Scenarii for endogenous liquidity crises

Antoine Fosset, Jean-Philippe Bouchaud & Michael Benzaquen
Ecole polytechnique & Capital Fund Management

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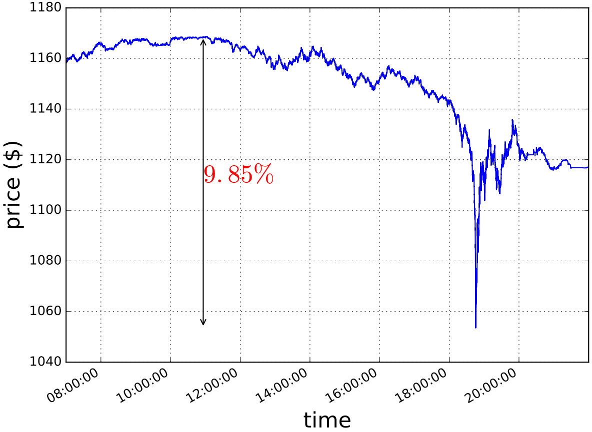


Figure: Price of the SPMini future contract during the flash crash, 6th of May 2010

The study of Joulin et *al.*[1] shows that anomalous price movements ($> 4\sigma$) on the scale of one minute:

- Only 5% are news related (*i.e.* exogenous).
- 95% are not news related (*i.e.* endogenous).

Necessity to have a **feedback** in the price formation mechanism in order to take into account the endogeneity!

[1] Armand Joulin, Augustin Lefevre, Daniel Grunberg, and Jean-Philippe Bouchaud. Stock price jumps: news and volume play a minor role.
Wilmott Magazine, September/October, 1–7, 2008

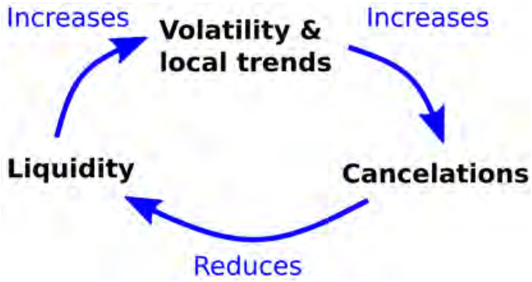
Figure: Flock of birds: collective behaviour emerging from a zero-intelligence feedback.

Liquidity: Number of orders in the order book.
Rate: Number of orders per unit time.
Limit order: Buy or sell the item at its specified price.
Market order: Buy or sell the item immediately, at the current best price.

Figure: Example of a real order book.

Necessity to put some **feedback** of **past price** on the price formation process.

Possible mechanism of feedback:



Feedback of past price changes on cancelations:

$$\lambda_t^c \propto \underbrace{\lambda_0^c}_{\text{base rate}} + \underbrace{\alpha_K \left(\int_0^t \sqrt{2\beta} e^{-\beta(t-s)} dp_s \right)^2}_{\text{feedback on past price changes}}$$

We would like to motivate this choice of feedback by looking at the data.

We aim to fit the following rate of events:

$$\lambda_t \propto \text{base rate} + \text{past trend contribution} + \text{square past trend contribution} \qquad (1)$$

Take home message

The **past square trend** dimishes the **future liquity** equaly at the bid and at the ask. This effect is mainly in the **cancelations**.
The **past trend** has a slight effect on the bid-ask imbalance (*i.e* difference of liquidity between the bid and ask).

Numerical simulations

- N size of the system *i.e.* the number of available price levels.
- T time of simulation.
- α_K feedback intensity: $\lambda_t^c \propto \alpha_0^c + \alpha_K \left(\int_0^t \sqrt{2\beta} e^{-\beta(t-s)} dp_s \right)^2$.
- $1/\beta$: time scale of the feedback.

Video of an order book

We measure the time τ_c of first liquidity crisis: first time when one side of the order book empties.

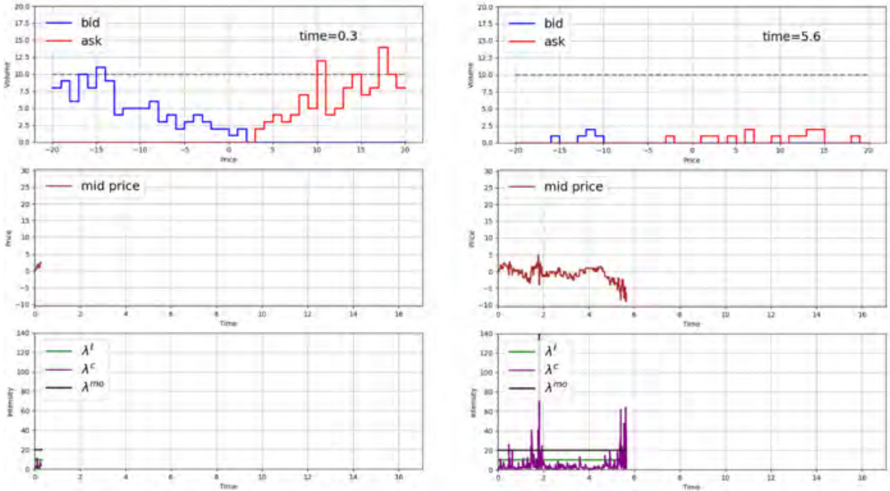


Figure: Snapshots of the order book. The left figure is taken at the beginning of the simulation and the right figure is taken during a period of high volatility. We can see that the liquidity almost dries out.

