Then, one day, it burns out, after which it has no value whatsoever.

While the light bulb is an extreme example (and often too short-lived to be considered capital), many assets appear to provide nearly constant service flows during their initial years. Automobiles are one example. Even though automobile resale values decline rapidly (depreciate) during the first three years of their service lives, two and three year old autos are often as nice looking and reliable as new ones. In other words, their services do not deteriorate very rapidly. Why, then, do their values depreciate? The depreciation reflects the buyers expectations of the *future* services the auto will provide. Buyers and sellers are evidently quite aware that a three year old auto will become unreliable much sooner than a new one, even if it is presently in "good condition".

<sup>19</sup> Suppose the price of a new asset in year t is \$1.00, and that a similar asset is improved by 5% by year t+1, and that the price of the improved asset is \$1.08. The new asset, like the old, will be weighted with an efficiency function of 1.00 the year it is created. In order to preserve the notion of a quantity index, we need to have the new asset count as 1.05 units of investment. Since we are measuring investment by deflating nominal expenditures, the price index must rise 3 percent between the two years in order to ensure this result.

<sup>20</sup> Robert Mall [1968] discusses the theoretical properties of the aggregate capital measure.

<sup>21</sup> This is also known as geometric decay, which is the discrete counterpart to exponential (Beta) decay. Harper [1983] concludes that geometric decay is in many cases unrealistic.

The distinction between depreciation and deterioration corresponds to the distinction between the value of a capital good and its service flow. The fundamental neoclassical assumption, that the value of an asset equals the discounted value of future services (rents), addresses precisely this issue. At BLS, what we have concluded from this is that, for productivity measurement, we want the specification of  $s_t$  to reflect an asset's efficiency profile and not its price profile. To emphasize that our measures are constructed with productivity measurement in mind, we have dubbed them "productive capital stocks." We sometimes refer to capital stocks constructed from age/price profiles as "wealth stocks".

When we settled on our procedures in 1983, we decided to use equation (1), rather than the simpler constant rate of deterioration model, because we felt the later was unrealistic for most asset types. The difficulty this left us with is that the best available datum on aging capital was often an estimate of its rate of depreciation. While we have found data related to services in a few cases<sup>22</sup>, they are fairly scarce. We have made use of estimates of service lives made by BEA in most cases. At BLS we use a flexible form<sup>23</sup> for the age/efficiency profile, and then use evidence on service lives, and depreciation rates to set the parameters of that form. We have selected forms which decline gradually early in an asset's life, and then more rapidly later in its life.

During the past couple of years, BEA has been doing research to change its service life estimates to conform with evidence on rates of economic depreciation which appear in the literature. While we have not yet received their final study, our plan is to use this information to adjust our service life estimates. Neoclassical theory predicts that each age/efficiency profile will have a specific associated age/price profile which is "dual" to it. At BLS, we hope to use BEA's information to adjust our age/efficiency profiles to ensure that they predict age/price patterns which are consistent with the new evidence assembled by BEA.

While equation (2) affords some flexibility, the assumption that efficiency is a fixed function of age is fairly rigid. Unfortunately, it is difficult to avoid this assumption owing to the paucity of data.<sup>24</sup>

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<sup>22</sup> We made use of some data on miles driven by commercial trucks by age in BLS [1983] as described on p. 44.

<sup>23</sup> We use a hyperbolic formula for efficiency,  $s_t = (L - \tau) / (L - B\tau)$ , Where L is the service life, t is the age of the asset and B is a parameter. By varying B, the graph of this function can take on various shapes. For B=1 we have "one hose shay" capital and for B=0 we have straight line deterioration. For  $0 < B < 1$  the function is "concave" to the origin and for  $B < 0$  the function is convex, similar to the shape of a geometric decay curve. BLS assumes B=.5 for equipment and B=.75 for structures. BLS also assumes that, for each type of asset, service lives are normally distributed about a mean because discards do not occur a fixed number of years after investment. A more extensive discussion of these subjects can be found in Appendix C of BLS(1983), in Harper [1983], and in Powers [1989].

