PISA

Equations and Inequalities

MAKING MATHEMATICS ACCESSIBLE TO ALL

Programme for International Student Assessment

OECD

PISA

Equations and Inequalities

MAKING MATHEMATICS ACCESSIBLE TO ALL

OECD
BETTER POLICIES FOR BETTER LIVES

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Foreword

PISA has long established that disadvantaged students tend to trail behind their privileged peers in their mathematics achievement – even if the achievement gap varies widely across countries. But that left open the question: to what extent can teachers and schools do something about this?

The PISA report, *Equations and Inequalities: Making Mathematics Accessible to All*, sheds light on this. While education systems have generally done well in providing equitable access to the *quantity* of mathematics education – in the sense that disadvantaged students spend about the same time in mathematics classes in school as their advantaged peers – the data show large differences in the *quality* of learning experiences between social groups. These inequalities result in a waste of talent.

While disadvantaged students tend to learn simple facts and figures and are exposed to simple applied mathematics problems, their privileged counterparts experience mathematics instruction that help them think like a mathematician, develop deep conceptual understanding and advanced mathematical reasoning skills.

These differences matter, because greater exposure to pure mathematics tasks and concepts has a strong relationship with higher performance in PISA, and the data suggest that exposing all students to challenging problems and conceptual knowledge in mathematics classes can have a large impact on performance. In addition, the relationship between the content covered during mathematics instruction at school and the socio-economic profile of students and schools is stronger in countries that track students early into different study programmes, that have larger percentages of students in selective schools, and that transfer less-able students to other schools.

On the one hand, the findings from this report are disappointing, in the sense that they show that mathematics education often reinforces, rather than moderates, inequalities in education. On the other hand, they show that high-quality mathematics education, and thus education policy and practice, are an essential part of the solution to redressing social inequality. Policy makers can develop more ambitious and coherent mathematics standards that cover core mathematical ideas in depth, increase connections between topics and align instructional systems with these standards. They can also reduce tracking and stratification and/or moderate their effects. Teachers can help students acquire higher-order mathematics knowledge and skills by replacing routine tasks with challenging open problems, support positive attitudes towards

mathematics, provide students with multiple opportunities to learn key concepts at different levels of difficulty, and offer tailored support to struggling students. Parents' expectations and attitudes towards mathematics matter too. And we can all do much better in monitoring and analysing not just students' learning outcomes, but students' opportunity to learn.

Andreas Schleicher
Director for Education and Skills

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Executive Summary

With numeracy skills needed more than ever in the work place, today's students must be able to compute fluently, engage in logical reasoning and use mathematics to tackle novel problems. However, PISA 2012 results show that only a minority of 15-year-old students in most countries grasp and can work with core mathematics concepts. On average, less than 30% of students across OECD countries understand the concept of an arithmetic mean, while less than 50% of students can work with the concept of a polygon.

"Opportunity to learn" refers to the content taught in the classroom and the time a student spends learning this content. Not all students, not even those in the same school, experience equal opportunities to learn. Reducing inequalities in access to mathematics is not an impossible task. PISA results show that performance disparities between socio-economically advantaged and disadvantaged students are largely linked to differences in students' familiarity with mathematics. Thus, raising disadvantaged students' opportunities to learn mathematics concepts and processes may help reduce inequalities and improve the average level of performance. This objective can be achieved through a more focused and coherent curriculum, a thorough evaluation of the effects of policies and practices that sort students by ability, and stronger support for teachers who teach heterogeneous classes.

Tracking and ability grouping affect students' exposure to mathematics and teachers' practices

Across OECD countries, socio-economic differences among students and schools account for around 9% – and in some countries, as much as 20% – of the variation in familiarity with mathematics concepts. Certain system-level policies, such as between-school tracking, academic selectivity or transferring students from one school to another because of low achievement or poor behaviour, are also associated with more unequal access to mathematics content. PISA 2012 results show that, across OECD countries, around 54% of the international differences in the impact of students' and schools' socio-economic status on students' familiarity with mathematics are explained by system-level differences in the age at which students are tracked into vocational or academic programmes.

Some countries have replaced between-school tracking with ability grouping within schools. Across OECD countries, more than 70% of students attend schools whose principal reported that students are grouped by ability for mathematics classes. But this type of ability grouping can reduce opportunities to learn for disadvantaged students just as much as between-school tracking does.

Postponing between-school tracking and reducing ability-grouping can reduce the influence of socio-economic status on students' opportunities to learn but it has an impact on teachers: they must be prepared to teach more heterogeneous classes. Teachers are generally committed to providing equal education opportunities: across OECD countries, about 70% of students attend schools where teachers believe it is best to adapt academic standards to the students' levels and needs. However, adapting instruction to each student's skills and needs while advancing learning for all students in the classroom is not easy. Teachers need more support to use pedagogies, such as flexible grouping or co-operative learning strategies, that increase learning opportunities for all students in mixed-ability classes.

Exposure to mathematics concepts and procedures matters for performance, but is not sufficient for higher-order thinking skills

PISA data confirm previous evidence that the effectiveness of instruction time closely depends on the quality of the disciplinary climate in the classroom. But, more than the amount of time, the content of instruction matters for performance.

Greater exposure to pure mathematics tasks and concepts (such as linear and quadratic equations) has a strong relationship with higher performance in PISA, even after accounting for the fact that better-performing students may attend schools that offer more mathematics instruction. In contrast, exposure to simple applied mathematics problems (such as working out from a train timetable how long it would take to get from one place to another) has a weaker relationship with student performance. This suggests that simply including some references to the real-world in mathematics instruction does not automatically transform a routine task into a good problem. Using well-designed, challenging problems in mathematics classes can have a large impact on students' performance.

The mastery of core concepts and procedures is a necessary component of mathematics learning, but is hardly sufficient for solving the most complex problems. PISA data show that frequent exposure to equations and formulas can make a difference to students tackling tasks that state the main terms of the problem and that require students to apply procedures they learned at school. But exposure to these procedures does not necessarily teach students how to think and reason mathematically. Introducing problem-solving strategies – such as teaching students how to question, make connections and predictions, conceptualise and model complex problems – requires time and is more challenging in disadvantaged schools. Restructured textbooks, teaching materials and dedicated training can help minimise the time needed to incorporate these teaching practices into an already full schedule.

Exposure to complex mathematics can influence students' attitudes

Exposure to relatively complex mathematics topics may undermine the self-beliefs of students who do not feel up to the task, while at the same time improving the attitudes and self-beliefs of those who are relatively well-prepared and ready to be challenged. On average across OECD countries, exposure to more complex mathematics concepts is associated with lower self-concept/higher anxiety among low-performing students, and with higher self-concept/lower anxiety among high-performing students. PISA finds that practices such as encouraging students to work in small groups, providing extra help to students when they need it, or reducing the mismatch between what is taught and what is assessed can improve students' self-beliefs and problem-solving skills. The data also show that students become more engaged with mathematics when they use computers in class. Moreover teachers can work with parents to improve students' attitudes towards mathematics, as PISA data suggest that parents can unknowingly transmit mathematics anxiety to their children.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

■ Table 0.1 [Part 1/2] ■
SNAPSHOT OF OPPORTUNITIES TO LEARN MATHEMATICS

Countries/economies where instruction time/exposure is **above** the OECD average
Countries/economies where instruction time/exposure is not statistically different from the OECD average
Countries/economies where instruction time/exposure is **below** the OECD average

	Time spent per week in regular school lessons in mathematics (minutes)		Exposure to applied mathematics		Exposure to pure mathematics	
	2012	Change between 2003 and 2012	Percentage of students who reported that they frequently encounter, at school, problems like "Working out from a train timetable how long it would take to get from one place to another"	Index	Percentage of students who reported that they frequently encounter, at school, equations like "$6x^2 + 5 = 29$"	Index
	Minutes	Dif.	%	Mean index	%	Mean index
OECD average	218	**13**	17.1	0.00	61.6	0.00
Chile	398	m	28.1	-0.03	55.4	-0.10
Canada	314	**91**	13.7	-0.10	59.5	-0.09
United Arab Emirates	311	m	18.1	0.07	58.4	-0.10
Portugal	288	**93**	7.3	-0.37	48.0	-0.35
Singapore	288	m	12.4	0.31	74.8	0.33
Peru	287	m	20.9	0.13	62.9	0.11
Tunisia	276	**26**	14.3	-0.20	46.7	-0.30
Macao-China	275	3	11.9	-0.11	68.3	0.21
Shanghai-China	269	m	14.2	0.18	67.0	0.06
Argentina	269	m	15.7	-0.16	50.4	-0.25
Hong Kong-China	268	-2	6.5	-0.14	64.4	0.15
Colombia	263	m	21.5	-0.16	42.5	-0.39
Qatar	259	m	26.1	0.09	50.1	-0.28
Israel	254	m	15.2	-0.39	65.4	0.03
United States	254	**33**	11.4	-0.08	65.5	0.09
Mexico	253	**18**	17.7	0.18	56.7	-0.03
Iceland	244	**-10**	23.6	0.20	72.3	0.23
Chinese Taipei	243	m	8.7	-0.11	59.6	-0.04
New Zealand	241	1	13.4	-0.05	48.4	-0.27
Australia	236	**6**	15.7	-0.10	51.1	-0.17
Japan	235	**18**	17.5	-0.18	69.4	0.19
Italy	232	**19**	11.7	-0.42	71.7	0.22
United Kingdom	230	m	18.8	0.03	62.0	0.02
Jordan	227	m	24.6	0.30	55.2	-0.22
Viet Nam	227	m	8.7	-0.23	68.0	0.17
Denmark	224	**18**	25.0	0.27	46.3	-0.37
Latvia	224	**10**	11.2	0.02	59.9	-0.01
Estonia	223	m	18.1	0.07	62.5	0.03
Russian Federation	222	**15**	25.4	0.18	75.0	0.29
Belgium	217	**21**	12.6	-0.23	62.6	-0.09
Brazil	215	4	25.8	0.05	38.1	-0.56
Korea	213	**-33**	24.3	0.40	79.4	0.43

Notes: The *index of exposure to applied mathematics* refers to student-reported experience with applied tasks at school, such as working out from a train timetable how long it would take to get from one place to another.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
The OECD average of the time spent per week in regular school lessons in mathematics in 2012 is based on all OECD countries. The corresponding OECD average reported in Table 1.6 is based on the OECD countries that participated in both PISA 2003 and PISA 2012.
Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the time spent in regular mathematics lessons.
Source: OECD, PISA 2012 Database, Tables 1.6, 1.9a and 1.9b.

StatLink http://dx.doi.org/10.1787/888933377644

■ Table 0.1 [Part 2/2] ■
SNAPSHOT OF OPPORTUNITIES TO LEARN MATHEMATICS

Countries/economies where instruction time/exposure is **above** the OECD average
Countries/economies where instruction time/exposure is not statistically different from the OECD average
Countries/economies where instruction time/exposure is **below** the OECD average

	Time spent per week in regular school lessons in mathematics (minutes)		Exposure to applied mathematics		Exposure to pure mathematics	
	2012	Change between 2003 and 2012	Percentage of students who reported that they frequently encounter, at school, problems like "Working out from a train timetable how long it would take to get from one place to another"	Index	Percentage of students who reported that they frequently encounter, at school, equations like "$6x^2 + 5 = 29$"	Index
	Minutes	Dif.	%	Mean index	%	Mean index
OECD average	218	**13**	17.1	0.00	61.6	0.00
Liechtenstein	211	-5	13.8	0.01	76.2	0.22
Spain	210	**34**	17.7	0.17	74.1	0.27
Indonesia	209	**-23**	20.2	0.05	53.5	-0.15
Greece	209	**22**	12.8	-0.41	67.5	0.05
Costa Rica	208	m	23.3	-0.37	57.1	-0.06
France	207	-1	15.9	-0.05	64.9	0.02
Switzerland	207	8	17.7	-0.02	62.7	0.01
Thailand	206	**-18**	11.6	0.40	53.0	-0.09
Luxembourg	205	**4**	20.0	-0.28	52.8	-0.25
Malaysia	201	m	10.7	0.00	59.8	-0.02
Norway	199	**33**	17.8	0.18	57.8	0.00
Poland	198	**-7**	21.2	0.48	61.8	0.09
Germany	197	**14**	15.4	0.06	68.9	0.13
Cyprus[1]	189	m	22.5	-0.17	60.4	-0.04
Ireland	189	-2	20.0	0.14	68.1	0.14
Kazakhstan	183	m	35.9	0.51	68.6	0.16
Czech Republic	182	**14**	11.0	-0.25	54.2	-0.09
Sweden	182	**17**	22.1	0.33	45.0	-0.25
Slovak Republic	181	**-18**	15.4	0.05	57.1	-0.11
Finland	175	**19**	21.1	0.23	61.3	0.00
Turkey	172	**-28**	17.0	-0.17	58.8	-0.10
Lithuania	172	m	16.6	0.19	65.3	0.13
Albania	171	m	16.6	0.22	69.5	0.15
Netherlands	171	**21**	6.8	0.22	64.6	-0.01
Romania	169	m	19.1	0.10	60.6	-0.07
Slovenia	160	m	17.7	0.04	67.2	0.20
Austria	156	**-10**	19.0	-0.03	63.8	-0.03
Uruguay	156	**-27**	12.5	-0.51	58.0	-0.06
Serbia	154	m	19.9	-0.24	60.5	-0.08
Hungary	150	**-13**	19.9	0.11	67.4	0.14
Croatia	147	m	17.6	-0.04	67.8	0.19
Montenegro	142	m	30.1	0.06	59.8	-0.09
Bulgaria	134	m	19.3	0.00	65.4	0.06

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Notes: The *index of exposure to applied mathematics* refers to student-reported experience with applied tasks at school, such as working out from a train timetable how long it would take to get from one place to another.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
The OECD average of the time spent per week in regular school lessons in mathematics in 2012 is based on all OECD countries. The corresponding OECD average reported in Table 1.6 is based on the OECD countries that participated in both PISA 2003 and PISA 2012.
Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the time spent in regular mathematics lessons.
Source: OECD, PISA 2012 Database, Tables 1.6, 1.9a and 1.9b.
StatLink http://dx.doi.org/10.1787/888933377644

■ Table 0.2 [Part 1/2] ■
SNAPSHOT OF FAMILIARITY WITH MATHEMATICS

Countries/economies where familiarity with mathematics is **above** the OECD average
Countries/economies where familiarity with mathematics is not statistically different from the OECD average
Countries/economies where familiarity with mathematics is **below** the OECD average

	Familiarity with mathematics						
		Arithmetic mean		Linear equation		Vectors	
	Index	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept
	Mean index	%	%	%	%	%	%
OECD average	0.00	30.8	29.4	12.8	41.8	34.9	20.3
Korea	1.34	52.4	13.2	0.9	69.0	34.4	2.7
Shanghai-China	1.12	7.4	68.3	50.1	8.5	7.1	74.7
Chinese Taipei	0.95	9.6	46.2	21.1	23.9	19.6	19.4
Spain	0.82	20.0	34.9	12.3	41.8	31.3	28.5
Japan	0.79	1.2	76.1	1.6	69.1	31.6	9.6
Macao-China	0.52	22.7	35.7	1.3	72.3	33.3	20.8
Hong Kong-China	0.50	15.8	44.7	31.7	28.4	45.1	13.0
Viet Nam	0.43	20.1	25.6	64.9	4.2	5.1	60.4
Latvia	0.41	5.2	62.9	3.3	49.1	43.4	8.9
Estonia	0.35	4.8	59.2	1.0	63.7	39.7	6.3
Hungary	0.33	33.4	19.4	5.4	52.8	7.2	45.6
Cyprus[1]	0.31	15.0	38.1	26.5	23.0	7.6	41.0
Greece	0.31	9.5	44.5	18.4	23.4	5.6	46.4
Czech Republic	0.26	8.7	52.3	2.7	59.5	48.6	11.8
Belgium	0.11	33.2	28.5	29.7	21.9	25.2	36.6
Finland	0.11	67.0	3.3	7.9	33.4	60.1	2.6
Turkey	0.10	4.7	49.3	6.4	26.4	4.6	42.1
Israel	0.10	20.6	46.0	16.4	53.9	65.7	10.0
France	0.09	38.0	21.3	10.5	44.3	24.8	48.9
Germany	0.09	50.4	17.3	6.2	63.6	42.0	14.4
Austria	0.05	53.4	14.8	10.9	51.3	28.5	30.1
Liechtenstein	0.04	60.0	10.8	16.2	50.7	38.3	27.3
United States	0.03	42.5	18.7	3.2	56.8	31.5	12.7
Singapore	0.02	35.8	26.0	2.4	62.6	15.1	44.0
Iceland	0.02	30.1	32.5	53.0	8.2	73.6	2.8
Slovak Republic	-0.04	11.7	47.1	4.5	57.0	51.1	12.3
Italy	-0.04	10.3	56.6	19.5	36.8	17.9	36.2
Slovenia	-0.06	15.5	39.6	2.2	64.2	17.1	28.9
Russian Federation	-0.07	2.3	74.2	1.5	70.8	2.8	65.1
Uruguay	-0.07	54.8	6.4	18.7	26.4	14.9	35.0
United Arab Emirates	-0.08	13.7	52.9	8.1	55.0	29.9	27.1
Canada	-0.10	45.3	14.6	5.8	55.6	32.4	13.2
Lithuania	-0.12	17.7	36.8	15.1	35.1	57.9	3.0

1. See note 1 under Snapshot Table 0.1 [Part 2/2].
Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Countries/economies are ranked in descending order of the index of familiarity with mathematics.
Source: OECD, PISA 2012 Database, Tables 1.7 and 1.8.
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■ Table 0.2 [Part 2/2] ■
SNAPSHOT OF FAMILIARITY WITH MATHEMATICS

Countries/economies where familiarity with mathematics is **above** the OECD average
Countries/economies where familiarity with mathematics is not statistically different from the OECD average
Countries/economies where familiarity with mathematics is **below** the OECD average

	Familiarity with mathematics						
		Arithmetic mean		Linear equation		Vectors	
	Index	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept	Percentage of students who have never heard the concept	Percentage of students who know well/ understand the concept
	Mean index	%	%	%	%	%	%
OECD average	0.00	30.8	29.4	12.8	41.8	34.9	20.3
Croatia	-0.14	9.8	49.3	1.4	72.0	3.5	55.9
Switzerland	-0.18	51.0	11.1	21.1	31.2	45.5	17.3
Portugal	-0.18	30.9	20.9	16.8	24.5	8.4	47.3
Bulgaria	-0.19	9.7	53.7	5.4	57.5	9.7	40.7
Serbia	-0.26	12.6	38.0	1.6	64.2	3.2	58.0
Poland	-0.27	1.8	65.7	20.0	15.8	16.3	21.6
Chile	-0.27	28.8	17.4	4.9	49.7	16.5	30.4
Denmark	-0.31	10.4	42.1	11.0	38.8	54.1	3.3
United Kingdom	-0.32	40.3	18.6	11.3	35.9	18.4	27.0
Australia	-0.34	43.2	15.5	9.2	47.1	31.1	12.9
Ireland	-0.34	38.6	22.1	11.8	38.0	58.1	4.0
Romania	-0.34	5.6	54.3	5.3	52.7	7.4	39.8
Jordan	-0.38	7.8	66.1	9.2	60.3	33.1	18.7
Costa Rica	-0.39	46.4	12.3	27.3	23.7	39.7	25.2
Tunisia	-0.40	12.2	46.3	47.6	12.3	33.2	19.6
Colombia	-0.42	21.9	18.2	12.6	28.4	25.5	26.4
Netherlands	-0.43	27.5	25.0	10.2	42.5	58.0	8.2
Montenegro	-0.47	24.9	22.4	3.9	59.5	9.0	44.6
Kazakhstan	-0.48	5.8	53.6	6.9	47.8	5.5	54.4
Mexico	-0.48	18.7	17.9	9.0	30.0	27.2	10.5
Sweden	-0.49	65.3	3.8	39.0	8.6	71.5	3.4
New Zealand	-0.53	49.2	10.2	13.0	36.7	34.0	13.0
Peru	-0.56	15.2	25.1	7.1	35.4	29.6	18.8
Brazil	-0.57	29.1	17.5	28.5	12.9	36.8	11.4
Luxembourg	-0.58	56.7	10.4	27.8	27.7	39.0	28.3
Argentina	-0.60	58.7	7.5	27.6	23.8	38.6	19.0
Albania	-0.62	5.6	52.7	6.6	42.6	3.1	58.3
Thailand	-0.72	5.4	31.0	3.4	34.9	16.3	22.8
Qatar	-0.83	19.1	35.9	15.3	44.3	27.8	24.5
Malaysia	-0.85	54.3	3.9	9.1	35.7	30.1	10.2
Indonesia	-0.90	5.0	27.2	8.6	19.6	20.2	11.1
Norway	m	m	m	m	m	m	m

Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Countries/economies are ranked in descending order of the index of familiarity with mathematics.
Source: OECD, PISA 2012 Database, Tables 1.7 and 1.8.
StatLink http://dx.doi.org/10.1787/888933377650

■ Table 0.3 [Part 1/2] ■

SNAPSHOT OF VARIATION IN OPPORTUNITIES TO LEARN MATHEMATICS, BY CHARACTERISTICS OF STUDENTS AND SCHOOLS

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is **below** the OECD average

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is not statistically different from the OECD average

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is **above** the OECD average

	Percentage of variation in familiarity with mathematics explained by students' and schools' socio-economic profile	Difference between socio-economically advantaged and disadvantaged students				Familiarity with mathematics (index)		
		Time spent per week in regular school lessons in mathematics (minutes)	Exposure to applied mathematics (index)	Exposure to pure mathematics (index)	Familiarity with mathematics (index)	Difference (boys-girls)	Difference (non-immigrant - immigrant)	Difference (attended pre-primary education - not attended)
	%	Dif.	Dif.	Dif.	Dif.	Dif.	Dif.	Dif.
OECD average	8.5	**7**	**0.23**	**0.44**	**0.45**	**-0.15**	**0.17**	**0.29**
Liechtenstein	24.5	-15	**0.36**	0.28	**0.60**	0.06	**0.48**	c
Hungary	21.4	3	0.07	**0.41**	**0.85**	**-0.25**	-0.07	c
Austria	18.6	-3	**0.11**	**0.51**	**0.77**	-0.07	**0.30**	0.23
Germany	16.3	-11	**0.11**	**0.44**	**0.61**	**-0.12**	**0.29**	**0.37**
Slovenia	15.3	**19**	**0.22**	**0.35**	**0.43**	**-0.15**	**0.13**	**0.11**
Belgium	14.4	**31**	**0.19**	**0.69**	**0.76**	**-0.09**	**0.33**	**0.51**
Chinese Taipei	13.7	**57**	**0.50**	**0.59**	**0.74**	**-0.17**	c	**0.34**
Netherlands	12.6	-10	0.06	**0.63**	**0.42**	**-0.08**	**0.25**	**0.26**
Korea	12.5	**24**	**0.55**	**0.42**	**0.63**	-0.11	c	0.05
Chile	12.4	-20	**0.22**	**0.50**	**0.59**	**-0.06**	-0.01	**0.32**
Slovak Republic	11.8	6	-0.10	**0.36**	**0.50**	**-0.22**	c	**0.51**
Brazil	11.6	**18**	**0.19**	**0.19**	**0.46**	**-0.12**	0.08	**0.18**
Switzerland	11.4	-15	**0.15**	**0.50**	**0.61**	-0.04	**0.31**	**0.44**
Croatia	11.2	**31**	0.08	**0.32**	**0.45**	**-0.16**	**0.11**	**0.17**
Japan	10.7	**53**	**0.33**	**0.40**	**0.33**	0.00	c	**0.94**
Italy	10.5	4	0.04	**0.38**	**0.40**	**-0.08**	**0.42**	**0.38**
Portugal	10.5	20	**0.36**	**0.66**	**0.74**	**-0.24**	0.15	**0.22**
Turkey	10.3	**37**	-0.01	**0.48**	**0.45**	**-0.37**	c	**0.25**
Thailand	10.2	**34**	**0.28**	**0.42**	**0.35**	**-0.26**	c	0.16
Serbia	10.1	**16**	-0.02	**0.26**	**0.43**	**-0.21**	**-0.14**	**0.14**
Uruguay	9.8	6	-0.05	**0.39**	**0.54**	**-0.18**	c	**0.30**
Bulgaria	9.2	16	**0.17**	**0.52**	**0.58**	**-0.34**	c	**0.22**
Singapore	8.7	**30**	**0.11**	**0.33**	**0.54**	**-0.20**	0.00	**0.58**
Luxembourg	8.4	3	**0.34**	**0.58**	**0.50**	-0.03	0.03	0.05
Czech Republic	7.9	4	0.04	**0.40**	**0.27**	**-0.12**	0.16	**0.30**
Spain	7.8	-4	0.07	**0.31**	**0.79**	**-0.21**	**0.44**	**0.48**
Romania	7.6	9	**0.22**	**0.50**	**0.59**	**-0.16**	c	**0.26**
Montenegro	7.6	**21**	0.14	**0.25**	**0.39**	**-0.15**	-0.04	**0.14**
Colombia	7.5	17	**0.27**	**0.18**	**0.39**	-0.03	c	**0.14**
Shanghai-China	7.4	11	**0.13**	0.09	**0.55**	**-0.15**	c	**0.85**
Peru	7.3	23	**0.43**	**0.51**	**0.47**	**-0.11**	c	**0.16**
United States	6.6	24	**0.31**	**0.36**	**0.60**	**-0.24**	-0.02	0.15
Australia	5.5	3	**0.37**	**0.62**	**0.34**	**-0.09**	**-0.22**	**0.19**

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the percentage of variation in familiarity with mathematics explained by students' and schools' socio-economic profile.
Source: OECD, PISA 2012 Database, Tables 2.2, 2.3, 2.4a and 2.10.
StatLink http://dx.doi.org/10.1787/888933377666

■ Table 0.3 [Part 2/2] ■

SNAPSHOT OF VARIATION IN OPPORTUNITIES TO LEARN MATHEMATICS, BY CHARACTERISTICS OF STUDENTS AND SCHOOLS

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is **below** the OECD average

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is not statistically different from the OECD average

Countries/economies where the strength of the relationship between socio-economic status and familiarity with mathematics is **above** the OECD average

	Percentage of variation in familiarity with mathematics explained by students' and schools' socio-economic profile	Difference between socio-economically advantaged and disadvantaged students				Familiarity with mathematics (index)		
		Time spent per week in regular school lessons in mathematics (minutes)	Exposure to applied mathematics (index)	Exposure to pure mathematics (index)	Familiarity with mathematics (index)	Difference (boys-girls)	Difference (non-immigrant - immigrant)	Difference (attended pre-primary education - not attended)
	%	Dif.	Dif.	Dif.	Dif.	Dif.	Dif.	Dif.
OECD average	8.5	**7**	**0.23**	**0.44**	**0.45**	**-0.15**	**0.17**	**0.29**
Lithuania	5.4	5	**0.20**	**0.28**	**0.23**	**-0.33**	0.11	**0.13**
Ireland	5.1	1	**0.28**	**0.44**	**0.35**	**-0.15**	0.03	0.05
United Kingdom	5.0	-8	**0.26**	**0.36**	**0.32**	**-0.15**	0.04	**0.33**
New Zealand	4.9	3	**0.56**	**0.72**	**0.33**	**-0.12**	**-0.10**	**0.21**
Russian Federation	4.8	**20**	**0.22**	**0.35**	**0.36**	**-0.20**	**0.19**	**0.21**
Poland	4.7	9	**0.24**	**0.30**	**0.41**	**-0.21**	c	**0.21**
Argentina	4.7	**65**	**0.24**	**0.35**	**0.31**	**-0.17**	**0.28**	**0.26**
Indonesia	4.4	27	**0.33**	**0.27**	**0.18**	**-0.04**	c	**0.14**
Costa Rica	4.2	**22**	**0.19**	**0.42**	**0.32**	**-0.09**	**0.23**	**0.16**
United Arab Emirates	4.1	-5	**0.42**	**0.55**	**0.28**	**-0.35**	**-0.42**	**0.33**
Qatar	3.9	-5	**0.24**	**0.48**	**0.32**	0.02	**-0.48**	**0.32**
Greece	3.7	10	-0.04	**0.48**	**0.41**	**-0.32**	**0.36**	**0.34**
Iceland	3.5	3	**0.53**	**0.40**	**0.33**	**-0.32**	**0.46**	**0.38**
Latvia	3.3	**13**	**0.20**	**0.43**	**0.31**	**-0.36**	**0.29**	-0.06
Kazakhstan	3.2	**37**	**0.18**	**0.25**	**0.22**	**-0.10**	**0.11**	**0.18**
Macao-China	2.8	8	**0.14**	0.05	**-0.27**	0.00	**-0.24**	**0.46**
Israel	2.7	18	0.13	**0.44**	**0.32**	**-0.16**	0.07	**0.66**
Sweden	2.7	-6	**0.45**	**0.40**	**0.26**	**-0.17**	**0.16**	**0.23**
Canada	2.6	11	**0.41**	**0.43**	**0.29**	**-0.18**	-0.04	0.07
Viet Nam	2.6	**21**	-0.02	**0.40**	**0.24**	**-0.19**	c	**0.22**
Tunisia	2.2	21	**0.30**	**0.50**	**0.12**	**-0.12**	c	**0.12**
Mexico	1.9	11	**0.15**	**0.23**	**0.18**	**-0.10**	**0.22**	**0.14**
Jordan	1.6	3	**0.55**	**0.54**	**0.33**	**-0.53**	-0.04	**0.25**
Finland	1.4	5	**0.36**	**0.40**	**0.23**	**-0.24**	**0.29**	0.10
Denmark	1.2	-1	**0.16**	**0.16**	**0.20**	-0.03	**0.21**	**0.42**
Hong Kong-China	1.2	8	**0.23**	**0.23**	**-0.24**	0.05	**-0.11**	**0.33**
Malaysia	0.6	**33**	**0.50**	**0.59**	**0.11**	**-0.07**	0.02	0.03
Estonia	0.6	4	**0.29**	**0.29**	**0.13**	**-0.21**	**0.23**	**-0.15**
Cyprus[1]	0.2	6	**0.41**	**0.54**	0.11	**-0.43**	**0.32**	**0.24**
Albania	m	m	m	m	m	-0.01	c	**0.14**
France	w	**18**	**0.32**	**0.54**	**0.64**	**-0.16**	**0.21**	**0.62**
Norway	m	2	**0.27**	**0.28**	m	m	m	m

1. See note 1 under Snapshot Table 0.1 [Part 2/2].

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another.

The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

Statistically significant values are indicated in bold.

Countries/economies are ranked in descending order of the percentage of variation in familiarity with mathematics explained by students' and schools' socio-economic profile.

Source: OECD, PISA 2012 Database, Tables 2.2, 2.3, 2.4a and 2.10.

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■ Table 0.4 [Part 1/2] ■

SNAPSHOT OF VARIATION IN FAMILIARITY WITH MATHEMATICS, BY STUDENTS' SOCIO-ECONOMIC STATUS

Countries/economies where familiarity with mathematics is **above** the OECD average
Countries/economies where familiarity with mathematics is not statistically different from the OECD average
Countries/economies where familiarity with mathematics is **below** the OECD average

	Percentage of students who know well/understand the concept								
	Arithmetic mean			Linear equation			Vectors		
	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)
	%	%	% dif.	%	%	% dif.	%	%	% dif.
OECD average	20.4	39.9	**19.5**	29.9	54.3	**24.5**	12.1	29.8	**17.7**
Bulgaria	31.7	72.5	**40.8**	35.8	75.3	**39.5**	19.7	60.7	**41.0**
Romania	36.9	74.7	**37.8**	37.8	72.6	**34.7**	28.5	54.3	**25.8**
Slovak Republic	28.9	63.0	**34.1**	41.7	72.0	**30.4**	5.3	21.6	**16.3**
Poland	48.8	82.1	**33.4**	10.0	22.0	**12.0**	12.1	33.4	**21.3**
Chinese Taipei	31.4	62.5	**31.1**	10.9	41.1	**30.1**	11.4	28.5	**17.1**
Croatia	35.3	66.1	**30.8**	61.6	83.1	**21.5**	45.3	67.7	**22.5**
Greece	30.5	60.3	**29.8**	16.6	33.2	**16.6**	31.6	63.4	**31.8**
Cyprus[1]	24.6	54.2	**29.7**	10.3	41.6	**31.2**	25.8	60.2	**34.4**
Serbia	24.0	53.6	**29.6**	53.4	77.2	**23.8**	47.2	70.7	**23.5**
Estonia	46.4	75.0	**28.6**	54.6	73.6	**19.0**	4.2	9.1	**4.9**
Russian Federation	57.3	85.9	**28.6**	55.8	84.2	**28.4**	51.6	78.8	**27.3**
Israel	29.8	58.4	**28.5**	41.1	67.2	**26.1**	6.0	16.6	**10.6**
Portugal	8.7	37.0	**28.4**	16.9	34.7	**17.8**	28.3	65.8	**37.4**
Czech Republic	39.4	66.9	**27.5**	46.5	70.8	**24.3**	4.7	21.7	**17.1**
Turkey	38.1	65.6	**27.5**	22.1	34.7	**12.6**	31.4	56.7	**25.3**
Spain	21.7	49.1	**27.5**	27.5	56.2	**28.7**	15.8	42.3	**26.5**
Kazakhstan	38.7	66.0	**27.3**	34.8	60.3	**25.6**	41.7	66.7	**25.0**
Shanghai-China	54.3	81.3	**27.0**	5.3	12.8	**7.5**	57.7	87.1	**29.4**
Slovenia	26.3	53.2	**26.9**	50.0	76.9	**27.0**	11.2	47.4	**36.2**
Denmark	29.1	55.9	**26.8**	26.6	52.8	**26.2**	1.7	6.6	**4.8**
Tunisia	34.5	59.4	**24.9**	9.1	17.7	**8.7**	16.6	23.2	**6.6**
Latvia	49.1	73.8	**24.8**	38.9	59.7	**20.8**	3.7	13.1	**9.5**
Lithuania	25.0	48.3	**23.3**	25.4	46.4	**20.9**	0.8	5.4	**4.6**
Singapore	14.9	37.9	**23.0**	45.3	78.4	**33.1**	27.6	57.7	**30.1**
Korea	3.9	26.5	**22.6**	51.0	84.5	**33.4**	0.7	5.6	**4.9**
Italy	45.2	67.4	**22.2**	25.0	49.6	**24.6**	25.9	47.3	**21.4**
Hungary	9.3	31.4	**22.1**	32.7	72.7	**40.0**	29.9	60.6	**30.7**
United States	9.8	31.6	**21.7**	42.6	71.4	**28.7**	7.5	17.7	**10.3**
Peru	15.0	36.6	**21.6**	23.6	50.5	**26.9**	9.6	30.0	**20.4**
Viet Nam	15.9	37.5	**21.6**	2.6	6.1	**3.5**	45.6	73.1	**27.5**
Brazil	10.3	30.6	**20.3**	7.7	21.0	**13.3**	5.7	23.2	**17.5**
Austria	5.5	25.5	**20.0**	30.1	71.2	**41.1**	15.2	49.7	**34.5**

1. See note 1 under Snapshot Table 0.1 [Part 2/2].
Note: Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the difference in the percentage of students who know well/understand the concept of arithmetic mean between socio-economically advantaged and disadvantaged students.
Source: OECD, PISA 2012 Database, Table 2.4b.
StatLink http://dx.doi.org/10.1787/888933377670

■ Table 0.4 [Part 2/2] ■

SNAPSHOT OF VARIATION IN FAMILIARITY WITH MATHEMATICS, BY STUDENTS' SOCIO-ECONOMIC STATUS

Countries/economies where familiarity with mathematics is **above** the OECD average
Countries/economies where familiarity with mathematics is not statistically different from the OECD average
Countries/economies where familiarity with mathematics is **below** the OECD average

	Percentage of students who know well/understand the concept								
	Arithmetic mean			Linear equation			Vectors		
	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)	Socio-economically disadvantaged students	Socio-economically advantaged students	Difference (advantaged - disadvantaged)
	%	%	% dif.	%	%	% dif.	%	%	% dif.
OECD average	20.4	39.9	**19.5**	29.9	54.3	**24.5**	12.1	29.8	**17.7**
Indonesia	20.4	40.2	**19.9**	14.9	26.9	**11.9**	8.2	15.4	**7.2**
Thailand	23.6	43.2	**19.7**	27.8	47.0	**19.2**	13.4	37.7	**24.3**
Ireland	12.3	31.9	**19.6**	25.7	51.4	**25.7**	3.3	5.1	**1.8**
Belgium	19.7	39.1	**19.3**	16.2	27.3	**11.1**	24.9	47.9	**23.0**
Japan	66.6	84.3	**17.8**	57.1	77.7	**20.6**	4.3	14.8	**10.5**
Iceland	26.3	42.7	**16.3**	5.3	13.9	**8.6**	2.0	5.4	**3.4**
United Kingdom	11.2	27.6	**16.3**	24.2	51.1	**26.9**	17.2	38.4	**21.2**
Jordan	57.6	72.6	**15.0**	48.5	68.7	**20.2**	14.2	23.7	**9.6**
Liechtenstein	4.0	18.8	**14.8**	34.6	62.6	**28.0**	18.2	36.3	**18.0**
Australia	8.4	23.1	**14.6**	30.1	63.9	**33.8**	8.7	18.1	**9.3**
New Zealand	3.5	17.9	**14.5**	20.5	54.3	**33.8**	5.8	22.6	**16.7**
Netherlands	19.3	32.9	**13.7**	29.8	59.1	**29.3**	5.4	12.5	**7.2**
Chile	12.1	25.3	**13.2**	31.3	70.4	**39.1**	15.0	46.9	**32.0**
France	17.0	30.1	**13.2**	36.3	54.8	**18.5**	27.3	72.1	**44.7**
Colombia	13.7	26.1	**12.4**	19.5	41.8	**22.3**	16.9	38.9	**22.0**
Luxembourg	6.0	17.8	**11.8**	16.7	37.4	**20.7**	12.6	45.3	**32.7**
Switzerland	6.4	17.7	**11.2**	19.3	46.7	**27.5**	11.8	26.1	**14.3**
Montenegro	17.0	28.1	**11.1**	47.0	69.4	**22.3**	33.8	55.2	**21.4**
Costa Rica	7.7	18.1	**10.4**	12.5	38.4	**25.9**	15.1	36.7	**21.6**
Canada	9.8	19.8	**10.0**	41.1	69.5	**28.5**	8.4	17.9	**9.5**
Mexico	14.4	23.9	**9.5**	21.2	42.6	**21.4**	5.6	17.4	**11.8**
Germany	13.5	22.2	**8.8**	48.0	72.9	**24.8**	11.6	17.4	**5.9**
Hong Kong-China	40.8	49.0	**8.2**	18.3	40.8	**22.5**	8.0	21.3	**13.4**
Macao-China	32.1	39.5	**7.4**	71.7	72.0	0.3	14.1	28.6	**14.5**
Qatar	29.0	35.8	**6.7**	30.0	49.1	**19.2**	15.6	30.4	**14.7**
Finland	1.8	6.4	**4.6**	22.3	45.0	**22.7**	1.3	4.3	**3.1**
Uruguay	3.9	8.2	**4.3**	16.3	38.5	**22.2**	20.8	51.4	**30.6**
United Arab Emirates	48.5	51.9	3.4	42.3	62.2	**19.9**	16.0	36.4	**20.4**
Argentina	5.8	8.9	**3.0**	17.3	32.4	**15.1**	11.2	27.0	**15.8**
Sweden	2.8	5.5	**2.8**	5.6	12.0	**6.4**	2.2	4.8	**2.6**
Malaysia	3.8	4.5	0.7	21.8	54.1	**32.4**	6.2	17.2	**11.0**
Norway	m	m	m	m	m	m	m	m	m

Note: Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the difference in the percentage of students who know well/understand the concept of arithmetic mean between socio-economically advantaged and disadvantaged students.
Source: OECD, PISA 2012 Database, Table 2.4b.
StatLink http://dx.doi.org/10.1787/888933377670

■ Table 0.5 [Part 1/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND HORIZONTAL STRATIFICATION

	Age at first tracking	Percentage of students in vocational schools			Percentage of students in schools with ability grouping for some or all classes		
		All students	Socio-economically disadvantaged students	Students who are less familiar with mathematics	All students	Students in socio-economically disadvantaged schools	Students in schools with overall less familiarity with mathematics
	Years	%	%	%	%	%	%
OECD average	15	14.5	19.8	21.3	74.1	78.2	79.1
Serbia	m	74.4	87.9	86.9	94.8	98.3	97.9
Croatia	14	70.1	89.7	90.4	92.0	99.1	100.0
Austria	10	69.3	79.2	89.5	28.1	62.9	57.9
Montenegro	15	66.0	81.5	76.6	93.1	95.6	92.9
Slovenia	14	53.2	74.9	74.7	49.5	50.6	38.9
Italy	14	49.6	68.1	65.6	75.9	80.8	80.1
Belgium	12	44.0	64.0	69.6	79.4	87.9	78.3
Bulgaria	13	40.8	55.2	48.6	93.1	91.9	92.9
Turkey	11	38.1	43.5	55.1	75.8	74.1	88.4
Chinese Taipei	15	34.5	49.9	41.7	80.5	83.6	75.9
Czech Republic	11	31.0	33.7	33.7	41.2	44.6	35.2
Colombia	15	25.2	19.3	17.6	93.6	89.4	94.4
Mexico	15	25.2	19.3	21.5	73.7	78.4	82.7
Japan	15	24.2	36.3	30.6	63.1	64.5	73.7
Netherlands	12	22.2	38.5	37.7	93.6	94.5	95.0
Shanghai-China	15	21.2	29.5	36.4	94.1	94.2	87.3
Indonesia	15	20.2	18.6	17.1	75.4	75.1	86.4
Korea	14	19.9	37.7	34.2	90.1	83.7	77.2
Thailand	15	19.6	21.4	26.0	76.3	69.7	77.7
Portugal	15	16.7	27.9	29.4	61.7	80.4	74.3
France	15	15.3	23.2	27.4	56.2	68.7	74.4
Luxembourg	13	14.5	16.0	14.3	67.9	80.6	86.0
Argentina	15	14.5	16.7	16.0	85.5	87.3	84.1
Hungary	11	14.3	30.4	31.7	76.7	72.6	73.9
Greece	15	13.5	22.5	24.8	18.6	32.0	34.1
Malaysia	15	13.3	13.4	13.8	95.9	97.7	100.0
Australia	16	10.9	14.1	14.1	98.4	99.5	99.4
Cyprus[1]	15	10.8	20.3	19.7	50.9	60.8	66.7
Switzerland	12	10.7	10.6	13.4	85.0	92.4	98.8
Costa Rica	m	9.1	8.1	5.7	60.4	50.9	47.6
Albania	15	8.4	m	8.3	99.9	m	100.0
Slovak Republic	11	8.2	13.2	14.6	71.6	70.4	77.7
Kazakhstan	m	7.7	8.1	7.6	97.6	100.0	100.0
Russian Federation	15.5	4.1	6.2	4.8	96.0	92.7	100.0

1. See note 1 under Snapshot Table 0.1 [Part 2/2].
Countries/economies are ranked in descending order of the percentage of students in vocational programmes.
Source: OECD, PISA 2012 Database, Tables 2.16, 2.17 and 2.19a.
StatLink http://dx.doi.org/10.1787/888933377680

■ Table 0.5 [Part 2/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND HORIZONTAL STRATIFICATION

