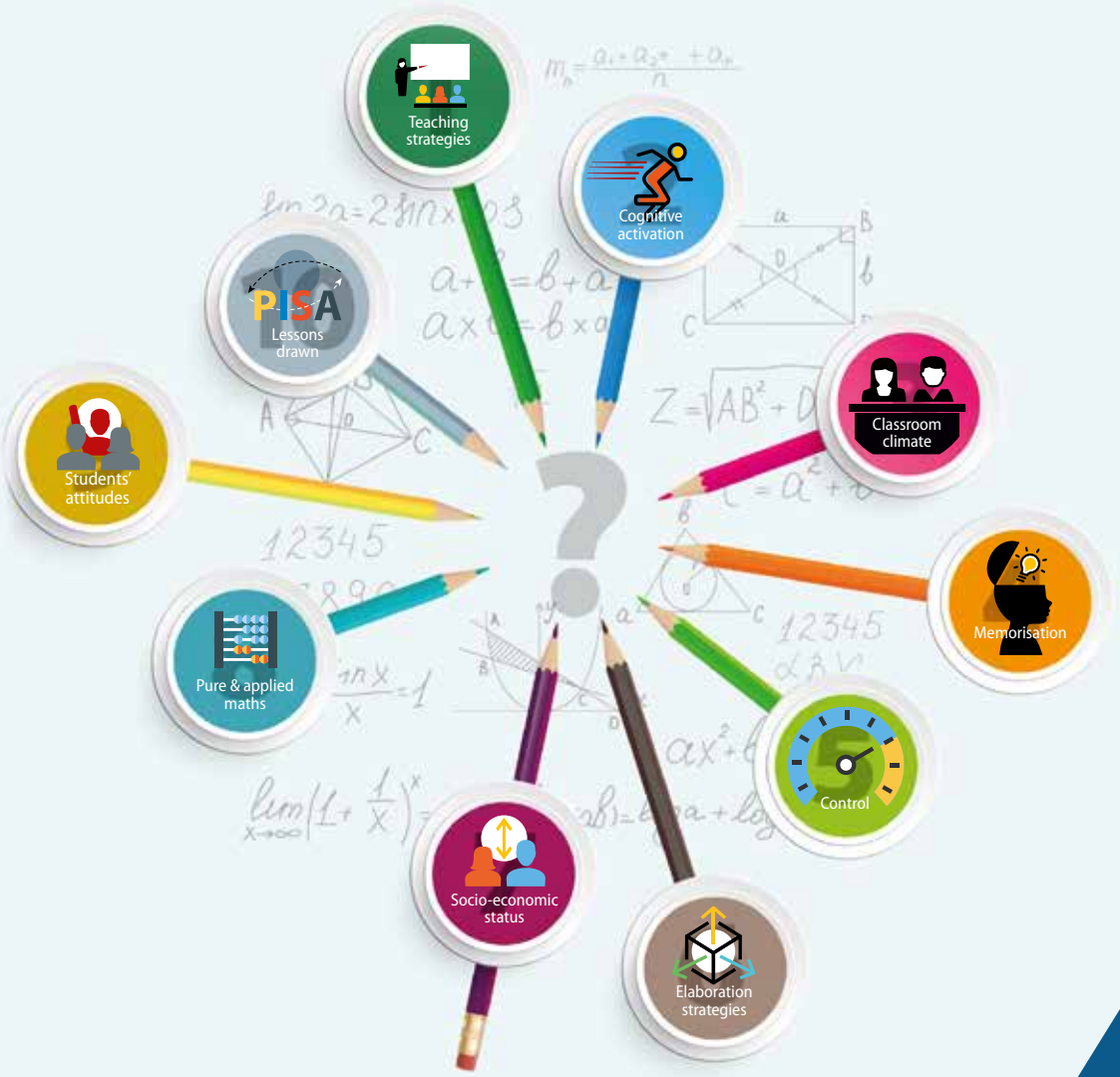

Ten Questions for Mathematics Teachers

... and how PISA can help answer them

OECD
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PISA

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Latvia was not an OECD member at the time of preparation of this publication. Accordingly, Latvia is not included in the OECD average.

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A teacher's guide to mathematics teaching and learning

Every three years, a sample of 15-year-old students around the world sits an assessment, known as PISA, that aims to measure how well their education system has prepared them for life after compulsory schooling. PISA stands for the Programme for International Student Assessment. The assessment, which is managed by the OECD, in partnership with national centres and leading experts from around the world, is conducted in over 70 countries and economies. It covers mathematics, science and reading.

PISA develops tests that are not directly linked to the school curriculum; they assess the extent to which students can apply their knowledge and skills to real-life problems. In 2012, the assessment focused on mathematics. The results provide a comparison of what 15-year-old students in each participating country can or cannot do when asked to apply their understanding of mathematical concepts related to such areas as quantity, uncertainty, space or change. As part of PISA 2012, students also completed a background questionnaire, in which they provided information about themselves, their homes and schools, and their experiences at school and in mathematics classes in particular. It is from these data that PISA analysts are able to understand what factors might influence student achievement in mathematics.

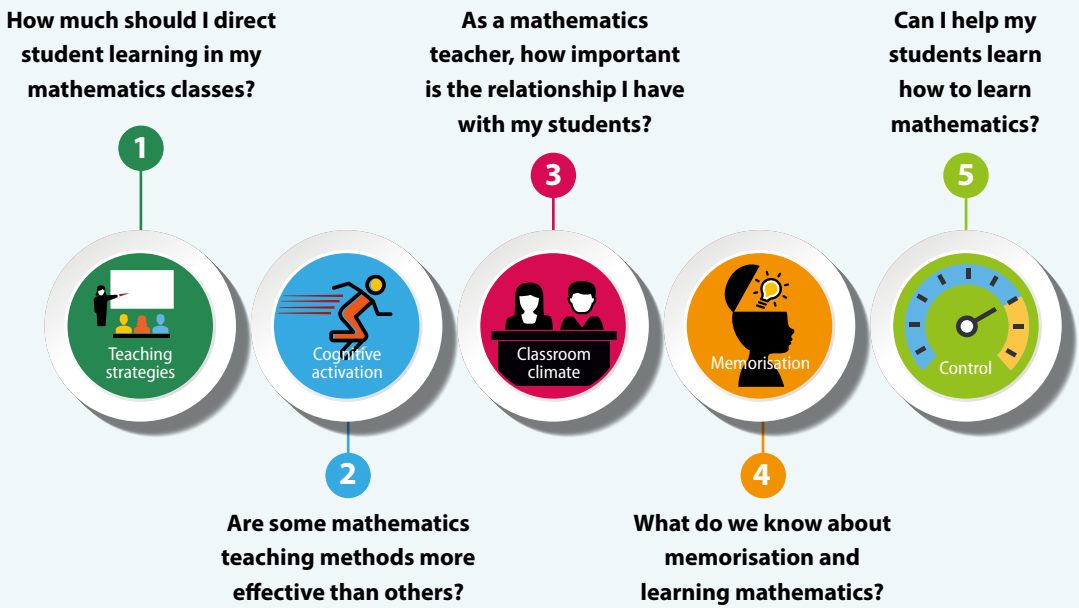
While many national centres and governments try to ensure that the schools and teachers participating in the assessments get constructive feedback based on PISA results, most of the key messages published in the PISA reports don't make it back to the classroom, to the teachers who are preparing their country's students every day. Until now.

USING PISA TO SUPPORT MATHEMATICS TEACHERS

The PISA student background questionnaire sought information about students' experiences in their mathematics classes, including their learning strategies and the teaching practices they said their teachers used. This information, coupled with students' results on the mathematics assessment, allow us to examine how certain teaching and learning strategies are related to student performance in mathematics. We can then delve deeper into the student background data to look at the relationships between other student characteristics, such as students' gender, socio-economic status, their attitudes toward mathematics and their career aspirations, to ascertain whether these characteristics might be related to teaching and learning strategies or performance. PISA data also make it possible to see how the curriculum is implemented in mathematics classes around the world, and to examine whether the way mathematics classes are structured varies depending on the kinds of students being taught or the abilities of those students.

This report takes the findings from these analyses and organises them into ten questions, listed below, that discuss what we know about mathematics teaching and learning around the world – and how these data might help you in your mathematics

Questions included in this report:

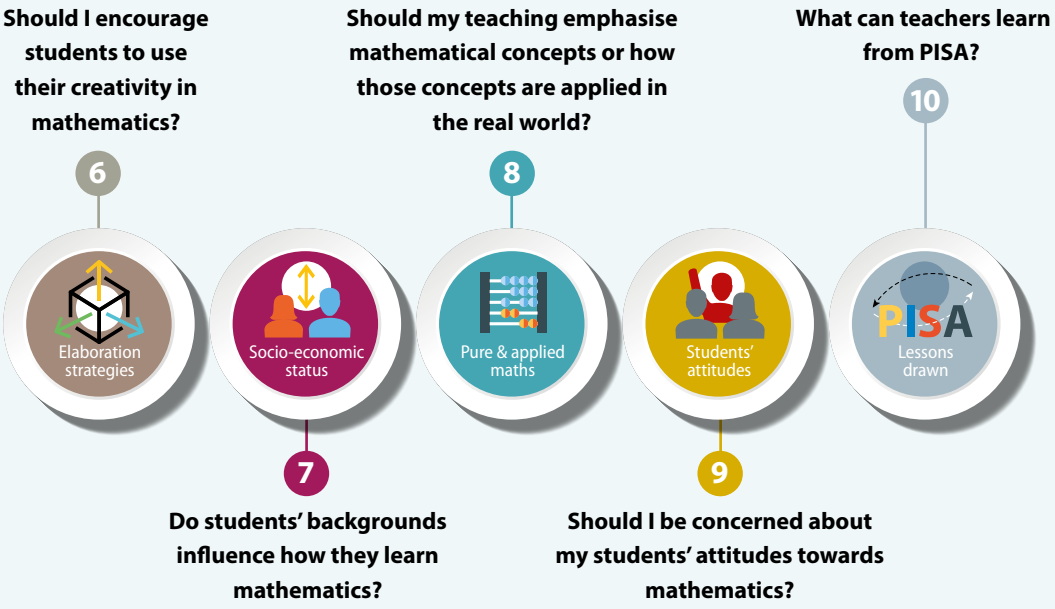


classes right now. The questions encompass teaching strategies, student learning strategies, curriculum coverage and various student characteristics, and how they are related to student achievement in mathematics and to each other. Each question is answered by the data and related analysis, and concludes with a section entitled "What can teachers do?" that provides concrete, evidence-based suggestions to help you develop your mathematics teaching practice.

WHAT DO WE MEAN BY TEACHING AND LEARNING STRATEGIES?

In simple terms, teaching strategies refer to "everything teachers do or should do in order to help their learners learn".[1] Also called teaching practices in this book, they can include everything from planning and organising lessons, classes, resources and assessments, to the individual actions and activities that teachers engage in during their classroom teaching.

Learning strategies are the behaviours and thoughts students use as they attempt to complete various tasks associated with the process of learning a new concept or acquiring, storing, retrieving and using information.[2]



You'll also find some data in this report from the Teaching and Learning International Survey, or TALIS, an OECD-led survey in which 34 countries and economies – and over 104,000 lower secondary teachers – took part in 2013. (Lower secondary teachers teach students of approximately the same age as the students who participate in PISA.) TALIS asked teachers about themselves, their teaching practices and the learning environment. These data provide information about how certain teaching strategies or behaviours might influence you as a teacher. In other words, could certain actions that you take actually improve your own feelings of self-confidence or your satisfaction with your work?

THE BOTTOM LINE

Teaching is considered by many to be one of the most challenging, rewarding and important professions in the world today. As such, teachers are under constant pressure to improve learning and learning outcomes for their students. This report tries to give you timely and relevant data and analyses that can help you reflect on how you teach mathematics and on how your students learn. We hope that you find it useful in your own development as a mathematics teacher.

ABOUT THE DATA

The findings and recommendations in this report are based on the academic research literature on mathematics education, on data from the PISA 2012 assessment and from the questionnaires distributed to participating students and school principals, and on teacher data from TALIS 2013. Keep in mind that the teaching and learning strategies discussed in this report were not actually observed; students were asked about the teaching practices they observed from their current teachers only, and teachers were asked to report on the strategies they use. PISA and TALIS are cross-sectional studies – data are collected at one specific point in time – and they do not – and cannot – describe cause and effect. For these reasons, the findings should be interpreted with caution.

The OECD average is the arithmetic mean of 34 OECD countries: Australia, Austria, Belgium, Canada, Chile, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. Latvia acceded to the OECD on 1 July 2016. It is not included in the OECD average.

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1. Hatch, E., and C. Brown (2000), Vocabulary, Semantics and Language Education, Cambridge University Press, Cambridge.

2. Dansereau, D. (1985), "Learning Strategy Research", in J. Segal, S. Chipman and R. Glaser (eds.), *Thinking and Learning Skills,* Lawrence Erlbaum Associates, Mahwah, New Jersey.

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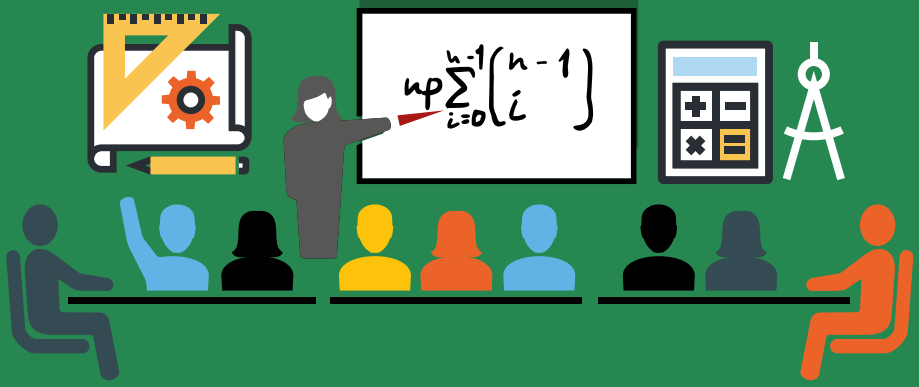
Teaching strategies

1 How much should I direct student learning in my mathematics classes?



The traditional view of a classroom that has existed for generations in schools around the world consists of students sitting at desks, passively listening as the teacher stands in the front of the class and lectures or demonstrates something on a board or screen. The teacher has planned the lesson, knows the content she needs to cover and delivers it to the students, who are expected to absorb that content and apply it to their homework or a test. This kind of "teacher-directed" instruction might also include things like lectures, lesson summaries or question-and-answer periods that are driven by the teacher. This form of teaching isn't limited to mathematics, necessarily, and it's a teaching strategy that everyone has experienced as a student at one time or another.

For decades now, educationalists have encouraged giving students more control over their own learning; thus student-oriented teaching strategies are increasingly finding their way into classrooms of all subjects. As the name indicates, student-oriented teaching strategies place the student at the centre of the activity, giving learners a more active role in the lesson than in traditional, teacher-directed strategies. These student-oriented teaching strategies can include activities such as assigning student projects that might take a week or longer to complete or working in small groups through which learners must work together to solve a problem or accomplish a task.

Which type of teaching strategy is being used to teach mathematics in schools around the world? And which one should teachers be using? Data indicate a prevalence of teacher-directed methods, but deciding how to teach mathematics isn't as simple as choosing between one strategy and another. Teachers need to consider both the content and students to be taught when choosing the best teaching strategy for their mathematics lessons.

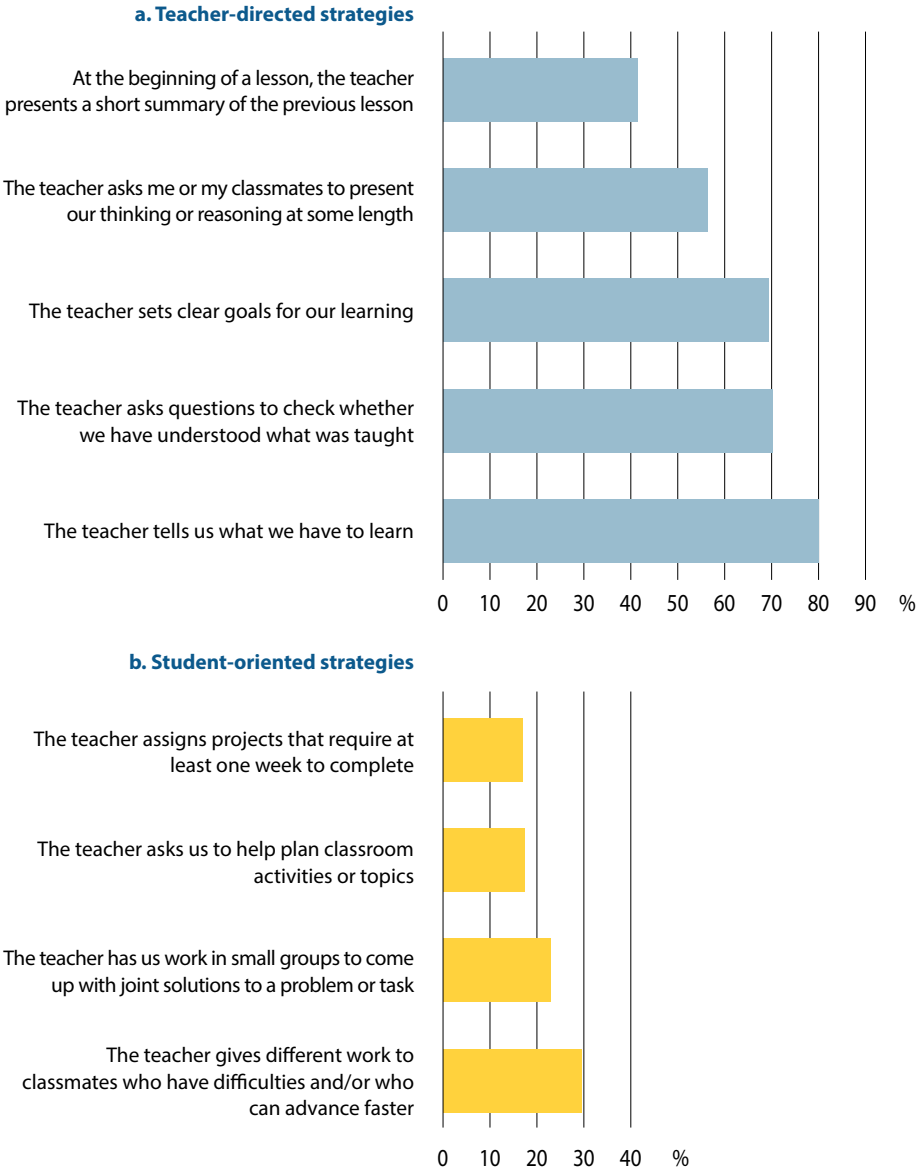
WHERE DOES MATHEMATICS TEACHING FALL IN THE TEACHER- VS. STUDENT-DIRECTED LEARNING DEBATE?

In PISA, students were asked about the frequency with which their teachers use student-oriented or teacher-directed strategies in their lessons. Findings indicate that today, teacher-directed practices are used widely. For instance, across OECD countries, eight out of ten students reported that their teachers tell them what they have to learn in every lesson, and seven out of ten students have teachers who ask questions in every lesson to check that students understand what they're learning.

On the other hand, the student-oriented practice that teachers most commonly use is assigning students different work based on their ability, commonly called differentiated instruction. However, according to students, this practice is used only occasionally, as fewer than one in three students in OECD countries reported that their teachers use this practice frequently in their lessons. Figure 1.1 shows the reported frequency of both teacher-directed and student-oriented instructional strategies for mathematics.

Figure 1.1 **Teacher-directed and student-oriented instruction**

Percentage of students who responded "in every lesson" or "in most lessons", OECD average



Note: The OECD average includes all member countries of the OECD except Latvia.

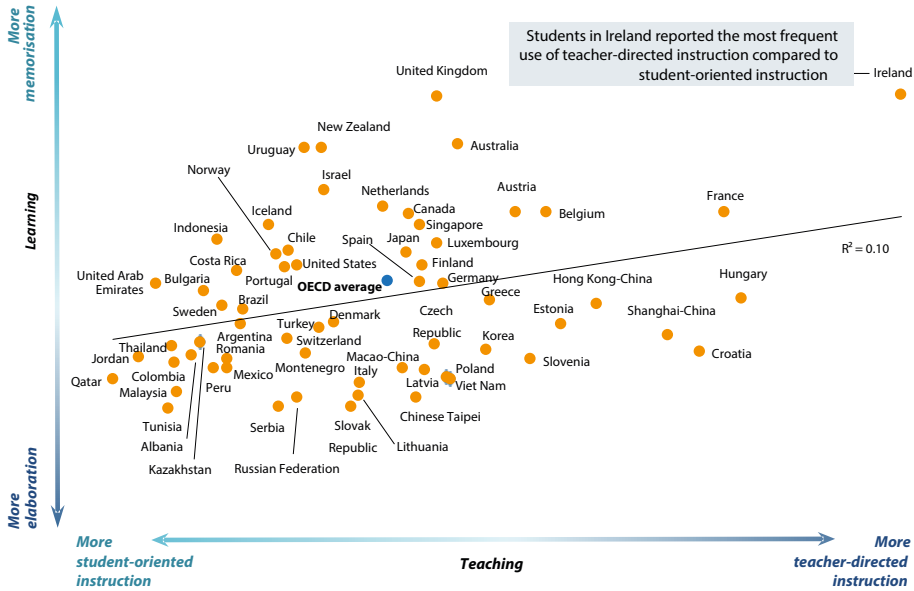
Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414750

The PISA survey also indicates that students may be exposed to different teaching strategies based on their socio-economic status or gender. For example, girls reported being less frequently exposed to student-oriented instruction in mathematics class than boys did. Conversely, disadvantaged students, who are from the bottom quarter of the socio-economic distribution in their countries, reported more frequent exposure to these strategies than advantaged students did. Teachers might have reasons for teaching specific classes in the ways they do; and other factors, such as student motivation or disruptive behaviour, might be at play too. Ideally, however, all students should have the opportunity to be exposed to some student-oriented strategies, regardless of their gender or social status. Also, when considering an entire country, the more frequently teacher-directed instruction is used compared with student-oriented instruction, the more frequently students learn using memorisation strategies (Figure 1.2).

