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PIAAC Numeracy: A Conceptual Framework

PIAAC Numeracy Expert Group

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PIAAC NUMERACY: A CONCEPTUAL FRAMEWORK

OECD Education Working Paper No. 35

By PIAAC Numeracy Expert Group

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ABSTRACT

Governments and other stakeholders have become increasingly interested in assessing the skills of their adult populations in order to monitor how well prepared they are to meet the challenges of the new information world. The current paper provides an overview of the conceptual framework for the assessment of numeracy developed for the OECD's Programme for the International Assessment of Adult Competencies (PIAAC). This builds on the numeracy framework developed for the Adult Literacy and Life Skills Survey (ALL). Numeracy is broadly defined and complemented with a definition of 'numerate behaviour'. Four facets of numerate behaviour are identified and described to guide the development of assessment tasks.

RÉSUMÉ

Les gouvernements et autres parties prenantes de la région OCDE et en dehors s'intéressent de plus en plus à l'évaluation des compétences de leur population adulte, dans un objectif de suivi de leur état de préparation pour faire face aux défis du nouveau monde de l'information. Le présent document fournit une vue d'ensemble du cadre d'évaluation de l'aptitude au calcul du Programme international pour l'évaluation des compétences des adultes (PIAAC), à partir des cadres d'évaluation existants et des connaissances développés par les enquêtes « International Adult Literacy Survey » (IALS) menée dans les années 1990 et « Adult Literacy and Life Skills Survey » (ALL) conduite en 2003 et 2006. La conceptualisation de l'aptitude au calcul en tant que compétence clé du cadre d'évaluation du PIAAC est détaillée et les différents contextes dans lesquels les individus y sont confrontés sont également décrits. En outre, le cadre d'évaluation présente le barème à utiliser pour l'évaluation directe de l'aptitude au calcul au sein du PIAAC et explique comment ce barème a été défini, le distinguant d'autres barèmes d'évaluations internationales similaires.

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PIAAC NUMERACY: CONCEPTUAL FRAMEWORK

INTRODUCTION

1. This document presents a framework for conceptualising numeracy and developing a scale for the direct assessment of adult numeracy as part of OECD's Programme for International Assessment of Adult Competencies (PIAAC). Numeracy as viewed here refers to adults' ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.

2. The conceptual framework and assessment issues discussed in this document were developed on the basis of several lines of work. This document builds on conceptual and assessment frameworks and cumulative wisdom developed in connection with prior surveys of adult skills, primarily the Adult Literacy and Life Skills Survey (ALL) and the International Adult Literacy Survey (IALS), but also surveys of school-age students (e.g., PISA, TIMSS). During preparation of this document by members of the PIAAC Numeracy Expert Group, a detailed review was conducted of professional and research literature in relevant areas such as on adult competencies, workplace skills, adult learning, or mathematics and statistics education. Ideas and feedback were also obtained from an international expert panel in 2006-2007 and all participating countries could react to drafts circulated for commentary during 2008.

3. This framework is organised in six parts which cover separate but interrelated issues:

Part 1: A rationale for assessment of numeracy in PIAAC

Part 2: Conceptual and theoretical foundations about adult numeracy

Part 3: Facets of numeracy

Part 4: Scale development: Principles, constraints, implementation

Part 5: Differences between PIAAC's numeracy and related constructs and scales

Part 6: Summary and further reflections

PART 1: A RATIONALE FOR ASSESSMENT OF NUMERACY IN PIAAC

1.1 Overview

4. PIAAC is a policy-driven initiative intended to provide policy makers and key stakeholders at the national and international levels with information that can inform policy-setting and planning of social interventions and programs. PIAAC's overarching goals (OECD, 2006) are twofold. First, to identify and measure differences within and across countries in "*literacy competencies for the information age – the interest, attitude, and ability of individuals to access, manage, integrate, and evaluate information, construct new knowledge, and communicate with others in order to participate effectively in the information age*". Second, to assess the relationship of adult competencies with economic and social outcomes believed to underlie both personal and societal success (e.g., earnings, employment, educational attainment, participation in further learning) and optionally with additional outcomes or processes at the individual level (e.g., health, social capital) or workplace level, and with transitions at key points over the lifespan, such as school-to-work and possibly other stages.

5. The OECD planning paper states (OECD, 2006) that PIAAC is expected to provide reliable, valid, and valuable information to policy makers, differentiate the performance of low-scoring adults in each participating country, as well as include technology-based measures that tap into higher-order reading and thinking skills. As such, the design for the direct assessment in PIAAC incorporates computer-based measures of competencies, yet also use paper and pencil means for adults who are unfamiliar or uncomfortable with computers.

6. PIAAC is further expected to enable continuity with and links to the two previous international adult assessments, the International Adult Literacy Survey (IALS) and the Adult Literacy and Life Skills Survey (ALL). Nineteen OECD countries[1] participated in either IALS or ALL, with nine participating in both. For these countries, it is paramount that PIAAC's direct assessment enables capitalisation on their previous investments and provides an indication of how adults' competencies have changed since the previous measurement point(s). Equally important to maximizing countries' past investments in adult learning, is adding value to IALS and ALL by seeking innovation by broadening what skills are measured as well as how they are measured. As the intent of PIAAC is to have its results linked to previous international adult assessments, PIAAC has been designed with a specification by OECD that 60% of the literacy and numeracy tasks will come from item pools used in ALL and IALS. As a result, this conceptual framework for assessing numeracy in PIAAC maintains conceptual and pragmatic links to the numeracy framework developed for ALL.

7. Finally, OECD has expressed a desire that PIAAC's direct assessment will be conceptually compatible with PISA. Although no direct statistical links or common items are necessarily expected between the two assessment programmes, such a linkage can provide policy-makers with information about the spectrum of competencies across different points of the lifespan, enable analyses of antecedents, consequences, and correlates of the distribution of competencies, and help to identify implications and relevant social interventions.

---

1 Countries that participated in IALS include: Australia, Belgium (Flemish Community), Czech Republic, Canada, Denmark, Finland, Germany, Great Britain, Hungary, Ireland, Italy, Korea, Netherlands, New Zealand, Norway, Poland, Sweden, Switzerland, and the United States. Countries that participated in the first stage of ALL include: Bermuda, Canada, Italy, the Mexican State of Nuevo Leon, Norway, Switzerland, and the United States. ALL's second stage included Australia, Korea, Hungary, New Zealand, and the Netherlands.

1.2 Rationale For Assessing Numeracy In PIAAC

8. 'Numeracy' is listed in the PIAAC overarching framework as a component of the broad set of 'literacy competencies'. Yet, numeracy should be viewed as a key competency which is *not* subsumed under 'literacy' as this later term has been traditionally defined regarding reading, writing, and comprehending the meaning of text or communicating through textual means. While literacy and numeracy in the traditional sense have some linkages, numeracy is a broad construct with a life of its own and it has a central and often quite distinct role in adults' lives. A later section reviews various perspectives which inform the conceptualisation of numeracy, leading to a definition of numeracy for PIAAC and description of facets or dimensions of numerate behavior.

9. This framework is founded on the assumption that a direct assessment of numeracy in PIAAC is an essential and worthwhile undertaking (Willms, 2006; Murray, 2006), for four separate but related reasons:

a) ***Numeracy is essential for adults and for the societies in which they live.*** Basic computational or mathematical knowledge has always been considered as part of the fundamental skills that adults need to possess to function well and be able to accomplish various goals in their everyday, work, and social life. Societies now present increasing amounts and wider range of information of a quantitative nature to citizens from all walks of life, in diverse contexts such as regarding health risk factors, school performance, or financial planning and insurance purchasing, to name just a few. As workplaces are becoming more concerned with involving all workers in improving efficiency and quality, the importance of numeracy skills is growing. Numeracy-related skills have been shown to be a key factor in labor market participation, sometimes even more so than literacy skills. Adults with lower skills in numeracy and literacy are more likely to be unemployed or require social assistance. Further, some numeracy skills are deemed essential for post-secondary education in many areas, including but not limited to hard sciences, engineering and technology. (Jones, 1995; Murnane, Willett & Levy, 1995; Hoyles, Wolf, Molyneux-Hodson, & Kent, 2002; Coulombe, Tremblay, & Marchand, 2004; Desjardins, Murray, Clermont & Werquin, 2005).

b) ***Public policy in most countries includes separate investments in literacy and numeracy.*** The separate acquisition of skills in these two fundamental areas is emphasised throughout both primary and secondary school systems, and in adult education or nonformal learning schemes. Countries expect that investment in literacy and numeracy will increase citizens' ability to act independently towards their own progress and income security, thereby reducing future social expenditures as well as contributing to citizens' participation in economic and social life in an information-laden society (European Commission, 1996). Numeracy has been shown to be associated with future learning which is important for re-training and for upgrading of skills as industries evolve (Marr & Hagston, 2007). The demands for higher numeracy performance will affect the employability of the labor force.

c) ***The policy and program responses are different for numeracy than for literacy.*** Efforts to improve literacy and numeracy levels of specific population groups are not implemented via the same mechanisms—they often require different experts, resources, and learning systems because of differences in the underlying knowledge components and learning trajectories. It is vital that nations have information about their workers' and citizens' numeracy, independently of other competency areas, in order to evaluate the human capital available for advancement, to plan school-based and lifelong learning opportunities, and to better understand the factors that affect citizens' acquisition and usage of numeracy (Johnston, & MacGuire, 2005).

d) ***Numeracy skill levels are not measured well by literacy measures.*** It is not possible to represent the *numeracy* levels in a population via people's performance on literacy measures that examine how well people read, process, and comprehend various types of texts and documents, or communicate about such texts. As explained later in more detail, numeracy involves, among other things, the handling of arithmetical processes, understanding of proportions and probabilistic ideas, understanding of numerical, geometric and graphical types and representations of quantitative information, critical interpretation of statistical or mathematical messages, ability to solve various types of quantitative problems, and other elements or processes that bear little relation to what is subsumed by literacy measures. (Coben, 2000; Gal, van Groenestijn, Manly, Schmitt, & Tout, 2005)

10. It follows that a direct assessment of numeracy in PIAAC can provide policy makers and other stakeholders with a sound basis for evaluating the distribution of the actual numeracy competence in the adult population.

PART 2. CONCEPTUAL AND THEORETICAL FOUNDATIONS

11. The conceptualisation of 'numeracy' in an international context is a challenging undertaking. Like literacy, the term numeracy has multiple meanings across countries and languages. In some countries the term numeracy relates to basic skills which school children are expected to acquire as a prerequisite to learning formal mathematics at higher grades. In other countries the term numeracy encompasses a broad range of skills, knowledge and dispositions that adults should possess but it does not necessarily relate to formal schooling (Baker & Street, 1994; NRDC, 2006). Finally, some countries do not even have a word such as numeracy; therefore, as part of educational or policy-oriented discourse in such countries, experts or translators either had to invent a special new word for it (e.g., 'Numeratie' in Canada, 'Numeralitet' in Denmark), or use other phrases such as "mathematical literacy", "functional mathematics", or terms equivalent to "computational ability". Such diversity in terminology, or the lack of an accepted term with which policy-makers feel comfortable, can complicate the communication with and among policy makers interested in PIAAC.

12. The range of meanings attached to the term numeracy and the lack of an equivalent term across languages may create miscommunications or gaps in expectations regarding what will be measured by a numeracy scale in PIAAC. This can affect the perceived policy relevance of a numeracy scale. Thus, attention has to be given to making sure that discussions regarding numeracy assessment in PIAAC are based on a consensus about the scope of the term and recognition of its centrality in a wide range of adult life circumstances.

13. However, it must be remembered that what will be measured by a numeracy assessment scale is *jointly* determined by two interrelated factors – by (1) a conceptual scheme describing numeracy and its elements, and (2) by an assessment scheme describing how the general conceptualisation of numeracy is operationalised and manifested in the nature and range of tasks used in the assessment scale and the mode of administration and scoring. Of course, the conceptualisation and assessment of adult numeracy involves many questions, such as: What are the key numeracy tasks which adults have to face in their lives? What facets or sub-domains are subsumed under 'adult numeracy'? What are the differences and commonalities

between numeracy and related key constructs such as Quantitative Literacy which was assessed in IALS or Mathematical Literacy which is assessed in PISA?

14. This part is organised in three sections. In section 1, the notion of "competence" as defined by OECD is outlined. In section 2, the contexts and situations in adults' lives which require numeracy are reviewed. Section 3 examines perspectives on the meaning of numeracy and prior definitions and conceptualisations of numeracy, leading to a definition of numeracy for PIAAC. Part 3 which follows further examines the dimensions of numerate behavior, including contexts, expected responses, content areas of mathematical information and ideas, and representations, as a way of operationalizing the numeracy construct for scale development. It also discusses enabling processes, both cognitive and non-cognitive or dispositional, which underlie numerate behavior. Subsequent parts outline principles for assessment of numeracy in PIAAC, and comment on differences and commonalities between PIAAC's numeracy and related constructs assessed in IALS and PISA.

2.1 Numeracy as a competence

15. The conceptualisation of numeracy in an assessment program which focuses on "literacy competencies for the information age" has to be congruent with the broader notion of "competence". Within OECD, prior work on the Definition and Selection of Competencies project (DeSeCo; see Rychen & Salganic, 2003) has defined competence as "the ability to meet individual or social demands successfully, or to carry out an activity or task". The DeSeCo view, which was adopted by OECD and also informed the design of assessment scales for PISA, places at the forefront how individuals function in the face of external demands that may stem from a personal or social context of action. DeSeCo (2002: 8-9) conceptualises competencies as internal mental structures, i.e., abilities, capacities or dispositions embedded in the individual:.

> *Each competence is built on a combination of interrelated cognitive and practical skills, knowledge (including tacit knowledge), motivation, value orientation, attitudes, emotions, and other social and behavioral components that together can be mobilised for effective action. Although cognitive skills and the knowledge base are critical elements, it is important not to restrict attention to these components of a competence, but to include other aspects such as motivation and value orientation.*

16. Further, DeSeCo (2002:7) argues that the terms "skills" and "competencies" are not synonyms. Skills designate an "ability to perform complex motor and/or cognitive acts with ease, precision, and adaptability to changing conditions", while competence designates "a complex action system encompassing cognitive skills, attitudes and other non-cognitive components".

17. The conceptualisation of numeracy discussed below, which is based on a review of scholarly literature and research findings, operates on two levels. It relates to numeracy as a construct describing a competence as defined above, and to numerate *behavior* which is the way a person's numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature. In this way, inferences about a person's numeracy are possible through analysis of performance on assessment tasks designed to elicit numerate behavior. In congruence with the above view of a competence, numeracy will be described as comprised both of cognitive elements (i.e., various knowledge bases and skills) as well as non-cognitive or semi-cognitive elements (i.e., attitudes, beliefs, habits of mind, and other dispositions) which together shape a person's numerate behavior.

2.2 Contexts and demands for numeracy

18. Once a view of numeracy as a competence as defined above is adopted, a discussion of what is encompassed by numeracy (and numerate behavior) has to start by identifying the nature of the contexts which contain mathematical[2] elements, or which include information of a quantitative nature, that adults face and which pose demands with which they have to cope. This in turn provides the basis for describing the knowledge elements and supporting processes which enable adults to cope with real-world numeracy tasks (Ginsburg, Manly & Schmitt, 2006), and can later help to form a road map which can guide the design and selection of tasks for inclusion in the numeracy assessment in PIAAC.

19. The literature pertaining to the uses of numeracy in the real world can be divided into three strands: literature on the roles of literacy and numeracy in adults' lives, on the mathematical demands of workplace and functional settings, and on educational perspectives on mathematical needs of school graduates and citizens. These areas are certainly intertwined but also offer complementary ideas, hence each is reviewed separately below.