	Age at first tracking	Percentage of students in vocational schools			Percentage of students in schools with ability grouping for some or all classes		
		All students	Socio-economically disadvantaged students	Students who are less familiar with mathematics	All students	Students in socio-economically disadvantaged schools	Students in schools with overall less familiarity with mathematics
	Years	%	%	%	%	%	%
OECD average	15	14.5	19.8	21.3	74.1	78.2	79.1
Israel	15	3.1	5.2	7.1	98.3	98.5	100.0
Chile	16	2.8	4.3	3.7	64.3	77.1	77.8
United Arab Emirates	15	2.7	1.6	4.4	86.2	91.9	81.7
Germany	10	2.0	3.3	4.9	68.1	82.5	84.2
Macao-China	15	1.6	3.0	1.8	66.1	56.8	77.1
Uruguay	11	1.4	2.0	1.9	91.1	93.3	97.0
United Kingdom	16	1.1	1.5	1.4	99.3	99.5	99.6
Latvia	16	0.9	1.2	0.7	82.2	88.4	88.2
Ireland	15	0.8	2.1	1.3	99.2	100.0	100.0
Spain	16	0.7	1.6	1.5	92.4	96.0	94.0
Lithuania	16	0.6	1.3	1.3	84.1	83.8	96.2
Estonia	15	0.4	1.0	0.0	89.1	82.1	91.5
Sweden	16	0.4	0.1	0.4	84.3	79.0	87.5
Poland	16	0.1	0.0	0.0	57.6	51.6	30.2
Brazil	15	0.0	0.0	0.0	81.6	80.2	83.2
New Zealand	16	0.0	0.0	0.0	98.7	99.4	100.0
Finland	16	0.0	0.0	0.0	64.5	51.6	60.2
Canada	16	0.0	0.0	0.0	92.9	94.6	94.9
Norway	16	0.0	0.0	m	45.8	59.5	m
Romania	14	0.0	0.0	0.0	90.3	86.7	91.7
Iceland	16	0.0	0.0	0.0	87.1	98.2	100.0
Qatar	15	0.0	0.0	0.0	91.6	92.8	93.5
Denmark	16	0.0	0.0	0.0	75.9	77.2	85.9
Liechtenstein	15	0.0	0.0	0.0	59.9	c	100.0
Jordan	16	0.0	0.0	0.0	81.7	85.3	92.2
Viet Nam	15	0.0	0.0	0.0	93.1	87.9	88.3
United States	16	0.0	0.0	0.0	93.9	94.9	79.9
Singapore	12	0.0	0.0	0.0	97.2	97.9	100.0
Tunisia	m	0.0	0.0	0.0	82.3	80.2	100.0
Hong Kong-China	15	0.0	0.0	0.0	91.0	97.7	100.0
Peru	16	0.0	0.0	0.0	86.8	84.4	83.8

Countries/economies are ranked in descending order of the percentage of students in vocational programmes.
Source: OECD, PISA 2012 Database, Tables 2.16, 2.17 and 2.19a.
StatLink http://dx.doi.org/10.1787/888933377680

■ Table 0.6 [Part 1/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND MATHEMATICS PERFORMANCE

Countries/economies where performance in the mathematics subscale is **above** the OECD average
Countries/economies where performance in the mathematics subscale is not statistically different from the OECD average
Countries/economies where performance in the mathematics subscale is **below** the OECD average

	Performance in mathematics, by content area				Score-point difference in mathematics performance associated with a one-unit increase in the index of:			Percentage of the difference in mathematics performance between socio-economically disadvantaged and advantaged students associated with different levels of familiarity with mathematics
	Change and relationships	Quantity	Space and shape	Uncertainty and data	Exposure to applied mathematics	Exposure to pure mathematics	Familiarity with mathematics	
	Mean score	Mean score	Mean score	Mean score	Score-point change	Score-point change	Score-point change	%
OECD average	493	495	490	493	**9**	**30**	**41**	18.8
Korea	559	537	573	538	**28**	**61**	**55**	33.7
New Zealand	501	499	491	506	**26**	**42**	**55**	14.4
Australia	509	500	497	508	**21**	**37**	**55**	20.7
Chinese Taipei	561	543	592	549	**27**	**47**	**51**	22.2
Switzerland	530	531	544	522	**10**	**36**	**50**	29.5
Liechtenstein	542	538	539	526	15	**33**	**49**	33.9
Hungary	481	476	474	476	2	**28**	**48**	29.0
Singapore	580	569	580	559	**8**	**44**	**48**	19.1
Germany	516	517	507	509	3	**35**	**48**	29.9
Slovenia	499	504	503	496	4	**28**	**48**	19.2
France	497	496	489	492	**20**	**33**	**47**	22.3
Italy	477	491	487	482	1	**31**	**47**	21.6
Portugal	486	481	491	486	**8**	**29**	**47**	26.3
Netherlands	518	532	507	532	2	**44**	**46**	22.5
Croatia	468	480	460	468	**10**	**26**	**45**	23.3
United States	488	478	463	488	**13**	**31**	**44**	27.4
Slovak Republic	474	486	490	472	**-10**	**30**	**43**	13.6
United Kingdom	496	494	475	502	**20**	**32**	**43**	15.3
Sweden	469	482	469	483	**10**	**20**	**43**	14.9
Belgium	513	519	509	508	**12**	**38**	**42**	28.2
Austria	506	510	501	499	**8**	**31**	**41**	31.3
Brazil	368	389	378	400	**4**	**9**	**40**	26.5
Peru	349	365	370	373	**5**	**33**	**40**	19.3
Poland	509	519	524	517	**12**	**26**	**40**	14.9
Canada	525	515	510	516	**15**	**28**	**40**	16.4
Luxembourg	488	495	486	483	**10**	**27**	**40**	17.6
Ireland	501	505	478	509	**16**	**28**	**40**	12.2
Qatar	363	371	380	382	2	**38**	**40**	19.3
Chile	411	421	419	430	**10**	**24**	**39**	22.7
Czech Republic	499	505	499	488	-4	**26**	**39**	13.6
Thailand	414	419	432	433	**12**	**30**	**39**	25.9
Serbia	442	456	446	448	-3	**17**	**38**	18.7
Uruguay	401	411	413	407	**-8**	**20**	**38**	15.5

Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Macao-China and Hong-Kong China are the only two economies where disadvantaged students report a higher familiarity with mathematics than advantaged students. In these two economies, eliminating the difference in familiarity between advantaged and disadvantaged students would increase the performance gap of disadvantaged students. This explains why the graph reports negative percentages for these two economies.
Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the score-point difference in mathematics performance associated with a one-unit increase in familiarity with mathematics.
Source: OECD, PISA 2012 Database, Tables 3.2a, 3.7 and 3.16.
StatLink http://dx.doi.org/10.1787/888933377691

■ Table 0.6 [Part 2/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND MATHEMATICS PERFORMANCE

Countries/economies where performance in the mathematics subscale is **above** the OECD average
Countries/economies where performance in the mathematics subscale is not statistically different from the OECD average
Countries/economies where performance in the mathematics subscale is **below** the OECD average

	Performance in mathematics, by content area				Score-point difference in mathematics performance associated with a one-unit increase in the index of:			Percentage of the difference in mathematics performance between socio-economically disadvantaged and advantaged students associated with different levels of familiarity with mathematics
	Change and relationships	Quantity	Space and shape	Uncertainty and data	Exposure to applied mathematics	Exposure to pure mathematics	Familiarity with mathematics	
	Mean score	Mean score	Mean score	Mean score	Score-point change	Score-point change	Score-point change	%
OECD average	493	495	490	493	**9**	**30**	**41**	18.8
Turkey	448	442	443	447	**-4**	**29**	**38**	19.6
Lithuania	479	483	472	474	**8**	**33**	**36**	9.7
Japan	542	518	558	528	**24**	**34**	**36**	13.2
Indonesia	364	362	383	384	**6**	**13**	**36**	14.9
United Arab Emirates	442	431	425	432	**10**	**36**	**36**	12.9
Bulgaria	434	443	442	432	-3	**28**	**35**	13.7
Shanghai-China	624	591	649	592	**-5**	2	**35**	11.0
Iceland	487	496	489	496	**12**	**31**	**34**	18.6
Spain	482	491	477	487	**-4**	**24**	**34**	23.1
Finland	520	527	507	519	**24**	**31**	**34**	11.3
Colombia	357	375	369	388	**7**	**15**	**34**	19.8
Israel	462	480	449	465	-4	**29**	**32**	7.4
Russian Federation	491	478	496	463	**4**	**29**	**32**	14.4
Montenegro	399	409	412	415	**5**	**24**	**30**	15.8
Greece	446	455	436	460	**-10**	**25**	**29**	9.4
Viet Nam	509	509	507	519	-2	**25**	**29**	7.9
Latvia	496	487	497	478	**7**	**29**	**28**	8.7
Estonia	530	525	513	510	**7**	**16**	**28**	5.1
Malaysia	401	409	434	422	**16**	**40**	**27**	3.6
Denmark	494	502	497	505	2	**7**	**26**	7.1
Mexico	405	414	413	413	**5**	**21**	**26**	7.0
Jordan	387	367	385	394	**8**	**28**	**24**	15.8
Cyprus[1]	440	439	436	442	**8**	**32**	**24**	2.1
Macao-China	542	531	558	525	-3	**17**	**23**	-21.0
Costa Rica	402	406	397	414	-3	**16**	**23**	7.6
Romania	446	443	447	437	**4**	**21**	**23**	11.1
Argentina	379	391	385	389	2	**17**	**21**	8.2
Kazakhstan	433	428	450	414	-2	**19**	**20**	7.9
Hong Kong-China	564	566	567	553	5	**38**	**18**	-6.0
Tunisia	379	378	382	399	1	**26**	**16**	3.1
Albania	388	386	418	386	-1	-3	-2	m
Norway	478	492	480	497	**15**	**30**	m	m

1. See note 1 under Snapshot Table 0.1 [Part 2/2].

Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another.

The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Macao-China and Hong-Kong China are the only two economies where disadvantaged students report a higher familiarity with mathematics than advantaged students. In these two economies, eliminating the difference in familiarity between advantaged and disadvantaged students would increase the performance gap of disadvantaged students. This explains why the graph reports negative percentages for these two economies.

Statistically significant values are indicated in bold.

Countries/economies are ranked in descending order of the score-point difference in mathematics performance associated with a one-unit increase in familiarity with mathematics.

Source: OECD, PISA 2012 Database, Tables 3.2a, 3.7 and 3.16.

StatLink http://dx.doi.org/10.1787/888933377691

■ Table 0.7 [Part 1/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND STUDENTS' ATTITUDES TOWARDS MATHEMATICS

Countries/economies where the percentage of students with positive attitudes towards mathematics is **above** the OECD average

Countries/economies where the percentage of students with positive attitudes towards mathematics is not statistically different from the OECD average

Countries/economies where the percentage of students with positive attitudes towards mathematics is **below** the OECD average

	Percentage of students who agreed or strongly agreed with the statement "I do mathematics because I enjoy it"	Percentage of students who disagreed or strongly disagreed with the statement "I am just not good at mathematics"	Percentage of students who agreed or strongly agreed with the statement "I often worry that it will be difficult for me in mathematics classes"	Change in the index of mathematics self-concept/anxiety associated with a one-unit increase in the index of familiarity with mathematics			
				Mathematics self-concept		Mathematics anxiety	
				Before accounting for performance in mathematics	After accounting for performance in mathematics	Before accounting for performance in mathematics	After accounting for performance in mathematics
	%	%	%	Index change	Index change	Index change	Index change
OECD average	38.1	57.3	59.5	**0.10**	**-0.10**	**-0.12**	**0.07**
Albania	70.3	39.4	66.8	**0.11**	**0.11**	**-0.26**	**-0.26**
Korea	30.7	42.6	76.9	**0.29**	0.04	**-0.14**	-0.04
Serbia	26.8	52.1	62.6	**0.19**	0.03	**-0.24**	**-0.09**
Jordan	64.9	48.9	77.5	**0.13**	0.03	**-0.09**	**-0.05**
Singapore	72.2	62.3	60.7	**0.17**	0.03	**-0.22**	**-0.06**
Chinese Taipei	40.3	39.9	71.5	**0.25**	0.02	**-0.12**	0.03
Turkey	52.7	47.6	66.7	**0.12**	0.01	**-0.18**	-0.04
United Arab Emirates	63.9	62.7	68.1	**0.11**	0.01	**-0.21**	**-0.05**
Viet Nam	67.4	75.5	72.1	**0.07**	0.00	**-0.08**	-0.01
Hong Kong-China	54.9	50.1	68.9	**0.06**	0.00	**-0.10**	**-0.05**
Peru	62.7	51.2	72.9	**0.09**	0.00	**-0.12**	-0.03
Israel	39.8	73.5	66.6	**0.08**	0.00	**-0.07**	0.02
Malaysia	73.4	48.3	76.6	0.05	-0.01	**-0.08**	-0.01
Romania	57.8	48.9	76.8	0.03	-0.02	**-0.14**	**-0.07**
Russian Federation	42.9	57.7	57.8	**0.09**	-0.02	**-0.11**	0.01
Colombia	51.3	56.5	64.4	**0.09**	-0.03	**-0.14**	-0.01
Montenegro	34.0	51.8	65.0	**0.10**	-0.03	**-0.13**	0.01
Spain	37.0	50.5	68.0	**0.14**	**-0.03**	**-0.08**	0.04
Italy	45.8	52.8	73.2	**0.16**	**-0.03**	**-0.10**	**0.06**
Mexico	52.8	47.0	77.5	**0.07**	**-0.04**	**-0.07**	0.03
Iceland	47.7	63.8	45.2	**0.18**	-0.04	**-0.24**	-0.04
Cyprus[1]	47.1	59.1	68.0	**0.08**	-0.04	**-0.11**	0.01
Shanghai-China	49.3	53.1	53.4	**0.06**	**-0.04**	**-0.11**	0.00
Bulgaria	39.2	43.7	70.2	**0.04**	**-0.04**	**-0.15**	0.01
Tunisia	58.0	45.2	79.4	0.02	-0.05	0.01	0.05
Costa Rica	47.5	55.8	72.4	**0.07**	-0.05	-0.05	0.05
Macao-China	42.3	51.6	70.4	0.04	**-0.05**	**-0.08**	0.01
Portugal	45.5	51.5	69.7	**0.17**	-0.05	**-0.11**	0.04
Brazil	56.4	44.0	71.4	**0.05**	**-0.06**	**-0.14**	0.01
Greece	51.7	56.5	72.7	**0.07**	**-0.06**	**-0.10**	0.03
Thailand	70.6	24.2	73.0	-0.03	**-0.07**	-0.04	0.01
Kazakhstan	72.6	63.0	55.2	-0.02	**-0.07**	-0.05	0.01

1. See note 1 under Snapshot Table 0.1 [Part 2/2].

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).

The *index of mathematics self-concept* is based on the degree to which students agreed with the statements: "I'm just not good in mathematics"; "I get good grades in mathematics"; "I learn mathematics quickly"; "I have always believed that mathematics is one of my best subjects"; and "In my mathematics class, I understand even the most difficult work".

The *index of mathematics anxiety* is based on the degree to which students agreed with the statements: "I often worry that it will be difficult for me in mathematics classes"; "I get very tense when I have to do mathematics homework"; "I get very nervous doing mathematics problems"; "I feel helpless when doing a mathematics problem"; and "I worry that I will get poor marks in mathematics".

The OECD average of the percentage of students who agreed or strongly agreed with the statement "I do mathematics because I enjoy it" is based on all OECD countries. The corresponding OECD average reported in Table 4.1 is based on the OECD countries that participated in both PISA 2003 and PISA 2012.

Statistically significant values are indicated in bold.

Countries/economies are ranked in descending order of the change in mathematics self-concept associated with a one-unit increase in familiarity with mathematics after accounting for performance in mathematics.

Source: OECD, PISA 2012 Database, Tables 4.1, 4.2, 4.3, 4.6 and 4.9.

StatLink http://dx.doi.org/10.1787/888933377700

■ Table 0.7 [Part 2/2] ■

SNAPSHOT OF THE RELATIONSHIP BETWEEN OPPORTUNITY TO LEARN AND STUDENTS' ATTITUDES TOWARDS MATHEMATICS

Countries/economies where the percentage of students with positive attitudes towards mathematics is **above** the OECD average

Countries/economies where the percentage of students with positive attitudes towards mathematics is not statistically different from the OECD average

Countries/economies where the percentage of students with positive attitudes towards mathematics is **below** the OECD average

	Percentage of students who agreed or strongly agreed with the statement "I do mathematics because I enjoy it"	Percentage of students who disagreed or strongly disagreed with the statement "I am just not good at mathematics"	Percentage of students who agreed or strongly agreed with the statement "I often worry that it will be difficult for me in mathematics classes"	Change in the index of mathematics self-concept/anxiety associated with a one-unit increase in the index of familiarity with mathematics			
				Mathematics self-concept		Mathematics anxiety	
				Before accounting for performance in mathematics	After accounting for performance in mathematics	Before accounting for performance in mathematics	After accounting for performance in mathematics
	%	%	%	Index change	Index change	Index change	Index change
OECD average	38.1	57.3	59.5	**0.10**	**-0.10**	**-0.12**	**0.07**
Hungary	27.5	53.7	62.0	**0.12**	**-0.08**	**-0.20**	0.03
Slovenia	27.1	54.7	61.3	**0.14**	**-0.08**	**-0.13**	0.03
Qatar	60.6	53.2	68.6	0.02	**-0.08**	**-0.15**	0.00
Latvia	38.6	59.1	57.1	**0.06**	**-0.08**	**-0.13**	-0.02
Indonesia	78.3	39.0	76.7	**-0.08**	**-0.08**	-0.05	0.01
Japan	30.8	45.9	70.4	0.02	**-0.09**	-0.02	**0.07**
Ireland	37.0	60.1	69.8	**0.11**	**-0.09**	**-0.14**	0.06
Australia	39.0	63.4	59.7	**0.19**	**-0.11**	**-0.18**	**0.08**
Canada	36.6	63.4	59.6	**0.15**	**-0.11**	**-0.17**	**0.06**
Croatia	20.9	55.1	66.4	**0.12**	**-0.11**	**-0.14**	**0.09**
United States	36.6	66.7	57.3	**0.12**	**-0.11**	**-0.16**	**0.08**
Poland	36.1	46.3	57.4	**0.19**	**-0.11**	**-0.22**	0.08
Finland	28.8	58.6	51.7	**0.14**	**-0.12**	**-0.11**	**0.07**
Chile	42.3	40.1	72.3	**0.10**	**-0.12**	**-0.09**	0.04
Estonia	38.1	50.5	53.8	**0.07**	**-0.12**	**-0.18**	0.01
France	41.5	57.7	64.5	**0.14**	**-0.12**	**-0.06**	**0.12**
Netherlands	32.4	62.6	36.9	0.01	**-0.12**	-0.05	0.08
Belgium	28.8	61.3	58.2	**0.04**	**-0.12**	-0.02	**0.14**
New Zealand	38.2	59.0	62.1	**0.11**	**-0.12**	**-0.16**	**0.09**
United Kingdom	40.8	67.5	47.3	**0.12**	**-0.13**	**-0.14**	**0.09**
Denmark	56.9	71.0	38.6	**0.09**	**-0.13**	**-0.14**	**0.07**
Slovak Republic	27.9	46.8	57.6	0.05	**-0.13**	**-0.10**	**0.11**
Uruguay	50.6	47.2	76.7	**0.06**	**-0.13**	**-0.12**	**0.07**
Czech Republic	30.3	57.6	55.3	**0.10**	**-0.13**	**-0.08**	**0.12**
Sweden	37.0	64.9	42.3	**0.09**	**-0.13**	**-0.11**	0.09
Argentina	37.9	37.8	80.0	-0.06	**-0.14**	**-0.08**	0.00
Lithuania	47.6	53.4	57.4	**0.07**	**-0.14**	**-0.17**	0.01
Luxembourg	35.3	61.3	55.9	0.00	**-0.15**	**-0.09**	**0.09**
Switzerland	48.5	65.8	49.2	0.05	**-0.16**	**-0.09**	**0.13**
Germany	39.0	64.9	53.2	0.04	**-0.24**	**-0.11**	**0.17**
Austria	23.8	63.1	55.4	-0.01	**-0.25**	-0.02	**0.22**
Liechtenstein	56.2	65.6	49.8	-0.10	**-0.32**	0.02	0.25
Norway	32.2	57.0	53.5	m	m	m	m

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).
The *index of mathematics self-concept* is based on the degree to which students agreed with the statements: "I'm just not good in mathematics"; "I get good grades in mathematics"; "I learn mathematics quickly"; "I have always believed that mathematics is one of my best subjects"; and "In my mathematics class, I understand even the most difficult work".
The *index of mathematics anxiety* is based on the degree to which students agreed with the statements: "I often worry that it will be difficult for me in mathematics classes"; "I get very tense when I have to do mathematics homework"; "I get very nervous doing mathematics problems"; "I feel helpless when doing a mathematics problem"; and "I worry that I will get poor marks in mathematics".
The OECD average of the percentage of students who agreed or strongly agreed with the statement "I do mathematics because I enjoy it" is based on all OECD countries. The corresponding OECD average reported in Table 4.1 is based on the OECD countries that participated in both PISA 2003 and PISA 2012.
Statistically significant values are indicated in bold.
Countries/economies are ranked in descending order of the change in mathematics self-concept associated with a one-unit increase in familiarity with mathematics after accounting for performance in mathematics.
Source: OECD, PISA 2012 Database, Tables 4.1, 4.2, 4.3, 4.6 and 4.9.
StatLink http://dx.doi.org/10.1787/888933377700

Reader's Guide

Data underlying the figures

The data tables are listed in Annex A and available on line at www.oecd.org/pisa.

Four symbols are used to denote missing data:

a The category does not apply in the country concerned. Data are therefore missing.

c There are too few observations or no observation to provide reliable estimates (i.e. there are fewer than 30 students or less than five schools with valid data).

m Data are not available. These data were not submitted by the country or were collected but subsequently removed from the publication for technical reasons.

w Data have been withdrawn or have not been collected at the request of the country concerned.

Country coverage

This publication features data on 64 countries and economies: 34 OECD countries (indicated in black in the figures) and 30 partner countries and economies (indicated in blue in the figures).

Calculating international averages

An OECD average was calculated for most indicators presented in this report. In most tables, the OECD average corresponds to the arithmetic mean of the respective country estimates. In cases where the OECD average is computed so as to be consistent across different categories within a table, this is indicated in a note at the bottom of the table.

Rounding figures

Because of rounding, some figures in tables may not exactly add up to the totals. Totals, differences and averages are always calculated on the basis of exact numbers and are rounded only after calculation. All standard errors in this publication have been rounded to one or two decimal places (i.e. the value 0.00 does not imply that the standard error is zero, but that it is smaller than 0.005).

Bolding of estimates

This report discusses only statistically significant differences or changes (statistical significance at the 5% level). These are denoted in darker colours in figures and in bold in tables.

Reporting student data

The report uses "15-year-olds" as shorthand for the PISA target population. PISA covers students who are aged between 15 years 3 months and 16 years 2 months at the time of assessment and who have completed at least 6 years of formal schooling, regardless of: the type of institution in which they are enrolled; whether they are in full-time or part-time education; whether they attend academic or vocational programmes; and whether they attend public, private or foreign schools within the country.

Reporting school data

The principals of the schools in which students were assessed provided information on their schools' characteristics by completing a school questionnaire. Where responses from school principals are presented in this publication, they are weighted so that they are proportionate to the number of 15-year-olds enrolled in the school.

Indices used in this report

Some analyses in this report are based on synthetic indices. Indices from student and school questionnaires summarise information from several related questionnaire responses into a single global measure. The construction of the following indices is detailed in the *PISA 2012 Technical Report* (OECD, 2014):

- *Index of exposure to applied mathematics*
- *Index of exposure to pure mathematics*
- *Index of familiarity with mathematics*
- *Index of instrumental motivation*
- *Index of intrinsic motivation*
- *Index of mathematics anxiety*
- *Index of mathematics self-concept*
- *Index of disciplinary climate*
- *PISA index of economic, social and cultural status (ESCS)*

Categorising student performance

This report uses a shorthand to describe students' levels of proficiency in the subjects assessed by PISA:

- **Top performers** *are those students proficient at Level 5 or 6 of the assessment.*
- **Low performers** *are those students proficient at or below Level 1 of the assessment.*
- **Highest achievers** *are those students who perform at or above the 90th percentile in their own country/economy.*
- **Lowest achievers** *are those students who perform below the 10th percentile in their own country/economy.*

Categorising students and schools according to their socio-economic profile

- **Socio-economically advantaged students** are those students whose value of the *PISA index of economic, social and cultural status* (ESCS) is at or above the 75th percentile in their own country/economy.
- **Socio-economically disadvantaged students** are those students whose value of the *PISA index of economic, social and cultural status* (ESCS) is below the 25th percentile in their own country/economy.
- **Socio-economically advantaged schools** are those schools whose average value of the *PISA index of economic, social and cultural status* (ESCS) is significantly higher than the average in their own country/economy.
- **Socio-economically disadvantaged schools** are those schools whose average value of the *PISA index of economic, social and cultural status* (ESCS) is significantly lower than the average in their own country/economy.

Categorising students and schools according to their familiarity with mathematics

- **Students who are more familiar with mathematics** are those students whose value of the *index of familiarity with mathematics* is at or above the 75th percentile in their own country/economy.
- **Students who are less familiar with mathematics** are those students whose value of the *index of familiarity with mathematics* is below the 25th percentile in their own country/economy.
- **Schools where students are more familiar with mathematics** are those schools whose average value of the *index of familiarity with mathematics* is significantly higher than the average in their own country/economy.
- **Schools where students are less familiar with mathematics** are those schools whose average value of the *index of familiarity with mathematics* is significantly lower than the average in their own country/economy.

Abbreviations used in this report

% dif.	Percentage-point difference	S.E.	Standard error
Dif.	Difference	OTL	Opportunity to Learn
ESCS	PISA index of economic, social and cultural status		
ISCED	International Standard Classification of Education		

Further documentation

For further information on the PISA assessment instruments and the methods used in PISA, see the *PISA 2012 Technical Report* (OECD, 2014).

StatLinks

This report uses the OECD StatLinks service. Below each table and chart is a url leading to a corresponding Excel™ workbook containing the underlying data. These urls are stable and will remain unchanged over time. In addition, readers of the e-books will be able to click directly on these links and the workbook will open in a separate window, if their Internet browser is open and running.

Note regarding Israel

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

Reference

OECD (2014), *PISA 2012 Technical Report,* PISA, OECD, Paris, www.oecd.org/pisa/pisaproducts/PISA-2012-technical-report-final.pdf.

1

Why Access to Mathematics Matters and How it Can be Measured

This chapter discusses the importance of mathematics knowledge for acquiring numeracy skills and developing problem-solving abilities. It presents the concept of "opportunity to learn" and argues that measuring opportunity to learn is of critical importance for international comparisons of curricula and student performance. An overview of the data on opportunity to learn in PISA 2012 shows that education systems differ greatly in the degree to which students are exposed to mathematics concepts and also in the way mathematics problems are formulated and presented to students.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

The teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students with routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking (Polya, 1973).

Countries repeatedly reform their mathematics curricula to make sure they are relevant to students and societies (Cai and Ni, 2011; Usiskin and Willmore, 2008). Over time, reforms have been based on various factors, including on two observations: both national and international assessments proved that too many students were completing compulsory schooling without being able to use basic mathematics; and the evidence often showed that disadvantaged students were relegated to mathematics courses that were poorer in content and quality – a violation of the principle that all students should be exposed to high-quality instruction.

What the data tell us

- Numeracy skills are used daily in many jobs and are important for a wide range of outcomes in adult life, from successful employment to good health and civic participation.
- In 2012, the average 15-year-old student in an OECD country spent 3 hours and 32 minutes per week in regular mathematics lessons at school; 13 minutes more per week than the average student did in 2003.
- On average across OECD countries, less than 30% of students reported to know well the concept of arithmetic mean; less than 50% of students reported to know well the concepts of polygon and divisor.
- There are large international differences in students' average familiarity with algebraic and geometric concepts. Students in Macao-China reported the most familiarity with algebraic concepts, while students in Shanghai-China had the most familiarity with geometric concepts.
- There is only a weak correlation between students' exposure to applied mathematics and to pure mathematics at the system level, suggesting that the two methods of instruction rarely complement each other.

International data on students' classroom experiences with mathematics are illuminating because they show that policy makers and experts in charge of reform tend to think about mathematics differently than students do (Schoenfeld, 1983; Brown et al., 2008). For the skilled mathematician, solving a mathematics problem is an exciting process of discovery and mental training; for many students towards the end of compulsory education, mathematics is a well-defined set of facts that must be rehearsed until it is learned (Echazarra et al., 2016).

Notwithstanding the good intentions of mathematics teachers, weaker students who are underexposed to the practice of mathematics problem-solving – in many cases, these are students from disadvantaged families – never get an opportunity to develop a "taste for, and some means

of, independent thinking" (Polya, 1973). Given the importance of mathematics reasoning for life, mathematics curricula need to be enriching and challenging also for those students who do not plan to continue their formal education after compulsory schooling and for those who have fallen behind, in knowledge and self-confidence, since primary school.

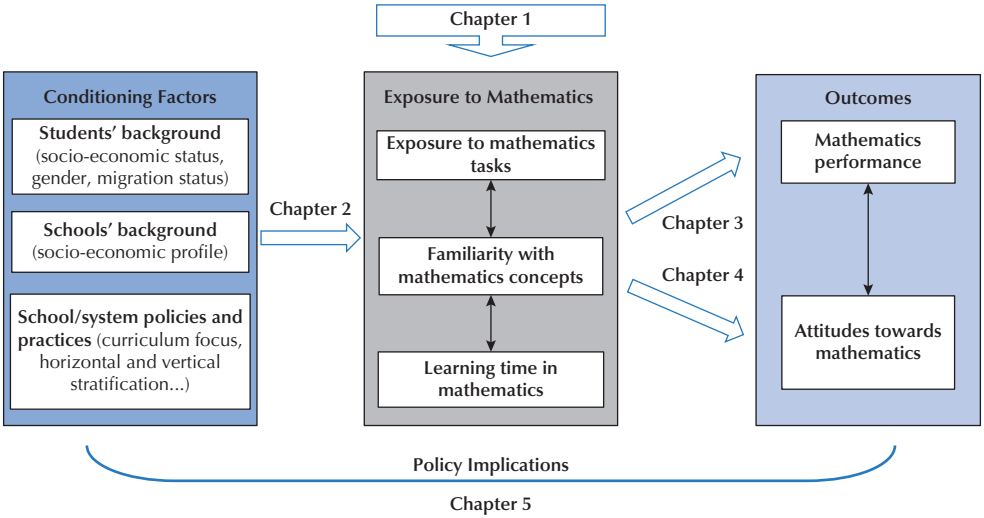
What these results mean for policy

- All students need mathematics for their adult life. Reducing socio-economic inequalities in access to mathematics content is thus an important policy lever for increasing social mobility.
- In many countries, the small share of students who reported that they know well and understand basic concepts signals the need to increase the effectiveness of mathematics teaching by focusing on key mathematics ideas and making more connections across topics.
- The large differences between the intended, the implemented and the achieved curriculum suggest the importance of regularly collecting data on students' exposure to mathematics content.
- International comparisons of curriculum standards, frameworks and teaching material can help countries to design reforms that increase the coherence of the mathematics curriculum.

Achieving equitable opportunities to learn involves not only the content and flexibility of the curriculum, but also how students from different socio-economic backgrounds progress through the system, how well learning materials match students' skills, and how teachers understand and manage the learning needs of diverse students. No matter how detailed and flexible the curriculum might be, mathematics teachers need to make difficult trade-offs to design mathematics lessons that are both accessible to weak students and challenging to bright ones.

This report uses data from PISA 2012 to describe students' opportunity to learn mathematics, including mathematics instruction time and the mathematics content to which students are exposed. It illustrates how students', schools' and systems' characteristics interact in affecting students' capacity to use the mathematics knowledge they acquire at school to solve real-world problems. Figure 1.1 shows the analytical framework of the report. This chapter introduces the concept of opportunity to learn, describes the metrics on content coverage and exposure developed for PISA 2012, and discusses how these metrics capture international differences in mathematics curricula. The second chapter takes one step back to examine student-, school- and system-level variables that can explain how these differences arise. The third chapter looks at how time spent on pure and applied mathematics tasks affects student performance in PISA, while the fourth chapter focuses on the relationship between content exposure and students' attitudes towards mathematics, such as mathematics self-concept and anxiety, which are closely related to mathematics performance. The fifth chapter discusses the policy implications of the preceding analyses.

■ Figure 1.1 ■
The analytical framework



THE IMPORTANCE OF MATHEMATICS SKILLS IN EVERYDAY LIFE

Mathematics teachers are accustomed to answering questions about the usefulness of what they teach. Not only students, but also parents and policy makers often worry about a mismatch between what is taught at school and the quantitative skills needed in everyday life. While it might be difficult to explain why students spend so much time learning algebra and geometry, mathematics is a core part of the curriculum for virtually every secondary student in the world. Is this justified? Should all students learn a significant amount of mathematics beyond what is needed to make simple calculations?

One of the rationales used to explain the central role of mathematics in global education curricula is the idea, dating back to Plato, that mathematics education enhances higher-order thinking skills. Those who are good at mathematics tend to be good thinkers, and those who are trained in mathematics learn to be good thinkers. According to this view, mathematics should be taught for its own sake, rather than to serve more concrete and practical aims.

Beyond the effects of mathematics training on some abstract mental faculties, there is a more intuitive and practical benefit from mastering mathematics at a reasonably good level: mathematics is a gatekeeper. The mathematics studied at school is the main entry point to quantitative literacy, and without solid quantitative skills a person cannot do many jobs. Exam scores in mathematics are, in fact, important factors in determining acceptance into higher education programmes leading to scientific and professional careers.

The demand for STEM (science, technology, engineering and mathematics) professionals has been continuously rising over recent years. For example, employment of STEM professionals across the European Union was approximately 12% higher in 2013 than it was in 2000, notwithstanding the effects of the economic crisis (European Parliament, 2015). Moreover, organisations compete for talent, and many of them now use rigorous quantitative assessments that test both verbal and mathematical ability when selecting employees (Schmitt, 2013).

The value of having quantitative skills has risen over recent years. Our societies are "drenched with data" (Steen, 2001), and the level of number skills needed to carry on daily life activities has increased. Understanding concepts such as "exponential growth" or "line of best fit", assessing the rate at which a variable is changing or knowing what to expect from the flip of a coin have become important for making informed judgements and choices. Computers have reduced the need for mechanical calculations, but the importance of understanding numbers has become even greater in the digital age. In fact, the more people can do with information technology in mathematics, the greater the need for their understanding of and their ability to critically analyse what they are doing (OECD, 2015).

Data from the Survey of Adult Skills, a product of the OECD Programme for the International Assessment of Adult Competencies (PIAAC), provide some tools for assessing the value of quantitative skills at work and in everyday life (Box 1.1). The survey assesses numeracy skills, defined as "the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life" (OECD, 2013a). Numeracy thus refers not just to the ability to perform basic calculations, but to a wide range of skills, such as being able to measure, use and interpret statistical information; understand and use shape, design, location and direction; and think critically about quantitative and mathematical information (Gal and Tout, 2014). The survey measures numeracy from adults' answers to a set of carefully designed and contextualised problems.

Box 1.1. **The Survey of Adult Skills (PIAAC)**

The Survey of Adult Skills is an international survey conducted as part of the Programme for the International Assessment of Adult Competencies (PIAAC). It measures the key cognitive and workplace skills needed for individuals to participate in society and for economies to prosper. The survey is conducted among adults aged 16 to 65. It assesses their literacy and numeracy skills, as well as their ability to solve problems in technology-rich environments, and collects a broad range of information, including how skills are used at work, at home and in the community.

The first round was conducted in 2011-2012 in 24 countries and subnational regions. Results of the second round, released in June 2016, include 9 additional countries.

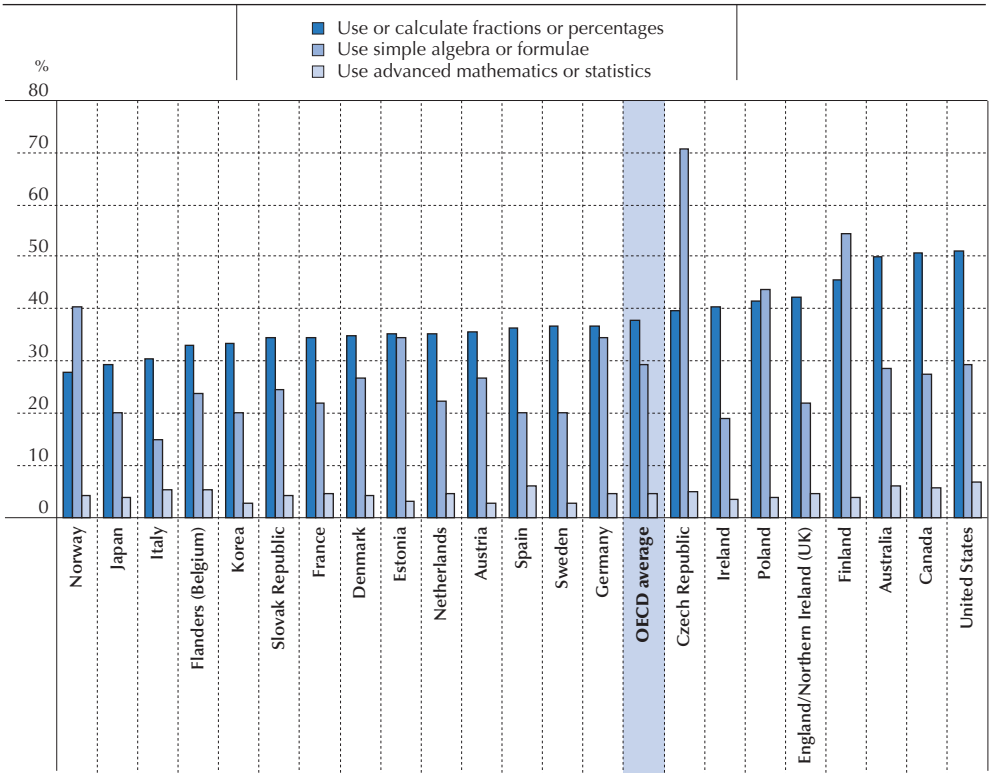
Source:
http://www.oecd.org/site/piaac/

Figure 1.2 shows the extent to which various numeracy skills are used at work, as assessed in the Survey of Adult Skills. On average across participating OECD countries, 38% of workers aged 16 to 65 use or calculate fractions, decimals or percentages, 29% use simple algebra or formulas, and 4% use advanced mathematics at work at least once a week. More than one in three workers in Estonia, Germany, Norway and Poland use algebra at work weekly or daily, as do more than one in two workers in the Czech Republic and Finland. The use of mathematics at work is not limited to the top-paying occupations. On average across OECD countries, 36% of workers in the highest earnings quartile use algebra at work, compared to 18% of workers in the bottom earnings quartile (Table 1.1b).

■ Figure 1.2 ■

Numeracy skills used at work

Percentage of workers who reported that they use these numerical skills at work at least once a week



Note: The OECD countries included in the analysis are: Australia, Austria, Flanders (Belgium), Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Poland, the Slovak Republic, Spain, Sweden, England/Northern Ireland (UK) and the United States.

Countries and economies are ranked in ascending order of the percentage of workers who reported that they use or calculate fractions or percentages at work.

Source: OECD, Survey of Adult Skills (PIAAC) (2012), Table 1.1a.

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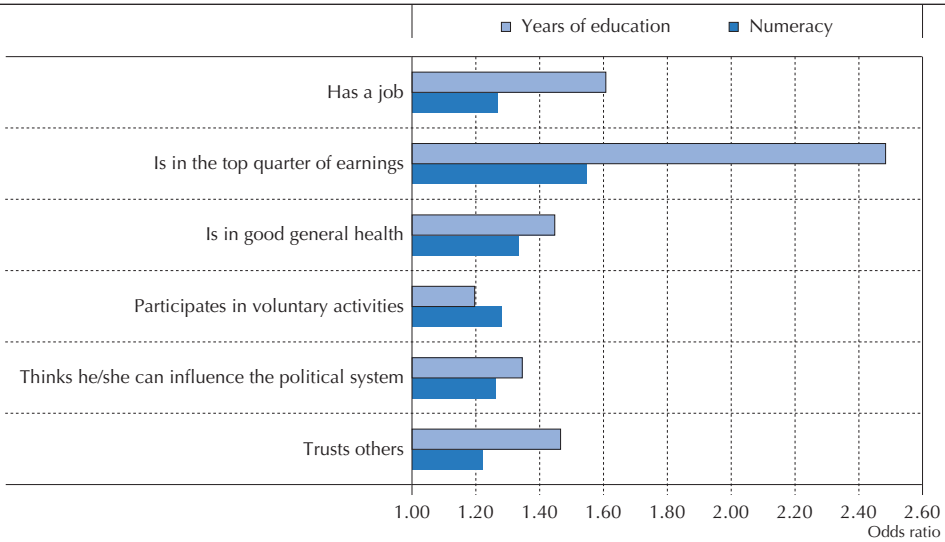
Important decisions in one's personal life, on the job, and in matters of public interest call for sophisticated quantitative reasoning (Schoenfeld, 2002). For example, perceptions about the levels of health risks are less accurate among individuals with low numeracy (Carman and Kooreman, 2014), and low numeracy constrains informed patient choice, reduces medication compliance and limits access to treatments (Nelson et al., 2008). Data from the Survey of Adult Skills show that higher numeracy skills are strongly correlated with other outcomes, such as participation in the labour market, income, good health, participation in volunteer activities, feeling that one has an influence on political life, and the level of trust in others (Figure 1.3). Adults performing 50 points higher than the mean on the survey's numeracy scale are 27% more likely to have a job and 55% more likely to earn high wages than adults performing at the mean. A numeracy score 50 points above the mean raises the odds of being employed to the same level as completing two additional years of education would do.

■ Figure 1.3 ■

Relationship between years of education and numeracy, and economic and social outcomes

Increase in the likelihood of the outcome related to an increase of one standard deviation in years of education or in numeracy; OECD average (22 countries)



How to read the chart: An odds ratio of 1.27 corresponding to the outcome "has a job" and "numeracy" means that an individual who scored one standard deviation higher than another on the Survey of Adult Skills (PIAAC) numeracy scale is 27% more likely to be employed.

Notes: "Years of education" has an average standard deviation of 3.7 years; "numeracy" has an average standard deviation of 51 points.

The OECD countries included in the analyses are: Australia, Austria, Flanders (Belgium), Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Poland, the Slovak Republic, Spain, Sweden, England/Northern Ireland (UK), and the United States.

Source: OECD, Survey of Adult Skills (PIAAC) (2012), Table 1.2.

StatLink *http://dx.doi.org/10.1787/888933376878*

These results from the Survey of Adult Skills show relationships, and cannot be interpreted as the causal effect of mathematics instruction on life outcome. However, the findings are consistent with a large literature showing that attending more advanced mathematics courses has an impact on labour market outcomes (Joensen and Nielsen, 2009; Levine and Zimmerman, 1995). In a study on students' earnings a decade after graduation in the United States, Rose and Betts (2004) find that the math curriculum is responsible for around 27% of the earnings gap experienced by students from lowest-income families relative to middle-income families.

THE RELATIONSHIP BETWEEN MATHEMATICS KNOWLEDGE AND MATHEMATICAL LITERACY

Many argue that the traditional mathematics curriculum fails students because it emphasises a type of mathematics that is radically different from the one used at the workplace (Steen, 2001). Problem solving at work is characterised by pragmatic approaches and techniques that are quick and efficient for specific types of tasks, while the formal mathematics taught at school strives for consistency and generality (Hoyles et al., 2010). This argument has gained popularity because it is not easy to define which mathematics content in the curriculum is most likely to help develop numeracy. Workplace mathematics is also a moving target: changes in society, in technology and in the practice of mathematics also shift the priorities among the many mathematics topics that can be useful for solving problems at work.

Are the differences between school mathematics and the numeracy skills used in life really so large? A look at the PISA performance of students with different levels of exposure to mathematics at school can help to answer this question. PISA assesses the mathematical literacy of students. Mathematical literacy is closely related to the concept of numeracy used in the Survey of Adult Skills,[1] even if it has a stronger connection with the mathematics knowledge acquired at school.

The mathematics framework of PISA defines mathematical literacy as:

"an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (OECD, 2013b).

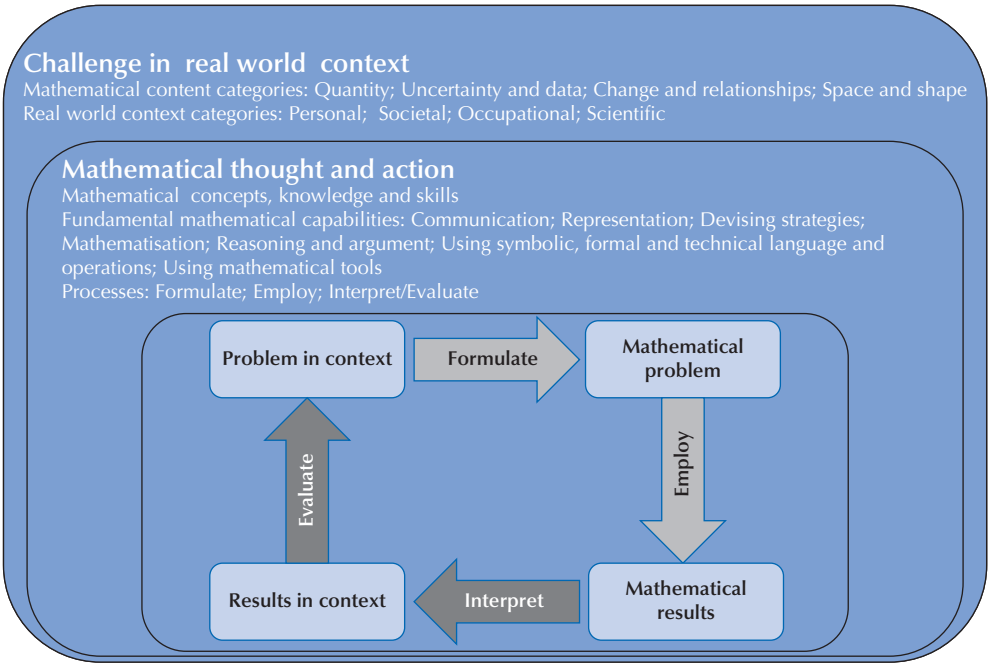
The focus of PISA has been less about what students know after studying a particular curriculum, and more on students' ability to use what they have learned at school to address authentic, real-life challenges and problems (OECD, 2013b; Cogan and Schmidt, 2015). In the description of what students should know and be able to do at different levels of achievement, the PISA mathematics framework refers to "big ideas" (core concepts); it does not specify algebra or geometry or any other specific facet of mathematics. But this does not mean that the structure of the mathematics curriculum, the mastery of concepts and the time spent on mathematics exercises do not matter for developing students' mathematical literacy.