Figure 1.2 **How teachers teach and students learn**

Results based on students' reports



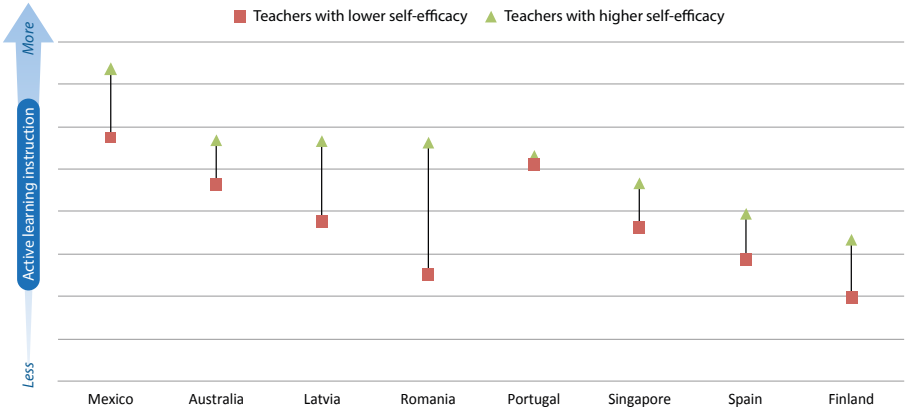
Source: OECD, PISA 2012 Database.

Statlink: http://dx.doi.org/10.1787/888933414765

WHICH TEACHERS USE ACTIVE-LEARNING TEACHING PRACTICES IN MATHEMATICS?

The TALIS study asked mathematics teachers in eight countries about their regular teaching practices. The study included four active-learning teaching practices that overlap in large part with student-oriented practices: placing students in small groups, encouraging students to evaluate their own progress, assigning students long projects, and using ICT for class work. These practices have been shown by many research studies to have positive effects on student learning and motivation. TALIS data show that teachers who are confident in their own abilities are more likely to engage in active-teaching practices. This is a somewhat logical finding, as active practices could be thought of as more "risky" than direct-teaching methods. It can be challenging to use ICT in your teaching or have students work in groups if you are not confident that you have the skills needed in pedagogy, content or classroom management.

Figure 1.3 **How teachers' self-efficacy is related to the use of active-learning instruction**



Notes: All differences are statistically significant, except in Portugal and Singapore.

Teachers with higher/lower self-efficacy are those with values above/below the country median.

The *index of active-learning instruction* measures the extent to which teachers use "information and communication technologies in the classroom", let "students evaluate their own progress", work with "students in small groups to come up with a joint solution to a problem" or encourage students to work on long projects.

The *index of self-efficacy* measures the extent to which teachers believe in their own ability to control disruptive behaviour, provide instruction and foster student engagement.

Countries are ranked in descending order of the frequency with which teachers with higher self-efficacy use active-learning instruction.

Source: OECD, TALIS 2013 Database.

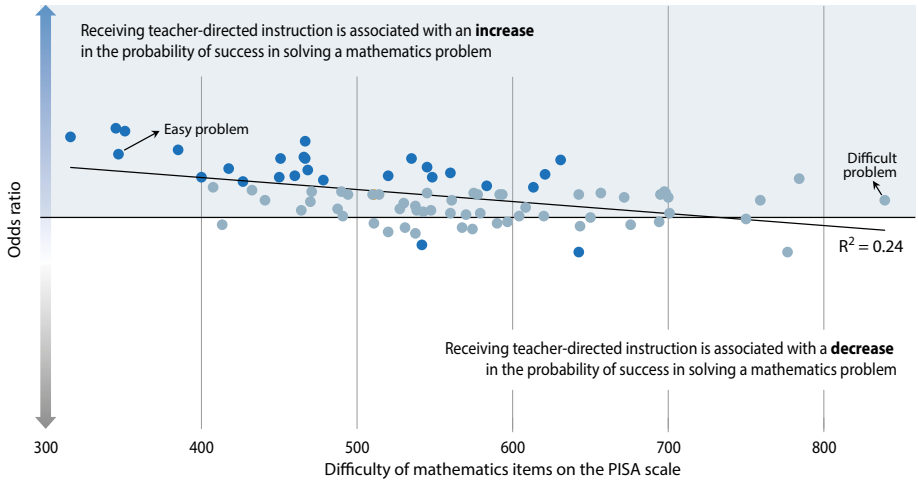
Statlink: http://dx.doi.org/10.1787/888933414779

HOW CAN A VARIETY OF TEACHING STRATEGIES BENEFIT STUDENT ACHIEVEMENT?

When looking at students' mean mathematics scores on the PISA assessment alongside their exposure to the teaching strategies discussed in this chapter, another reason for using a variety of teaching strategies emerges. Let's look first at the most commonly used teaching practices in mathematics, teacher-directed strategies. The data indicate that when teachers direct student learning, students are slightly more likely to be successful in solving the easiest mathematics problems in PISA. Yet as the problems become more difficult, students with more exposure to direct instruction no longer have a better chance of success. Figure 1.4 shows the relationship between the use of teacher-directed strategies and students' success on mathematics problems of varying difficulty.

Figure 1.4 **Teacher-directed instruction and item difficulty**

Odds ratio, after accounting for other teaching strategies, OECD average



Notes: Statistically significant odds ratios are marked in a darker tone.

Chile and Mexico are not included in the OECD average.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414786

Therefore, just as one teaching method is not sufficient for teaching a class of students with varying levels of ability, a single teaching strategy will not work for all mathematics problems, either. Past research into the teaching of mathematics supports this claim too, suggesting that teaching complex mathematics skills might require different instructional strategies than those used to teach basic mathematics skills.[1] More recent research furthers this argument, saying that more modern teaching methods, such as student-oriented teaching strategies, encourage different cognitive skills in students.[2]

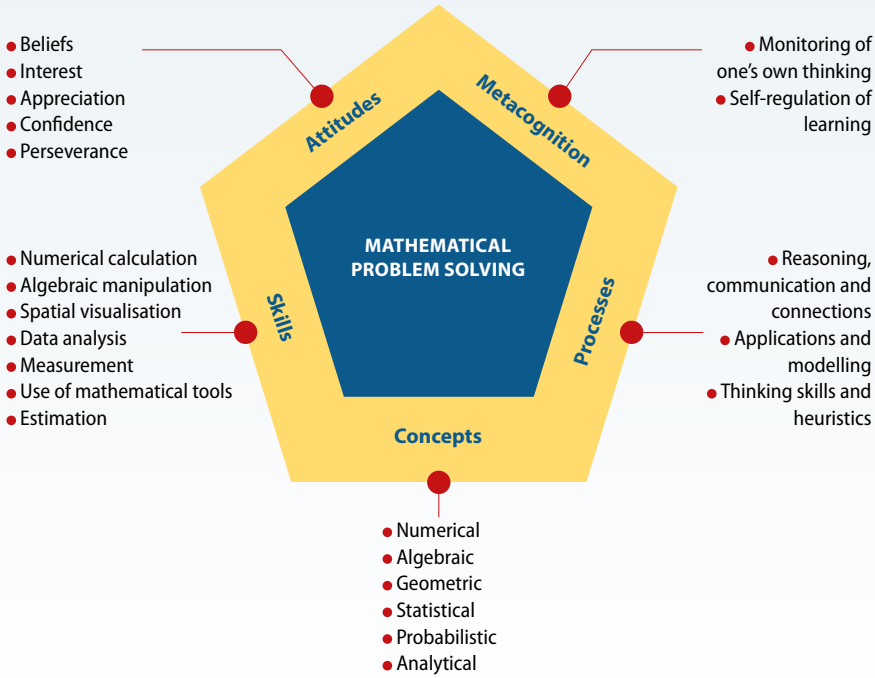
Some countries, such as Singapore, are taking this research to heart and are designing mathematics curricula that require teachers to use a variety of teaching strategies (Box 1.1). Yet rather than doing away with more traditional, teacher-directed teaching methods altogether, these methods should be used in tandem. In other words, teachers need a diverse set of tools to teach the breadth of their mathematics curriculum and to help students advance from the most rudimentary to the most complex mathematics problems.

Box 1.1 TEACHING AND LEARNING STRATEGIES FOR MATHEMATICS IN SINGAPORE

The objective of the mathematics curriculum in Singapore is to develop students' ability to apply mathematics to solve problems by developing their mathematical skills, helping them acquire key mathematics concepts, fostering positive attitudes towards mathematics and encouraging them to think about the way they learn. To accomplish this objective, teachers use a variety of teaching strategies in their approach to mathematics. Teachers typically provide a real-world context that demonstrates the importance of mathematical concepts to students (thereby answering the all-too-common question: "Why do I have to learn this?"). Teachers then explain the concepts, demonstrate problem-solving approaches, and facilitate activities in class. They use various assessment practices to provide students with individualised feedback on their learning.

Students are also exposed to a wide range of problems to solve during their study of mathematics. In this way, students learn to apply mathematics to solve problems, appreciate the value of mathematics, and develop important skills that will support their future learning and their ability to deal with new problems.

Singapore Mathematics Curriculum Framework



Source: Ministry of Education, Singapore

WHAT CAN TEACHERS DO?

Plan mathematics lessons that strive to reach all levels of learners in a class.
The benefits of differentiating instructions for students of different abilities are widely acclaimed across the research literature of all subject areas. Teachers should take this into consideration when planning mathematics lessons. Make sure each lesson or unit contains extension activities that are available for those students who finish their work quickly or are ready to move on to more challenging subjects. Think about planning time during each week for you – or your more advanced students – to offer support to those learners who might be struggling. Propose research or project-based problems that provide a variety of activities and roles for students with different abilities and interests.

Provide a mix of teacher-directed and student-oriented teaching strategies.
In mathematics especially, it is easy for teachers to rely on a textbook in their lessons, using it as a guide to explaining concepts to students and then assigning the exercises supplied by the publisher as students homework. This kind of lesson only provides teacher-directed instruction to students, and doesn't allow for much student input into their own learning. (It also doesn't account for differences in students' abilities and motivation.) Try to move beyond the textbook-provided lectures and homework and add new activities to lessons that allow students to work together or use new tools, such as technology and games, to cement their understanding of mathematical concepts.

Let the difficulty of the mathematics problem guide the teaching strategy.
When you are thinking about which strategies to use to reach different students in your class, spend a moment thinking about the strategies that work best for problems of different levels of difficulty. You may want to reserve your teacher-directed lessons for simpler mathematical concepts, and research other strategies for teaching more difficult concepts.

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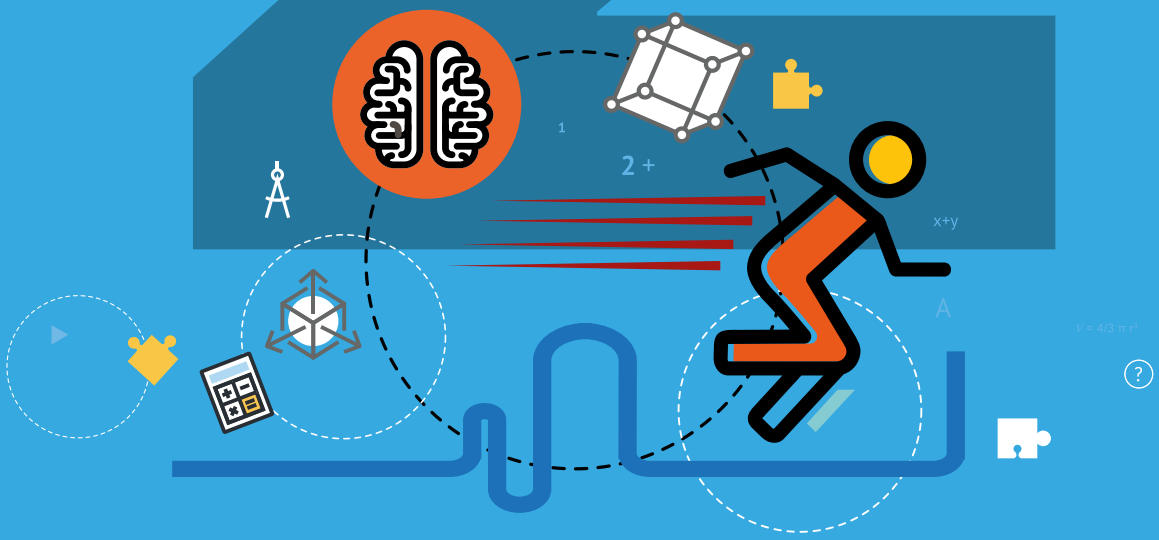
Cognitive activation

2 Are some mathematics teaching methods more effective than others?



It's so easy, as a teacher, to forget how important it is to give students – and ourselves – the time to think and reflect. With the pressures of exams, student progress, curriculum coverage and teacher evaluations constantly looming, it is often easier to just keep moving through the curriculum, day by day and problem set by problem set. Teachers may have become accustomed to teaching a certain way throughout their careers without taking a step back and reflecting on whether the teaching methods they are using are really the best for student learning. It's time for all of us to stop and think.

As the previous chapter discusses, using a variety of teaching strategies is particularly important when teaching mathematics to students with different abilities, motivation and interests. But student data indicate that, on average across PISA-participating countries, the use of cognitive-activation strategies has the greatest positive association with students' mean mathematics scores.[1] These types of teaching strategies give students a chance to think deeply about problems, discuss methods and mistakes with others, and reflect on their own learning. Teachers should understand the importance of this kind of teaching and should have a strong grasp of how to use these strategies in order to give learners the best chance of success in mathematics.

WHAT IS COGNITIVE ACTIVATION IN MATHEMATICS TEACHING?

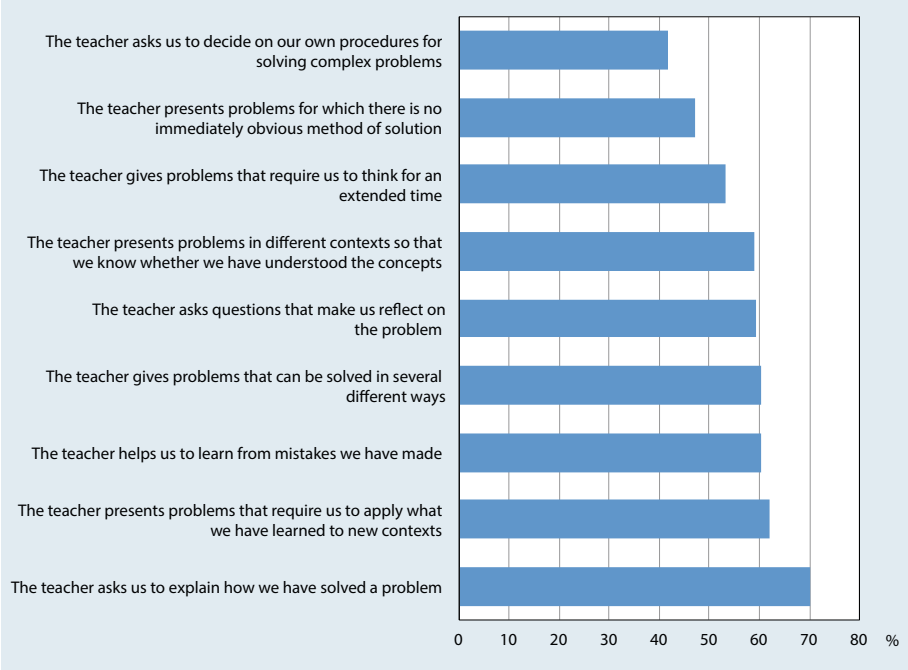
Cognitive activation is, in essence, about teaching pupils strategies, such as summarising, questioning and predicting, which they can call upon when solving mathematics problems. Such strategies encourage pupils to think more deeply in order to find solutions and to focus on the method they use to reach the answer rather than simply focusing on the answer itself. Some of these strategies will require pupils to link new information to information they have already learned, apply their skills to a new context, solve challenging mathematics problems that require extended thought and that could have either multiple solutions or an answer that is not immediately obvious. Making connections between mathematical facts, procedures and ideas will result in enhanced learning and a deeper understanding of the concepts.[2]

HOW WIDELY USED ARE COGNITIVE-ACTIVATION STRATEGIES?

The good news is that, across countries, cognitive-activation strategies are frequently used in mathematics teaching (Figure 2.1). Data indicate that the most frequently used practice in this category is asking students to explain how they solved a problem. Over 70% of students around the world reported that their teachers ask them to do this in most lessons or in every lesson.

Figure 2.1 **Cognitive-activation instruction**

Percentage of students who reported their teachers use cognitive-activation strategies "in every lesson" or "most lessons", OECD average



Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414798

In addition, more than 50% of students across the surveyed countries also reported that their teachers use other cognitive-activation strategies, such as those that require students to apply or recognise concepts they have learned in different contexts, reflect on how to solve a problem – possibly for an extended time – or learn from their own mistakes.

HOW CAN THE USE OF COGNITIVE-ACTIVATION STRATEGIES BENEFIT STUDENT ACHIEVEMENT?

PISA data indicate that across OECD countries, students who reported that their teachers use cognitive-activation strategies in their mathematics classes also have higher mean mathematics scores. The strength of the relationship between this type of teaching and student achievement even increases after the analyses also take into account teachers' use of other teaching strategies in the students' mathematics classes. As Figure 2.2 shows, when students' exposure to cognitive-activation instruction increases, their performance improves.

The use of cognitive-activation teaching strategies makes a difference no matter how difficult the mathematics problem. In fact, the odds of student success are even greater for more challenging problems. Students who are more frequently exposed to cognitive-activation teaching methods are about 10% more likely to answer easier items correctly and about 50% more likely to answer more difficult items correctly.

IN WHAT ENVIRONMENT DOES COGNITIVE ACTIVATION FLOURISH?

Studies in education as well as data collected from PISA give us a picture of the kinds of schools and classrooms in which cognitive activation thrives. Students in academically-oriented schools (as opposed to vocational schools) reported more exposure to cognitive-activation strategies. Socio-economically advantaged students reported more exposure to these strategies than disadvantaged students; and when cognitive-activation strategies are used, the association with student performance is stronger in advantaged schools than in disadvantaged schools (Figure 2.3).

If these strategies are so beneficial, why isn't every teacher using them more frequently? PISA data suggest that certain school and student characteristics might be more conducive to using cognitive-activation strategies. These types of teaching strategies emphasise thinking and reasoning for extended periods of time, which may take time away from covering the fundamentals of mathematics. Thus, using cognitive-activation strategies might be easier in schools or classes in which students don't spend as much time focusing on basic concepts. It might also be difficult for a teacher to use cognitive-activation strategies in a class that is frequently disrupted by disorderly student behaviour (see here for more information on how classroom climate can affect the teaching of mathematics).

Figure 2.2 **Mathematics performance and cognitive-activation instruction**

Score-point difference in mathematics associated with more frequent use of cognitive-activation instruction

Notes: Statistically significant values before accounting for other teaching strategies are marked in a darker tone. All values after accounting for other teaching strategies are statistically significant.

Other teaching strategies refer to the PISA indices of teacher-directed, student-oriented and formative-assessment instruction.

The *index of cognitive-activation instruction* measures the extent to which students reported that teachers encourage them to acquire deep knowledge through instructional practices such as giving students problems that require them to think for an extended time, presenting problems for which there is no immediately obvious way of arriving at a solution, and helping students to learn from the mistakes they have made.

Countries and economies are ranked in ascending order of the score-point difference in mathematics performance, after accounting for other teaching strategies.