2.2.1. The roles of literacy and numeracy in adults' lives.

20. The purposes served by adults' numeracy may parallel those served by adults' literacy, and further, people's numeracy may at times relate to or even depend in part on literacy skills or other lifeskills. Work to describe the purposes served by adults' literacy and numeracy skills has been conducted in several countries. In Australia, for example, Kindler et al., (1996) reported on four such purposes: *literacy for self-expression, literacy for practical purposes, literacy for knowledge, and literacy for public debate*. In the USA, the National Institute for Literacy has sponsored efforts to define critical skill areas. As part of its Equipped for the Future initiative, four broad types of purposes were identified (Stein, 1995): *Literacy for access and orientation in the world, literacy as voice to one's ideas and opinions, literacy for independent action, solving problems and making decisions as a parent, citizen and worker, and literacy as a bridge to further learning and to keep up with a rapidly changing world*.

21. Work has also been done in adult education contexts to identify different purposes and functions of using mathematical knowledge. In Australia, for example, one key project (Kindler et al. 1996), pointed to four broad categories regarding the uses of numeracy: *Numeracy for practical purposes* addresses aspects of the physical world that involve designing, making, and measuring. *Numeracy for interpreting society* relates to interpreting and reflecting on numerical and graphical information in public documents and texts. *Numeracy for personal organisation* focuses on the numeracy requirements for personal organisational matters involving money, time and travel. *Numeracy for knowledge* describes the mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings or assumptions.

22. A scheme developed by Steen (1990), a noted mathematics educator, outlines five dimensions of numeracy:

- *Practical*, focused on mathematical and statistical knowledge and skills that can be put to immediate use to cope with tasks in daily life

2 The term "mathematical" is used here as inclusive of situations where statistical or probabilistic information may appear or where statistical thinking or statistical literacy are required as well. Such usage is made for brevity and convenience only. It is acknowledged that statistics is not a branch of mathematics, and that statistical reasoning and statistical literacy have unique elements, concepts and processes which are not mathematical in nature (Moore & Cobb, 2000).

- *Professional*, focused on the mathematical skills required in specific jobs
- *Civic*, focused on benefits to society
- *Recreational*, related to the role of mathematical ideas and processes in games, puzzles, sports, lotteries, and other leisure activities
- *Cultural*, concerned with mathematics as a universal part of human culture (and related to appreciation of mathematical aspects such as in cultural or artistic artifacts)

23. Overall, the purposes regarding literacy and numeracy appear to agree and suggest that adults need to be able to apply their numeracy and literacy skills to tasks with a social or personal purpose in both informal and more formal contexts (NRDC, 2006). Such perspectives supplement Bishop's (1988) proposal that there are six modes of mathematical actions that are common in all cultures and pertain both to children and adults: counting, locating, measuring, designing, playing and explaining.

2.2.2. Numeracy in the workplace and in functional settings.

24. Mathematical and statistical skills that are important in adults' work have been described in large-scale efforts to define "core skills" or "key competencies" that workers should have, usually in response to the need to maintain economic competitiveness and improve employability of adults and school graduates. In addition, several projects looked specifically at the mathematical skills of workers in a range of occupational groups or workplace clusters.

25. Basic computational knowledge has always been considered as part of the fundamental skills that adults need to possess, but recent skills frameworks claim that workers need to possess a much broader range of mathematical skills. Examples exist in many countries and the following selective description from the United States is indicative of the nature of such efforts. Following earlier research by a task force of the American Society of Training and Development (Carnevale, Gainer, & Meltzer, 1990), the U.S. Secretary of Labor's Commission on Achieving Necessary Skills (SCANS) (Packer, 1997) has differentiated between mastery of basic arithmetical skills and much broader and flexible understanding of principles and underlying ideas subsumed under the notion of mathematical skills (SCANS, 1991, p. 83):

- *SCANS arithmetical skills:* Performs basic computations; uses basic numerical concepts such as whole numbers and percentages in practical situations; makes reasonable estimates and arithmetic results without a calculator; and uses tables, graphs, diagrams and charts to obtain or convey quantitative information.
- *SCANS mathematical skills:* Approaches practical problems by choosing appropriately from a variety of mathematical techniques; uses quantitative data to construct logical explanations for real world situations; expresses mathematical ideas and concepts orally and in writing; and understands the role of chance in the occurrence and prediction of events.

26. Based on a later survey of employers, industry trainers, and educators, among others, Forman & Steen (1999) similarly argued that quantitative skills desired by employers are much broader than mere facility with the mechanics of addition, subtraction, multiplication, and division and familiarity with basic number facts; they also include some knowledge of statistics, probability, mental computation strategies, some grasp of proportional reasoning or modeling relationships, and broad problem-solving and communication skills about quantitative issues.

27. Work on mathematical skills and their use in specific workplaces has been conducted over the last decade in both the manufacturing and service sectors in several countries such as the UK, Australia, USA, and others (e.g., Buckingham, 1997; Bessot & Ridgway, 2000; Hoyles et al., 2002; Fitzsimons, 2005; Skills Australia, 2005). Overall, these studies complement the SCANS study and suggest that employees need to possess a range of specific mathematical skills or knowledge, such as the following key (but not the only) examples:

- skills in both fast and accurate computations but also estimation, and knowing when each skill is required and why
- ability to deal with proportions and percents
- understanding measurement concepts and procedures
- working with or creating simple formulas
- a sense for the use of models and modeling in foreseeing future needs
- understanding of basic statistical concepts and displays

28. In addition, on a broader and less technical level, these studies argue that workers need to be able to make decisions in the face of uncertainty in real situations, prioritise actions and make choices regarding the approach to handling different tasks, depending on changing external demands. As well, there is a need for workers to be able to communicate with other workers or clients or understand written documentation (e.g., through text or with tables, charts, and graphs) about issues such as quantities, schedules, variation over time, results of quantitative projections, or analysis of different courses of action in this regard. Such findings echo the earlier distinctions made by the SCANS analysis between the need to attend both to basic arithmetical skills and more elaborate and complex mathematical skills in the workplace, but also highlight some areas where specific literacy and communication skills are intertwined with numeracy skills.

29. An important research literature has also accumulated over the last decades regarding the ways in which people use mathematical skills or cope with mathematical tasks in both formal (i.e., school-based) and informal (i.e., everyday, workplace) contexts (e.g., Rogoff & Lave, 1984; Resnick, 1987; Saxe, 1988; Carraher, Schliemann, & Carraher, 1988; Scribner & Sachs, 1991; Nunes, 1992; Presmeg, 2007). While too complex to discuss in detail here (see Greeno, 2003, for one of several reviews of this literature), among other things these studies highlight the situatedness of mathematical knowledge used in functional contexts and the need for actors in different contexts to develop situation-specific mathematical procedures and know-how. Further, numerous researchers (e.g., Straesser, 2003; Wedege, 2003; Williams & Wake, 2007) have argued, based on ethnographic analyses of workers' activities in diverse industries, that important portions of the mathematical activities at work are made "invisible" to occasional observers as well as to the workers themselves, or are disguised as nonmathematical. Various factors have been posited as causing this phenomenon, such as the encapsulation of many mathematical activities into routines or automated procedures; the use of tools and instruments or information technology (e.g., spreadsheets); the normative use of job-specific linguistic terms that are different than traditional school terms; or the division of labor among different workers.

30. Based on such and related findings, many projects have argued that mathematical skills as used in the workplace are often different and broader in scope than what is traditionally taught in school mathematics, but also take on different forms depending on the specific work context (Marr, & Hagston, 2007). Overall, the above suggests that what employers, training and employment specialists, and researchers know about the mathematical or statistical demands of different occupations may be

incomplete. Further, projections of future skills demands usually focus on shifts in demand for *workers* with given skills, e.g., how many more engineers, technicians, or call-center operators will be required with a given mixture of currently-defined skills (Karoly, 2007), *not* on the changing future numeracy skills that workers will require.

31. Most sources discuss future skills required in occupational or job-market sectors, not to skills required in family, civic, or community contexts. Yet, with the accessibility of high-speed internet connections in homes, more adults are able to connect to, search, and make use of an increasing array of information sources, including many with quantitative components, such as regarding health, personal finances, comparative shopping, sports, education, official statistics, and more. Many service organisations increasingly open more opportunities for customers not only to *access* information, but also *interact and take action*, through Internet websites and other technology-based devices (e.g., ATM machines, 'smart' cellular phones, GPS navigation systems), thus involving adults in new types of transactions and activities. These changes are important because they blur the distinction between "work" and "non-work" situations and their skill demands in the area of numeracy.

32. Given the above, the conceptualisation of numeracy for PIAAC was derived with reference to the types of numeracy demands as depicted earlier in this subsection. Further, a working assumption has been made that it is not feasible to employ assessment items that are very workplace-specific (e.g., couched in the context of a single workplace or occupation) because mathematics or statistics as used in this context may not be visible or familiar to most other adults (Hoyles et al., 2002).

2.2.3. Educational perspectives on numeracy and informed civic participation.

33. A growing dialogue about the goals and impact of mathematics education in schools has intensified in recent years. This is in part due to economic pressures and industry expectations on the one hand, but also due to the realisation that mathematical knowledge and skills serve multiple and separate gateway functions on the other hand. Specifically, mathematical competencies affect chances of entry into key occupational tracks (mainly in science, technology, and economics) and may affect employability and labor-force participation, underlie some important aspects of civic participation, and may impact on the possibilities of certain population groups for social equality and mobility. While the dialogue about these issues admittedly overlaps to some extent the points raised earlier in discussing the roles of literacy and numeracy in society, it is worth elaborating upon because it brings forward some additional points and broadens the understanding of contexts where demands on adults' numeracy exist.

34. Various arguments have been forwarded over the last few decades to support a broadening of the conceptions regarding the mathematical skills and knowledge that school graduates should possess, and the ways in which learned knowledge serves adults (Ernest, 2004). Educators working both with school students and adults increasingly aim to assist learners in developing mathematical concepts and skills in ways that are personally meaningful but also functional. Such approaches usually assume that there is often more than one right way to cope with a real-world functional task, and that adults require access to a repertoire of strategies for solving functional problems. Adults' personal methods of using mathematics are encouraged and valued. This is often a significant difference from traditional (pre-reform) school-based mathematics teaching, within which school students were often expected to solve a problem following the one correct method or algorithm, introduced by the teacher.

35. Several decades ago, ideas already began emerging in different countries that since mathematics is an essential aspect of society, mathematics education in schools should be derived from or prepare learners for broad real-life situations in family, work, community, and other contexts (National Council of Teachers of Mathematics, 2000; Willis, 1990), beyond employers' desire to focus mostly on practical or job-specific numeracy skills. Two early influential examples are the recommendations of the Cockcroft

Committee in the UK (Department of Education and Science/Welsh Office, 1982), and Freudenthal's work in the Netherlands which has led to the Realistic Mathematics Education movement (Heuvel-Panhuizen & Gravemeijer, 1991). Over the last two decades, various countries (e.g., Australia, UK) have adopted adult education frameworks which give explicit attention to numeracy skills. For example, in the UK, the Adult Numeracy Core Curriculum aims to move learners through up to five levels of demand, where the expectation is that at the highest one (level 2), an adult:

> *Understands mathematical information used for different purposes and can independently select and compare relevant information from a variety of graphical, numerical and written material – e.g. compare data using mean and median, work out discounts as fractions and percentages of amounts, work out distances and lengths from scale drawings (Gillespie, 2007: p.4).*

36. Educators have also paid much attention to the importance of quantitative literacy in civic and social contexts, and argued that mathematical knowledge is a crucial part of a common fabric of communication indispensable for modern civilised society, in part because it is the language of science and technology. Thus, it has been claimed (National Research Council, 1989) that understanding of public discussions and reports about socially important topics such as health and environmental issues is impossible without using the language of mathematics.

37. Further, it has been claimed that in a society in which the media constantly present information in numerical or graphical form to all citizens, the ability to interpret quantitative and statistical messages is vital for all adults (Paulus, 1995; Steen, 1997). It is essential for all adults to possess the ability to critically reflect on quantitative information encountered in various media sources and documents (Frankenstein, 1989), and to understand how to be a careful or critical consumer of statistical arguments of various kinds (Gal, 2002; Utts, 2003; Watson & Callingham, 2003).

38. Indeed, the dialogue about the various demands on adults' knowledge has been reflected in part in the emphasis in PISA on the assessment of mathematical literacy and science literacy. Such constructs pertain, broadly speaking, to school students' readiness for entering adults' life contexts; it is indicative that they have been chosen to be the focus of assessment rather than more traditional notions of formal knowledge in mathematics or science areas which were assessed primarily in earlier studies.

2.2.4 More on numeracy situations and demands.

39. The discussion above suggests that numeracy is required so that people can effectively cope with or respond to a range of situations that are embedded in a life stream with real, personal meaning to them. The situations that call for activation of the numeracy competency can be situated in a hypothetical "numeracy task space", defined by dimensions such as the nature of the required response, the number and characteristics of the quantitative elements in the situation, or the extent and nature of literacy processes involved. Based on Gal (2000), below are described three key types of situations which illustrate the range of numeracy demands placed on adults.

1. ***Generative situations*** demand that people count, quantify, compute, or otherwise manipulate numbers, concrete objects, visual elements, and so forth, to create/generate new numbers or estimates. Examples are calculating the total price of products while shopping, finding the number of boxes in a crate, measuring the area of a room to be painted in order to calculate the amount of materials needed to do the job, reading a menu and computing the cost of a specified meal, filling out an order form for a product, figuring out travel times between train stations based on a timetable, and so forth. Generative situations include computational or quantitative literacy tasks (Kirsch et al, 1993), but certainly go beyond them, such as when measurements have to be made regarding length, volume, time, etc.

The numerical information in many types of generative situations may be evident in the situation itself (e.g., real objects to be arranged, sorted, counted, or measured; a graph on a computer display). Yet, numerical information may also be communicated through text or be embedded in different types of text; hence, such situations may also involve language skills to varying degrees. In generative situations, tools such as a hand-held calculator, a computer-based application, or a measuring tape or ruler may ease the mechanics of performing needed calculations or increase accuracy, although a person may choose not to use them. Even when such tools are used, a person still needs to know how to use them efficiently and effectively, and his or her operations are at times more likely to be correct or accurate if the person has alternative strategies (e.g., mental calculations) to check on accuracy and completeness of actions performed.

2. ***Interpretive situations*** demand that people make sense, and grasp the implications, of messages that contain information of a mathematical or statistical nature but that *do not involve direct manipulation of numbers*. An example is being faced, when reading the newspaper, with a report of results from a recent opinion poll based on a small sample, and having to decide if to take as valid a generalisation made by the writer about differences between two populations. Paulus (1995) describes many mathematical statements made in the media that call for careful consideration of their validity; other examples can be added where references to proportions, averages, samples, bias, correlation, risk, or causality are discussed or implied, such as in the context of genetic or medical counseling, or understanding of statistical process control displays.

   In simple interpretive situations, such as when one has to read nutrition information or a drug label to decide if a product contains certain ingredients (e.g., sugar, allergens, contaminants) whose dosage exceeds an allowed limit, the response can be assessed as correct or incorrect. Yet in more complex interpretive situations, the response expected is usually the creation of an *opinion*, and to create this opinion one needs to invoke and answer a set of critical before the information or arguments presented are accepted as credible or valid. In such cases, the response, i.e., opinion, has to be judged in terms of its reasonableness and the quality of the arguments or evidence on which it is based. Thus an opinion cannot always be easily classified as “right/wrong” or "accurate/inaccurate", as with responses to many generative situations.

3. ***Decision situations*** demand that people locate and consider multiple pieces of information in order to determine a course of action, typically in the presence of conflicting goals, constraints, or uncertainty. Two key subtypes here are: *Optimisation tasks*, which require the identification of optimal ways to use resources such as money or supplies, or schedule personnel or time (see SCANS, 1991); and *Choice tasks*, which require a choice among alternatives, such as which of several apartments to rent, which pension or health insurance plan to join, whether to undergo a surgical medical procedure which has known probabilities of certain side effects.

   It is important to note that optimisation and choice tasks can be part of a broader *problem-solving process*, where alternatives have to be generated and then evaluated. Thus, what is being termed here a *decision* situation can at times also be viewed as a *problem-solving* situation.

