

# Taking better account of top incomes when measuring inequality levels and trends

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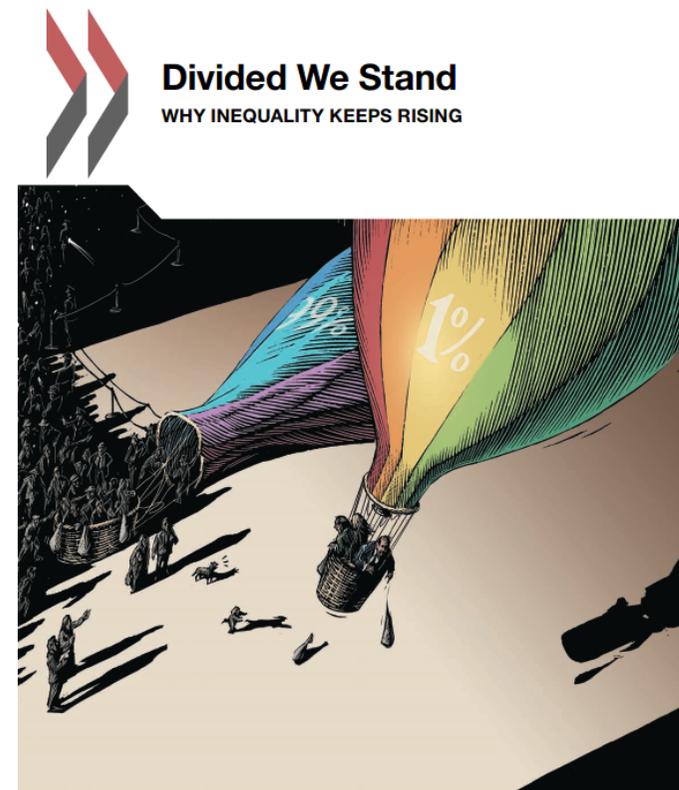
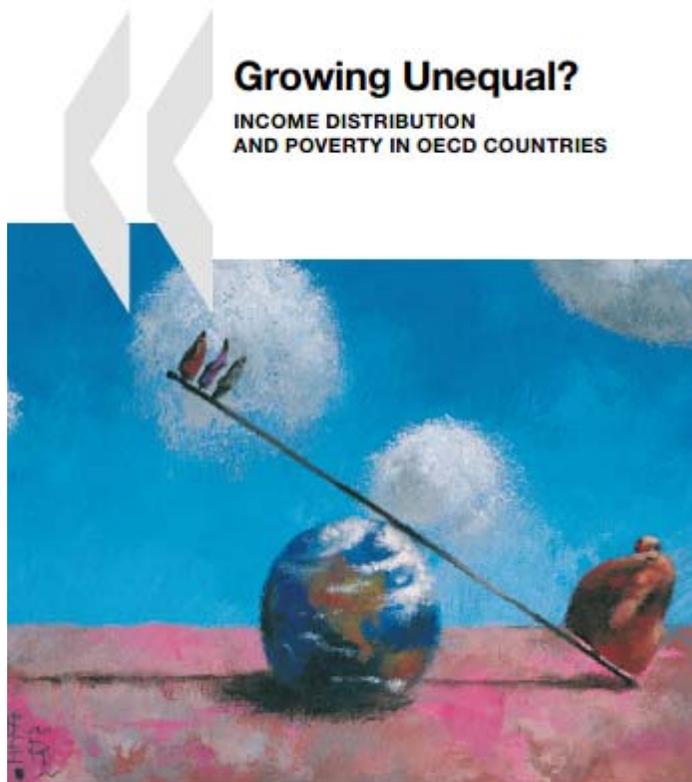
2<sup>nd</sup> Meeting of Providers of OECD Income Distribution Data, Paris, 19 February 2016

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# Has income inequality increased?

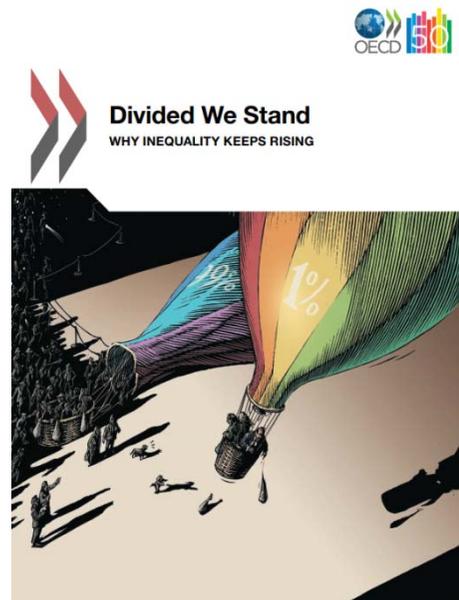
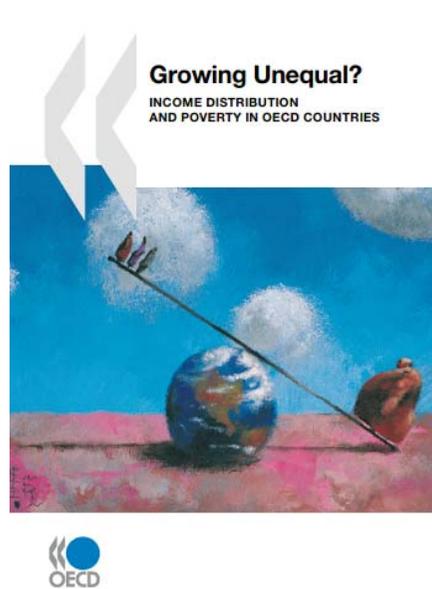
- The one-word answer is **yes**, at least for most rich countries
- **Survey data** are the principal source of evidence for these findings in the work by OECD and the basis of the estimates of most national statistical agencies (including e.g. USA, UK but not “register” countries)
- How accurate are the survey-based estimates of levels and trends?



## Two main sources of inequality data

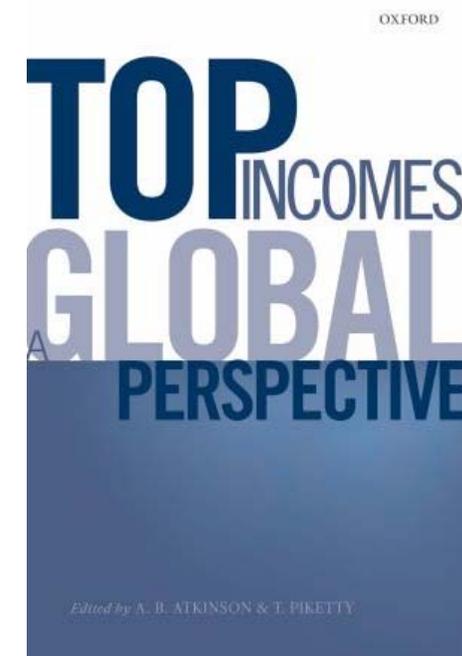
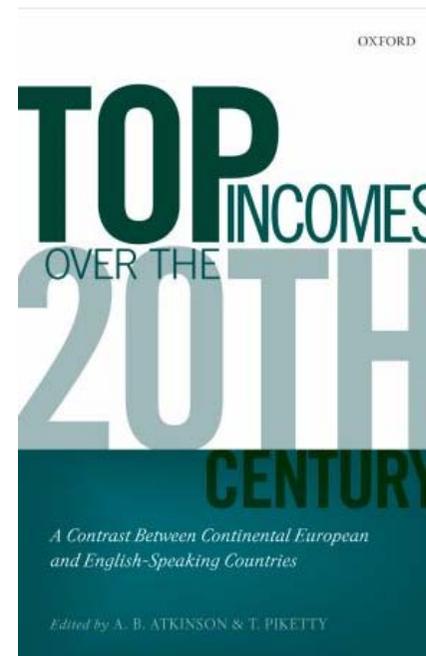
### Household surveys