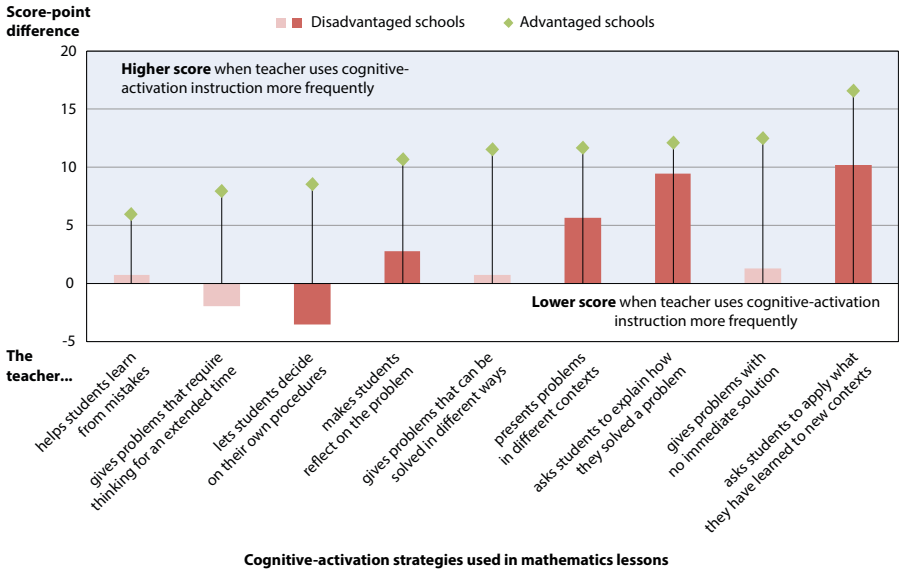
Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414800

The OECD teacher survey, TALIS, also suggests that teachers' own collaboration with colleagues makes a difference in the teaching strategies they use and can even influence student performance (Box 2.1).

Figure 2.3 **Cognitive-activation strategies and students' performance in mathematics, by schools' socio-economic profile**

Score-point difference in mathematics associated with the use of each cognitive-activation strategy, OECD average



Notes: Statistically significant values for disadvantaged schools are marked in a darker tone. All values for advantaged schools are statistically significant.

Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* is statistically lower (higher) than the mean index across all schools in the country/economy.

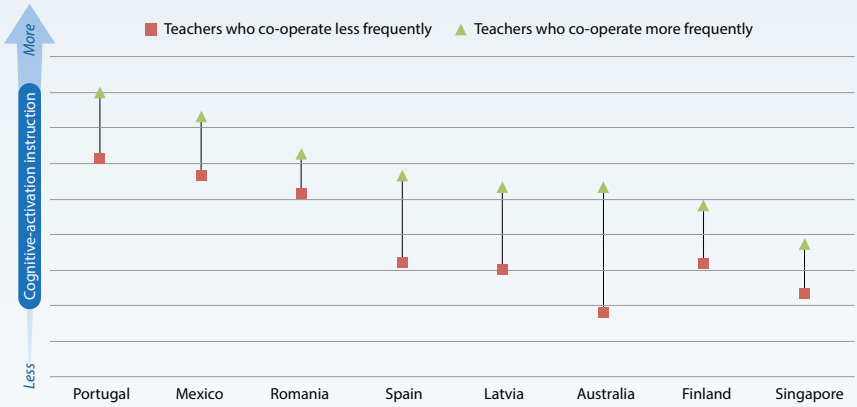
Source: OECD, PISA 2012 Database, adapted from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377210

Box 2.1 THE RELATIONSHIP BETWEEN TEACHER CO-OPERATION AND USE OF COGNITIVE-ACTIVATION STRATEGIES IN MATHEMATICS

Data from the TALIS 2013 teacher survey demonstrated that teachers who collaborate with colleagues reap many benefits themselves, such as higher levels of job satisfaction and confidence in their own abilities as teachers. The impact of teacher collaboration on mathematics teaching practices was examined when TALIS 2013 data was combined with data from the PISA 2012 assessment. The analyses indicated that the more a mathematics teacher co-operates with colleagues from the same school, the more likely he or she is to regularly use cognitive-activation practices in teaching mathematics. The figure below shows the relationship between teachers' reported collaboration with fellow teachers and their use of cognitive-activation practices in their mathematics classes.

Figure 2.4: **How teacher co-operation is related to the cognitive-activation instruction**



Notes: All differences are statistically significant, except in Mexico and Romania.

Teachers who co-operate more/less are those with values above/below the country median.

The *index of cognitive-activation instruction* measures the extent to which teachers challenge their students, such as by expecting them to "think about complex problems" or encouraging them "to solve problems in more than one way".

The *index of teacher co-operation* measures the frequency with which teachers "observe other teachers' classes and provide feedback" or "teach jointly as a team in the same class".

Countries are ranked in descending order of the extent to which teachers who co-operate more frequently use cognitive-activation instruction.

Source: OECD, TALIS 2013 Database.

Statlink: http://dx.doi.org/10.1787/888933414810

WHAT CAN TEACHERS DO?

Use cognitive-activation strategies. Data indicate that the use of these strategies is related to improved student achievement for problems of all levels of difficulty, and that these strategies are especially effective as problems become more challenging. This makes sense: students should be able to learn from their mistakes, work together, and reflect on problems that are both simple and more advanced.

Find ways to use cognitive-activation strategies in all of your classes. Challenging students might be easier in quiet classrooms with more advanced students, but you can also see it the other way round: challenging and "activating" your students may be the most effective way of creating a positive learning environment in your classroom. There are also ways to encourage students to be creative and critical in seemingly disorganised environments. Genuine creative and critical thinking often blooms in less-structured settings, for instance when students are asked to work in small groups, debate with their peers or design their own experiments.

Look at what the research says about how students best learn mathematics. Many teachers will have studied how students learn mathematics during their initial teacher education, but that may have been years ago. Teachers may have developed other teaching habits tailored to the curriculum or to the culture of the school, some of which could be enriched by incorporating the findings of new research. It is worth refreshing your knowledge of the research in teaching and learning of mathematics to make sure your beliefs are aligned with your teaching practices.

Collaborate with other teachers. Collaborating with your colleagues, both inside and outside of school, can help you acquire new learning tools and gain confidence in using them. Your students will benefit as a result.

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1. Echazarra, A., et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Papers, No. 130, OECD Publishing, Paris.

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Classroom climate

As a mathematics teacher, how important is the relationship I have with my students?

Every teacher has great teaching days. These are the days when your lesson works, and the students are motivated to learn and are engaged in class activities. Think back to your last great teaching day: how was the learning environment in your classroom? Did you continually have to discipline students because of their behaviour? Were students late for class or causing other disruptions? Or were learners staying on task, actively participating and treating you and their peers with respect? This kind of positive classroom climate, with minimal interference, gives teachers more time to spend on teaching, and makes those great teaching days possible. Teachers don't have to spend time addressing disruptions, and the classroom becomes an environment in which learning can take place. What's more, the quality of the learning environment is not only related to how teachers are able to teach, but also how they feel about their jobs and their own abilities as teachers.

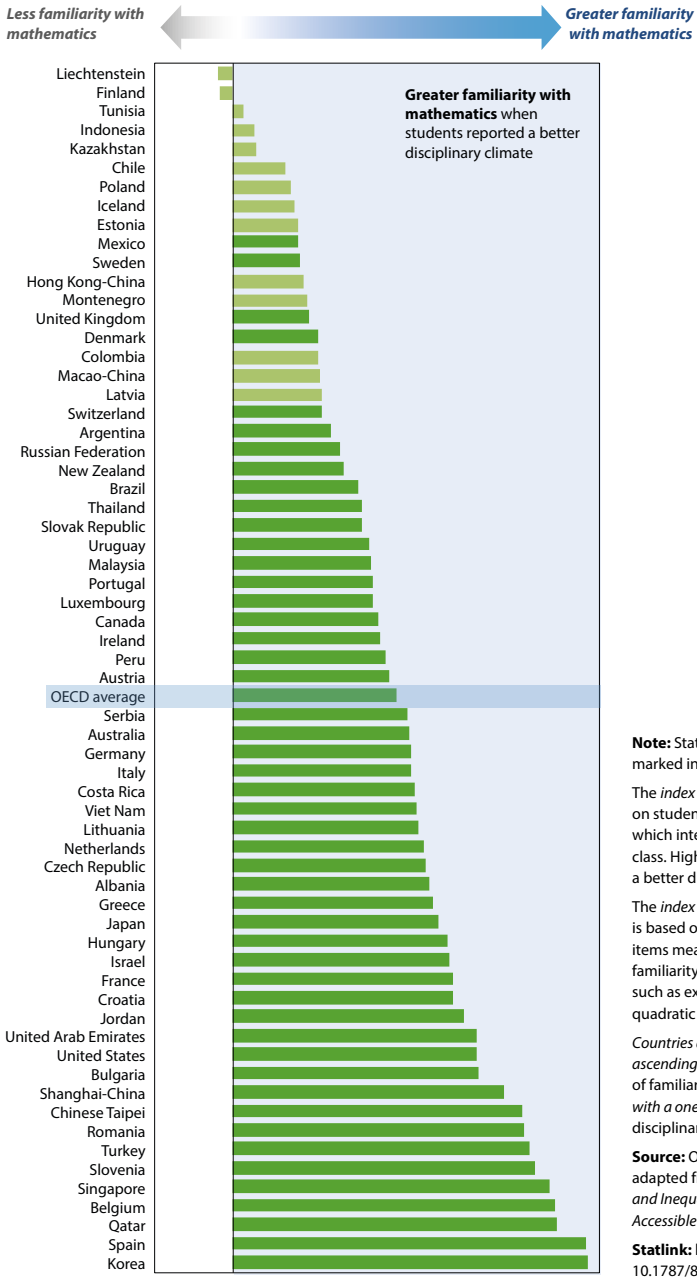
WHAT IS A GOOD CLASSROOM ENVIRONMENT FOR MATHEMATICS TEACHING AND LEARNING?

A positive classroom climate, good classroom management and strong relationships between teachers and learners should be considered prerequisites for high-quality teaching. In general, more teaching, and presumably learning, occurs when there is a positive school environment, including support from teachers and good classroom management. In addition, the disciplinary climate of the classroom is related to what and how teachers are able to teach. For example, it might be easier for teachers to use cognitive-activation strategies, such as encouraging students to be reflective in their thinking, in classrooms where students stay on task and disruptions are kept to a minimum.

PISA data suggest a link between the behaviour of students in a class and their overall familiarity with mathematics in general. As Figure 3.1 indicates, in most countries, a better disciplinary climate is related to greater familiarity with mathematics, even after comparing students and schools with similar socio-economic profiles.

Figure 3.1 **Disciplinary climate and familiarity with mathematics**

Change in students' familiarity with mathematics associated with a better disciplinary climate in class



Note: Statistically significant values are marked in a darker tone.

The *index of disciplinary climate* is based on students' reports of the frequency with which interruptions occur in mathematics class. Higher values on the index indicate a better disciplinary climate.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts, such as exponential function, divisor and quadratic function.

Countries and economies are ranked in ascending order of the change in the index of familiarity with mathematics *associated with a one-unit increase in the* index of disciplinary climate.

Source: OECD, PISA 2012 Database, adapted from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377232

This finding is especially important as students' familiarity with mathematics and their access to mathematics content at school can affect not only their performance in school but also their social and economic situation later in life. PISA data show large variations within countries in students' awareness of and access to mathematical content in schools; some of these variations could stem from the quality of the classroom learning environment.

HOW DOES THE LEARNING ENVIRONMENT IN MY CLASSROOM INFLUENCE MY TEACHING AND MY STUDENTS' LEARNING?

Whether students feel supported and listened to by their teachers is important to their schooling experience for many reasons, both social and academic. In mathematics, there appears to be a link between how a teacher teaches and the relationships he or she has with students. According to PISA data, students say that their teachers are more likely to use all teaching practices if there is a better disciplinary climate (except for student-oriented strategies), a system of classroom management in place, and students feel supported by their teachers and have good relations with them.[1] Other PISA findings also show that the disciplinary climate in mathematics lessons and student performance go hand-in-hand.[2]

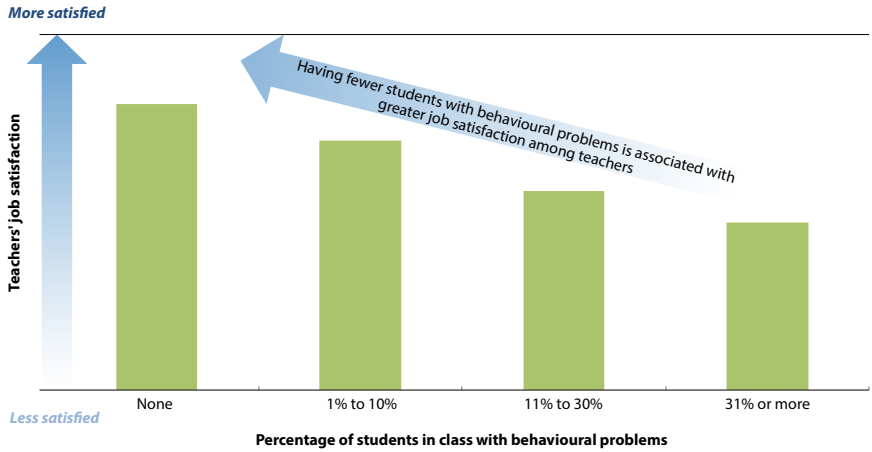
It's not just students who benefit from improvements in classroom management and more positive relationships between teachers and learners; teachers themselves profit in many ways. TALIS 2013 asked teachers about both the climate of their classroom and their relationships with their students. Their responses revealed important connections between the quality of the learning environment and teachers' job satisfaction, as well as their confidence in their own abilities as teachers. For example, as Figure 3.2 shows, on average across countries, teachers' job satisfaction is lower when there are higher percentages of students in their classes with behavioural problems. In many countries, having more students with behavioural problems is also associated with teachers feeling less confident about their own teaching abilities.

These results are perhaps understandable. Dealing with challenging classrooms of students all day can be difficult and might make teachers feel more negative towards their job, school or chosen career. Such demanding classes might also cause a teacher to question his or her own abilities, especially in the area of classroom discipline. But having strong, positive relationships with students can help. TALIS data also indicate that the detrimental effects that challenging

classrooms have on teachers' job satisfaction are mitigated when teachers also report having strong interpersonal relationships with their students.

Figure 3.2 **Teachers' job satisfaction and students with behavioural problems**

Lower secondary teachers' job satisfaction by the percentage of students with behavioural problems



Notes: Data on students with behavioural problems are reported by teachers and refer to a randomly chosen class they currently teach in their weekly timetable.

To assess teachers' job satisfaction, TALIS asked teachers to indicate how satisfied they feel about their job (on a four-point scale ranging from "strongly disagree" to "strongly agree") by responding to a number of statements about their work environment ("I would like to change to another school if that were possible"; "I enjoy working at this school"; "I would recommend my school as a good place to work"; and "All in all, I am satisfied with my job") and the teaching profession ("The advantages of being a teacher clearly outweigh the disadvantages"; "If I could decide again, I would still choose to work as a teacher"; "I regret that I decided to become a teacher"; and "I wonder whether it would have been better to choose another profession").

The analysis is based on the average of the countries participating in the TALIS survey.

Source: OECD, TALIS 2013 Database.

Statlink: http://dx.doi.org/10.1787/888933414826

WHAT CAN TEACHERS DO?

Focus time and energy on creating a positive classroom climate. If classroom management and discipline are of particular concern to you, find a way to get additional support. Speak to or observe other teachers in your school to learn successful classroom-management strategies. Ask your school leadership if you can look for ongoing professional development on this issue.

Invest time in building strong relationships with your students. This is particularly demanding for those teachers who see upwards of 150 students each day, but it could make a difference to both your students' learning and your teaching – not to mention your own well-being as a teacher. Students want to feel that their teachers treat them fairly, listen to them and will continue teaching them until they understand the material. In addition, learning about students' lives outside of school might help you to connect topics in mathematics with real-world situations that are meaningful to your students.

References

1. Echazarra, A., et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Papers, No. 130, OECD Publishing, Paris.

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Memorisation

4

What do we know about memorisation and learning mathematics?

Every mathematics course involves some level of memorisation. *The area of a circle is pi times radius squared. The square of the hypotenuse is equal to the sum of the square of the other two sides.* As teachers, we encourage our students to commit some elements, such as formulas, to memory so that they might be effortlessly recalled to solve future mathematics problems. PISA data suggest that the way teachers require students to use their memory makes a difference. Are we asking students to commit information to memory and repeatedly apply it to many similar problems? Or do we expect our students to memorise, understand and apply the concepts they have learned to problems in different contexts? Data indicate that students who rely on memorisation alone may be successful with the easiest mathematics problems, but may find that a deeper understanding of mathematics concepts is necessary to tackle more difficult or non-routine problems.

HOW PREVALENT IS MEMORISATION AS A LEARNING STRATEGY IN MATHEMATICS?

Teachers and students alike are familiar with the technique of memorisation: to learn something completely so that it can later be recalled or repeated. In mathematics classes, teachers often encourage students to use their memories through activities such as rehearsal, routine exercises and drills. To find out how students around the world learn mathematics, PISA asked them which learning strategy best described their own approach to the subject. Students were asked whether they agreed with statements that corresponded to memorisation strategies.

PISA findings indicate that students around the world often use memorisation to learn mathematics. On average in almost every country, when students were asked about the learning strategies they use, they agreed with one of the four possible memorisation-related statements (Figure 4.1). These statements are listed in Box 4.1.

That most students use memorisation to a greater or lesser degree is not surprising, given that memorisation does have some advantages as a learning strategy,

Box 4.1 MEASURING THE USE OF MEMORISATION STRATEGIES IN MATHEMATICS LEARNING

To calculate how often students use memorisation strategies, they were asked which statement best describes their approach to mathematics using four questions with three mutually exclusive responses to each: one corresponding to a memorisation strategy, one to an elaboration strategy (such as using analogies and examples, or looking for alternative ways of finding solutions) and one to a control strategy (such as creating a study plan or monitoring progress towards understanding). The *index of memorisation*, with values ranging from 0 to 4, reflects the number of times a student chose the following memorisation-related statements about how they learn mathematics:

a) When I study for a mathematics test, I learn as much as I can by heart.
b) When I study mathematics, I make myself check to see if I remember the work I have already done.
c) When I study mathematics, I go over some problems so often that I feel as if I could solve them in my sleep.
d) In order to remember the method for solving a mathematics problem, I go through examples again and again.

Statement a) assesses how much students use rote learning, or learning without paying attention to meaning. The remaining three statements come close to the ideas of drill, practice and repetitive learning.

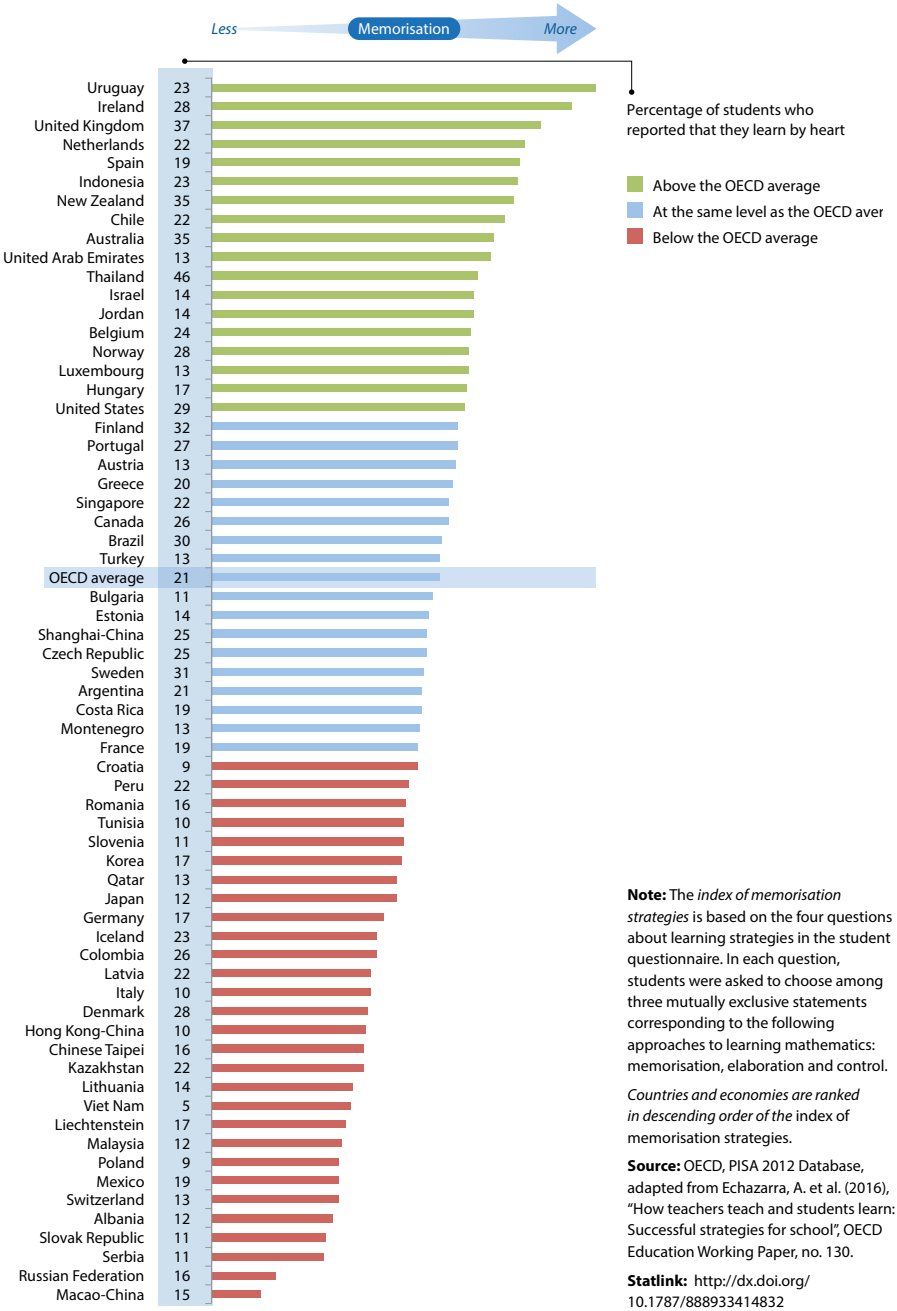
particularly when it is not just mechanical memorisation. Memorising can lay the foundation for conceptual understanding by giving students concrete facts on which to reflect. It can also lead to mathematics "automaticity", speeding up basic arithmetic computations and leaving more time for deeper mathematical reasoning.