   As with interpretive situations, a response to a decision situation has to be assessed in terms of its reasonableness, because the response may be based on multiple pieces of quantitative information (e.g., timetables, financial figures, statistical trends, event probabilities), the

response may be created through a process that involves both generative and interpretive steps, and the response is shaped after a person evaluates the quality of the final decision or choice against external contextual criteria. An example is when a small business owner has to compare financial information from several banks to decide which loan schedule is the best or most manageable, given the current financial situation and anticipated costs and revenues of the business. The assessment of the reasonableness of a response in a decision situation may be further complicated, beyond what happens in an interpretive situation, because the response in a decision situation may also be shaped by a person's subjective preferences and value system, assumptions he or she makes about future trends or event probabilities, and other factors.

40. To be sure, the three types of numeracy situations described above are not mutually exclusive, and other cases may exist, possibly of a hybrid nature. Further, in considering the implications of these and other types of situations for the numeracy competency required of adults, it is important to keep in mind the impact of evolving technologies. As has been argued and documented by many sources (Expert Group on Future Skills Needs, 2007; Gatta, Appelbaum, & Boushey, 2007; Karoly, 2007), and summarised in the PIAAC planning documents, adults are presented with ever-increasing amounts of information of a quantitative nature through Internet-based or technology-based resources. More so than in prior decades, more types of quantitative information are more readily available, but this information has to be located, selected or filtered, interpreted, at times questioned and doubted, and analyzed for its relevance to the responses needed, whether generative, interpretative, or decision-oriented.

2.3 Towards a definition of numeracy for PIAAC

41. Reaching a consensus on a definition of numeracy that can fit an international programme of assessment is a challenging undertaking. First, as noted above there are various country-specific connotations for numeracy, if such a term at all exists in a local language. Second, there are overlapping or competing constructs such as quantitative literacy, mathematical literacy, functional mathematics, and so forth (Hagedorn, Newlands, Blayney, & Bowles, 2003). Third, an attempt to discuss the definition and meaning of numeracy is complicated by the fact different stakeholders already view it from within a given lens imposed by the historical and cultural aspects, whether organisational, social, economic, or linguistic, of the systems within which they operate. For example, some of the existing conceptions of numeracy were developed by educators working in delivery systems for school children, while other stakeholders link the term numeracy only to adult-related competencies.

42. With the above in mind, and using the conceptions of competence and of the contexts for numeracy presented earlier as a backdrop, the remainder of this section is organised as follows: First, a review of some of the many perspectives on numeracy is presented so as to portray the key ideas that prior workers and scholars have addressed when discussing numeracy. Next, a definition of numeracy for PIAAC is presented, followed by a discussion of the facets of numerate behavior, including key content areas of mathematical knowledge and other cognitive and non-cognitive enabling processes and factors which take part in or affect numerate behavior. Finally, differences between numeracy and the constructs assessed in PISA and IALS are examined, in particular mathematical literacy and quantitative literacy.

2.3.1 Perspectives on [adult] numeracy

43. Formulation of what numeracy encompasses have evolved since the term was introduced in the 1959 Crowther Report in England and Wales. Maguire and O'Donoghue (2003) have recently reviewed and organised conceptions of numeracy from several countries (Ireland, Canada, USA, UK, the Netherlands, Denmark and Australia) along a continuum of increasing levels of complexity or sophistication. *Formative* conceptions view numeracy as related to basic arithmetic skills. *Mathematical*

conceptions consider numeracy in a contextualised way, as a broader set of mathematical knowledge and skills (beyond basic computations) of relevance in everyday life. Finally, *integrative* conceptions consider numeracy as a multifaceted, sophisticated construct incorporating not only mathematics but also communicative, cultural, social, emotional, and personal elements which interact and pertain to how different people function in their social contexts. (Coben, 2000; Condelli, Safford-Ramus, Sherman, Coben, Gal, & Hector-Mason, 2006).

44. At this time, formative conceptions which view numeracy as basic computational facility are often associated with how numeracy is viewed in connection with goals of primary schooling, and reflected in how numeracy is defined when classifying literacy/numeracy levels worldwide (UNESCO, 1997). Most extant conceptions which adult education, workplace training, and national and international assessments have adopted fall at different points across the mathematical and integrative phases described by Maguire and O'Donoghue. Below are four different but related views of numeracy, the first pair from the UK, and the second pair from Australia. These definitions illustrate that conceptions evolve over time and that variability can be noticed even within the same national system.

> *[numeracy is]…an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life…[and] an ability to have some appreciation and understanding of information, which is presented in mathematical terms, for instance graphs, charts or tables or by reference to percentage increase or decrease (From the Cockcroft report: Department of Education and Science/Welsh Office (1982), p. 11).*
>
> *The ability to use mathematics at a level necessary to function at work and in society in general…understand and use mathematical information; calculate and manipulate mathematical information; interpret results and communicate mathematical information (From the UK government's Skills for Life strategy to improve standards of adult literacy and numeracy in England, DfEE, 2001, p.3).*
>
> *Numeracy is the mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's own development and of one's community (Beazley committee, 1984, Australia).*
>
> *Numeracy involves abilities that include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles (Queensland Department of Education, 1994, Australia).*

45. An interesting case study of defining numeracy is offered by Lindenskov and Wedege (2001). Based on their work in adult and mathematics education in Denmark, they have imported numeracy from English-speaking countries and introduced a new term, Numeralitet, with a conceptual framework that was later adopted by the Danish Ministry of Education. According to this perspective, it is essential to distinguish between what numeracy is, or ought to be, from the individual's and from society's points of view. Lindenskov and Wedege (2001) advocate a societal view, whereby numeracy is seen as a competence that involves a dynamic interaction between functional mathematical skills and conceptions and operations on the one hand, and a series of activities and various types of data and media on the other. They argue that this skill- and activity-based view should be coupled with the understanding that in principle all people need to have this competence, and that numeracy is a competence determined by society and technology and that it changes in time and space along with social change and technological development.

46. Other views of numeracy, usually developed by adult education experts, focus on the role of adults as reflective communicators and critical consumers of information in society who are involved in the exchange and interpretation of messages encountered in media or in political and community contexts (Frankenstein, 1989). Johnston (1994) argues:

> *"To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in school or university mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real-world, with all its diversity" (Johnston, 1994).*

47. The definition quoted from the UK's Cockcroft Committee (1982) has been quite influential in that its conception of numeracy implied it is an ability to cope with various functional tasks in real-world contexts as well as interpretive tasks, but also pointed to the centrality of underlying supporting non-cognitive components. These key ideas are reflected, albeit with different terminologies and foci, in other views of numeracy. Another important commonality is the presence of mathematical elements or ideas in real situations, and the notion that these can be used or addressed by a person in a goal-oriented way, dependent on the needs of the individual within the given context, i.e., home, community, workplace, societal action, etc.

2.3.2 Numeracy-literacy connections

48. Several scholars and projects have pointed to the need to consider *literacy* when discussing numeracy, as the two are related and can affect each other (Baker & Street, 1994). Examples are when quantities are described in words and not in numbers, or appear within surrounding text whose interpretation is essential to understanding what is being required in terms of computations, or when there is a need to understand the mathematical relationships described in simple phrases, e.g., realizing that "four more than" is a different relationship than "four times as much."

49. An important aspect of the literacy-numeracy linkage has been acknowledged early on by the Kirsch & Mosenthal construction of literacy as comprised of Prose, Document, and Quantitative dimensions (Kirsch, Jungblut, & Mosenthal, 1998). Accordingly, adults skills in dealing with arithmetical operations embedded in text have been assessed by the Quantitative Literacy scale in IALS and several prior national studies. However, other areas where literacy and numeracy are linked do exist and need to be recognised, such as in the context of interpretation of statistical arguments in media articles (Gal, 2002a), or comprehending financial information which pertains to planning one's retirement pensions or medical benefits. Thus, while it is possible to define numeracy in general terms without invoking literacy, as all the definitions quoted above have done, the structure of the tasks and demands in adults' lives shows that these areas cannot be considered as mutually exclusive. Mathematical or statistical information is carried by or embedded in text in some, but certainly not all, contexts in which adults have to function. To the extent this happens, one's performance on numeracy tasks will depend not only on formal mathematical or statistical knowledge but possibly also on literacy-related factors such as vocabulary, reading comprehension, reading strategies, or prior literacy experiences.

2.3.3 Numeracy and numerate behavior in the ALL survey.

50. The above ideas informed the conceptualisation of numeracy for the Adult Literacy and Life Skills Survey (ALL), which was developed in 1998-2000 by an international team (Gal, van Groenestijn, Manly, Schmitt, & Tout, 2005). This was the first time the construct of numeracy had to be defined in a comparative assessment context and not purely in an educational context. Cognizant of the complexity and multi-faceted nature of the numeracy construct, the ALL team developed a three-tier conceptualisation which attempted to reflect key perspectives of numeracy on the one hand, but also enable operationalisation of the construct in an assessment scale on the other (Tout, 2006). The three tiers are a

brief definition of numeracy, a more elaborate definition of numerate behavior, both presented below, and a detailed listing of components of the facets of numerate behavior (see Gal et al., 2005).

> Numeracy is the knowledge *and skills required to effectively manage and respond to the mathematical demands of diverse situations.*
>
> Numerate behavior is observed *when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors, and processes.*

51. Both the brief and elaborate definitions shown above were seen by the ALL numeracy team to be required, given the needs of a comparative assessment. A brief definition is essential to simplify communication with various stakeholders, such as policy-makers and experts. However, as with most brief definitions of complex constructs, the language used is broad and abstract, hence the definition cannot be explicit about what a numerate person can do and what behavior to observe in an assessment. With this in mind, the more detailed definition of numerate behavior was developed as a way to emphasise four key facets or dimensions which were seen by the ALL numeracy team as underlying numerate behavior, as follows:

- *Contexts*: The range of external demands (e.g., work, home, etc.);
- *Responses*: What a person can do in response to the external demands (e.g., compute, interpret, communicate, etc.);
- *Mathematical ideas/content*: The informational content which the context carries or enables access to, which can be seen as mathematical (or statistical) in nature and hence of interest in the context of an assessment of numeracy (e.g., numbers, proportions, measurements, statistical concepts, etc);
- *Representations*: The different ways in which the mathematical (or statistical) information exists or is conveyed to the person in the given context (e.g., text, numbers, graphs, etc.)

52. The advantage of using a more elaborate definition of numerate behavior was that it is more explicit about what to examine in an assessment, and thus serves as a springboard for developing an actual specification for an assessment scale. It is important to also note that the definition of numerate behavior points to the presence of both cognitive and non-cognitive factors which underlie or enable effective numerate behavior. Ideally, coverage of both cognitive and non-cognitive aspects of numerate behavior is essential in order to generate a full picture regarding the competence of numeracy.

2.3.4 Definition of numeracy for PIAAC.

53. The development of the conceptualisation and definition of numeracy for PIAAC went through several stages of work and consultation. An expert panel appointed to develop the overall assessment design for PIAAC presented in summer 2006 tentative recommendations regarding all competencies to be assessed in PIAAC (OECD, 2006) and then proposed to define numeracy as: *"The ability to use, apply, and communicate mathematical information"*. Various perspectives on numeracy and its assessment were later examined by participants at the Canada-OECD Expert Technical Workshop on Numeracy, which met in November 2006 in Ottawa; a tentative working definition of numeracy was then proposed for PIAAC and included in a draft framework circulated for external review (Gal, 2007). Further development of the

numeracy framework has been undertaken by the Numeracy Expert Group for PIAAC appointed in April 2008, which released a revised framework for review by all participating countries in October 2008.

54. In general, work on the development of a numeracy framework for PIAAC, together with the assessment scale and related item pool, has been conducted with two somewhat conflicting objectives in mind. One objective is the need to maintain compatibility with the conceptualisation of numeracy in the ALL survey, given the need for PIAAC to provide trend data related to ALL results. For this reason PIAAC was designed with a specification that 60% of the literacy and numeracy tasks that will be employed in the final assessment scale will come from item pools used in ALL and IALS. The other objective is the need to extend the ALL definition in light of PIAAC's overarching conceptualisation of "literacy competencies in the information age", and consider new or emerging uses of numeracy in the adult world.

55. Taking all the above into consideration, numeracy has been defined for PIAAC as follows:

> **Numeracy** *is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life*

56. This definition captures essential elements in numerous conceptualisations of numeracy in the extant literature; it is compatible with the definition used for ALL and appears to provide a solid basis from which to develop an assessment scale for PIAAC with its emphasis on competencies in the information age. The inclusion of "engage" in the definition signals that not only cognitive skills but also dispositional elements, i.e., beliefs and attitudes, are necessary for effective and active coping with numeracy situations. It is also important to note that while the definition of numeracy for PIAAC has been developed in the context of an assessment programme, it has been crafted so as to contribute to public dialogue regarding the goal of educational and social interventions focused on developing adult competencies in general, and adult numeracy and related mathematical and statistical skills and dispositions in particular.

57. However, since numeracy is a broad, multifaceted construct referring to a complex competency, the definition of numeracy given above should not be considered by itself, but should be coupled with a more detailed definition of *numerate behavior* and with further specification of the facets of numerate behavior. This pairing is essential in order to enable operationalisation of the construct of numeracy in an actual assessment, thereby contributing to the assessment's validity and interpretability, and in order to further broaden the understanding of key terms appearing in the definition itself. Consequently, a definition of numerate behavior similar in general terms to the one used for the ALL survey, but shorter, has been adopted for PIAAC:

> **Numerate Behavior** *involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways.*

Table 1: Numerate behavior – key facets and their components

Numerate behavior involves managing a situation or solving a problem…

1. **in a real context:**
   - everyday life
   - work
   - societal
   - further learning
2. **by responding:**
   - identify, locate or access

- act upon, use: order, count, estimate, compute, measure, model - interpret - evaluate / analyze - communicate 3. **to mathematical content/ information/ ideas:** - quantity & number - dimension & shape - pattern, relationships, change - data & chance 4. **represented in multiple ways :** - objects & pictures - numbers & mathematical symbols - formulae - diagrams & maps, graphs, tables - texts - technology-based displays
Numerate behavior is founded on the activation of several enabling factors and processes: - mathematical knowledge and conceptual understanding - adaptive reasoning and mathematical problem-solving skills - literacy skills - beliefs & attitudes - numeracy-related practices and experience - context/world knowledge

58. The definition of numerate behavior pertains to four facets: *Contexts, Responses, Mathematical content/information/ideas, Representations*. Table 1 lists the components of the four facets, and these components are explained in more detail the next section. Table 1 is based on the original description of the facets of numerate behavior developed for the ALL survey, but some changes have been implemented, such as the addition of "access" and of "evaluate/analyze" as possible responses, the merging of the content categories of "change" and "pattern and relationship", or the reference to "technology-based displays" as another representation mode.

59. It should be noted that the bottom part of Table 1 also lists several enabling factors and processes elaborated in section 3.5, whose activation underlies numerate behavior. Most of these enabling factors and processes appeared in the ALL conceptual framework, but some changes were introduced, such as the positioning of "adaptive reasoning and mathematical problem-solving" as a separate enabling factor. Overall, the definition of numerate behavior presented earlier, together with the details in Table 1 and the further explanations below, provided a roadmap for the development of a numeracy scale for PIAAC.

PART 3. FACETS OF NUMERACY

3.1 Facets of numerate behavior

60. This section[3] elaborates on the facets of numerate behavior and their components, listed in Table 1. The discussion of the first facet, context, mainly revisits ideas which appeared in section 2.2 above regarding contexts and demands for numeracy. Elaborations on the other facets are based on materials and ideas in the bibliographic sources mentioned earlier, and the analysis of the components of adult numeracy by Ginsburg et al. (2006) which was based on an integrative review of multiple numeracy frameworks from several countries (see also Hagedorn et al, 2003). This section has also benefited from the positions presented in a report of the UK's National Research and Development Centre for Adult Literacy and Numeracy (NRDC, 2006), background papers prepared for the OECD-Canada Expert workshop on numeracy (Nov. 2006, Ottawa) and suggestions made by workshop participants, external reviews of earlier drafts of this framework, and professional perspectives of PIAAC's Numeracy Expert Group.