Mathematical literacy and mathematics knowledge – defined as familiarity with mathematics concepts and procedures – are, in fact, not separate but intertwined. Mathematics content areas and concepts have been developed over time as a means to understand and interpret natural and social phenomena (OECD, 2013b). Exposure to this codified content helps students to understand the underlying structure of real problems, shaping what they see and how they behave when they encounter new situations related to those they have previously abstracted and codified (Roterham and Willingham, 2010).

Figure 1.4 shows a simplified version of the stages through which students use the mathematics they learn at school to solve real-life problems. In the first stage, the student takes advantage of his or her knowledge of mathematics first to recognise the mathematical nature of a problem and then to formulate the problem in mathematical terms. The downward-pointing arrow in Figure 1.4 depicts the work undertaken as the problem-solver uses mathematical concepts, procedures, facts and tools to obtain the results. This stage typically involves mathematical reasoning, manipulation, transformation and computation. Next, the results need to be interpreted in terms of the original problem. These processes of formulating, employing and interpreting mathematics draw on the problem-solver's knowledge about individual topics and on a range of fundamental mathematics capabilities.

■ Figure 1.4 ■

The PISA model of mathematical literacy



Exposure to mathematics content helps students to navigate through the processes of formulating, employing and interpreting mathematics. However, becoming mathematically literate requires more than just acquiring knowledge and practicing. Students have to learn to recognise how mathematics can help them deal with situations, solve problems and make sound judgements. The challenge for schools, beyond selecting *which* fundamentals to teach, is *how* to teach these fundamentals in a way that improves students' problem-solving abilities. Teachers not only have to carefully select the content of their lessons, but they also have to tailor the delivery of this content to suit the different capacities of students.

PISA 2012 included detailed information on the types of mathematics students had the opportunity to learn. In an assessment focusing on mathematics skills for life, this information provides a unique opportunity to better understand the relationship between the mathematics taught in school and that used outside of school.

THE CONCEPT OF OPPORTUNITY TO LEARN

The opportunity to learn (OTL) concept refers to the notion that what a student learns at school is related to the content taught in the classroom and the time a student spends learning this content (Cogan and Schmidt, 2015; Schmidt and Maier, 2009). The most quoted definition of OTL comes from Husen's report of the 1964 First International Mathematics Study (FIMS): "whether or not students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967, pp. 162-163, cited in Burstein, 1993). Research on opportunity to learn started as an afterthought in FIMS when analysts became concerned that not all the tested students had the same opportunities to study a particular topic or to learn how to solve a particular type of problem presented in the assessment (Floden, 2002).

Carroll's (1963) model of school learning provides a strong theoretical basis for the analysis of OTL. The model expresses key factors of learning, including aptitude and ability, in the metric of time, so that the crucial question is no longer "*What* can this student learn?" but "*How long* will it take this student to learn?". The following relationship describes the elements of the model:

$$\text{Learning} = f\left[\frac{\left(\begin{array}{c}\textbf{\textit{Opportunity to learn}}\ \textit{or}\\ \textit{Time allocated for learning}\end{array}\right) * \left(\begin{array}{c}\textbf{\textit{Perseverance}}\ \textit{or}\\ \textit{Percentage of time actually spent engaged in learning}\end{array}\right)}{\left(\begin{array}{c}\textbf{\textit{Aptitude}}\ \textit{or}\\ \textit{Time actually needed to learn}\end{array}\right) * \left(\begin{array}{c}\textbf{\textit{Quality of instruction}} * \textbf{\textit{Ability}}\ \textit{to understand}\end{array}\right)}\right]$$

Aptitude, ability and perseverance are student characteristics, while opportunity to learn and the quality of instruction are mainly controlled by teachers within the conditions established by the education system. After Carroll, several authoritative reviews of research concluded that time spent on content and the way in which time is organised are primary factors influencing student achievement (Carroll, 1989; Scheerens and Bosker, 1997; Marzano, 2003). Within a short period of time, OTL had a profound impact on the thinking of researchers and practitioners alike (Marzano, 2003).

The school curriculum defines the intended objectives of the education system in terms of content coverage and time allocated to topics. Beyond the *intended* curriculum, what matters for students' learning is the *implemented* curriculum, or the content actually delivered by the teachers. The existence of a single coherent mathematics curriculum delivered by all teachers is nothing more than a myth: discrepancies between the intended and the delivered curriculum exist across all education systems (Floden, 2002; Schmidt et al., 1997; Schmidt et al., 2001). Even when highly structured textbooks are used, teachers make independent choices regarding which topics will be covered and to what extent (Doyle, 1992; Valverde et al., 2002). Teachers might depart from the intended curriculum because some of their students are not sufficiently prepared to absorb the content of overly ambitious and lengthy textbooks, or because the curriculum itself dissuades teachers from sticking closely to its plans. For example, teachers might omit some material because they know that the students' future teachers will have to cover the same material again. Starting from what is taught in classrooms and how it is taught, the *achieved* curriculum – what students actually learn – is, in turn, related to students' ability, aptitude and attitudes towards learning.

Students' opportunity to learn depends on both the intended and the implemented curriculum. Students may not be exposed to certain mathematics concepts because these concepts are not included in the curriculum or because teachers may not cover them. Data collected as part of the 2011 Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2012) show that a core set of topics is covered in the intended curriculum of most countries. However, large differences across countries exist in the allocation of these topics to different grades, and in the percentage of teachers who actually teach the topic in each grade (Table 1.3). The percentage of students who are taught basic topics, like fractions, in grade 8 is relatively low (less than 50% in most participating countries), consistent with the fact that fractions are supposed to be covered in the early grades (in most countries, fractions are not expected to be covered after grade 7). In contrast, linear equations and formulas for perimeters, areas and volumes are expected to be covered in the eighth grade in almost all participating countries. But in Hong Kong-China, Japan, Norway, Slovenia, Sweden, Chinese Taipei and Ukraine – where linear equations are part of the eighth-grade curriculum – less than 50% of students in grade 8 are taught them. Teachers may decide not to cover a certain topic with some students or to cover it in earlier or later grades, especially when the curriculum allows for such flexibility.

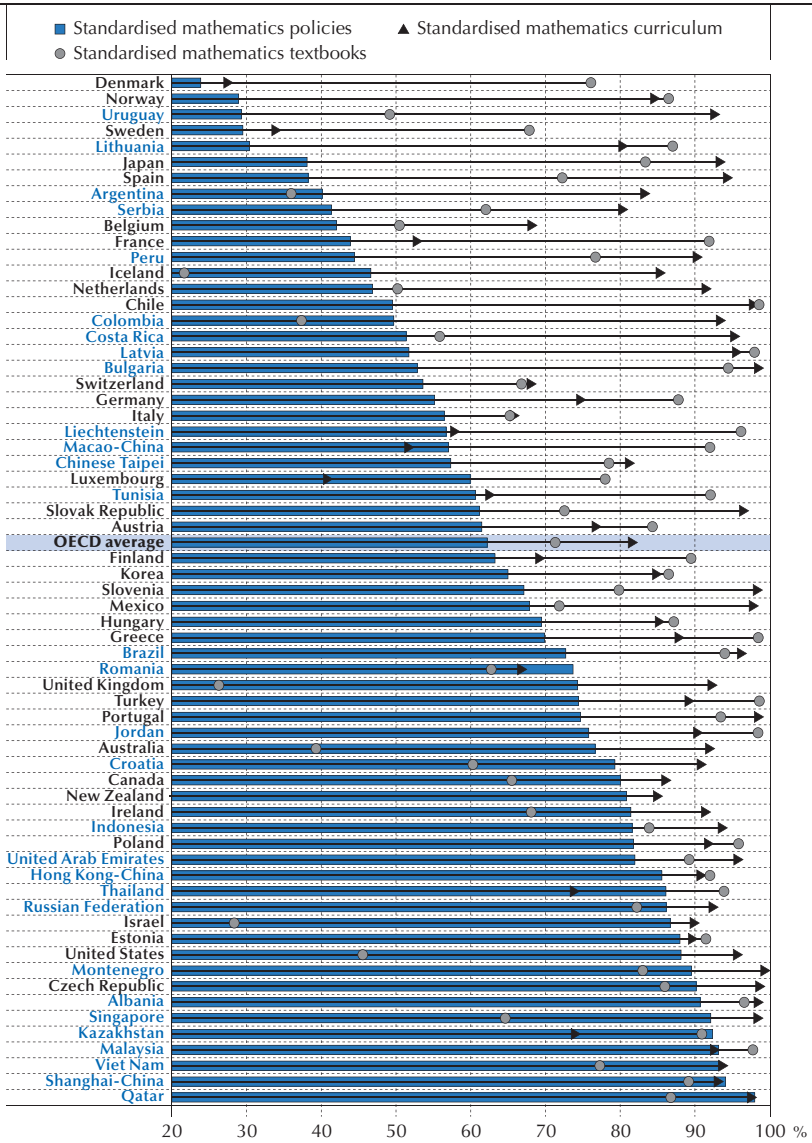
Standardisation policies – such as using a common curriculum across all classes in a school – can limit the freedom of teachers to define the content of their instruction. Figure 1.5 shows that there are large differences across countries in the level of standardisation of mathematics teaching. Around 60% of students in OECD countries are in schools that adopt standardised mathematics policies with shared instructional materials accompanied by staff development and training. These policies are relatively rare in the Nordic countries, but relatively common in several Asian countries and economies. In all countries and economies but Denmark, Luxembourg and Sweden, the majority of students attends schools where teachers are required to follow a mathematics curriculum that specifies the content to be covered each month.

Textbooks are a key link between the intended and the implemented curriculum. Textbooks influence which topics are likely to be covered by teachers, in which order and through

■ Figure 1.5 ■

Use of standardised practices for curriculum and teaching

Percentage of students in schools that practice standardised policies for mathematics teaching, curriculum and textbooks



Note: A standardised policy for mathematics consists of a school curriculum with shared instructional materials accompanied by staff development and training. A standardised curriculum specifies content that mathematics teachers should follow at least monthly. All measures are reported by the school's principal.

Countries and economies are ranked in ascending order of the percentage of students in schools that use standardised mathematics policies.

Source: OECD, PISA 2012 Database, Table 1.5.

StatLink http://dx.doi.org/10.1787/888933376885

which pedagogical strategies (Freeman and Porter, 1989; Grossman and Thompson, 2008; Johansson, 2005; Reys et al., 2003; Stathopoulou, Gana and Chaviaris, 2012). Adopting a single textbook for all mathematics classes in a school is a common practice in most countries (Figure 1.5), but at the same time teachers have a major role in selecting textbooks. On average across OECD countries, 77% of students attend schools where teachers choose textbooks, while principals and school governing boards are less involved (Table 1.4). Only in Greece, Jordan, Luxembourg and Malaysia over 80% of students attend schools where the national education authority chooses which textbooks are used in the school.

MEASURING OPPORTUNITY TO LEARN IN PISA

For international comparisons, measures of OTL are relevant in two ways: as a possible factor leading to international differences in achievement, and as indicators of cross-national and within-countries differences in the implemented curriculum. If OTL is not taken into account in cross-national comparisons, its effects might be mistakenly attributed to other characteristics of students or education systems (Schmidt et al., 2014). A clear international picture of the similarities and differences in the content students are given the opportunity to learn provides each country with a context for considering curriculum reforms and evaluating equity in access to learning opportunities.

There are two main approaches to measuring OTL. The first, adopted in early studies, such as the First International Mathematics Study (FIMS), measures students' exposure to content at the classroom level through a teacher survey. The second, used in PISA 2012, presents exemplar problems to test-takers, asking them whether they have seen anything similar during their school lessons. Both approaches have advantages and shortcomings. Teachers' reports are generally more accurate descriptions of the delivered curriculum. Students' reports can provide more reliable measures of the time students are actually engaged in learning the topic, under the assumption that students can objectively establish the similarity between what they do in class and what they see in the problems presented in the questionnaire.

The student questionnaire in PISA 2012 included several questions on the degree to which students encounter various types of mathematics problems in their courses, how familiar they are with certain formal mathematics content, and how frequently they are taught to solve specific mathematics tasks. Responses to these questions were used to construct a number of OTL measures and indices, as detailed in Box 1.2.

Based on students' self-reports, the data show substantial variation across education systems in students' exposure to mathematics content. These international differences emerge clearly from the simplest measure of OTL in PISA – the time students reported spending in mathematics classes each week. In 2012, the average 15-year-old student in an OECD country spent 3 hours and 32 minutes per week in mathematics lessons (Figure 1.6). However, behind this average lie great variations among school systems. While 15-year-old students in Canada spent more than 5 hours per week in mathematics lessons, students in Hungary spent 2 hours and 30 minutes per week.

Box 1.2. **Measures of Opportunity to Learn in PISA 2012**

PISA 2012 assessed Opportunity to Learn mathematics through a number of measures:

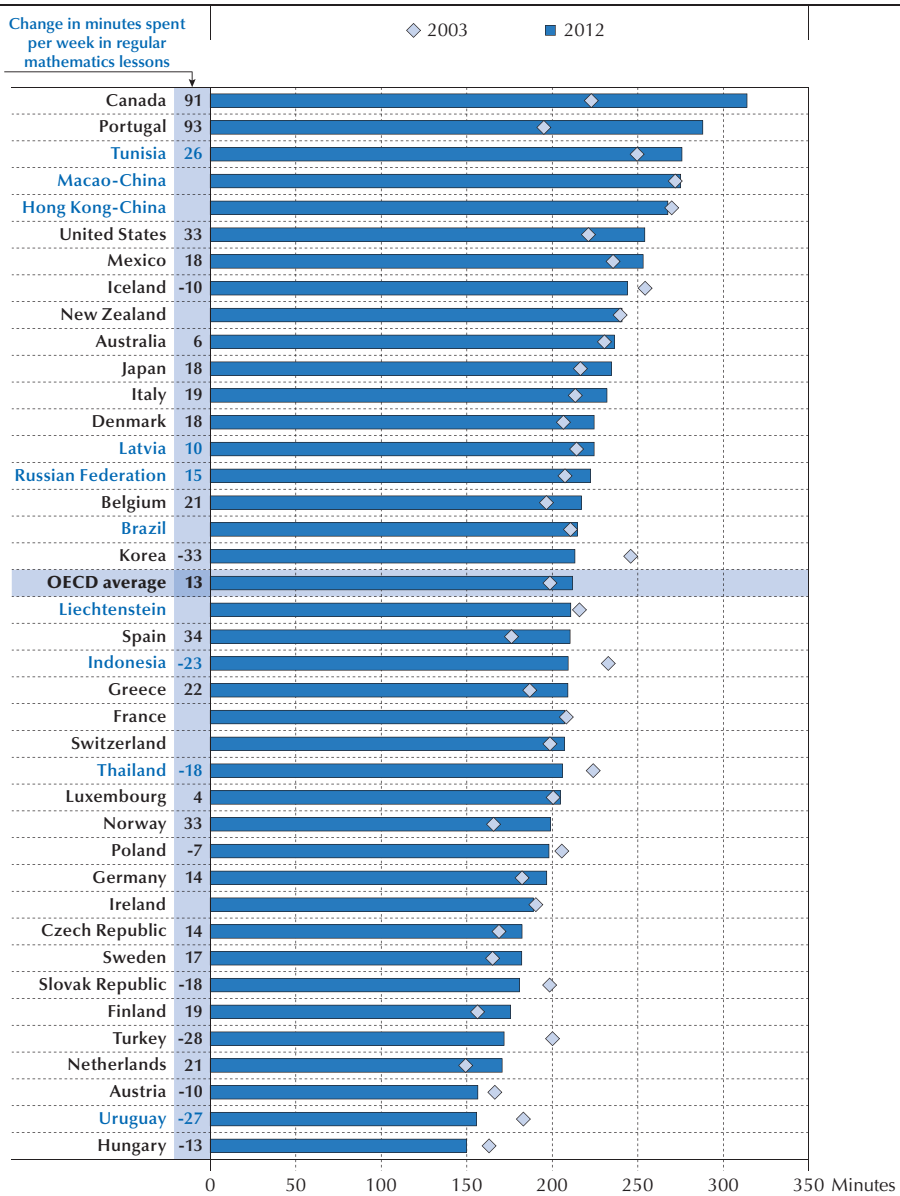
- **Time** spent per week in regular mathematics lessons, in minutes.
- **Exposure** to different types of mathematics tasks during time in school (Question 1 at the end of this chapter), which was scaled to derive two indices (both indices are normalised to have an OECD average of 0 and a standard deviation of 1):
  - The *index of exposure to applied mathematics* refers to student-reported experience with applied tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
  - The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school that require knowledge of algebra (linear and quadratic equations).
- **Familiarity with mathematics concepts:** Question 2 (reproduced at the end of this chapter) asked students to judge how familiar they were with 13 mathematics concepts. Replies were used to create the *index of familiarity with mathematics,* which was normalised to have an OECD average of 0 and a standard deviation of 1. Part of the analysis contained in this report looks at familiarity with:
  - Algebra, measured as the average student's familiarity with the concepts of exponential function, quadratic function and linear equation.
  - Geometry, measured as the average student's familiarity with the concepts of vector, polygon, congruent figure and cosine.

The question about familiarity also included three foils, i.e. non-existing pseudo-concepts. Responses indicating that students heard of these concepts or knew them well were considered to indicate overclaiming. The *index of familiarity with mathematics* used in this report is corrected for overclaiming.

- Frequency of experience with **specific mathematics tasks** in mathematics lessons and in tests, including the following:
  - Algebraic word problems (Question 3a, reported at the end of this chapter), such as: "Ann is two years older than Betty and Betty is four years older than Sam. When Betty is 30, how old is Sam?".
  - Procedural tasks (Question 3b), such as solving a linear equation or finding the volume of a box.
  - Pure mathematics problems (Question 3c), such as determining the height of a pyramid using geometrical theorems, and solving a problem with prime numbers.
  - Contextualised mathematics problems (Question 3d), such as interpreting a trend in a chart.

■ Figure 1.6 ■

Change between 2003 and 2012 in the time spent per week in mathematics classes



Notes: Statistically significant changes between 2003 and 2012 in the time spent per week in regular mathematics lessons are shown next to the country/economy name.
Only countries with comparable data for both PISA 2003 and PISA 2012 are included.
Countries and economies are ranked in descending order of the time spent in mathematics classes per week in 2012.
Source: OECD, PISA 2012 Database, Table 1.6.
StatLink *http://dx.doi.org/10.1787/888933376897*

Time spent in mathematics classes has increased over the past decade. Across OECD countries, students in 2012 spent an average of 13 minutes more per week in mathematics classes than their counterparts did in 2003. In some countries, the average time spent in regular mathematics classes increased much more than that. In Canada and Portugal, for example, students in 2012 spent 1.5 hours more in mathematics classes than their counterparts in 2003 did, and students in Norway, Spain and the United States spent at least 30 minutes more. The amount of time students spent in mathematics lessons increased by more than 15 minutes in another 11 countries and economies. Only in Korea, which had the fifth longest mathematics class time in 2003, did that class time shrink over the period – by more than 30 minutes.

Education systems differ substantially not only in the time allocated to mathematics teaching, but also in how this time is allocated to different topics. PISA asked students how familiar they are with certain formal mathematics content, including such topics as quadratic functions, radicals and the cosine of an angle (see Box 1.2 for a description of the *index of familiarity with mathematics*). On average across OECD countries, less than 30% of 15-year-old students reported to know well and understand the concept of arithmetic mean; less than 50% of students reported to know well and understand the concepts of divisor and polygon (Table 1.7).

Students in Hong Kong-China, Japan, Korea, Macao-China, Shanghai-China, Spain and Chinese Taipei are most familiar with mathematics concepts in general (Table 1.8). More specifically, Figure 1.7 shows that students in Japan, Macao-China and Singapore reported greater familiarity with the algebraic concepts of linear equation, quadratic function and exponential function. Most students in Shanghai-China reported frequent exposure to the geometric concepts of vector, polygon, congruent figure and cosine. The high levels of exposure to advanced mathematics concepts among Asian students is partly due to the academically oriented mathematics curricula in those countries/economies (Morris and Williamson, 2000), to the emphasis on advanced mathematics courses in teacher-training programmes (Ding et al., 2013), and to a culture of high-stakes examinations that requires teachers to cover all the topics students will need to know for their future tests (Yang, 2014).

At the other end of the spectrum, the majority of students in Sweden reported that they had either never encountered or had encountered only once or twice these algebraic and geometric concepts. In several countries, students reported greater familiarity with algebra than with geometry, or vice versa. For example, while 15-year-old students in Greece were among the most frequently exposed to geometry, they lagged behind the OECD average in exposure to algebra.

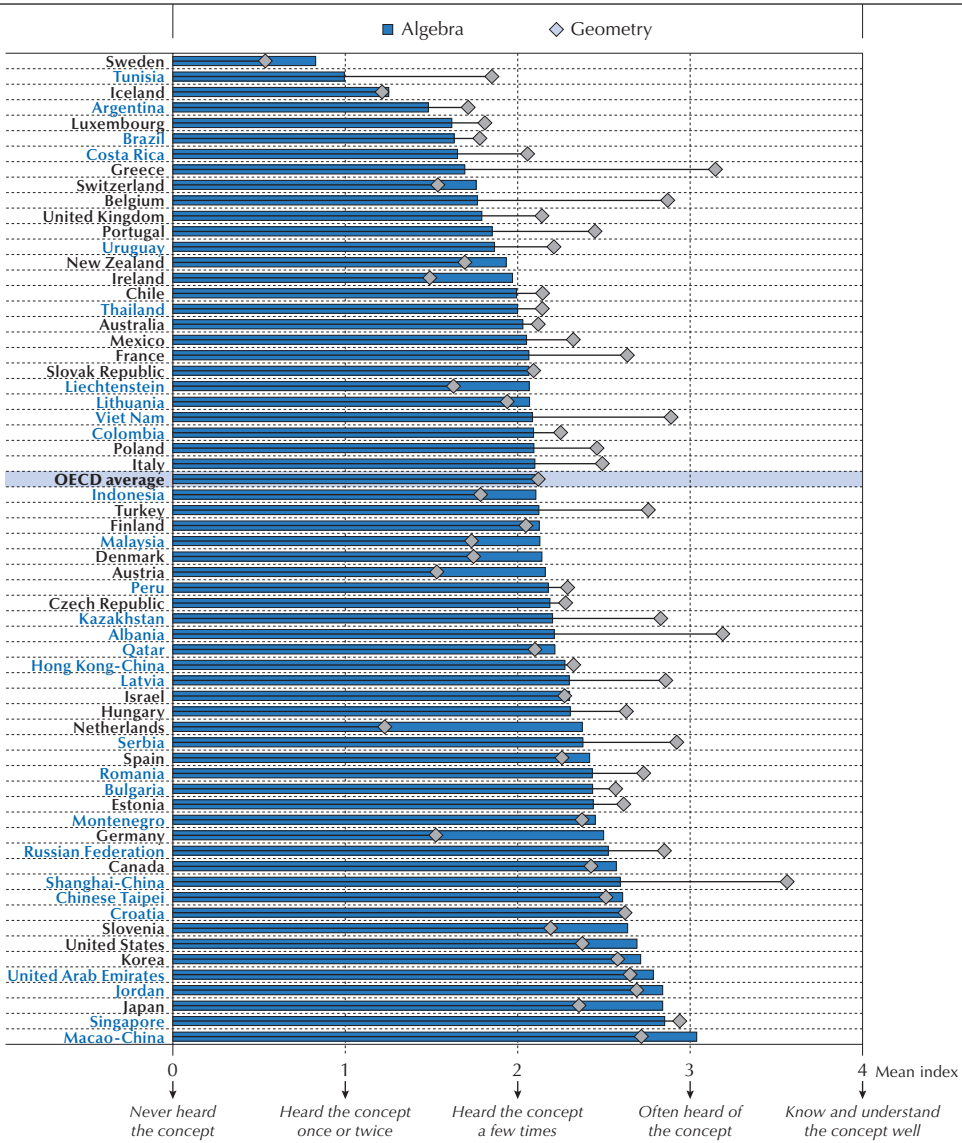
Another set of questions in PISA 2012 was intended to determine whether the teaching of mathematics was more oriented towards pure or applied mathematics (see Question 1 at the end of this chapter). Students' responses to these questions were used to derive the two indices of exposure to pure mathematics and exposure to applied mathematics (Box 1.2).

Students in Korea, the Russian Federation, Singapore and Spain reported the most frequent exposure to pure mathematics at school. Students in Kazakhstan, Korea, Poland and Thailand reported the greatest exposure to applied mathematics (Figure 1.8). Across education systems, there is only a weak relationship between average exposure to applied mathematics and average

■ Figure 1.7 ■

Students' familiarity with algebra and geometry

Self-reported knowledge of mathematics concepts



Notes: Familiarity with geometry is measured as the average student's familiarity with the concepts of vector, polygon, congruent figure and cosine. Familiarity with algebra is measured as the average student's familiarity with the concepts of exponential function, quadratic function and linear equation.

Countries and economies are ranked in ascending order of average familiarity with algebra.

Source: OECD, PISA 2012 Database, Table 1.8.

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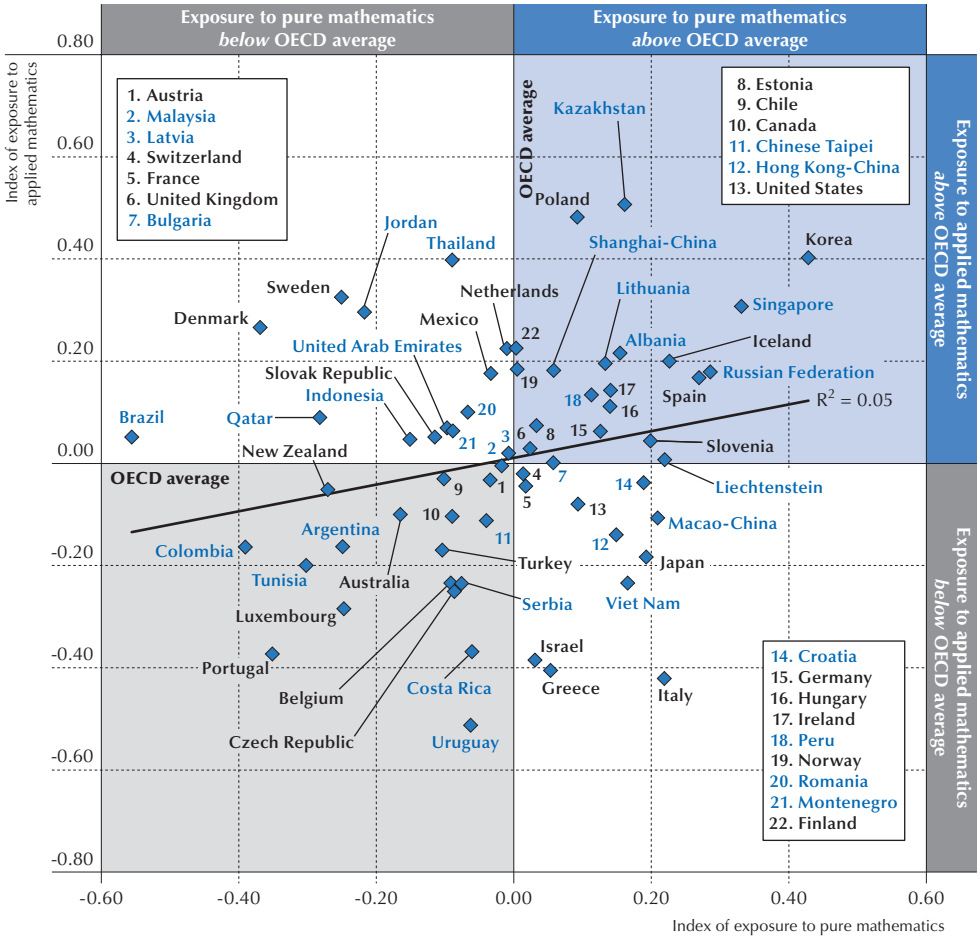
■ Figure 1.8 ■

Relationship between exposure to applied mathematics and exposure to pure mathematics



Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks requiring knowledge of algebra (linear and quadratic equations).
The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
Source: OECD, PISA 2012 Database, Table 1.9a.
StatLink http://dx.doi.org/10.1787/888933376914

exposure to pure mathematics. Several education systems, including those in Greece, Hong Kong-China, Italy, Japan, Macao-China, the United States and Viet Nam, devote more time to pure mathematics problems than to applied problems, while the opposite is observed in Brazil, Denmark, Jordan, Mexico, Montenegro, Qatar, Romania, the Slovak Republic, Sweden, Thailand and the United Arab Emirates (Table 1.9a).

A suitable balance between formal and applied content has been one of the most contentious issues in the public debate on mathematics education. "Maths wars" have raged between those who think that "underlying ideas must be elevated above the examples that illustrate them" (Munson, 2010) and those who believe that "algorithms are harmful" and children should be left free "to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks" (Kamii and Dominick, 1998: 132). This debate has focused on the structure, presentation and type of problems included in mathematics textbooks; on the extent to which all students should learn mathematics, the type of mathematics they should learn, and the types of problems that are suitable for them to work on as they learn it; and on the type of representations emphasised for student learning and problem solving (Goldin, 2008; Schoenfeld, 2004).

The alternating fortunes of the advocates of "traditional" and "reform" mathematics have influenced curriculum changes, the direction of pedagogical innovations and the content of in-service or pre-service teacher training (Klein, 2003; Schoenfeld, 2004). Some mathematics curricula have tried to reach a middle ground between the two extremes, emphasising the importance of both a high level of mathematics rigour and of opportunities to use mathematics in real-life contexts. In Germany, for instance, the ability to construct models to interpret and understand real problems is one of six compulsory competencies in the new national "Educational Standards" for mathematics (OECD, 2011).

Exposure to mathematics tends to increase as students move to higher grades in schools, but this progression varies across different mathematics content (Figure 1.9). The indices of exposure to pure mathematics and familiarity with mathematics show clear progressions as students advance through the school system. The progression is steeper for familiarity with mathematics because the 13 mathematics concepts included in the measure cover an exhaustive range of material at different levels of difficulty, while the *index of exposure to pure mathematics* is based on a set of algebraic concepts (linear and quadratic equation) of average difficulty for 15-year-old students.

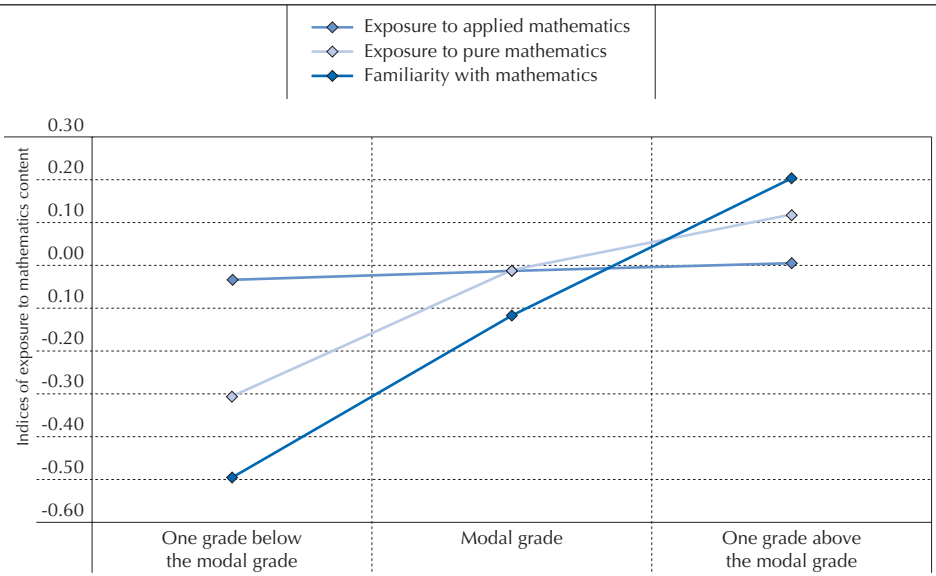
By contrast, students in lower and higher grades reported similar levels of exposure to applied mathematics. This may be because the *index of applied mathematics* in PISA is based on students' reports of exposure to relatively simple contextualised tasks that require basic numeric skills. Different patterns of exposure to applied mathematics, depending on the students' grade level, are observed across countries. Students in the Netherlands are relatively frequently exposed to applied mathematics, and this exposure increases among older students. By contrast, in the Czech Republic and the Slovak Republic, teachers in higher grades tend to focus less on the types of applied mathematics tasks that are presented in the PISA questionnaire (Table 1.10).

It is difficult to teach mathematics as both general and concrete. Research suggests that, to achieve this, several different representations (e.g. numerical, verbal, symbolic and graphical) of concepts and phenomena are essential, as are the links and transitions between these representations (e.g. Janvier, 1987). The questions on opportunity to learn in PISA 2012 tried to illustrate international differences in the way mathematics problems are presented to students (Box 1.2).

■ Figure 1.9 ■

Exposure to mathematics content in class, by grade

OECD average (23 countries)



Notes: The modal grade is defined in each country as the grade with the largest number of students tested in PISA.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school that require knowledge of algebra (linear and quadratic equations).
The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The OECD average for exposure to pure, applied and familiarity with mathematics is calculated only for countries with a valid number of students in the three grades (one grade below the modal grade, the modal grade and one grade above the modal grade).
Source: OECD, PISA 2012 Database, Table 1.10.
StatLink http://dx.doi.org/10.1787/888933376925

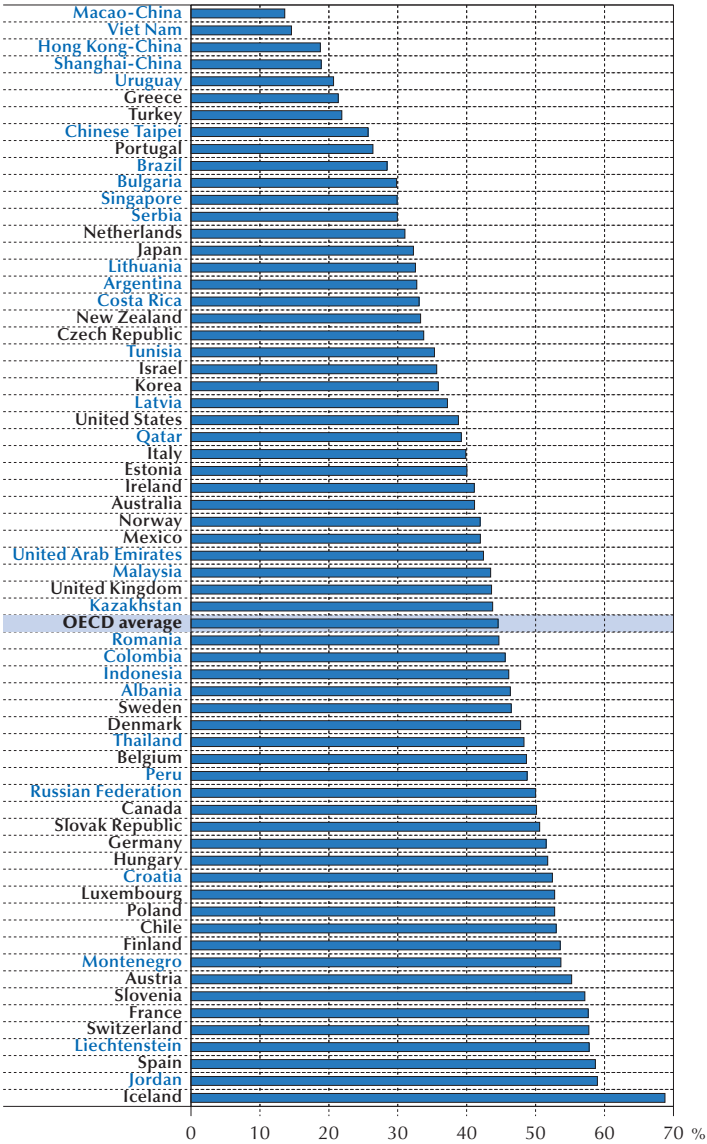
Word problems are used consistently throughout the mathematics curriculum. They are often developed by teachers who wish to connect the mathematics tasks to students' experiences more directly, and to provide contexts to which students can more easily relate (see Question 3a at the end of the chapter). On average across OECD countries, 87% of students see this type of problem at least sometimes in their mathematics lessons, and 79% see these problems at least sometimes in their assessments (Table 1.11a). In the East Asian economies of Hong Kong-China, Macao-China, Shanghai-China and Viet Nam, less than 20% of students are frequently exposed to algebraic word problems (Figure 1.10a).

Another question asked students to describe the extent to which they encounter contextualised mathematical problems, similar to those used in PISA, during their mathematics lessons and assessments (see Question 3d at the end of this chapter). This type of problem requires students to apply mathematics knowledge to find a solution to a problem that arises in everyday life or

■ Figure 1.10a ■

Exposure to algebraic word problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to algebraic word problems during their mathematics lessons



Note: An example of an algebraic word problem is the following: "Ann is two years older than Betty and Betty is four years older than Sam. When Betty is 30, how old is Sam?"

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to algebraic word problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11a.

StatLink http://dx.doi.org/10.1787/888933376935

work, such as interpreting a trend in a chart. Most mathematics teachers make limited use of PISA-type mathematics problems in their lessons. Only 21% of students reported seeing this type of problem frequently at school (Figure 1.10b), and 45% reported seeing these problems only sometimes (Table 1.11b). Applied mathematics problems requiring interpretation and reasoning in a real-life context are even more rarely used in assessments.

PISA-type mathematics problems often require a skill in "mathematics modelling" – making connections between the real world and mathematics. Mathematics modelling has been discussed and recommended most intensely during the past few decades (Blum and Borromeo Ferri, 2009); however, it is rarely applied in everyday school practice, possibly because it is more difficult both for students and teachers than the replication of routine exercises. Several high-performing countries and economies are among those where students are less likely to report exposure to the kinds of contextualised mathematics problems like those included in the PISA test (Figure 1.10b). This does not mean that exposure to contextualised tasks has a negative effect on performance: rather, it is more likely that contextualisation is used to facilitate access to complex mathematics concepts of students with a weaker knowledge base (see also Table 3.8b on the relationship between exposure to contextualised tasks and performance within countries). Teaching mathematics is complex, and there are other factors that influence performance more than the amount of real-life connections students make during a task (Mosvold, 2008). Moreover, effectively applying contextualised problems in the classroom significantly depends on teachers' ability to support students' capacity to transfer what they learned in a specific context to similar problems in different contexts (see Box 1.3).

Formalised tasks that require applying procedural knowledge (such as those presented in Question 3b at the end of this chapter) are most commonly used in mathematics instruction. PISA shows that around 68% of students in OECD countries see this type of problem frequently in their mathematics lessons (Figure 1.10c), and another 25% of students are sometimes exposed to these problems (Table 1.11c). Almost 90% of students reported solving these problems as part of their assessments at least sometimes (Table 1.11c). At the system level, countries and economies whose students reported frequent exposure to procedural mathematics problems also frequently use algebraic word problems in mathematics classes (Table 1.12).

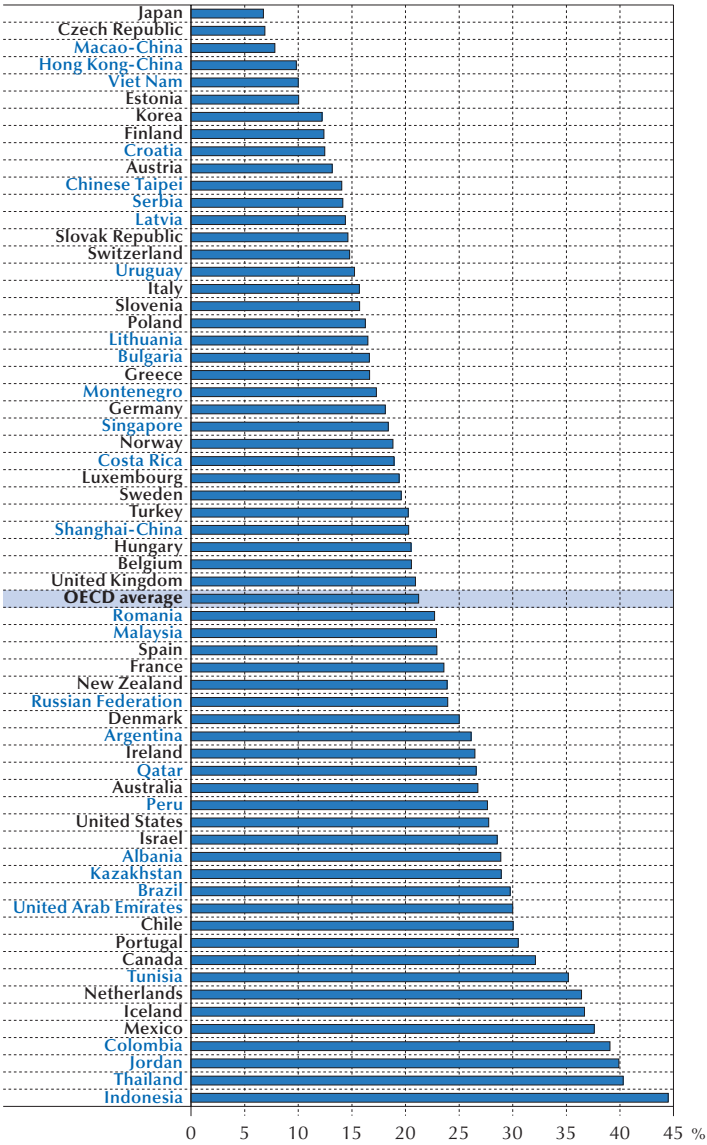
The dominance of procedural mathematics compared with modelling is problematic if students fail to establish the connection between procedures and concepts. For example, students often look at the operational side of equations arriving at the solution with no real understanding of the concept of the equation (Niss, 1987). In the long-standing debate about the relationship between procedural and conceptual knowledge, there is a prevalent view that instruction should develop conceptual knowledge before focusing on procedural knowledge (Grouws and Cebulla, 2000; NCTM, 2000, 2014). A recent analysis of the evidence further suggests that conceptual understanding and procedural fluency are equally important as interdependent strands of mathematical proficiency (Rittle-Johnson et al., 2015). Both contribute to the long-term development of problem-solving skills.

Pure mathematics problems are also examined in the PISA student questionnaire (see Question 3c at the end of this chapter). These problems require a foundation of conceptual knowledge

■ Figure 1.10b ■

Exposure to contextualised mathematics problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to contextualised mathematics problems during their mathematics lessons



Note: Contextualised mathematics problems require the application of mathematics knowledge to find an answer to a problem that arises in everyday life or work, such as interpreting a trend in a chart.

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to contextualised mathematics problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11b.

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Box 1.3. **Advantages and possible costs of contextualised mathematics**

Mathematics and science teachers at all levels are frequently encouraged to incorporate concrete, meaningful, real-world examples into their lessons when teaching new material (Rivet and Krajcik, 2008). First, concrete examples are easier to process than more abstract representations and connect the learner's existing knowledge with new, to-be-learned knowledge. For instance, a mathematics instructor teaching simple probability theory may present probabilities by rolling a six-sided die. Second, tasks embedded in real-life contexts have high motivational power; students are most easily engaged with problems that are taken from their everyday lives (Hiebert et al., 1996). Well-designed real-life tasks can also encourage the idea that mathematics is a useful discipline (Trafton et al., 2001).

Despite these advantages, research suggests that concrete examples may also come with a cost. For example, any information presented that is not essential tends to distract learners from the relevant content, leading to poorer recall for that material (the "seductive details effect"; Day et al., 2015; Harp and Mayer, 1998). Grounding mathematics using concrete contexts can thus potentially limit its applicability to similar situations in which just the surface details are changed, particularly for low-performers. In a series of experiments with undergraduate and high school students, Kaminski et al. (2008) found that learning one, two or three concrete examples resulted in little or no transfer, whereas learning one generic example resulted in significant transfer. On these grounds, the benefits of contextualised problems exceed their costs only if the tasks are very well designed (e.g. minimising unnecessary distractions) and if teachers address the transfer problem, for instance by presenting a concrete example and then a generic example for the same topic.

