OECD CLIs for G7 countries
A comparison of the Hodrick-Prescott (HP) filter and the Multivariate Direct Filter Approach (MDFA)

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Motivation

- To test for potential improvements over the current OECD CLI methodology (HP-filter applied to each CLI component and uniform aggregation).

- Expected benefits with the MDFA:
  – 1. Better longitudinal behaviour: real-time filter characteristics are tailored to track the target series.
  – 2. Better cross-sectional behaviour: filter characteristics are chosen component by component (i.e.: non-uniform weighting scheme).

Baseline exercise

- Target series: band-pass filtered GDP (GDP-BP)
  Symmetric ideal band-pass filter truncated at +/- 30 observations from the centre, with cut-offs at 2 and 10 years
- Information set: OECD CLI components
- Testing real-time accuracy:
  - The MDFA is tuned to minimize filter error (MSE) with respect to the target series 6 months ahead (T+6).
  - The HP-filter is tested against the same target series. The same filter applies to all CLI components, which are equally weighted.
- Testing revision properties:
  - The MDFA is recalibrated to fit T+5, T+4, …, T-30 variants of the target series.
  - HP-filter values are calculated on a span of 61 observations, and the 'least' asymmetric value shifted by 6 months is used in comparisons with T+5, T+4, …, T-30 values of the target series

(M)DFA methodology (1/2)

- Purpose of the (M)DFA: to minimize discrepancies between an ideal filter and a filter which can be implemented in practice (real-time and/or sample length constraints).
- Ideal filter of a time series x_t: $y_t \equiv \sum_{j=-\infty}^{+\infty} \gamma_j x_{t-j}$
- Implementable filter: $\widehat{y_t} = \sum_{j=k_1}^{k_2} b_j x_{t-j}$
- We want to minimize discrepancies between the two filters using the mean-square norm:

$$\min_{b_j} E[(y_t - \widehat{y_t})^2]$$

- Useful notations:

$$\Gamma(\omega) \equiv \sum_{j=-\infty}^{+\infty} \gamma_j e^{-ij\omega} \text{ and } \widehat{\Gamma}(\omega) \equiv \sum_{j=k_1}^{k_2} b_j e^{-ij\omega}$$

(M)DFA methodology (2/2)

Univariate case (DFA):
Discrepancies are assessed in the frequency-domain using the identity:

$$E[(y_t - \hat{y}_t)^2] = \int_{-\pi}^{+\pi} \left|\Gamma(\omega) - \hat{\Gamma}(\omega)\right|^2 S_x(\omega) d\omega$$

In practice, because $S_x(\omega)$ is unknown, the optimization program becomes:

$$\min_{b_j} \frac{2\pi}{T} \sum_{k=-T/2}^{+T/2} \left|\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)\right|^2 \left|\Xi_{T,x}(\omega_k)\right|^2$$

$\Xi_{T,x}(\omega_k)$ is the discrete Fourier transform of process x: $\Xi_{T,x}(\omega_k) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} x_t e^{-it\omega_k}$

Multivariate case (MDFA): m additional series $\{w_n\}_{n=1}^{m}$ to approximate the ideal filter

$$\min_{b_j, c_j^n} \frac{2\pi}{T} \sum_{k=-T/2}^{+T/2} \left|\Gamma(\omega_k)\Xi_{T,x}(\omega_k) - \left(\hat{\Gamma}_x(\omega_k)\Xi_{T,x}(\omega_k) + \sum_{n=1}^{m} \hat{\Gamma}_{w_n}(\omega_k)\Xi_{T,w_n}(\omega_k)\right)\right|^2$$

Closed-form solutions exist for unknown parameters $\left\{b_j\right\}_{j=k_1}^{k_2}$ and $\left\{c_j^n\right\}_{j=k_1}^{k_2}$

Key results

Table 1: Real time and final MSE performance

	Real time			**Final**		
	MDFA	**HP**	**% improvement over HP**	**MDFA**	**HP**	**% improvement over HP**
DEU	0.63	1.16	46%	0.18	0.5	64%
FRA	0.57	1	43%	0.17	0.41	59%
USA	0.51	0.88	42%	0.19	0.39	51%
GBR	0.87	1.25	30%	0.43	0.56	23%
ITA	0.69	1.15	40%	0.23	0.51	55%
CAN	0.62	1.13	45%	0.13	0.63	79%
JPN	0.82	1.42	42%	0.17	0.49	65%

Decomposition of the filter performance: longitudinal vs. cross-sectional gains

Table 5: Error decomposition into - longitudinal and cross sectional effects

	Correlation			MSE		
	MDFA	MDFA (uniform)	HP	MDFA	MDFA (uniform)	HP
DEU	0.63	0.35	0.12	0.63	0.84	1.16
USA	0.56	0.44	0.12	0.51	0.53	0.88
FRA	0.76	0.43	0.28	0.57	0.83	1
GBR	0.45	0.22	0.09	0.87	1	1.25
ITA	0.64	0.33	0.21	0.69	0.9	1.15
CAN	0.6	0.44	-0.21	0.62	0.69	1.13
JPN	0.59	0.51	0.09	0.82	0.92	1.42

Table based on real time performance, measured by MSE and correlation (in the time domain).

