Global Teaching InSights

Technical Report

Section I: Study background

3 Curriculum mapping

Eckhard Klieme

This chapter outlines the process involved in finding a common focal teaching and learning unit in each participating country's/economy's mathematics curriculum. The chapter has three sections: the first explains why the Study did focus on teaching and learning within a single subject matter unit of quadratic equations; the second describes the process of defining and developing the focal unit for the Study; and the third explains how measurement instruments for the focal unit were developed.

Introduction

The OECD *Global Teaching InSights: A video study of teaching* (results from the TALIS Video Study project, and is hereafter cited in this chapter as "the Study" or "GTI") uses a single topic to study teaching and learning across multiple classrooms. A focus on teaching and learning within a single subject matter unit means that the Study's measures of teaching and learning (test, questionnaires, video observations) are tightly aligned. This approach provides a greater power for testing the relationships between teaching practices and student learning.

This chapter explains the challenge, as well as the process to define quadratic equations as the focus of the Study. The focus on quadratic equations allows for more instructionally sensitive measures of teaching and student outcomes in an international setting. In what follows, the chapter describes the instrument development for the focal unit across participating countries/economies.

A focus on learning within a single subject matter unit

Global Teaching InSights (resulting from the TALIS Video Study project) uses a "single topic, micro-genetic design" to study content-specific teaching and learning across multiple classrooms. By selecting a single topic to be taught in a large sample of participating classrooms, pre- and post-tests can be designed to be instructionally sensitive, observational protocols can be tailored to the content, and finally the relationship between teaching and student learning can be analysed in detail, with appropriate control of pre-knowledge and other input factors. By focusing a single unit of instruction, which covers just a few lessons, the processes generating student learning can be studied on a "micro" level – whereas large-scale assessments such as PISA provide "macro" views of teaching and learning practices.

While this study design potentially provides greater power for testing the relationships between teaching practices, classroom practices and student learning outcomes at an international scale (Praetorius et al., 2019[1]), so far it has only been used to study the relationship between teaching and learning within a single country (e.g. (Decristan et al., 2015[2]): teaching on floating and sinking in German primary schools) or to compare two countries (Klieme, Pauli and Reusser, 2009[3]): teaching the Pythagorean theorem in Germany and Switzerland or across three countries (Fischer et al., 2014[4]): teaching the concept of electricity in Finland, Germany and Switzerland).

The challenge in a global context is finding a common curricular topic. A curriculum mapping was thus undertaken at the outset of the Study to identify potential focal topics on the basis of the protocols of international assessments, previous comparative work on mathematics curricula, the curriculum for mathematics in the participating countries and economies, and textbooks used in participating countries and economies. The criteria for the selection of a focal topic for the Study included:

- inclusion in the curriculum in all countries for essentially 100% of students
- implementation close to age 15, in grade 8 or higher
- consists of a reasonable number of lessons (6-12)
- requires a pedagogical approach with a well-defined starting point that introduces a core mathematical concept, while still having opportunities for rich mathematical activities (e.g. application, modelling and transfer), for deep mathematical thinking (e.g. argumentation, proof) and for working with different representations.

Decision on focal unit based on curriculum mapping

The following five steps describe how the teaching and learning focal topic for Global Teaching InSights Study was determined:

Step 1: Identification of potential mathematics units

A total of 23 potential focal units for the Study were initially identified on the basis of content rubrics from international assessments, previous comparative work on mathematics curricula and the most comprehensive U.S. Common Core standards for mathematics (Common Core State Standards Initiative, 2016[5]). These included units from algebra, geometry, numbers, functions and probability.

Step 2: Identification of common mathematics units across participating countries/economies

Each participating country/economy indicated whether, when and for how many lessons these 23 units are included in their national mathematics curriculum, learning standards and textbook guidelines. Only ten of the 23 units are part of the compulsory mathematics curriculum, implemented with students close to the age of 15 years, and taught with a reasonable number of lessons in all participating countries/economies.