WHO USES MEMORISATION THE MOST?

There are many reasons why students use particular learning strategies, or a combination of them, when learning mathematics. Among students who mainly use memorisation, drilling or repetitive learning, some may do so to avoid intense mental effort, particularly if they are not naturally drawn to mathematics, are not familiar with more advanced problems, or do not feel especially confident in their own abilities in the subject. To some extent, PISA results support this hypothesis. They indicate that, across OECD countries, persevering students, students with positive attitudes, motivation or interest in problem solving and mathematics, students who are more confident in their mathematics abilities, and students who have little or no anxiety towards mathematics are somewhat less likely to use memorisation strategies. Boys, too, are less likely than girls to use these

Figure 4.1 **Students' use of memorisation strategies**

Based on students' self-reports



Note: The *index of memorisation strategies* is based on the four questions about learning strategies in the student questionnaire. In each question, students were asked to choose among three mutually exclusive statements corresponding to the following approaches to learning mathematics: memorisation, elaboration and control.

Countries and economies are ranked in descending order of the index of memorisation strategies.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

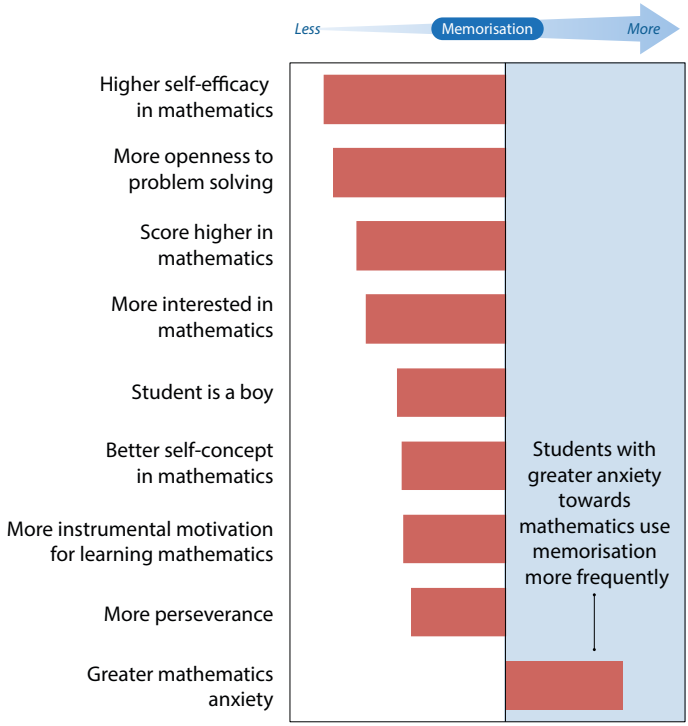
Statlink: http://dx.doi.org/10.1787/888933414832

strategies; in fact, in no education system did boys report more intensive use of memorisation when learning mathematics than girls (Figure 4.2).

When looking at students' self-reported use of memorisation strategies across countries, the data also show that many countries that are amongst the highest performers in the PISA mathematics exam are not those where memorisation strategies are the most dominant. For example, fewer students in East Asian countries reported that they use memorisation as a learning strategy than did 15-year olds in some of the English-speaking countries to whom they are often compared. These findings may run against conventional wisdom, but mathematics instruction has changed considerably in many East Asian countries, such as Japan. (Box 4.2).

Figure 4.2 **Who's using memorisation?**

Correlation with the index of memorisation, OECD average



Note: All coefficient correlations are statistically significant.

Source: OECD, PISA 2012 Database.

Statlink: http://dx.doi.org/10.1787/888933414846

Box 4.2 RECENT REFORMS IN MATHEMATICS TEACHING IN JAPAN

Mathematics teaching in Asian countries has historically been regarded as highly traditional, particularly by many western observers. Whether accurate or not, the typical image of Japanese education often includes highly competitive entrance exams, cram schools and rote memorisation.

However, Japanese education has gradually evolved into a system that promotes the acquisition of foundational knowledge and skills and encourages students to learn and think independently, which is one of the ideas behind the "Zest for Living" reform. In Japanese education today, academic and social skills refer to the acquisition of basic and foundational knowledge and skills; the ability to think, make decisions and express oneself to solve problems; and being motivated to learn.[1] For example, the policy "Period for Integrated Studies", which asks teachers and schools to develop their own cross-curricular study programmes, encourages students to participate in a range of activities, including volunteer activities, study tours, experiments, investigations, and presentations or discussions, with the aim of developing students' ability to recognise problems, learn and think independently and improve their problem-solving skills.

WILL MEMORISATION HELP OR HURT MY STUDENTS' PERFORMANCE IN MATHEMATICS?

Some experts in mathematics education consider memorisation to be an elementary strategy that is better suited to solving routine problems that require only a shallow understanding of mathematics concepts.[2] PISA results reinforce this view. They show that students who reported using memorisation strategies are indeed successful on easier mathematics tasks. For example, one of the easiest mathematics problems in the PISA 2012 assessment was a multiple-choice question involving a simple bar chart. Some 87% of students across PISA-participating education systems answered this question correctly. Students who reported that they use memorisation strategies to learn mathematics had about the same success rate on this easy item as students who reported using other learning strategies.

Although memorisation seems to work for the easiest mathematics problems, its success as a learning strategy does not extend much beyond that. According to the data, as problems become more challenging, students who use memorisation are less likely to be able to solve them correctly. Results are even worse for the most challenging mathematics problems. Only 3% of students answered the most difficult question on the 2012 PISA exam correctly. Solving this problem required multiple

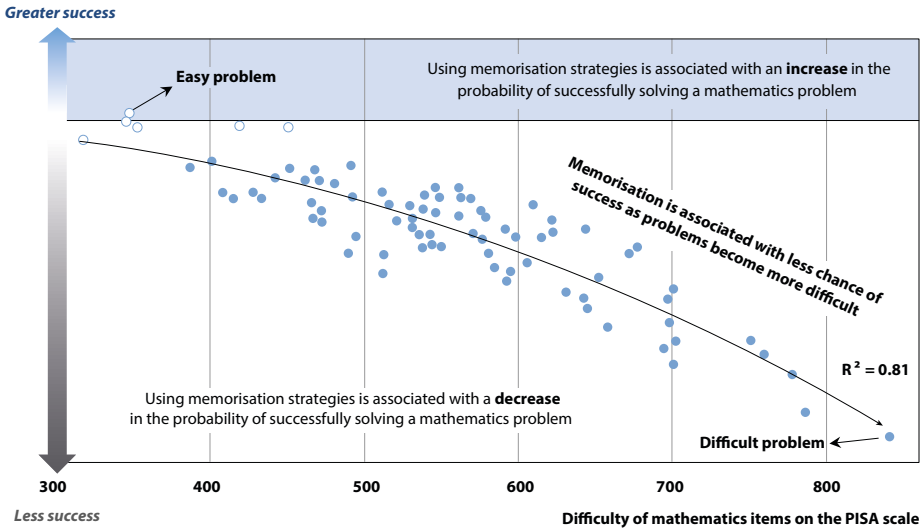
steps and involved substantial geometric reasoning and creativity. An analysis of PISA results shows that students who reported using memorisation the most when they study mathematics – those who chose the memorisation-related statement for all four questions – were four times less likely to solve this difficult problem correctly than students who reported using memorisation the least (Figure 4.3).

Indeed, PISA results indicate that no matter the level of difficulty of a mathematics problem, students who rely on memorisation alone are never more successful in solving mathematics problems. This would suggest that, in general, teachers should encourage students to go beyond rote memorisation and to think more deeply about what they have learned and make connections with real-world problems.

But PISA results also show a difference in students' performance based on the types of memorisation activities used. Students who practice repetitive learning (drilling) are more successful in solving difficult problems than those who simply learn something by heart (rote memorisation). Repetitive learning can ease students'

Figure 4.3 **Memorisation strategies and item difficulty**

Odds ratio across 48 education systems



Notes: Statistically significant odds ratios are marked in a darker tone.

Chile and Mexico are not included in the OECD average.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414854

anxiety towards mathematics by reducing the subject to a set of simple facts, rules and procedures that might seem less challenging for the least-confident students to master. Drilling can also free up time for more advanced mathematics by gradually reducing the mental effort needed to complete simple tasks.

WHAT CAN TEACHERS DO?

Encourage students to complement memorisation with other learning strategies. Memorisation can be used for some tasks in mathematics, such as recalling formulas or automating simple calculations to speed up problem solving. This will help students free up time for deeper thinking as they encounter more difficult problems later on. However, you should encourage your students to go beyond memorisation if you want them to understand mathematics, and solve real complex problems later in life.

Use memorisation strategies to build familiarity and confidence. Students may practice or repeat certain procedures as this helps consolidate their understanding of concepts and builds familiarity with problem-solving approaches. These activities don't have to be boring; teachers can find free interactive software or games on line to make such practice activities more interesting to students.

Notice how your students learn. Learners who are less confident in their own mathematical abilities or more prone to anxiety may rely too much on memorisation. Urge those students to use other learning strategies as well by helping them make connections between concepts and real-world problems and encouraging them to set their own goals for learning mathematics. Also, remember that the way you teach concepts and assess students' understanding can influence how students approach mathematics.

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Control

5 Can I help my students learn how to learn mathematics?

Learning mathematics is an important skill that is vital for students' success both in school and later in life. Perhaps even more important than the content students take away from their schooling is the act of simply "learning to learn". Throughout their school experience, students adopt learning strategies that they then apply throughout their lives. Learning strategies are an integral part of acquiring knowledge and can be defined as the thoughts and actions that students use to complete learning tasks.

A teacher's role is to recommend or encourage the use of specific learning strategies that are the most beneficial for individual learners or the problem at hand. While no one learning strategy is perfect for all learners and all situations, PISA results indicate that students benefit throughout their schooling when they control their learning. Students who approach mathematics learning strategically are shown to have higher success rates on all types of mathematics problems, regardless of their difficulty.

WHAT ARE CONTROL STRATEGIES IN MATHEMATICS?

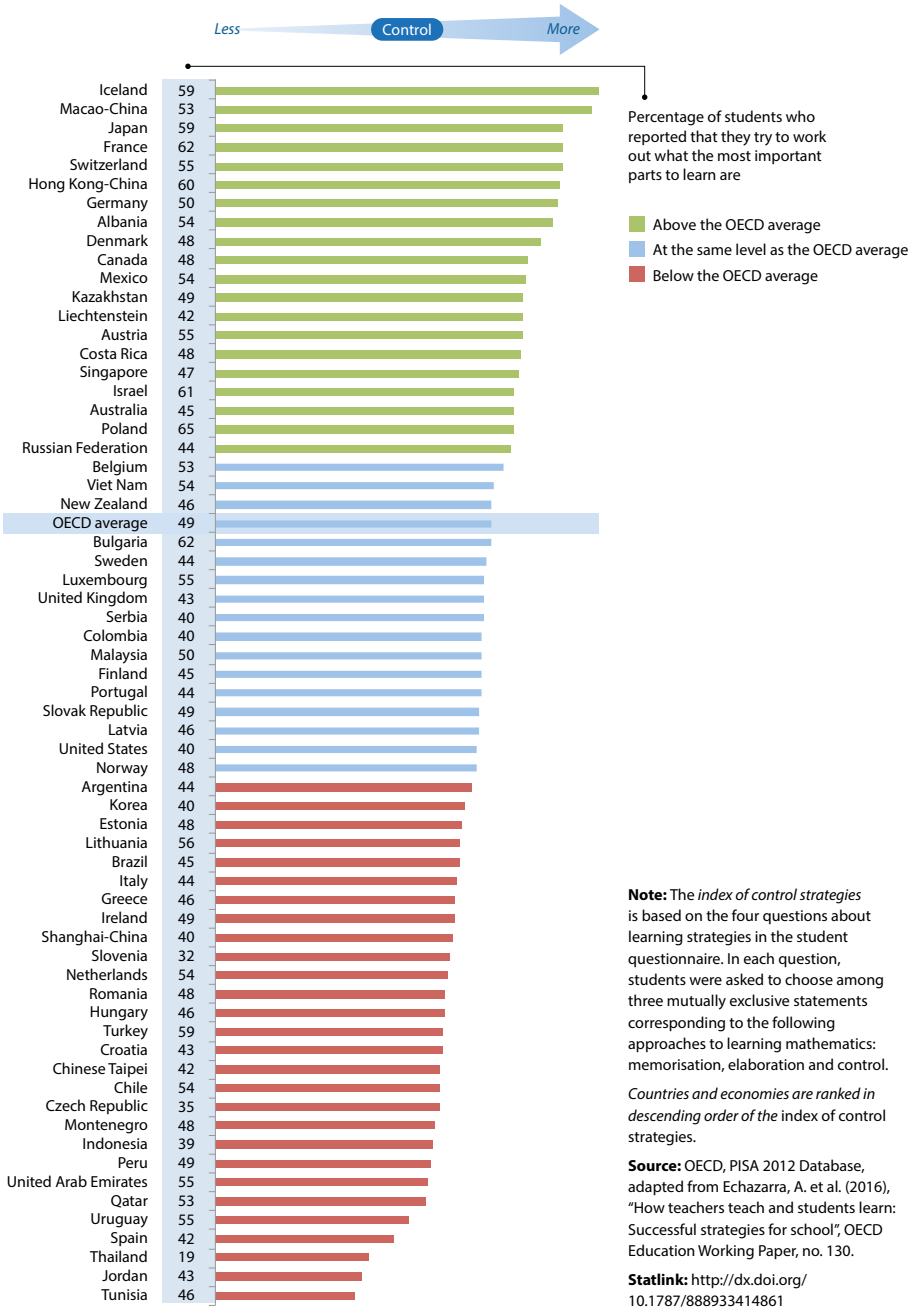
Learning strategies referred to as "control strategies" are exactly what their name implies: by allowing students to set their own goals and track their own learning progress, these methods help learners control their own learning. This approach includes activities such as organising material, creating a study plan and reflecting on the learning strategies used, and is related to concepts such as efficiency, strategic learning, self-regulation and metacognition.

PISA asked students questions that measured their use of control strategies in mathematics (Box 5.1). The PISA results overwhelmingly show that students around the world tend to use control strategies to learn mathematics more than memorisation or elaboration strategies (see Question 6 of this report) (Figure 5.1).

What these data reinforce is the idea that most students do have a strategy for their learning, which makes sense if we assume that students want to pass their exams. Many research studies show evidence that students are strategic in acquiring whatever surface and deep understanding is needed in order to complete homework or pass exams (Hattie, 2009).[1] Interestingly, Figure 5.1 shows

Figure 5.1 **Students' use of control strategies**

Based on students' self-reports



Note: The *index of control strategies* is based on the four questions about learning strategies in the student questionnaire. In each question, students were asked to choose among three mutually exclusive statements corresponding to the following approaches to learning mathematics: memorisation, elaboration and control.

Countries and economies are ranked in descending order of the index of control strategies.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414861

that students in better-performing school systems are also more likely to report using control strategies in learning mathematics.

Box 5.1 MEASURING THE USE OF CONTROL STRATEGIES IN MATHEMATICS LEARNING

To calculate how often students use control strategies, students were asked which statement best describes their approach to mathematics using four questions with three mutually exclusive responses to each: one corresponding to a control strategy, one to a memorisation strategy (such as performing routine exercises and drilling) and one to an elaboration strategy (such as using analogies and examples, or looking for alternative ways of finding solutions). The *index of control*, with values ranging from 0 to 4, reflects the number of times a student chose the following control-related statements about how they learn mathematics:

a) When I study for a mathematics test, I try to work out what the most important parts to learn are.
b) When I study mathematics, I try to figure out which concepts I still have not understood properly.
c) When I study mathematics, I start by working out exactly what I need to learn.
d) When I cannot understand something in mathematics, I always search for more information to clarify the problem.

All four statements try to measure how systematic students are in planning learning, tracking progress and identifying material that might be important, unfamiliar or difficult. Statements a) and c) look at students' efficiency in learning, while statements b) and d) capture how effective they are.

WHAT IS THE BENEFIT OF STRATEGIC LEARNING IN MATHEMATICS?

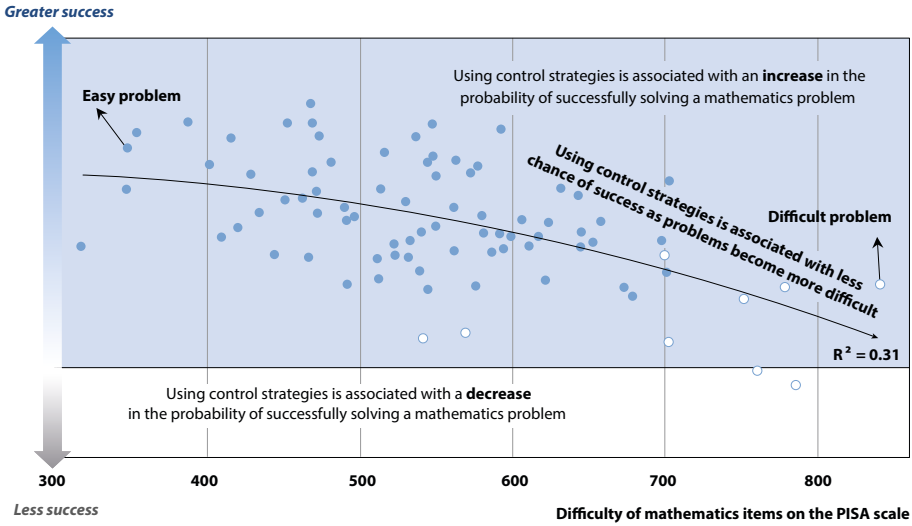
The findings from PISA also indicate a link between the use of control strategies and student performance. Specifically, students who use control strategies more frequently score higher in mathematics than students who use other learning strategies. What's more, control strategies work equally well for nearly all mathematics problems, except the most difficult ones (Figure 5.2).

Control strategies might not be as effective for solving the most complex mathematics problems because too much control and strategic learning might hinder students from tapping their creativity and engaging in the deep thinking needed to solve them. What is taught in mathematics class and how learning is assessed might also limit the effectiveness of these strategies. Research suggests that the success of these strategies depends on what is being asked of students by

their teachers, schools and education systems. In other words, when students are only being assessed on surface-level knowledge of concepts, they won't venture into deeper learning of mathematics on their own.

Figure 5.2 **Control strategies and item difficulty**

Odds ratio across 48 education systems



Notes: Statistically significant odds ratios are marked in a darker tone.

Chile and Mexico are not included in the OECD average.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

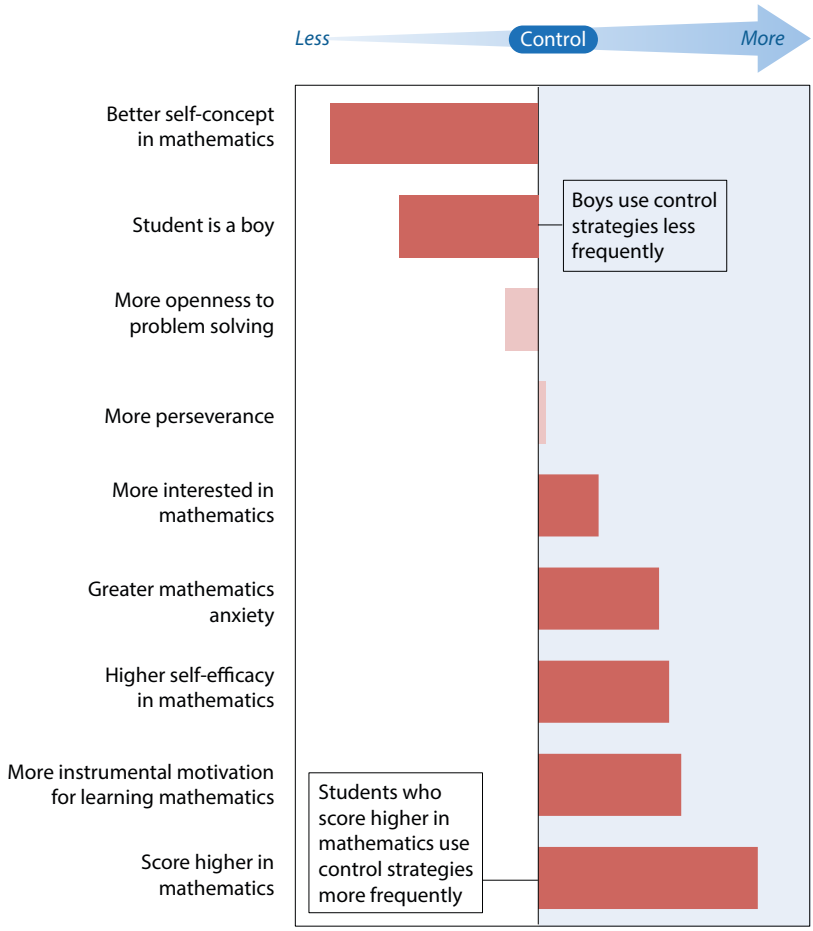
Statlink: http://dx.doi.org/10.1787/888933414878

IF CONTROL STRATEGIES ARE SO SUCCESSFUL, WHY SHOULDN'T I ENCOURAGE STUDENTS TO USE ONLY THESE LEARNING STRATEGIES AND NOTHING ELSE?