<sup>24</sup> Ball and Harper [1990] were able to relax this assumption and others using a database on dairy and beef cattle assembled by the U.S. Department of Agriculture. It is hard to imagine finding the right data to do similar work for equipment or structures.

### Aggregation of assets of different types

Once we have estimated productive capital stocks for numerous types of assets owned by an industry, we face the problem of aggregating these into a measure of total capital service inputs. We assume that the capital services of each type of asset are proportional to its stock. However, the proportionality constants are likely to be different for each type of stock. Therefore, we must consider what the appropriate weights are rather than simply adding stocks together.

Neoclassical theory again comes into play on this issue. We first assume that property income,  $\Psi_t$ , is the total rent received from the various assets in each time period,  $t$ :

$$\Psi_t = \sum_i C_{i,t} K_{i,t} \quad (3)$$

where  $K_{i,t}$  is the productive stock of the  $i$ th asset and  $C_{i,t}$  is its rental price. We already have estimates of the stocks and, as mentioned earlier, of property income.

In order to estimate the rental prices we draw on the fundamental neoclassical premise that the price,  $P_{i,t}$ , of an asset, equals the discounted stream of future rents it will generate. From this premise, we can derive a formula for the rental price:

$$C_{i,t} = P_{i,t} r_t + P_{i,t} \delta_{i,t} - \Delta P_{i,t} \quad (4)$$

where  $\delta_{i,t}$  is the rate of depreciation,  $\Delta P_{i,t} = P_{i,t} - P_{i,t-1}$  is the rate of increase in the price of new goods of that type, and  $r_t$  is the assumed discount rate. The price,  $P_{i,t}$ , is taken to be the price index used in deflating investment goods. The rate of depreciation is estimated from the age/price profile which neoclassical theory assigns to the assumed age/efficiency profile.<sup>25</sup>

The single remaining unknown,  $r_t$ , is *solved for* by substituting rental price equations (4) into the formula for the sources of property income, equation (3). This ensures a set of rental prices which precisely accounts for property income. In turn this ensures consistency of the capital model with the notion that asset rents account for property income. As discussed earlier, this notion is also used to rationalize using property income in constructing capital's weight for MFP measurement.

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<sup>25</sup> Equations relating these two profiles are presented by Hall [1968].

The rate of return which the investor uses to discount future rents while making investment decisions is sometimes called an *ex-ante* rate of return. The *ex-ante* rate of return is presumably a "hurdle" rate of return used by firms as a criterion for investing. Under certain conditions, it might be appropriately measured as the market rate for some class of bonds. However, the rate,  $r_t$ , which we use reflects the property income actually realized and is called an *ex-post* rate of return. It is noteworthy that the neoclassical theory which rationalizes the model we use assumes perfect foresight.<sup>26</sup> As such, there should be no difference between the *ex-ante* and *ex-post* rates of return. However, in reality expectations often are not met and the result is a difference between these two rates of return.<sup>27</sup>

The neoclassical premise from which equation (4) is derived is consistent with rational behaviour by "investors". The rental price formula happens to correspond to the *user cost* of capital, that is, a formula which classifies the capital costs faced by a "production manager" into various types. It is often easiest to understand it from this production manager perspective. Thus the cost of utilizing a machine for one year would include interest costs net of any appreciation and would also include depreciation costs.

Empirically, the key issue is that depreciation rates differ widely among assets. Rates for equipment (particularly computers) are relatively fast. Rates for structures are relatively slow and depreciation of land is negligible. So the approach we use *gives more weight in the aggregate* to a dollar's worth of equipment than to a dollar's worth of structures owing to the equipment's higher rental price. The rationale for doing so is provided by neoclassical theory and can readily be understood from the investor's perspective. When investing in a building, the investor knows she can earn her returns over many years. However when she invests in a computer, she knows the asset will be of only marginal usefulness in a few years. So she must expect a dollar's worth of computer to yield services more "intensely" than a dollar's worth of building in order to justify the levels of investment in each which she selects.