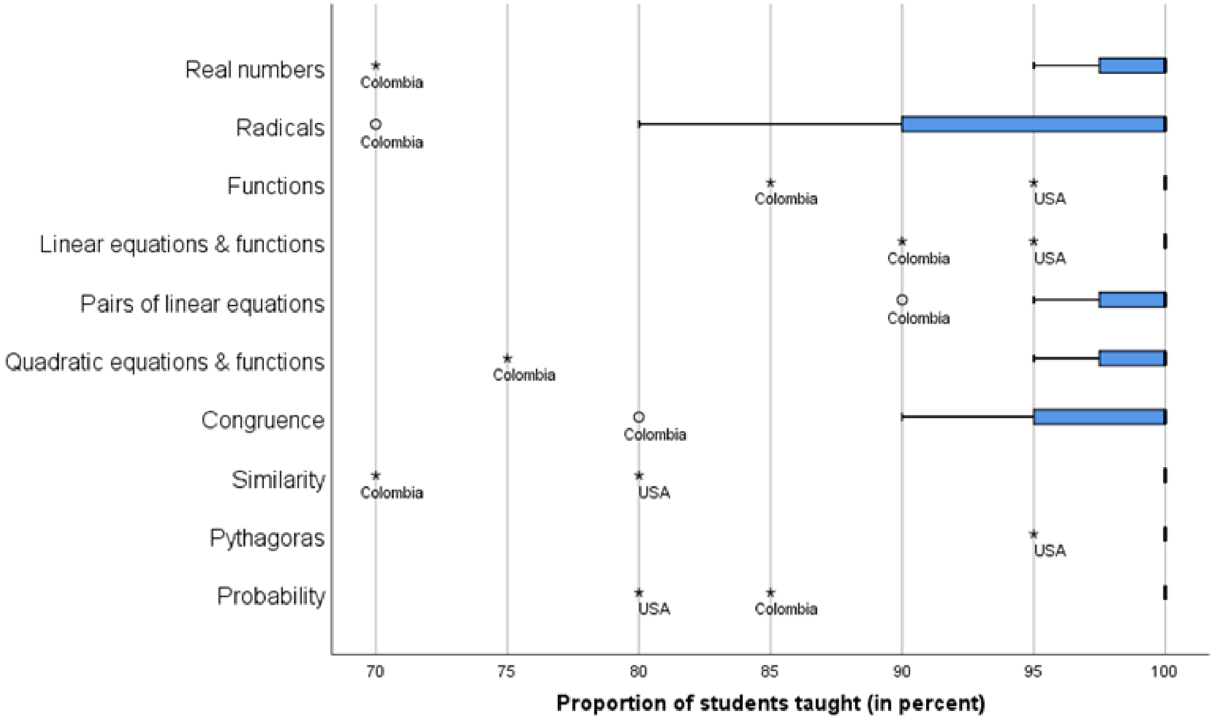
These ten topics and their respective areas of mathematics are: numbers (real numbers, radicals); functions (introducing the concept of functions); algebra (linear equations and functions, pairs of simultaneous linear equations, quadratic equations with or without functions); geometry (congruent figures, similarity, Pythagorean theorem); and statistics and probability (chance processes and the concept of probability). Figure 3.1 to Figure 3.4 show the variation of the above mentioned criteria across participating countries and economies for each of the ten mathematic topics. Most countries fall within the respective boxes, while outliers are identified as such.

Then, consideration was given to pedagogical aspects such as whether the topic requires a well-defined starting point that introduces a core mathematical concept, whilst still having opportunities for rich mathematical activities (e.g. application, modelling and transfer), for deep mathematical thinking (e.g. argumentation, proof), and for working with different representations.