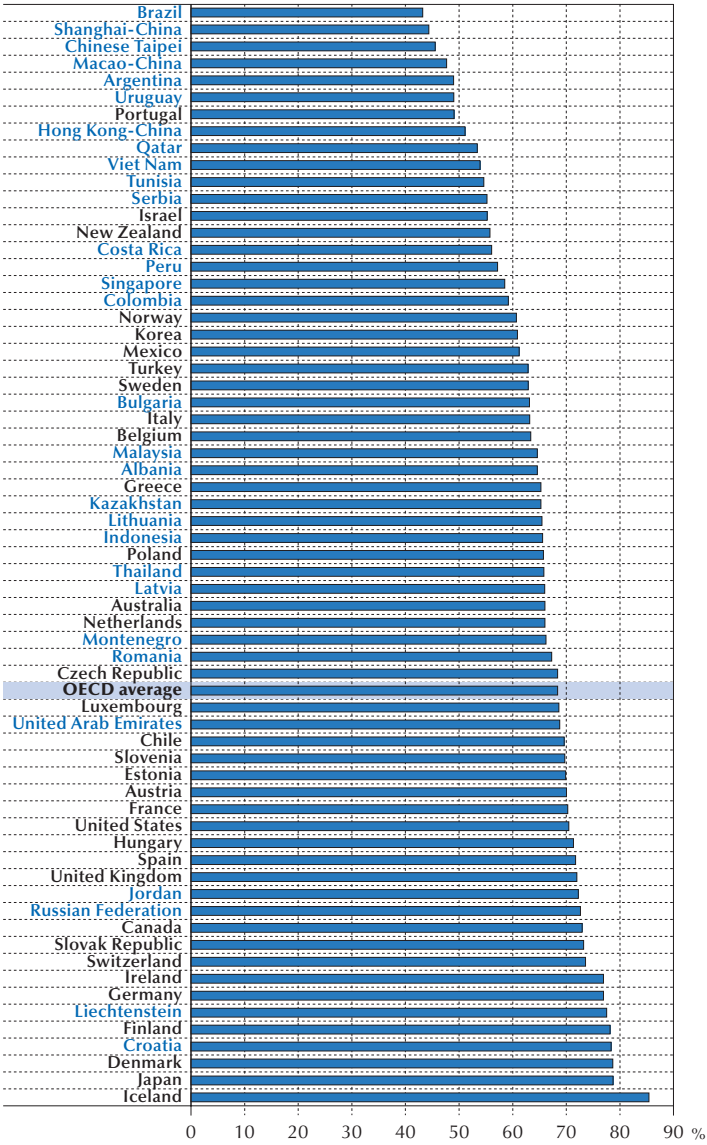
Mathematics teaching in the Netherlands has traditionally had the highest amount of connections to real-life (Hiebert et al., 2003); most textbook problems present some kind of real-life context (Mosvold, 2008). Several other countries have taken initiatives to increase the frequency and to improve the quality of real-life mathematics tasks students tackle in class. For example, the Singapore Mathematics Assessment and Pedagogy Project (SMAPP) developed a new assessment system that includes real-life mathematics, producing contextualised tasks that teachers can use in their lessons. According to the SMAPP framework, a good task should: include links to real life using relevant data; connect to the curriculum; assess multiple competencies and content knowledge; enrich student experiences; and include scaled levels of difficulty. The tasks were developed by a team of mathematicians, reviewed by teachers, and then revised after testing in real lessons. Japan recently revised its "Course of Study" and introduced mathematical activities with stronger connections with real-life problems.

and the use of procedures that are not automatised, but rather require conscious selection, reflection and sequencing of steps. Three out of four students across OECD countries see this type of problem either frequently or sometimes in their mathematics lessons (Table 1.11d), and two out of three students solve these problems at least sometimes in the tests they take at school. Students in Finland, Norway and Sweden are less exposed to this type of task than students in other countries and economies.

■ Figure 1.10c ■

Exposure to procedural mathematics tasks during mathematics lessons

Percentage of students who reported that they are frequently exposed to procedural mathematics tasks during their mathematics lessons



Note: Solving a linear equation or finding the volume of a box are examples of procedural mathematics tasks.
Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to procedural mathematics tasks during mathematics lessons.
Source: OECD, PISA 2012 Database, Table 1.11c.
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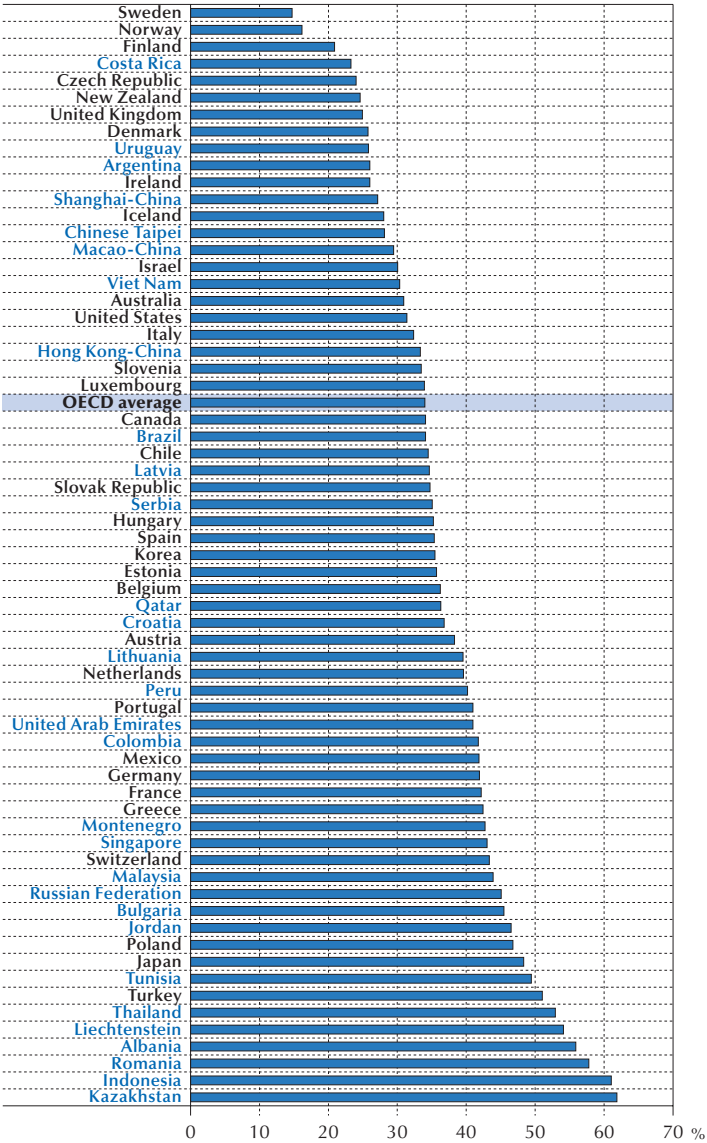
■ Figure 1.10d ■

Exposure to pure mathematics problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to pure mathematics problems during mathematics lessons



Note: Pure mathematics problems require the use of mathematics knowledge to draw conclusions, without referring to any practical application (e.g. determining the height of a pyramid using geometrical theorems).

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to pure mathematics problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11d.

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PISA provides substantial data on the international variation in the intensity, topic coverage and representation of mathematics instruction. These data show remarkable differences between education systems in the opportunity to learn mathematics. The value of these data for education policy emerges when they are used in combination with information on student performance on the PISA assessment of mathematics (Chapter 3), and when the analysis moves beyond country means to look at how opportunity to learn is distributed among students of different socio-economic status (Chapter 2). If a solid knowledge of mathematical concepts is necessary to solve non-routine mathematics problems and to apply mathematics in complex contexts outside the classroom, then socio-economic differences in access to mathematics knowledge will perpetuate differences in student performance – and in later social and economic outcomes – that are linked to socio-economic status.

QUESTIONS USED TO MEASURE OPPORTUNITY TO LEARN IN PISA 2012

The PISA 2012 student questionnaire contains six questions on opportunity to learn mathematics. Box 1.2 explains how responses were scaled and combined into several indices. These questions are shown below.

EXPOSURE TO PURE AND APPLIED MATHEMATICS

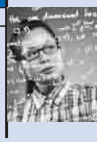
This question asks students to report on the frequency with which they have encountered specific applied and pure mathematics tasks during mathematics lessons. Students' responses to the items a) through f) in this question were scaled to produce the *index of exposure to applied mathematics* and responses to the items g) through i) were used for the *index of exposure to pure mathematics*.

Question 1

How often have you encountered the following types of mathematics tasks during your time at school?

(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
	Applied mathematics tasks				
a)	Working out from a <train timetable> how long it would take to get from one place to another.	☐1	☐2	☐3	☐4
b)	Calculating how much more expensive a computer would be after adding tax.	☐1	☐2	☐3	☐4
c)	Calculating how many square metres of tiles you need to cover a floor.	☐1	☐2	☐3	☐4
d)	Understanding scientific tables presented in an article.	☐1	☐2	☐3	☐4
e)	Finding the actual distance between two places on a map with a 1:10,000 scale.	☐1	☐2	☐3	☐4
f)	Calculating the power consumption of an electronic appliance per week.	☐1	☐2	☐3	☐4
	Pure mathematics tasks				
g)	Solving an equation like: $6x^2 + 5 = 29$	☐1	☐2	☐3	☐4
h)	Solving an equation like: $2(x + 3) = (x + 3)(x - 3)$	☐1	☐2	☐3	☐4
i)	Solving an equation like: $3x + 5 = 17$	☐1	☐2	☐3	☐4



FAMILIARITY WITH MATHEMATICS

This question evaluates students' familiarity with different mathematical concepts covered in the mathematics curriculum.

Question 2

Thinking about mathematical concepts: how familiar are you with the following terms?
(Please tick only one box in each row.)

		Never heard of it	Heard of it once or twice	Heard of it a few times	Heard of it often	Know it well, understand the concept
a)	Exponential Function	☐1	☐2	☐3	☐4	☐5
b)	Divisor	☐1	☐2	☐3	☐4	☐5
c)	Quadratic Function	☐1	☐2	☐3	☐4	☐5
d)	Linear Equation	☐1	☐2	☐3	☐4	☐5
e)	Vectors	☐1	☐2	☐3	☐4	☐5
f)	Complex Number	☐1	☐2	☐3	☐4	☐5
g)	Rational Number	☐1	☐2	☐3	☐4	☐5
h)	Radicals	☐1	☐2	☐3	☐4	☐5
i)	Polygon	☐1	☐2	☐3	☐4	☐5
j)	Congruent Figure	☐1	☐2	☐3	☐4	☐5
k)	Cosine	☐1	☐2	☐3	☐4	☐5
l)	Arithmetic Mean	☐1	☐2	☐3	☐4	☐5
m)	Probability	☐1	☐2	☐3	☐4	☐5

STUDENTS' EXPOSURE TO DIFFERENT KINDS OF MATHEMATICS PROBLEMS

The following four questions explore students' experience with different types of mathematics problems at school. They include a brief description of the type of problem and two examples of mathematics problems for each type. The students had to read each problem but did not have to solve it.

Question 3a: Algebraic word problems

The box is a series of problems. Each requires you to understand a problem written in text and perform the appropriate calculations. Usually the problem talks about practical situations, but the numbers and people and places mentioned are made up. All the information you need is given. Here are two examples:

1. <Ann> is two years older than <Betty> and <Betty> is four times as old as <Sam>. When <Betty> is 30, how old is <Sam>?

2. Mr <Smith> bought a television and a bed. The television cost <$625> but he got a 10% discount. The bed cost <$200>. He paid <$20> for delivery. How much money did Mr <Smith> spend?

We want to know about your experience with these types of word problems at school. Do not solve them!

(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3b: Procedural mathematics problems

Below are examples of another set of mathematical skills.

1) Solve 2x + 3 = 7.

2) Find the volume of a box with sides 3m, 4m and 5m.

We want to know about your experience with these types of problems at school. Do not solve them!

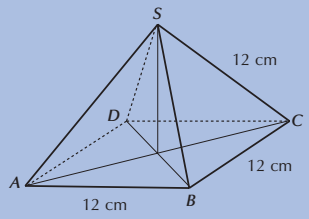
(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3c: Pure mathematics problems

In the next type of problem, you have to use mathematical knowledge and draw conclusions. There is no practical application provided. Here are two examples.

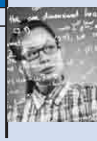
1) Here you need to use geometrical theorems:



Determine the height of the pyramid.

2) Here you have to know what a prime number is:

If n is any number: can $(n+1)^2$ be a prime number?



We want to know about your experience with these types of problems at school. Do not solve them!

(Please tick only one box in each row.)

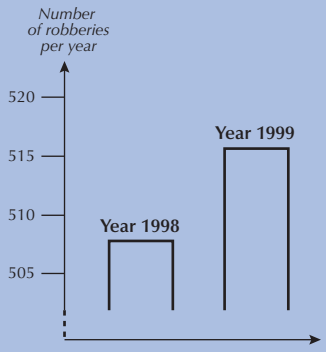
		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons?**	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3d: Contextualised mathematics problems

In this type of problem, you have to apply suitable mathematical knowledge to find a useful answer to a problem that arises in everyday life or work. The data and information are about real situations. Here are two examples.

Example 1

A TV reporter says "This graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Do you consider the reporter's statement to be a reasonable interpretation of the graph?

Give an explanation to support your answer.

Example 2

For years the relationship between a person's recommended maximum heart rate and the person's age was described by the following formula:

Recommended maximum heart rate = 220 – age

Recent research showed that this formula should be modified slightly. The new formula is as follows:

Recommended maximum heart rate = 208 – (0.7 × age)

From which age onwards does the recommended maximum heart rate increase as a result of the introduction of the new formula? Show your work.

We want to know about your experience with these types of problems at school. Do not solve them!

(Please check only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Note

1. The OECD Survey of Adult Skills defines numeracy as the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life (OECD, 2012).

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2

Variations in Students' Exposure to and Familiarity with Mathematics

Students' exposure to mathematics varies within countries even more than between countries. This chapter first explores how access to mathematics content varies by socio-economic status and other student characteristics, such as gender, immigrant background and attendance at pre-primary school. It then analyses the extent to which school- and system-level factors – including student sorting and teaching resources and practices – can produce segregation in opportunities to learn mathematics based on students' socio-economic status.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

Lack of access to mathematics content at school can leave young people socially and economically disadvantaged for life. Who gets to learn mathematics, and the nature of the mathematics that is learned, have an impact on education systems, social cohesion and productivity. Education systems that fail to provide the same opportunities to all students can end up reinforcing, rather than beginning to dismantle, social inequalities. When education is no longer a pathway to individual fulfilment and social mobility, talent is wasted and feelings of injustice grow. Failing disadvantaged youth at school can also pave the way for a wide range of social problems later on, including poverty, poor health and crime (Schoenfeld, 2002; OECD, 2012).

This chapter focuses on how opportunities to learn mathematics vary within countries. Across all education systems, socio-economically disadvantaged young people have less access to mathematics content. The results confirm the findings of extensive research on the strong links between socio-economic advantage, mastery of mathematics and perseverance in secondary school mathematics (Crosnoe and Schneider, 2010).

Students of higher socio-economic status have an advantage from the beginning: they tend to have access to high-quality learning opportunities, both formal and informal. At an early age, they often attend better resourced and better organised elementary schools, and they have access to after-school programmes that enrich their learning experience as they approach high school (Downey, von Hippel and Broh, 2004; Entwisle, Alexander and Olson, 2005; Lareau, 2011; NICHD Early Child Care Network, 2005). The parents of these students tend to have greater experience navigating through the education system, which makes it more likely that their children will pursue higher education and succeed in the labour market (Morgan, 2005).

What the data tell us

- Across OECD countries, around 9% of the variation in familiarity with mathematics within countries is explained by students' socio-economic status and by the concentration of socio-economic advantage in certain schools. Socio-economic differences among students and schools account for less than 1% of the variation in Estonia and Malaysia and for more than 20% of the variation in Hungary and Liechtenstein.
- Around 70% of the students who have at least one tertiary-educated parent reported that they know well or have often heard of the concept of linear equation; only 52% of students whose parents have only primary education as their highest level of attainment so reported.
- Around 54% of the variation among OECD countries in the impact of students' and schools' socio-economic status on students' familiarity with mathematics is explained by system-level differences in the age at which students are tracked into vocational or general/academic programmes or schools.
- Ability grouping is more prevalent in disadvantaged schools than in advantaged schools, on average across OECD countries.
- On average across OECD countries, teachers' use of cognitive-activation strategies is associated with greater familiarity with mathematics among students in socio-economically advantaged schools; but this is not the case in disadvantaged schools.

PISA data show large variations across countries in the association between socio-economic status (Box 2.1) and students' familiarity with mathematics, suggesting that the organisation of education systems can either mitigate or reinforce inequalities in access to knowledge.[1] The mechanisms in place for selecting students in schools according to their perceived ability and preparation seem to play a significant role in reducing opportunities to learn mathematics among disadvantaged students.

What these results mean for policy

- As exposure to, and familiarity with, mathematics are strongly correlated with students' socio-economic status, education systems and policies should be designed with the aim of giving all students equal opportunity to learn mathematics concepts and to practice challenging mathematics tasks.
- In order to give all students equal opportunity to learn mathematics, tracking should be delayed and/or struggling students should be offered individualised instruction tailored to their needs.
- More professional training in teaching in multicultural settings should be provided to teachers, particularly to those who work in disadvantaged schools.
- All students would benefit from teaching practices that emphasise mathematics reasoning and problem solving; but policy makers, school authorities and teachers should ensure that such practices do not take time away from covering key mathematics concepts, especially for socio-economically disadvantaged students.

VARIATIONS IN ACCESS TO MATHEMATICS CONTENT WITHIN COUNTRIES

Catering to the needs of a diverse student body and ensuring consistently high standards across schools represent formidable challenges for any school system (OECD, 2004). Variations in opportunity to learn mathematics within countries can be related to a variety of factors, including regional differences in the socio-economic and cultural characteristics of the communities that are served by schools; the quality of the school staff or the education policies implemented in some schools and not in others; the distribution of human and financial resources available to schools; and system-level factors, such as the way students are grouped, according to their academic potential and interests, in specific programmes (OECD, 2013a).

Figure 2.1 shows large international differences in the extent to which students' self-reported knowledge of mathematics varies within each country. The total length of the bars indicates the observed variation in the *index of familiarity with mathematics*. The variation in familiarity with mathematics is more than four times greater in Liechtenstein and Spain than in Indonesia. Across OECD countries, around 86% of the country-level variation in familiarity with mathematics can be traced to differences across students who attend the same school, while around 14% can be ascribed to differences across students who attend different schools. In Austria, Germany, Hungary, Liechtenstein and Qatar, at or over a quarter of the variation is due to differences across schools. Denmark, Finland, Malaysia, Sweden and Tunisia are the most comprehensive

systems, where there is less than 5% variation in students' familiarity with mathematics observed among schools.[2]

Box 2.1. **What is socio-economic status and how it is measured in PISA?**

Socio-economic status in PISA is a broad concept that summarises many different aspects of a student, school or system. A student's socio-economic status is estimated by an index, the *PISA index of economic, social and cultural status* (ESCS), which is based on such indicators as parents' education and occupation, the number and type of home possessions that are considered proxies for wealth, and the educational resources available at home. The index is built to be internationally comparable (see the *PISA 2012 Technical Report*, OECD, 2014a). Students are considered socio-economically advantaged if they are among the 25% of students with the highest ESCS in their country or economy; socio-economically disadvantaged students are those among the 25% of students with the lowest ESCS. Schools are defined as socio-economically advantaged (disadvantaged) if the average ESCS of students in the school is statistically significantly above (below) that of the average school.

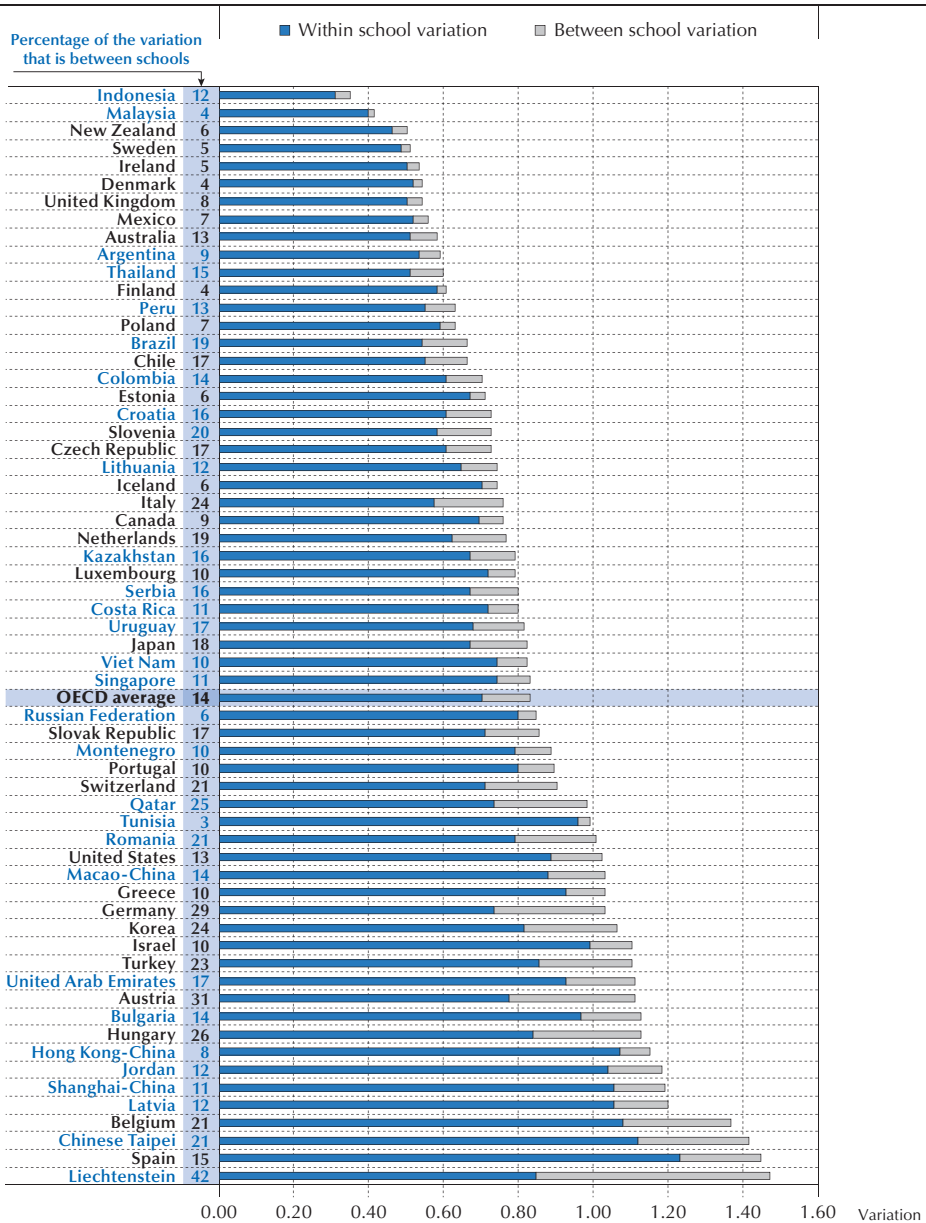
PISA consistently finds that socio-economic status is associated with performance at system, school and student levels. These patterns reflect, in part, the inherent advantages in resources that relatively high socio-economic status provides. However, they also reflect other characteristics that are associated with socio-economic status but that are not measured by the PISA index. For example, at the system level, high socio-economic status is related to greater wealth and higher spending on education. At the school level, higher socio-economic status is associated with a range of characteristics of a community that might be related to student performance, such as a safe environment and the availability of quality educational resources, such as public libraries or museums. At the individual level, socio-economic status may be related to parents' attitudes towards education, in general, and to their involvement in their child's education, in particular.

Source:
OECD (2013a).

Across OECD countries, around 4% of the variation in students' familiarity with mathematics within countries is explained by the socio-economic status of students; this percentage more than doubles when also taking into account the socio-economic composition of the schools – that is, the concentration of students with a similar socio-economic status who attend the same schools (Figure 2.2).[3] The cumulative effect of a student's socio-economic status and the concentration of socio-economic advantage in a school is particularly large in Austria, Hungary and Liechtenstein. In Portugal and Spain, the socio-economic profile of a school adds little to the effect related to an individual student's socio-economic status, suggesting that disadvantaged students lag behind other students in access to mathematics no matter which school they attend. The opposite pattern is observed in the Czech Republic, the Netherlands and Japan, where segregation by socio-economic status occurs mostly between schools.

■ Figure 2.1 ■

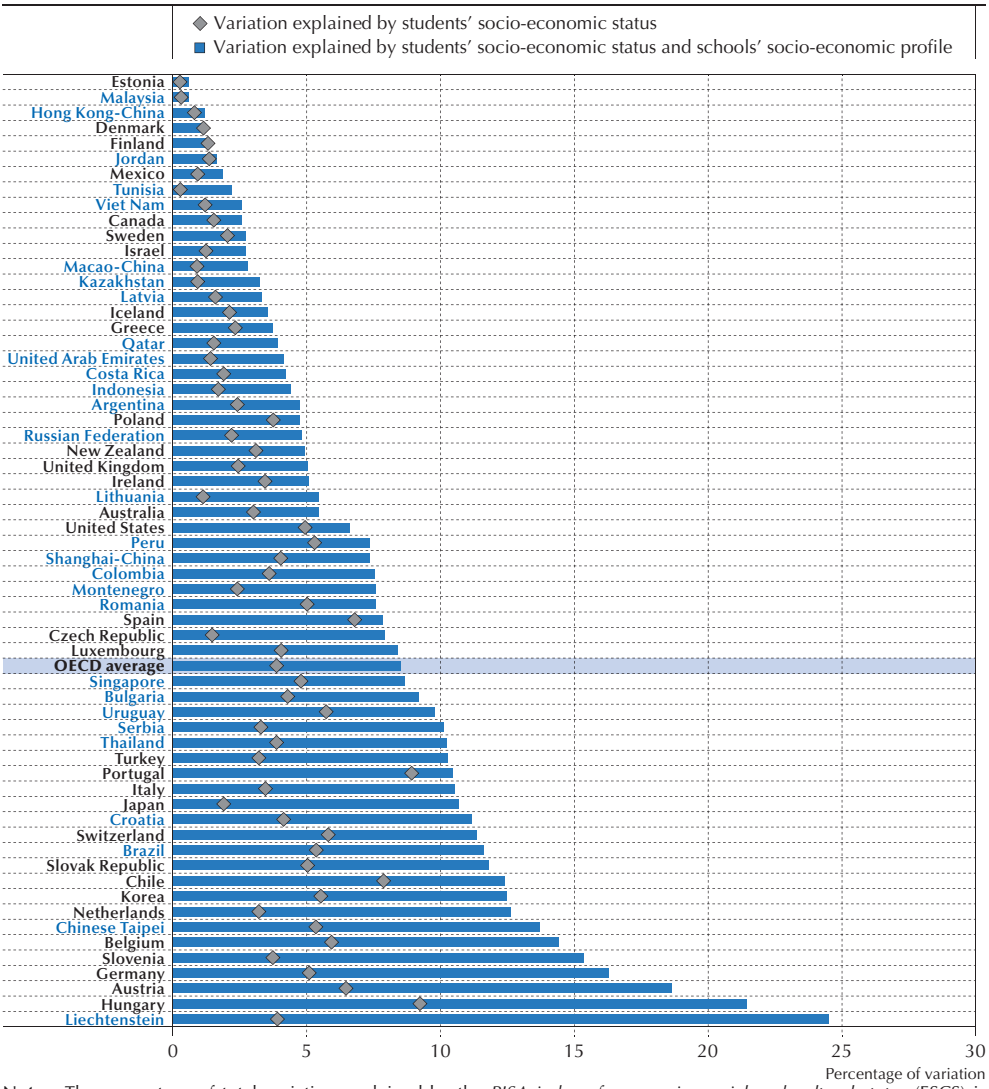
Variation in familiarity with mathematics, within and between schools



Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Countries and economies are ranked in ascending order of the total variation (within and between) in the index of familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.1.
StatLink http://dx.doi.org/10.1787/888933376973

■ Figure 2.2 ■

Variation in familiarity with mathematics explained by students' and schools' socio-economic profile



Notes: The percentage of total variation explained by the *PISA index of economic, social and cultural status* (ESCS) is estimated through a linear model. The relationship between familiarity with mathematics and ESCS is statistically significant in all countries and economies.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Estimates for France based on the schools' socio-economic profile were deleted at the request of the country. Students' socio-economic status explains 6.7% of the variation in familiarity with mathematics within the country.

Countries and economies are ranked in ascending order of the percentage of variation in familiarity with mathematics explained by students' socio-economic status and by schools' socio-economic profile.

Source: OECD, PISA 2012 Database, Table 2.2.

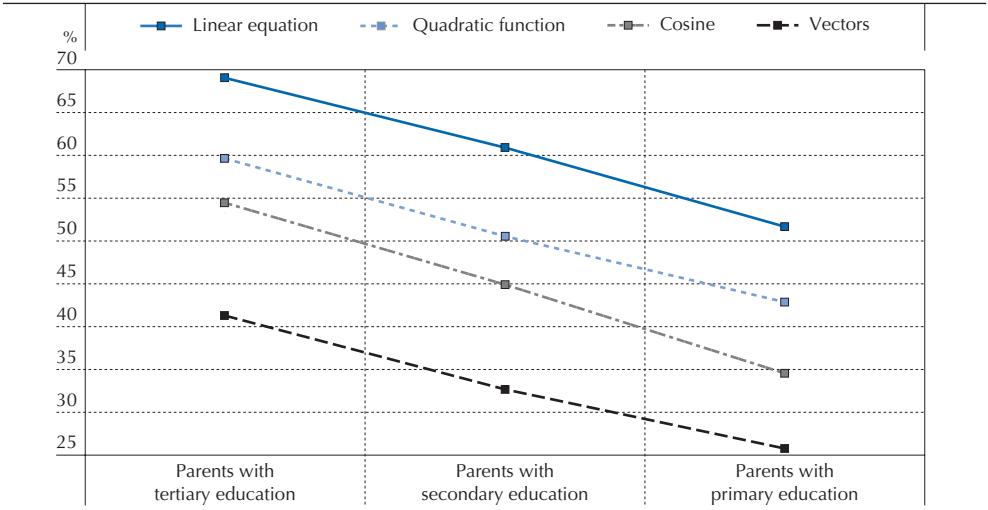
StatLink http://dx.doi.org/10.1787/888933376988

Figure 2.3 shows the magnitude of the differences in familiarity with mathematics associated with students' socio-economic status. Around 70% of the students who have at least one tertiary educated parent reported that they know well or have often heard of the concept of linear equation, on average across OECD countries; only 52% of students whose parents have only primary education as their highest level of attainment so reported. Similarly, around 55% of students with highly educated parents and only 35% of students with low-educated parents reported that they know well or have frequently been exposed to the geometric concept of cosine.

■ Figure 2.3 ■

Familiarity with mathematics concepts, by parents' highest level of education

Percentage of students who reported that they know well or have often heard the concept, OECD average



Source: OECD, PISA 2012 Database, Table 2.5a.
StatLink *http://dx.doi.org/10.1787/888933376994*

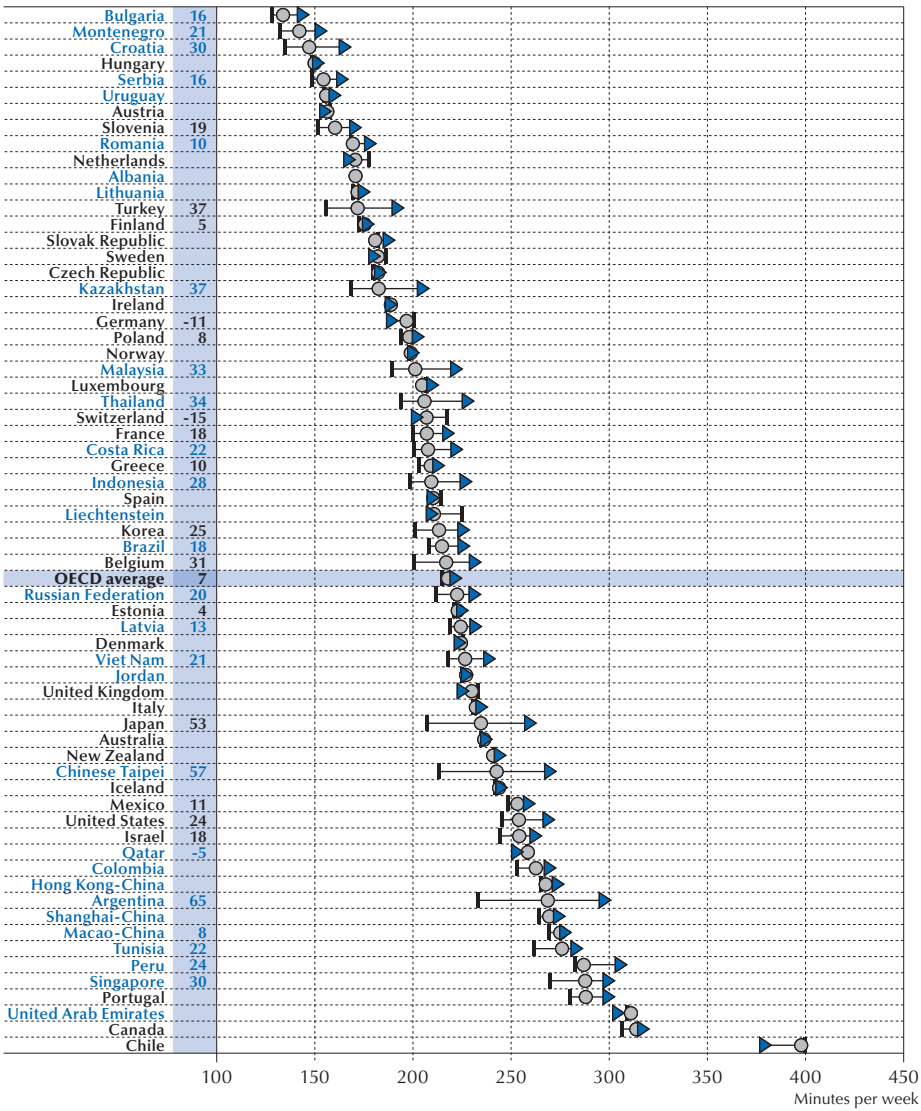
This gap in opportunities to learn mathematics is not strongly related to the time students spend in mathematics classes (Figure 2.4). In fact, in 2012, disadvantaged students spent only seven minutes less per week in mathematics courses at school than advantaged students did (equivalent to one-tenth of a standard deviation), on average across OECD countries. There were only a few exceptions: in Argentina, Japan and Chinese Taipei, disadvantaged students spent around one hour less in mathematics classes per week than advantaged students did.

Rather than the amount of time spent on mathematics, it is how that time is used that influences the difference in familiarity with mathematics between advantaged and disadvantaged students. Figures 2.5a and 2.5b show clearly that disadvantaged students have less exposure to both the applied and the pure mathematics tasks included in PISA (see Chapter 1 for the definition of these indices). In Iceland, Jordan, Korea, New Zealand and Chinese Taipei, the difference in exposure to applied mathematics tasks between advantaged and disadvantaged students is more than 0.5 units (equivalent to half of a standard deviation of the OECD average) (Figure 2.5a).

■ Figure 2.4 ■

Mathematics learning time at school, by students' socio-economic status

Average minutes per week of mathematics instruction in class



Notes: Disadvantaged students are defined as those students in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS). Advantaged students are students in the top quarter of ESCS.

Only statistically significant differences in mathematics learning time between advantaged and disadvantaged students are shown next to the country/economy name.

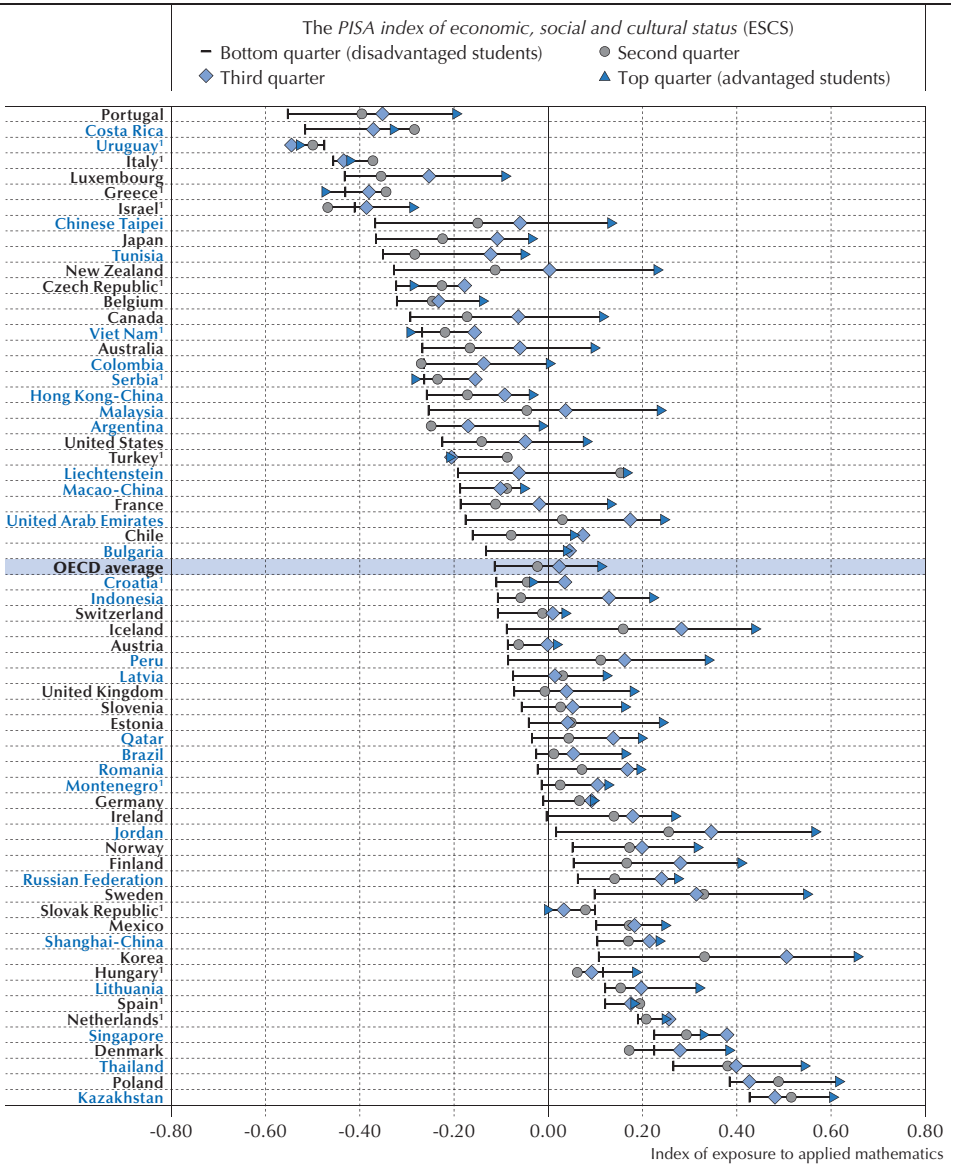
Countries and economies are ranked in ascending order of the minutes per week spent learning mathematics at school for all students.

Source: OECD, PISA 2012 Database, Table 2.3.

StatLink http://dx.doi.org/10.1787/888933377006

■ Figure 2.5a ■

Exposure to applied mathematics, by students' socio-economic status



1. The difference between the top and the bottom quarters of the *PISA index of economic, social and cultural status* (ESCS) is not statistically significant.

Note: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

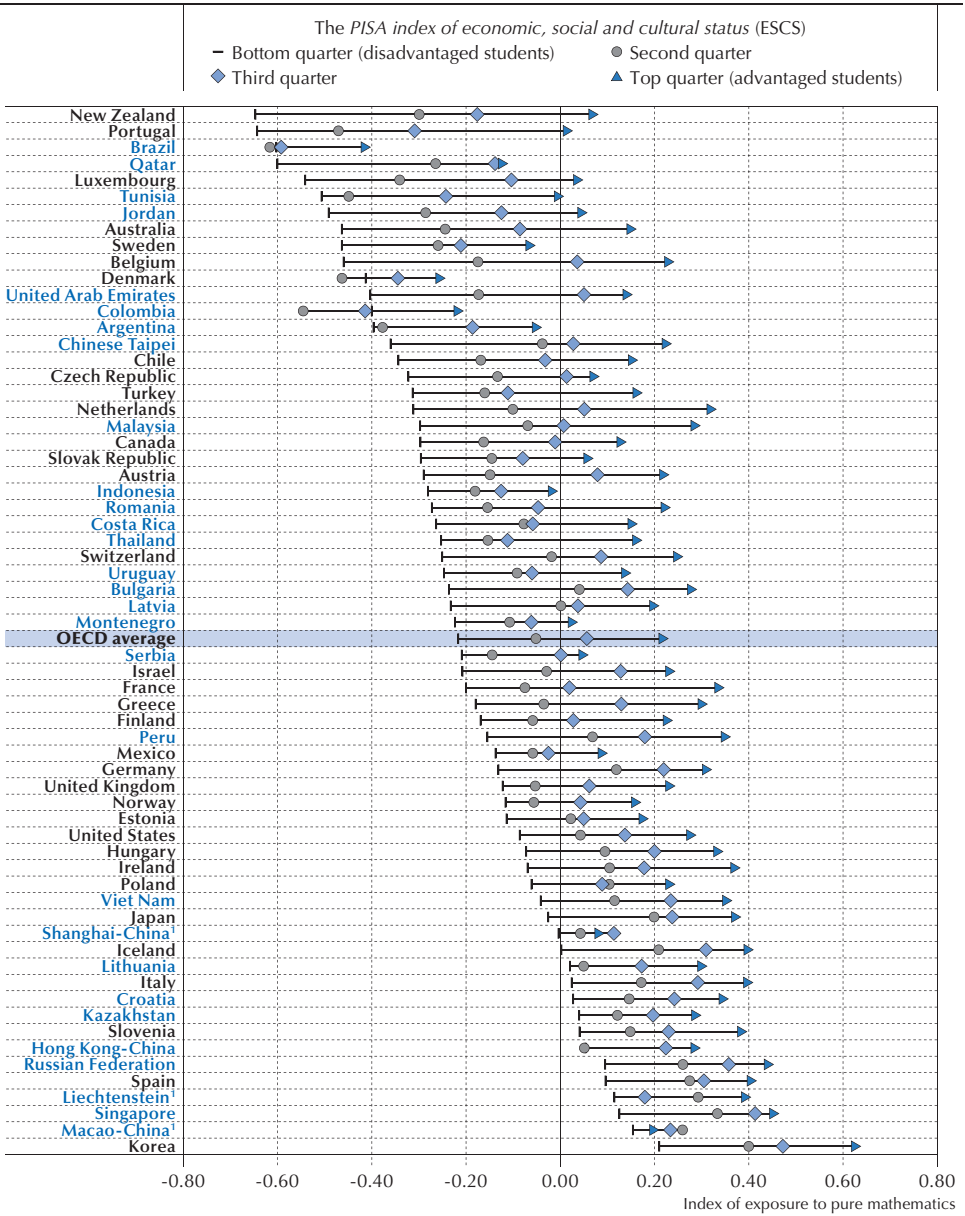
Countries and economies are ranked in ascending order of the average index of exposure to applied mathematics *of students in the bottom quarter of ESCS.*

Source: OECD, PISA 2012 Database, Table 2.4a.

StatLink http://dx.doi.org/10.1787/888933377010

■ Figure 2.5b ■

Exposure to pure mathematics, by students' socio-economic status



1. The difference between the top and the bottom quarters of ESCS is not statistically significant.

Note: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

Countries and economies are ranked in ascending order of the average index of exposure to pure mathematics *of students in the bottom quarter of ESCS.*

Source: OECD, PISA 2012 Database, Table 2.4a.

StatLink http://dx.doi.org/10.1787/888933377022

The difference in exposure to pure mathematics tasks (functions and equations) is even larger and is statistically significant in all countries and economies except Macao-China, Liechtenstein and Shanghai-China (Figure 2.5b). On average across OECD countries, there is a difference of 0.44 of a standard deviation in the *index of exposure to pure mathematics* between advantaged and disadvantaged students. To put this magnitude in perspective, the average difference between students in the modal grade and students one grade below is 0.29 of a standard deviation (Chapter 1, Table 1.10). In Belgium and New Zealand, the difference in the *index of exposure to pure mathematics* between advantaged and disadvantaged students is over two-thirds of a standard deviation. In 49 of 63 countries and economies with available data, socio-economically disadvantaged students had less than the OECD average exposure to pure mathematics (Figure 2.5b).