Just as one teaching strategy doesn't work for every student or every mathematical concept, control strategies aren't appropriate for every student or every problem all the time. And there are some reasons to avoid using control strategies too frequently. These strategies are somewhat associated with students having greater anxiety towards mathematics and less self-confidence in their abilities in mathematics (Figure 5.3). And the use of control strategies could become a problem when tackling the most challenging mathematics problems. To solve these problems, students need to be more reflective, confident and creative. In these instances, other learning strategies, such as elaboration strategies, have been shown to be more appropriate and effective.

Figure 5.3 **Who's using control strategies**

Correlation with the index of control strategies, OECD average



Note: Statistically significant odds ratios are marked in a darker tone.

Source: OECD, PISA 2012 Database.

Statlink: http://dx.doi.org/10.1787/888933414881

However, the negative characteristics associated with control strategies don't seem to affect students' performance on exams; and encouraging students to plan their study time and track their progress could provide them with helpful learning strategies to use in other subjects as well (Box 5.2).

Box 5.2 **TEACHING TO SUPPORT METACOGNITION IN MATHEMATICS**

Encouraging students to think about their own learning in mathematics can help them monitor progress and reveal any difficulties they might be having. One way to support metacognition in mathematics is to engage students in activities that promote classroom communication. For example, rather than just stating that answers to problems are "right" or "wrong", have students provide explanations for their solutions and work with peers to uncover where errors might lie, why they are errors, and how to correct them. Have students compare different methods for solving problems and explain the benefits and drawbacks of each. This kind of thinking and reasoning not only aids student learning, but gives teachers insight into students' mathematical thinking. It also helps students become more comfortable expressing their thoughts and concerns about mathematics, which could alleviate some of the anxiety that can be associated with the subject.

Source: National Research Council (2005)[2]

WHAT CAN TEACHERS DO?

Make sure that your own teaching doesn't prevent students from adopting control strategies. When teachers adopt certain teaching practices, they may be inadvertently reinforcing the use of certain learning strategies. For example, by giving homework that includes mathematics drilling exercises, you might be encouraging students to use memorisation over control strategies (Question 4 in this report discusses the negatives and positives of memorisation).

Familiarise yourself with the specific activities in the category of "control strategies". Once you understand what constitutes a control strategy in mathematics, you can work to incorporate related activities into your teaching and encourage your students to use similar strategies themselves. For example, you might have your students work in groups to create a study plan for an upcoming exam and monitor their own progress.

Encourage students to reflect on how they learn. Provide students with opportunities to discuss their problem-solving procedures with you and with their peers. Helping students develop a language with which to express their mathematical thinking can also help you better target any support you provide to your students.

References

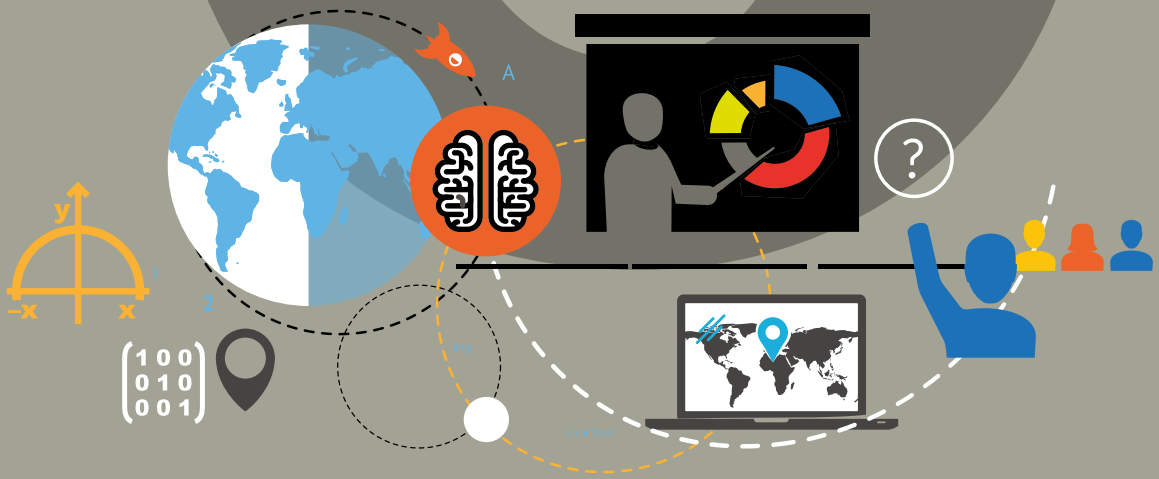
1. Hattie, J. (2009), *Visible Learning: A Synthesis of Over 800 Meta-analyses Relating to Achievement*, Routledge, London and New York.

2. National Research Council (2005), *How Students Learn: History, Mathematics and Science in the Classroom, Committee on How People Learn*, A Targeted Report for Teachers, M.S. Donovan and J.D. Bransford (Eds), Division of Behavioural and Social Sciences and Education, The National Academies Press, Washington, D.C.

Elaboration strategies

6

Should I encourage my students to use their creativity in mathematics?



As a teacher of mathematics, how many times have you heard students complain, "Why do I have to learn this? When am I ever going to use it outside of class?" Sometimes it's not easy for students to immediately grasp connections between what they learn at school and real-world problems, between different school subjects, or between new and already acquired knowledge. Education research extols the benefits of making these types of connections and exploring different ways of solving problems in mathematics.[1] This is exactly what happens when students use elaboration strategies in their learning. Approaching mathematics in this way seems to pay off, as students who use elaboration strategies are more successful in solving the most difficult mathematics problems. However, elaboration strategies don't work for every mathematics problem, and PISA data show that when students use a combination of learning strategies in mathematics, they are even more successful.

WHAT ARE ELABORATION STRATEGIES IN MATHEMATICS?

The learning strategies known as elaboration strategies encourage students to make connections among mathematics tasks, link students' learning to their own prior knowledge and real-life situations, and find different ways of solving a problem. These learning strategies include developing analogies and examples, brainstorming, using concept maps, and finding different ways of solving problems. Elaboration strategies are especially useful in helping students understand new information in mathematics and retain this information over the long term.

PISA asked students questions that measured their use of elaboration strategies in mathematics (Box 6.1). Findings indicate that around the world, fewer students reported using elaborations strategies to learn mathematics, as compared with memorisation or control strategies.

As Figure 6.1 shows, in only a third of the countries surveyed did the average student say that he or she uses elaboration strategies in at least one of the four

Box 6.1 MEASURING THE USE OF ELABORATION STRATEGIES IN MATHEMATICS LEARNING

To calculate how often students use elaboration strategies, students were asked which statement (below) best describes their approach to mathematics using four questions with three mutually exclusive responses to each: one corresponding to an elaboration strategy, one to a memorisation strategy (such as performing routine exercises and drilling) and one to a control strategy (such as creating a study plan or monitoring progress towards understanding). The *index of elaboration*, with values ranging from 0 to 4, reflects the number of times a student chose the following elaboration-related statements about how they learn mathematics:

a) When I study for a mathematics test, I try to understand new concepts by relating them to things I already know.
b) When I study mathematics, I think of new ways to get the answer.
c) When I study mathematics, I try to relate the work to things I have learned in other subjects.
d) I think about how the mathematics I have learned can be used in everyday life.

Statements a), c) and d) are directly connected to the definition of elaboration: they measure the extent to which students make connections between the task at hand, prior knowledge and their real-life experience. Statement b) reflects the idea of seeking alternatives, a creative process that is inherent in the process of elaboration.

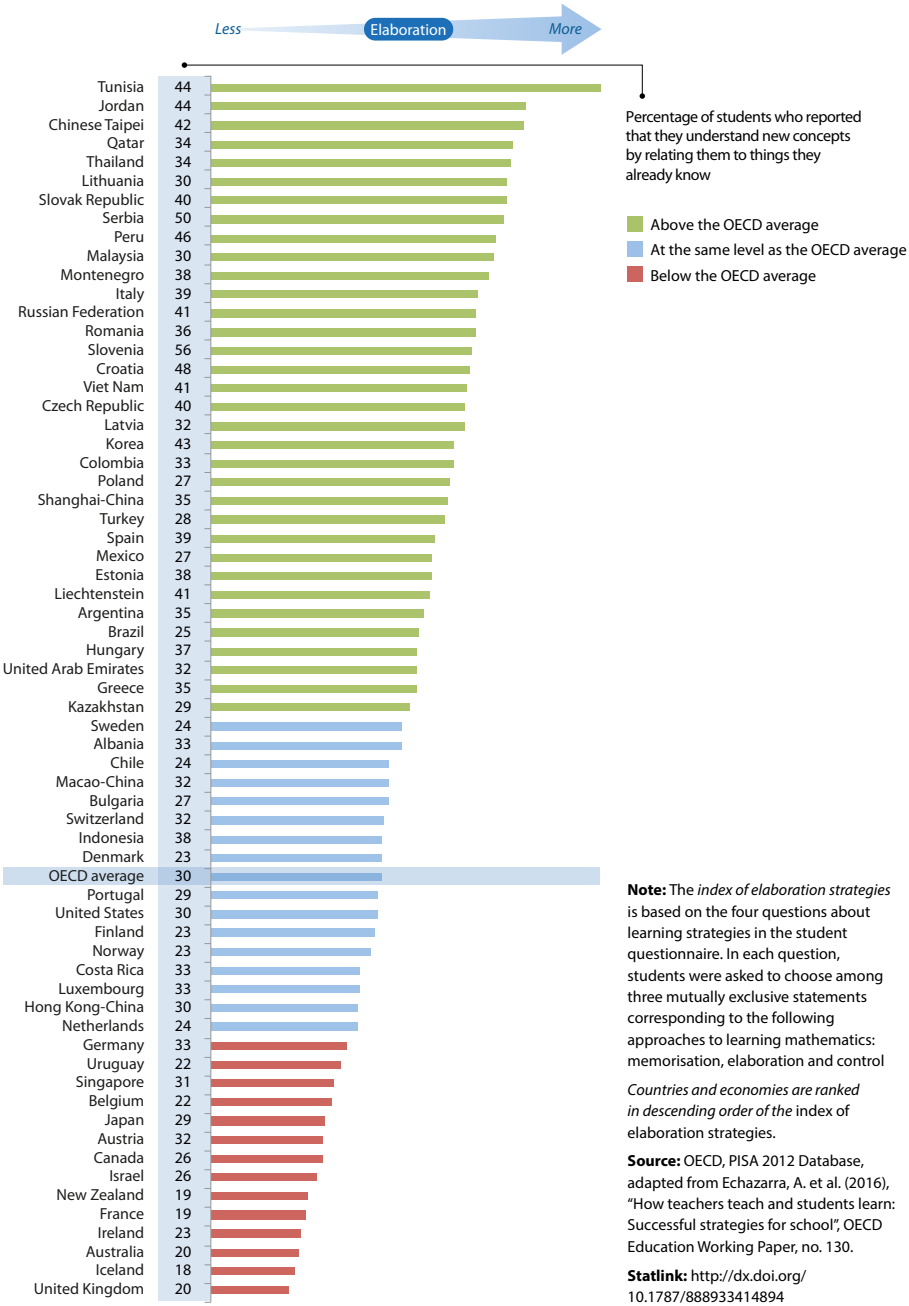
questions on learning strategies. This is surprising given how positively the research on mathematics education views elaboration-type learning strategies. The results also reveal interesting differences among countries. For example, as Figure 6.1 shows, elaboration strategies are infrequently used in English-speaking countries, including Australia, Canada, Ireland, New Zealand and the United Kingdom.

HOW DOES THE USE OF ELABORATION STRATEGIES RELATE TO STUDENT SUCCESS IN MATHEMATICS?

Students who use elaboration strategies tend to be confident in their mathematical abilities, interested in mathematics and not anxious about the subject. Yet these positive attitudes towards mathematics don't necessarily translate into overall better performance on tests. PISA results indicate no notable difference in overall performance between students who use elaboration strategies and those who don't. In fact, students who use elaboration strategies are often less successful in correctly solving the easiest mathematics problems than students who use other learning strategies, including memorisation.

Figure 6.1 **Students' use of elaboration strategies**

Based on students' self-reports



Note: The *index of elaboration strategies* is based on the four questions about learning strategies in the student questionnaire. In each question, students were asked to choose among three mutually exclusive statements corresponding to the following approaches to learning mathematics: memorisation, elaboration and control

Countries and economies are ranked in descending order of the index of elaboration strategies.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

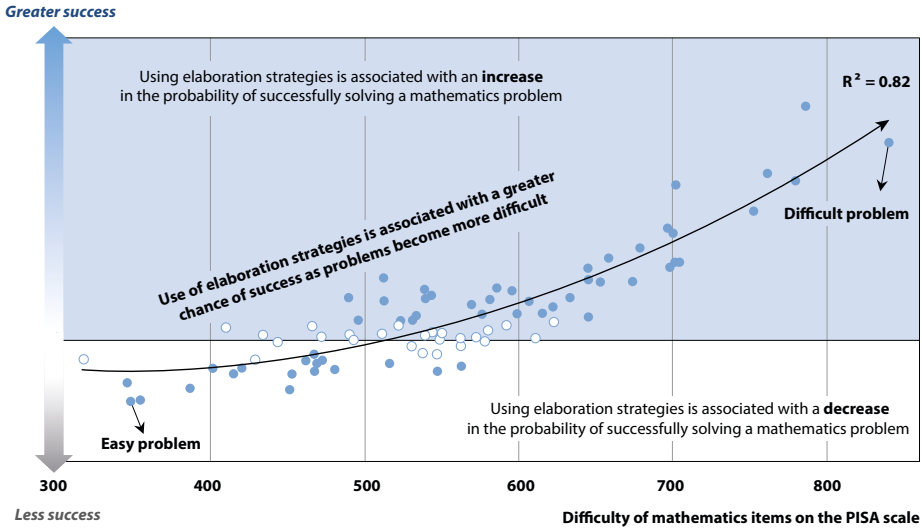
Statlink: http://dx.doi.org/10.1787/888933414894

However, elaboration strategies seem to work best on the most difficult PISA items (Figure 6.2). On average across OECD countries, students who agreed with the four statements related to elaboration were about three times more likely to succeed on the most difficult items as students who always chose other learning strategies. Even among students with similar self-confidence in mathematics or similar levels of anxiety towards mathematics, using elaboration strategies more frequently is related to greater success on the most difficult problems.[2] This makes sense, as the most challenging mathematics problems often require deeper and creative thinking about the best way to solve the problem, which elaboration strategies promote.

Another advantage of elaboration strategies is that they seem to benefit socio-economically advantaged and disadvantaged students equally. PISA results show that both groups of students perform better on the most challenging problems when they are able to make connections and look for alternative ways of finding solutions.

Figure 6.2 **Elaboration strategies and item difficulty**

Odds ratio across 48 education systems



Note: Statistically significant odds ratios are marked in a darker tone.

Chile and Mexico are not included in the OECD average.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

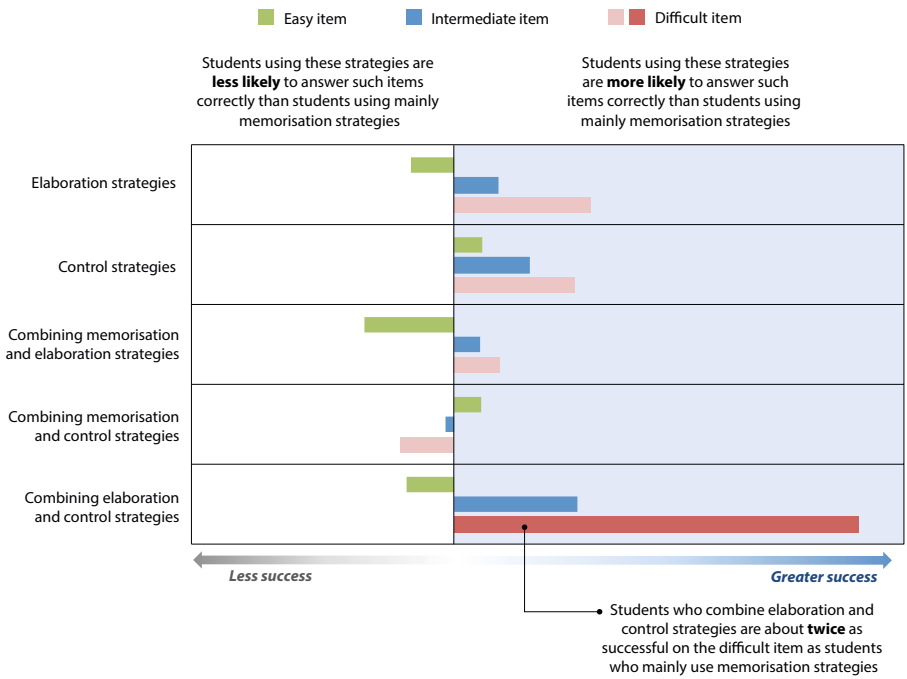
Statlink: http://dx.doi.org/10.1787/888933414903

WHICH LEARNING STRATEGY IS OPTIMAL IF I WANT STUDENTS TO BE SUCCESSFUL ON ALL TYPES OF PROBLEMS?

There is no one-size-fits-all learning strategy for all students and all problems. Research into mathematics education suggests that the best learning approach is one that combines various learning strategies. Although we know how each of the three learning strategies analysed in PISA (memorisation, control and elaboration) perform individually, PISA also seeks to understand how each strategy works when used in combination with another (Figure 6.3).

Figure 6.3 **Pure and mixed learning strategies and success on mathematics problems**

Odds ratio of succeeding on PISA mathematics items compared to using mainly memorisation strategies, across 48 education systems



Note: Odds ratios for the easy and intermediate items are not statistically significant. Statistically significant odds ratios for difficult items are marked in a darker tone.

Source: OECD, PISA 2012 Database.

Statlink: http://dx.doi.org/10.1787/888933414913

As the data show, students who combine memorisation and control strategies – or use only control strategies – perform best on easy mathematics problems. But students who use a combination of elaboration and control strategies have the most success overall on intermediate and difficult items. In fact, students who use both of these strategies are nearly twice as successful on difficult PISA problems as those students who use mainly memorisation strategies.

Applying some creativity to problem solving can do no harm to students who are primarily strategic and efficient learners, in the same way that some strategic thinking and focus can benefit students who prefer to learn by making connections and seeking alternative ways of finding solutions. Thus teachers shouldn't necessarily think about which strategy they emphasise for a particular mathematics problem or concept. Rather, they should ensure that students are familiar with a range of learning strategies and that they understand when to apply each one – individually or as part of a combination – to the mathematics problems they encounter.

WHAT CAN TEACHERS DO?

Emphasise the use of elaboration strategies on challenging tasks. Success on the most difficult mathematics problems can be improved by encouraging students to use elaboration strategies more intensively. Making connections, brainstorming and thinking creatively about the best way to solve a problem become necessary on problems for which an immediate solution is not obvious.

Challenge all of your students, without raising mathematics anxiety. Most teachers believe that students need to be constantly challenged. Elaboration strategies challenge all students, regardless of their socio-economic background, to relate problems to their own prior knowledge and life experience and find new ways of solving problems. Students who use these learning strategies also exhibit lower levels of anxiety towards mathematics and higher self-concept in mathematics.

Develop versatile learners. A good learner is a flexible learner who can use and combine strategies, depending on the task at hand and the context in which the learning occurs. Encourage a combination of learning strategies, particularly control and elaboration strategies. This provides students with enough direction and strategic thinking for easier mathematics problems and enough motivation and creativity for the most complex problems.

Create assessments that challenge students to prepare them for deeper learning. Teachers should develop homework and exams that challenge students to go beyond surface-level learning, and help them rise to the challenge.

References

1. Caine, R., and G. Caine (1991), *Making Connections: Teaching and the Human Brain, Association for Supervision and Curriculum Development, Alexandria,* Virginia.

2. Echazarra, A., et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Papers, No. 130, OECD Publishing, Paris.

Socio-economic status

Do students' backgrounds influence how they learn mathematics?