- E.g. OECD IDD

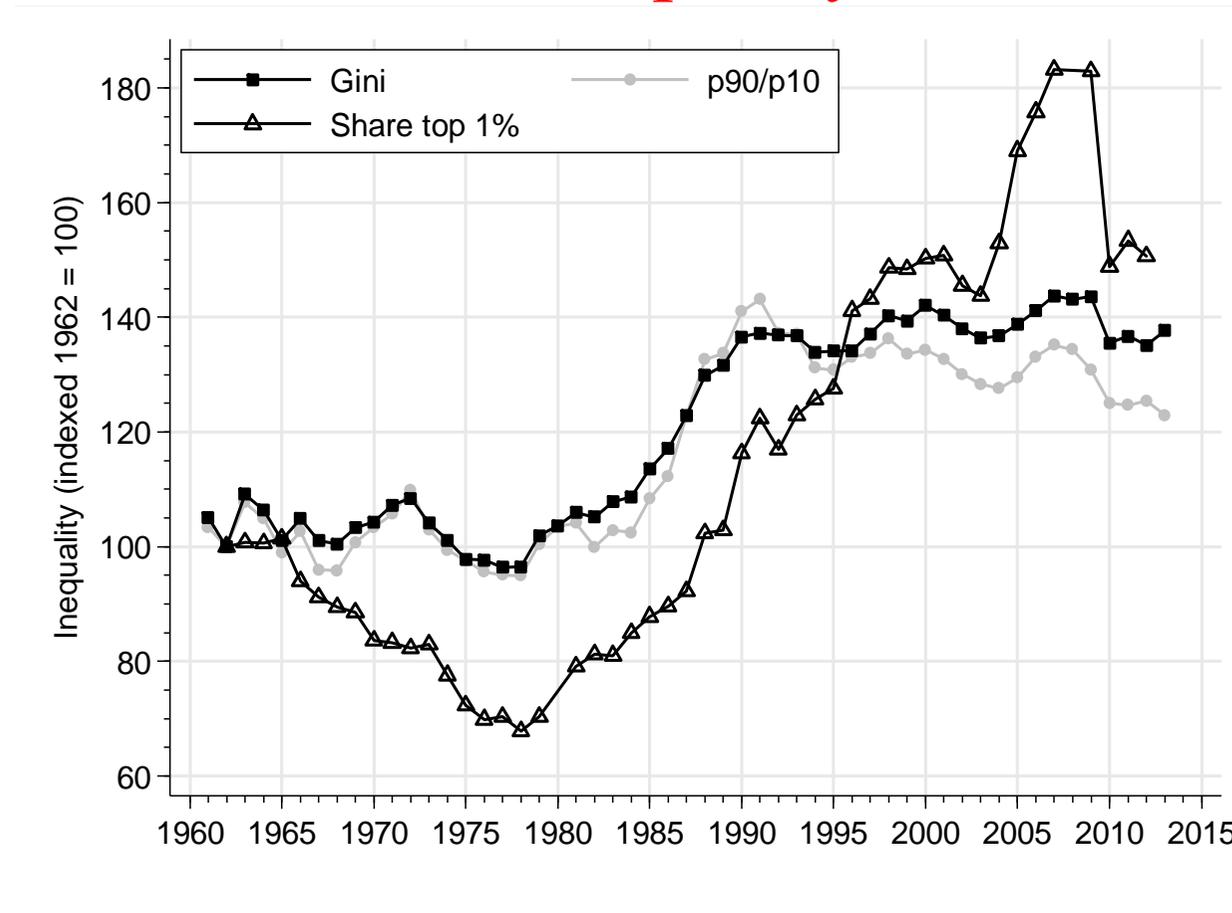


### Income tax returns

- E.g. World Top Incomes Database (WTID) project

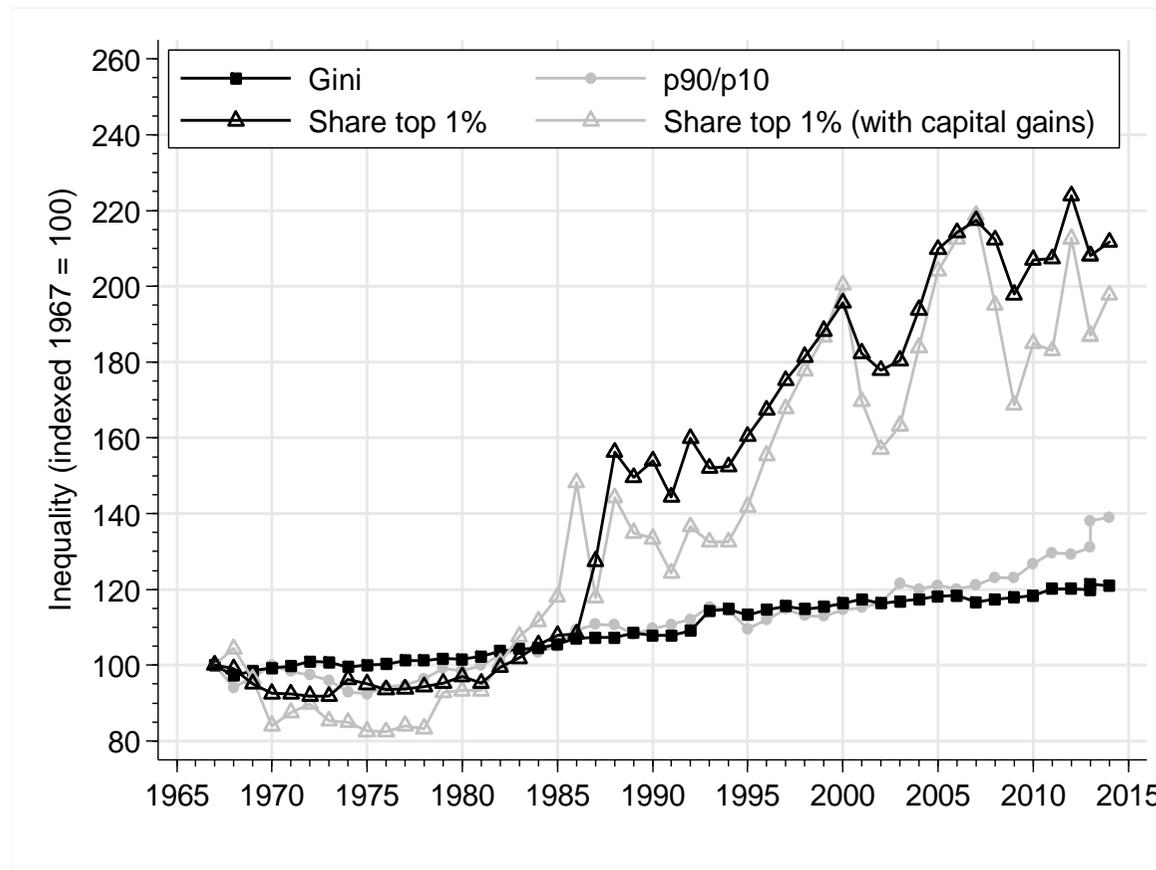


# Surveys and tax return data provide different estimates of inequality trends: UK



Notes: The Gini and  $p90/p10$  measures are based on household survey data using the same definitions as employed by the UK's official income distribution statistics (source: authors' derivations from the spreadsheet accompanying Belfield et al. 2015). The top 1% share measure is based on tax return data (source: authors' derivations from Alvaredo et al. 2015). The data sources and income definitions that each series employs are discussed further in Section 2 of Burkhauser et al. (2016)

# Surveys and tax return data provide different estimates of inequality trends: USA



Notes: The Gini and  $p90/p10$  measures are based on Current Population Survey data (source: authors' derivations from De-Nevas Walt and Proctor, 2015, Table A-2). The two top 1% share measures are based on IRS tax return data (source: authors' derivations from the World Top Incomes Database). There were substantial changes in CPS survey design in 1993 and 2013, and the IRS tax return estimates are affected by the 1986 Tax Reform Act: see Atkinson et al. (2011) and Burkhauser et al. (2012) for detailed discussions. However, between the mid-1990s and 2012, there were no such changes hindering comparability of trends across the two sources. Over this period, the increase in inequality as measured by the change in the top 1% share was substantially greater than the increased measured by the Gini coefficient or  $p90/p10$ .