These data raise questions about the effectiveness of the time disadvantaged students spend studying mathematics at school. Given a similar investment of time, disadvantaged students still reported having less knowledge about key mathematics concepts, spending less time solving equations and engaging less in relatively simple applied mathematics tasks. What do these students do and learn during the many hours they spend in mathematics classes? Can the knowledge gap be traced to other student characteristics, or is it more strongly linked to how schools and school systems are organised and how they teach mathematics?

INDIVIDUAL STUDENT CHARACTERISTICS AND ACCESS TO MATHEMATICS CONTENT

Gender differences in opportunity to learn mathematics

In most countries, mathematics and mathematics-related fields are indisputably male-dominated. There is no innate reason why girls should not be able to do as well as boys in mathematics; most empirical studies find no gender difference in standardised mathematics scores upon entry to school (Fryer and Levitt, 2010). However, in most of the countries and economies that participate in PISA, girls do worse than boys in mathematics, on average, particularly among high-performing students (OECD, 2014b). Differences in perceived ability in mathematics and mathematics anxiety are major factors behind the gender gap in mathematics performance, and have been shown to predict later achievement and occupational choices (Chapter 4, [Bandura et al., 2001; Dweck, 2007; Eccles, 2007; Häussler and Hoffmann, 2002]).

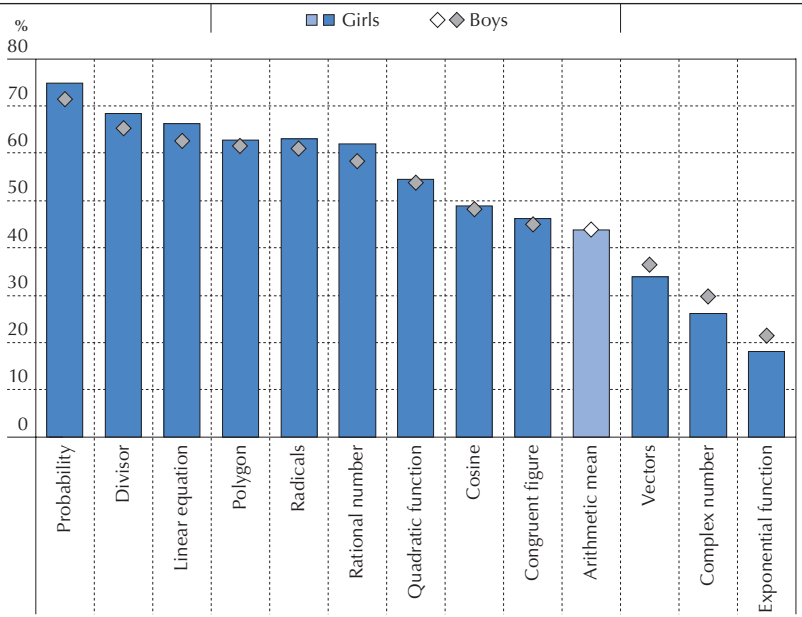
Gender disparities in mathematics achievement might also result from differences in the opportunities boys and girls have to practice their mathematics skills. PISA data show that boys and girls have different opportunities to develop mathematics skills outside of school. For example, girls are less likely than boys to play chess, program computers, take part in mathematics competitions, or do mathematics as an extracurricular activity (OECD, 2015a).

What about opportunities to learn mathematics during school time? Figure 2.6 shows that girls were more likely than boys to report that they often heard of and/or know well most mathematics concepts, even though gender differences are not large in most countries. Girls are more likely than boys to report being familiar with mathematics, particularly with concepts to which most 15-year-olds have been exposed. For example, on average across OECD

■ Figure 2.6 ■

Familiarity with mathematics concepts, by gender

Percentage of students who reported that they know well or have often heard the concept, OECD average



Note: Statistically significant gender differences are marked in a darker tone.
Countries and economies are ranked in descending order of boys' familiarity with the concept.
Source: OECD, PISA 2012 Database, Table 2.5b.
StatLink http://dx.doi.org/10.1787/888933377036

countries, 75% of girls and 71% of boys reported a high level of familiarity with the concept of probability. This difference in favour of girls is over 15 percentage points in Jordan, Thailand and the United Arab Emirates (Table 2.5b). By contrast, 30% of boys and 26% of girls are familiar with the more advanced concept of complex numbers. Boys' advantage in familiarity with this concept is over 10 percentage points in Germany, Liechtenstein and Luxembourg. These patterns of exposure reflect the broader picture of gender differences in mathematics, with boys excelling at the top and struggling at the bottom of the performance distribution.

While girls appear to be more likely than boys to have encountered pure mathematics tasks, such as solving quadratic and linear equations, gender differences in self-reported experience with applied mathematics tasks are generally small; in fact, in the large majority of countries and economies there is no difference in boys' and girls' exposure to such tasks (OECD, 2015a). Differences in girls' and boys' likelihood to repeat grade or be enrolled in a vocational rather than an academic programme explain only a small part of the gender differences in students' familiarity with mathematics concepts and with pure mathematics tasks (OECD, 2015a).

Immigrant students' familiarity with mathematics concepts

In most PISA-participating countries and economies, foreign-born students score lower in mathematics than students without an immigrant background, and students who were born in the country in which they sat the PISA test, but whose parents were born outside the country, perform somewhere between the two (OECD, 2015b). On average across OECD countries, immigrant students are 1.7 times more likely than students without an immigrant background to perform in the bottom quarter of the mathematics performance distribution (OECD, 2013a). The performance gap between the two groups of students tends to be smaller in mathematics than in reading, suggesting that language comprehension is one of the most serious hurdles for immigrant students.

Immigrant students are much less familiar with the mathematics concepts that they are expected to learn in secondary school (linear equations, exponential functions, divisors and quadratic functions) than students without an immigrant background. The gap in the self-reported familiarity with mathematics concepts between foreign-born students and students without an immigrant background is particularly large (more than half a standard deviation) in Italy and Spain, two of the OECD countries that saw the largest increase in immigration over the past decade (Table 2.7). In most countries, students who were born in the country in which they sat the PISA test, but whose parents weren't, reported greater familiarity with mathematics than students who were not born in the country. This suggests that late arrival might reduce the opportunities to learn mathematics content, or increase the mismatch between what was learned in the country of origin and what is learned in the destination country.

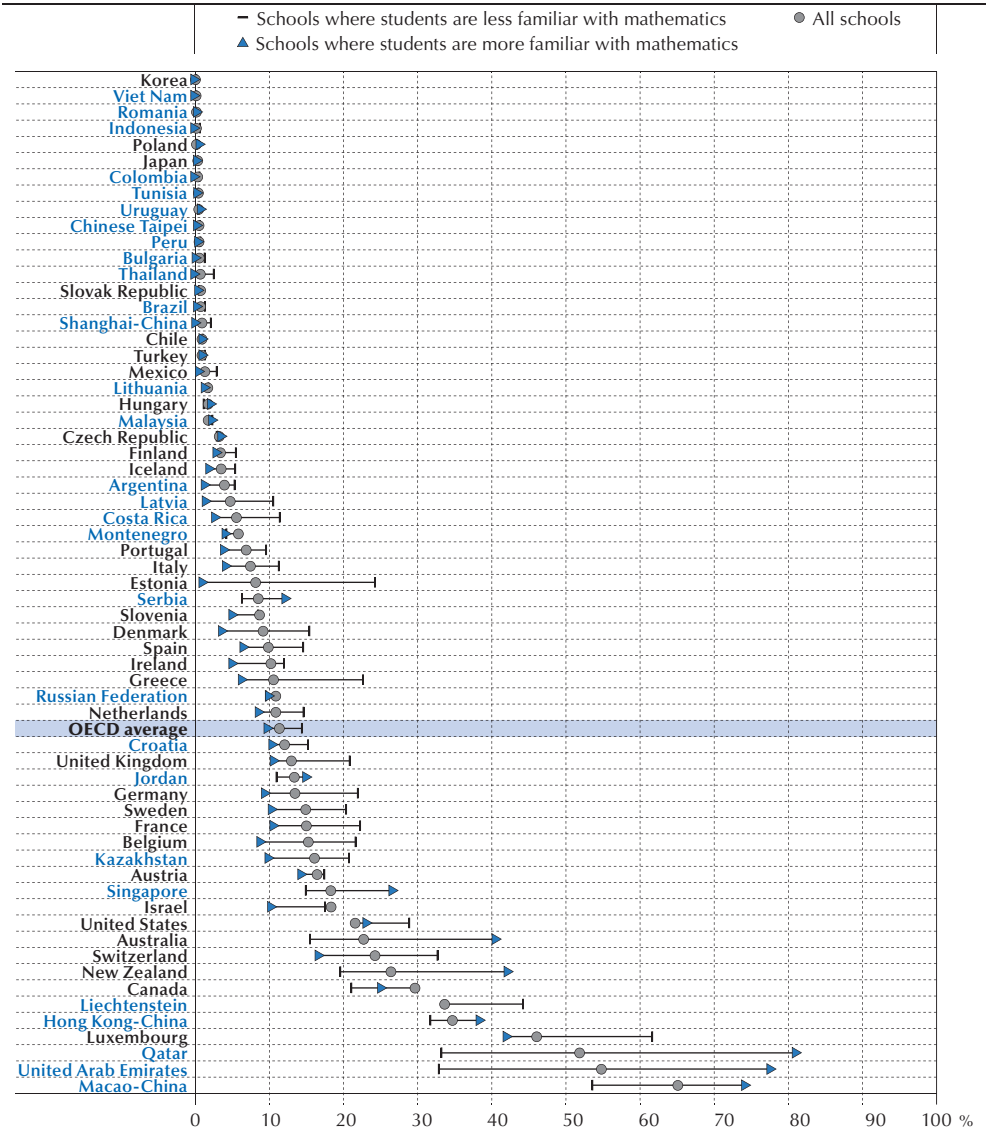
Differences in the quality of instruction and in the depth and coverage of curricula across countries of origin and destination can lead to gaps in students' readiness to learn advanced mathematics material. Immigrant students, and particularly refugees, are also likely to have spent considerable time out of school as they were making their way from their country of origin to their host country. At least one in six immigrant students who attend school in an OECD country lost more than two months of school at least once in his or her life (OECD, 2015b). But apart from these differences, the high concentration of immigrant students in disadvantaged schools in host countries might explain why these students are not familiar with certain mathematics concepts. Immigrant students are often concentrated in schools that suffer from high turnover rates among teachers, less effective learning time, and low-quality educational resources (OECD, 2013a). In these contexts, immigrant students are less likely to overcome their initial learning disadvantages.

Figure 2.7 shows that immigrant students tend to be concentrated in schools where students reported less exposure to mathematics concepts. On average across OECD countries, 14% of students in schools whose students reported relatively less familiarity with mathematics are immigrant students, as are 10% of the students in schools with greater average familiarity with mathematics. In Greece, almost 1 in 4 students in schools where the reported familiarity with mathematics concepts is low is an immigrant student, as is only 1 in 16 students in schools with greater average familiarity with mathematics.

A strong relationship between the concentration of immigrant students in a school and the school's average familiarity with mathematics concepts is also observed in Estonia, Liechtenstein,

■ Figure 2.7 ■

Percentage of immigrants in schools, by school-level familiarity with mathematics



Notes: Schools where students are less (more) familiar with mathematics are defined as those where the students' average level on the *index of familiarity with mathematics* is statistically significantly below (above) the average across all schools in the country/economy.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).

Countries and economies are ranked in ascending order of the percentage of students with an immigrant background in all schools.

Source: OECD, PISA 2012 Database, Table 2.6.

StatLink http://dx.doi.org/10.1787/888933377049

Luxembourg and Switzerland. In these countries, the difference in the percentage of immigrant students between schools with less and more familiarity with mathematics is larger than 15 percentage points. These differences reflect both the level of skills among immigrant students in their respective host countries and the concentration of students with knowledge deficits who attend the same school (Figure 2.7).

Teachers and school administrators face the challenge of teaching increasingly multiethnic and multilingual classes. Many of them recognise that handling cultural diversity in class is difficult and requires preparation. On average across the 21 OECD countries that participated in the OECD Teaching and Learning International Survey (TALIS) in 2013,[4] 12% of teachers reported they needed professional development for teaching in multicultural settings (OECD, 2015b). This feeling of unpreparedness was shared by 27% of teachers in Italy and 33% of teachers in Mexico.

Many believe that mathematics is a subject free of the influence of culture, beliefs and values, and it can be taught even in the absence of a common language, because it is, in itself, a universal language. In reality, cultural beliefs about mathematics affect teaching practices and influence immigrant students' participation in the classroom and learning (Gorgorió and Planas, 2005). Immigrant students might differ not only in their background knowledge, but also in the strategies they use to solve problems. For example, mathematics teachers can choose among many different representations of the algorithm of division, and this choice is often culture-specific. Teachers who are not fully aware of these differences in approaches to mathematics problems or who "play down" cultural differences, arguing for general notions of ability and equity (Abreu, 2005), are ill-equipped to build on their students' knowledge and experience with mathematics.

On average across OECD countries, only 4% of students are in schools whose principal reported that ethnic heterogeneity is a serious obstacle to learning (Table 2.8). Not surprisingly, principals of socio-economically disadvantaged schools (that is, schools where the average socio-economic status of students is statistically significantly below that of the average school in the country/economy) are much more likely than principals of advantaged schools to report that ethnic diversity hinders learning very much. This is particularly the case in Belgium, where ethnic heterogeneity is perceived as a serious obstacle to learning by principals in 20% of disadvantaged schools. These perceptions reflect the fact that immigrant students, who have, arguably, the largest learning and linguistic deficits, tend to be concentrated in disadvantaged schools. They also make it clear that disadvantaged schools need more support so they can start regarding ethnic differences as a resource for learning, rather than an obstacle to learning (OECD, 2015b).

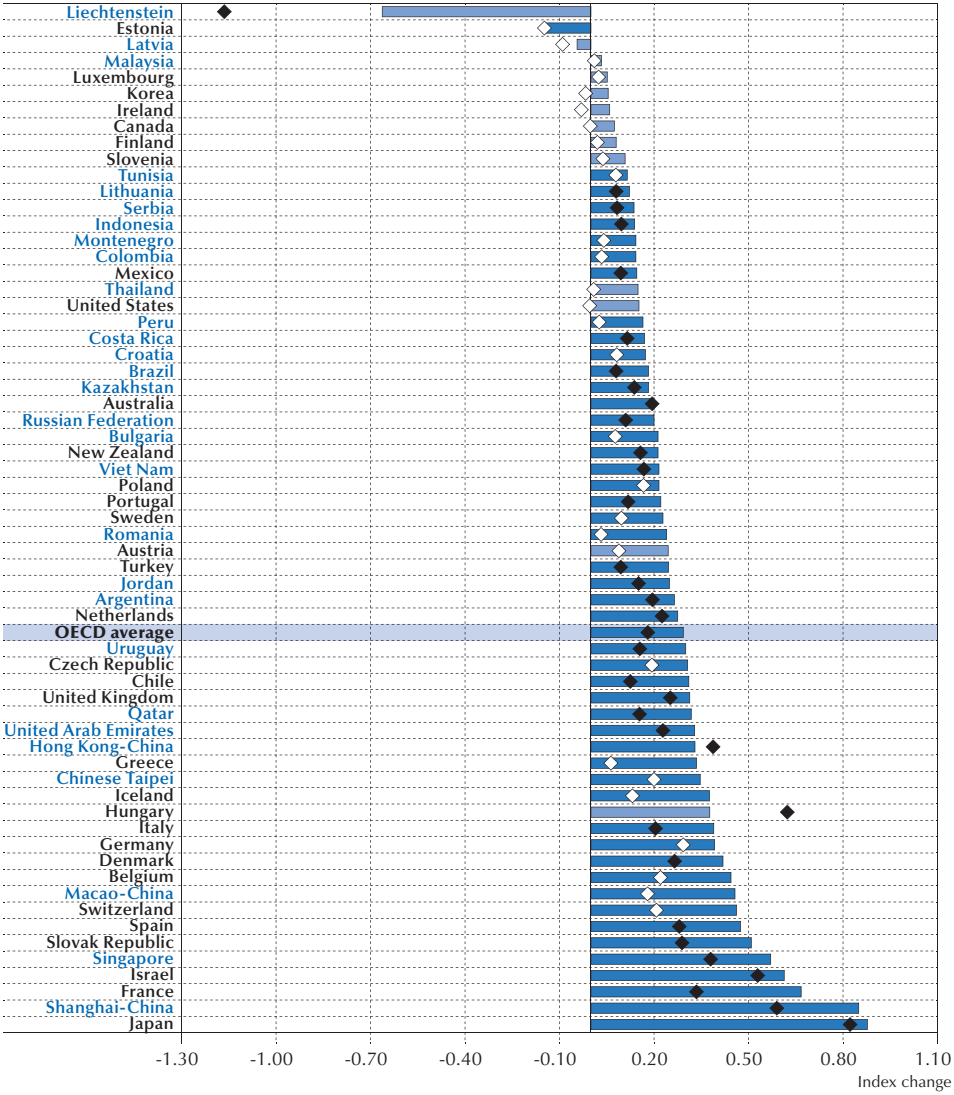
Attendance at pre-primary education and familiarity with mathematics

Very young children have the potential to learn mathematics that is complex and sophisticated (Sarama and Clements, 2010), and pre-primary education can help children with their first steps towards mathematical literacy. Identifying the relationship between pre-primary education and later performance in school is challenging, because attendance at pre-school is often correlated with socio-economic advantage. When disadvantaged children enter pre-school, they already lag behind advantaged children because they are likely to have had fewer play opportunities at home to explore patterns, shapes and spatial relations; compare magnitudes; and count objects.

■ Figure 2.8 ■

Familiarity with mathematics and attendance at pre-primary education

Change in the index of familiarity with mathematics associated with attendance at pre-primary education



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).
"Students' characteristics" include students' gender, socio-economic status, immigrant background and language spoken at home.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the change in the index of familiarity with mathematics *associated with attendance at pre-primary education before accounting for students' characteristics.*

Source: OECD, PISA 2012 Database, Table 2.9b.

StatLink http://dx.doi.org/10.1787/888933377056

A growing body of experimental studies have overcome this analytical challenge and shown that policy interventions in early childhood education can yield large returns in general, and be particularly effective for children in low-income families (Heckman and Carneiro, 2003; Blau and Currie, 2006; Cunha et al., 2006).

At age 15, students who had attended pre-primary education report greater familiarity with mathematics than those who had not attended (Figure 2.8). The knowledge advantage of students who had attended pre-primary education remains substantial (around one-fifth of a standard deviation, on average across OECD countries) even after accounting for other student characteristics, such as their gender, socio-economic status, and immigrant background. Pre-primary education makes a large difference in those countries, like Hungary, where attendance at these programmes is almost universal. On average across OECD countries in 2012, 89% of disadvantaged students and 96% of advantaged students had attended at least one year of pre-primary education (Table 2.9a).

Unfortunately, many disadvantaged children attend pre-schools that are not of high quality. For example, evidence from the United States shows that children from poor neighbourhoods are more likely than children from wealthier communities to be taught by teachers with fewer qualifications (Clifford et al., 2005; Sarama and Clements, 2010). The approaches used for teaching mathematics in pre-primary schools might make a difference for building a sturdy base for further learning. In pre-primary mathematics activities, the content is usually not the main focus, but is embedded in a fine-motor or reading activity. Experimental evidence suggests that a lack of explicit attention to mathematics concepts and procedures, along with a lack of intention to engage in mathematics practice, results in insufficient opportunities to build strong cognitive foundations (Clements and Sarama, 2011). The same evidence indicates that interventions that provide early experience with numbers, space, geometry, measurement, and the processes of mathematical thinking can be particularly effective for children from poorer communities.

PARENTS' PREFERENCES, SCHOOL SELECTIVITY AND OPPORTUNITY TO LEARN MATHEMATICS

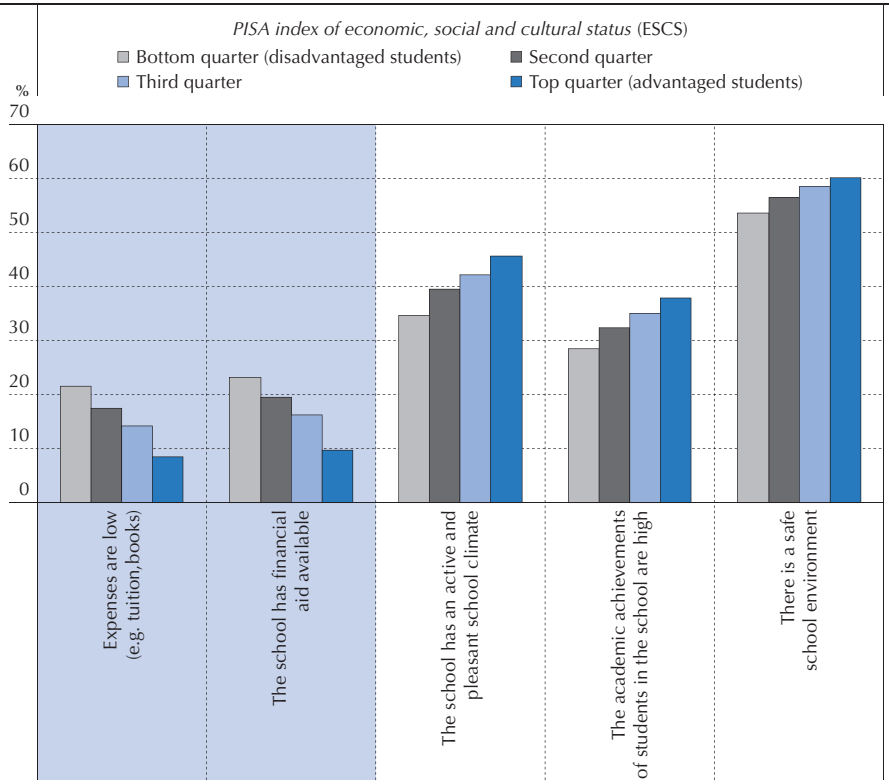
In recent decades, reforms in many countries have tended to give a greater say in school choice to parents and students (Heyneman, 2009). Parents' background and preferences are important for school choice, especially in school systems with early tracking and where school selection is not based on achievement. Parents' criteria for choosing schools are a component of the effect of socio-economic status on opportunity to learn, as wealthier parents tend to have access to the information needed to select the best schools for their children.

Even if all parents would like their child to attend the best schools, not everyone can afford to care only about school quality when choosing a school. Figure 2.9 shows how, across the 11 countries and economies where the parent questionnaire was distributed, disadvantaged parents tended to assign relatively greater importance to financial considerations when choosing a school for their child.

■ Figure 2.9 ■

Parents' preferences for schools, by socio-economic status

Percentage of students whose parents reported that the following criteria are "very important" in choosing a school for their child; average across 11 countries/economies



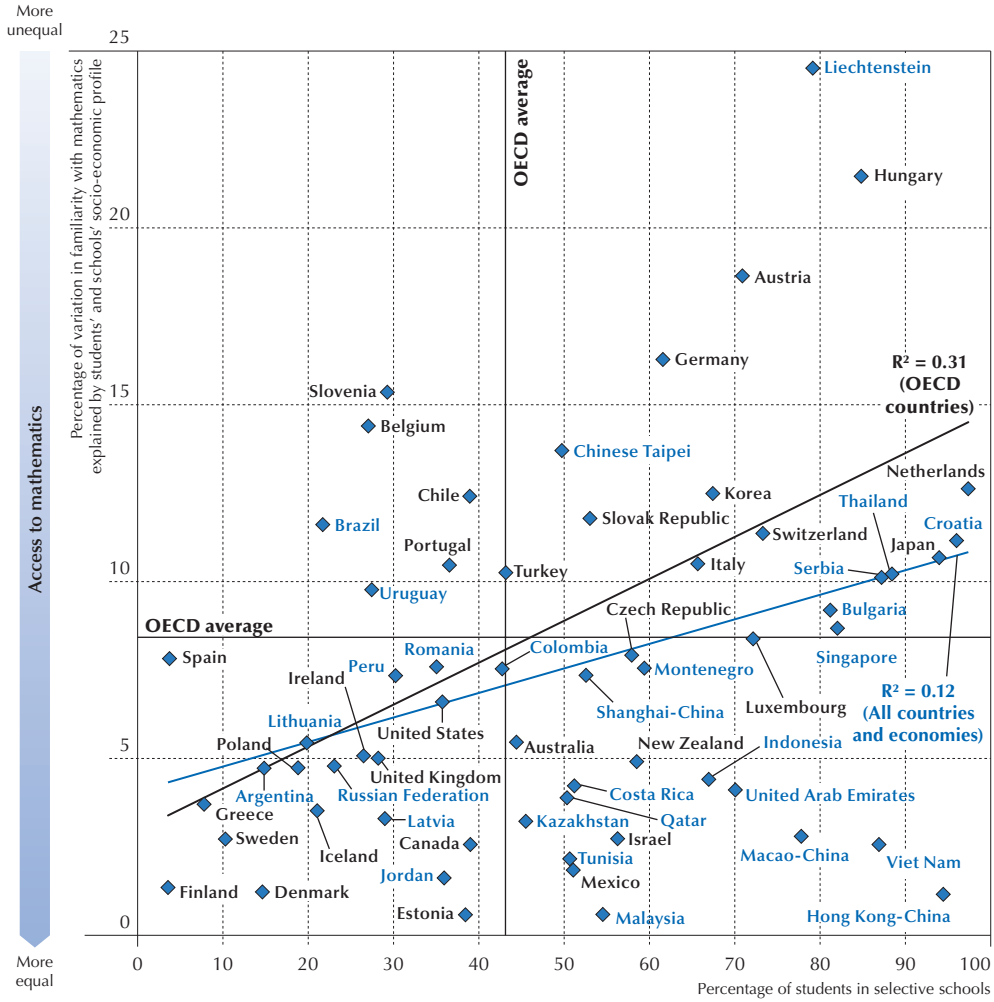
Notes: Parents' reports on their criteria for choosing schools for their children, by students' socio-economic status.
Only the following countries and economies with available data from the parent questionnaire are shown: Belgium (Flemish community), Chile, Croatia, Germany, Hong Kong-China, Hungary, Italy, Korea, Macao-China, Mexico and Portugal.
Source: OECD, PISA 2012 Database, Table 2.11.
StatLink http://dx.doi.org/10.1787/888933377063

Schools' practices of selecting students by academic achievement have a similar effect of reinforcing inequalities. Schools that select students for admittance based on their academic performance tend to show better average performance; but in systems with more academically selective schools, the impact of students' and schools' socio-economic profile on student performance is stronger (OECD, 2013b). Moreover, selective education systems are also linked with greater inequality in social outcomes later in life (Burgess, Dickson and Macmillan, 2014).

School selectivity is also associated with more unequal opportunities to learn mathematics. As shown in Figure 2.10, in Croatia, Hong Kong-China, Japan and the Netherlands, over 90% of students attend selective schools, i.e. schools where student academic performance and/or recommendations from feeder schools are always considered for admission.

■ Figure 2.10 ■

Academic selectivity and equity in familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Selective schools are defined as those schools where student academic performance and/or recommendations of feeder schools are always considered for admission.
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates a stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.12.
StatLink http://dx.doi.org/10.1787/888933377079

Across OECD countries, 31% of the variation in the association between students' socio-economic status and their familiarity with mathematics concepts is explained by the percentage of students in selective schools. (Across all participating countries and economies, 12% of the variation is so explained).

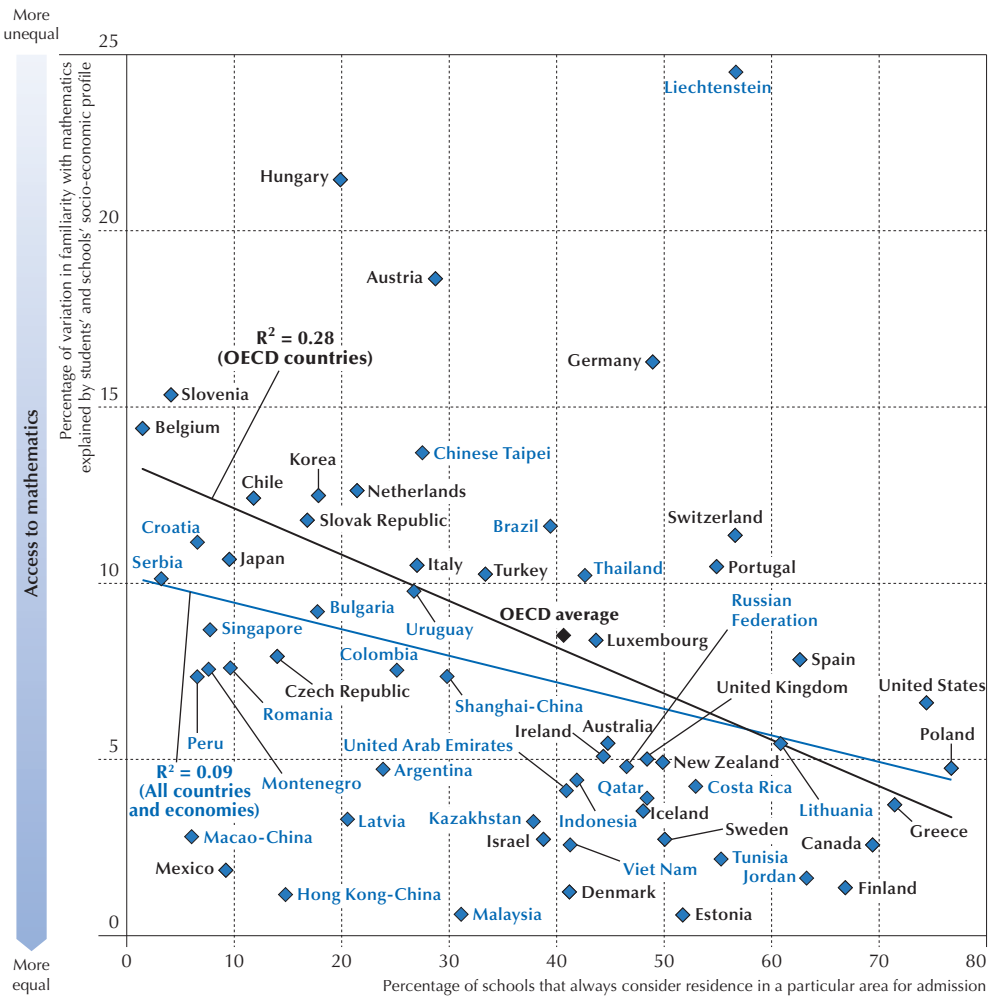
Admission requirements based on residence make school choice less dependent on families' socio-economic status, particularly if residential segregation is not pervasive. Figure 2.11 shows that over 70% of students in Greece, Poland and the United States attend schools that always consider residence for admission. Across OECD countries, a higher percentage of students in schools that always consider residence for admission is related to a weaker impact of socio-economic status on familiarity with mathematics. Schools' admission requirements account for around 28% of the between-country differences in equity in access to mathematics (defined as the within-country variation in the *index of familiarity with mathematics* explained by the students' and school's socio-economic profile). Again, the relationship between admission requirements based on residence and equity in opportunities to learn mathematics is weaker when looking at all countries and economies, possibly because, in partner countries and economies, people with similar backgrounds tend to live in the same areas to a greater extent than do people in OECD countries.

EQUITY IN OPPORTUNITY TO LEARN AND SORTING STUDENTS

Most school systems aim to improve the effectiveness of teaching by sorting students into relatively homogeneous groups according to their level of achievement. PISA gathers information on how schools and school systems group and select students, known as vertical and horizontal stratification (Figure 2.12). Vertical stratification describes the ways in which students progress through the school system. It is affected by policies governing the age at entry into the school system and grade repetition. Horizontal stratification refers to differences in instruction within a grade or education level, between or within schools, according to students' interests and performance.

Vertical and horizontal stratification are two sides of the same coin: they create opportunities to choose which type of education should be provided to which students. These decisions are based on various factors, not purely on students' abilities. When students are young, they are still in the process of developing their academic potential. These choices are also driven by subjective beliefs: the beliefs that students and their parents hold about their education needs, and the beliefs that teachers, school administrators and regulators hold about the costs of mixing students with different abilities and levels of preparedness in the same classroom. As students navigate this complex system, they face points at which decisions must be made – actively or passively, by the students, their parents or the school – about the next move, with each move affecting the subsequent one (Crosnoe and Schneider, 2010; Morgan, 2005).

In those school systems that sort students into different types of secondary schools or tracks (e.g. vocational or academic), a student's socio-economic status tends to have a strong impact on which school or track is selected. Other systems may have fewer of these distinct "branching points" of sorting, but differences in opportunity to learn related to socio-economic status are observed nonetheless. For example, disadvantaged students tend to select less academically challenging mathematics courses, especially when those courses are elective (Csikszentmihalyi and Schneider, 2000).

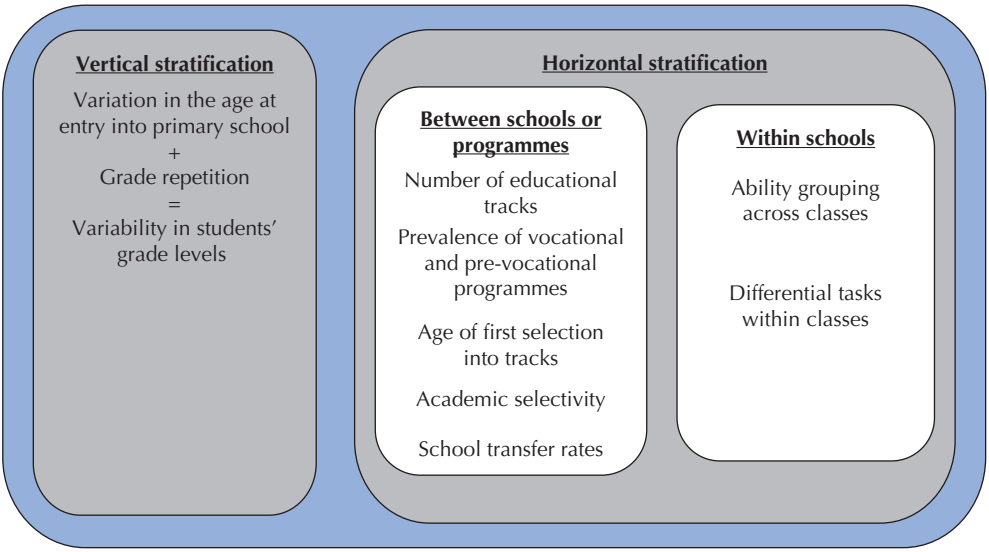


Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates a stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.13.
StatLink http://dx.doi.org/10.1787/888933377086

The cumulative effect of socio-economic status on access to mathematics content throughout a student's career can be easily illustrated through PISA data. PISA assesses students who are between the ages of 15 years 3 months and 16 years 2 months. In several education systems, it is thus possible to observe both students who are immediately before, and students who are

■ Figure 2.12 ■

Selecting and grouping students



Source: OECD (2013).

immediately after, one key branching point: the transition from lower secondary education (ISCED 2) to upper secondary education (ISCED 3).

As students progress from lower to upper secondary education, their familiarity with mathematics becomes more dispersed and more correlated with their socio-economic status (Table 2.14). Figure 2.13 shows that the impact of students' socio-economic status on their familiarity with mathematics concepts tends to be stronger among students in the first year of an upper secondary programme than among students in the last year of a lower secondary programme, after taking into account whether they had repeated a grade at least once in primary or secondary school.[5] This result confirms that the more choices and "permanent" transitions are allowed in a school system, the stronger the impact of socio-economic status on opportunity to learn.

The solution to this dilemma cannot be a fully inflexible system, with no scope for horizontal shifts; this would limit incentives to excel at school and dramatically constrain the right to express preferences. Rather, education systems can weaken the link between socio-economic status and opportunity to learn by becoming more flexible (granting real opportunities to change tracks and courses) and more objective (making track and course placement more dependent on achievement and students' interests rather than on parents' preferences and background).

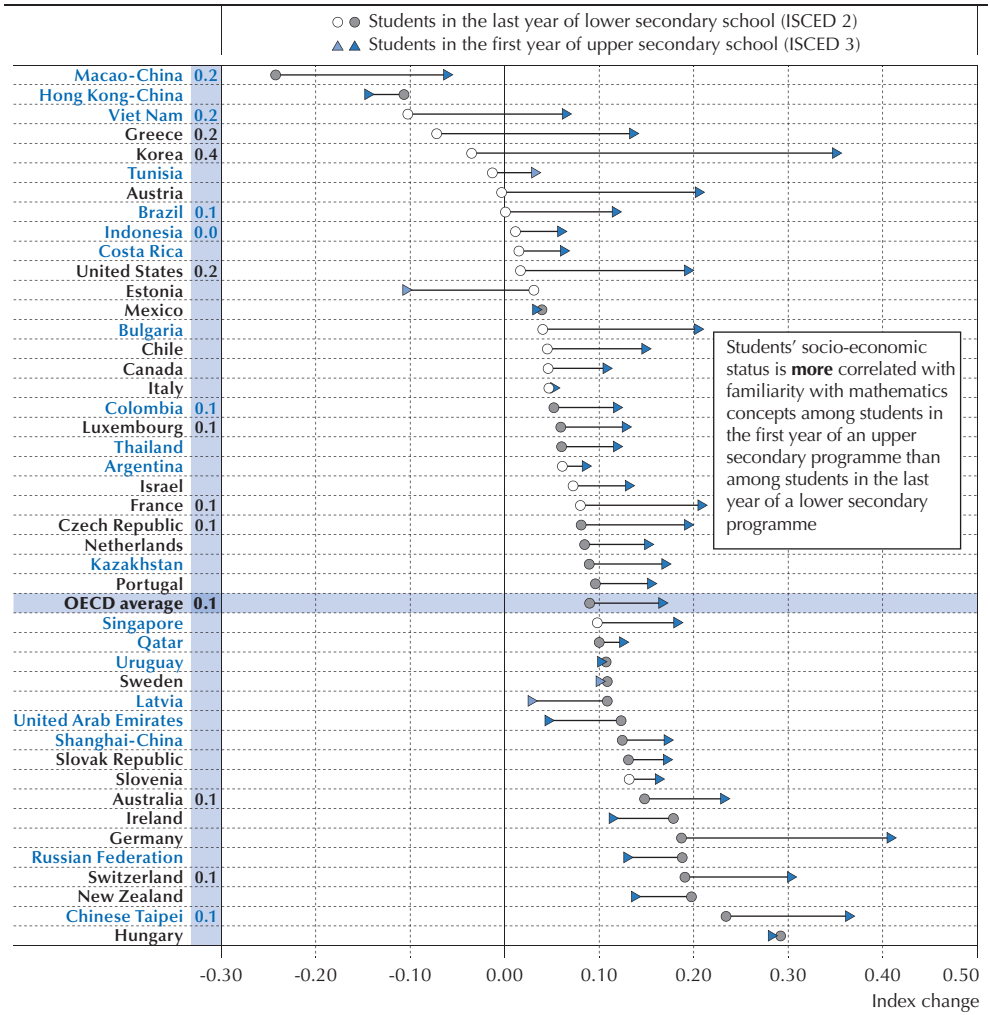
Vertical stratification through grade repetition

Grade repetition, a form of vertical stratification, is used in many systems to give low-performing students a second chance to master their coursework. On average across OECD countries,

■ Figure 2.13 ■

Familiarity with mathematics and students' socio-economic status, by level of education

Change in the index of familiarity with mathematics associated with a one-unit change in the PISA index of economic, social and cultural status (ESCS)



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The analysis takes into account grade repetition (i.e. whether students reported that they had repeated a grade at least once in primary, lower secondary or upper secondary school).
Statistically significant values are marked in a darker tone.
Only statistically significant index change differences between students in upper secondary school (ISCED 3) and students in lower secondary school (ISCED 2) are shown next to the country/economy name.
Countries with available data are shown.
Countries and economies are ranked in ascending order of the index change of students in the last year of lower secondary school (ISCED 2).

Source: OECD, PISA 2012 Database, Table 2.14.
StatLink http://dx.doi.org/10.1787/888933377092

13% of students reported that they had repeated a grade at least once in primary, lower secondary or upper secondary school (OECD, 2013c: Figure IV.2.2). Grade repetition might be used not so much to help students who are lagging behind, but rather as a stigmatising, and possibly discriminatory, form of punishment for inappropriate behaviour in class (National Research Council, 1999).

Many studies have shown that grade repetition is not necessarily beneficial for students. In fact, it may increase the probability that students drop out, stay longer in the school system, or spend less time in the labour force (Allen et al., 2009; Alexander, Entwisle and Dauber, 2003; Ikeda and García, 2014; Jacob and Lefgren, 2009; Manacorda, 2010). It is, moreover, costly to education systems, because of the expense of providing an additional year of education for a student, and to the wider society, as it delays a student's entry into the labour market (OECD, 2011).

Previous PISA analysis has shown that grade repetition is negatively related to equity in education. Systems where more students repeat a grade tend to show a stronger impact of students' socio-economic status on their performance (OECD, 2013c: Figure IV.1.4). At the same time, retention rates depend significantly on socio-economic factors (Corman, 2003). On average across OECD countries, socio-economically disadvantaged students are 1.5 times more likely to have repeated a grade than advantaged students who perform at the same level (OECD, 2013c). Immigrant students are almost twice as likely as students without an immigrant background to have repeated a grade, after accounting for both performance and socio-economic status (OECD, 2015b).

Figure 2.14 shows that, across OECD countries, grade repetition is negatively related to equity in access to mathematics. Around 38% of the variation in the impact of students' socio-economic status on their familiarity with mathematics concepts can be explained by differences in the proportion of students who had repeated a grade during their school career. (Across all PISA-participating countries and economies, the association is weaker).

The relationship between grade repetition and equity in opportunities to learn observed across OECD countries does not necessarily imply a causal link, as grade repetition might, in some systems, be a response to, rather than a cause of, differences in students' level of preparedness related to socio-economic inequalities. But given the lack of any solid evidence that repeating a grade improves mastery of mathematics concepts, the economic and social costs of retention are hard to justify.

Horizontal stratification between and within schools and programmes

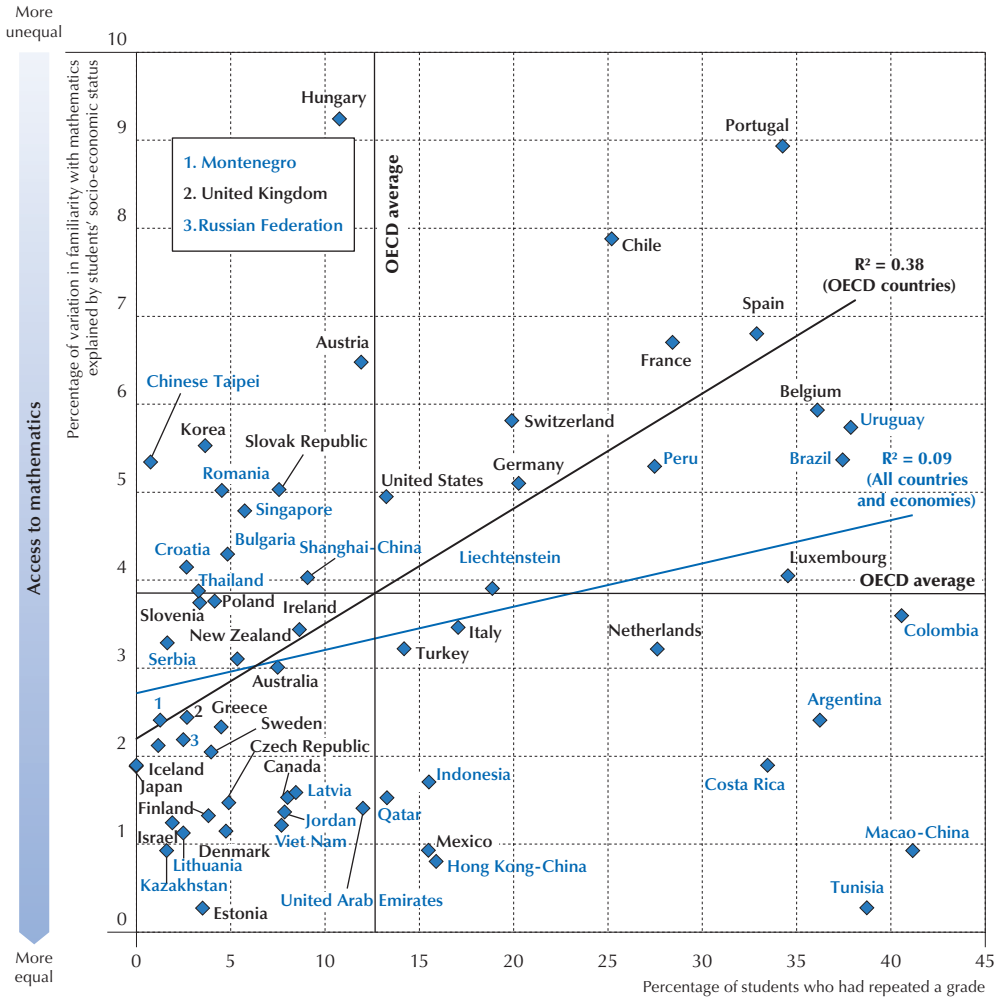
Various forms of horizontal stratification have been associated with greater inequality in education, as the goal of differentiating curricula by students' achievement level often translates into segregating students by their socio-economic status (Hanushek and Woessmann, 2010; van de Werfhorst and Mijs, 2010; see also Box 2.2).