3.1.1 Facet 1: Contexts.

61. People try to manage or respond to a numeracy situation because they want to satisfy a purpose or reach a goal. Four types of contexts where demands on people's numeracy may appear are described below. These are not mutually exclusive and may involve the same underlying mathematical themes.

a. **Everyday life.** The numeracy tasks that occur in everyday situations are often encountered in personal and family life, or revolve around hobbies, personal development, and interests. Representative tasks are handling money and budgets, comparison shopping, personal time management, making decisions involving travel, planning holidays, mathematics involved in hobbies like quilting or wood-working, playing games of chance, understanding sports scoring and statistics, reading maps, and using measurements in home situations such as cooking or home repairs.

b. **Work-related.** At work, one is confronted with quantitative situations that often are more specialised than those seen in everyday life. In this context, people may develop good skills in managing situations that might be narrower in their application of mathematical themes. Representative tasks are completing purchase orders, totaling receipts, calculating change, managing schedules, budgets, and project resources, using spreadsheets, organizing and packing different shaped goods, completing and interpreting control charts, making and recording measurements, reading blueprints, tracking expenditures, predicting costs, and applying formulas.

c. **Societal or community.** Adults need to know about trends and processes happening in the world around them (e.g., regarding crime, health issues, wages, pollution) and may have to take part in social events or community action. This requires that adults can

3 Selected text portions in section 3.1, as well as in subsections 3.1.3 – 3.1.5, were adapted and expanded from the numeracy framework developed in 1998-2000 for the ALL survey, and used here with permission of Statistics Canada. The final version of the ALL numeracy framework is presented by Gal, van Groenestijn, Manly, Schmitt, & Tout (2005).

read and interpret quantitative information presented in the media, including statistical messages and graphs. Also, they may have to manage situations like organizing a fund-raiser, realizing the fiscal effect of community programs, or interpreting the results of a study about a health issue.

d. **Further learning.** A numeracy competency may enable a person to participate in further study, whether for academic purposes or as part of vocational training. In either case, it is important to be able to know some of the more formal aspects of mathematics that involve symbols, rules, and formulas and to understand some of the conventions used to apply mathematical rules and principles.

62. It should be emphasised that performance in all of the above contexts is based on a combination of cognitive and non-cognitive elements, and thus requires that we think of numeracy as a competence as defined earlier, not just as possession of a set of technical skills or know-how. For example, engagement in further learning of mathematical topics, whether in formal or informal contexts, involves willingness to start such learning in the first place as well as perseverance in such learning. For such engagement to occur, an adult is required to have positive beliefs and attitudes about mathematics and about oneself as a person capable to cope with mathematical tasks.

3.1.2 Facet 2: Responses.

63. In different types of real-life situations, people may have to react with diverse types of responses, grouped below under three broad headings: *Identify, locate, or access; Act upon or use;* and *Interpret, evaluate/analyse, communicate.* It should be noted that while these types of responses are described separately, in real life they may co-occur in a dynamic fashion, and may vary from simple to more complex. The extent to which response types will vary in complexity or co-occur in an integrated way depends on various aspects of the situation at hand discussed later, such as the density of the information available, presence of distracting information, transparency of the task, literacy demands, number of steps and iterations involved. Further, responses are shaped by the interaction between situational demands on the one hand and the goals, skills, dispositions, and prior practices and experiences of the person on the other hand.

a. **Identify, locate, or access**. In virtually all situations, people have to identify, locate or access some mathematical information present in the task or situation confronting them that is relevant to their purpose or goal. When it exists alone, this response type often requires only low level mathematical understanding or application of simple arithmetic skills. Usually, however, this response type is subsumed or co-occures with the other types of responses listed below.

b. **Act upon or use.** In situations described earlier (see 3.2.4) as "generative", people have to perform actions on the mathematical information which can be identified in the situation, or use known mathematical procedures and rules. Acting upon or using encompasses arithmetical operations such as when counting, doing calculations "in the head", with pen and paper or with a calculator. Acting upon or using may also involve ordering or sorting, estimating, figuring out an area or volume of a certain object in an approximate way, or using various measuring devices to generate needed mathematical information of a more exact nature. Finally, acting upon may involve using (or developing) a formula which serves as a model of a situation or a process.

c. **Interpret, evaluate/analyse, communicate.** This response type encompasses three separate but related responses, described below in more detail:

- ***Interpret.*** Some situations do not demand any direct manipulation or action on available quantitative information, but the interpretation of the meaning and implications of given information of a mathematical or statistical nature. Further, in such situations, described earlier as "interpretive", the person in the situation may need to not only interpret mathematical or statistical information but also make a judgment or create an opinion, such as about trends, changes, or differences described in a graph or in a text appearing in a newspaper article or advertisement. It should be emphasised that interpretive responses may be in reaction not to information that is numerical (i.e., figures or statistical data), but to broader mathematical or statistical concepts possibly expressed in oral, textual, or visual manner, including ideas such as rate of change, proportions, shape of distribution, samples, bias, correlation, probability or risk, or causality.

- ***Evaluate/analyse.*** This response category is in part an extension of the Interpret response type described above, which was used in ALL. It accommodates responses that may be more likely in situations requiring a person to analyze a problem and in so doing evaluate the quality of the solution against some criteria or contextual demands, and if needed cycle again through the interpretation, analysis and evaluation stages. Such situations may be encountered in various contexts, including in dynamic or information-rich technology environments, or those termed earlier 'decision situations'. Examples are when, as part of developing a solution to demands of a given situation, an adult has to process raw quantitative information through technology-enabled channels (e.g., sift through a website of a national statistical office), or retrieve and integrate information from multiple sources after evaluating their relevance to the task at hand (e.g., compare information from different sources regarding costs of competing courses of action).

- A common feature that pertains both to "interpret" and "evaluate/analyze" responses is that the judgments or opinions expected in the situation may need to be critical in nature. Examples are when a person needs to question the validity of the data or information presented, identify gaps between the available information and the conclusions presented by a source (e.g., a journalist or politician), or reflect about the proposed implications of the data, both for himself or herself as an individual or for the wider community. As a result, these two responses types have an inherent overlap between them.

- Communicate. In addition to the responses listed above, a person may have to represent and communicate about the mathematical information given, describe the results of one's actions or interpretations to someone else, or explain and justify the logic of one's analysis or evaluation. This can be done via oral or written means (ranging from presenting a simple number, a word, all the way to a detailed explanation), through a drawing (a diagram, map, graph) or by generating a computer-based display (e.g., by referencing a spreadsheet-based chart showing the results of "what if" scenarios), and various combinations of these and other modes of communication and illustration.

64. Problem-solving is *not* seen as a separate response type, but rather assumed to be part of the demands set forth by the external situation - as is implied in Table 1, the goal of numerate behavior is managing a situation involving a numeracy task or *solving* a numeracy-related problem. Hence, several response types described above may be called upon and co-occur when people have to solve numeracy-

related problems, especially novel ones. Such responses may be aided or organised by more generalised skills of adaptive reasoning and problem-solving, examined later on as part of "enabling processes" that underlie numerate behavior.

Note about assessment of certain response types:

65. The ideas above describe key ways in which people may respond to mathematical/statistical tasks embedded in a range of real-life situations. However, one needs to distinguish between a conceptual framework (discussed in this section) and an assessment framework (discussed later on). Not all real life numeracy tasks can necessarily be simulated well in a specific assessment. Further, the ability of an assessment to actually *capture, evaluate, and score* responses associated with numerate behavior ultimately depends on the technical aspects of that assessment. While the computer-based assessment platform chosen for PIAAC offers many advantages, the design specifications adopted by participating countries pose limitations on skills assessments in PIAAC, due to the computer-based environment employed for the assessment, the need for immediate scoring of responses given the adaptive testing process necessary for efficient ability estimation, and restrictions on testing time per respondent which are typical in large scale household surveys. These realities necessitate the use of short separate tasks, exclude extended problem tasks, and prohibit the use of most types of numeracy tasks that require respondents to communicate via free-form text input. Specifically, tasks requiring communication-based responses, such as when adults have to explain interpretations of given information, or describe their evaluation or analysis of a situation *or their thinking about that situation*, could hardly be used in the direct assessment of all skills targeted by PIAAC. Such tasks do comprise an important, inseparable part of the landscape of adult numeracy situations and are an inherent part of the conceptual framework of adult numeracy, yet very few could be included in the item pool for the first cycle of PIAAC.

3.1.3 Facet 3: Mathematical content/information/ideas.

66. Mathematical information can be classified in several ways and on different levels of abstraction. One approach is to refer to fundamental “big ideas” in mathematics. Steen (1990), for example, identified six broad categories: Quantity, Dimension, Pattern, Shape, Uncertainty, and Change. Rutherford & Ahlgren (1990) described networks of related ideas: Numbers, Shapes, Uncertainty, Summarizing data, Sampling, and Reasoning. Dossey (1997) categorised the mathematical behaviors of quantitative literacy as: Data representation and interpretation, Number and operation sense, Measurement, Variables and relations, Geometric shapes and spatial visualisation, and Chance. More broadly, many curriculum frameworks around the world in one way or another refer to these key areas, albeit using somewhat different terminologies and with somewhat different groupings (e.g., NCTM, 2000).

67. Based on such and related classifications, the ALL numeracy framework (Gal et al., 2005) defined five areas of mathematical content and ideas that characterise the mathematical demands faced by adults: *Quantity & number; Dimension & shape; Data & chance; Pattern, functions, & relationships; Change*. In PIAAC the first three areas were retained, while the last two areas listed for ALL were united to create a single area called “Patterns, relationships and change”. Such a change was deemed sensible because these two areas are sufficiently related. Indeed, Ginsburg et al, 2006, in their analysis of the components of numeracy, subsumed “change” as part of a unified domain termed “patterns, functions, and algebra.” Uniting the two areas has to advantages: it enables PIAAC to focus on domains that are more distinct from each other, as well as maintains better conceptual compatibility with PISA, an issue discussed in Section 5.

68. With the above in mind, four key areas of mathematical content, information and ideas are covered by the numeracy assessment in PIAAC and are briefly summarised below.

a. **Quantity and Number.** *Quantity* is described by Fey (1990) as an outgrowth of people's need to quantify the world around us, using attributes such as: numbers of features or items; costs and charges for goods and services; size (e.g. length, area, and volume); temperature, humidity, and pressure of our atmosphere; populations and growth rates of species; revenues or profits of companies, etc. *Number* is fundamental to quantification and different types of number constrain quantification in various ways: whole numbers can serve as counters or estimators; fractions, decimals and percents as expressions of greater precision, parts or comparisons; and positive and negative numbers as directional indicators. In addition to quantification, numbers are used to put things in order and as identifiers (e.g., telephone numbers or zip codes). There is also the requirement to operate on such quantities and numbers (the four main operations of +, –, x, ÷ and others such as squaring). Facility with quantity, number, and operation on number requires a good "sense" of magnitude. Contextual judgment comes into play when deciding how precise one should be or which tool (calculator, mental math, a computer) to use. Money and time management, the ubiquitous mathematics that is part of every adult's life, depends on a good sense of number and quantity. A basic level numeracy task might be figuring out the cost of one can of soup, given the cost of 4 for $2.00; a task with a higher cognitive demand could involve more complex numbers such as when figuring out the cost when buying 0.283 kg of cheese at 12.95 Euros per kg.

b. **Dimension and shape**. *Dimension* includes "big ideas" related to one, two, and three dimensions of "things" (using spatial and numerical descriptions), projections, lengths, perimeters, areas, planes, surfaces, location, etc. Facility with each dimension requires a sense of "benchmarks" and estimation, direct measurement and derived measurement skills. *Shape* is a category describing real images and entities that can be visualised (e.g., houses and buildings, designs in art and craft, safety signs, packaging, snowflakes, knots, crystals, shadows and plants),in both two and three dimensions . Direction and location are fundamental qualities called upon when reading, interpreting or sketching maps and diagrams. This content area requires an understanding of units and systems of measurement, both informal and standardised such as the Metric and Imperial systems. A basic numeracy task in this fundamental aspect could be shape identification whereas a complex task might involve describing the change in the capacity of an object when one dimension is changed.

c. **Pattern, relationships, and change.** It is frequently written that mathematics is the study of patterns and relationships. *Pattern* is seen as a wide-ranging concept that covers patterns encountered all around us, such as those in musical forms, nature, traffic patterns, etc. It is argued by Senechal (1990) that our ability to recognise, interpret, and create patterns is the key to dealing with the world around us. The human capacity for analyzing and identifying patterns and relationships undergirds much mathematical thinking. *Relationships* and *change* relate to the mathematics of how things in the world are associated or develop. Individual organisms grow, populations vary over time, prices fluctuate, and objects traveling speed up and slow down. Some characteristics or values can change directly in proportion or relation to another change, whilst other characteristics may change in the opposite direction or in a different way. Change and rates of change help provide a narration of the world as time marches on. The ability to generalise and to characterise relationships between variables is a crucial gateway to understanding basic economic, political or social analyses. This domain includes the ability to develop and/or use a mathematical formula between the different variables involved in a situation, alongside the need to be able to understand, use and apply a

sense of proportional reasoning. A lower level numeracy task may ask someone to use a familiar formula such as that for calculating the area of a square or rectangle. More demanding tasks involving relationships and change may require using formulae such as for calculating compound interest or one's BMI (Body Mass Index). Or, tasks could require using an electronic spreadsheet or a Web-based dedicated calculator (applet) for exploring "what if" scenarios related to different interest rates, or to different levels of weight loss or weight gain, and their impact on one's long-range savings or health risk levels, respectively.

d. **Data and chance.** Data and chance encompass two related but separate topics. *Data* covers "big ideas" such as variability, sampling, error, or prediction, and related statistical topics such as data collection, data displays, and graphs. Modern society demands that adults interpret and produce organisers of data such as frequency tables, pie charts, graphs and to sort out relevant from irrelevant data. *Chance* covers "big ideas" related to probability, subjective probability, and relevant statistical methods. Few things in the world are 100% certain; thus the ability to attach a number that represents the likelihood of an instance is a valuable tool whether it has to do with the weather, the stock-market, or the decision to board a plane. In this mathematical category, a simple numeracy skill might be the interpretation of a simple pie chart; a more complex task would be to infer the likelihood of an occurrence, such as predicting the weather, based upon past information.

3.1.4 Facet 4: Representations of mathematical information.

69. Mathematical information in a situation may be available or represented in many forms. It may appear as concrete objects to be counted (e.g., people, buildings, cars, etc.) or as pictures of such things. It may be conveyed through symbolic notation (e.g., numerals, letters, and operation or relationship signs). Sometimes, mathematical information will be conveyed by formulae, which are a model of relationships between entities or variables. Mathematical information may be encoded in visual displays such as a diagram or chart; graphs and tables may be used to display aggregate statistical or quantitative information (by displaying objects, counting data, etc.). Similarly, a map of a real entity (e.g., of a city or a project plan) may contain information that can be quantified or mathematised. Last but not least, textual elements may carry much mathematical information or affect the interpretation of mathematical (and statistical) information, as explained further below.

70. A person may have to extract mathematical information from various types of *texts*, either in prose or in documents with specific formats (such as in tax forms). Two different kinds of text may be encountered in numeracy tasks. The first involves mathematical information represented in textual form, i.e., with words or phrases that carry mathematical meaning. Examples are the use of number words (e.g., "five" instead of "5), basic mathematical terms (e.g., fraction, multiplication, percent, average, proportion), or more complex phrases (e.g., "crime rate increased by half") which require interpretation, or coping with double meanings (or with differences in mathematical and everyday meanings of the same terms). The second involves cases where mathematical information is expressed in regular notations or symbols (e.g., numbers, plus or minus signs, symbols for units of measure, etc.), but is surrounded by text that despite its non-mathematical nature also has to be interpreted in order to provide additional information and context. An example is a bank deposit slip with some text and instructions in which numbers describing monetary amounts are embedded, or a parking ticket specifying an amount of money that has to be paid by a certain date due to a parking violation, but also explaining penalties and further legal steps that will be enacted if the fine is not paid by a certain date.