We've all heard that students' home and family lives are vital to their success in school. The environment in which a young person is raised, the resources they have access to, the education of their parents – all of these can influence students' performance in school and later in life. In many countries, public schooling was created to be a great equaliser. Modern schooling was designed to give students from all backgrounds an equal opportunity for success in school and in society. If schools are designed to promote equality, should teachers consider the socio-economic backgrounds of their students when deciding how or what to teach them?

PISA analyses examined how much exposure students have to mathematical concepts and whether their familiarity with mathematics is related to their own backgrounds. Findings indicate that students' exposure to mathematical concepts differs according to their socio-economic status.

ARE SOCIO-ECONOMICALLY DISADVANTAGED STUDENTS EXPOSED LESS FREQUENTLY TO MATHEMATICS CONTENT?

National or local governments often mandate how many hours students have to be in school each day. Those regulations may even extend to the classroom, suggesting or dictating how many hours students need to spend in class for a particular subject each week. Thus, students from different socio-economic backgrounds might spend the same number of hours per week in mathematics classes, but their results could differ considerably. Socio-economically disadvantaged students may not be exposed to the same mathematics content as advantaged students if they are more likely to attend schools with a less-challenging mathematics curriculum (such as vocational schools), if they are sorted into classes or ability groups where less-advanced mathematics is taught, or if they end up in schools or classes with poorer disciplinary climates and poorer learning environments. It is the content presented during instruction time and the quality of instruction that matter.

PISA data show large disparities across countries in the extent to which students' self-reported familiarity with mathematics varies within each country. One of the areas of variation is in students' exposure to both pure mathematics and applied

mathematics. Pure mathematics tasks include using functions and equations, whereas applied mathematics requires students to use their knowledge of mathematics to solve problems that they may encounter in everyday life (see Question 8 of this report). According to PISA, disadvantaged students are less frequently exposed to both applied and pure mathematics when compared with their more advantaged peers (Figures 7.1a and 7.1b). On average across OECD countries, about 65% of socio-economically advantaged students, but only 43% of disadvantaged students, reported that they know well or have often heard of the concept of quadratic function.

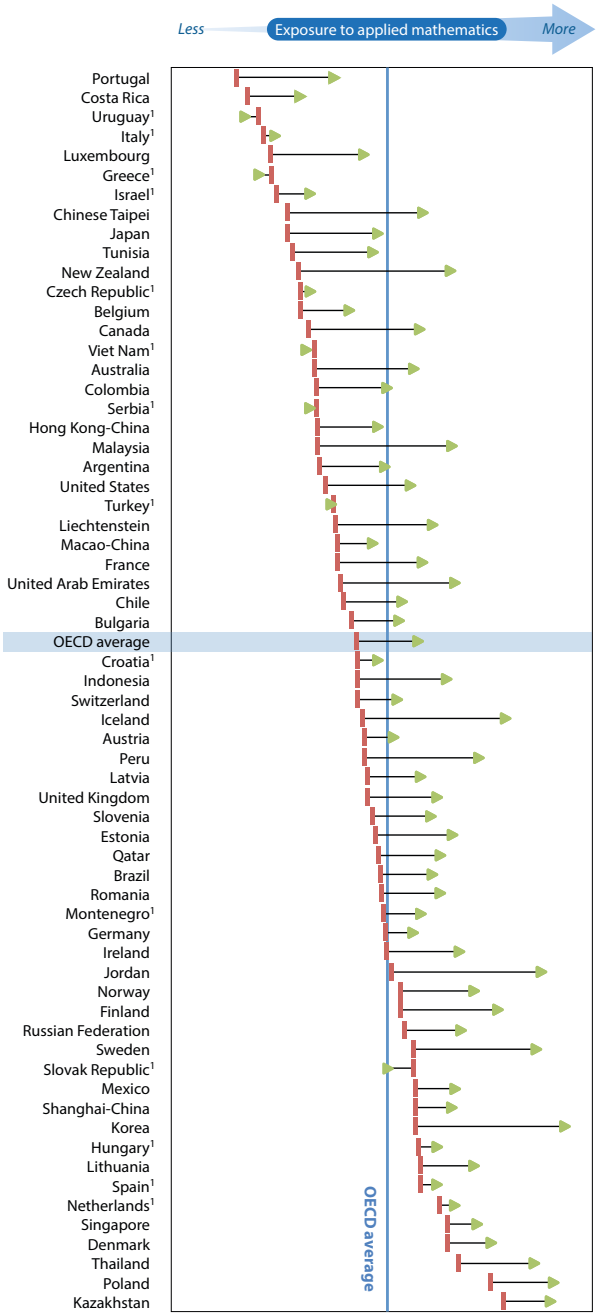
What do these data tell us about the effectiveness of the time that disadvantaged students spend studying mathematics at school? Given a similar investment of time, disadvantaged students still reported hearing of key mathematical concepts less often, spending less time solving equations and engaging less frequently in relatively simple applied mathematics tasks. The question that policy makers, schools and teachers should be asking themselves now is: what do these students do during the many hours they spend in mathematics class that is different from what students with more advantaged backgrounds do?

Part of the answer lies in how school systems are organised and how they sort students into different schools and tracks; but another part is related to what schools and teachers are doing to make sure that any gaps in mathematics knowledge and understanding among disadvantaged students are filled and not allowed to widen from year to year.

Teachers are generally committed to providing equal education opportunities; they know that not all students are equally ready to progress at the same speed. Across OECD countries, about 70% of students attend schools where teachers believe it is best to adapt academic instruction to the students' levels and needs. However, adapting lessons to each student's skills and needs, while simultaneously advancing learning for all students in the classroom, is complicated. There is always a risk of watering down the standards of instruction, with severe consequences for the future of disadvantaged students.

Additional support for teachers in disadvantaged schools would be beneficial, as discussed in Box 7.1, and it might be necessary to offer struggling students more individualised support. Teachers need more support to use pedagogies, such as flexible

Figure 7.1a **Exposure to applied mathematics, by students' socio-economic status**



The *PISA index of economic, social and cultural status:*

- Bottom quarter (disadvantaged students)
- Top quarter (advantaged students)

1. The difference between the top and the bottom quarters of the *PISA index of economic, social and cultural status* is not statistically significant.

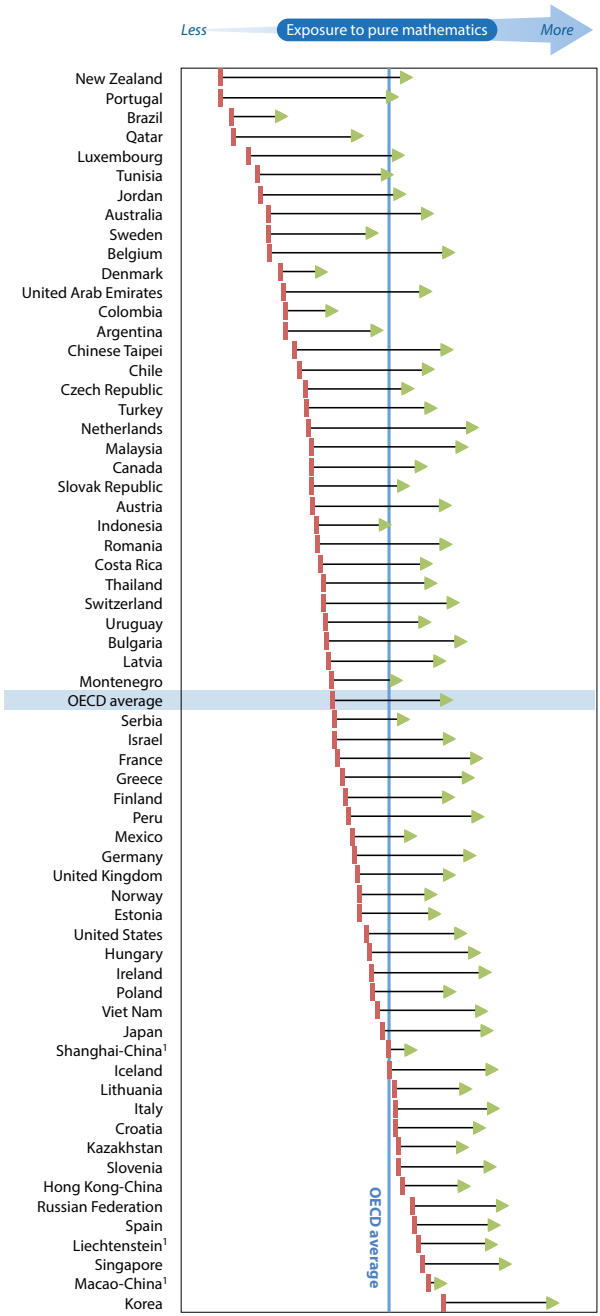
Note: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable, how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Countries and economies are ranked in ascending order of the average index of exposure to applied mathematics of students in the bottom quarter of the PISA index of economic, social and cultural status.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377010

Figure 7.1b **Exposure to pure mathematics, by students' socio-economic status**



The *PISA index of economic, social and cultural status:*

- Bottom quarter (disadvantaged students)
- Top quarter (advantaged students)

1. The difference between the top and the bottom quarters of the *PISA index of economic, social and cultural status* is not statistically significant.

Note: The *index of exposure* to pure mathematics measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

Countries and economies are ranked in ascending order of the average index of exposure to pure mathematics of students in the bottom quarter of the PISA index of economic, social and cultural status.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377022

Box 7.1 SUPPORTING TEACHERS IN DISADVANTAGED SCHOOLS

Teachers in schools with larger proportions of students from socio-economically disadvantaged backgrounds often face many additional challenges when compared with their peers in more advantaged schools. In addition to the curriculum design issues discussed here, teachers of less-advantaged students might also have to work with fewer educational resources, larger classes or more disruptive students. It's no wonder, then, that teachers in these circumstances often report lower job satisfaction than their peers who don't have to work in these conditions.

The TALIS 2013 survey looked at the elements that might be related to teachers' job satisfaction across all schools and in schools where the student population is less advantaged. Findings indicate that in high-needs schools (where more than 30% of the student population is composed of second-language learners, students with special needs or socio-economically disadvantaged students), teachers express less dissatisfaction with their jobs when they are involved in collaborative networks with their peers. Schools are encouraged to adopt practices that expand teacher networks for exchanging ideas, resources and expertise.

grouping or co-operative learning strategies, that increase learning opportunities for all students in mixed-ability classes.

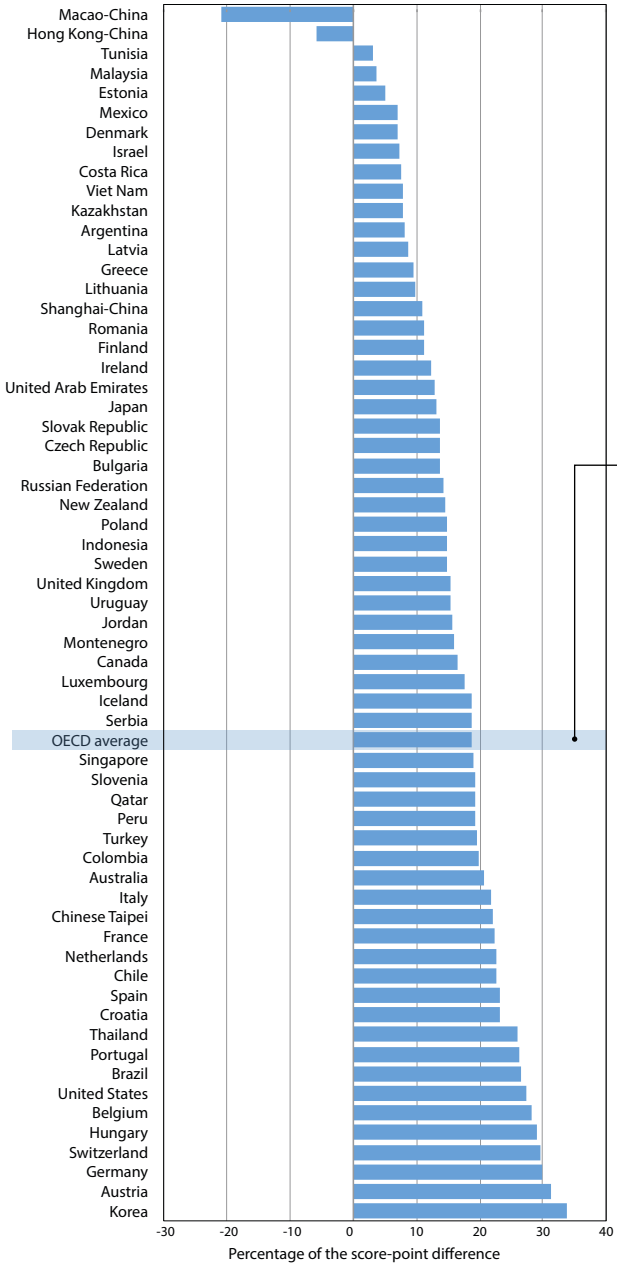
DOES THIS DIFFERENCE IN EXPOSURE TO MATHEMATICS AFFECT THE PERFORMANCE OF DISADVANTAGED STUDENTS?

The short answer to this question is "yes": students' level of familiarity with mathematical concepts is related to the performance gap between advantaged and disadvantaged students. In fact, across countries, 19% of the performance difference between socio-economically advantaged and disadvantaged students can be explained by their differing levels of exposure to mathematics (Figure 7.2). The strong relationship between performance and exposure to mathematics runs in both directions. Infrequent exposure to relevant concepts and tasks certainly limits students' capacity to solve complex problems; but students' performance in mathematics might also influence how frequently they are exposed to mathematics content, as teachers adapt their lessons to meet – and expand – their students' abilities.

When we look at the data in more detail, it becomes apparent that the differences in performance between advantaged and disadvantaged students vary with the difficulty of the mathematics problems. While disadvantaged students lag behind their more advantaged peers across all items tested, they fall further behind on

Figure 7.2 **Differences in performance related to familiarity with mathematics**

Percentage of the score-point difference between advantaged and disadvantaged students explained by different familiarity with mathematics



Across OECD countries, 19% of the difference in mathematics scores between advantaged and disadvantaged students is explained by disadvantaged students being less familiar with mathematics

Notes: Socio-economically advantaged (disadvantaged) students are defined as those students in the top (bottom) quarter of the *PISA index of economic, social and cultural status*.

In Hong Kong-China and Macao-China, the percentage is negative because disadvantaged students reported greater familiarity than advantaged students. In these economies, eliminating the difference in familiarity would widen the performance gap between these two groups of students.

Countries and economies are ranked in ascending order of the percentage of the performance gap between advantaged and disadvantaged students explained by familiarity with mathematics.

Source: OECD, PISA 2012 Database, adapted from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377436

the most difficult items. On average across OECD countries, a disadvantaged student is 1.3 times less likely to solve the easiest problems on the PISA exam, but more than 3 times less likely to correctly solve the most difficult items. Once students' relative lack of familiarity with these mathematical concepts is taken into account, the performance gap related to socio-economic status narrows across all PISA mathematics problems, regardless of the level of difficulty.

Problem solving, modelling and application of mathematical concepts make lessons more demanding, for both teachers and students. Weaker students – and particularly disadvantaged students – are less confident in their mathematics abilities and tend to prefer more external direction. These students might need additional support in, for example, identifying the intended mathematical ideas embedded in contextualised problems, or describing those ideas to the rest of the class. That said, mathematics teachers should not be discouraged from integrating problem solving in their instruction when teaching weaker classes. Students who are less familiar with mathematics can still participate if the teacher builds a supportive relationship with students, conducts individualised tutoring sessions, builds on what students know, preserves equity among students in the classroom, and makes explicit the desired classroom norms. Formal and informal teacher networks can be useful platforms for sharing experiences and ideas.

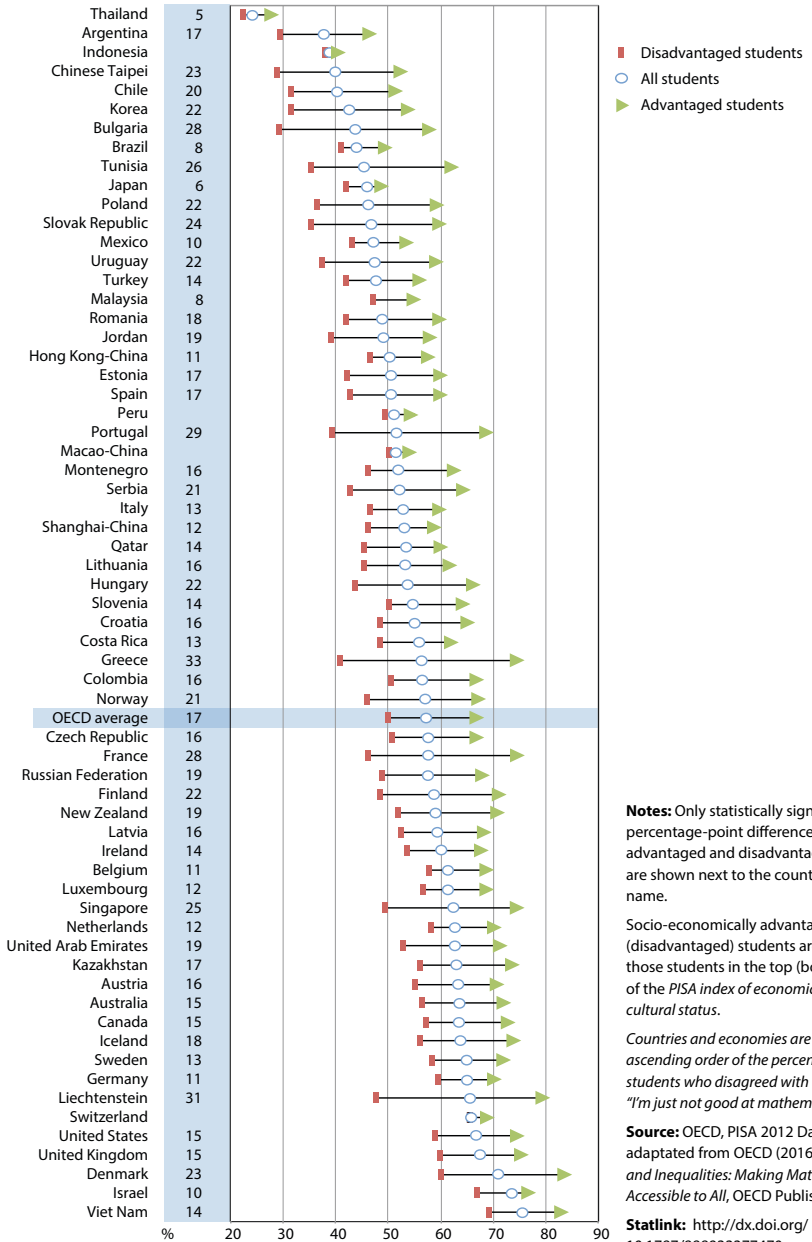
WHAT ELSE DOES A STUDENT'S SOCIO-ECONOMIC STATUS INFLUENCE?

Unfortunately, socio-economic disadvantage has a negative relationship with more than just performance in mathematics. It also influences students' attitudes towards mathematics as a whole. PISA results indicate that disadvantaged students are much more likely than their advantaged peers to have a negative view of their own capabilities in mathematics (Figure 7.3). The feelings of "not being any good" at mathematics are likely linked to these students' poorer performance in the subject.

A student's negative feelings about his or her own mathematical abilities can have a wide-reaching impact. For example, students who have low self-concept in mathematics might also have feelings of anxiety towards the subject. And these negative attitudes can carry forward into adult life, even affecting students' expectations for their future career. Data indicate that students with negative attitudes towards mathematics are also less likely to expect to pursue a career in the sciences as adults. In fact, only 13% of disadvantaged students and 28% of advantaged students across OECD countries reported that they expect to work as a professional in the fields of science, technology, engineering or mathematics.

Figure 7.3 **Mathematics self-concept, by students' socio-economic status**

Percentage of students who reported that they "disagree" or "strongly disagree" with the statement "I am just not good at mathematics"



Notes: Only statistically significant percentage-point differences between advantaged and disadvantaged students are shown next to the country/economy name.

Socio-economically advantaged (disadvantaged) students are defined as those students in the top (bottom) quarter of the *PISA index of economic, social and cultural status*.

Countries and economies are ranked in ascending order of the percentage of all students who disagreed with the statement "I'm just not good at mathematics".

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377470

WHAT CAN TEACHERS DO?

Review the curriculum you are covering for the year. Either with other teachers in your mathematics department or on your own, take note of the curriculum you plan to cover in your mathematics classes and compare it against one or more of the following benchmarks:

– National standards in mathematics for the age level

– Curriculum plans from a neighbouring school (possibly with a more advantaged student population)

– The topics that were covered in previous years in your class or other classes in your school.

Examine the extent to which you adapt coverage and pace your teaching to your students' level of preparation, and think about whether you can find ways to bridge the knowledge gap among the weakest students without watering down the content. Consider how you can improve the transition from one topic to the next and highlight connections across topics. Find areas in which you might streamline content to provide more time to focus on big mathematical ideas and to fill knowledge gaps among your weaker students.