Selection through between-school tracking

Although tracking is widely used – either to sort students into vocational or academic programmes or to base entry into a school on achievement – there is little support for the idea that it positively affects learning (Michaelowa and Bourdon, 2006). In fact, there is considerable international

■ Figure 2.14 ■

Grade repetition and equity in familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by the students' socio-economic status. A higher percentage indicates as a stronger impact of socio-economic status on students' familiarity with mathematics.

Source: OECD, PISA 2012 Database, Table 2.15.

StatLink http://dx.doi.org/10.1787/888933377108

Box 2.2. **Trends in between-school and within-school selection**

Both between-school and within-school selection aim to differentiate curricula by students' achievement levels and are widely practiced in OECD countries. In several countries, within-schools ability grouping has increased with the decline or the postponement of between-school tracking.

Since the 1960s a number of developed countries have started reforming their systems by delaying the age of streaming into schools with different orientations or by creating comprehensive schools, including Finland, France, Germany, Norway, Poland, Spain, Sweden, the United Kingdom and the United States (Ariga et al., 2005; Heidenheimer, 1974; Lucas, 1999; Pischke and Manning, 2006; Pekkarinen, Uusitalo and Kerr, 2009). At the same time, ability grouping or other forms of within-school tracking have become more common in some of the same countries, such as France, Germany, the United Kingdom and the United States (Duru-Bellat and Suchaut, 2005; Feinstein and Symons, 1999; Lucas, 1999; Kämmerer, Köller and Trautwein, 2002).

Internationally, countries with the highest rates of course-by-course tracking are Anglophone countries (Australia, Canada, New Zealand, the United Kingdom and the United States), the countries with moderate rates are Nordic and other comprehensive systems (Iceland, Norway, Poland, Spain and Sweden), and the countries with low rates are Denmark and Finland, as well countries practicing primarily academic/vocational streaming (Austria, Germany, Greece and Japan).

Source:

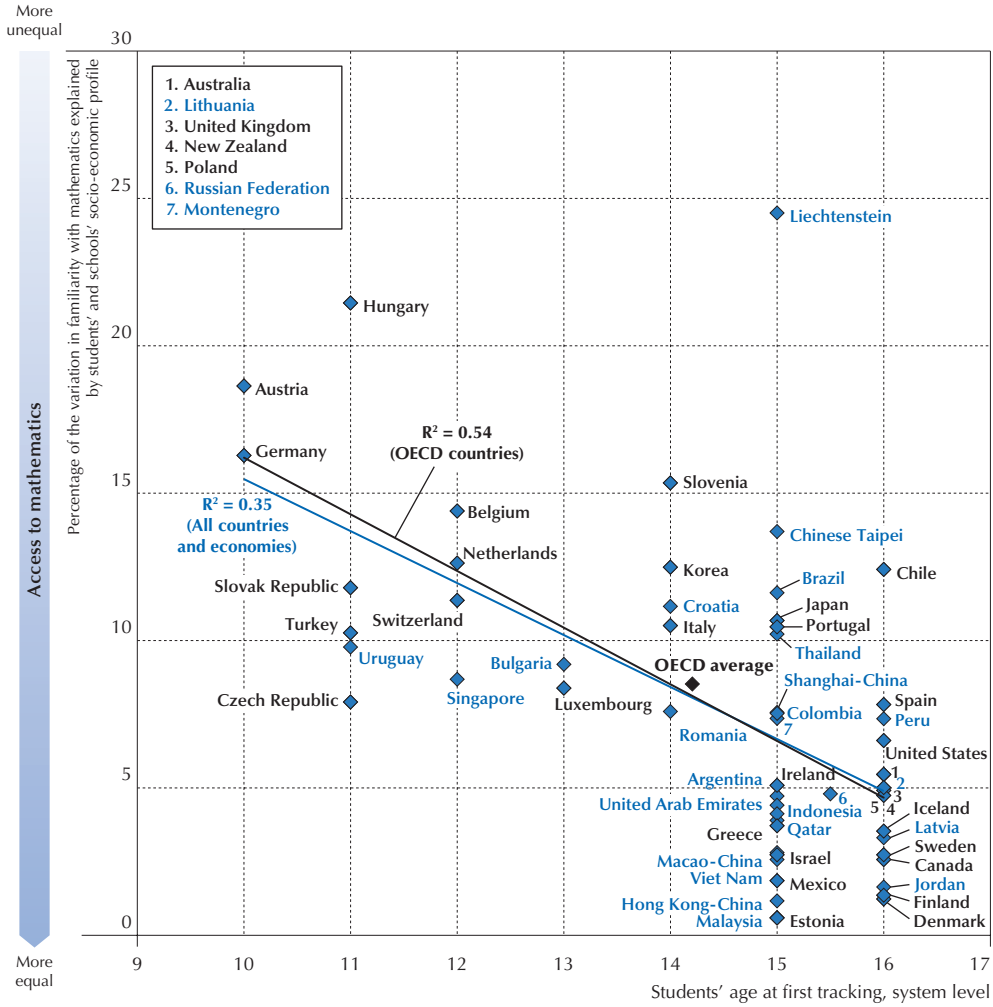
Chmielewski (2014).

evidence that tracking, especially early tracking, is associated with inequality in education, both in student performance (Hanushek and Woessmann, 2006) and in the extent to which individual student achievement and other life outcomes, such as enrolment in tertiary education and earnings in the labour market, reflect family background (Ammermüller, 2005; Brunello and Checchi, 2007; Ferreira and Gignoux, 2014; Horn, 2009; Schütz, Ursprung and Woessmann, 2008; Woessmann et al., 2009). Previous PISA analysis has also shown a negative association between early tracking and equity in education at the system level (OECD, 2013c).

Similarly, PISA 2012 data show that early tracking is related to inequalities in opportunities to learn mathematics. The relationship between the age at which a student is tracked and equity in access to mathematics is strong: on average across OECD countries, 54% of the variation in the impact of students' and schools' socio-economic profile on students' familiarity with mathematics is explained by system-level differences in the age at which students are first sorted into academic or vocational programmes (Figure 2.15). Across all participating countries and economies, the relationship is somewhat weaker, but still 35% of the variation in equity in access to mathematics is explained by differences in the age at which students are first tracked. In countries like Austria and Germany that start tracking students very early, heterogeneity in overall opportunity to learn is also quite large (as measured by the total variation in familiarity with mathematics within the country, Figure 2.1).

■ Figure 2.15 ■

Age at first tracking and equity in familiarity with mathematics

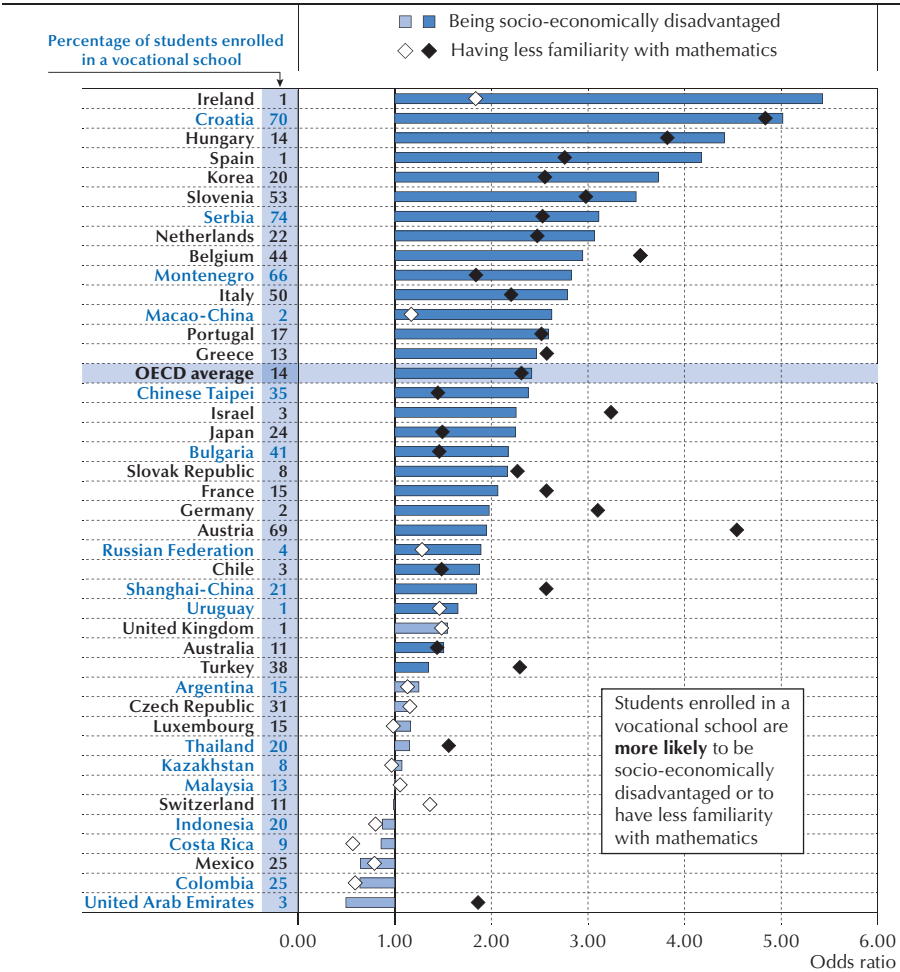


Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in *the index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.16.
StatLink http://dx.doi.org/10.1787/888933377117

■ Figure 2.16 ■

Concentration in vocational schools of disadvantaged students and students with less familiarity with mathematics

Relationship between enrolment in vocational schools and likelihood of having less familiarity with mathematics and/or being socio-economically disadvantaged



How to read the chart: An odds ratio of 2 for socio-economic status means that a student enrolled in a vocational school is twice as likely to be disadvantaged as a student who is not enrolled in a vocational school. Similarly, an odds ratio of 0.5 for socio-economic status means that a student enrolled in a vocational school is 50% less likely to be disadvantaged than a student who is not enrolled in a vocational school.

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).

Students with less familiarity with mathematics are students in the bottom quartile of the *index of familiarity with mathematics*.

Disadvantaged students are students in the bottom fourth of the *PISA index of economic, social and cultural status* (ESCS).

Students enrolled in a modular programme are not considered to be enrolled in a vocational school.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in descending order of the odds ratio for disadvantaged socio-economic status.

Source: OECD, PISA 2012 Database, Table 2.17.

StatLink http://dx.doi.org/10.1787/888933377124

The age at which a student is first tracked matters because younger students are more dependent upon their parents, and socio-economically advantaged parents might be in a better position to promote their children's best interests. As students grow older, more information about their abilities is available for educators to make objective evaluations, and for students to know what type of education better suits their preferences and career expectations.

The effect of tracking on equity of opportunity to learn is also related to the share and composition of students attending different types of schools. Almost one in seven students, on average across OECD countries, and more than one in two students in Austria, Croatia, Montenegro, Serbia and Slovenia, attend vocational schools (Table 2.17).

Figure 2.16 shows that, in most countries, students attending a vocational school are disproportionately more likely to be disadvantaged and to have less familiarity with mathematics than students attending an academic school. Students in Croatia, Hungary, Ireland, Korea, the Netherlands, Serbia, Slovenia and Spain who attend vocational schools are more than three times as likely to come from a disadvantaged background as students attending academic schools. Moreover, students in Austria, Belgium, Croatia, Germany, Hungary and Israel who attend a vocational school are over three times as likely to have less familiarity with mathematics as students who attend an academic school.

Ability grouping in different courses within the same school

Grouping students by ability within schools is another way of addressing students' differences in readiness to learn. In several countries, within-school ability grouping has increased with the decline or postponement of between-school tracking (Box 2.2).

PISA 2012 asked school principals to indicate the extent to which differences in students' abilities within classes hinder learning. Figure 2.17 shows that disruption to learning related to differences in students' abilities was reported more frequently by principals in disadvantaged schools than by those in advantaged schools. More than 30% of students in Chile, Croatia, Greece, Thailand and Uruguay attend disadvantaged schools whose principals reported that differences in students' abilities seriously hinder learning.

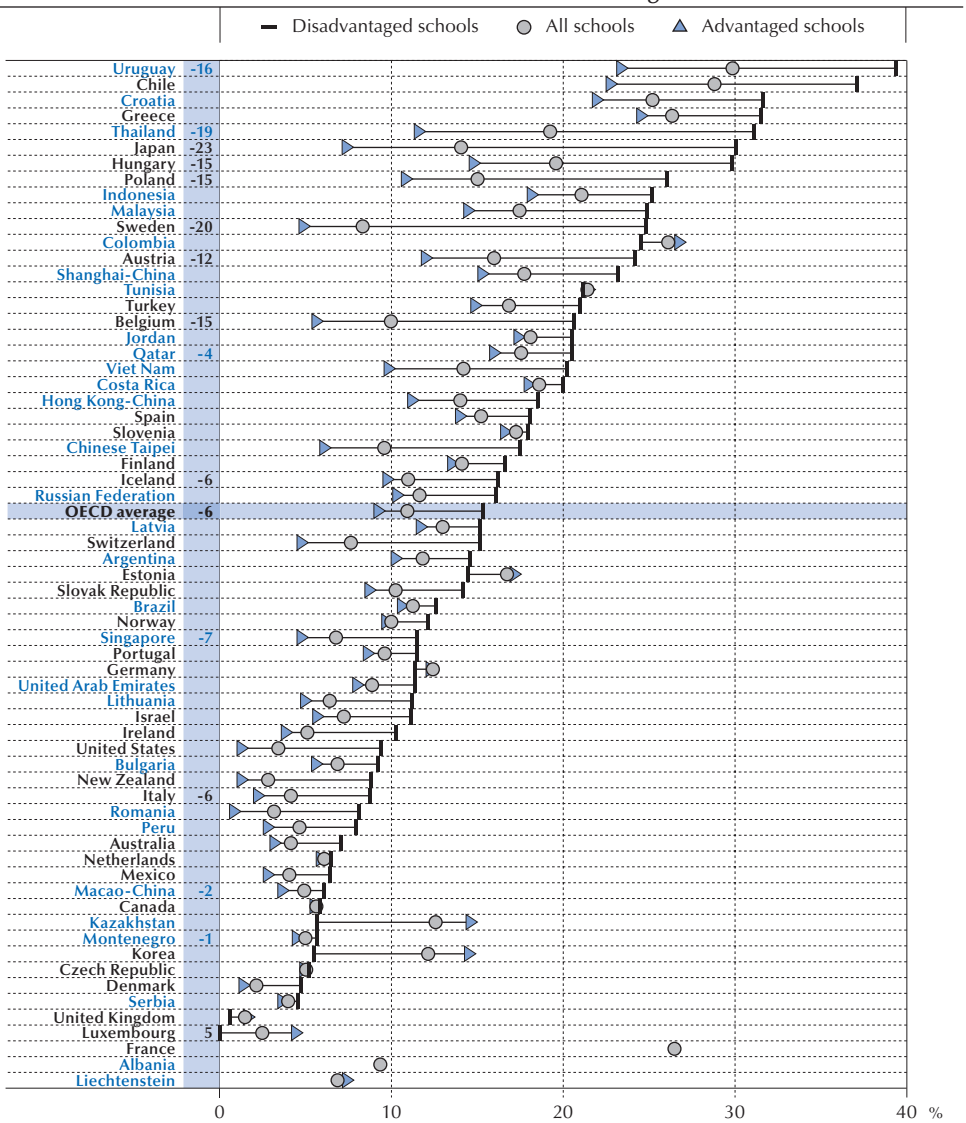
Ability grouping is relatively widespread across OECD countries, with more than 70% of students attending schools whose principal reported that students are grouped by ability for mathematics classes (Figure 2.18a). Over 95% of students in Australia, Ireland, Israel, Kazakhstan, Malaysia, New Zealand, the Russian Federation, Singapore and the United Kingdom attend such schools.

The effect of ability grouping within schools on achievement is unclear. While most studies find positive effects on the performance of high-achieving students, the effects among low-achieving students are open to debate (Argys, Rees and Brewer, 1996; Betts and Shkolnik, 2000; Collins and Gan, 2013; Figlio and Page, 2002; Zimmer, 2003). Moreover, ability grouping seems to reinforce socio-economic inequalities, as does between-school tracking: socio-economically disadvantaged students are disproportionally represented in less-able groups (Braddock and Dawkins, 1993; Oakes, 2005). Indeed, in a study of 20 education systems, Chmielewski (2014) finds that

■ Figure 2.17 ■

Effects of ability differences on the learning environment

Percentage of students in schools whose principal reported that ability differences within classes hinder learning a lot



Notes: Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in descending order of the percentage of students in disadvantaged schools whose principal reported that ability differences within classes hinder learning a lot.
Source: OECD, PISA 2012 Database, Table 2.18.
StatLink http://dx.doi.org/10.1787/888933377131

the relationship between students' socio-economic status and mathematics achievement is even stronger with ability grouping than with tracking between vocational and academic schools.

Data from PISA 2003 show that the type of mathematics classes that students attend as part of their sorting by ability is also related to their socio-economic status. Out of nine countries with available data, socio-economically advantaged students in Australia, Germany, Greece, Hungary, Iceland, Korea, the United Kingdom and the United States were more likely to report that they were attending advanced mathematics classes than disadvantaged students (Table 2.20a). After taking students' mathematics performance into account, advantaged students in Hungary, Korea and the United States were more than 50% more likely to take advanced mathematics than disadvantaged students; advantaged students in Greece and Iceland were more than twice as likely to take advanced mathematics as disadvantaged students after accounting for mathematics performance (Table 2.20b).

Figure 2.18a shows that, on average across OECD countries, ability grouping is more prevalent in disadvantaged than advantaged schools. In Austria, Chile, Croatia, Germany, Iceland, Luxembourg, Mexico, Portugal and Switzerland the difference in the percentage of students attending disadvantaged and advantaged schools that group students by ability is at least 10 percentage points.

Moreover, Figure 2.18b shows that, on average across OECD countries, the practice of grouping students by ability is associated with less familiarity with mathematics. In Austria and Switzerland, students in schools that practice ability grouping in all or some classes are less familiar with mathematics by more than 40% of a standard deviation compared to students in schools that do not group students by ability.

It is unclear, however, whether ability grouping is segregating low-achieving students even further or whether it reflects the schools' level of academic selectivity and is actually used to provide greater assistance to low-performing students in disadvantaged schools. Figure 2.18b also shows that, when comparing students of the same gender and socio-economic status who attend schools with similar socio-economic profiles, the negative association between ability grouping and familiarity becomes weaker or not statistically significant. These results suggest that the relationship between ability grouping and familiarity with mathematics is negative largely because ability grouping is used more often in disadvantaged schools, and because it may be used more as a way to give students who have less familiarity with mathematics more practice, rather than as a way to provide advanced education to gifted students.

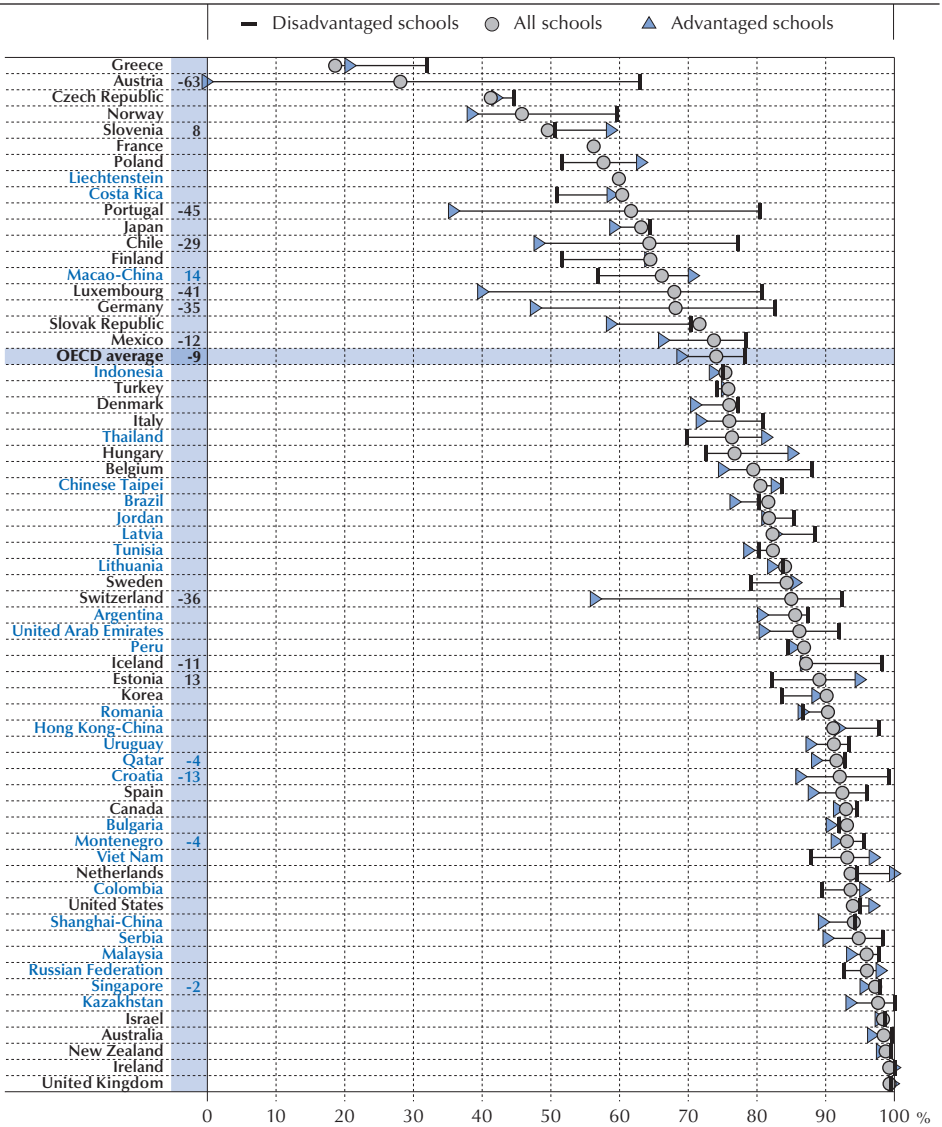
Teaching heterogeneous classes

Teachers are generally committed to providing equal education opportunities; but adapting instruction to each student's skills and needs while advancing learning for all students in the classroom is no small feat. The simplest strategy available to teachers is to use easier mathematics tasks whenever they teach weaker students. According to principals' reports, in most countries and economies at least half of all students attend schools where teachers believe that it is

■ Figure 2.18a ■

Prevalence of ability grouping, by schools' socio-economic profile

Percentage of students in schools whose principal reported that students are grouped by ability for mathematics classes

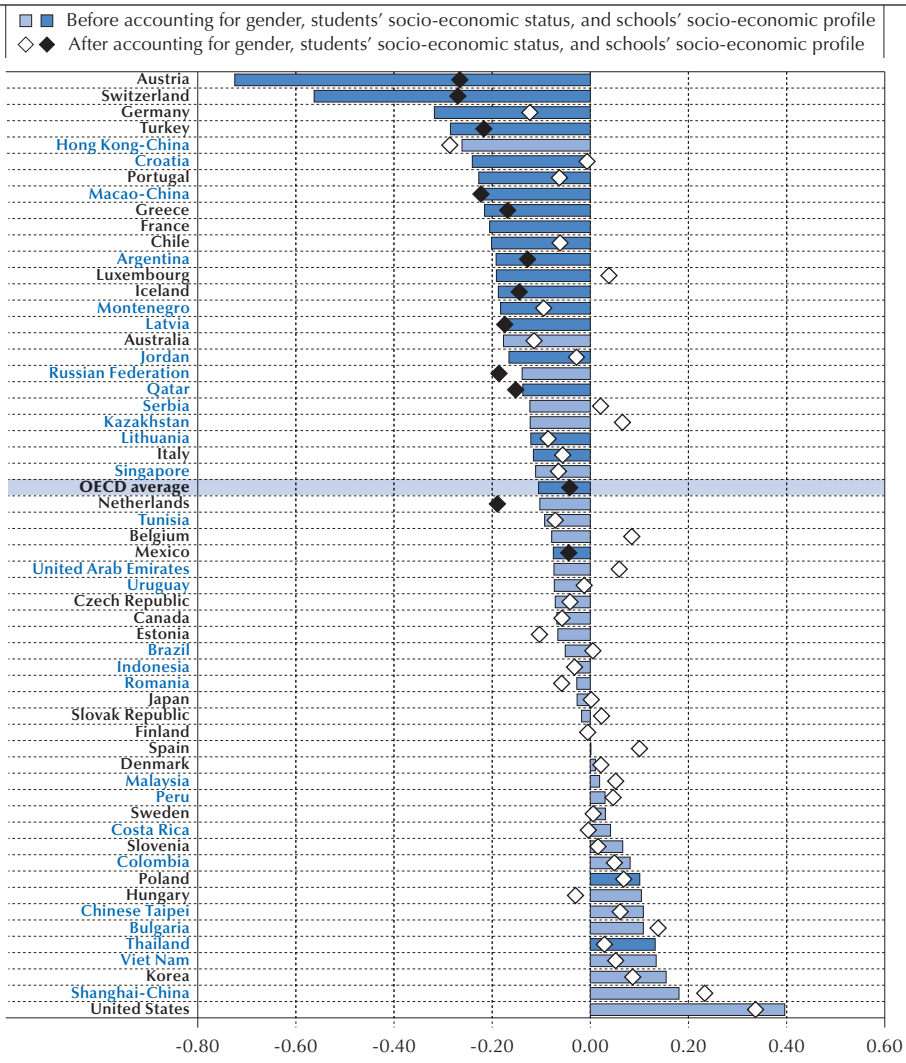


Notes: Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the percentage of students in all schools.
Source: OECD, PISA 2012 Database, Table 2.19a.
StatLink http://dx.doi.org/10.1787/888933377148

■ Figure 2.18b ■

Ability grouping and students' familiarity with mathematics

Change in students' familiarity with mathematics associated with the school's practice of grouping students by ability in some or all classes



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).
For each student, the school's average familiarity with mathematics is calculated as the average value on the index for all the other students in the school.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the change in familiarity with mathematics associated with the school's practice of grouping students by ability, before accounting for gender, students' socio-economic status, and schools' socio-economic profile.

Source: OECD PISA 2012 Database, Table 2.19b.

StatLink http://dx.doi.org/10.1787/888933377150

best to adapt academic standards to the students' levels and needs (on average across OECD countries, about 70% of students attend such schools [Figure 2.19]). Differences in teaching cultures and heterogeneity within classes probably explain why teachers in Montenegro, the Russian Federation and most Asian economies are more open to the idea of adapting academic standards than teachers in Austria, Germany and Luxembourg. But teachers' beliefs are also shaped by the environment in which they work, and in the large majority of countries, teachers in disadvantaged schools are much more willing to adjust their academic standards than teachers in advantaged schools.

Another way of teaching students of different abilities within the same class is to assign them different tasks. About 30% of students, on average across OECD countries, reported that teachers in their school differentiate between students when assigning tasks (Figure 2.20). Again, task differentiation is more frequently practiced in disadvantaged than advantaged schools. In Austria, Bulgaria, Germany, the Netherlands, Portugal, Romania, Serbia, the Slovak Republic, Slovenia and the United Arab Emirates, the difference between students in advantaged and disadvantaged schools who reported that their mathematics teacher differentiates tasks according to students' abilities is at least 20 percentage points (Figure 2.20). Assigning different tasks based on students' abilities can better address the needs of low performers, but might, at the same time, prevent low-achieving students from having the same opportunities to learn as higher-achieving students.

Selection through transfers

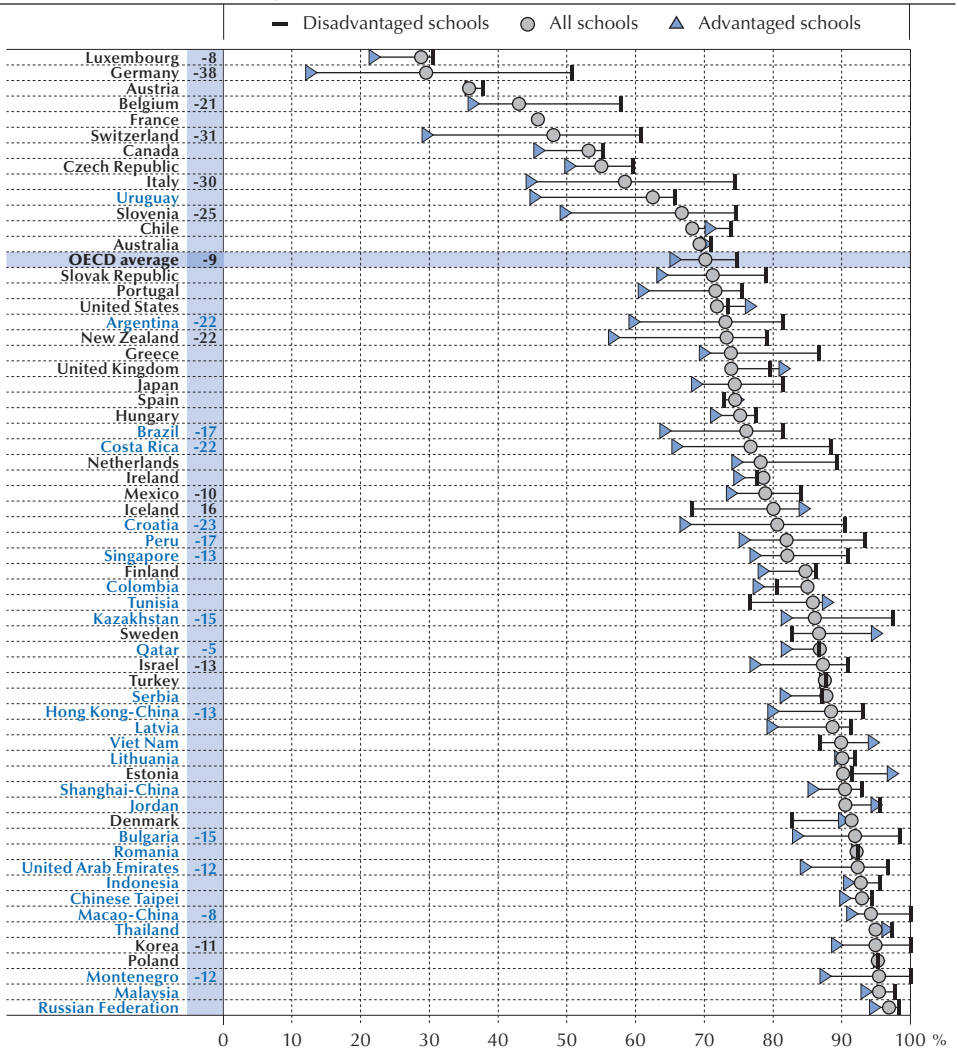
A much more radical way of separating students by ability consists in transferring low-achieving students to other schools (OECD, 2013c: Figure IV.2.6). This highly segregating practice is used more frequently than what might be expected. Over 70% of students in Austria, Macao-China, Slovenia and Chinese Taipei attend schools whose principals reported that they would transfer low-achieving students to another school (Table 2.23). The objectives of this policy might be either to preserve the learning environment of schools already struggling with low performers and poor results on standardised tests, or to assign students with special needs to specially equipped schools.

At the system level, the extent to which students' and schools' socio-economic profile affects familiarity with mathematics is positively related to the practice of transferring low-achieving students. As shown in Figure 2.21, across OECD countries, 42% of the variation in the impact of students' and schools' socio-economic status on students' familiarity with mathematics concepts is explained by the percentage of students in schools that are likely to transfer low-performing students (across all participating countries and economies, 16% of the variation is so explained). This association is fairly easy to explain: young people who are pushed out of a school (or strongly encouraged to leave) are disproportionately poor. The fewer learning opportunities and the social stigma that come from a forced transfer to another school can lead to early dropout and social exclusion (Books, 2010).

■ Figure 2.19 ■

Teachers' beliefs about the need to adapt academic standards to ability

Percentage of students in schools where teachers reported that they believe that it is best to adapt academic standards to the students' levels and needs



Notes: The figure reports the percentage of students in schools whose principal agreed or strongly agreed that there is consensus among mathematics teachers that it is best to adapt academic standards to students' levels and needs.

Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.

Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of students in all schools whose principal agreed or strongly agreed that there is consensus among mathematics teachers that it is best to adapt academic standards to students' levels and needs.

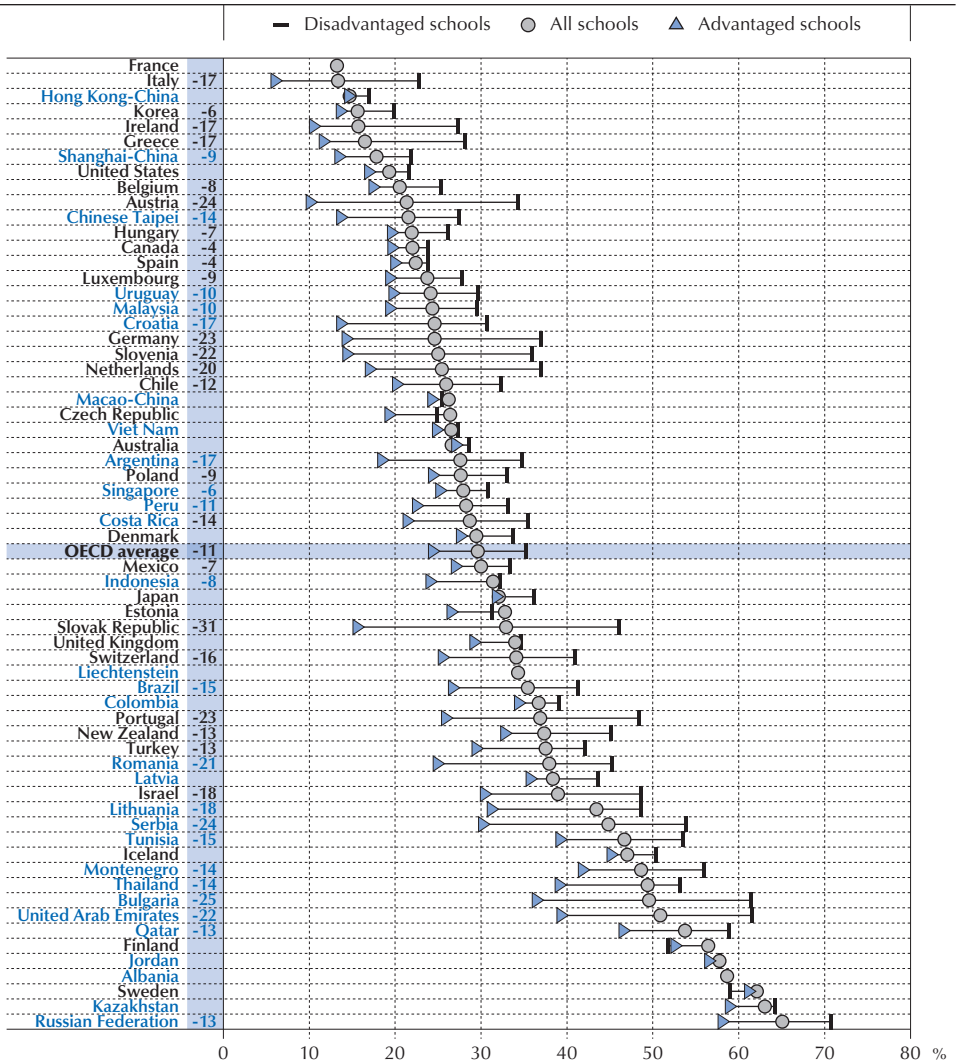
Source: OECD, PISA 2012 Database, Table 2.21.

StatLink http://dx.doi.org/10.1787/888933377168

■ Figure 2.20 ■

Teachers assigning different tasks to students based on ability, by schools' socio-economic profile

Percentage of students who reported that teachers in their school differentiate between students when assigning tasks



Notes: Task differentiation by teachers is measured on the basis of students' self-reports.

Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.

Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.

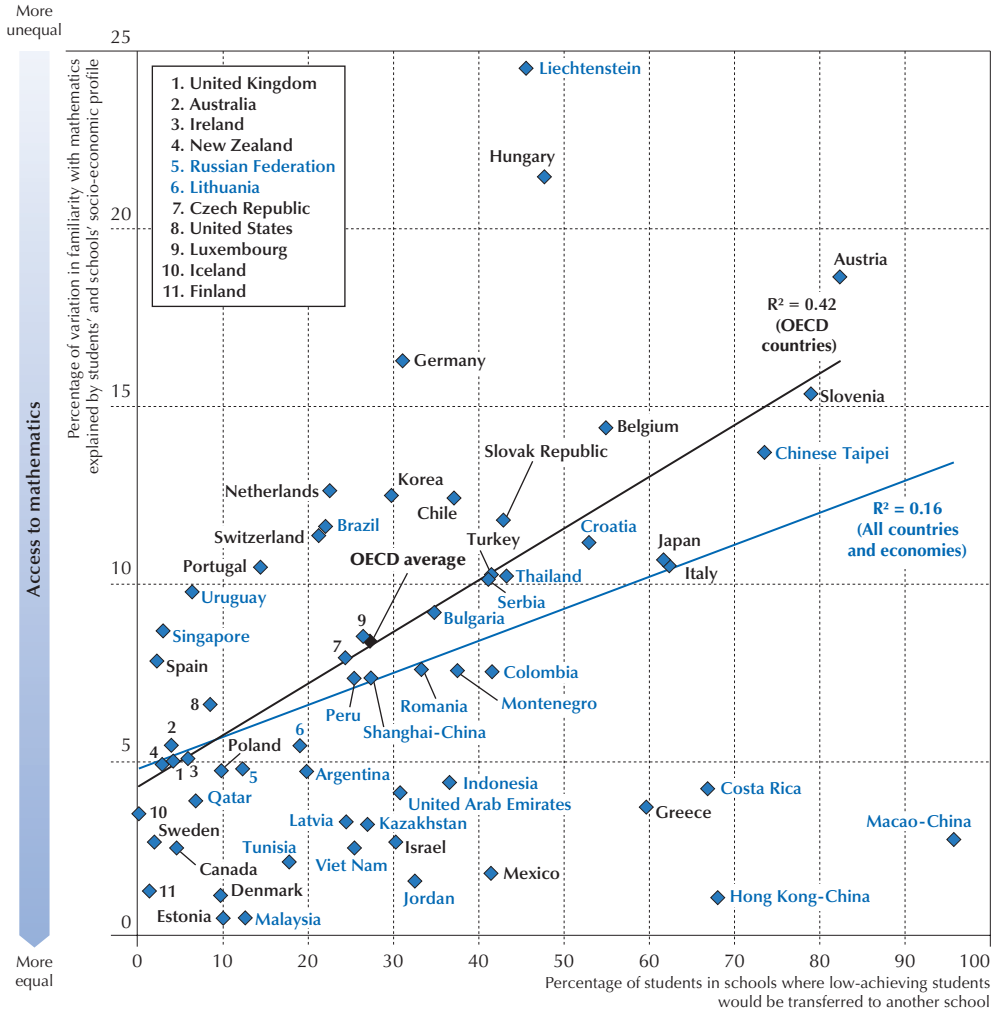
Countries and economies are ranked in ascending order of the percentage of students in all schools where teachers differentiate between students when giving tasks.

Source: OECD, PISA 2012 Database, Table 2.22.

StatLink http://dx.doi.org/10.1787/888933377173

■ Figure 2.21 ■

School transfers and equity in familiarity with mathematics



Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Source: OECD, PISA 2012 Database, Table 2.23.
StatLink http://dx.doi.org/10.1787/888933377186

HOW EQUITY IN OPPORTUNITY TO LEARN IS RELATED TO TEACHING RESOURCES AND PRACTICES

Teachers can influence equity in access to mathematics content not only by grouping students of similar ability and by assigning different tasks to students, based on their ability, but also more directly: through the quantity and quality of the tasks, and by engaging in certain teaching practices. Effective teaching is the most important in-school factor influencing strong academic performance (Chetty, Friedman and Rockoff, 2014; Rivkin, Hanushek and Kain, 2005). Low-achieving students and disadvantaged students stand to gain the most from highly qualified teachers (Gamoran, 1993; Nye, Konstantopoulos and Hedges, 2004), but they are often paired with the least-skilled teachers (Lankford, Loeb and Wyckoff, 2002).

Historically, schools serving poor communities face staffing problems and high rates of teacher turnover. Some teachers might quit because they prefer to work with more advantaged students (Hanushek, Kain and Rivkin, 2004), but most teachers in disadvantaged schools leave because of disciplinary problems, weaker collegial relationships, poor leadership, high student turnover, and general safety concerns that are more pervasive in disadvantaged schools (Gregory, Skiba and Noguera, 2010). Students in disadvantaged schools are thus more likely than their peers in wealthier schools to experience inconsistent staffing from one year to the next and to be taught by teachers who are new to their school and, often, new to the profession (Simon and Johnson, 2015).

Figure 2.22a shows that most countries allocate an equal or larger number of teachers per student in disadvantaged schools than in advantaged schools, even though differences tend to be small. On average across OECD countries, there is one additional student per teacher in advantaged schools than in disadvantaged schools. The main exceptions are Brazil and Turkey, where there are about 7-8 more students per teacher in disadvantaged than in advantaged schools.

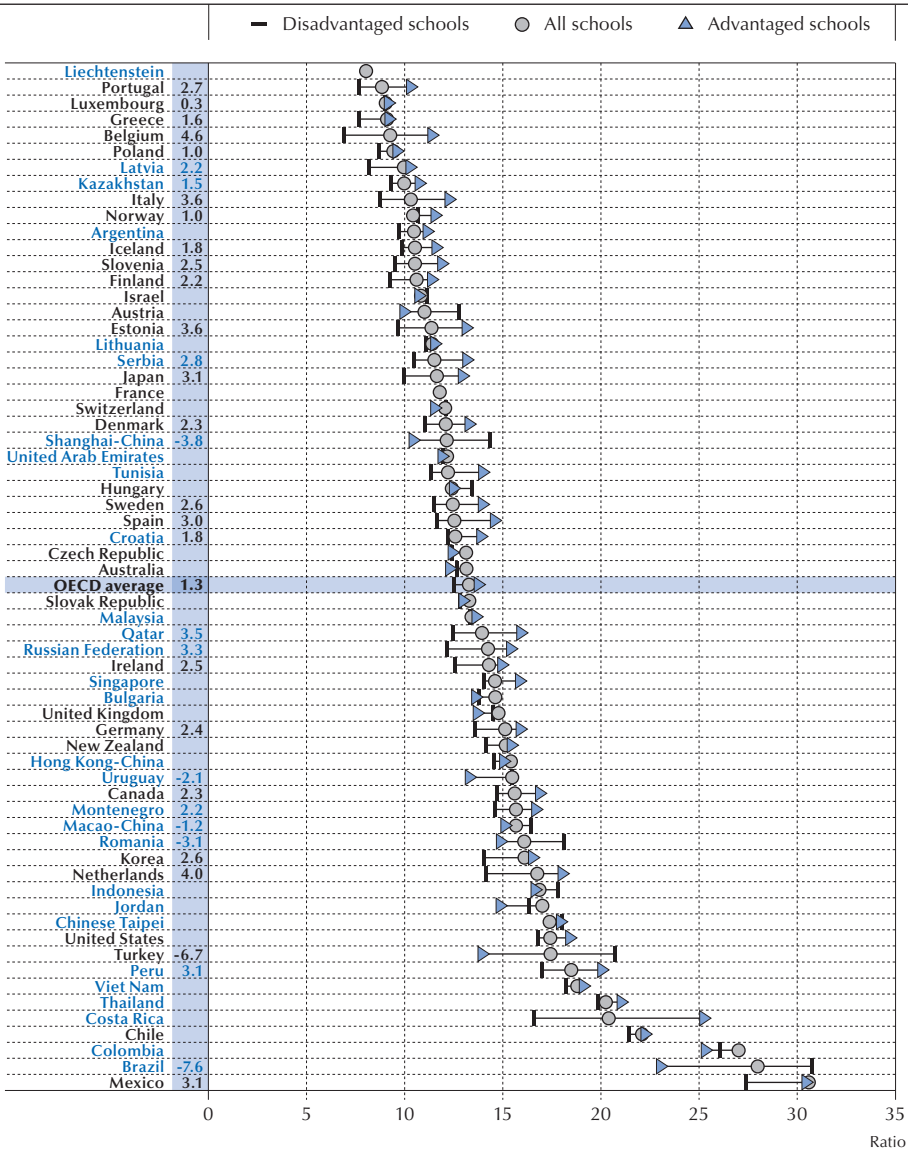
Even though disadvantaged schools have a (slightly) lower student-to-teacher ratio, mathematics teachers in disadvantaged schools tend to be less qualified. Figure 2.22b shows that the percentage of students in schools whose teachers majored in mathematics is generally higher in advantaged schools than in disadvantaged schools. On average across OECD countries, the share of qualified mathematics teachers in advantaged schools is eight percentage points larger than in disadvantaged schools, potentially exacerbating inequalities in opportunities to learn. By contrast, in Finland, Iceland, Macao-China, Spain and the United Arab Emirates, disadvantaged schools have a higher percentage of qualified teachers than advantaged schools.