# The Problem

Under-coverage of top incomes in  
household surveys

# Under-coverage at top has 2 components

1. Unreliability of the top incomes that *are* found in the survey data
    - Under-reporting, outliers and sparse observations
    - Right-censoring ...
  2. Not covering some parts of the top income range at all
    - Non-contact, refusal, etc.
    - Truncation ...
- NB you can't reweight obs that aren't present
  - NB methods treating the problem as one of right-censoring can't deal with truncation
    - Examples include: Burkhauser et al. (2012), Alfons et al. (2013), Ruiz & Wolozsko (2015)

# Under-coverage of the US CPS

- Atkinson, Piketty, Saez (*JEL*, 2011) show the under-coverage of top incomes by CPS relative to IRS data
  - This is despite the best efforts of Burkhauser et al. (*REStat*, 2012) to “fix” CPS top-coding problems by fitting GB2 distribution to right-censored data (NB like-for-like income definitions applied in both sources)

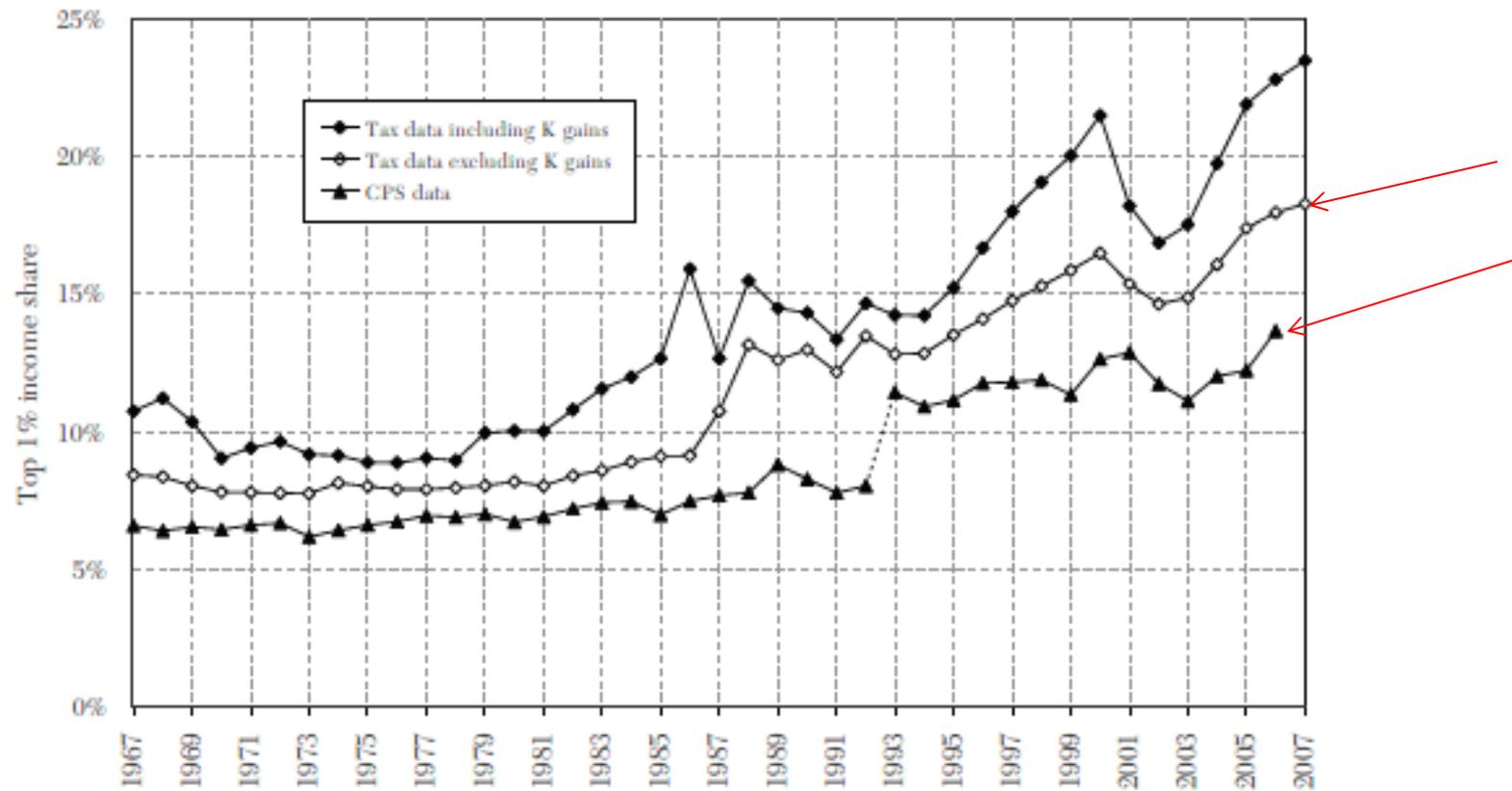
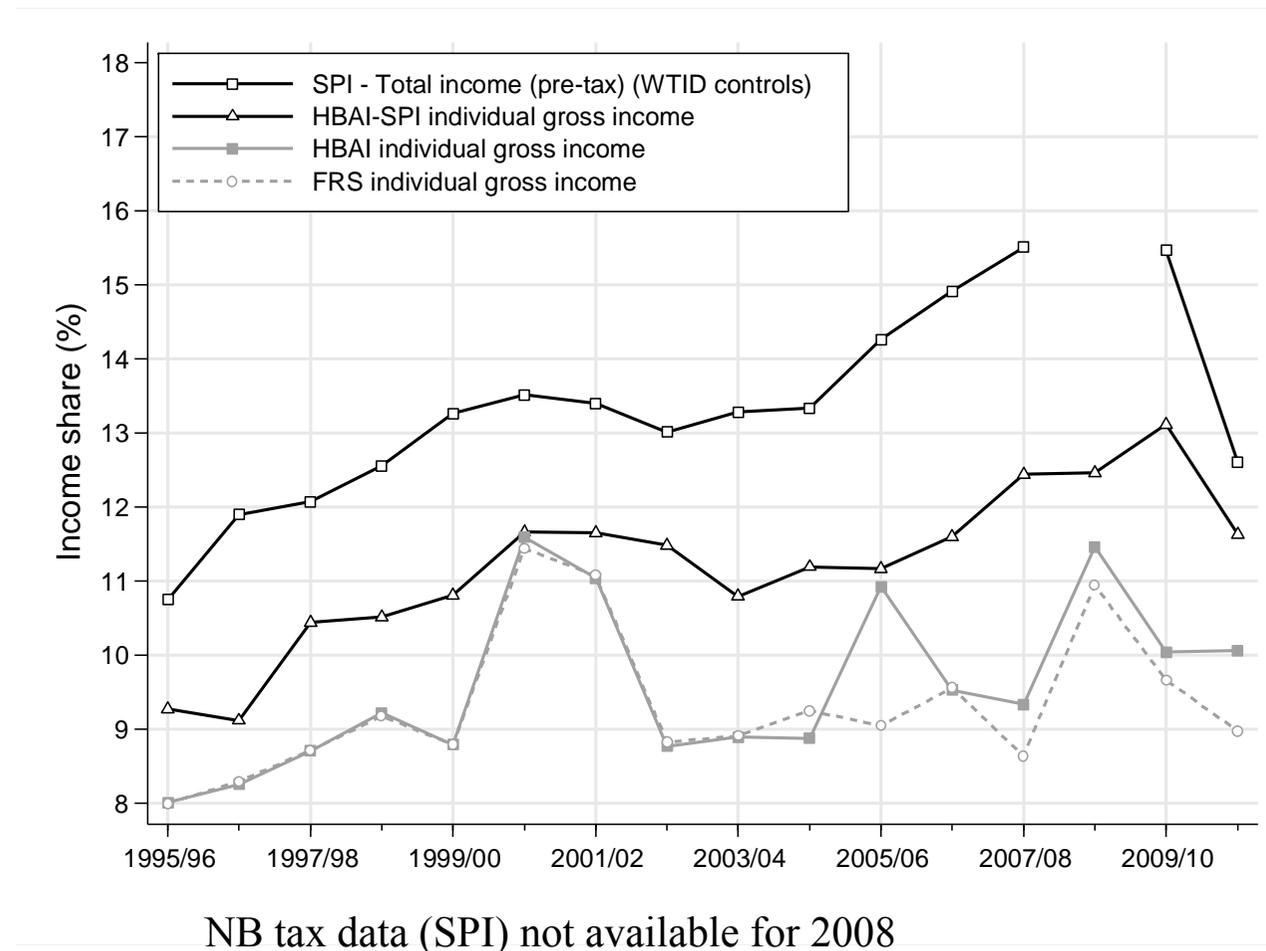


Figure 5. Comparing Top 1 Percent Income Share from Tax and CPS Data

# Under-coverage of the UK FRS

- Burkhauser et al. (2016) show the under-coverage of top incomes by Family Resource Survey variants, including the DWP's HBAI-SPI series, compared to personal income tax return data (Survey of Personal Incomes, SPI). NB same income definitions applied in both sources