Don't shy away from challenging mathematics topics. All students, regardless of their ability or socio-economic background, should be challenged in mathematics. While your students may not grow up to become mathematicians, they still need to know how to reason mathematically to be successful later in life. Be sensitive to the fact that challenging mathematics problems can increase anxiety in lower-performing students. Offer extra support for those students, but don't avoid difficult topics or problem solving altogether. Try to use tasks and problems that stimulate engagement by referring to experiences your students have personally lived through. Encourage active participation from all students by building supportive relationship with students, conducting individualised tutoring sessions whenever possible, building on what students know, preserving equity among students in the classroom, and making explicit the desired classroom norms.

Make your students aware of the importance of mathematics for their future careers, particularly students from disadvantaged backgrounds. Either with your school's career specialist, others in your department or on your own, spend some time talking to students about which careers rely on mathematics or reasoning skills. Many students can't make the connection between the problems they solve in class and real-life work. If students can better understand how mathematics might benefit their future, they might have more interest in the subject and continue to pursue it after compulsory schooling has ended.

Pure & applied mathematics

Should my teaching emphasise mathematical concepts or how those concepts are applied in the real world?

In many countries, the way mathematics is taught has changed a great deal since today's teachers or parents of school-age children were in school themselves. But glancing through a mathematics textbook today, you are still likely to see mathematics that looks much different from that which might be used in a modern workplace. The mathematics used in the workplace is centred around problem solving, using pragmatic approaches and techniques that are efficient for a variety of tasks. The mathematics that is taught in schools is often thought to be consistent and general, which may explain why students are often unsure how certain concepts they are learning might relate to real-world problems.

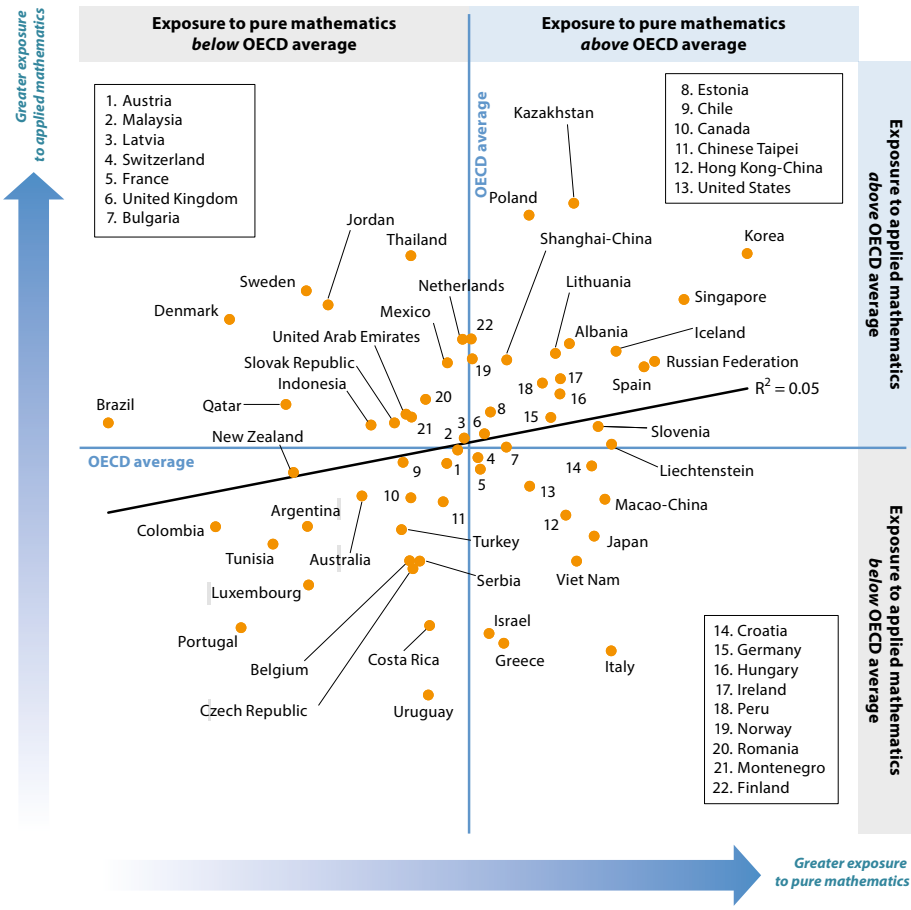
Debates about the best way of teaching mathematics have raged for decades, but it is generally agreed that students should be able to confidently and effortlessly perform some mathematical functions while being able to apply concepts they have learned to new or real-world problems. In other words, students should be competent and flexible in their mathematical abilities. What helps develop these skills in students? PISA data uncover a link between students' exposure to different types of mathematics and students' performance on the PISA assessment.

DO CURRICULA FOCUS ON PURE OR APPLIED MATHEMATICS?

Mathematics educators often disagree as to whether it is more important to teach "pure" mathematics or "applied" mathematics in schools. The teaching of pure, or formal, mathematical concepts focuses on the rules of mathematics separate from the world around us, often emphasising equations or formulas. Applied mathematics, on the other hand, allows learners to apply mathematical concepts and models to solve problems in the real world. It is the mathematics that is mainly used in various branches of science, engineering and technology.

Around the world, mathematics curricula vary by country as to how often students are exposed to these different types of mathematics. Figure 8.1 shows that there are large differences across countries in students' exposure to pure and applied mathematics in school. Across education systems, there is only a weak relationship between average exposure to applied mathematics and average exposure to pure mathematics, which implies that in most countries, the two methods of instruction coexist.

Figure 8.1 **Relationship between students' exposure to pure and applied mathematics, by country**



Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks requiring knowledge of algebra (linear and quadratic equations).

The *index of exposure to applied mathematics* measures student-reported experience with applied mathematical tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All,* OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933376914

HOW DOES EXPOSURE TO PURE AND APPLIED MATHEMATICS RELATE TO STUDENT PERFORMANCE?

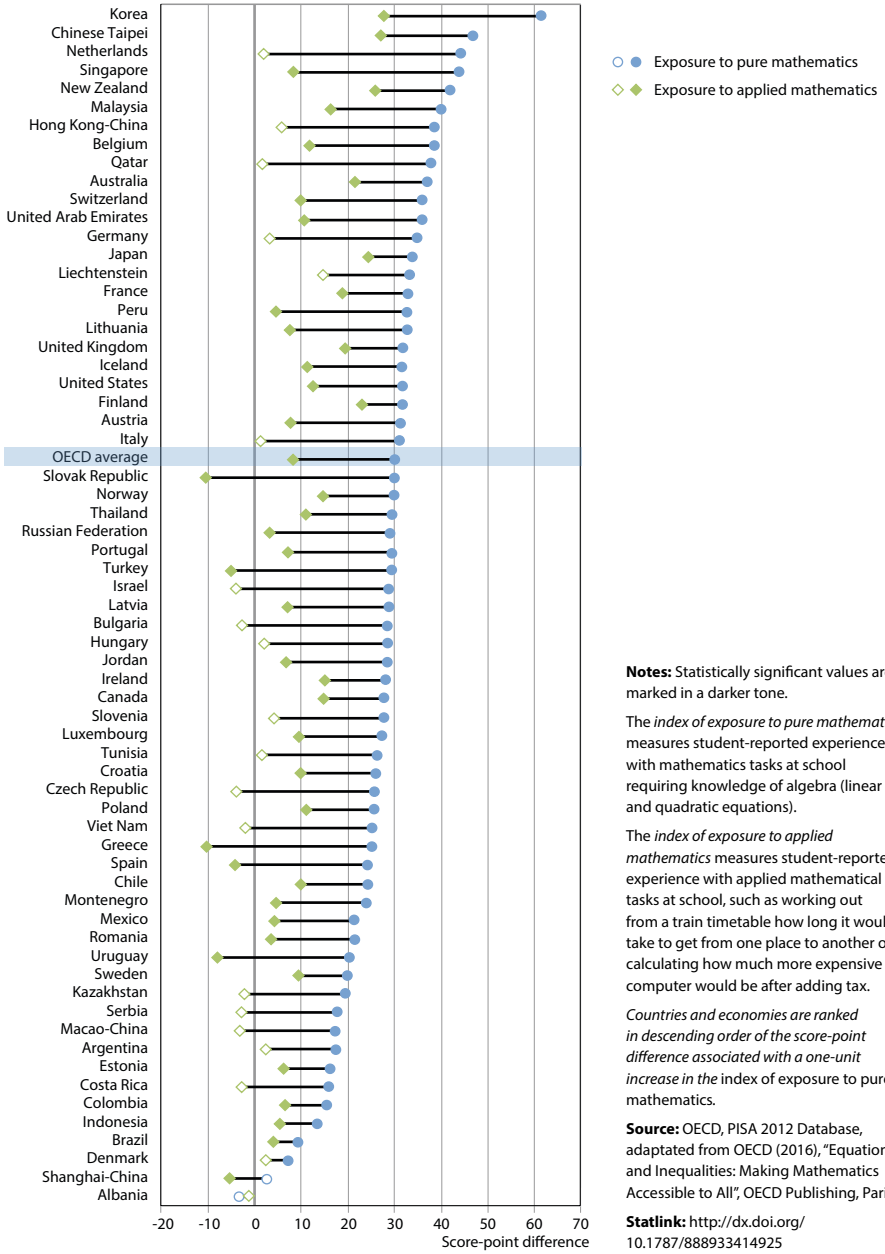
PISA data indicate that more frequent exposure to mathematical concepts and procedures is associated with better mathematics performance (Figure 8.2). The relationship between exposure to pure mathematics and performance is strong and is seen across all PISA countries and economies. Further analyses of the data show that greater exposure to pure mathematics increases the chances that a student will be a top performer in mathematics, and reduces the chances that he or she will be a low performer.

There is also a relationship between exposure to applied mathematics and student performance, but it is not as strong as that with pure mathematics and it isn't observed in all countries. In fact, in some countries, greater exposure to applied mathematics is related to poorer student performance. This could be because, in PISA, students have to report their level of exposure to simple applied mathematics tasks (such as working out from a train timetable how long it would take to get from one place to another), and low-performing students are more likely than high-performing students to have been exposed to these types of tasks.

These simple applied tasks often used at school are routine mathematics tasks "dressed up" in the words of everyday life, and do not require any deep thinking and modelling skill. More involved and multi-faceted problem solving in different contexts is more likely to be beneficial, because it can teach students how to question, make connections and predictions, conceptualise, and construct models to interpret and understand real situations.

Figure 8.2 **Relationship between exposure to pure mathematics and mathematics performance**

Score-point difference in mathematics performance associated with greater exposure to pure or applied mathematics



Notes: Statistically significant values are marked in a darker tone.

The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

The *index of exposure to applied mathematics* measures student-reported experience with applied mathematical tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Countries and economies are ranked in descending order of the score-point difference associated with a one-unit increase in the index of exposure to pure mathematics.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), "Equations and Inequalities: Making Mathematics Accessible to All", OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933414925

WHAT DOES THIS MEAN FOR MY TEACHING?

Knowledge of mathematics terminology, facts and procedures is beneficial for performance on mathematics tasks in general, and especially useful for more challenging problems. But it takes more than content knowledge and practice to be successful at solving problems. Students still need to be able to think and reason mathematically. PISA analyses looked at two difficult problems from the 2012 assessment, one that required students to answer a question using a specific formula (DRIP RATE Question 1) and one that asked students to engage in complex reasoning using a formula that they should know but that is not referred to in the text (REVOLVING DOOR Question 2). The second question required students to be able to model a real situation in mathematical form, which requires a high level of skill in mathematics (see Box 8.1 on the following page for the full text of both problems).

PISA data show that familiarity with mathematical concepts explains a much larger share of the variation in performance on DRIP RATE Question 1 – a question that mostly require the application of procedural knowledge – than on REVOLVING DOOR Question 2, which requires students to engage in more advanced reasoning.

What this suggests is that exposure to formal mathematics can improve students' performance, but only to a point. Just being familiar with mathematical concepts might not be enough to solve problems that require in-depth thinking and reasoning skills.

Several other skills are central to mathematics proficiency. These include the ability to use a wide range of mathematics strategies; the ability to reason using mathematical ideas and to communicate one's reasoning effectively; the ability to use the knowledge and time at one's disposal efficiently; and the disposition to see mathematics as useful and worthwhile, coupled with a belief in one's own abilities. The most effective mathematics teachers cover the fundamental elements of the mathematics curriculum and still find the time to expose students to problems and activities that exercise all of these abilities.

Box 8.1 TWO DIFFICULT ITEMS FROM THE PISA 2012 ASSESSMENT

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, *D*, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

Question 1: DRIP RATE

A nurse wants to double the time an infusion runs for.

Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

.....

.....

SCORING

QUESTION INTENT:

Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant

Mathematical content area: Change and relationships

Context: Occupational

Process: Employ

Full Credit

Explanation describes both the direction of the effect and its size.

It halves

It is half

D will be 50% smaller

D will be half as big

Box 8.1 *continued*

Partial Credit

A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.

D gets smaller [no size]

There's a 50% change [no direction]

D gets bigger by 50% [incorrect direction but correct size]

No Credit

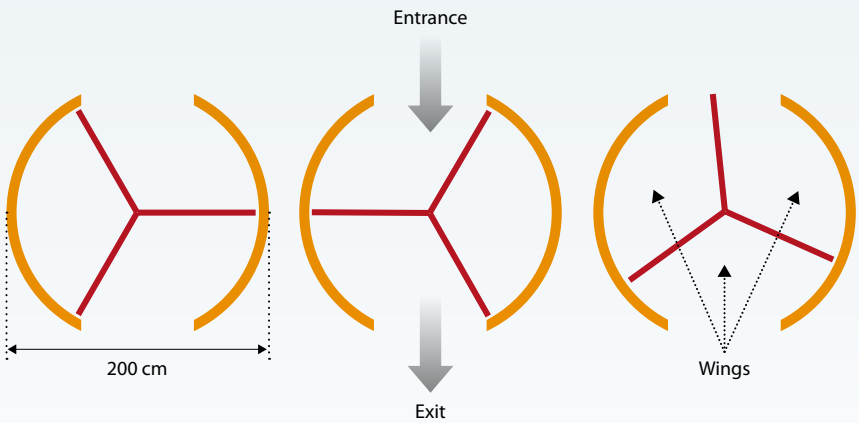
Other responses.

D will also double [both the size and direction are incorrect]

Missing.

REVOLVING DOOR

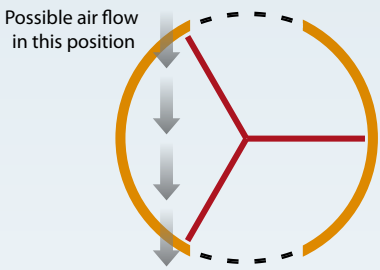
A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Box 8.1 *continued*

Question 2: REVOLVING DOOR

The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram below.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ..... cm

SCORING

QUESTION INTENT:

Description: Interpret a geometrical model of a real life situation to calculate the length of an arc

Mathematical content area: Space and shape

Context: Scientific

Process: Formulate

Question format: Constructed response expert

Difficulty: 840.3 score points

Full Credit

Answers in the range from 103 to 105. [Accept answers calculated as 1/6th of the circumference ($\frac{100\pi}{3}$). Also accept an answer of 100 only if it is clear that this response resulted from using $\pi = 3$. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]

No Credit

Other responses.

209 [states the total size of the openings rather than the size of "each" opening]

Missing.

WHAT CAN TEACHERS DO?

Cover core mathematics ideas in sufficient depth and show how they are related. Students often don't understand how the mathematics they are learning in school might be used in the real world. In addition, the order of topics presented in many mathematics textbooks doesn't make it clear how certain concepts are related to each other. Work with colleagues in your department to teach the curriculum in a way that makes these connections clearer for students. When students understand the relationships among the topics, they stop seeing mathematics as a laundry-list of formulas to memorise, and start to make sense of what they learn. In addition, when students understand why concepts are important for their future life or possible careers, they might become more interested in mathematics.

Don't just cover the fundamentals of the curriculum. Teachers should of course cover the fundamental elements of the mathematics curriculum but still find time to expose students to problems that promote conceptual understanding and activate their cognitive abilities. To do this, it might be worthwhile to increase your use of problem solving as a method of teaching mathematics. Problem solving can be used to introduce core mathematical concepts through lessons involving exploration and discovery. It will prepare students for some of the more complex reasoning that is involved in more difficult mathematics problems.

Provide students with a variety of applied problems to solve. Teaching today's mathematics curricula, which are thought to be general, often makes it challenging for students to apply this knowledge to concrete problems. Students need to be exposed to several different representations of concepts in order to develop the skills needed to translate between the real world and world of mathematics, and vice versa. Give students a variety of problems that includes contextualised problems in which students need to apply knowledge to find a solution to a problem encountered in everyday life. Pedagogies such as project- or problem-based learning present students with real-world problems that they have to solve, often as a team, applying the skills they have just learned.

9

Should I be concerned about my students' attitudes towards mathematics?

Every student has a favourite subject in school – and a least favourite. The reasons for certain feelings about school subjects might have to do with the teacher, the teaching or a student's performance in the subject, among other factors. A student's attitude towards a particular subject influences their motivation, their success in school and their future career choices. Data from PISA indicate that both positive attitudes towards mathematics and a student's confidence in his or her own abilities in mathematics are closely linked to the student's problem-solving abilities. In short, teachers should be concerned about students' attitudes towards mathematics and should take steps to increase students' positive feelings, self-confidence and interest in mathematics when needed.

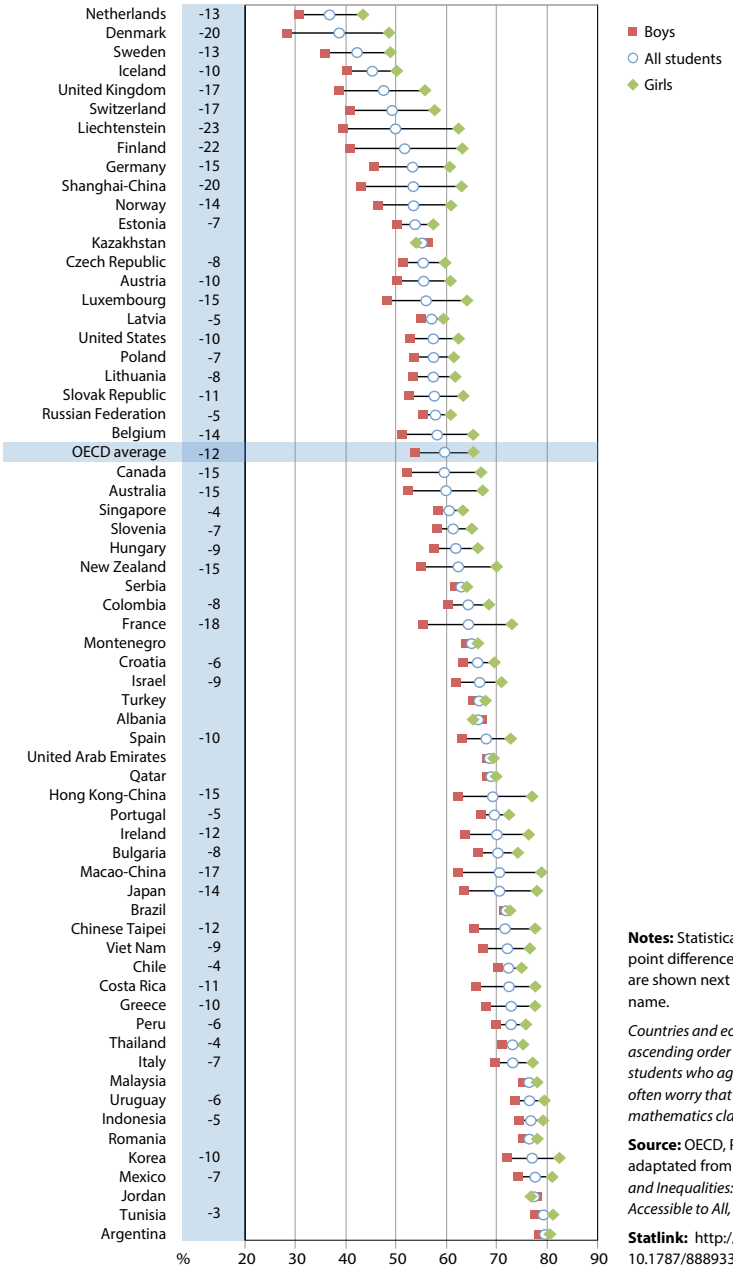
HOW DO STUDENTS FEEL ABOUT MATHEMATICS?

It's safe to say that for the majority of students across PISA countries, mathematics is not their favourite subject. Only 38% of students reported that they study mathematics because they enjoy it. On average across OECD countries, 43% of students believe that they are not good at mathematics. These negative feelings about their own mathematics ability can shape students' actions in mathematics, especially when they are confronted with challenging problems. In addition, students with negative views about their own abilities are more likely to report feeling anxious towards mathematics. Anxiety is detrimental as it often prevents students from demonstrating their real abilities in mathematics. On average, 59% of students often worry that mathematics classes will be difficult for them. Among these students, girls are more likely than boys to report anxiety towards mathematics. As Figure 9.1 shows, across OECD countries, 65% of girls are concerned that they will have difficulties in mathematics classes, as compared with 54% of boys.