### *Accounting for Taxes in Aggregation by Asset Type*

We also account for the effects of tax laws on rental prices. The U.S- tax laws have varied from year to year, but they have generally "favored" investments in equipment by their tendency to make the effective cost of using equipment

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<sup>26</sup> Berndt and Fuss [1986] and Hulten [1986] showed that the appropriate weight for capital in MFP measurement is based on property income. The "shadow price" of capital computed from data on property income is consistent with an *ex-post* rate of return. Fluctuations in "capacity utilization" manifest themselves as fluctuations in the shadow price of capital and in the *ex-post* rate of return.

<sup>27</sup> Unrealized expectations present practical problems for the empirical implementation of the model. Harper, Berndt and Wood [1989] explored these problems. One of their conclusions was that the *ex-post* rate of return is indeed to be preferred in MFP measurement. Another was that the price revaluation term,  $\Delta p_{t,t}$ , is appropriately measured as the *expected* rate of price revaluation.

lower than the cost for structures. The laws accomplish this by allowing an "investment tax credit" for equipment and also by allowing depreciation deductions for equipment over very short periods of time.

The rental price formula used by BLS includes, in addition to the elements in equation (4): a term which adds in property taxes assessed on the particular asset type, and a factor which adjusts the rental price for the effects on the returns to capital of the corporate income tax rate, depreciation deductions, and credits.<sup>28</sup> This tax-adjusted rental price formula can be derived from the neoclassical assumption that the price of an investment good equals the discounted value of its future services. The assumption is applied to a price which is adjusted for any tax credit and to a stream of "after-tax" future income.

Since its introduction in the productivity literature<sup>29</sup>, the tax-adjusted rental price has become widespread in MFP measurement for the U.S. and other countries<sup>30</sup>. The idea of an after-tax rental price has also developed an important new use: the analysis of the likely effects of proposed changes to the tax laws.<sup>31</sup>

### *Aggregation of Industry Capital Inputs*

Once we have measures of the capital service inputs of industries, they can be used to construct measures for more aggregate sectors. In aggregating capital across industries, the weights we use are the industry's shares in the aggregate sector's property income- Since property income is a part of each industry's value added, it is additive across industries. Therefore the industry shares within a sector add up to one.

Use of property income in aggregating industry capital is consistent with its use as capital's share in industry MFP measurement and with its use as the total rental cost for the various assets deployed by the industry. It is also consistent with the goal of an aggregate production model which has been developed, within a neoclassical framework, from outputs and inputs of the industries of the sector.<sup>32</sup>

Use of property income to construct weights for aggregation has some empirical significance in cases where some industries are earning higher rates of return than others. Investors in all industries may face similar ex-ante interest rates and so in theory capital would be allocated to industries in such a way that the rental price of a particular type of asset is the same in all industries. However, from the ex-post perspective, some industries earn

<sup>28</sup> Specification provided in BLS [1983] on p. 49.

<sup>29</sup> Robert Hall [1967] originated this line of research.

<sup>30</sup> Examples are Christensen, Jorgenson, and Cummins [1980], Dean, Darro and Neef [1990] and Lysko [1995].

<sup>31</sup> A whole area of literature has emerged. Major volumes on this subject include Hulten [1981], Landau and Jorgenson [1986] and Jorgenson [1996].

<sup>32</sup> Such a model was first proposed by Domar [1961].

higher rates of return than others. By giving more weight to the industries earning higher returns, we are effectively saying that their assets are generating more capital services. So, the notion of capital services motivates the procedures we use in aggregation of assets over vintages, asset types, and industries. This is why we call our capital input measure a measure of capital services.