Decomposition of the filter performance: Accuracy, Timeliness and Smoothness

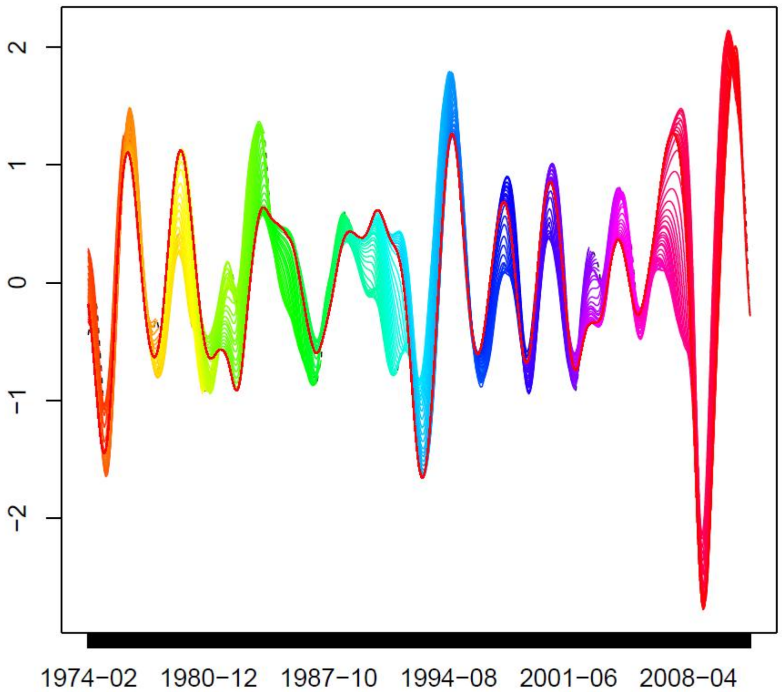
Table 4: Error decomposition into - accuracy, smoothness, timeliness

	MDFA				HP			
	Accuracy	Smoothness	Timeliness	Total	Accuracy	Smoothness	Timeliness	Total
DEU	0.26	0.01	0.2	0.46	0.09	0.03	0.86	0.99
FRA	0.17	0	0.04	0.22	0.05	0.02	0.78	0.85
USA	0.31	0.01	0.2	0.53	0.14	0.03	1.07	1.24
GBR	0.05	0	0.04	0.1	0.08	0.03	0.74	0.85
ITA	0.34	0	0.13	0.47	0.15	0.02	1.11	1.28
CAN	0.34	0.01	0.16	0.51	0.17	0.04	1.46	1.68
JPN	0.27	0	0.1	0.37	0.23	0.01	1.01	1.26

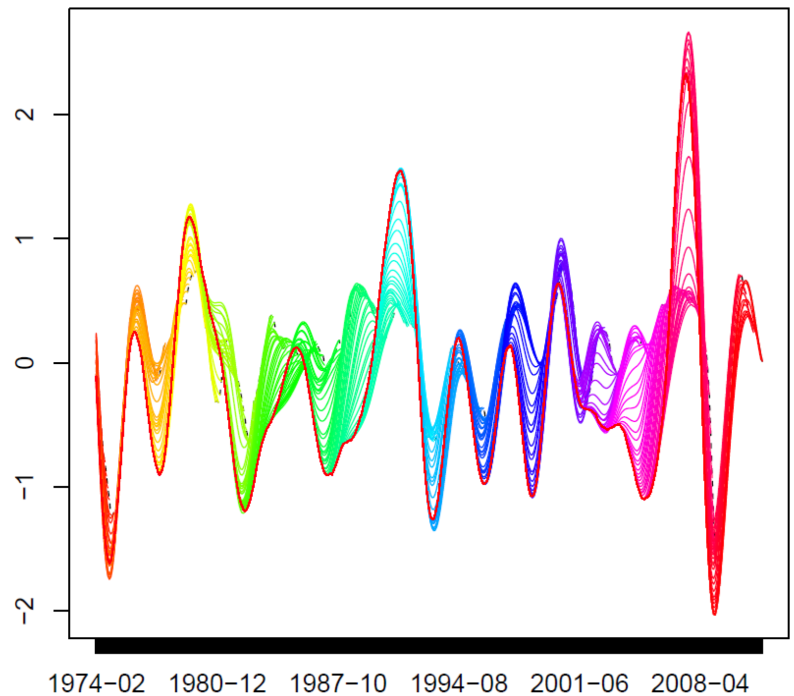
Table based on real time performance, measured by MSE (in the frequency domain).