PISA data also show that girls are less likely than boys to report that they intend to take additional mathematics courses after the end of compulsory schooling and pursue a career in science, mathematics, technology or engineering. If we want to encourage more girls into mathematics-related fields, we need to pay attention to their attitudes towards mathematics in school.

Figure 9.1 **Mathematics anxiety, by gender**

Percentage of students who reported that they "agree" or "strongly agree" with the statement "I often worry that it will be difficult for me in mathematics classes"



Notes: Statistically significant percentage-point differences between boys and girls are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of all students who agreed with the statement "I often worry that it will be difficult for me in mathematics classes".

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All,* OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377487

A student's feelings about mathematics have an impact on more than just their current performance in mathematics classes. A lack of confidence in their own mathematics abilities could influence students' choices for their future education or career. Given this, many countries have started to include developing positive attitudes towards mathematics as an objective of their national mathematics curriculum. Box 9.1 provides some examples from around the world.

Box 9.1 DEVELOPING POSITIVE MATHEMATICS ATTITUDES AS A CURRICULUM OBJECTIVE

Some countries have incorporated improving students' attitudes towards mathematics in recent revisions to their national mathematics curricula. The national mathematics curricula in Australia, Hong Kong-China, Korea and Singapore include specific text about developing positive student attitudes towards mathematics, in addition to focusing on mathematics skills. While these countries produce some of the world's highest-performing students in mathematics, their governments recognise that performance alone is not enough. In Korea, for example, while international assessments have consistently demonstrated high achievement in mathematics, students also expressed little interest and low self-confidence in mathematics. The government of Korea took measures to reduce and rearrange some of the content in their national mathematics curriculum in order to provide more time for creative and self-directed activities so that students might become more interested and motivated in their mathematics studies.[1]

WHAT CAN INFLUENCE STUDENTS' FEELINGS TOWARDS MATHEMATICS?

We have learned that students' feelings towards mathematics can be shaped by their socio-economic status, and that gender also plays a role in whether students might feel anxious towards mathematics. Teachers should certainly be aware of this as they teach their students, but there are other factors related to teaching practice that are influential as well. We discussed the benefits and drawbacks to student performance of exposing students to applied and pure mathematics. Data also suggest that, across students of similar ability, those who are more exposed to complex mathematics are more likely to worry that they will get poor grades in mathematics class. In addition, students who perform worse in mathematics reported greater anxiety with more exposure to complex mathematical concepts. This makes sense: if a student is already performing poorly in mathematics, he or she might naturally worry when faced with more difficult problems.

Also not surprisingly, if students are tested on something they don't feel they have practiced very often in class, they are more likely to feel anxious. As Figure 9.2

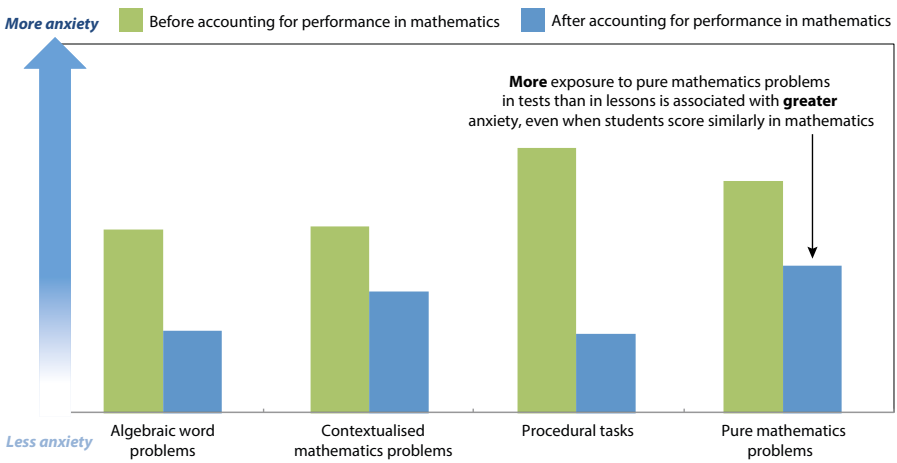
shows, on average across countries, students who are more frequently exposed to a certain type of mathematics task in tests rather than in their classes are more anxious than students who have more opportunities to practice the task during class before sitting the test.

WHAT CAN HELP?

First, it is important to use competition and rankings within the class judiciously, because students' beliefs in their own abilities are strongly influenced by social comparisons with their peers. In almost all of the countries surveyed, students who reported less familiarity with mathematics than the average student at their

Figure 9.2 **Mathematics anxiety and the mismatch between what is taught and what is tested**

Change in students' anxiety towards mathematics associated with more frequent exposure to mathematics tasks during tests than during lessons, OECD average



Notes: All values are statistically significant.

The figure compares students who are exposed less frequently to mathematics tasks in tests than in lessons to students who are exposed more frequently to mathematics tasks in tests than/as in lessons.

The *index of mathematics anxiety* is based on the degree to which students agreed with the statements: I often worry that it will be difficult for me in mathematics classes; I get very tense when I have to do mathematics homework; I get very nervous doing mathematics problems; I feel helpless when doing a mathematics problem; and I worry that I will get poor marks in mathematics.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All,* OECD Publishing, Paris.

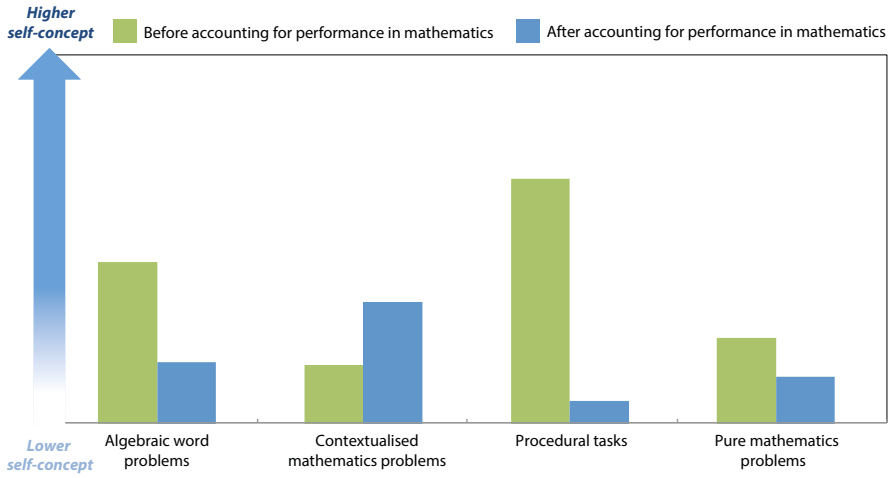
Statlink: http://dx.doi.org/10.1787/888933377548

school also reported feeling less confident in their mathematical abilities. Second, teachers might consider increasing their students' exposure to problem solving in real–world contexts. As Figure 9.3 indicates, students who reported that they are frequently exposed to the kinds of contextualised problems prevalent in applied mathematics, such as those used in PISA, tend to be more positive about their own capabilities in mathematics.

Understandably, traditional assessments, such as timed tests, can also impose additional stress on students. Research shows that when students are required to take mathematics tests under timed, high-stakes conditions, their anxiety can adversely affect their performance on these tests.[2] Offering students a chance

Figure 9.3 **Relationship between exposure to mathematics tasks in class and students' self-concept**

Change in students' self-concept in mathematics associated with frequent exposure to mathematics tasks during lessons, OECD average



Notes: All values are statistically significant.

The *index of mathematics self-concept* is based on the degree to which students agree with the statements: I'm just not good in mathematics; I get good grades in mathematics; I learn mathematics quickly; I have always believed that mathematics is one of my best subjects; and In my mathematics class, I understand even the most difficult work.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All,* OECD Publishing, Paris.

Statlink: http://dx.doi.org/10.1787/888933377533

to practice for high-stakes tests in less formal – and potentially less stressful – circumstances is already a relatively common practice in many schools and can help ease anxiety. Formative assessments, or providing students ongoing, informal feedback on their progress, can also help alleviate some students' worry about mathematics. PISA data indicate that some common communication strategies in class, such as telling students what they have to learn, what is expected of them and informing them of their progress, are related to lower levels of mathematics anxiety among students.

PISA data also suggests that the way a teacher teaches can influence how a student feels about mathematics. For example, students who reported that their teachers encourage them to work in small groups have more confidence in their own capabilities in mathematics. When computers are used in mathematics lessons, students reported greater motivation for learning mathematics. Thus, although some student characteristics might be responsible for students' attitudes towards mathematics, teachers' practices can be influential too.

WHAT CAN TEACHERS DO?

In addition to what you teach, think about whom you teach and how you teach. This is relevant for all students, but especially for students from disadvantaged backgrounds and girls. Teachers should be aware of which elements of their teaching might trigger anxiety or reduce their students' self-confidence and consider alternative teaching methods. These could involve the use of more real-world applications for the mathematical concepts they teach, as discussed here. Consider also making mathematics personally relevant to learners by providing them with problems that relate to their own interests or experiences. Using problems that are relevant to students helps them see a reason for learning a certain topic or concept and may increase their motivation for learning it.

Prepare students for what to expect on math tests. Having students sit practice tests in advance of sitting high-stakes exams is not new, but it can help students be more comfortable in the conditions in which they sit these exams. Teachers and schools might also consider reducing the time pressure around high-stakes exams for certain students, when possible. In addition, explaining to students what they can expect on exams and providing clear feedback on their progress in mathematics can also reduce their anxiety.

Explore innovative teaching tools for mathematics. Technology, including dynamic graphical, numerical and visual technology applications, can help students visualise mathematics problems while increasing their motivation or interest in the topic. Online teacher forums include many free mathematics tools, and common desktop software packages used in schools include mathematics formula tools and even advanced graphing functions. But in all school systems that participated in the TALIS survey, teachers cited improving their ICT skills as one of the most important priorities for their professional development. Teachers need to be confident, themselves, in using these tools to ensure that they are adding value to the concepts being presented, rather than simply providing an interesting distraction for students.

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Lessons drawn

10

What can teachers learn from PISA?

With school budgets ever shrinking in many countries, it is often difficult for teachers to participate in professional development activities. The teacher and school leadership have to factor in time away from class, the cost of a teacher to cover the day's classes, course fees and travel costs, where appropriate. Add to that the teacher's own time preparing for a substitute teacher, worries about what's happening while you're away and trying to get students back on track when you return, and it's understandable why teachers and their leadership would want to minimise the time that teachers take away from their teaching duties.

What this means is that teachers spend the vast majority of their time in school – their own school – and don't often get to experience and learn from the work of other professions outside of teaching. Indeed, the TALIS 2013 survey indicated that only 13% of teachers, on average across countries, had participated in observation visits to businesses or other local, non-education organisations, and only 14% had attended in-service training courses on site in a business or other public organisation. Many teachers today increasingly rely on free resources (such as this guide) that they can access on line, from home or school, and that offer lesson plans or other guidance to help teachers with their planning, their own professional development or with their future lesson activities.

This guide was created to provide mathematics teachers with professional development in the form of recommendations and ideas for your teaching based on evidence from the PISA mathematics assessment. But PISA is more than just the assessment, charts and league tables. The OECD has over 15 years of experience working with research partners, policy makers, schools and teachers from over 65 countries around the world to create the PISA assessment and its accompanying questionnaires. Years of work go into developing these materials and producing the reports that cover each cycle.

Thus, while teachers might not be able to visit the OECD, or our research partners, for professional development to learn first-hand how these vast international assessments are designed and conducted, this chapter provides some of this

information for you. There are some useful lessons that teachers and schools can draw from the thinking behind the development of the PISA mathematics assessment. These lessons are offered as a complement to what you learn in professional development activities, particularly in school-based activities, and within professional communities of practice.[1]

WHAT HAS PISA TAUGHT US?

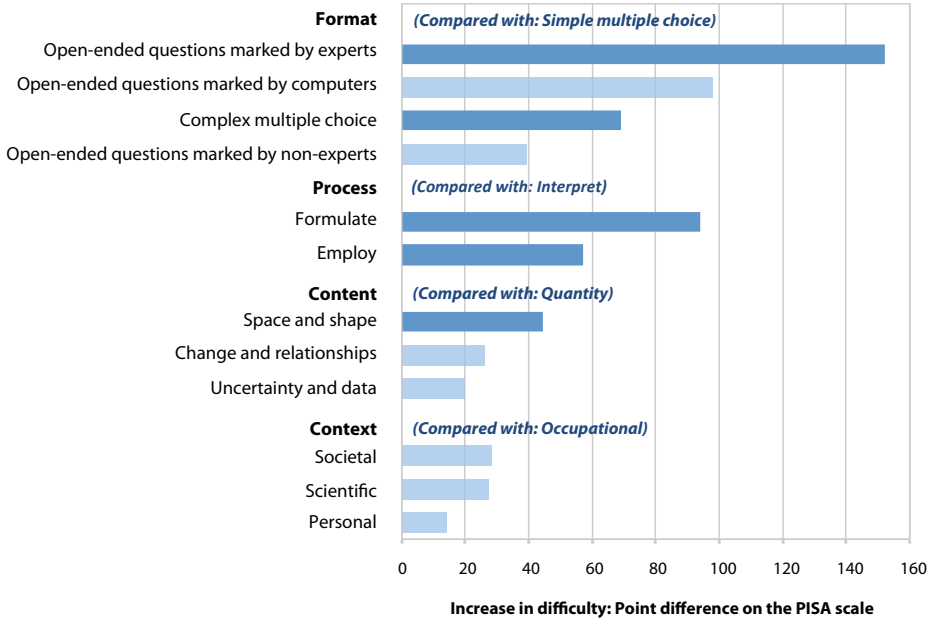
Develop balanced assessments. Measuring a broad concept like mathematical literacy using an international standardised test requires a wide variety of questions. These questions need to be asked in different formats, be located in various contexts and be related to several content areas. In order to assess the proficiency of a student at the end of schooling, assessments need to cover the full mathematics modelling cycle (formulate, employ and interpret) as well as the range of skills for a "typical" 15-year-old.

PISA results show that the types of questions on these assessments matter, as does the design of the assessment as a whole. For example, open-ended questions in PISA, especially those coded by experts, are typically more difficult for students than multiple-choice questions (Figure 10.1). Balanced assessments also help us learn more about student performance across a wide range of problems and the factors that influence performance. PISA data tell us that students who mainly use memorisation strategies in their learning tend to perform worse on mathematics questions that require formulating a problem, compared with problems that ask students to use formulas or interpret results. We can only learn this by including both types of mathematics problems on the assessment.

RECOMMENDATION

Make sure your teaching and assessments are balanced so that students can develop all the skills they will need for their future learning. Use multiple types of assessments, including oral tests, collaborative problem-solving and long-term projects, in addition to traditional written exams. Even standardised tests can be used occasionally to compare the performance of your class with students from other classes, schools, districts, cities and countries. Take advantage of questions from PISA that have been made public by the OECD or from PISA for Schools exams to serve this purpose.

Figure 10.1 **Describing the difficulty of PISA mathematics items**



Note: Statistically significant regression coefficients are marked in a darker tone.

Source: OECD, PISA 2012 Database, adapted from Echazarra, A. et al. (2016), "How teachers teach and students learn: Successful strategies for school", OECD Education Working Paper, no. 130.

Statlink: http://dx.doi.org/10.1787/888933414931

Focus on students' abilities and skills. PISA focuses on the competencies that 15-year-old students are likely to need in the future and evaluates what they can do with what they have learned in school. For mathematics in particular, these skills include representing, devising strategies, mathematising, reasoning and developing arguments, using symbolic, formal and technical language and operations and using mathematical tools. But the skills that are important for today's learners go beyond those that are mathematics-specific; they include adaptability, communication, problem solving and using information and communication technologies. All of these skills are necessary to be successful in mathematics, and many of them apply to other subjects as well.

RECOMMENDATION

Yes, teachers are constrained by curricula and standards in a way that PISA is not, but it's still worth asking yourself: "What is important for citizens to know and be able to do in situations that involve mathematics?" Even within the confines of your curriculum, this kind of thinking could help you decide which topics to present to your students – and how to present them – so that your students are better prepared for future learning and life outside of school. Many teachers around the world already emphasise abilities and skills in mathematics with the purpose of assessing both students' content knowledge (what they know) and applied knowledge (what they can do with what they know). Again, reading some assessment questions released by PISA might give you additional ideas for your class.

Be fair. This is not intended to sound pedantic; teachers don't need the OECD to tell them that they need to treat students fairly. But as this guide has shown, sometimes the way a teacher teaches or develops assessments could give advantages to a particular group of students without the teacher realising it. The PISA assessments are carefully designed to avoid giving an advantage to a particular education system or a social group. Mathematics problems, for instance, cannot be set in contexts that are unfamiliar to some young people (for example, rural students may not be familiar with subway systems). To guarantee that no given location is at a disadvantage, PISA asked countries to select the questions they believed would be easier for their students in the PISA 2012 test, based on context. The student performance on these questions was then compared with their overall performance on the test. No consistent bias was found; some countries performed better in the questions they considered easier for their students, but many others performed worse in their preferred questions.

RECOMMENDATION

Teach and assess students in ways that are fair and inclusive for everyone, regardless of gender, socio-economic background or ability. This guide provides several different recommendations related to fairness based on various student characteristics; but teaching in a more inclusive manner can be as simple as explaining content using different perspectives, evaluating students in different ways, and always taking into consideration students' background.

Collaborate with others. The OECD could not design and implement an international assessment on such a scale for over 15 years by working alone from its offices in Paris. PISA is the result of the ongoing collaboration among the OECD, national governments, research partners and education experts from all over the world. Decisions about the scope and nature of the PISA assessments and the background information collected are taken by leading experts in participating countries. Governments, guided by the OECD, oversee these decisions based on shared, policy-driven interests.

RECOMMENDATION

Teachers do not teach in a vacuum. Every day you have classrooms of students who can inform your teaching, and colleagues down the hall or in the staff room who have different – and perhaps complementary – expertise and experience. In a supportive school environment, you are not solely responsible for your students' success. Listen to your students, collaborate with other teachers, participate in school decision-making, communicate with parents and learn from experts in your field.

Innovate, innovate, innovate. Since the first cycle of PISA in 2000, the OECD has never stopped learning and innovating on the original PISA design. New subject domains have been added, such as problem-solving and collaborative problem-solving, digital literacy and financial literacy. Different background questionnaires have been created to learn more about cross-curricular skills, students' career expectations, whether – and what – students read for enjoyment, student well-being and teachers themselves. With vast amounts of data from all of this work, the OECD also continually explores new ways to disseminate results and discovers new questions about learning and schooling that beg for answers. PISA has also adapted to the digital world by transitioning from a paper-based test to an interactive digital assessment, despite the considerable difficulties involved in making the switch.

RECOMMENDATION

Don't let the constraints of a national curriculum or national exams limit your or your students' creativity. It is possible to innovate with tools and pedagogies. New approaches to teaching are tried and tested all the time, with varying degrees of success. If you're nervous, read up on strategies that have been successful for other teachers but might be new to you, or participate in innovation networks. Once you're more confident with the risks and rewards associated with innovation in teaching, you'll be the one developing new strategies and resources for your colleagues to try.

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ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

The OECD is a unique forum where governments work together to address the economic, social and environmental challenges of globalisation. The OECD is also at the forefront of efforts to understand and to help governments respond to new developments and concerns, such as corporate governance, the information economy and the challenges of an ageing population. The Organisation provides a setting where governments can compare policy experiences, seek answers to common problems, identify good practice and work to co-ordinate domestic and international policies.

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Ten Questions for Mathematics Teachers
… and how PISA can help answer them

Every three years, the Programme for International Student Assessment, better known as PISA, evaluates 15 year-old students around the world to determine how well their education system has prepared them for life after compulsory schooling. Once the results are published, the media rush to compare their countries' positions in the international league tables. Government policy makers, journalists and academic researchers mine the report to find out how successful education systems elicit the best performance from their students while making access to high-quality education more equitable. But sometimes the key messages don't make it back to the teachers who are preparing their country's students every day.

Ten Questions for Mathematics Teachers… and how PISA can help answer them aims to change that. This report delves into topics such as, "How much should I encourage my students to be responsible for their own learning in mathematics?" or "As a mathematics teacher, how important is the relationship I have with my students?". It gives teachers timely and relevant data and analyses that can help them reflect on their teaching strategies and how students learn.

Content:

Introduction: A teacher's guide to mathematics teaching and learning
Question 1: How much should I direct student learning in my mathematics classes?
Question 2: Are some mathematics teaching methods more effective than others?
Question 3: As a mathematics teacher, how important is the relationship I have with my students?
Question 4: What do we know about memorisation and learning mathematics?
Question 5: Can I help my students learn how to learn mathematics?
Question 6: Should I encourage students to use their creativity in mathematics?
Question 7: Do students' backgrounds influence how they learn mathematics?
Question 8: Should my teaching emphasise mathematical concepts or how those concepts are applied in the real world?
Question 9: Should I be concerned about my students' attitudes towards mathematics?
Question 10: What can teachers learn from PISA?

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