3.2 Enabling processes: cognitive and non-cognitive

71. People's numeracy competence is revealed through their responses (i.e., identifying, interpreting, acting upon, evaluating, and communicating) to the mathematical information or ideas that may be represented in a situation or that can be applied to the situation at hand. It is clear that numerate behavior will involve an attempt to engage with a task and not delegate it to others or deal with it by intentionally ignoring its mathematical content. Numerate behavior, however, depends not only on cognitive skills or knowledge bases, but also on several enabling factors and processes listed in Table 1 (NRDC, 2006; Tout, 2006).

72. Specifically, the enabling processes involve integration of mathematical knowledge and conceptual understanding with broader reasoning, problem-solving skills, and literacy skills. Further, numerate behavior and autonomous engagement with numeracy tasks depend on the dispositions (beliefs, attitudes, habits of minds, etc), and prior experiences and practices that an adult brings to each situation. These are briefly discussed below. Most of these enabling factors and processes have also been described by Kilpatrick (2001) as part of his analysis of the construct of mathematical literacy, and further examined and deemed relevance for description of adult numeracy in a recent analysis by Ginsburg et al. (2006).

3.2.1 Mathematical knowledge and conceptual understanding.

73. The notion of conceptual understanding refers to *an integrated and functional grasp of mathematical ideas* (Kilpatrick, Swafford & Findell, 2001: 118). Ginsburg et al (2006) suggest that the two aspects of conceptual understanding, i.e., it being integrated and functional, frame the ability to think and act numerately and effectively, and that across different numeracy frameworks in different countries, equivalent terms are used such as “meaning making,” “relationships,” “model,” and “understanding.” Conceptual understanding can help learners produce reasonable estimates that can help them catch computational errors, or realise that an exact product is not necessary, but an estimate is enough for the purpose. Ginsburg et al (2006) further explain that conceptual understanding permits one to be free from relying on memory for all methods and procedures, i.e., an adult can think about the meaning of the task and “construct or reconstruct” a representation that both illustrates what it means and suggests a method for solution. As an example they state that a fundamental conceptual understandings include interpreting and visualizing 23 x 13 as the repeated addition of 13 objects, 23 times (one could arrive at an accurate answer by adding groups), or as a 23 by 13 rectangular array (one could count the elements in the array).

3.2.2 Adaptive reasoning and problem-solving skills.

74. Throughout life, adults develop or apply diverse strategies to manage their quantitative situations. Some strategies may be based on prior formal learning, while others may be self-invented or adapted to fit the situation at hand. To solve computational problems or to manage certain quantitative tasks, people have to re-construct reality in a mathematical way, for example, model or mathematise. They can do so either on their own or in discussion with other people. Problem-solving strategies may include, e.g., extracting relevant information from the task/activity; rewriting/restating the task; drawing pictures, diagrams or sketches; guessing and checking; making a table; and/or generating a concrete model or representation (Kilpatrick, 2001; Ginsburg et al., 2006).

3.2.3 Literacy skills.

75. The ability to read, write, and talk are important skills in undertaking a numeracy task or activity or communicating the outcomes of working on such tasks. In cases where “mathematical representations” involve text, one's performance on numeracy tasks will depend not only on formal mathematical or

statistical knowledge but also on reading comprehension and literacy skills, reading strategies, and prior literacy experiences. For example, following a computational procedure described in text (such as the instructions for computing shipping charges or adding taxes on an order form) may require special reading strategies, as text is very concise and structured. Likewise, analyzing the mathematical relationships described in words requires specific interpretive skills, as in the simple case of recognizing the similarity of "the price doubled" and "the priced was twice as high", but the different meanings in "production levels were constant over the last five years" and "production levels constantly increased over the last five years".

3.2.4 Context/world knowledge.

76. Proper interpretation of mathematical information or quantitative messages by adults depends on their ability to place messages in a context and access their world knowledge, as well as rely on their personal experiences and practices, noted further below. World knowledge also supports general literacy processes and is critical to enable "sense-making" of any message. For example, adults' ability to make sense of statistical claims or media-based graphs will depend on information they can glean from the message about the background of the study or data being discussed. When interpreting statistical claims made by journalists, advertisers and the like, context knowledge is the main determinant of the reader's familiarity with sources for variation and error, helps to imagine why a difference between groups can occur (as in a medical or educational experiment), or what alternative interpretations may exist for reported findings about an association or correlation between certain variables. Likewise, world knowledge is a prerequisite for enabling critical reflection about statistical messages and for understanding the implications of the reported findings.

3.2.5 Beliefs and attitudes.

77. Research literature suggests that the ways in which a person responds to a numeracy task, including overt actions as well as internal thought processes and the adoption of a critical stance, depend not only on knowledge and skills but also on negative attitudes towards mathematics, beliefs about one's mathematical skills, habits of mind, and prior experiences involving tasks with mathematical content (Lave, 1988; Schliemann & Acioly, 1989; Saxe, 1991). In some cultures, some adults, including highly educated ones, decide that they are not "good with numbers" or have other sentiments or self-perceptions usually attributed to negative prior experiences they have had as pupils of mathematics (Tobias, 1993). Such attitudes and beliefs stand in contrast to the desired sense of "at-homeness with numbers" (Cockcroft, 1982) and can interfere with one's motivation to develop new mathematical skills or to tackle math-related tasks, and may also affect test performance (McLeod, 1992).

78. In real-world contexts, adults with a negative mathematical self-concept may elect to avoid a problem with quantitative elements, address only a portion of it, or prefer to delegate a problem, e.g., by asking a family member or a salesperson for help. Such decisions or actions can serve to reduce both mental and emotional load (Gal, 2000). Yet, such actions may fall short of autonomous engagement with the mathematical demands of real-world tasks, carrying negative consequences, e.g., not being able to fully achieve one's goals.

3.2.6 Numeracy-related practices and experiences.

79. Research suggests that, for adults as well as for children, mathematical knowledge develops both in and out of school (e.g., Schliemann & Acioly, 1989; Saxe, 1991; Lave, 1998). Saxe and his colleagues have written about the importance of cultural practice in the development of mathematical thinking and how such practices profoundly influence an individual's cognitive constructions and mathematical ideas, depending, e.g., on the artifacts or tools they use, the nature of the measurement systems in their culture, the counting or calculating devices (abacus, calculator) they use, the distribution of work among family

members, or general patterns and types of social activity. Further, the frequency of engaging with mathematical tasks or of exposure to mathematical or statistical information or displays, whether at work, home, when shopping, or in other contexts, is of much interest. Engagements or practices in this regard can be both the result of a certain skill level, but also the cause of observed skill levels, or at a minimum a factor influencing observed skill level apart from prior formal schooling.

80. The ideas above suggest that numerate behavior does not rely only on mathematical knowledge or related reasoning and problem-solving skills acquired as part of formal learning in a school context. Both attitudes and beliefs as well as numeracy-related practices and world knowledge are important enabling processes and may influence adults' ability to act in a numerate way. Therefore, scales assessing selected attitudes and beliefs about mathematics, and numeracy-related practices in work, everyday, and other settings, have been developed for PIAAC's Background Questionnaire (BQ). Information collected by such scales can help to explain differences in performance among adults, further inform our understanding of factors that affect skill acquisition and retention or motivation for further learning, and explain the links between numeracy and covariates such as participation in further learning or employment/unemployment status.

PART 4. SCALE DEVELOPMENT: PRINCIPLES, CONSTRAINTS, IMPLEMENTATION

81. The operationalisation of the construct of numeracy in a large-scale assessment scale is affected by many factors which shape the extent to which the theoretical construct can be fully addressed by the actual collection of items used in the direct assessment. This part first describes general expectations in assessing adult numeracy gleaned from prior work on assessing adults' mathematical skills and the theoretical foundations reviewed above, followed by an outline of design constraints that affect the development of a scale for direct assessment of cognitive skills in PIAAC. Based on these foundations, an outline is presented of design principles that guide the assessment of numeracy in PIAAC, and further details on a supporting scheme regarding factors that affect task complexity (or item difficulty) which is of importance both for task design as well as interpretation of results regarding numeracy in PIAAC.

4.1 General ideas about shaping tasks for assessing adult numeracy

4.1.1 Task authenticity and realism.

82. Numerous authors have highlighted the need to retain in assessments of adults' numeracy the authenticity of assessment tasks and make them as similar as possible to the way adults encounter mathematics in different life contexts. Ginsburg et al (2006: 9), for example, claim:

> *"If one accepts the premise that 'realistic' is not the same as 'real', a serious question is raised about the extent to which 'efficient', short-response standardised test items are valid measures of a person's numeracy when the items are not structured to elicit the practices an adult actually employs in a real situation".*

83. It follows that differential performances can occur when assessment is divorced from, as opposed to contextualised in, realistic settings (Lave, Murtagh, & de la Rocha, 1984). Problem-solving in contextualised real life and work activities may differ from solving school-like problems (Resnick, 1987;

Greeno, 2003). Thus, assessments of adult numeracy have to aim for a high degree of realism and authenticity in both stimuli and tasks presented to respondents. The desire to retain authenticity, however, may at times be at odds with the need to establish cultural appropriateness of tasks and stimuli and reduce context effects. Tasks deemed as authentic and valid in the context of one country or culture may be unfamiliar to a smaller or larger degree in another cultural context. This is a traditional problem in cross-cultural testing that has challenged generations of test specialists.

84. Arguably the problem of authenticity and cultural appropriateness is lessened when testing pupils in schools, such as in PISA, because test designers can use conventional mathematical terminology, formulae, symbols, and so forth; this helps school-age assessments to standardise the demands from respondents by conveying the mathematical information embodied in different situations in consistent ways regardless of the cultural context. However, testing of adults' numeracy presents more challenges because many will not remember formal school-based notations or terminology. In countries where a sizable proportion of the population are immigrants or speak multiple home-based languages, the gaps between mother tongues and school-based mathematical linguistic conventions may further affect performance on some numeracy tasks. Thus, attention has to be given to linguistic and cultural factors when adapting items for adult assessments.

4.1.2 Task format and coding.

85. Another aspect of importance in designing assessment of adult numeracy is task format, i.e., forced-choice (or multiple choice) format versus a constructed-response format where respondents communicate in their own words the answers to tasks or questions given as part of the direct assessment, or otherwise are free to choose how to respond and are not limited to a specific and small set of given responses as in multiple-choice tests. Some of the key arguments for using constructed-response formats in adult numeracy assessment are that in most real-life situations, adults have flexibility in how they choose to respond to given tasks. Many real life tasks call for approximate answers or estimates rather than for accurate results, or for opinions or judgments that adults have to express in their own words. Further, there is long-standing awareness regarding the limited ability of forced-choice items to reflect reasoning or problem-solving processes and arguments that underlie the choice of a particular response. Thus, the use of multiple-choice items undermines the ability to assess the extent to which adults can "communicate mathematically". On the other hand, the coding of the constructed responses as being correct or incorrect can be more complicated and require much further training of coders, while items where respondents have to choose from among a limited set of possible answers sometimes (but certainly not always) offer advantages in terms of cost, speed, and reliability of coding.

4.1.3 Usage of calculators and other tools or objects.

86. The assessment of numeracy, whether by paper and pencil tasks or their computer-based equivalents, has to take into account that the practice of numeracy in everyday or work situations also involves the use of certain objects and artifacts. First we should examine the use of hand-held calculators, which by now are inexpensive and would be widely available to adults from all walks of life in many countries. Calculators have been included for quite some years now in school curricula so should be familiar to many adults. Thus, hand-held calculators are tools which are part of the fabric of numeracy life in many cultures. Increasingly, respondents in large scale tests are allowed, sometimes even expected, to use calculators. However, we still see discussions as to whether an assessment should be conducted with, or without, allowing respondents to use calculators or other technological tools.

87. It follows that adults should be given access to a calculator as part of an assessment of numeracy skills, and they can then choose if and how to use it. That said, when a calculator is made available, it is not possible to know what exactly the respondent does with it or for what it is being used in each task, e.g.,

does the respondent use it to compute a result which is then given as an answer to the task, or use it to verify results that were first obtained by mental or manual (written) calculation? It is also difficult to document problems in using a hand-held calculator without having an examiner looking all the time "over the shoulder" of the respondent and intruding into the respondents' work process. Thus, while making a calculator available during testing is paramount, collecting information about its usage presents many challenges. Yet, without information about the purpose of usage, it is difficult to analyze whether the usage of a calculator helps adults cope with certain numeracy tasks, or to conclude what might be the educational or policy implications.

88. In addition to a calculator, other tools or objects could be used in certain assessment tasks. The use of a ruler or measuring tape, whether in a metric or imperial (inches) system are part of contexts where adult numeracy competence is manifested, both in certain work setting and parallel home settings (e.g., carpentry, construction, home remodeling projects). Further, the use of objects that can be counted or manipulated (coins, beans) can shed more light on the ability of low-ability or low-literacy individuals to handle certain everyday situations involving simple quantitative information. (The use of other more sophisticated objects, such as a computer spreadsheet, of course can also fit under the assessment of numeracy, but in PIAAC is taken under the framework of the Problem Solving domain).

89. The use of a calculator, ruler, or objects such as coins is in principle desired in an assessment of adult numeracy skills. Yet, actual implementation in a large-scale assessment carries both psychometric, operational, and cost implications when there is a need to test thousands of adults in their homes in multiple countries in a standard and efficient manner. For these reasons, in PIAAC's first cycle respondents are able to use hand-held calculators as well as paper (printed) rulers that have both metric and imperial measurements. However, it is not possible to use other types of objects such as country-specific coins due to the heterogeneity in this regard.

4.2 PIAAC approach to assessment

90. The PIAAC assessment design involves using a household survey methodology which assumes that overall testing time per respondent is around 60-80 minutes. During this time, there is a need to administer a short core test (screener), present direct assessment items in one or more of the different competency domains (Literacy, Numeracy, Problem-solving), and collect information about the respondents' background and various correlates of interest via the background questionnaire (BQ). The design for the direct assessment in PIAAC incorporates computer-based measures of competencies when possible, yet also use written ("paper and pencil") assessment booklets for adults who are unfamiliar or uncomfortable with computers.

4.2.1 Adaptive testing.

91. To increase assessment efficiencies, the direct assessment is administered to the majority of respondents via a computer platform, TAO, which uses a computer-based adaptive testing process. The adaptive testing process means that tasks (i.e., stimuli and questions about them) are shown on a computer screen, the respondents answer on the computer, and their *answers are automatically* (immediately) *scored as correct or incorrect*, without human judges or coders being involved in interpretation of responses. This automatic scoring is essential because adaptive testing is based on the cumulative performance on tasks; at various points during the assessment TAO decides, based on decision rules stored by the computer program, what additional assessment tasks (at higher or lower difficulty levels) to select from a pool of assessment items for presentation to the respondent.

92. The key advantage of an adaptive testing scheme is that is can achieve the best estimate of each respondent's ability level, using a smaller number of assessment items than in a traditional test design

where respondents have to answer all questions included in the test, from easiest to most difficult. Thus, adaptive testing can enable deeper and more accurate assessment of respondents ability level, while reducing response burden and the chance respondents will face many tasks which are above their ability level and hence cause frustration.

93. However, the assessment of numeracy in the first cycle of PIAAC is constrained in several ways because of the assessment design. Firstly, the overall testing time per respondent does not allow inclusion of extended problems or lengthy simulations of complex authentic numeracy tasks, although it is recognised that ability to solve complex or extended numeracy problems is an inherent part of the numeracy competency. In order to cover all facets of the numeracy construct in the limited time available, the use of a larger number of short tasks is prescribed.

94. Secondly, the need to score all responses automatically limits the type of assessment tasks that can be used. While the TAO system allows respondents to provide an answer in several different modes (e.g., numeric entry, clicking on an area of the screen, choosing from pull-down menus), in its present stage of development it cannot accept most types of free-form text-based answers because of the huge possible diversity in how respondents may enter their answers. The limitations stem from the difficulty to automatically code (i.e., designate an answer as correct or incorrect) *free-form* responses in dozens of languages while accommodating various grammatical and syntactical structures, as well as overcoming typing mistakes which are naturally expected when people type text into a computer. Examples are when respondents:

- write number ranges or estimates which have multiple mathematically-equivalent representations, such as "a quarter", "0.25", "1/4", "1 in 4", or “around five to six”, "1.00 to 6.00";

- provide explanations of how a certain result was reached (“subtracted six from the sum of 30", "I did 30 - 6");

- describe their interpretation of given information such as in a simulated media statement;

- write justifications for their answers, or list arguments supporting their conclusions.