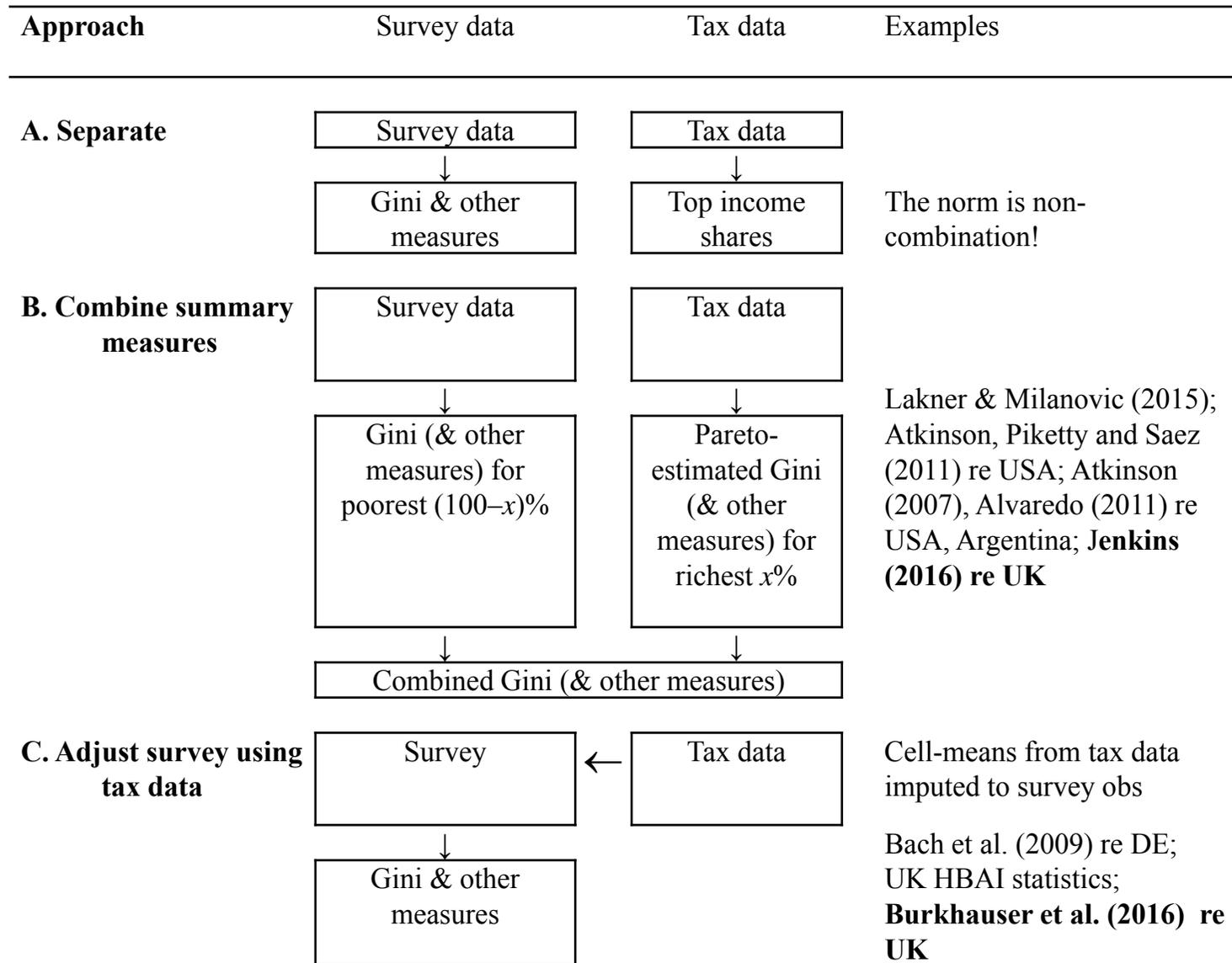


# Solutions

Two approaches to addressing  
under-coverage at the top

# Approaches to addressing under-coverage at the top

- Combine the 2 sources (approaches B, C); don't use separately (A)



## Approaches to addressing under-coverage at the top

The ‘combining’ approaches (B, C) require both sources to:

- a) refer to the same population; and
- b) use the same ‘income’ definitions
  - Preferably the broader ‘Canberra’ definitions, as in household survey (cf. Burtless review of Piketty, *JPAM* 2015)
    - Currently not possible given limitations of tax data (except maybe in countries with extensive register data?)
  - With different definitions, we have an “apples + bananas” non-comparability problem
- More positively: **with survey data, one can do a cross-walk from survey to tax data definitions and population = gross (i.e. pre-tax) income among adults (UK definition)**
  - UK: Burkhauser et al. (2016), Jenkins (2016)
  - DE: Bach et al. (2009) matching SOEP and tax data records

## Approach B: combining estimates

Estimate inequality and mean for top incomes (tax data) and for non-top incomes (survey data), and then derive a combined inequality estimate for all incomes

## Approach B: combine inequality measures

- Formulae for Gini derived by Atkinson (2007) assuming top group infinitesimal in size; extended by Alvaredo (2011) , not making this assumption
  - Inequality decomposition for two non-overlapping population subgroups (top and non-top incomes):
  - Total inequality = Within-group Inequality + Between-group Inequality = (weighted sum of top-group Inequality and bottom-group Inequality) + (Inequality between top and bottom groups)
  - Can estimate inequality for top group by fitting Pareto distribution to top incomes in each year and deriving implied Gini and mean income for that group
    - Especially useful if only have grouped top income data
- Method extends to other additively decomposable inequality indices too (SPJ)

## Approach B: implementation issues

- What distribution to fit to top incomes?
  - Pareto (type I) is ubiquitous: “fits well”, single shape parameter, easy to derive statistics from parameter
    - Most Pareto I GoF “tests” are in fact “tests” of Paretianity / heavy-tailedness rather than Pareto I per se (see e.g. Cirillo 2013)
  - Other distributions are also possible and “fit the data”
    - E.g. Pareto II (Generalized Pareto Distribution) with 2 shape parameters
- Even if we assume a Pareto I tail, what is the income threshold above which this is assumed to hold?
  - Pareto I shape parameter varies a lot depending on threshold and hence so too do estimates of top inequality and top mean
  - There are some statistical methods for choosing the threshold
  - The threshold indicated is often much higher than  $p_{90}$  or  $p_{95}$
  - Choice also depends on the income range in which survey and tax data densities are no longer similar (this varies over time)

## Similar densities at the top? UK survey & tax data

Red: Survey of Personal Incomes (SPI, tax data)