Teaching practices as well as teachers' qualifications affect students' opportunity to learn. PISA asked students how often their mathematics teachers engage in cognitive-activation strategies, that is instructional practices involving challenging tasks, the activation of prior knowledge and higher-level thinking (Lipowsky et al., 2009). In particular, PISA asked students how often their mathematics teachers adopt the following practices:

- ask questions that make students reflect on the problem
- give problems that require students to think for an extended time

■ Figure 2.22a ■

Number of students per teacher, by schools' socio-economic profile



Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.

Only statistically significant differences in the ratio between advantaged and disadvantaged schools are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the number of students per teacher in all schools.

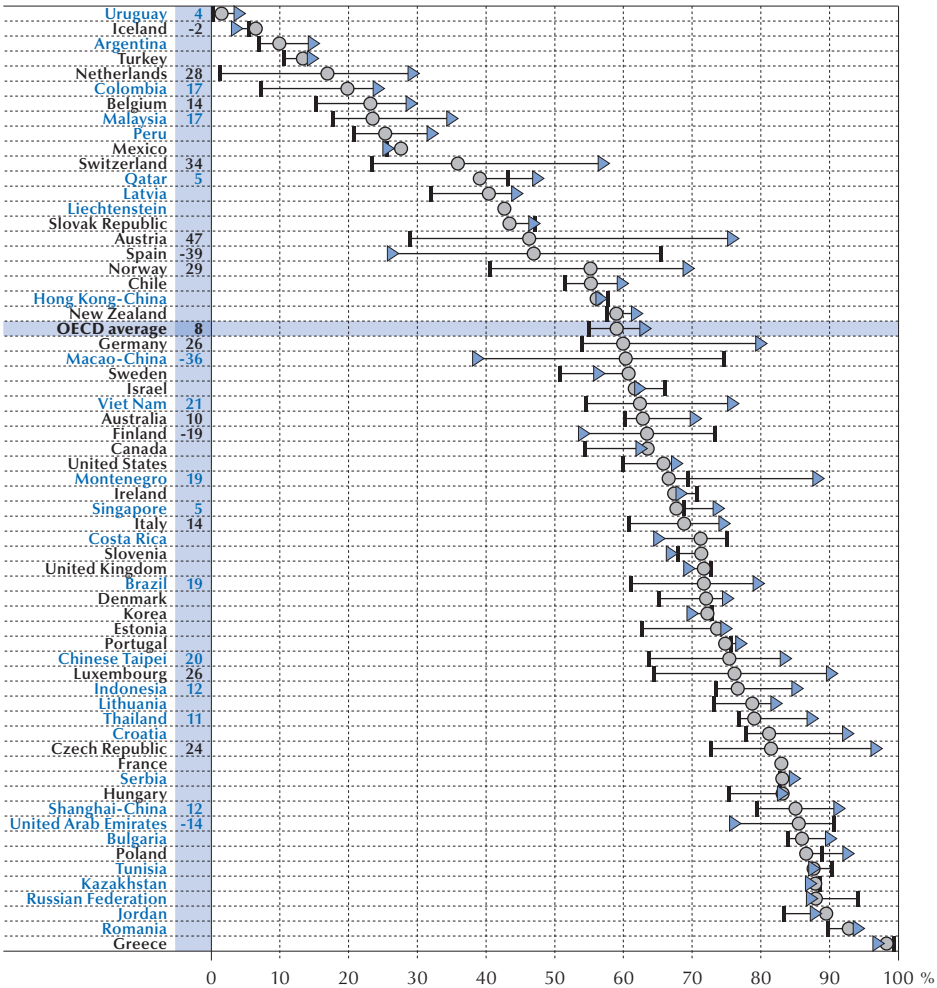
Source: OECD, PISA 2012 Database, Table 2.24.

StatLink http://dx.doi.org/10.1787/888933377197

■ Figure 2.22b ■

Percentage of qualified mathematics teachers, by schools' socio-economic profile

Percentage of students in schools whose principal reported that mathematics teachers are qualified



Notes: Qualified mathematics teachers are those teachers with a major in mathematics (ISCED 5A). The percentage of qualified mathematics teachers are reported by the school's principal.

Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.

Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of qualified mathematics teachers in all schools.

Source: OECD, PISA 2012 Database, Table 2.24.

StatLink http://dx.doi.org/10.1787/888933377200

- ask students to decide on their own procedures for solving complex problems
- present problems for which there is no immediately obvious method of solution
- present problems in different contexts so that students know whether they have understood the concepts
- help students to learn from mistakes they have made
- ask students to explain how they have solved a problem
- present problems that require students to apply what they have learned to new contexts
- assign problems that can be solved in several different ways.

Previous analysis of PISA data shows that students who indicated that their mathematics teacher uses cognitive-activation strategies reported particularly high levels of perseverance and openness to problem solving, were more likely to favour mathematics as a field of study over other subjects, and/or were more likely to regard mathematics as more necessary to their prospective careers than other subjects (OECD, 2013b).

Cognitive-activation strategies tend to be used more often in socio-economically advantaged schools than in disadvantaged schools (Table 2.25a). This is especially the case for strategies that require problem-solving skills and go beyond simple coverage of the curriculum. For instance, on average across OECD countries, the share of students whose teachers assign problems with no obvious solutions is seven percentage points larger in advantaged schools than in disadvantaged schools; and the share of students whose teachers require that they apply what they have learned to new contexts is five percentage points larger in advantaged than in disadvantaged schools. By contrast, "helping students to learn from mistakes they have made" is a strategy reported more often in disadvantaged schools than in advantaged schools, possibly because this strategy is more frequently used to help low-achieving students.

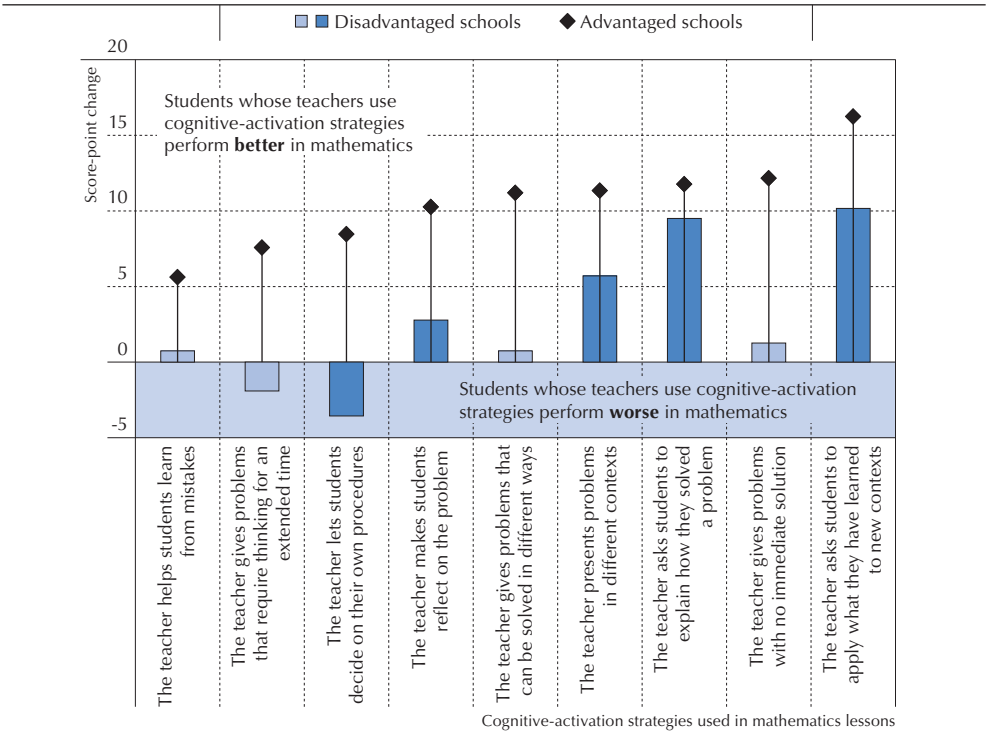
Previous research has shown a positive association between the use of cognitive-activation strategies and mathematics achievement (Echazarra et al., 2016; Lipowsky et al., 2009). What is the relationship between cognitive-activation strategies and opportunity to learn? And how does it vary according to schools' socio-economic profile? Figure 2.23a shows the change in mathematics performance associated with exposure to these strategies, while Figure 2.23b shows the change in familiarity with mathematics associated with the strategies. On average across OECD countries, the use of cognitive-activation strategies is associated with higher scores, in both advantaged and disadvantaged schools. However, in disadvantaged schools, only four out of nine such strategies are associated with better mathematics performance, while all strategies are associated with better performance in advantaged schools. Moreover, the effect on performance is larger in advantaged schools then in disadvantaged schools.[6]

Differences in the effect of cognitive-activation strategies according to schools' socio-economic profile are even more striking when looking at familiarity with mathematics. On average across OECD countries, the effect of cognitive-activation strategies on opportunity to learn mathematics is mixed in advantaged schools. Some strategies are associated with greater familiarity while

■ Figure 2.23a ■

Teachers' use of cognitive-activation strategies and students' performance in mathematics, by schools' socio-economic profile

Change in mathematics score associated with mathematics teachers' use of cognitive-activation strategies, OECD average



Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.

Statistically significant values for disadvantaged schools are marked in a darker tone. All values for advantaged schools are statistically significant.

Source: OECD, PISA 2012 Database, Table 2.25b.

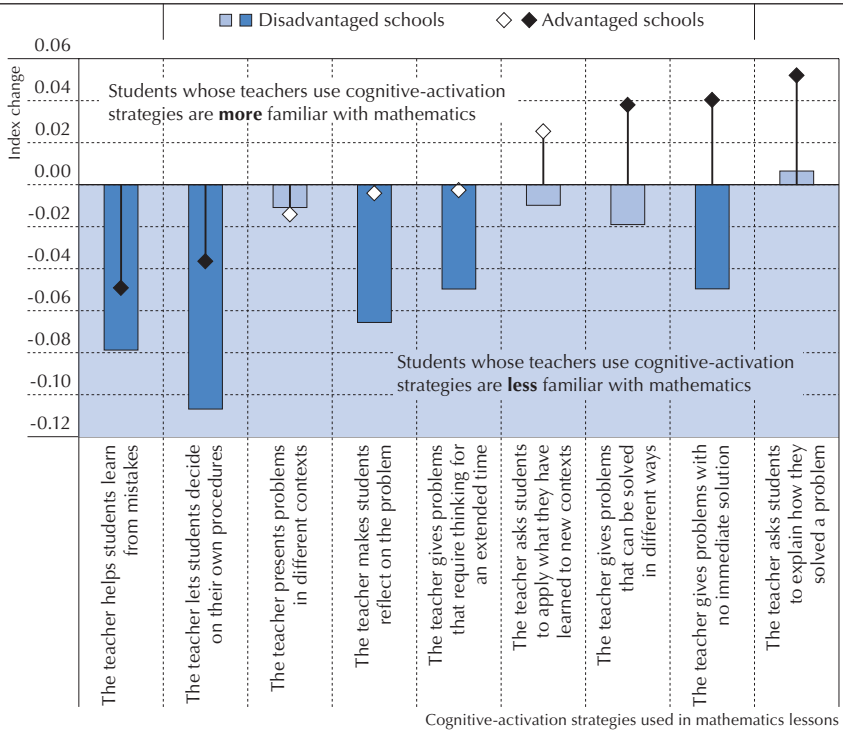
StatLink *http://dx.doi.org/10.1787/888933377210*

other strategies are related to less or no change in familiarity. However, on average across OECD countries, no cognitive-activation strategy is associated with greater familiarity with mathematics among students in disadvantaged schools. Students in disadvantaged schools who are exposed to five out of nine of these strategies are less familiar with mathematics than students who are not exposed; and the remaining four strategies are not related to any significant change in familiarity with mathematics.

■ Figure 2.23b ■

Teachers' use of cognitive-activation strategies and students' familiarity with mathematics, by schools' socio-economic profile

Change in the index of familiarity with mathematics associated with mathematics teachers' use of cognitive activation strategies, OECD average



Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Statistically significant values are marked in a darker tone.
Source: OECD, PISA 2012 Database, Table 2.25c.
StatLink http://dx.doi.org/10.1787/888933377223

Overall, these results suggest that teachers use cognitive-activation strategies to deepen the curriculum content and support the development of problem-solving abilities among students in advantaged schools. By contrast, in disadvantaged schools, it appears that there might be a price to pay for using strategies that emphasise thinking and reasoning for an extended time: less material is covered.

Why is it so difficult to use cognitive-activation strategies in disadvantaged schools? One reason is that these strategies might be more effective with students who already have a sound background in conceptual and procedural mathematics. Another reason might be that teachers may not be able to make students reflect on problems or assign problems that require thinking for an extended time in classrooms where there is noise and disorder. These difficulties should not discourage mathematics teachers in disadvantaged schools from adopting cognitive-activation strategies and problem solving. The time cost of these strategies can in fact be minimised by choosing well-framed problems and encouraging positive classroom behaviour.

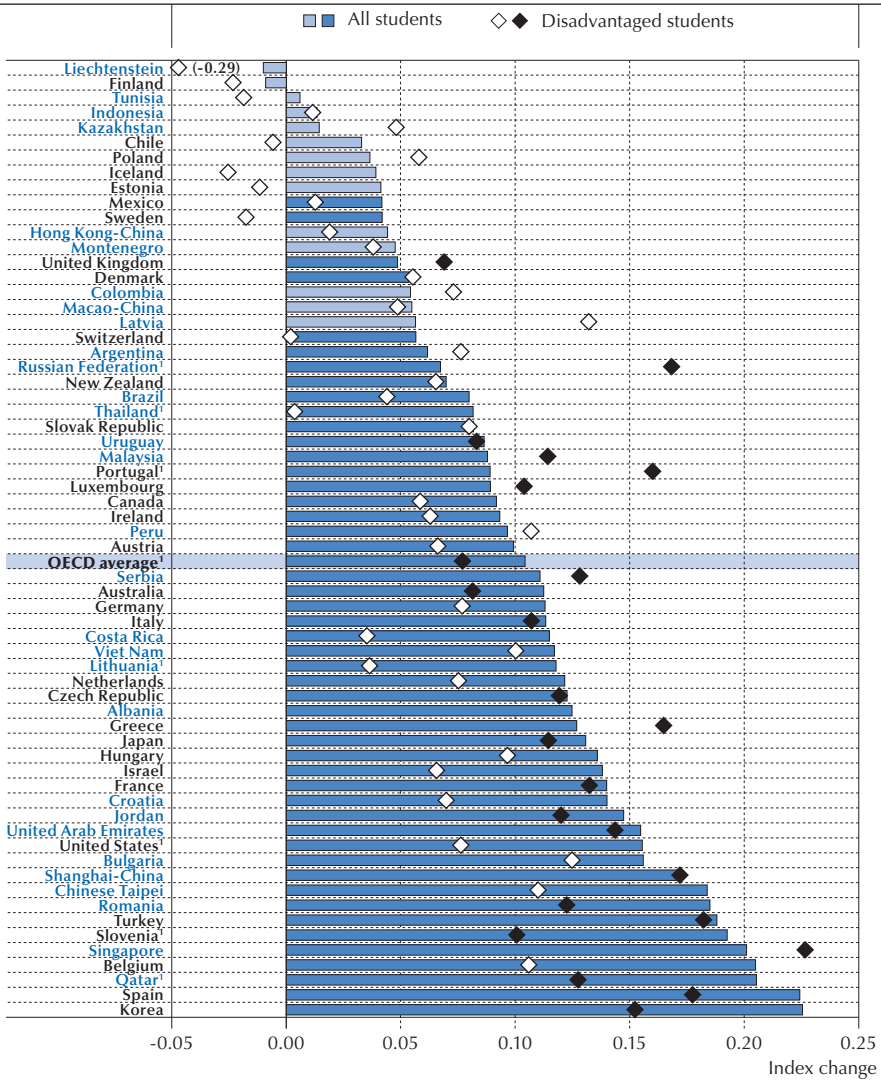
Previous PISA analysis showed that the disciplinary climate is positively correlated to a school's socio-economic profile (OECD, 2013b). In most countries and economies, better disciplinary climate is related to greater familiarity with mathematics, even after comparing students and schools with similar socio-economic profiles (Table 2.26). Moreover, Figure 2.24 shows that, on average across OECD countries, the association between disciplinary climate and familiarity with mathematics is weaker among disadvantaged students than among students in general, possibly because students with more positive attitudes towards mathematics benefit more from a favourable learning environment.

The preceding analyses show that access to mathematics is unequally distributed across individuals, schools and systems. Familiarity with mathematics is strongly related to students' socio-economic status, and the organisation of most education systems tends to reinforce socio-economic inequalities in access to mathematics. Selecting students into more homogenous groups through grade repetition, between-school tracking, academic admission requirements and school transfers is associated not only with a more unequal achievement distribution, but also with more unequal access to mathematics content, on which mathematical literacy is based. This suggests that alternative and more individualised approaches should be considered to provide struggling students with instruction tailored to their ability and needs (see Chapter 5).

■ Figure 2.24 ■

Disciplinary climate and familiarity with mathematics, by students' socio-economic status

Change in the index of familiarity with mathematics associated with a one-unit change in the index of disciplinary climate



1. The difference between disadvantaged and all students is statistically significant.

Notes: The *index of disciplinary climate* is based on students' reports of the frequency with which interruptions occur in mathematics class. Higher values on the index indicate a better disciplinary climate.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the change in familiarity with mathematics associated with a one-unit change in the index of disciplinary climate *for all students.*

Source: OECD, PISA 2012 Database, Table 2.26.

StatLink http://dx.doi.org/10.1787/888933377232

Notes

1. As discussed in Chapter 1, PISA 2012 data offer various measures of exposure to mathematics and familiarity with mathematics. Most of the analysis presented in Chapter 2 focuses on familiarity with mathematics because it better reflects the effect of cumulative opportunities to learn over students' school career – rather than just recent exposure – and because PISA gauges familiarity over a larger set of items than exposure, offering more statistical robustness and variation.

2. The relatively large within-school variations in the *index of familiarity with mathematics* might be partly explained by a certain degree of subjectivity in the interpretation of the questions, for example in how students define what is frequent and what is rare.

3. Equity in access to mathematics (the percentage of the variation in familiarity explained by students' and schools' socio-economic profile) is computed through a single-level linear regression for consistency with the definition of equity in education (the percentage of the variation in mathematics performance explained by students' socio-economic status) used in previous analyses of PISA 2012 data (OECD, 2013a). The correlation between equity in access to mathematics content and system-level indicators of stratification presented in this chapter is robust to an alternative definition of equity computed through a two-level model (as in Figure 2.1).

4. The OECD Teaching and Learning International Survey (TALIS) is conducted among teachers and leaders of mainstream schools in representative samples of schools. In 2013, 34 countries surveyed teachers in their primary, lower secondary and upper secondary schools. TALIS asks teachers and schools about their working conditions and learning environments. It covers such themes as initial teacher education and professional development; what sort of appraisal and feedback teachers get; the school climate; school leadership; and teachers' instructional beliefs and pedagogical practices. In 2013, some countries also chose to gain additional insights by conducting the survey in schools that participated in the 2012 Programme for International Student Assessment (PISA).

5. The difference in the impact of socio-economic status as students progress through school is probably underestimated, because we cannot observe those students who drop out of school between lower and upper secondary school. Dropout rates are generally much higher among relatively disadvantaged students.

6. These results need to be interpreted with some caution because it is not possible to distinguish whether the association is due to a genuine effect of teaching on achievement, or whether it is due to different uses of these teaching strategies according to students' abilities.

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3

Exposure to Mathematics in School and Performance in PISA

This chapter analyses how opportunity to learn mathematics influences students' performance in PISA and their capacity to solve the most challenging PISA tasks. The results show that exposure to pure mathematics has a strong association with performance that tends to increase as the difficulty of mathematics problems increases. Socio-economically disadvantaged students, who have fewer opportunities to learn how to use symbolic language, acquire fluency in procedures and build mathematic models, lack some of the essential skills needed to solve mathematics problems.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

It is hard to find two scholars holding the same view of how mathematics should be taught, but there is a general agreement among practitioners about the final goal: mathematics should be taught "as to be useful" (Freudenthal, 1968; Gardiner, 2004). In other words, mathematics should help students build competent and flexible performance on "demanding" tasks (Schoenfeld, 1994; Schoenfeld, 2004). Competent performance implies that mathematical operations are fast and effortless; flexible performance means not just solving familiar problems, but also being able to tackle novel problems built on the same principles (Rosenberg-Lee and Lovett, 2006).

Are mathematics curricula structured and implemented in ways that help students to develop competence and flexibility for demanding tasks? An analysis of PISA data can help to answer this question. PISA challenges students to solve a set of problems that might be encountered in real life, that are of greater or lesser difficulty and that do not look like those presented in mathematics classes at school. By analysing how students who have been exposed to mathematics to varying degrees perform on different PISA tasks, this chapter provides new evidence on whether students can apply the mathematics they learn at school.

While PISA data cannot establish cause and effect, the analysis shows a positive relationship between exposure to pure mathematics and performance. This is not only because "smarter" students may be concentrated in the same schools. Rather, students in the same school who are more frequently exposed to pure mathematics tend to do better in PISA. The chapter also looks at the relative strengths and weaknesses of countries and students, particularly disadvantaged students, across different areas of mathematics.

What the data tell us

- On average across OECD countries, student performance on mathematics tasks requiring familiarity with algebraic operations improved between 2003 and 2012, while performance on tasks with a focus on geometry deteriorated.
- In Austria, Croatia, Korea, Romania, Shanghai-China and Chinese Taipei, re-allocating one hour of instruction from reading to mathematics is associated with an improvement in mathematics performance, compared with reading performance, of more than 10 score points. However, the effect of such changes in instruction time on performance is not statistically significant in the majority of the other countries and economies.
- Students' exposure to pure mathematics tasks and concepts has a strong relationship with performance in PISA; and the association is stronger for more challenging PISA tasks. In contrast, exposure to simple applied mathematics problems has a weaker association with student performance.
- Around 19% of the performance difference between socio-economically advantaged and disadvantaged students can be attributed to disadvantaged students' relative lack of familiarity with mathematics concepts, on average across OECD countries. Disadvantaged students perform relatively worse on those tasks that require a mastery of symbolic and technical operations and on tasks that test their ability to build mathematic models of reality.

What these results mean for policy

- Students need to be exposed to mathematics content for a sufficient amount of time, but what matters the most is using instruction time effectively.
- Greater exposure to formal mathematics content improves performance – up to a point. All students should be exposed to a curriculum that is coherent across topics and over time, and focuses on key mathematics ideas, so that students can build solid foundations in mathematics.
- Greater familiarity with mathematics may not be sufficient for solving the most complex mathematics problems. Students also need to be exposed to problems that stimulate their reasoning abilities and promote conceptual understanding, creativity and problem-solving skills.
- Disadvantaged students would benefit most from any policy that increases their opportunities to develop not only procedural mathematics skills, but also skills in mathematical modelling.

MATHEMATICS CURRICULA AND PERFORMANCE ON DIFFERENT CONTENT AREAS OF PISA

Not only does PISA assess students on their performance in mathematics, reading and science, but it can also describe students' performance on four distinct mathematical content areas, the "big ideas" that nourish the growing branches of mathematics (Steen, 1990; OECD, 2013a):

- ***Change and relationships***: Tasks related to *change and relationships* require students to use suitable mathematical models to describe and predict change. They often require the application of algebra.
- ***Space and shape***: Tasks related to *space and shape* entail understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. *Space and shape* is the "big idea" most closely related to geometry.
- ***Quantity***: Tasks related to *quantity* involve applying knowledge of numbers and number operations in a wide variety of settings.
- ***Uncertainty and data***: Tasks related to *uncertainty and data* involve knowledge of variation in processes, uncertainty and error in measurement, and chance. This area has a strong connection to probability and statistics.

The PISA test items are split almost evenly across the four content areas (OECD, 2013a).

The four content areas are related to broad parts of the mathematics curriculum found in all PISA countries and economies. Students typically do better on items in which underlying concepts, formats and contexts are familiar to them, than on items in which these aspects are not as familiar. As such, the relative performance of countries across the four content areas (the PISA mathematics content subscales) reflects differences in course content available to 15-year-old

students, curriculum priorities, and item difficulty (OECD, 2014). Among other factors, weaker relative performance in a content area might signal an imbalance in the curriculum, which could lead to curriculum reform (Cosgrove et al., 2004).

Shanghai-China significantly outperforms all other countries/economies on each of the mathematics content-area subscales. In relative terms, Shanghai-China performs extraordinarily well on the *space and shape* subscale and less markedly so on the *change and relationships* subscale (Figure 3.1). Several other Asian countries are relatively strong in those tasks that require students to apply geometry.

There are greater international differences in contents and emphasis with geometry curricula than with arithmetic and algebra (French, 2004). Curriculum descriptions collected for the 2011 Trends in International Mathematics and Science Study (TIMSS) show that several Asian economies expose students to advanced spatial mathematics at early grades. For example, in Hong Kong-China, the relationship between three-dimensional shapes and their two-dimensional representations are introduced to students in grade 7, when they are around 13 years old. In Chinese Taipei, students practice the "translation, reflection and rotation" of figures when they are taught the topic of "quadratic function" in grade 9, when they are around 15 years old[1].

At the other end of the spectrum, Ireland performs relatively worse on PISA items in the *space and shape* content area (Figure 3.1). This relative weakness among Irish students might reflect differences between the PISA content area *space and shape*, which focuses more on visualisation skills, and the Irish Junior Certificate Geometry, which emphasises traditional Euclidian geometry (Shiel, 2007).

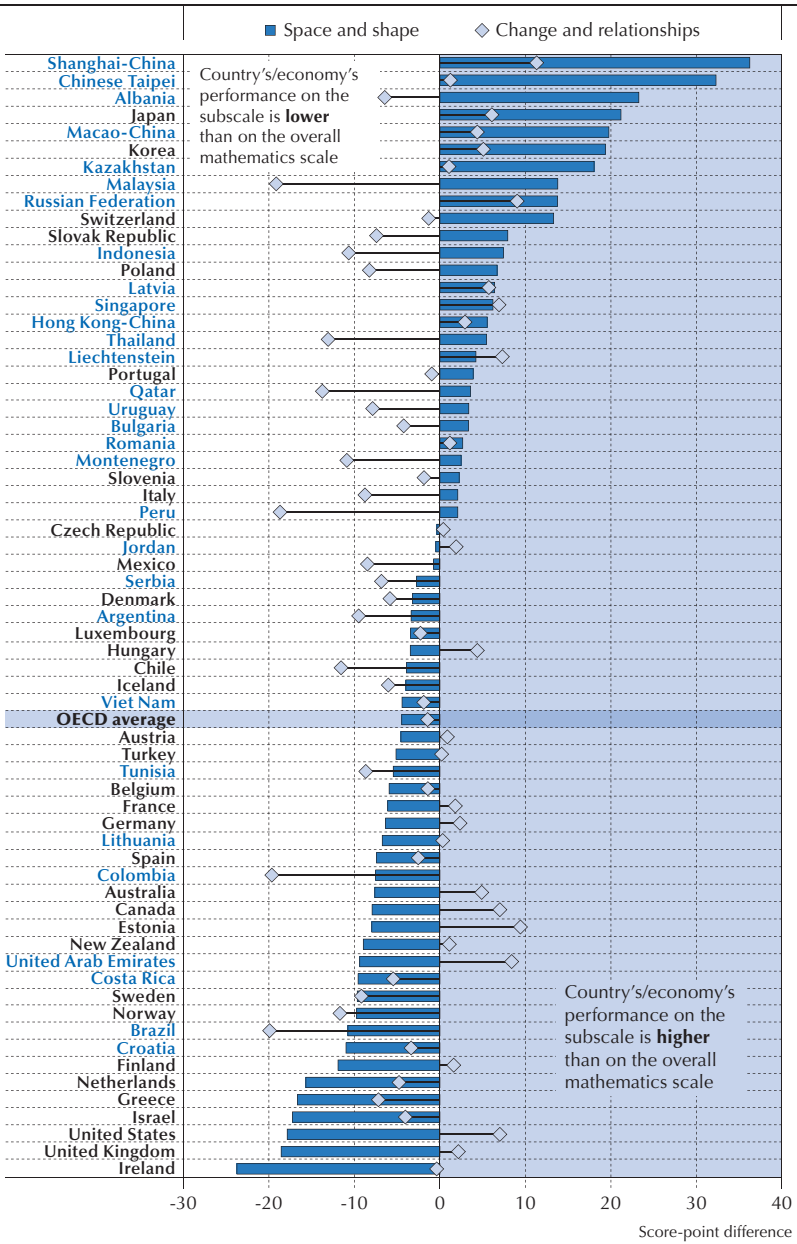
Differences in countries' relative performance should not be solely attributed to variations in how curricula are organised across the different areas of algebra, geometry, quantity and statistics; they may also reflect other characteristics of the individual tasks, such as the task's level of difficulty. An item's difficulty can be described by the percentage of students who responded correctly to it. The analysis in the rest of the chapter and of the report will refer to a logarithmic transformation of this percentage (a logit), where positive logits mean that more than half respondents answered correctly and negative logits mean that fewer than half respondents answered correctly.[2]

Figure 3.2 shows that the items classified in the content areas *space and shape* and *change and relationships* are, on average, much more difficult than the items in the areas of *quantity* and *uncertainty and data*.[3] The item REVOLVING DOOR Question 2 (see the full text of the question at the end of this chapter) is a *space and shape* item and is over 520 points more difficult on the PISA scale than CHARTS Question 1 (see the end of this chapter), an *uncertainty and data* item. The relatively higher performance among Asian countries and economies on tasks requiring greater knowledge of geometry and algebra can thus be explained by Asian students' greater capacities to solve more challenging problems, such as REVOLVING DOOR.

■ Figure 3.1 ■

Performance on the different mathematics content subscales

Score-point difference between the overall mathematics scale and each content subscale



Countries and economies are ranked in descending order of the difference between the overall mathematics score and the score on the mathematics subscale space and shape.

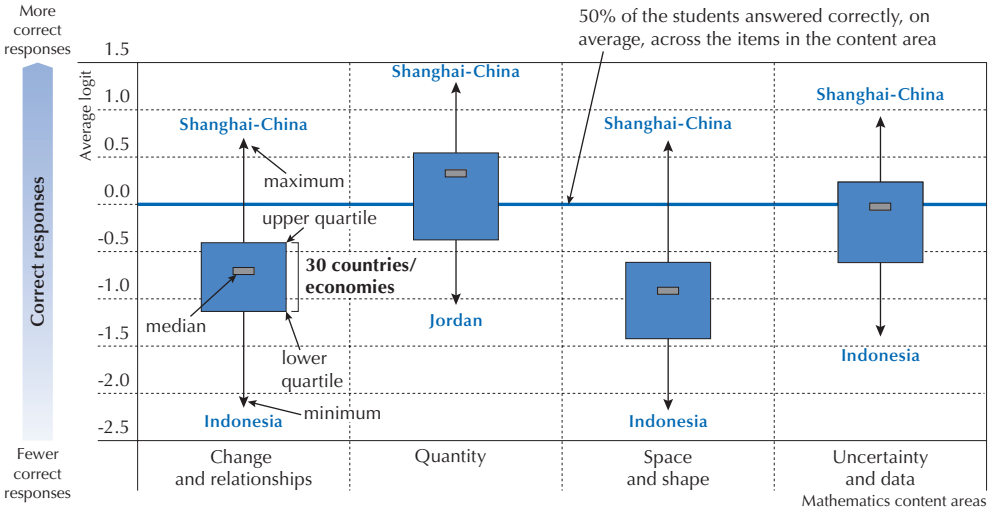
Source: OECD. PISA 2012 Database. Table 3.1.

StatLink http://dx.doi.org/10.1787/888933377248

■ Figure 3.2 ■

Difficulty of PISA tasks, by content area

Variation across all countries and economies



How to read the chart: The figure is a box-and-whisker plot showing the distribution of average logit values in 62 participating countries and economies with available data. For example, in the content area *change and relationships*, Indonesia has the minimum logit value (-2.20), while Shanghai-China has the maximum logit value (0.72) across all countries and economies. A quarter of the countries and economies have logit values included betwen the minimum and the lower limit of the box (-1.13) and a quarter of countries and economies have logit values above the upper limit (-0.41). Half of the countries and economies have logit values included between the lower and upper limit of the box (between -1.13 and -0.41); the horizontal bar represents the median value across all countries and economies (-0.71).

Note: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.

Source: OECD, PISA 2012 Database, Table 3.2b.

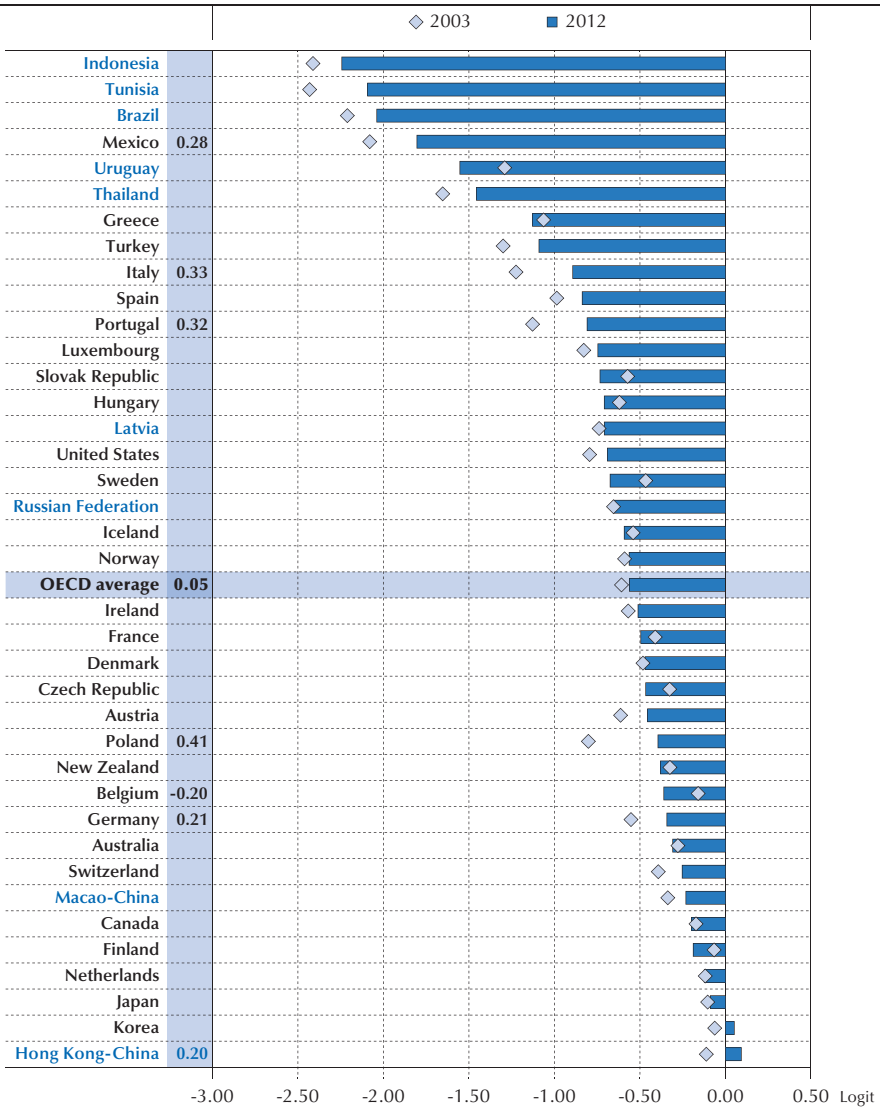
StatLink http://dx.doi.org/10.1787/888933377254

Looking at how the performance of countries has evolved across different content areas of mathematics reveals interesting trends, possibly related to changes in the focus of mathematics instruction. Figures 3.3a-d show trends in OECD countries' performance on the 31 mathematics items that were used in both the 2003 the 2012 assessments. On average across OECD countries, the percentage of students who were able to answer correctly the questions related to *change and relationships* increased, remained virtually the same for the questions related to *uncertainty and data*, but decreased for questions related to *quantity* and, more significantly, *space and shape*.

Performance on items related to *change and relationships* improved substantially in Italy, Poland and Portugal (over 0.4 logits in Poland, corresponding approximately to an increase of 7 points in the percentage of students answering correctly to those items, see Tables 3.3a and 3.3b). The Czech Republic, France, the Slovak Republic, Sweden and Uruguay saw a large deterioration in their students' performance on items related to *space and shape* that were tested in both 2003 and 2012 (0.5 logits in Uruguay, corresponding approximately to a decrease of 7 points

■ Figure 3.3a ■

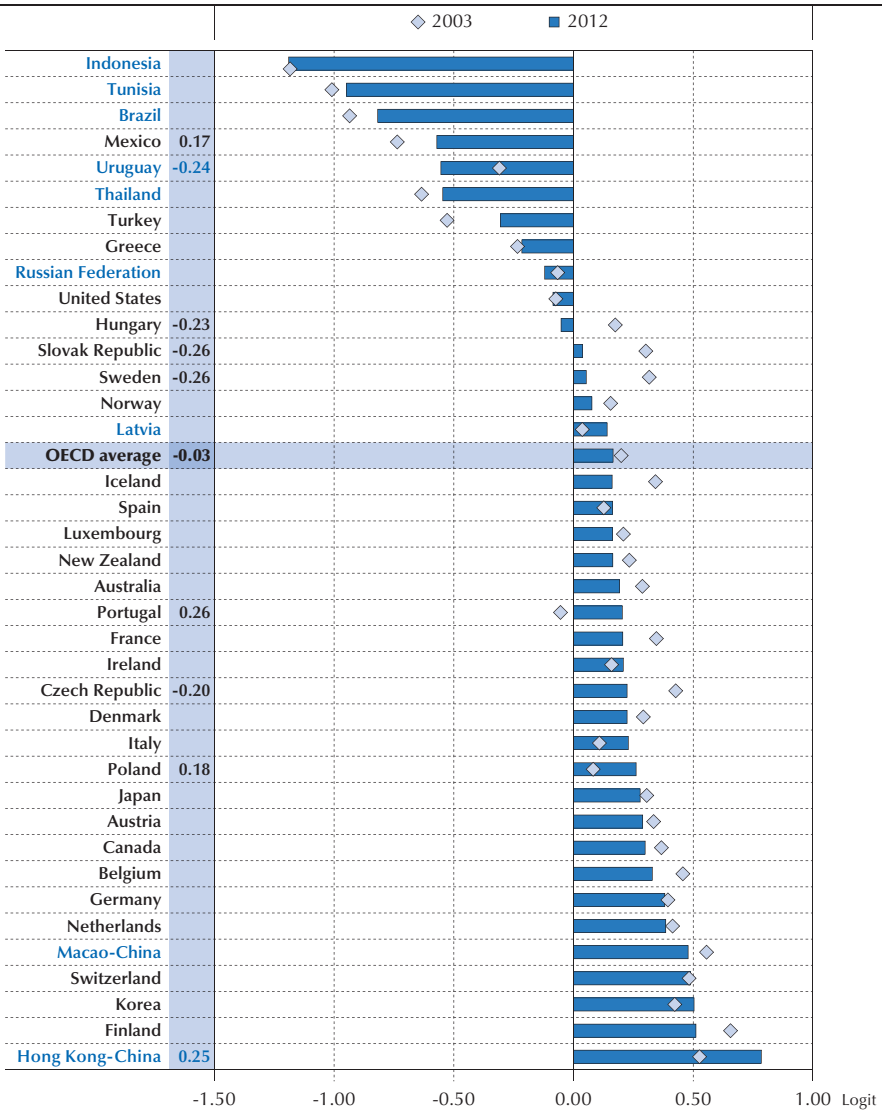
Change between 2003 and 2012 in mathematics performance across content area *change and relationships*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty. The logit coefficients are calculated over the 7 items in *change and relationships* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area change and relationships *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377262

■ Figure 3.3b ■

Change between 2003 and 2012 in mathematics performance across content area *quantity*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.

The logit coefficients are calculated over the 8 items in *quantity* assessed both in 2003 and 2012.

Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.

The OECD average is calculated as the average of 29 countries.

Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.

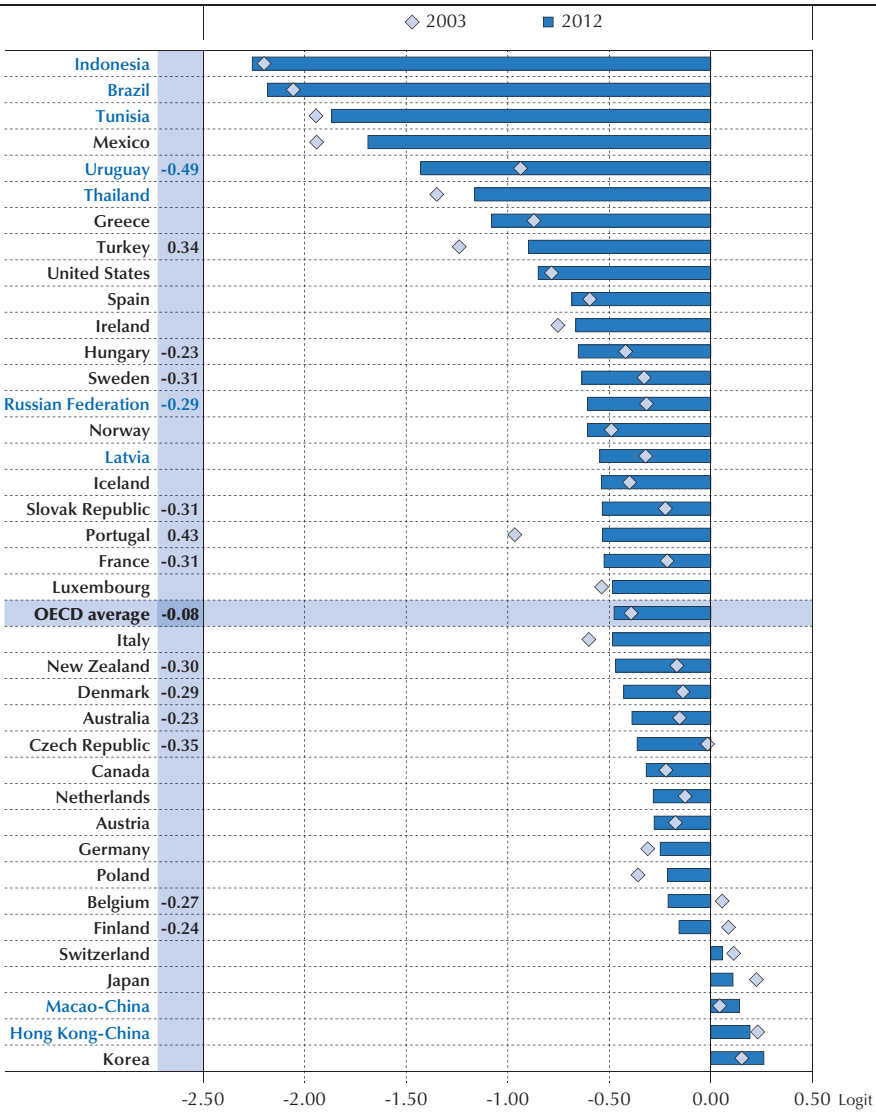
Countries and economies are ranked in ascending order of the average logit of questions in the content area quantity *in 2012.*

Source: OECD, PISA 2012 Database, Table 3.3a.