95. As a result of the restrictions discussed above, certain types of numeracy tasks, especially those involving interpretation or evaluation/analysis with communication responses, receive only partial or slight coverage in the first cycle of PIAAC. With this in mind, as part of the search for ways to circumvent somewhat the limitation on text-processing in the computer-based testing environment, in a few numeracy questions respondent may be asked to provide an explanation for a response by choosing from pre-designed encapsulated texts, so as to simulate the way a person provides a justification for an answer in real life. However, such experimental solutions are partial at best and have their own limitations. It is thus hoped that in future cycles of PIAAC, some of the current technical limitations will be resolved, allowing for broader coverage of more aspects of the numeracy construct. In addition, it should be noted that respondents uncomfortable or unfamiliar with using computers are directed to the paper-based branch of the assessment. While the tasks they encounter in that portion of the test are mostly duplicates of computer-based items, a few do require respondents to communicate interpretations or explanations of their reasoning about some tasks via free-form written answers, thus helping to expand the coverage of the numeracy construct slightly beyond what is possible in the computer-based assessment alone.

4.2.2 Item pools and scaling.

96. The intent of PIAAC is to have its results linked to previous international adult assessments. Therefore, the general PIAAC design requires that 60 percent of the literacy and numeracy tasks will come from item pools used in ALL and IALS. These former items serve as linking items, and in addition new items were developed for PIAAC that can fit the computer-based adaptive testing requirements and constraints. Overall, the items for numeracy assessment are expected to enable reporting of respondents' performance in a manner similar to the one used in ALL and IALS, which scaled raw ability scores in the range 0-500, but mainly focused on reporting performance on five ability levels with the following tentative boundaries:

- Level 1: raw score of 0 – 225 (lowest level)
- Level 2: raw score 226 – 275
- Level 3: raw score 276 – 325
- Level 4: raw score 326 – 375
- Level 5: raw score 376 – 500 (highest level)

4.3 Principles for assessing numeracy in PIAAC

97. The development of numeracy assessment for PIAAC has been based on a number of general principles or guidelines listed below. These principles reflect the cumulative literature on large-scale assessment of mathematical skills and adult numeracy (Gal et al., 2005; Gillespie, 2004; Murat, 2005), and various background documents and positions prepared as part of the planning of PIAAC (e.g., Gal, 2006; Jones, 2006; Murray, 2006; Tout, 2006), the general ideas listed earlier in this section, as well as the known technical limitations in the first cycle of PIAAC:

a. *Items should cover as many aspects as possible within each of the four facets of the numeracy competency.* Items should require the activation of a broad range of skills and knowledge included in the construct of numeracy, as portrayed in the conceptual framework depicted in Table 1. Specifically, all four areas of mathematical content, information, or ideas (Facet 3 in Table 1) should be covered, with relative proportions as follows:

- 30% Quantity & Number
- 25% Dimension & Space
- 20% Patterns, Relationships, Change
- 25% Data & Chance

b. *Items should aspire to maximal authenticity and cultural appropriateness.* Tasks should be derived from real-life stimuli and pertain to all types of contexts or situations (i.e., everyday life, work, societal, further learning) that can be expected to be of importance or relevant in the countries participating in PIAAC. Item content and questions should appear purposeful to respondents across cultures, although it must be acknowledged

that in a large-scale assessment such as PIAAC, not all items and contexts can be personally familiar to all adults within any one country, let alone across all countries .

c. *Items should have a free-response format, to the extent feasible by the computer platform used for administering the direct assessments in PIAAC.* Items should be structured to include a stimulus (e.g., a picture, drawing, visual display) and one or more questions, the answers to which the respondent communicates via the modes available within TAO, primarily: numeric entry, click, highlight a region of the stimulus, usage of various pull-down menus. (Text entry is limited to very specific words or sometimes a simple number due to the concerns listed above regarding the inability to score text entries with keying/typing errors, and the presence of multiple ways to express the same mathematical entities in words and/or numbers). In addition, items allowing a free-form response will be used in the paper-and-pencil portion of PIAAC, which some respondents will take, allowing for some expansion of possible responses, beyond those presently afforded within the computer platform.

d. *Items should spread over different levels of ability.* Items should span the range of ability levels anticipated within PIAAC participants, from low-skilled individuals (which are of interest in countries where policies and educational programs may be earmarked for low-skill populations), all the way to those with advanced competencies.

That said, it should be recognised that the need to reduce the number of items to be administered in any one domain has led designers of past assessments (IALS and ALL for adults, PISA for school students), as well as in PIAAC, to include few very easy items (i.e., items at level 1) and few very hard items (i.e., items at Level 5). Instead, respondents will be classified as at Level 1 if they could not do well on Level 2 tasks. Likewise, those classified at Level 5 will be those who performed well on Level 4 items and on the few real Level 5 items. It follows that a more detailed assessment of the specific skills that Level 1 respondents have requires a separate diagnostic assessment, such as the assessment of component literacy skills planned for PIAAC.

Given the above, to enable the adaptive testing process reach a an efficient estimation of respondents' ability levels, the following distribution of items at the different difficulty levels is likely to be sought for constructing the numeracy item pool for the Main assessment, based on the results of the field-test (pilot) in 2010:

- Level 1: 10% of the numeracy items
- Level 2: 25% of the numeracy items
- Level 3: 30% of the numeracy items
- Level 4: 25% of the numeracy items
- Level 5: 10% of the numeracy items

e. *Items should represent the different response types*. However as mentioned already, certain types of numeracy response types, especially those requiring the use of interpretation, evaluation, analysis and communication, will receive only partial or slight coverage in the first cycle of PIAAC due the the computer based assessment platform and its constraints at this stage. Therefore for PIAAC the response types of

Interpret, Analyse/evaluate and Communicate have been collapsed into a single response type. It is hoped that in future cycles of PIAAC, some of the current technical limitations will be resolved, allowing for better coverage of more aspects of the numeracy response facet. Given the above, the following distribution of items requiring the different types of response types will be sought for constructing the numeracy item pool for the main PIAAC direct assessment:

- 10% Identify, locate or access
- 50% Act upon, use: order, count, estimate, compute, measure, model
- 40% Interpret, evaluate/analyze, communicate

f. *Items should vary in the degree to which the task is embedded in text*. Some items should be embedded in or include relatively rich texts, while others should use little or no text. This distribution aims to reflect the different levels of text involvement in real-world numeracy tasks, as well as reduce overlap with the literacy scale.

g. *Items should be efficient.* To allow for coverage of many key facets of the numeracy competency, the inclusion of a large number of diverse stimuli and questions will be needed. However, in light of testing time constraints, the use of short tasks is necessitated, precluding items that can simulate extended problem-solving processes or that require a lengthy open-ended response.

h. *Items should be adaptible to unit systems across participating countries.* Items should be designed so that their underlying mathematical demands are as consistent as possible across countries, regarding language and mathematical conventions. For example, items should be designed so that different currency systems or different systems of measurement (metric or Imperial) could be applied to the numbers or figures used. Items should retain equivalency with respect to their mathematical or cognitive demands after being translated.

4.4 Factors explaining item/task complexity

98. In planning an assessment, it is of course desirable to be able to understand what it measures. Assessment designers assume that when engaged with the assessment items (including tasks, questions, stimuli, etc), respondents activate cognitive processes and rely on stored knowledge and learned skills which are part of the construct being measured. Thus, differential performance levels can be accounted for by the underlying cognitive knowledge bases and other enabling processes. It follows that it is useful to have a theoretical model or set of assumptions regarding what factors cause certain tasks to be harder or more complex than others, so that the assessment results can be correctly interpreted. A model or scheme of factors affecting task complexity can also help when linking the assessment results to possible social (or educational) interventions, i.e., point to the skills that are lacking and have to be further developed in the population (Brooks, Heath, & Pollard, 2005).

99. Prior seminal work by Kirsch and Mosenthal (e.g., Kirsch, 2001) and earlier projects has pointed to several key factors which account for task difficulty when considering arithmetic items or items involving text comprehension, primarily readability, type of match, plausibility of distractors, operation specificity ('transparency'), and type of calculation and number of steps. The Kirsch & Mosenthal work has informed the design of assessment tasks for IALS and other surveys, and the interpretation of their results. In designing the ALL numeracy scale, the ALL Numeracy team has attempted to advance the Kirsch and

Mosenthal complexity scheme and develop tentative assumptions regarding factors which affect difficulty of multiple types of new tasks introduced to measure the numeracy construct which were beyond those encompassed by the more focused construct of Quantitative Literacy in IALS. Examples are items involving percents, knowledge of measurement and spatial reasoning, statistical concepts, and so forth.

100. The developers of the Mathematical Literacy scale for PISA (2006) also recognised multiple factors affecting item difficulty, such as the kind and degree of interpretation and reflection required by the problem, the kind of representation skills required, or the kind and level of mathematical skill required, e.g., single-step vs. multi-step problems, or more advanced mathematical knowledge, complex decision-making, and problem solving and modeling skills, or the kind and degree of mathematical argumentation required. Further factors that are assumed to affect difficulty both in PISA, ALL and other surveys relate to the degree of familiarity with the context, and the extent to which tasks require reproduction of known procedures and steps or present novel situations requiring non-routine and perhaps more creative responses. It should be noted that the PISA description of complexity factors seems quite compatible with that of ALL, although some of the terminology is different, and published PISA reports do not explain in detail how it was used to guide the design of specific items.

101. The complexity scheme for numeracy used in ALL (Gal et al., 2005) has been instrumental for the item development and scale construction stages of that study, especially in that it helped to evaluate in advance if items will span different difficulty levels. Given that PIAAC's numeracy assessment is founded on the principles developed for ALL and that the PIAAC numeracy assessment scale uses over two dozen linking items used in ALL, the ALL complexity scheme has been adopted as an analytic tool for item development and interpretation for PIAAC as well. Further details about this scheme are provided in Appendix 1, which is adapted from Gal et al. (2005).

PART 5: DIFFERENCES BETWEEN PIAAC'S NUMERACY AND RELATED SCALES

102. To gain a better understanding of what is measured in the numeracy domain in PIAAC, it is important to discuss the differences between numeracy and related constructs targeted in international assessments, such as quantitative literacy and mathematical literacy. As will be seen, the differences are more a matter of degree rather than these constructs being totally different from each other - after all in one way or another they all pertain to some aspects of people's mathematical knowledge. Further, it must be pointed out that the differences emerge in more clarity when looking not at definitions (i.e., the conceptual level) but at their *operationalisation* (i.e., the assessment scale design, constraints on assessment, and the actual assessment tasks).

5.1 Adult assessments

103. Let us first examine some conceptions developed in international surveys of adult skills. A framework developed by Kirsch and Mosenthal (see Kirsch, Jungblut, & Mosenthal, 1998) to describe adults' literacy skills, including aspects of adult's quantitative skills, has been widely implemented in multiple national and international assessment projects, most recently the International Adult Literacy Survey (IALS; see Statistics Canada and OECD, 1996, 1997). The IALS framework made use of three literacy scales—Prose Literacy, Document Literacy, and Quantitative Literacy—to operationalise its

conception of literacy. The PIAAC domain of numeracy is most closely related to the Document Literacy (DL) and Quantitative Literacy (QL) scales, defined as follows.

> DL: *The knowledge and skills required to locate and use information contained in various formats (including job applications, payroll forms, transportation schedules, maps, tables, and graphics).*
>
> QL: *The knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials (such as balancing a check book, figuring out a tip, completing an order form, or determining the amount of interest on a loan).*

104. QL tasks as well as some DL tasks have addressed important aspects of people's mathematical knowledge and skills. For example, DL tasks required respondents to identify, understand, and interpret information given in various lists, tables, charts and displays; this information sometimes included quantitative information, such as numbers or percents. QL tasks required respondents to apply *arithmetical* operations learned mostly in elementary grades; these tasks did not require respondents to cope with other types of mathematical information (e.g., measurements, shapes) or with information whose processing does not require comprehension of text. In addition, tasks used in both QL and DL scales called for a limited range of responses, i.e., exact computations or specific types of interpretations. Such tasks and responses are important by themselves, yet they represent only a subset of the much wider range of tasks and responses that are typical of many everyday and work tasks, such as sorting, measuring, estimating, conjecturing, or using models (e.g., formulas). Thus it can be concluded that QL and DL cover a subset of the dimensions and ideas captured by numeracy for PIAAC.

5.2 School-age assessments

105. In the context of international assessments of school-age students, a central construct is mathematical literacy. This term first appeared as part of the second TIMSS, where students in their final year of secondary schooling (usually 12th grade) were assessed not only on mathematical knowledge but also on "mathematics literacy" (Mullis et al, 1998:43), defined as follows:

> *Mathematics literacy items address number sense, including fractions, percentages, and proportionality. Algebraic sense, measurement, and estimation are also covered, as are data representation and analysis. Several of the items emphasise reasoning and social utility. A general criterion in selecting the items was that they should involve the types of mathematics questions that could arise in real-life situations and that they are contextualised accordingly.*

106. This definition illustrates the connection between conceptualisation and operationalisation as two building blocks of the construct. But it can also be seen that the construct of mathematics literacy was defined through its facets, without there being a general definition.

107. PISA (2006) defines mathematical literacy as follows:

> *Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.*

108. This definition shows some overlap and consistency with the conception of numeracy used in the present framework as well as with broader conceptions of *literacy* as adopted by IALS, ALL, and PIAAC.

Table 2: Mathematical content areas covered by PIAAC and PISA

PIAAC	PISA
- Quantity & number	- Quantity
- Dimension & shape	- Space and shape
- Data & chance	- Uncertainty
- Pattern, relationships, & change	- Change & relationships

109. Table 2 lists the four key content areas covered by the numeracy assessment in PIAAC, in comparison with the four content areas of mathematical literacy covered in PISA. While using somewhat different terminologies, the two schemes of mathematical content refer to quite similar domains overall and point to conceptual compatibility between PIAAC and PISA.

110. There are several additional similarities between PISA's Mathematical Literacy assessment of school students of age 15 and PIAAC's Numeracy assessment of adults, such as:

- PISA examines how students of age 15 cope with tasks embedded in four contexts: personal, educational/occupational, public and scientific, which are quite similar to those contexts discussed in section 2.4.1 above regarding adult numeracy

- The PISA assessment framework for mathematical literacy highlights PISA's focus on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms, and demanding "the ability to apply those skills in a less structured context, where the directions are not so clear, and where the student must make decisions about what knowledge may be relevant and how it might usefully be applied" (PISA, 2006; p 72).

- The description of the four types of responses (i.e., identifying, locating or accessing; acting upon, using; interpreting; and communicating), bear some resemblance to the five steps in the mathematisation cycle posited in the PISA framework as involved in solving real-life problems. These steps involve, among other things: identifying mathematical information in a situation after trimming away non-essential elements in reality, solving a mathematical problem while switching between representations and using formal operations or technical language as needed, linking mathematical solutions and making sense of them in light of the real situation, or explaining results.

- The enabling processes that support numerate behavior, discussed above, relate to the eight key competencies described in the PISA framework, such as thinking, reasoning, modeling, problem-posing and problem-solving, communication, representation, and using symbolic and technical language.

111. While there are numerous similarities between the framework for PIAAC numeracy and the PISA framework for mathematical literacy, differences also emerge, in part due to the different environments for which these frameworks have been developed. PISA relates to school-based populations, and while it is interested in students' performance on real life problem, a basic underlying assumption is that the performance is to be based on skills and dispositions acquired in a schooling context. As a result, descriptions of students' desired actions or underlying cognitive processes in operation are couched in a school-based environment. Indeed, an arguable cursory examination of published PISA items shows that there is room to further examine the extent to which items use realistic contexts and stimuli. Some items use formal symbolism that reflect an expectation for formal knowledge of what was taught in schools, yet

such knowledge is less (or not) available to adults who have been out of formal school environment for years.