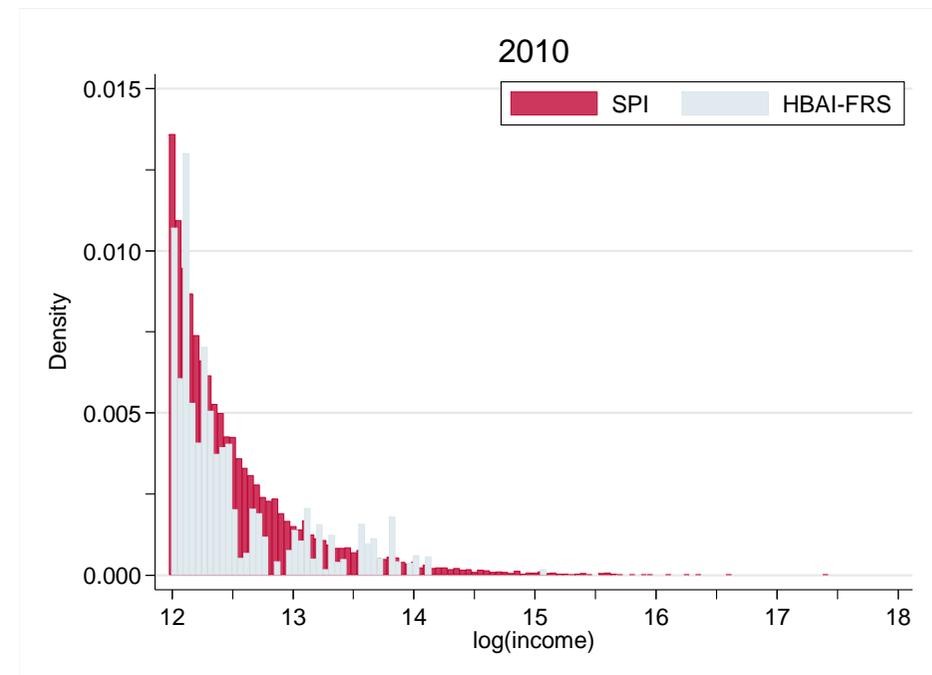
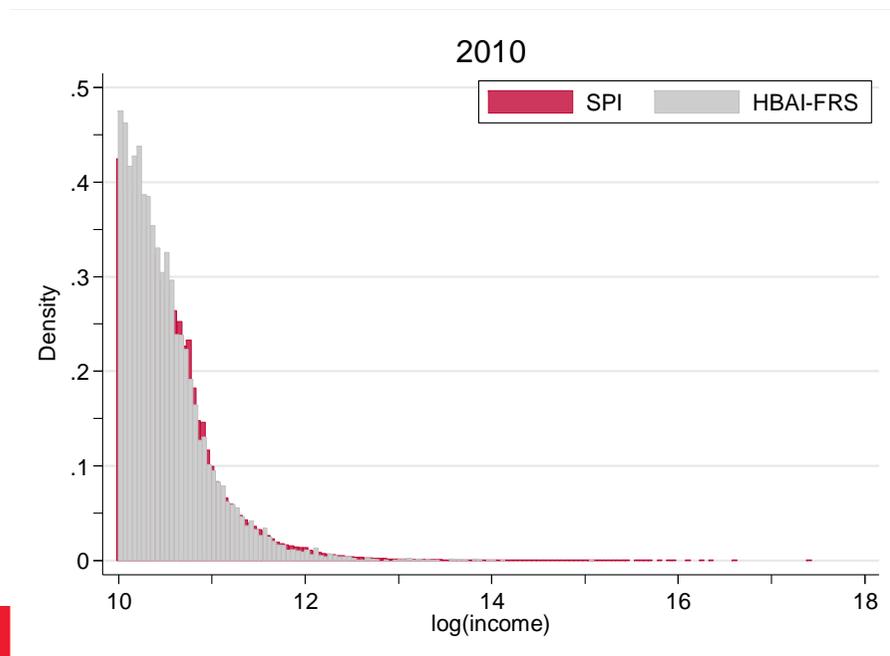
Grey: HBAI-FRS (survey data)

Same income definition: individual gross income (£ p.a. 2011 prices)

NB  $\exp(10) = 22,026$ ,  $\exp(11) = 59,874$ ,  $\exp(11.5) = 98,716$ ;  $\exp(12) = 162,755$

Cf. SPI data:  $p_{90} = 40,690$ ;  $p_{95} = 53,777$ ;  $p_{99} = 114,368$ ;  $p_{99.5} = 163,524$

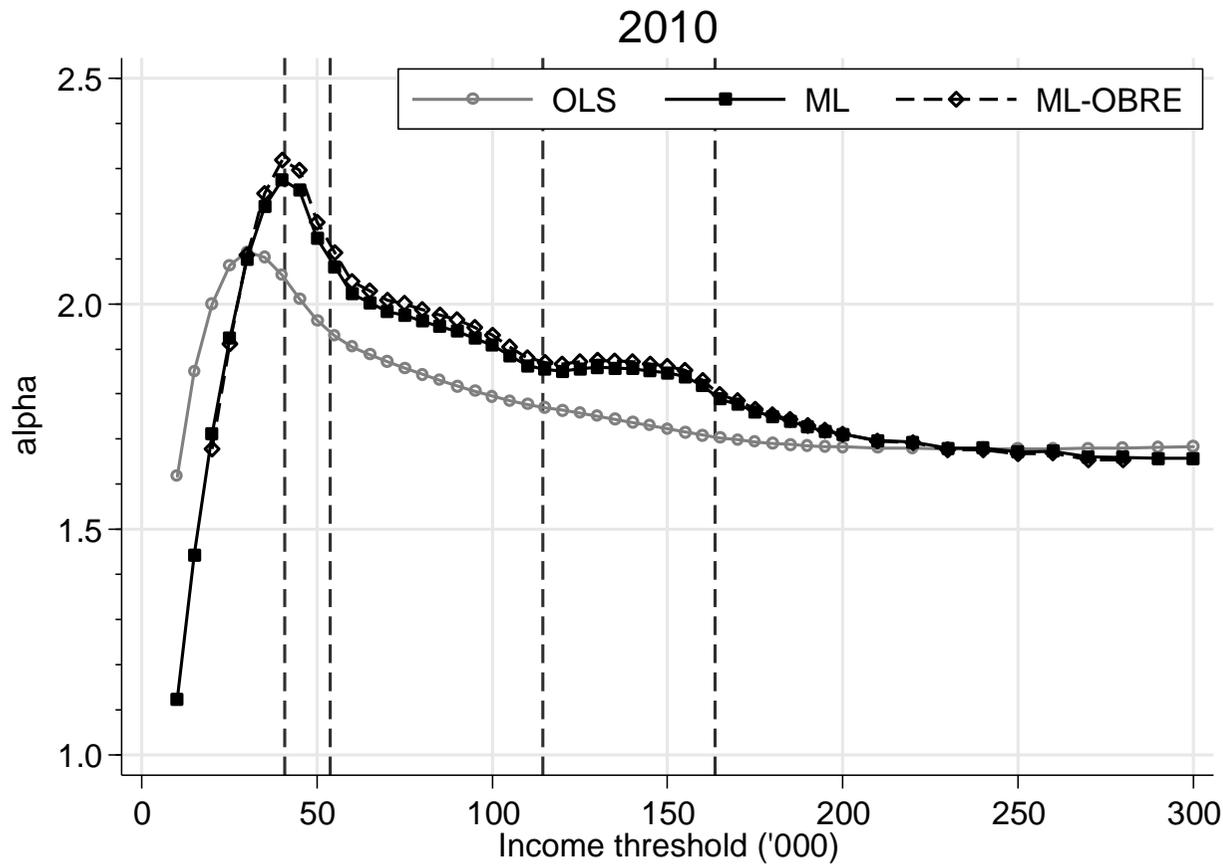
- Survey and tax data distributions appear to be fairly similar up to at least  $p_{95}$ ?
- Similar results for other years (1995 through 2010)



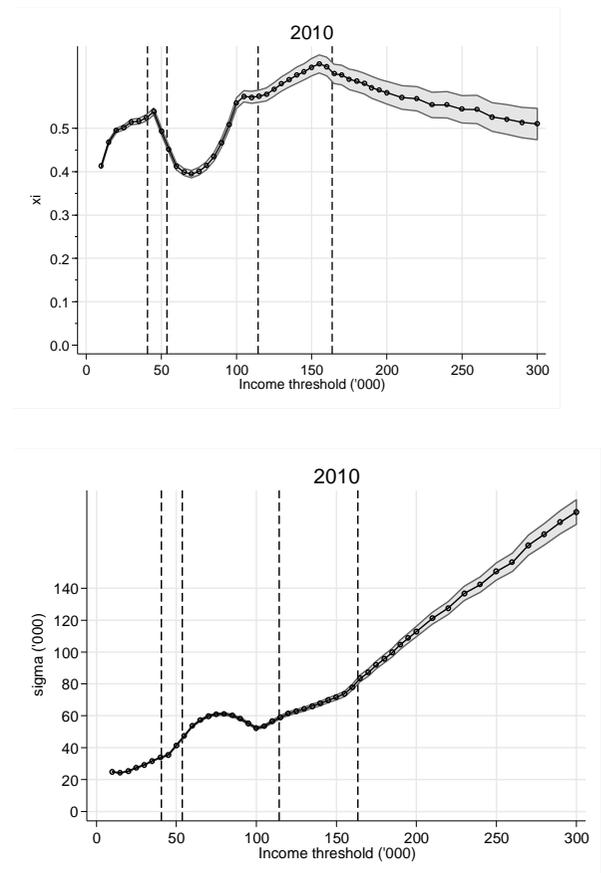
# Pareto I shape parameter varies with threshold

- Vertical lines show, from left to right:  $p_{90}$ ,  $p_{95}$ ,  $p_{99}$ ,  $p_{99.5}$  (UK SPI data)
- ML estimator preferred to OLS on statistical grounds
  - ML-OBRE, which accounts for “non-robustness” (influence of outliers), provides very similar estimates to ML
- Parameter stability (of a kind) above  $p_{99}$ ?
- Similar results for other years (1995 through 2010), and GPD:

## Pareto I



## Pareto II (GPD)



## ‘Optimal’ threshold is well above $p90$ (UK SPI)

- Kolmogorov-Smirnov criterion: choose threshold to minimize “distance” between fitted and actual (Clauset et al., *SIAM Review*, 2009)
- UK SPI (tax) data
- Optimal thresholds: “x” (Pareto I, ML); “X” (Pareto I, ML-OBRE) “y” (Pareto II)

Year	$\geq p95$ & $< p99$	$\geq p99$ & $< p99.5$	$\geq p99.5$
1995	xX	y	
*1996	y	x	
1997		xXy	
1998		y	xX
1999		y	xX
2000		y	xX
2001		y	xX
2002		y	xX
2003		y	xX
2004	X	y	x
2005	xXy		
2006	xXy		
2007	xXy		
2009	y		xX
2010		y	xX

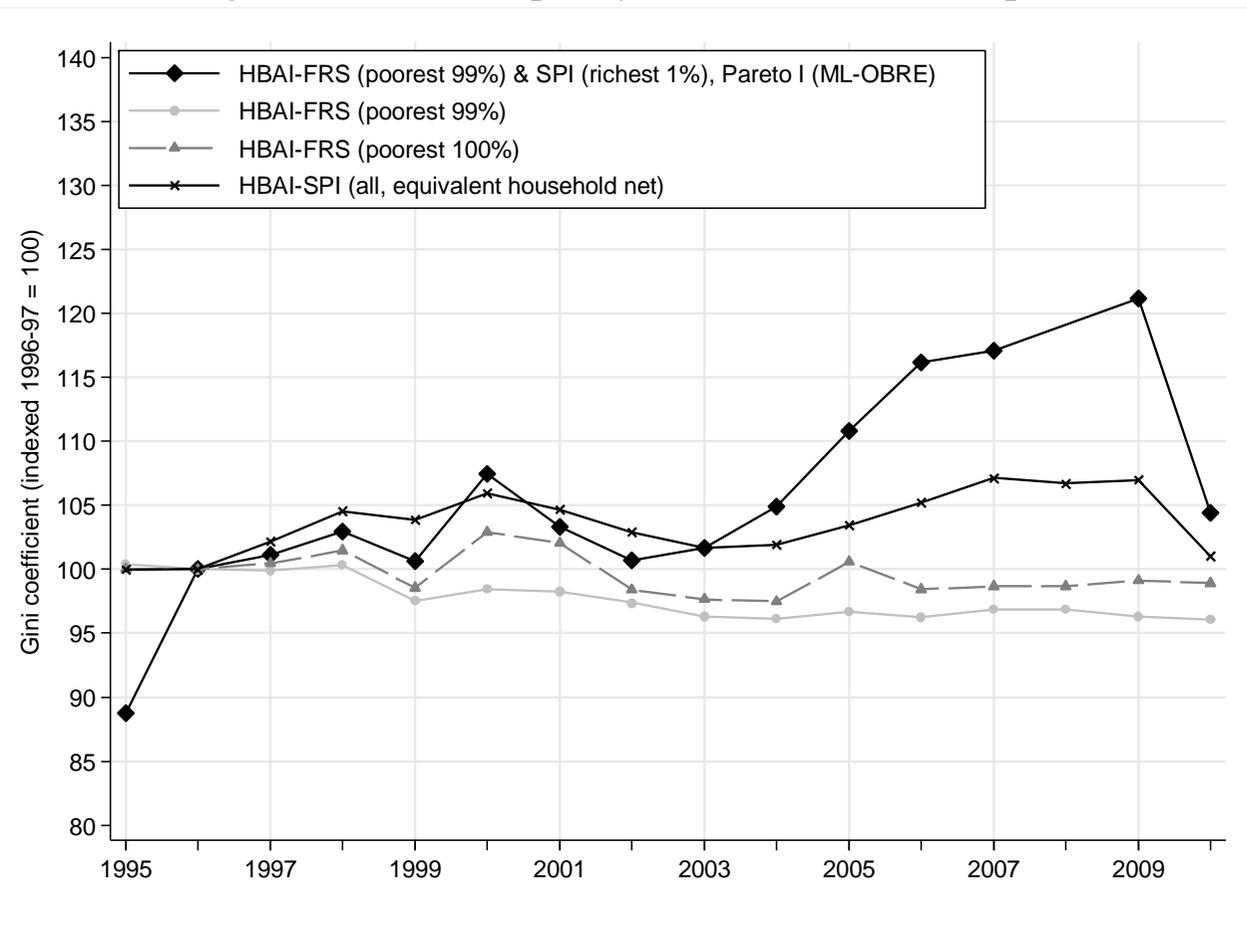
\*: ML-OBRE optimal  
threshold is  $\geq p75$  &  $< p90$

# Approach B: trends in Gini, UK

- Like-for-like income definition (& population) in survey and tax data: individual gross income (all persons aged 15+)
- Fit Pareto I distribution to SPI data using threshold =  $p99$  in the survey data
- Combine estimates of Gini from poorest 99% in survey (“HBAI-FRS”) and Gini for richest 1% in the tax data (SPI)
- Result: combined estimates show greater rise in inequality than official DWP report estimates (HBAI-SPI series):

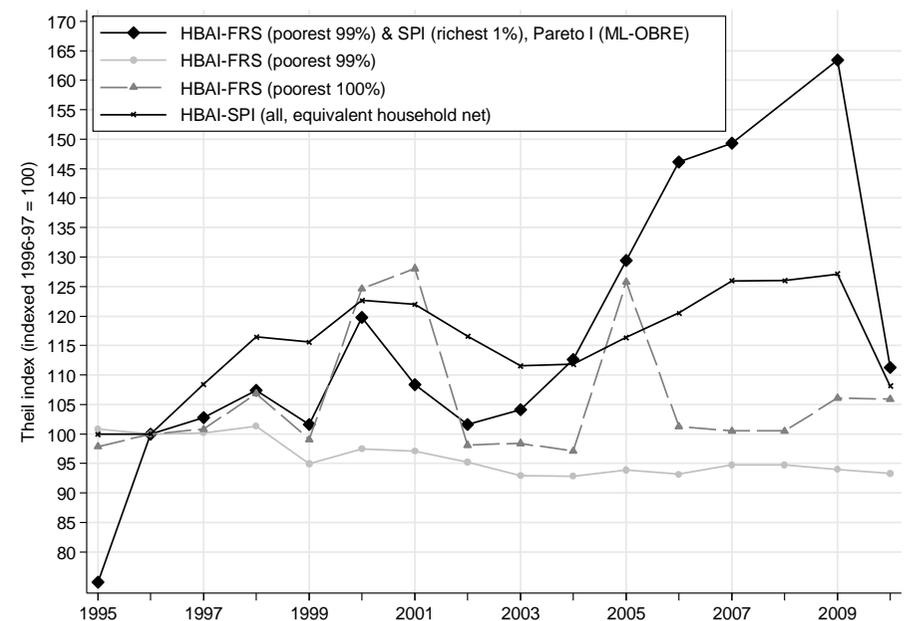
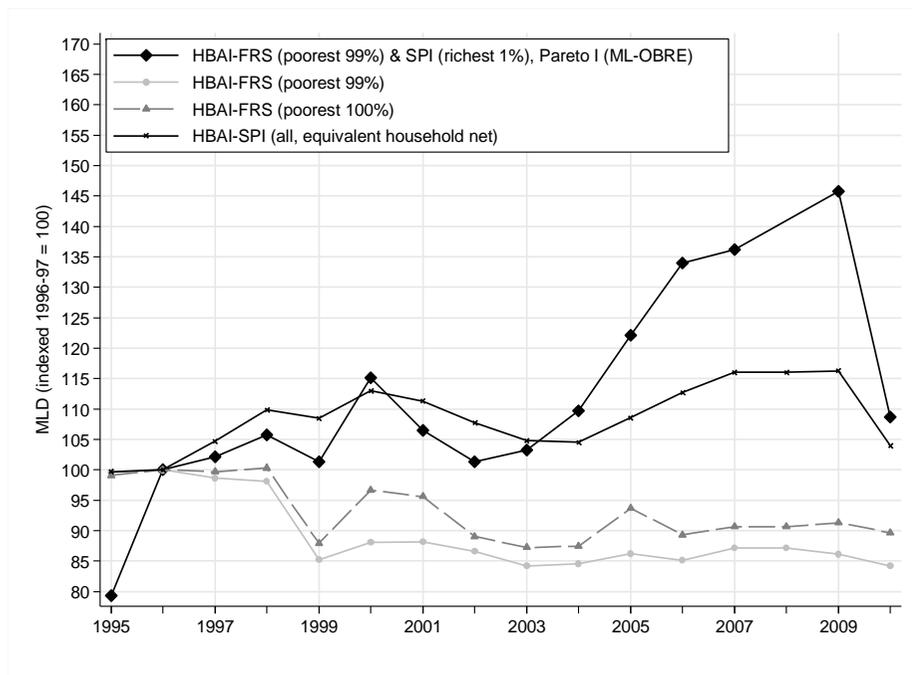
NB no tax data  
for 2008