Step 3: Selection of two mathematics topics: Linear and quadratic equations

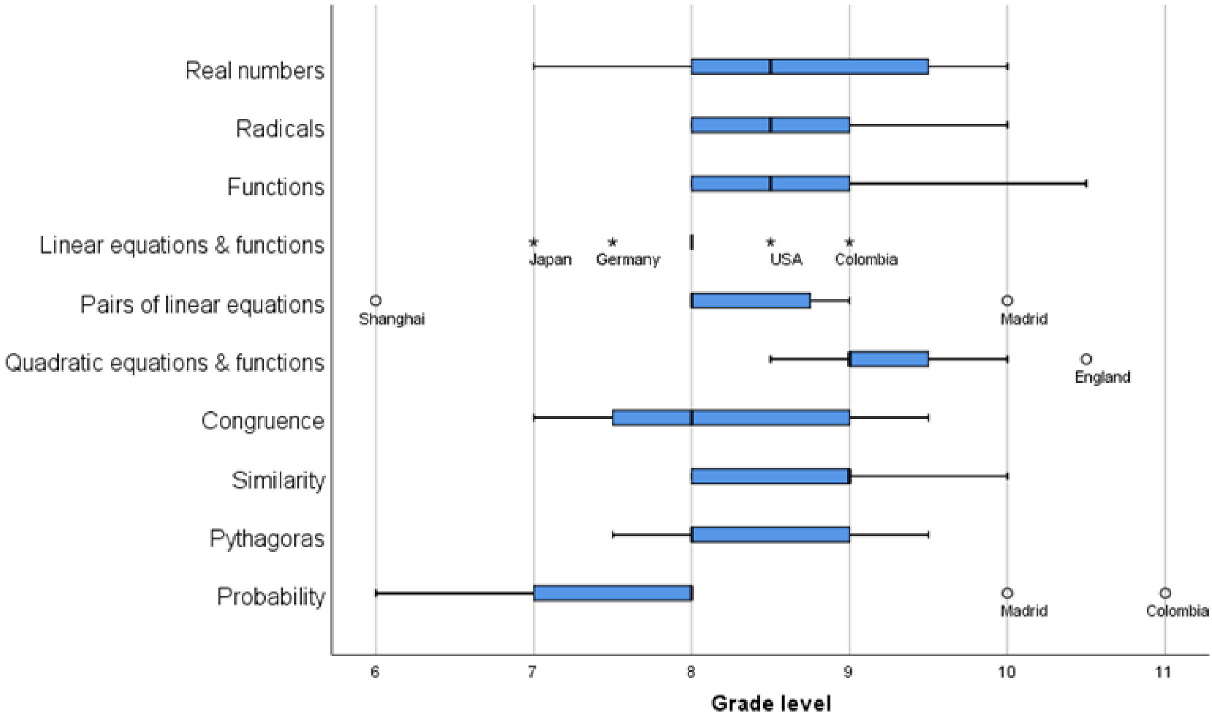
At the end of the curriculum-mapping exercise, two algebra topics were selected: pairs of simultaneous linear equations and quadratic equations. While there was a large variation across countries in the coverage of topics in other areas of mathematics, algebra was considered one of the most important and common areas in lower secondary mathematics curriculum. The broader topic of “quadratic equations and functions” was narrowed down to focus on algebraic content, with or without some use of quadratic functions in between, to allow for better comparability across countries/economies.

Figure 3.1. Percentage of students who are studying selected mathematics content any time in secondary school: Variation across participating countries/economies by topic



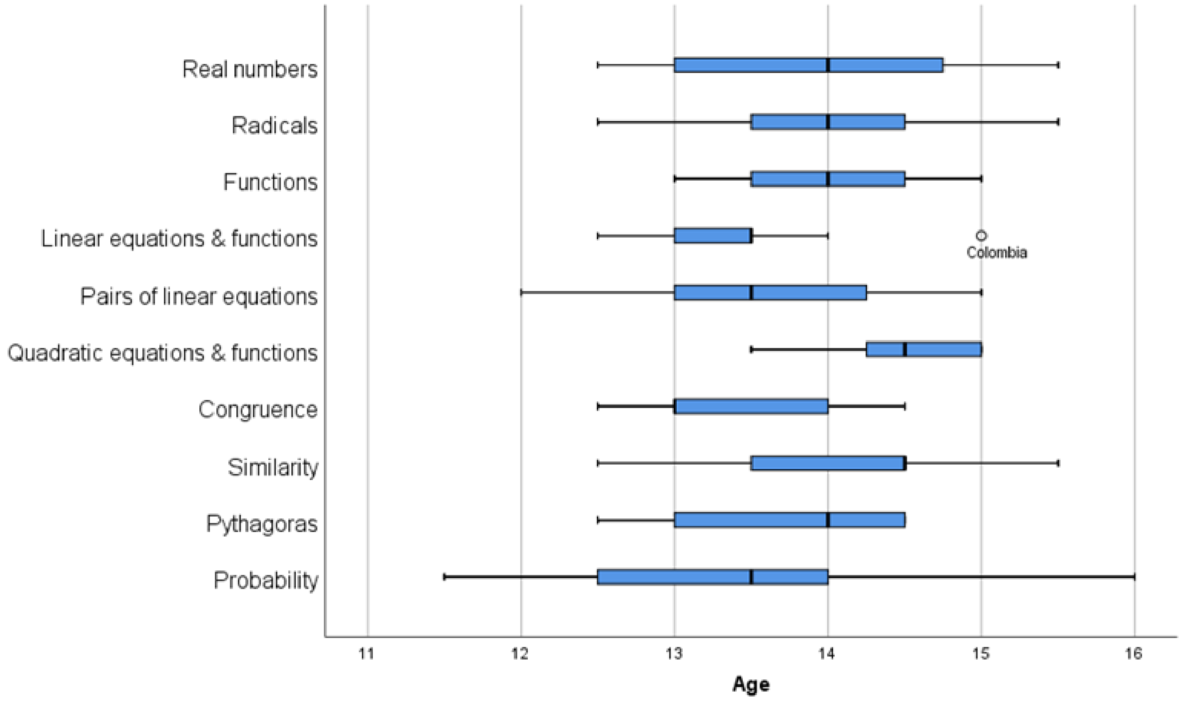
Source: OECD, Global Teaching InSights Database.