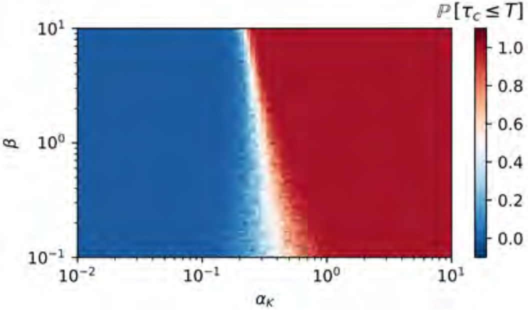


Figure: Stability map: Crisis probability $\mathbb{P}[\tau_c \leq T]$ for $T = 200$, $N = 280$, $\lambda_0^\ell = 10$, $\lambda_0^c = 1$ and $\lambda_0^m = 20$. The blue region corresponds to a stable order book, whereas the red region corresponds to liquidity crises

Numerical simulations show that we have an exact **phase transition**. For an infinite order book ($N = T = +\infty$), if $\alpha_K < \alpha^*$ there is no crisis and if $\alpha_K > \alpha^*$ there are crises. α^* is the critical point.

To make this scenario consistent, we have two possibilities:

- Real financial markets would have to sit below, but very close to the critical point and his critical point is attractive (see Self-Organized Criticality).
- The feedback parameters α_K is time dependent and occasionally visit the unstable phase.

Key ingredient: **spread opening events trigger more spread opening events**.

Model definition:

- Number of spread opening events before t: S_t^+
- Rate of spread closing event: λ_0^-
- Rate of spread opening event: $\lambda_t^+ = \lambda_0^+ + \epsilon X_t^2 =$ base rate $+$ feedback
- Number of spread opening events on time scale $1/\beta$: $X_t := \int_0^t \beta e^{-\beta(t-s)} dS_s^+$.

Let's call τ_c the time of liquidity crisis.

- If $\epsilon > 0$ there is always a liquidity crises.
- If $\epsilon > 0$ is small enough, before the liquidity crisis the spread seems stable: it is **metastable**.
- $\log \mathbb{E}[\tau_c] \approx \frac{1}{\beta\epsilon} \log \frac{1}{\epsilon\lambda_0^+}$ for $\epsilon > 0$ small enough.

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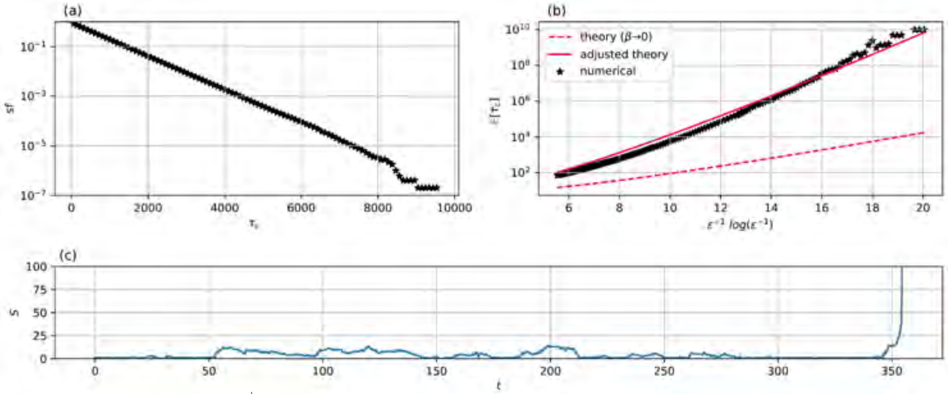
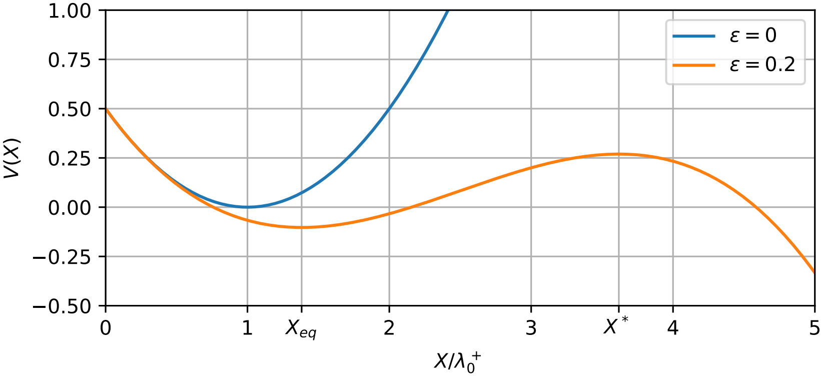


Figure: Set of parameters: $\lambda_0^+ = 1$, $\lambda_0^- = 0.5$ and $\epsilon = 0.2$. (a) Survival function (sf) of the time of metastability which is found to be exponential. (b) Evolution of the average metastability time with ϵ. The dotted red curve is the continuous time prediction. The plain red curve is obtained by multiplying the term in the exponential by a empirical factor 2.5. (c) Typical metastable trajectory.

Physical interpretation of metastability: the "particule" tries to minimize V. When it is trapped in X_{eq}, it needs a fluctuation large enough to cross the barrier $V(X^*) - V(X_{eq})$ and go beyond X^*.



Conclusions

On empirical data, the **past square trend** dimishes the **future liquity**.
Two scenarii of liquidity crises: one from a **phase transition**, the other from a **metastable description**.

Outline

We could also design a empirical test that could help discriminating between the second order phase transition and activation scenarii.
We also would design a **complete protocol to estimate** the price feedback and use it to **predict liquidity crises**.

- Armand Joulin, Augustin Lefevre, Daniel Grunberg, and Jean-Philippe Bouchaud.
  Stock price jumps: news and volume play a minor role.
  Wilmott Magazine, September/October, 1–7, 2008.
- Antoine Fosset, Jen-Philippe Bouchaud, and Michael Benzaquen.
  Endogenous liquidity crises.
  Arxiv 1912.00359, 2019.

Thank you !