NB problems of  
interpreting inequality  
fall after 2009:  
“forestalling”  
reaction to  
increase in top MTR  
from 40% to 50%,  
April 2010



# Approach B: trends in MLD and Theil, UK

- Derive and combine estimates as for Gini, except that now use Mean Logarithmic Deviation (MLD = GE(0)) and Theil index (GE(1))
- Result: combined estimates show greater rise in inequality than official DWP report estimates (HBAI-SPI series)
- Result: rise in inequality is greater for more top-sensitive index (Theil versus MLD)



## Approach C: combining data

Use cell-mean imputations from tax data to replace top income obs in the survey data, and then calculate inequality index from combined data

## Approach C: UK SPI-adjustment

The data used to calculate UK official “HBAI” estimates contain an ‘SPI-adjustment’ to “improve the quality of data on very high incomes and combat spurious volatility” (DSS Working Group, 1996: 23)

1. Replacement of a small number of “very rich” FRS respondents’ individual gross incomes in year  $t$  by cell-mean imputations ‘projected’ from tax data (SPI) for year  $t-1$ 
  - Distinction between pensioner and non-pensioner households. In mid-1990s, about 0.2% of (weighted) individuals had incomes SPI-adjusted; proportion increased steadily in the early 2000s; since 2008/09, fixed at c. 0.5 %
  - Benefit-unit and household incomes recalculated post-imputation
2. Recalibration of FRS weights to better gross-up to population (shift in weight towards top income holders)
  - Introduced first in 1992 (after DSS ‘Stocktaking’ report 1991), when HBAI used Family Expenditure Survey data, and originally imputed *tax/benefit unit* income from SPI, later changing to imputing *individual* incomes reflecting the change to individual taxation from 1990 and thence SPI data collection

## C: Burkhauser et al. (2016) “SPI2” adjustment

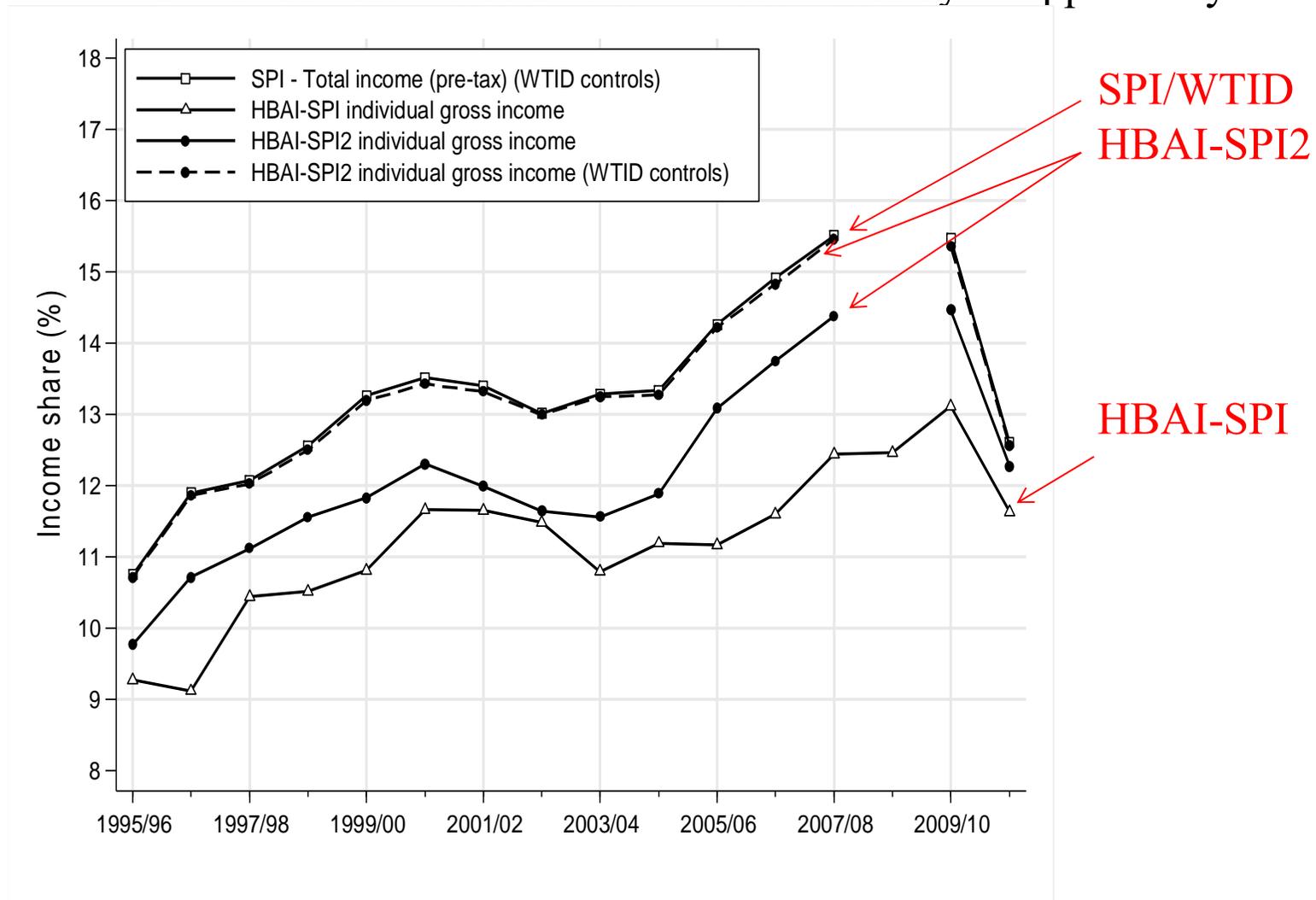
- More extensive systematic application of DWP approach
- Make adjustments in the HBAI data in order to obtain top 1% income shares (and top 0.5% and 0.1% income shares) that are fully consistent with the WTID benchmark
- Then explore distributional trends, cross-walking between tax data and survey data definitions, and exploiting survey ‘flexibility’ (change receipt unit definition; summary index)

### “SPI2” adjustment:

1. Rank individuals in the SPI according to total pre-tax income (TI)
2. Group individuals, with each group the size of 1/1000<sup>th</sup> of the total adult population (as given by WTID control totals) and group mean income
3. Repeat steps 1 to 2 with the HBAI data using our derived measure of (non-SPI’d) individual gross income
4. Replace individual income in the HBAI by the mean income of the same group in the SPI for the 10 top income groups (i.e. the top 1 per cent)
5. Total pre-tax income for the top 1 per cent is now the same in the HBAI and in the WTID/SPI