Figure 3.2. Students' grade level when taught selected mathematics content: Variation across participating countries/economies by topic



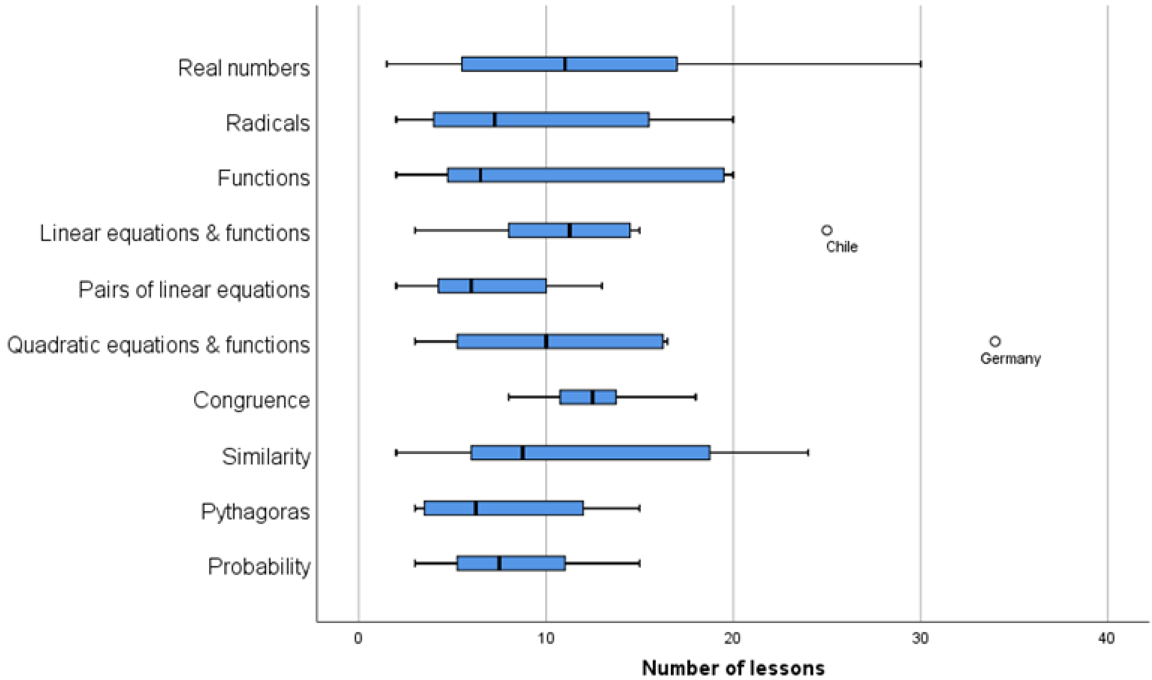
Note: England reported "year" instead of "grade"; year 11 is equivalent to grade 10 in other countries.
Source: OECD, Global Teaching InSights Database.

Figure 3.3. Student age when taught selected mathematics content: Variation across participating countries/economies by topic



Source: OECD, Global Teaching InSights Database.

Figure 3.4. Number of lessons in the target grade level for teaching selected mathematics content: Variation across participating countries/economies by topic



Source: OECD, Global Teaching InSights Database.

Step 4: Looking into the differences in the implementation of two mathematics topics

Once the two potential topics were identified, the work focused on scoping differences on how linear and quadratic equations and functions are implemented in each participating country and economy. National Project Managers (NPMs) provided details on what subtopics are covered in each grade level of the national curriculum as well as how are these framed. Where national curricula did not exist or were too vague, exemplary curriculum materials were used as additional source of information. For example, NPMs were given the following list of subtopics for quadratic equations and were asked to indicate whether, when and for how many lessons or hours these were taught, and to provide illustrative textbook examples of them.