Revisions characteristics

HP filter revision sequence



MDFA revision sequence

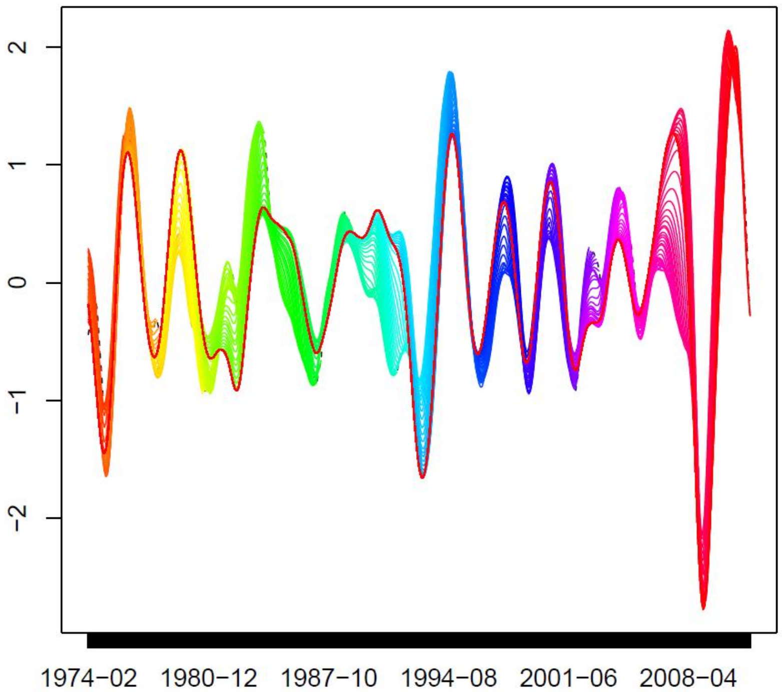


Case study: Germany

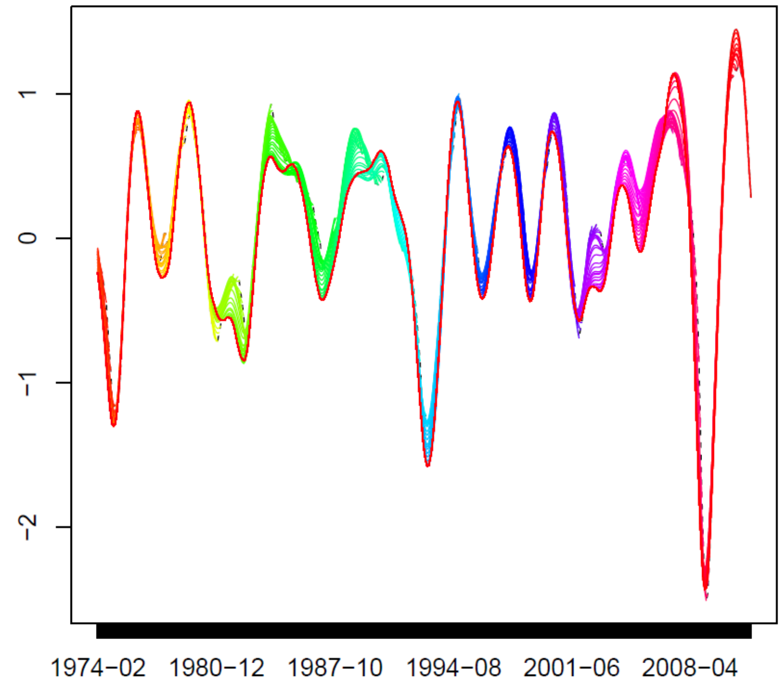
Revisions characteristics

(a way to better assess trade-offs between accuracy and revision patterns)

HP filter revision sequence



MDFA revision sequence when MDFA is tuned for the HP based CLI



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Way forward

- Identification of components in the MDFA context (interesting to contrast the MDFA with the output of factor models)
- Focus on turning-point characteristics
- Test implementation for production