112. It can be assumed that adults, much more so than 15-year olds, have personal experiences and ways of coping with everyday situations which are different than those of school-age students. Hence, the types of responses envisioned of adults tested in PIAAC, and the explanations for underlying enabling or causative factors (such as "literacy skills") are not couched in a "mathematical problem-solving" culture. For this reason, task realism in PIAAC assessment may play a somewhat different role than in PISA. As a result, the numeracy framework for PIAAC, while being informed by the established literature on school mathematics, certainly goes beyond it, and at times uses different terms and ideas that are based on additional literatures.

113. Note should also be taken of the role of *literacy* (in the narrow, technical sense of reading and writing) in PISA's mathematical literacy. Despite the inclusion of the term "literacy" in "Mathematical Literacy", the PISA mathematical assessment does *not* seem explicitly interested either in tasks where mathematical information is embedded in text, or in the influence of literacy skills on mathematical performance, described earlier in this framework. In creating the PISA Mathematical Literacy scale, relatively little effort appears to have been taken to control the literacy content of tasks; as a result, very high correlation were obtained between reading literacy and mathematical literacy scale scores; this in turn was one (but not the only) contributing factor towards a situation whereby countries scoring high on one scale usually (but not always) scored high on other scales. In contrast, the designers of the ALL numeracy scale sought to reduce the literacy demands of at least some of the numeracy items. This in turn contributed to numeracy scores having lower correlations with Document Literacy scores in ALL, compared to the relatively high correlations between Quantitative and Document Literacy scores in IALS.

5.3 Further issues in comparing large-scale assessments

114. The comparison of assessment frameworks, even in a single domain such as mathematics, can be a complex undertaking. This was demonstrated by a recent project of the National Center on Education Statistics (NCES) in the United States, which aimed to compare the mathematics frameworks and items for three large scale assessments, the National Assessment of Education Progress, TIMSS, and PISA (Neidorf, Binkley, Gattis & Nohara, 2006). The project involved comparisons of frameworks along multiple dimensions or topics, such as the mathematics content and process skills to be assessed, the main content areas included and the set of subtopics covered in each, calculator use policy, and so forth. The analysis of the commonalities and differences between the three assessments also included comparisons of hundreds of items, in terms of the mathematics content covered, performance expectations for different grade levels, the complexity of different tasks (e.g., the extent to which they require application of routinised versus novel approaches), cognitive processes underlying different items, item formats, and item contexts. This project required the work of a panel of a dozen experts over several days.

115. One of the general specifications for PIAAC's numeracy is that there would be conceptual continuity with Mathematical literacy as viewed by PISA. While the two constructs are related they should *not* be viewed as identical, for reasons explained above. Each construct has somewhat different operational implications at the level of assessment design and item content. For example, PISA items are somewhat different than PIAAC items due to differences in authenticity of tasks, the contexts from which tasks are drawn (and the role of technology and tools in them), and in particular the increased use of formal mathematical symbolisations, which many adults are normally not familiar with after leaving school. Further, numeracy implies the need for greater attention to dispositional aspects of the competence.

116. It follows from the above that the PIAAC numeracy scale is compatible and shares much common ground with PISA's mathematical literacy scale, as well as with ALL's numeracy and to some

extent with IALS QL and DL scales. Yet, the nature of the commonalities and differences cannot be fully analyzed at this stage without further investment in a more fine-grained analysis and some systematisation of terminology, thereby creating simpler bridges between PIAAC and PISA. Eventually, the meanings that can be attached to assessment results from PIAAC in the area of numeracy, and the degree of overlap between PIAAC's numeracy and PISA's mathematical literacy, depend not only on the conceptualisation of numeracy in PIAAC, but probably more so on a host of other factors discussed earlier. It must be kept in mind that an analysis of commonalities and differences between PIAAC's and PISA's mathematical scales cannot be conducted only on the conceptual level - it needs to also consider the characteristics of the actual assessment tasks. Such characteristics include task realism, density of text in stimuli, and so forth, as well as the constraints on assessment (i.e., what questions can be asked, what coding is possible, etc) imposed by the features of the computer platform for implementing PIAAC's direct assessments.

PART 6: SUMMARY AND FURTHER REFLECTIONS

117. Given the increasing need for adults to continuously adapt to changing citizenship and workplace demands (European Commission, 1996; Coben, O'Donoghue & FitzSimons, 2000), the assessment of numeracy is essential so that countries have a solid basis from which to design social interventions and effective lifelong learning opportunities that can improve competencies (OECD, 2006). Accordingly, numeracy has been conceptualised in this document as a broad construct that pertains to adults' level of coping with a diverse range of numeracy tasks couched in real-world contexts.

118. This document has reviewed literature and research regarding competencies (part 1), perspectives on the complex meanings associated with numeracy and related constructs, and the numeracy demands faced by adults in various contexts (part 2). Based on this background, later subsections in part 2 presented a definition of numeracy as a basis for developing the assessment scale for PIAAC. These theoretical foundations then served as the basis for discussing (part 3) facets of numerate behavior related to *Contexts, Responses, Mathematical content/information/ideas, and Representations*, each with several components. This document has also emphasised the importance of assessing dispositions and practices as an integral part of the numeracy competence, given that they affect performance on numeracy tasks and can correlate with various variables of interest. Later parts elaborated on assessment principles and design constraints inherent in PIAAC's assessment plan and computer-based platform (part 4), and examined commonalities and differences between PIAAC and other scales (part 5) in order to help readers understand how to interpret PIAAC results pertaining to numeracy and connect them with results regarding mathematical literacy (PISA) or Quantitative literacy (IALS).

119. At the *conceptual* level, the definition of numeracy in this framework is in general compatible with the ALL conceptualisation, yet introduces some advances of a modest scope that go beyond what existed in ALL. These changes bring the conceptualisation of numeracy presented in this document closer to the conception of "literacies in the information age" employed by PIAAC, while allowing for more compatibility with the PISA definition of mathematical literacy, as desired by OECD. The changes in the conceptualisation of numeracy were also introduced with a long-range view in mind, to enable the accommodation of new types of numeracy-related tasks and demands faced by adults in the information age in future cycles of PIAAC, while maintaining a common conceptual definition across assessment cycles.

120. It should be recognised that some modest overlap exists between key constructs measured in PIAAC, i.e., Document literacy, Numeracy, and Problem-Solving in Technology-Rich Environments, on the conceptual level *but* also separately at the scale or item level. In particular, numeracy includes as one of multiple sub-areas the ability to read and interpret quantitative information in graphs and tables, yet this is also subsumed as one part of Document Literacy. The existence of such an overlap is sensible and expected. After all, many real-world tasks, such as interpretive tasks which do not require the manipulation of numbers, or tasks involving quantitative statements embedded in text, require adults to *integrate* the use of numeracy and literacy skills (Kirsch et al., 1993; Gal, 2002a). Likewise, numeracy involves the ability to solve multi-step or extended problems couched in a technology context, not just short simple tasks, and some numeracy-related content is encountered when adults handle problems involving financial, scheduling, or other everyday tasks. Hence tasks chosen for the Problem-solving scale may also touch on numeracy topics. The presence of overlap between scales, however, should not be a cause for concern or a reason for restricting the conceptual definition of numeracy. The construct of numeracy stand on its own and reflects an authentic part of the adult world. In addition, one must also consider that "assessment drives instruction": Excluding some areas from the *conceptualisation* of numeracy to reduce overlap or inter-scale correlations may limit what delivery systems in, e.g., educational or workplace training contexts, will target when trying to develop desired competencies.

121. As emphasised earlier, what is measured by a numeracy assessment scale (direct assessment) and the associated BQ items (regarding numeracy-related practices, attitudes, beliefs, and so forth) is determined not only by a conceptual framework describing numeracy and its facets and enabling factors - it is also determined by an *assessment* framework. Such a framework describes how the general conceptualisation of numeracy is operationalised and manifested in the nature and range of tasks used in the actual assessment, and specifies what limitations may be created due to the mode of task administration and of scoring.

122. Overall, an assessment is a complex dynamic system that combines conceptual and technical elements. Eventually, the outcomes of an assessment, and the reliability, validity, and usefulness of the findings and their interpretations, are influenced by criteria and values placed by the assessment designers, and by the many choices they make about task design and methodology. The implementation of the numeracy conceptual framework in the first assessment cycle of PIAAC is influenced by practical limitations of the computer-based platform, which prohibited the use of certain types of open-ended, communication and evaluative type items. In addition, the need to cover a broad construct within the short timeframe available for assessment, a typical situation in large-scale household-based assessments, has forced the use of short tasks with machine scorable response formats, further limiting the use of extended problems. It is hoped that these limitations will gradually lessen as greater sophistication in computer platforms will enable more flexibility in task design, question-posing and response formats in future cycles of PIAAC.

ANNEX

FACTORS AFFECTING COMPLEXITY OF NUMERACY ITEMS

1A. This appendix describes a scheme of factors that account for the difficulty of different numeracy assessment tasks. The scheme was developed for the Adult Literacy and Life Skills Survey by the ALL Numeracy team. The following text is copied (with permission) from the ALL Technical Report, see Gal et al., 2005. This scheme was found useful to inform item development, i.e., help in the creation of items that spread over a range of difficulty levels. Results from the ALL pilot study showed that predicted difficulty of items used by the scheme described below was highly correlated with observed difficulty (r = 0.79). Because of the recursive nature of the testing of this scheme (e.g., the same individuals wrote the scheme and rated the complexity of items), caution should be exercised in further interpretive use of the present version. While further validation is needed, the scheme in its current state nonetheless appears to also be a possible useful tool for interpretation of testing results.

Previous research on task complexity

2A. In IALS, three factors were found to be the principal components of task difficulty (regarding literacy or text-based tasks): plausibility of distractors, type of match required, and type of information required. The difficulty of the Quantitative Literacy tasks appeared to be a function of several other factors:

1. The particular arithmetic operation required to complete the task

2. The number of operations needed to perform the task

3. The extent to which the numbers are embedded in printed materials

4. The extent to which an inference must be made to identify the type of operation to be performed (i.e. problem transparency; see below)

3A. The IALS QL difficulty factors overall fit those used in large-scale assessments of mathematical skills (with children), which often make use of three or four factors:

1. *The mathematical concepts involved:* number systems and number sense, spatial and geometrical topics, functions and algebra, chance/statistics topics, etc. Concepts that are related to topics taught in lower grades are considered easier.

2. *The complexity of operations*: addition, subtraction, multiplication, and division, as well as dealing with whole numbers, with decimals, and with percents. Operations that are related to topics taught in lower grades are considered easier.

3. *The number of operations*: one-step problems are considered easier than multi-step problems.

4. *Problem transparency*: This factor is sometimes relevant; it refers to the extent to which the problem situation includes clearly identified numbers or entities and the extent to which it is clear what operations or actions to perform. To the extent that these are not clear or transparent, respondents have to extract needed information by applying comprehension and inference strategies, making the task more complex.

4A. There are other adult-related assessment projects on which to draw to develop the levels of complexity. Both the Essential Skills Research Project and the Applied Numeracy sub-test of the Work Keys test battery (American College Testing, 1997) use a two-factor model of complexity in their description of numeracy levels. The first factor, "operations required;" is seemingly straightforward and refers to the difficulty of operations called for. However, this is complicated by the level of difficulty of the numbers being manipulated: computations that include fractions and decimals are usually more difficult than those with whole numbers.

5A. The Essential Skills model spells out two sequences of complexity on this factor: *Operations* and *Translation* of information (sometimes called 'problem transparency').

Operations

1. Only the simplest operations are required and the operations to be used are clearly specified. Only one type of mathematical operation is used in the task.

2. Only relatively simple operations are required. The specific operations to be performed may not be clearly specified. Tasks involve one or two types of mathematical operation. Few steps of calculations are required.

3. Task may require a combination of operations or multiple-applications of a single operation. Several steps of calculation are required. (More advanced operations may call for multiplication or division.)

4. Tasks involve multiple steps of calculation.

5. Tasks involve multiple steps of calculation. Advanced mathematical techniques may be required (e.g., percents, ratios, proportions).

Translation (Problem Transparency)

1. Only minimal translation is required to turn the task into a mathematical operation. All the information required is provided.

2. Some translation may be required or the numbers needed for the solution may need to be collected from several sources. Simple formulae may be used.

3. Some translation is required but the problem is well defined.

4. Considerable translation is required.

5. Numbers needed for calculations may need to be derived or estimated; approximations may need to be created in cases of uncertainty and ambiguity. Complex formulae, equations or functions may be used.

6A. Two considerations prompted us to question the appropriateness of using mathematics-related frameworks (from Essential Skills or elsewhere) as the sole source for development of a complexity scheme for items assessing *adults'* ability to cope with real-world numeracy tasks. First, effective coping with many real-world quantitative problems depends upon people's ability to make sense of and interact with different types of texts. This is hardly recognised by the Essential Skills model. Hence, it was essential to add difficulty factors that acknowledge the inherent links between literacy and numeracy, quite similar to those used in IALS.

7A. Another, albeit a more restricted consideration, is that the ordering of complexity of tasks by the type of operation performed may not be as clear with adults as it may be with children. Such ordering in school-based assessments is predicated on traditional school curricula, where more advanced topics are learned at higher grades. However, adults are known to use a lot of invented strategies, perhaps more so, and more efficiently so, than children. Multiplication or division problems, which can prove relatively hard for some young people, may be solved by seemingly simpler strategies, such as by repeated addition or repeated subtraction; complex numbers may be broken down in ways that ease mental load, and so forth. In addition, adults' familiarity with everyday contexts, such as with monetary entities, facilitates their performance with some seemingly advanced concepts. For example, specific benchmark values of fractions and percents, such as 1/2, 1/4, 50%, or 25%, are familiar to many people; as a result, they may be easier to manage than expected, violating curriculum-based ordering of difficulty. Hence, an *overall complexity level* has to be used, in order to weight these "inconsistencies" in ordering of difficulty levels proposed in other schemes.

Complexity factors in the ALL survey

8A. The above literature review suggests that a framework of factors affecting the complexity of numeracy tasks should not only address factors related to the numerical and textual aspects of tasks, but should also address other issues. It should treat separately the number of operations and the type of operations from the *type of mathematical (or statistical) information to be processed*, which may involve numbers explicitly but also other types of mathematical information. In so doing, the desired framework of complexity factors should take into account the broad scope of the definition of numeracy, i.e., reflect the variation within contexts, the range of mathematical ideas/content, the types of possible responses, and the types of representations that cut across adult life contexts.

Table 1A. Complexity Factors - Overview

Aspects	Category	Range
Textual aspects	1. Type of match/problem transparency	Obvious/explicit to embedded/hidden
	2. Plausibility of distractors	No distractors to several distractors
Mathematical aspects	3. Complexity of Mathematical information/data	Concrete/simple to abstract/complex
	4. Type of operation/skill	Simple to complex
	5. Expected number of operations	One to many

9A. With the above considerations in mind, five key factors have been identified that are predicted to affect, separately and in interaction, the difficulty level of numeracy tasks to be used in the ALL survey. These five "complexity factors" are outlined in Table 1A and are organised in two sets: two factors that address mainly textual aspects of tasks, and three factors that address the mathematical aspects of tasks. These five factors are listed separately for clarity of presentation, but in actuality are *not* independent of

each other and do interact in complex ways. Each factor is examined in some detail below, followed by a later subsection that describes the calculation of an overall complexity level for each item, taking into account all five factors.

Type of Match/Problem Transparency.

10A. This is a combination of the factor of Problem Transparency outlined above, and of an IALS factor called Type of Match. Problem Transparency is a function of how well the mathematical information and tasks are specified and includes aspects such as how apparently the procedure is set out, how explicitly the values are stated, etc. Type of Match refers to the process that a respondent has to use to relate the requested action in the question to the information in the task or text, which can range from a simple action of locating or matching to more complex actions that require the respondent to perform a number of searches through the information given. This measure of complexity for a numeracy task incorporates the degree of text embeddedness of the mathematical information.