0. Pre-Knowledge: Binomial formula $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$, $(a+b)(a-b) = a^2 - b^2$
1. Understanding $ax^2 + bx + c = 0$ as a specific type of equation
2. Distinguishing different cases depending on values of a, b, c
3. Transforming different kinds of quadratic equations into the standard form
4. Knowing and applying the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
5. Knowing and applying the formula for the reduced quadratic equation $x = -p/2 \pm \sqrt{(p^2/4 - q)}$
6. Vieta's rule: factorising by inspection
7. Understanding different cases: no solution, one solution, two solutions
8. Applying quadratic equations to everyday problems
9. Graphing functions of type $y = ax^2 + bx + c$
10. Types of parabolic functions (depending on values of a, b, c)
11. Solving applied problems graphically
12. Understanding that the solutions of a quadratic equation are the roots of the corresponding quadratic function

Table 3.1. Curriculum differences in teaching quadratic equations across participating countries and economies

Country/economy	Grade level	Age	Number of lessons[1] per term	Time of year	Subtopics included[2]
B-M-V (Chile)	11	16	13 15 12	May May-June June	9, 10, 11 12, 1, 2, 3 4, 5, 6, 8
Colombia	9	14	(16) (24)	July-Nov[3] July-Nov[4]	1, 4 7, 9, 2, 3
England (UK)	9, 10 or 11		(20-24) (4)	anytime afterwards	1, 2, 6, 3[5], 4[5], 7, 8 or 11[6] 9, 11
Germany*	9 or 10	14-15	15	Nov-Dec[7]	1 or 9, 12, 3, 4, 8 or 11
K-S-T (Japan)	9	14	13 11	June-July Sept-Oct	1, 2, 3, 4, 5, 8 9[8], 11
Madrid (Spain)	9	14	6 4	November April	2, 3, 4, 6, 7, 8 9, 10, 11, 12
Mexico	9	14	8-10 6	Oct-Nov January	1, 2, 3, 6, 8 4, 7, 8

Country/economy	Grade level	Age	Number of lessons[1] per term	Time of year	Subtopics included[2]
Shanghai (China)	8	14	10	Sept-Oct	1, 3, 4, 7, 8
United States*	9	14	13 13	second quarter second quarter	9, 10, 11 1, 2, 3, 4, 7, 8

Notes: Blue font represents other content, taught before or within the same school year; all others (in black font) represent the focal unit, quadratic equations, to be included in the Study.
* The United States only participated on the Study's pilot phase.
[1] Numbers in brackets refer to hours, not lessons; [2] Subtopics ordered as implemented in the curriculum; [3] >85% of population; [4] 60-75% of population; [5] to higher level students only; [6] Kinematics; [7] >85% of population, excluding lowest track; [8] $y= ax^2$ only.
*Germany refers to a convenience sample of volunteer schools.
B-M-V (Chile) refers to Bíobio, Metropolitana and Valparaíso.
Source: OECD, Global Teaching InSights Database.

Table 3.1 provides an overview of the curriculum data provided by NPMs for the grade level where most of the sub-topics were taught. Data may differ from Figures 3.1 to 3.4, since there was a restriction to mainly algebraic content. In England and Germany*[1], the grade level when quadratic equations are introduced is determined on the school and state level, respectively; therefore, adjacent grade levels were included. A similar analysis was provided for the alternative topic, simultaneous linear equations.

Step 5: Decision on the focal topic for the Study

Quadratic equations was chosen as the focal topic. The decision was made after considering that the age of students being taught quadratic equations would be closer to the PISA-target age of 15, there was less variation between countries on this topic compared to the teaching of simultaneous linear equations, and a common list of subtopics could be identified for quadratic equations. Also, students in PISA 2012 reported high levels of exposure to algebraic tasks, and experience with algebra, in particular with quadratic equations, was rather strongly correlated with achievement (OECD, 2016[6]).