### *Index Number Formulas*

In all of the BLS MFP work, we do aggregation using *chained superlative index numbers*<sup>33</sup> wherever possible. The BLS introduced the use of superlative index numbers in 1983 with its private business MFP measures. At the time, Tornqvist index numbers were used to aggregate asset productive stocks, at the sectoral level, and then to combine capital inputs with labor hours. BLS has since expanded the use of superlative indexes. We first use them to aggregate asset productive stocks within each of 56 industries. We also use them to aggregate the hours of laborers with different skills into a measure of labor *input*. As a final step, we use them to combine capital and labor inputs. BLS also uses new measures of output from BEA which have been reformulated to use superlative index numbers.<sup>34</sup>

### **Conclusion**

These rather complex procedures turn out to make a significant difference in the empirical assessment of the role of capital in U.S. economic growth. The growth rate of a traditional capital stock measure (the directly aggregated real stock of wealth) for the U.S. private nonfarm business sector from 1948-1994 was 3.0 percent per year as shown in Table I. However, the BLS measure of capital input grew 3.9 percent for the same period!<sup>35</sup> This is an astonishing difference over such a long period as it implies that capital services have grown 49 percent more than the capital stock. The main source of this difference is a long-term trend in the proportion of the capital stock accounted for by service-intensive short-lived equipment.

A traditional capital measure, which overlooks this important shift, gives a distorted picture of economic growth. Output grew at a 3.4% annual rate from 1948-94. As such, the ratio of capital service inputs to output *grew* 0.5 percent per year. However, the ratio of traditional capital stock to output appears to have *declined* at a 0+4 percent annual rate. Furthermore, output per hour grew at a 2.1 percent rate during the same period. In our MFP work, we have determined that growth in capital services per labor hour accounts for about 0.8 percent per year of that growth. Had we instead used a traditional capital-labor ratio, we would have accounted for only 0.5 percent per year of that growth.

<sup>33</sup> The term "superlative index *number*" was coined by W. Erwin Diewert [1976] to describe index number formulas which generate aggregates consistent with flexible specifications of the production function.

<sup>34</sup> Dean, Harper, and Sherwood [1996] discuss the use of superlative index numbers in the BLS productivity program.

<sup>35</sup> BLS[1996] reports our most recent set of MFP numbers and many details of the calculations. More detail are available from the author.

Table 1. Trends in Multifactor Productivity and Constituent Series for the U.S. Private Nonfarm Business Sector (Compound Annual Rates of Change)

|   | 1948-<br>1994 | 1948-<br>1973 | 1973-<br>1990 | 1990-<br>1994 |
|---|---------------|---------------|---------------|---------------|
| <i>BLS Measures</i>                                       |               |               |               |               |
| Multifactor Productivity . . . . .                        | 1.1           | 1.9           | 0.1           | 0.3           |
| Capital Services (Ks) . . . . .                           | 3.9           | 4.1           | 4.2           | 2.0           |
| Productive Capital Stock (KD) . . . . .                   | 3.0           | 3.1           | 3.2           | 1.6           |
| KS/Output Ratio . . . . .                                 | 0.5           | -0.1          | 1.4           | 0.0           |
| KS/Labor Ratio . . . . .                                  | 2.2           | 2.6           | 2.1           | 0.4           |
| Contribution of KS/Labour to MFP . . . . .                | 0.8           | 0.9           | 0.7           | 0.1           |
| <i>Alternatives Based on Traditional Capital Measures</i> |               |               |               |               |
| Multifactor Productivity . . . . .                        | 1.4           | 2.2           | 0.5           | 0.3           |
| Stock of Wealth (KW) . . . . .                            | 3.0           | 3.3           | 3.1           | 1.4           |
| KW/Output Ratio . . . . .                                 | -0.4          | -1.8          | 0.4           | -0.7          |
| KW/ Labour Ratio . . . . .                                | 1.4           | 1.8           | 1.0           | -0.2          |
| Contribution of KW/Labor to MFP . . . . .                 | 0.5           | 0.6           | 0.3           | 0.1           |

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The labor input figures used here are hours of all persons adjusted for the effects of changes in the education and experience of the work force.

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