11A. In easy tasks, the type of information (e.g., numerical values) and the operations needed are apparent and obvious from the way the situation is organised. In more difficult ones, the values must be located or derived from other values; the operations needed may have to be discovered by the performer, depending on his or her interpretation of the context and of the kind of response expected. As well, numeracy situations may involve text to varying degrees, and this text may be of different degrees of importance. There may be a situation where there is little or no text. Some situations may involve pure quantitative information that is to be interpreted or acted upon with virtually no text or linguistic input. In other words, the performer derives all the information needed to respond from the objects present in the situation or from direct numerical displays.

12A. At a higher level, some textual or verbal information may be present alongside the mathematical information. The text can provide background information about the problem situation, or some instructions. For example, a bus schedule, cooking instructions, and a typical school-type word problem all involve some text and some numbers. Still other situations would be heavily text-based or may not involve any numbers or mathematical symbols at all, just plain text. The task will contain mathematical or statistical information that a person needs to understand and, in some cases, act upon, but it will be much less transparent. It may be heavily embedded in dense text or may require using information from a number of sources within or even outside the text/task.

13A. This factor requires that a task will be analyzed in terms of the questions: *How difficult is it to identify and decide what action to take?* and *How many literacy skills are required?*

Plausibility of distractors.

14A. This variable is literacy related, even though it can involve mathematical components. In general, literacy tasks are easiest to process when there are no plausible distractors in the text, that is, there is no other information in the text that meets any of the requirements of the task. At higher levels of difficulty, tasks can involve irrelevant information both within the question as well as within the text. In terms of mathematical information, a low level of plausible distractors would mean that no other mathematical information was present apart from that requested, making the numbers or data required easy to identify. At a higher level, there may be either some other mathematical information in the task (or its text) that could be a distractor, or the mathematical information given or requested could occur in more than one place. A higher level of complexity could also mean that outside information (e.g. the knowledge of a formula) may be needed to answer the question.

15A. This factor requires that a task will be analyzed in terms of the questions: *How many other pieces of mathematical information are present?*, and *Is all the necessary information there?*

Complexity of Mathematical Information.

16A. Some situations present a person with simple mathematical information, such as concrete objects (to be counted), simple whole numbers, or simple shapes or graphs. At lower skill levels, the information will be more familiar, whereas at higher levels, the information may be less familiar. Situations will be more difficult to manage if they involve more abstract or complex information, such as very large or very small numbers, unfamiliar decimals or percents, information about rates, or dense visual information, as in a diagram or complex table.

17A. This factor requires that a task will be analyzed in terms of the question: *How complex is the mathematical information that needs to be manipulated or managed?*

Type of Operation/Skill.

18A. Some situations require simple operations, such as addition or subtraction, or simple measurement (e.g., finding the length of a shelf), or recognition of shape. These are usually easier to analyze mathematically than situations that require multiplication or division, and than situations that require using exponents. While the difficulty of recognizing and carrying out the operation implied by a situation (be it additive, multiplicative, etc.) has direct bearing on task complexity, there may be exceptions that occur when alternative approaches are obvious. There are some tasks that combine both interpretive and generative skills and may involve a deeper conceptual understanding than merely carrying out a procedure. Other more complex tasks may involve an explanation of one's reasoning. The interpretation of information appearing in graphs, for example, becomes more complex if comparisons, conjecturing, or "reading beyond the information given" is required.

19A. This factor requires that a task will be analyzed in terms of the question: *How complex is the mathematical action that is required?*

Expected Number of Operations.

20A. Tasks that require acting upon the mathematical information given may call for one application (step) of an operation, or for one action (e.g., literal reading of information in a table, or measurement). More complex tasks will demand more than one operation, which may be the same or similar to one another, such as the steps involved in multiple passes on the data or text. Still more complex tasks are those that involve the integration of several different operations.

21A. This factor requires that a task will be analyzed in terms of the question: *How many steps and types of steps are required?*

Overall Complexity Level

22A. It is possible to estimate the overall difficulty level of a specific item by first scoring the item on each of the five factors of complexity, according to the levels described in Appendix 2, and then summing together the scores for each factor. Figure 1 below explains the process; Appendix 2 describes each level of the five factors in detail. The total summary score can range between 5 (easiest) and 19 (most difficult).

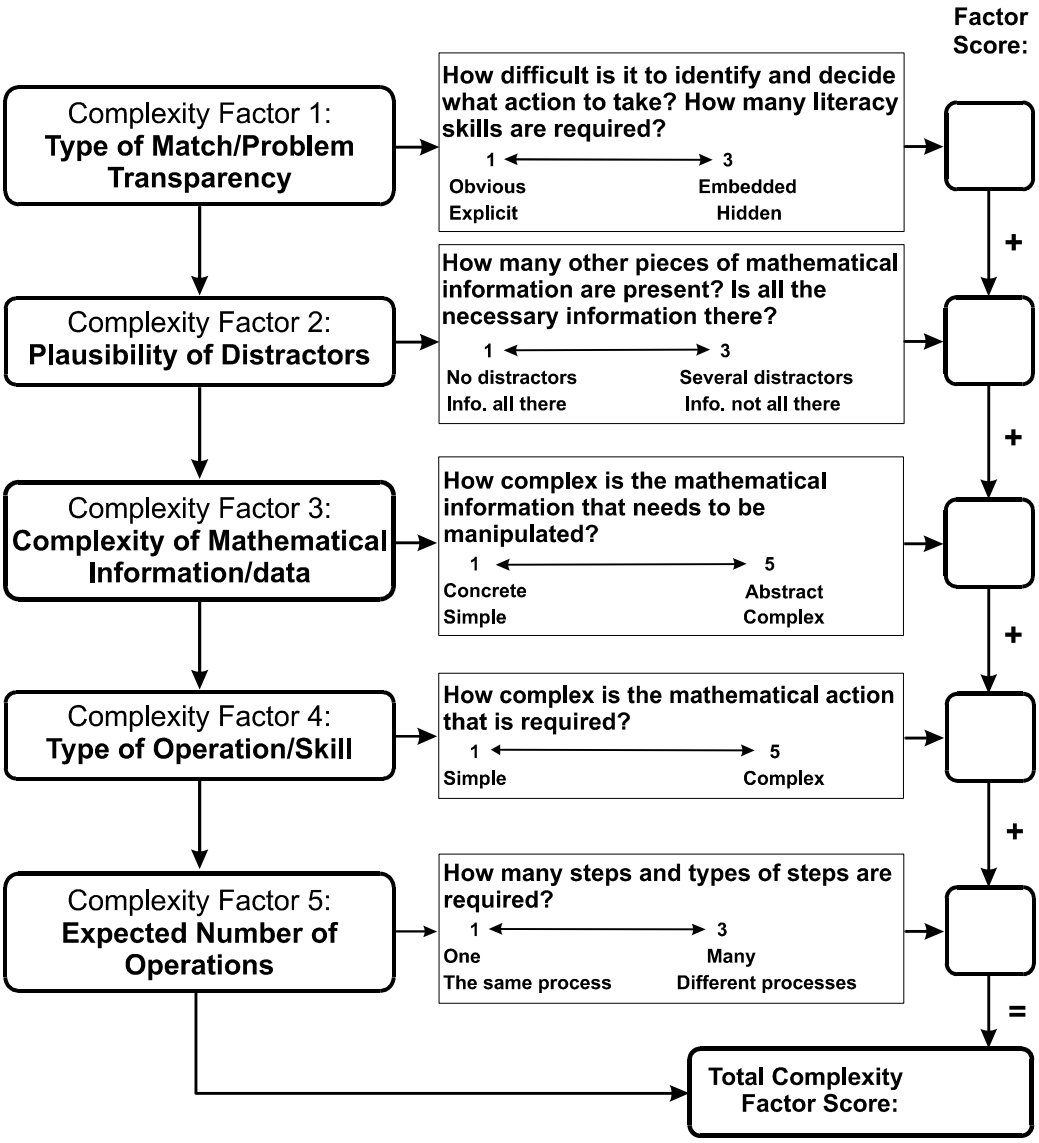
23A. The estimation process outlined in Figure 1 suggests that each factor has a separate contribution to an item's overall difficulty or complexity. However, it can be hypothesised that as tasks become more

complex, actual performance on items may increasingly depend not only on each factor by itself, but also on the interplay or interaction between them. Hence, the computational process suggested in Figure 1 can provide only approximate information about an item's anticipated difficulty level.

24A. Further, the difficulty of a task cannot in some cases be predicted without taking into account characteristics of the person who interacts with the task. The same task may be more difficult for some individuals and less difficult for other individuals, depending on factors such as their familiarity with the context in which a task is situated, knowledge of formal mathematical notations, background world knowledge, as well as general literacy, problem-solving, and reasoning skills. For example, it could be predicted that a task that involves the composition of a fertilizer would be more difficult for an urban apartment dweller than for a rural farmer whereas a task that uses a bus schedule would be more difficult for the farmer. For the above reasons, the prediction of the difficulty of a task in isolation of detailed knowledge about the respondent himself can only be an estimate.

25A. Despite the above limitations, the scheme of complexity factors developed for numeracy assessment in ALL comprises a theoretical contribution. It provides a conceptual basis for predicting the different levels of complexity of a broader range of items well beyond those involving arithmetic operations only. Indeed, this scheme was highly correlated with observed difficulty ($r = 0.79$). Because of the recursive nature of the testing of this scheme (e.g., the same individuals wrote the scheme and rated the complexity of items), caution should be exercised in further interpretive use of the present version. While further validation is needed, the scheme in its current state nonetheless appears to also be a possible useful tool for interpretation of testing results.

Figure 1A. Complexity Flowchart



SCORING FOR EACH OF THE COMPLEXITY FACTORS

Complexity Factor 1. Type of match/Problem transparency How difficult is it to identify and decide what action to take? How many literacy skills are required?		
Score 1	**Score 2**	**Score 3**
In the question and the stimulus, the information, activity or operation required: - is clearly apparent and explicit—and all required information is provided - is specified in little or no text, using familiar objects and/or photographs or other clear, simple visualisations - is about locating obvious information or relationships only - closed question—not open-ended	In the question and the stimulus, the information, activity or operation required: - is given using clear, simple sentences and/or visualisations where some translation or interpretation is required - is located within a number of sources within the text/activity. - fairly closed question	In the question and the stimulus, the information, activity or operation required: - is embedded in text where considerable translation or interpretation is required and/or - may need to be derived or estimated from a number of sources within or outside the text/activity and/or - the information or action required is not explicit or specified - more complex, open-ended task

Complexity Factor 2. Plausibility of distractors How many other pieces of mathematical information are present? Is all the necessary information there?		
Score 1	**Score 2**	**Score 3**
- no other mathematical information is present apart from that requested—no distractors	- there is some other mathematical information in the task that could be a distractor - the mathematical information given or requested can occur in more than one place - may need to bring to the problem simple information or knowledge from outside the problem.	- other irrelevant mathematical information appears - mathematical information given or requested appears in several places. - necessary information or knowledge is missing, so outside information or knowledge needs to be brought in

Complexity Factor 3. Complexity of mathematical information/answer required How complex is the mathematical information that needs to be manipulated?				
score 1	**score 2**	**score 3**	**score 4**	**score 5**
<u>Context</u> Based on very concrete, real life activities, familiar to most in daily life.	Based on common, real life activities.	Based on real life activities, but less often encountered.	Based on real life activities but unfamiliar to most	Based on abstract ideas or unfamiliar activity in a context new to most.
<u>Quantity</u> <u>Whole numbers</u> to 1,000 <u>Fractions, decimals, percents</u> - benchmark fractions ($^1/_2$, $^1/_4$, $^3/_4$) - decimal fraction for a half only (0.5) and equivalent as a percentage (50%)	- large whole numbers including millions - other benchmark fractions, like $^1/_3$ and $^1/_{10}$ - common decimals, like 0.1, 0.25 to 2 decimal places - common whole number percents, like 25% and 10%.	- large whole numbers including billions - other fractions - decimals to 3 decimal places (other than money) - all whole number percents	- negative integers - all remaining fractions, decimals and percentages	- all remaining types of rational (and some irrational) numbers including directed numbers
<u>Pattern and relationship</u> - very simple whole number relations and patterns	- simple whole number rates and ratios - whole number relations and patterns	- rates and ratios - relations and patterns including written everyday generalisations	- complex ratios, relations, patterns - simple formula	- formal mathematical information such as more complex formulae, knowledge of relationships between dimensions or variables, etc

Measures/ Dimension/Space - standard monetary values - common everyday measures for length (whole units) - time (dates, hours, minutes) - simple, common 2D shapes - simple localised maps or plans (no scales)	- everyday standard measures for length, weight, volume , including common fraction and decimal units - common 3D shapes and their representation via diagrams or photos - common types of maps or plans with visual scale indicators	- other everyday measures (area included) including fraction and decimal values - more complex 2D and 3D shapes, or a combination of 2 shapes - area and volume formulae - common types of maps or plans with ratio type scales	- all kinds of measurement scales - complex shapes or combinations of shapes	
Chance/Data - simple graphs, tables, charts with few parameters and whole number values - simple whole number data or statistical information in text	- graphs, tables, charts with common data including whole number percents—whole number scales in 1s, 2s, 5s or 10s - data or statistical information including whole number percents	- graphs, tables, charts with more complex data (not grouped data) - more complex data or statistical information including common average, chance and probability values - scales: more complex whole number, fractional or decimal	- complex graphs, tables or charts including grouped data - complex data or statistical information including probabilities, measures of central tendency and spread	

Complexity Factor 4. Complexity of Type of operation/skill How complex is the mathematical action that is required?				
score 1	**score 2**	**score 3**	**score 4**	**score 5**
Communicate no explanation - a single simple response required (orally, or in writing)	- no explanation - a simple response required (orally, or in writing	- simple explanation of a (level 1 or 2) mathematical process required (orally, or in writing) -	- explanation of a (level 3) mathematical process required (orally, or in writing)	- complex, abstract and generative reasoning or explanation required
Compute - a simple arithmetical operation (+, -, x, ÷) with whole numbers or money	- calculating common fraction, decimal fraction and percentages of values - using common rates (e.g. $/lb.); time calculations; etc - changing between common equivalent fraction, decimal and percent values, including for measurements e.g. $^1/_4$ kg = 0.250kg	- more complex applications of the normal arithmetical operations such as calculating with fractions and more complex rates, ratios, decimals, percentages, or variables - simple probability calculations	- applications of other mathematical operations such as squares, square roots, etc	- more advanced mathematical techniques and skills e.g. trigonometry
Estimate	- estimating and rounding off (when requested) to whole number values or monetary units	- estimating and rounding off to requested number of decimal places	- making a contextual judgment re whether a found answer is realistic or not and changing the answer to the appropriate correct rounded (but not necessarily mathematically correct) answer.	
Use formula/ model	- evaluating a given formula involving common operations (+, -, x, ÷)		- developing/creating and using straight forward formulae - using strategies such as working backwards or backtracking (e.g. 15% of ? = $255)	- generative reasoning - using and interpreting standard algebraic and graphical conventions and techniques

Measure - knowing common straight forward measures - naming, counting, comparing or sorting values or shapes	- visualizing and describing shapes, objects or geometric patterns or relationships - making and interpreting standard measurements using common measuring instruments	- using angle properties and symmetry to describe shapes or objects - estimating, making and interpreting measurements including interpolating values between gradations on scales - converting between standard measurement units within the same system	- calculating measures of central tendency and spread for non-grouped data - converting between non-standard measurement units within the same system - counting permutations or combinations	- converting between measurements across different systems
Interpret - locating/identifying data in texts, graphs and tables - orientating oneself to maps and directions such as right, left, etc	- reading and interpreting data from texts, graphs and tables - following or giving straight forward directions	- interpolating data on graphs - calculating distances from scales on maps	- generating, organising, graphing non-grouped data - extrapolating data - reading and interpreting trends and patterns in data on graphs, including slope/gradient	- graphing grouped data - calculating measures of central tendency and spread for grouped data

Complexity Factor 5. Expected number of operations How many steps and types of steps are required?		
score 1	**score 2**	**score 3**
one operation, action or process	application of two or three steps, the same or similar operation, action or process	integration of several steps covering more than one different operation, action or process

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