The curriculum mapping revealed significant differences across participating countries and economies. For example, Kumagaya, Shizuoka and Toda (Japan) (hereafter "K-S-T [Japan]"), and Shanghai (China) tend to teach algebraic operations independent from graphical material, while Germany* and the United States integrate algebra with graphical representations and everyday application. Also, schools can teach the focal unit at any time throughout the year in some countries and economies (e.g. Germany*), while the timing is highly standardised in others (e.g. K-S-T [Japan] and Shanghai [China]). These cultural differences had to be accounted for through sampling and fielding procedures as well as in analysis and reporting.

Box 3.1 describes the final definition of the focal unit across all countries/economies.

Box 3.1. Final definition of the focal unit on quadratic equations

Prerequisite (taught **before** the start of the unit, even one or two years before)

- Binomial formula $(a \pm b)^2 = a^2 \pm 2ab + b^2$

Core unit: Quadratic equations

- Start: Some example of a quadratic equation is shown with the goal to "solve for x"
- Includes: Solving quadratic equations of a general form such as $ax^2 + bx + c = 0$
    - either through factorising (e.g. Vieta's rule)
    - or through the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
    - or through the reduced quadratic equation $x = -p/2 \pm \sqrt{(p^2/4 - q)}$
- End: application of quadratic equations to everyday problems

Additional topic-related activities

- Different cases depending on values of a, b, c may or may not be discriminated.
- Different kinds of quadratic equations may be transformed into a standard form.
- Graphs (of quadratic functions) may or may not be shown and used for approximate solutions.

Developing instruments for the focal unit across participating countries/economies

The focus of the Study on the same curricular unit across participating countries and economies aimed at facilitating international comparability. All the instruments - such as the observation protocol, the artefact-rating manual, student tests, teacher and student post-questionnaires explicitly referred to the selected focal curricular unit. In each country/economy, the target grade level for sampling in the Study's Main Survey was determined by the grade level identified in Table 3.1.

The instructions for the first part of the students' post-questionnaire can provide an example of how instruments referred to the focal unit. Students read: "You have just finished the unit on the topic of <quadratic equations> (<approximately the last x lessons>). Your views and experiences as an important actor in the classroom are of vital interest to us. While answering the following questions, please always think about your learning during the unit on the topic of <quadratic equations>." Expressions in brackets were adapted to the specific practices of each participating country and economy. Thus, student responses on teaching quality as well as non-cognitive outcomes, such as their interest in mathematics and self-efficacy for mathematics, were explicitly related to the unit on quadratic equations.

The instruments were also designed to capture differences on actual curriculum implementation of the focal unit across participating countries and economies. While all the lessons were to focus on the teaching of the focal unit (quadratic equations), the specific solution methods used were expected to vary both within and among participants. For example, students learn to solve quadratic equations through graphical representations in some countries but not in others. To capture differences in the coverage of subtopics of the focal unit, teachers documented coverage in the teacher "log", students reported on their experience with eleven types of problems related to the focal unit, and raters assessed subtopic coverage in the artefacts.

Overall, the curriculum mapping and data collected in the fielding showed differences on what students were taught across participating countries and economies. Despite the focus on the same curricular unit, the remaining curriculum differences need to be considered in the interpretation of the results of the Study.

References

Common Core State Standards Initiative (2016), *Mathematics Standards*, http://www.corestandards.org/Math/ (accessed on 21 February 2020). [5]

Decristan, J. et al. (2015), "Embedded formative assessment and classroom process quality", *American Educational Research Journal*, Vol. 52/6, pp. 1133-1159, http://dx.doi.org/10.3102/0002831215596412. [2]

Fischer, H. et al. (2014), *Quality of Instruction in Physics Comparing Finland, Germany and Switzerland*, Waxmann, Münster, Germany. [4]

Klieme, E., C. Pauli and K. Reusser (2009), "The Pythagoras Study. Investigating effects of teaching and learning in Swiss and German mathematics classrooms", in Janik, T. (ed.), *The Power of Video Studies in Investigating Teaching and Learning in the Classroom*, Waxmann, Münster, Germany. [3]

OECD (2016), *Equations and Inequalities: Making Mathematics Accessible to All*, PISA, OECD Publishing, Paris, https://dx.doi.org/10.1787/9789264258495-en. [6]

Praetorius, A. et al. (2019), "Methodological challenges in conducting international research on teaching quality using standardized observations", in Suter, L., E. Smith and B. Denman (eds.), *The SAGE Handbook of Comparative Studies in Education*, SAGE Publishing, Thousand Oaks, CA. [1]

Notes

[1] Germany* refers to a convenience sample of volunteer schools.