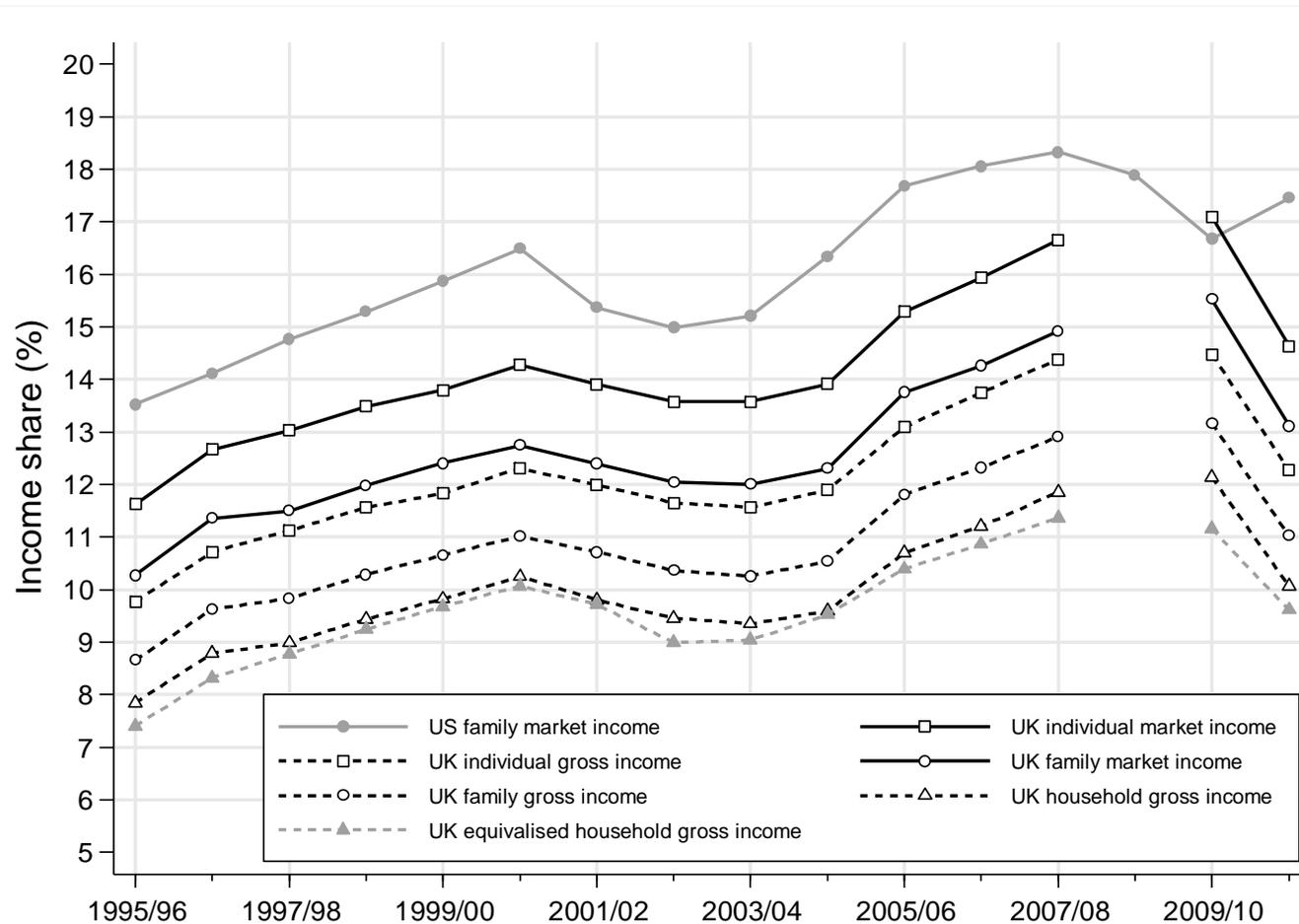
## Approach C: top 1% share

- HBAI-SPI2 matches SPI/WTID estimates exactly (by construction)
- HBAI-SPI under-estimates SPI/WTID estimates by ~ 2 ppt each year



## Approach C: Burkhauser et al. (2016), top 1% share, changing definitions of income recipient and 'income'

- Changing recipient unit definition affects estimates of levels, not trends
- Change to market income to compare with US WTID top 1% estimates
  - 'Parallel' trends through to 2007/08 (NB UK 'forestalling' problem from 2009/10)



## Inequality increase is greater when take better account of top incomes (and use more top-sensitive indices)

Inequality index	Data set	Income definition (equivalized household, all persons)	Percentage change			
			1995/96 to 2001/02	2001/02 to 2004/05	2004/05 to 2007/08	1995/96 to 2007/08
Gini coefficient	HBAI-SPI2	Gross	5.1	-2.5	7.3	10.1
	HBAI-SPI	Gross	5.2	-2.5	4.0	6.6
	HBAI-SPI	Net	4.7	-2.6	5.1	7.1
Mean logarithmic deviation (MLD)	HBAI-SPI2	Gross	12.2	-5.2	16.9	24.3
	HBAI-SPI	Gross	12.4	-5.3	9.1	16.1
	HBAI-SPI	Net	11.6	-6.1	11.0	16.4
Theil index	HBAI-SPI2	Gross	21.3	-5.0	24.9	44.0
	HBAI-SPI	Gross	22.5	-8.0	10.7	24.8
	HBAI-SPI	Net	22.0	-8.3	12.7	26.0
Half squared coefficient of variation	HBAI-SPI2	Gross	59.7	-7.2	61.1	138.9
	HBAI-SPI	Gross	62.5	-19.0	13.8	49.8
	HBAI-SPI	Net	59.8	-18.4	16.5	52.0

Note: it is not possible for us to derive *net* household income using HBAI-SPI2

## Take-home points (1)

- “Non-coverage” of top incomes by surveys has 2 components: (i) under-reporting etc. by respondents; (ii) missing top incomes altogether
  - Issue (ii) appears important according to WTID benchmarks based on tax return data (at least for UK and USA)
- If we want to address (ii), then a “combination” approach is needed, using information from both survey and tax/register data
  - Two approaches to combination: B (estimates) and C (data)
  - Both require like-for-like definitions of ‘income’ etc., in both sources – which typically means cross-walking from the survey to tax data definitions
- UK implementation suggests substantially larger recent rise in income inequality than does official survey-based estimates

## Take-home points (2)

Can the approaches be applied more generally?

- What you can do depends on the data sources available, the variables made available in them (and, related, national definitions of e.g. ‘tax unit’)
- Many choices to make in implementation of either approach, e.g. which Pareto model and which threshold (B), or how many cell-means to impute (C)

Things that data providers can help with:

- Routinely report inequality among poorest 99%, 95%, 90% as well as *p99*, *p95*, *p90*
- Methodological work on under-coverage, especially on prevalence of top incomes missing altogether from surveys, and about the ‘non-overlaps’ in income distribution densities in surveys and tax data
  - Reliability of the very lowest incomes also needs study!
- Ensure public-use survey data contain sufficiently detailed variables to do the cross-walking

## Selected references

- Jenkins, S. P. (in preparation). ‘Pareto distributions, top incomes, and recent trends in UK income inequality’, invited plenary lecture at Conference on the ‘Dynamics of Inequalities and their Perceptions’, Aix-Marseilles University, Marseilles, 26–27 May 2016
- Burkhauser, R. V., Héroult, N., Jenkins, S. P., and Wilkins, R. (2016), ‘What has been happening to UK income inequality since the mid-1990s? Answers from reconciled and combined household survey and tax return data’, NBER Working Paper w21991 (<http://www.nber.org/papers/w21991>), IZA Discussion Paper No. 9718 (<http://ftp.iza.org/dp9718.pdf>), ISER Working Paper 2016-03 (<https://www.iser.essex.ac.uk/research/publications/working-papers/iser/2016-03>).
- Alfons, A., Templ, M. and Filzmoser, P. (2013). Robust estimation of economic indicators from survey samples based on Pareto tail modelling. *Journal of the Royal Statistical Society, Series C*, 62 (2), 271–286.
- Atkinson, A. B., Piketty, T., and Saez, E. (2011). Top incomes in the long run of history. *Journal of Economic Literature*, 49 (1), 3–71.
- Bach, S., Corneo, G. and Steiner, V. (2009). From bottom to top. The entire income distribution in Germany, 1992–2003. *Review of Income and Wealth*, 55 (2), 303–330.
- Burkhauser, R.V., Feng, S., Jenkins, S. P., and Larrimore, J. (2012). Recent trends in top income shares in the USA: reconciling estimates from March CPS and IRS tax return data, *Review of Economics and Statistics*, 94 (2), 371–388.
- Cirillo, P. (2013). Are your data really Pareto distributed? *Physica A*, 392, 5947–5962
- Clauset, A., Shalizi, C. R., and Newman, M. E. J. (2009). Power-law distributions in empirical data. *SIAM Review*, 51 (4), 661–703
- Department for Work and Pensions (2015). *Households Below Average Income An Analysis of the Income Distribution 1994/95 – 2013/14*. London: Department for Work and Pensions
- Ruiz, N. and Woloszko, N. (2015). What do household surveys suggest about the top 1% incomes and inequality in OECD countries? *OECD Economics Department Working Papers*, No. 1265, <http://dx.doi.org/10.1787/5jrs556f36zt-en>