StatLink http://dx.doi.org/10.1787/888933377278

■ Figure 3.3c ■

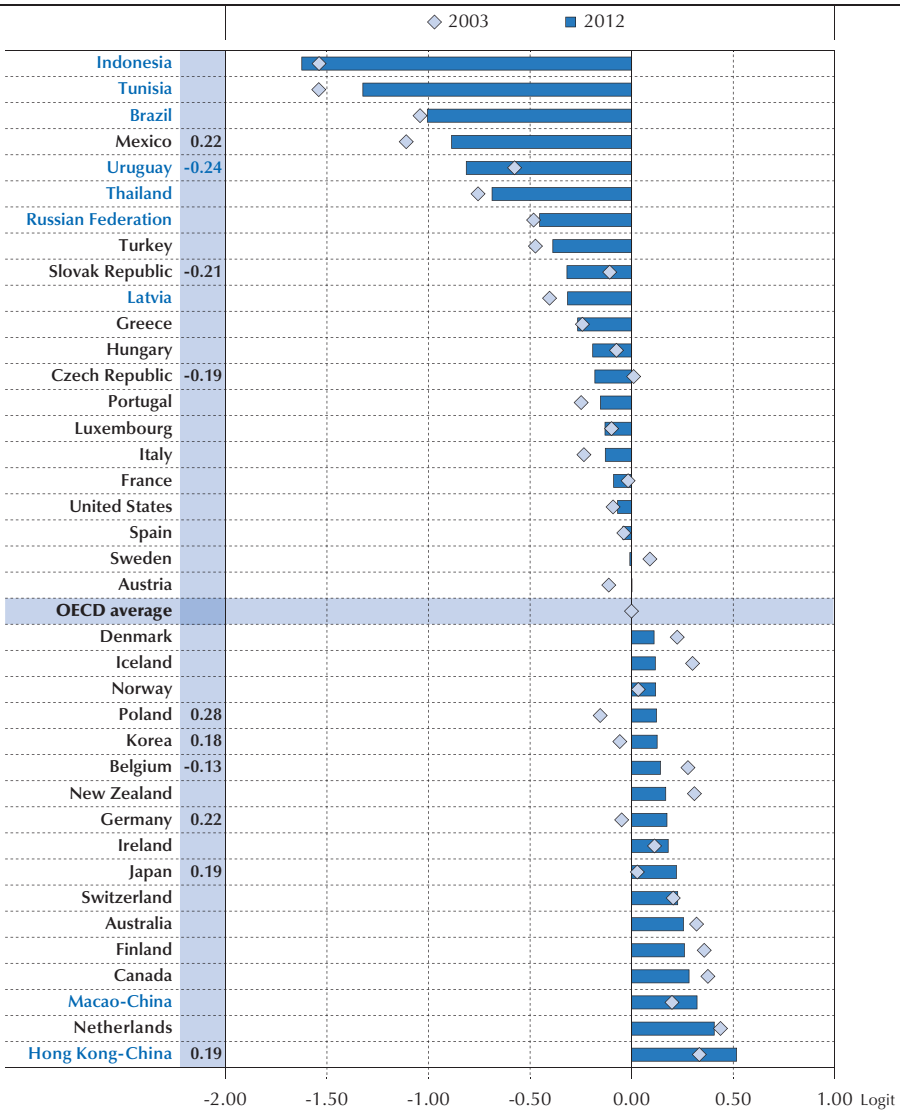
Change between 2003 and 2012 in mathematics performance across content area *space and shape*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty. The logit coefficients are calculated over the 9 items in *space and shape* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area space and shape *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377284

■ Figure 3.3d ■

Change between 2003 and 2012 in mathematics performance across content area *uncertainty and data*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty. The logit coefficients are calculated over the 7 items in *uncertainty and data* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statisically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area uncertainity and data *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377292

in the percentage of students answering those items correctly, see Tables 3.3a and 3.3b). The deterioration in performance on tasks in the *space and shape* area is possibly a consequence of a de-emphasis on geometry in mathematics curricula in some countries (Lehrer and Chazan, 1998). This negative trend deserves further consideration, as geometry is often the only visually oriented mathematics that students are offered. Students who receive training in the analysis of shapes develop their abilities to "see" the end product in their mind's eye, inspect its individual elements, and make sufficiently good conjectures about their relationships. All of these skills are essential, not only for scientists but for everyone in their daily lives.

VARIATIONS IN OPPORTUNITY TO LEARN AND PERFORMANCE IN MATHEMATICS

Variations in time spent learning

The amount of time that students spend learning is a basic component of their opportunity to learn (Carroll, 1963; Gromada and Shewbridge, 2016). PISA 2012 data based on students' reports show that, on average across OECD countries, students spend about 3 hours and 38 minutes per week in mathematics class, 3 hours and 35 minutes in language-of-instruction class, and 3 hours and 20 minutes in science class, though the time varies considerably within countries (OECD, 2013b, Table IV.3.21).

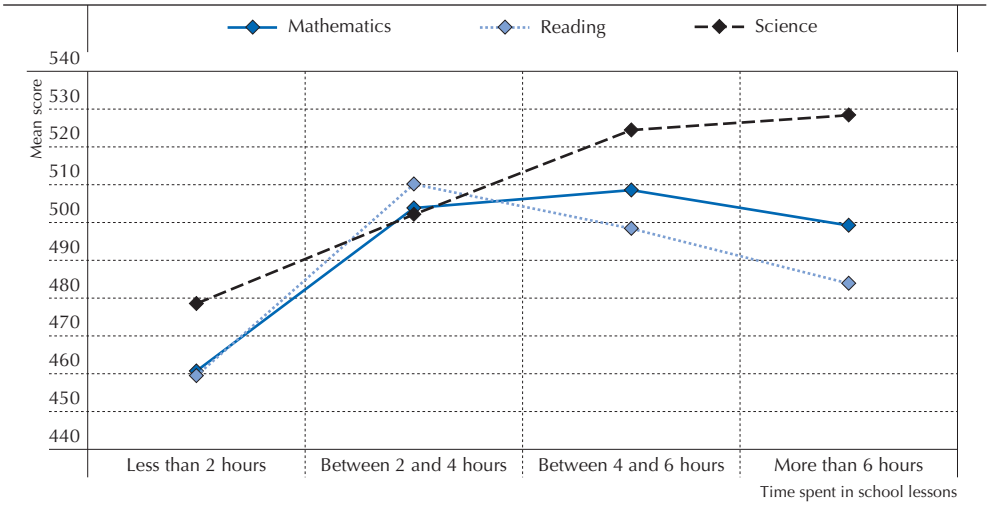
On average across OECD countries, learning time in regular mathematics lessons is positively correlated to student performance, even after accounting for various student and school characteristics, including socio-economic status (OECD, 2013b, Table IV.1.12c). However, Figure 3.4 shows that this relationship is not linear. An increase in class time of up to four hours per week is associated with a large improvement in performance in the three PISA subjects. After that threshold, longer instruction time is associated with smaller improvements in science performance and a deterioration in reading performance. More than six hours per week of class time is also associated with a slight deterioration in mathematics performance. PISA 2006 data also show that performance in mathematics and reading starts deteriorating moderately after six or more hours of instruction per week (OECD, 2011). One possible explanation for the difference across the three subjects is that the students who spend a long time in regular school science lessons choose to do so in enrichment courses, because they are interested in science and attend schools with the resources and facilities to offer such courses, while students who spend a long time in regular school mathematics or language-of-instruction lessons are obliged to do so for remedial purposes.

Besides remedial education, other school- and student-level differences can affect the time-achievement relationship. For instance, better performing students may be more likely to be enrolled in academic school tracks, to receive higher-quality instruction, and to be sorted into better classroom or school environments where they also receive longer instruction time, thus making it difficult to say whether longer time increases performance or whether better performing students receive longer instruction for other reasons. A number of studies have attempted to pin down the causal effect of instruction time on achievement by accounting for characteristics of students and schools (Lavy, 2015; Rivkin and Schiman, 2015) or by looking for variations in learning time not attributable to students', parents' or schools' behaviour, such as those related to school reforms or to unscheduled school closing due to snow (Bellei, 2009; Lavy, 2012;

■ Figure 3.4 ■

Relationship between performance and time spent in school lessons

OECD average



Note: The OECD average performance in each subject is calculated only for countries with a valid score across all four time brackets.
Source: OECD, PISA 2012 Database, Table 3.4a.
StatLink *http://dx.doi.org/10.1787/888933377309*

Marcotte, 2007; Marcotte and Hemelt, 2008; Pischke, 2007). They have generally found a positive relationship between instruction time and performance.

Following an approach similar to that of Rivkin and Schiman (2015), the analysis reflected in Figure 3.5 tries to isolate the link between instruction time and performance by showing how the score-point difference between mathematics and reading performance changes when the difference in instruction time for the two subjects increases by one hour.[4] These estimates are based on differences in time and performance between subjects among students enrolled in the same school and grade, so as to reduce possible influences coming from the fact that better performing students get sorted into schools and grades providing longer instruction time in mathematics.

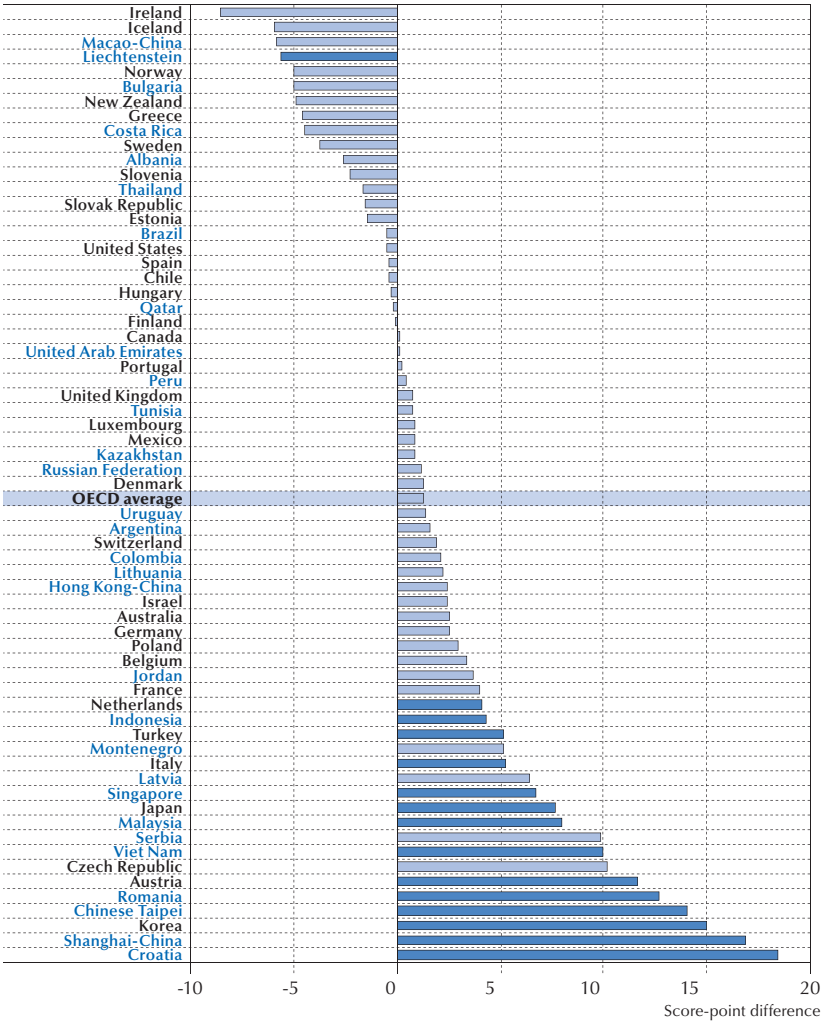
Figure 3.5 shows that mathematics performance improves compared with reading performance when the difference in instruction hours between mathematics and reading increases by one hour (that is, when students receive one hour more of instruction in mathematics per week than instruction in reading). This is observed in Austria (an improvement of 12 score points), Croatia (18 points), Indonesia (4 points), Italy (5 points), Japan (8 points), Korea (15 points), Malaysia (8 points), the Netherlands (4 points), Romania (13 points), Shanghai-China (17 points), Singapore (7 points), Chinese Taipei (14 points), Turkey (5 points) and Viet Nam (10 points).

Overall, this analysis of the relationship between instruction time and performance suggests that re-allocating time across subjects (moderately) increases students' performance in PISA only in a minority of countries and economies but that it does not automatically affect performance

■ Figure 3.5 ■

Mathematics performance and instruction time, after accounting for school characteristics

Score-point difference between mathematics and reading performance associated with a one-hour difference between mathematics and reading instruction time



Notes: The chart shows how the score-point difference between mathematics and reading performance changes when the difference in the amount of time devoted to mathematics with respect to reading instruction increases by one hour. In the Netherlands, for example, student performance in mathematics improves by four points compared with reading performance if the difference between the hours of mathematics and the hours of reading classes increases by one hour. The differences in performance and instruction time are calculated as averages for students in the same school and grade, and account for the observable and unobservable characteristics of schools that might influence the relationship between hours of instruction and student performance.

Statistically significant score-point differences are marked in a darker tone.

Countries and economies are ranked in ascending order of the effect of an additional hour of mathematics instruction on performance in mathematics compared with reading performance.

Source: OECD, PISA 2012 Database, Table 3.5.

StatLink http://dx.doi.org/10.1787/888933377313

on a large scale. In other words, the relationship between instruction time and performance turns out to be relatively weak after taking into consideration that better schools provide more instruction time. These results are not surprising considering that the PISA questionnaire provides information on the instruction time allocated to subjects, but not on the time that students spend engaged in learning.

If the quality of time spent learning in the classroom is poor, longer instruction time will not translate into greater opportunity to learn (Gromada and Shewbridge, 2016). More positive classroom environments – better student behaviour and good teacher-student relations – appear to augment the benefit of additional instruction time (Rivkin and Schiman, 2015). Figure 3.6 shows the distribution of mathematics performance and of the *index of disciplinary climate* by the time per week students spend in mathematics class.[5] In Korea, students who are exposed to mathematics for a longer time score higher and enjoy a positive learning climate. This suggests that good class discipline allows long instruction hours to be more productive. By contrast, the longer the time students in Switzerland spend in mathematics classes, the more they report poor performance and a poor classroom climate, suggesting that low-performing students spend more hours in the classroom, but these extra hours may fail to improve their performance because time is lost in noise and disorder.

In addition to regular classes, students are increasingly offered the opportunity to attend programmes providing additional instruction in school subjects outside of regular classes (Kidron and Lindsay, 2014). In PISA 2012, students reported information about the time they spend in after-school lessons offered at their school, at their home or somewhere else. On average across OECD countries, 38% of students reported that they attend after-school lessons in mathematics, 27% attend after-school lessons in the language of instruction, and 26% attend such lessons in science (OECD, 2013b: Table IV.3.25).

For all subjects, there is a negative correlation between performance and the time spent studying after school, on average across OECD countries (Figure 3.7). Again, this relationship should not be interpreted as causal, as low-performing students are more likely to participate in after-school remedial courses or personal tutoring. Japan and Korea are exceptions, as students in these countries who spend more hours in after-school mathematics lessons are also high performers in mathematics (Table 3.4b). In these two countries, after-school supplementary courses are often intended to help students master academic subjects, improve their performance, and ultimately earn good scores on high-stake tests, such as the competitive college entrance examination (Park, 2013).

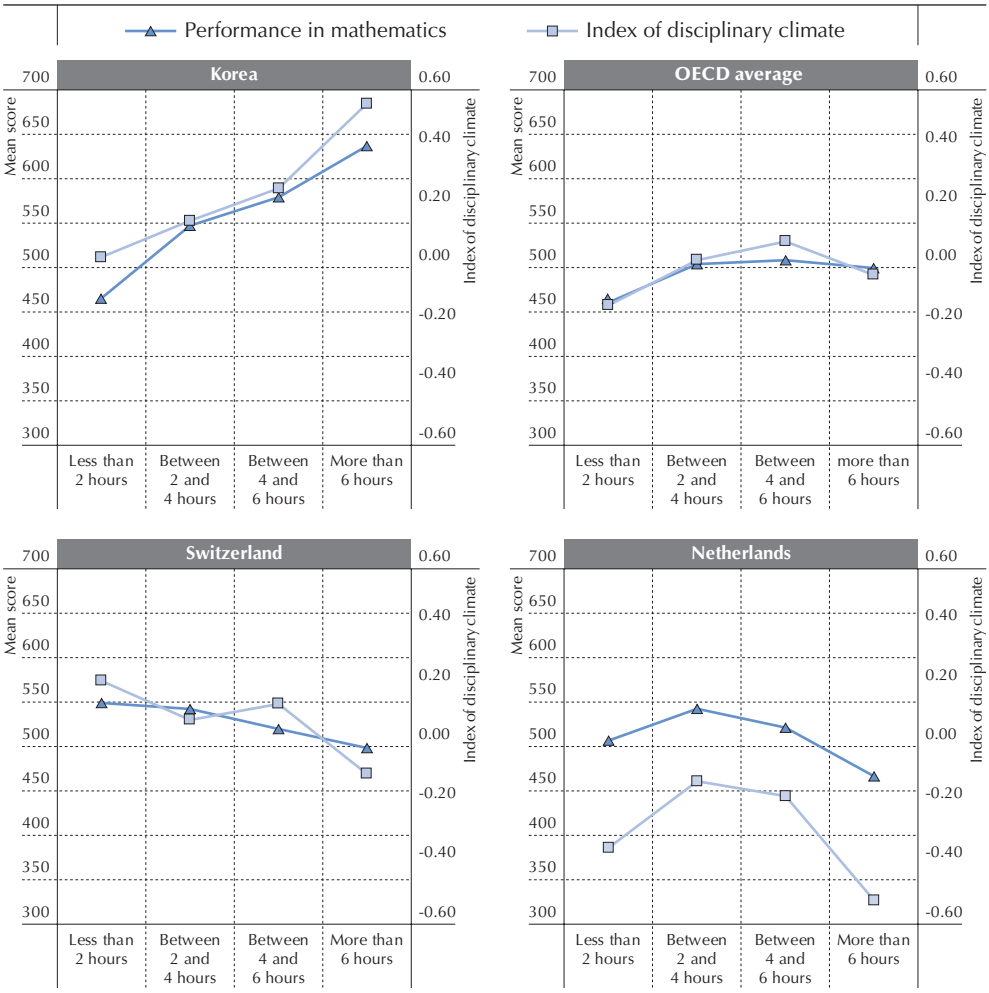
Variations in exposure to and familiarity with mathematics

Opportunity to learn refers not only to the time a student spends learning given content, but also, and more importantly, to the content taught in the classroom. International and country-level research has highlighted a positive association between content coverage and achievement in mathematics (Dumay and Dupriez, 2007; Rowan, Correnti, and Miller, 2002; Schmidt et al., 2001; Schmidt et al., 2011) and science (Sousa and Armor, 2010). This section extends previous analyses of PISA 2012 data investigating the link between exposure to and familiarity with mathematics on the one hand and mathematics performance on the other (OECD, 2014; Schmidt, Zoido and Cogan, 2014; Schmidt et al., 2015).

■ Figure 3.6 ■

Time spent in mathematics lessons, performance in mathematics and disciplinary climate

In hours per week



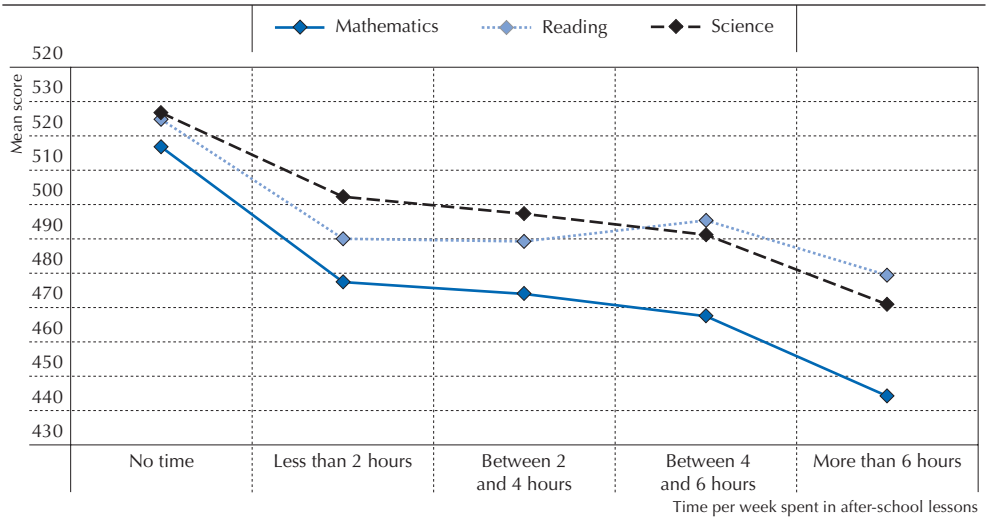
Notes: The *index of disciplinary climate* summarises students' reports on the frequency of noise, disorder and inactivity in the classroom due to disciplinary issues.
The OECD average of the *index of disciplinary climate* and of mathematics performance is calculated only for countries with a valid score across all four time brackets.
Source: OECD, PISA 2012 Database, Tables 3.4a and 3.6.
StatLink http://dx.doi.org/10.1787/888933377320

PISA data show that students' mathematics performance is positively associated with their exposure to pure and applied mathematics as well as with their familiarity with mathematics concepts.

■ Figure 3.7 ■

Relationship between performance and time spent in after-school lessons

In hours per week, OECD average



Notes: After-school lessons include lessons in subjects that students are also learning at school, on which they spend extra learning time outside of normal school hours. The lessons may be given at their school, at home or somewhere else.
The OECD average performance in each subject is calculated only for countries with a valid score across all the five time brackets.
Source: OECD, PISA 2012 Database, Table 3.4b.
StatLink *http://dx.doi.org/10.1787/888933377330*

First, Figure 3.8a shows that more frequent exposure to pure mathematics concepts is associated with better mathematics performance. On average across OECD countries, a one-unit increase in the *index of exposure to pure mathematics* is associated with an increase of 30 score points in mathematics performance. The link between exposure to pure mathematics and achievement is particularly strong in Korea, the Netherlands, New Zealand, Singapore and Chinese Taipei, where a one-unit increase in exposure to pure mathematics is related to an increase of more than 40 score points in mathematics performance.

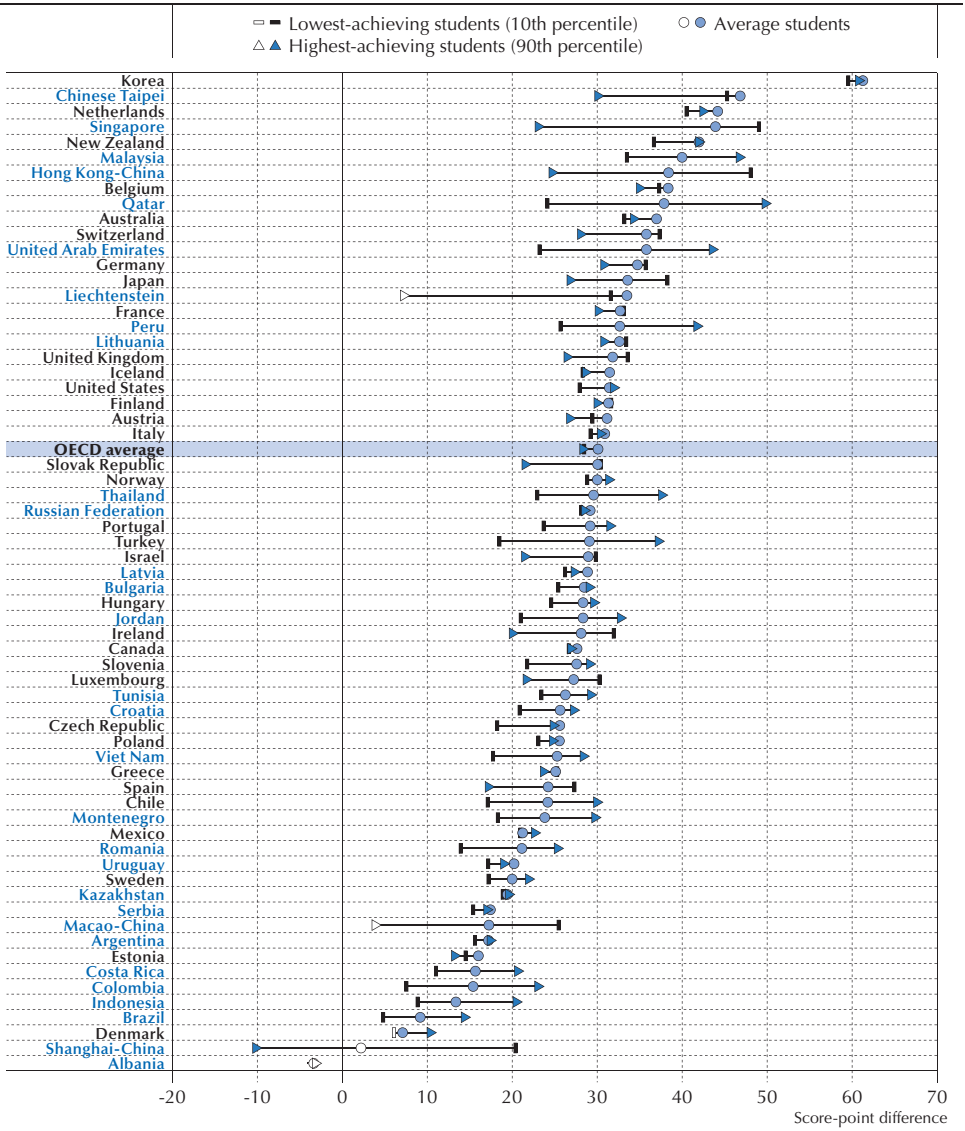
Second, more frequent exposure to applied mathematics is also related to mathematics performance in most countries (Figure 3.8b), even though the effect is weaker than that linked to exposure to pure mathematics. On average across OECD countries, a one-unit increase in the *index of exposure to applied mathematics* is associated with an increase of about 9 score points in mathematics performance. The effect is strongest in Australia, Finland, Japan, Korea, New Zealand, Chinese Taipei and the United Kingdom (more than 20 score points) while it is negative in Greece, Shanghai-China, the Slovak Republic, Spain, Turkey and Uruguay.

The association between exposure to applied mathematics and mathematics performance is weaker than that between pure mathematics and mathematics performance (or even negative). This may be due to reverse causality. The mathematics tasks PISA uses to measure exposure to

■ Figure 3.8a ■

Relationship between exposure to pure mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of exposure to pure mathematics

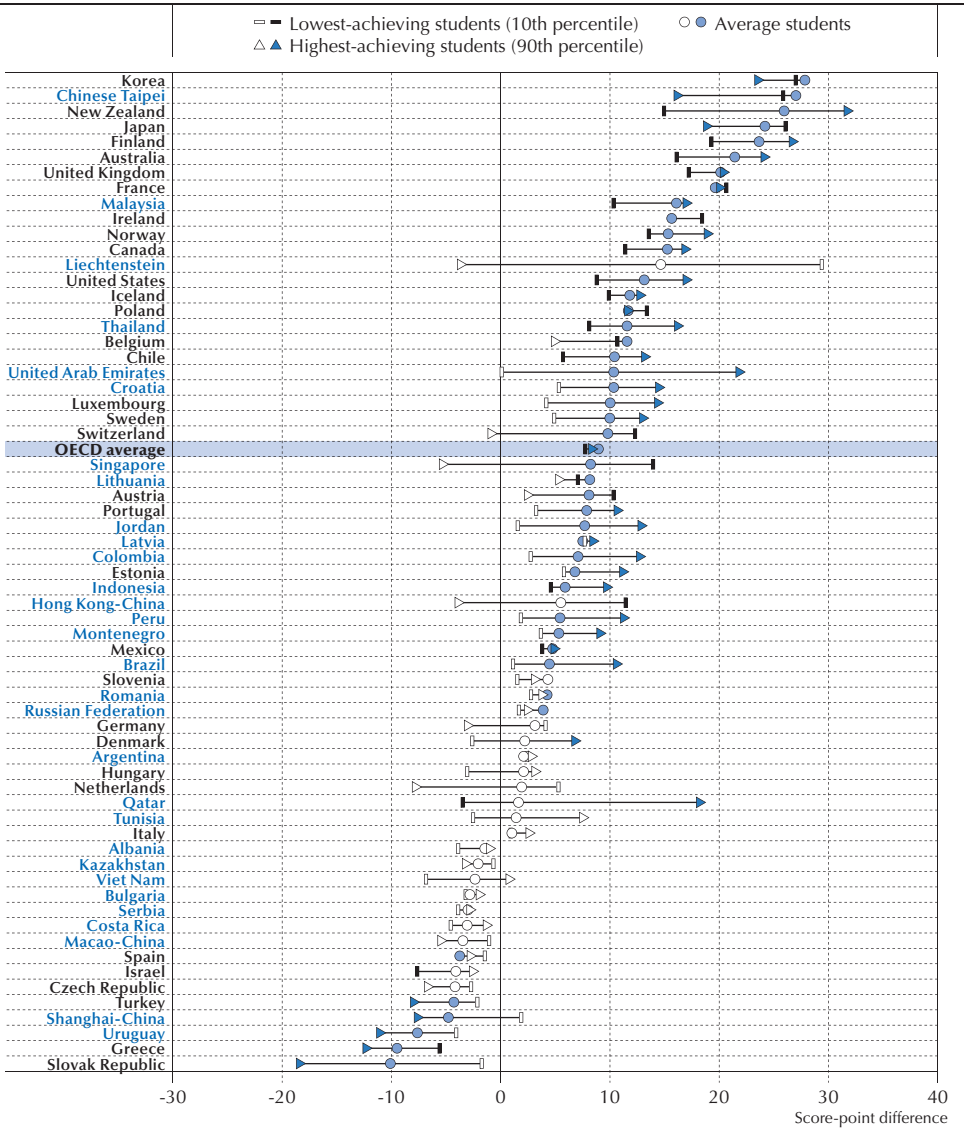


Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of exposure to pure mathematics *for the average students.*
Source: OECD, PISA 2012 Database, Table 3.7.
StatLink http://dx.doi.org/10.1787/888933377349

■ Figure 3.8b ■

Relationship between exposure to applied mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of exposure to applied mathematics



Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of exposure to applied mathematics *for the average students.*

Source: OECD, PISA 2012 Database, Table 3.7.

StatLink http://dx.doi.org/10.1787/888933377355

applied mathematics are relatively easy for 15-year-old students (e.g. working out from a train timetable how long it would take to get from one place to another), and low-performing students are more likely than higher-achieving students to have been exposed to this type of problem.

Third, mathematics performance is also related to greater familiarity with mathematics concepts (Figure 3.8c), the measure that better captures the cumulative opportunity to learn mathematics content over a student's career. On average across OECD countries, a one-unit increase in the *index of familiarity with mathematics* (equivalent to the difference between having heard of a series of mathematics concepts "often" and "a few times"; see Chapter 1) corresponds to a 41 score-point increase in mathematics performance. This association is stronger (an increase of more than 50 score points) in Australia, Korea, New Zealand and Chinese Taipei.

PISA asked students how frequently they are exposed to specific problems during mathematics lessons or assessments, including algebraic word problems, procedural tasks, contextualised mathematics problems, and pure mathematics problems (see Box 1.2 in Chapter 1 and questions at the end of Chapter 1 for some examples). Tables 3.8a-d show that students who are frequently exposed to these problems in mathematics classes perform better in mathematics than students who are never exposed to them. Across the four types of tasks, the relationship between exposure and performance is strongest for procedural tasks and weakest for contextualised mathematics tasks, on average across OECD countries. This confirms that exposure to pure mathematics is more closely related to performance in PISA than exposure to applied mathematics is. The results suggest that the use of real-life examples is not enough to transform routine problems into challenging mathematics problems that build mathematics literacy. Students – and low-achieving students in particular – might also have problems in transferring what they learn in a specific context to other contexts (see Box 1.3 in Chapter 1). When interpreting these results, it is important to keep in mind that it is much easier to measure exposure to pure mathematics than to measure exposure to applied and contextualised mathematics, given that applied mathematics problems are, by nature, more ambiguous and diverse.

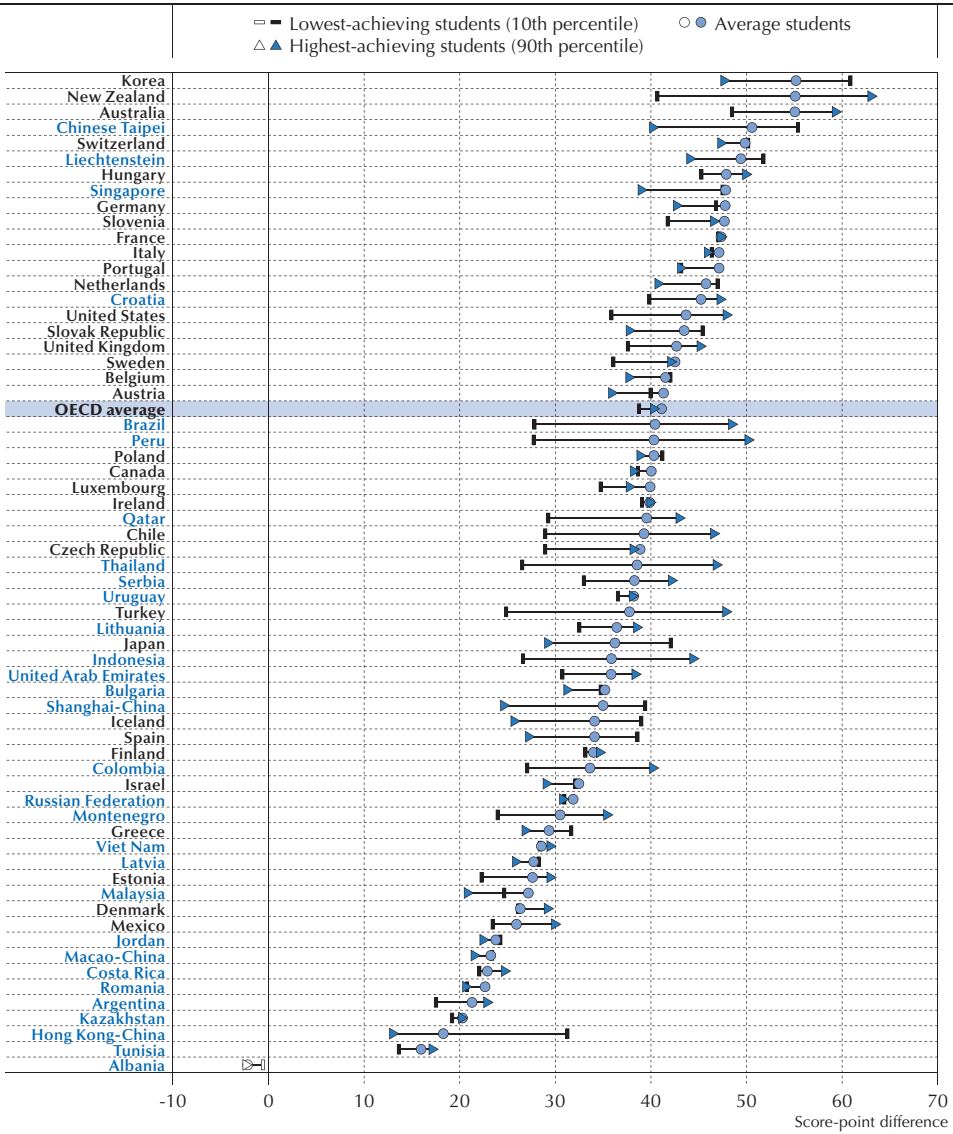
In most countries and economies, the association between opportunity to learn and mathematics performance is stronger among high-achieving students than among low-achieving students (Figures 3.8a, 3.8b and 3.8c). In Brazil, New Zealand, Peru, Thailand and Turkey, the effect of familiarity with mathematics on performance is more than 20 score points larger for the 10% of students with the highest scores than for the 10% of students with the lowest scores. In these countries, it is possible that high-achieving students can better profit from what they are taught but it may also be that they are exposed to a more advanced curriculum (Table 3.7).

But in a number of countries and economies, the reverse is true: the association between exposure to/familiarity with mathematics and mathematics performance is stronger among low achievers than among high achievers. For instance, in Hong Kong-China, Liechtenstein, Macao-China, Shanghai-China, Singapore and Chinese Taipei, the difference in performance related to frequent exposure to pure mathematics is at least 15 score points larger among low achievers than among high achievers (Figure 3.8a and Table 3.7). Perhaps, in these countries and economies, mathematics concepts are taught in such an accessible way that low-performing students benefit even more than high-performing students do, thus suggesting that the organisation of curriculum and teaching

■ Figure 3.8c ■

Relationship between familiarity with mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).
The OECD average for familiarity with mathematics concepts is thus calculated as the average of 33 countries.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of familiarity with mathematics *for the average students.*
Source: OECD, PISA 2012 Database, Table 3.7.
StatLink http://dx.doi.org/10.1787/888933377366

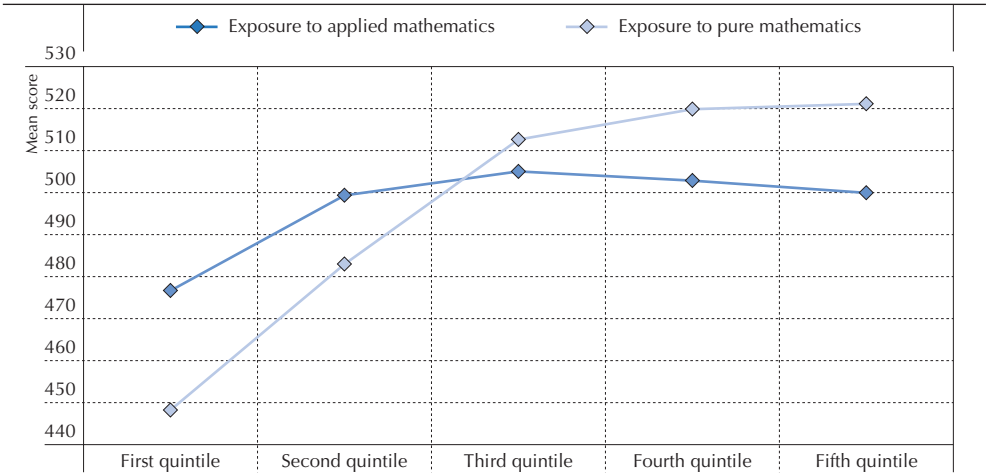
can help to narrow the achievement gap. Chapter 5 will discuss these policies and provide more examples.

The link between exposure to mathematics content and performance varies by the frequency of exposure. Figure 3.9 shows that more frequent exposure to pure mathematics is associated with smaller improvements in performance than less frequent exposure is. Figure 3.9 also shows that mathematics performance improves slightly between the first and the third quintiles of exposure to applied mathematics and decreases slightly among students who reported more frequent exposure (the fourth and fifth quintiles). Again, the slightly negative association between performance and a very frequent exposure to applied mathematics is unlikely to mean that greater exposure to applied mathematics reduces performance; it may rather come from the fact that the mathematics tasks PISA uses to measure exposure to applied mathematics are relatively easy and may be used to make mathematics accessible to low-performing students.

■ Figure 3.9 ■

Performance in mathematics, by exposure to applied and pure mathematics

OECD average



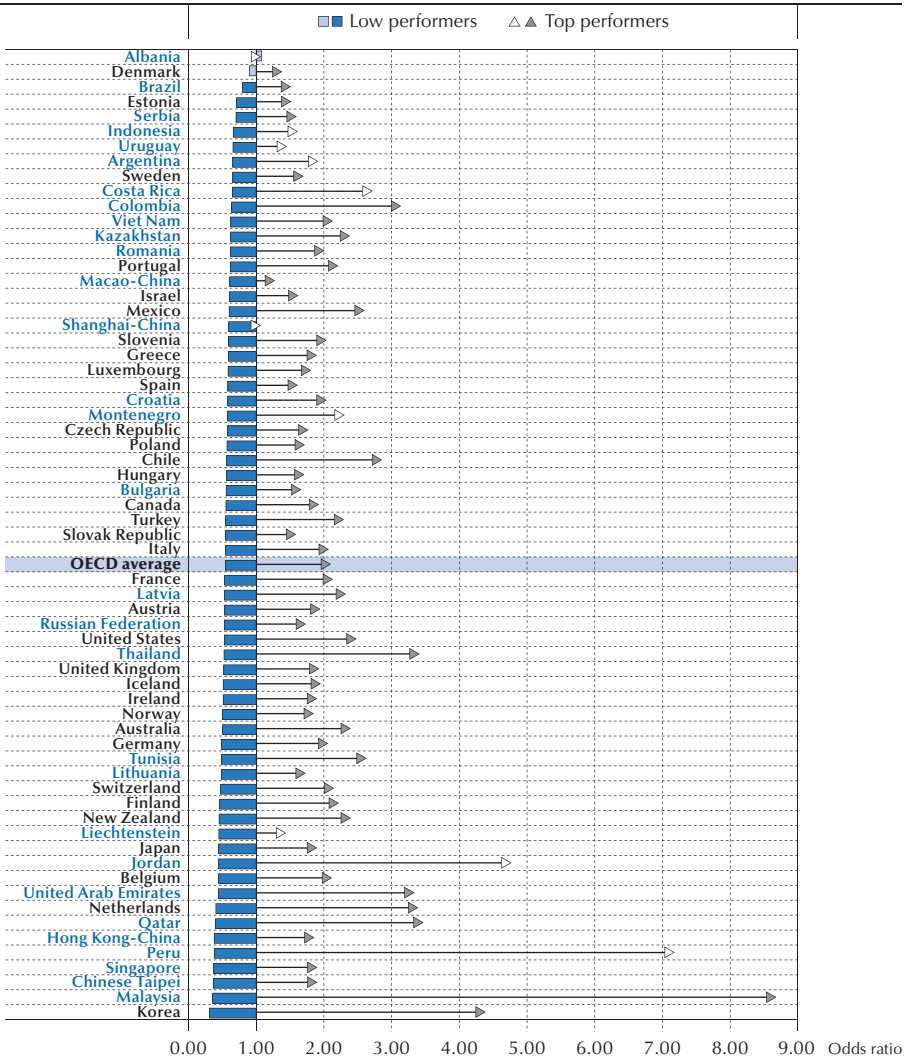
Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Source: OECD, PISA 2012 Database, Table 3.9.
StatLink http://dx.doi.org/10.1787/888933377377

Not only is exposure to mathematics associated with average performance, it is also related to a student's chances of being a low performer (that is, of performing at or below Level 1 on the PISA mathematics scale) or a top performer (that is, of performing at or above Level 5). Figure 3.10 shows that, for an average student in an OECD country, a one-unit increase in the *index of exposure to pure mathematics* doubles the likelihood of being a top performer,

■ Figure 3.10 ■

Exposure to pure mathematics and the likelihood of top and low performance

Change in the likelihood of low and top performance associated with a one-unit change in the index of exposure to pure mathematics



How to read the chart: An *odds ratio* of two for top-performance means that a one-unit increase in the *index of exposure to pure mathematics* doubles the probability that the student is a top performer in mathematics. Similarly, an odds ratio of 0.5 for low performance means that a one-unit increase in the *index of exposure to pure mathematics* reduces the probability of low performance by half.

Notes: Low performers are students who score below proficiency Level 2. Top performers are students who score at or above proficiency Level 5.

Values that are statistically significant are marked in a darker tone.

Countries and economies are ranked in descending order of the effect of exposure to pure mathematics on the likelihood of low performance.

Source: OECD, PISA 2012 Database, Table 3.10.

StatLink http://dx.doi.org/10.1787/888933377389

and reduces by half the probability that this student is a low performer. In Colombia, Korea, Malaysia, the Netherlands, Qatar, Thailand and the United Arab Emirates, a one-unit increase in the *index of exposure to pure mathematics* triples the likelihood that a student is a top performer.

The positive association between exposure to mathematics and performance in PISA does not necessarily indicate a causal link, as the direction of causality is not clear. Greater exposure to mathematics may improve performance, but at the same time, students with greater mathematical ability and motivation may choose – or be sorted into – schools that offer them greater exposure to mathematics.

This causality problem can be mitigated by analysing how different levels of exposure to mathematics influence the performance of students who attend the same school, as this excludes school factors that might influence the relationship. PISA randomly selects students of a given age within each school, so for the majority of PISA countries and economies it is possible to compare students who attend different grades within the same school. As shown in Chapter 1 (Figure 1.9), students in higher grades are more frequently exposed to pure mathematics. To investigate the relationship between exposure to pure mathematics and performance, the analysis will look at differences in exposure between students who attend different grades within the same school. This method accounts for differences across schools that might have an influence on the results presented so far (such as the fact that better performing students may choose schools that offer them more mathematics instruction) but does not account for ways in which students may be sorted within schools (for example, through ability grouping within the school).

Figure 3.11 shows that, across OECD countries, students who attend a higher grade have higher mathematics performance by 29 score points than students in the same school who attend a lower grade because they are more exposed to pure mathematics (by one index point).[6] Across students in the same school, the improvement in performance associated with greater exposure is larger than 50 score points in Korea, Luxembourg, Malaysia, Qatar and Spain. This implies that offering all students the opportunity to be exposed to a coherent curriculum does matter for mathematics performance.

Familiarity with mathematics and problem-solving skills

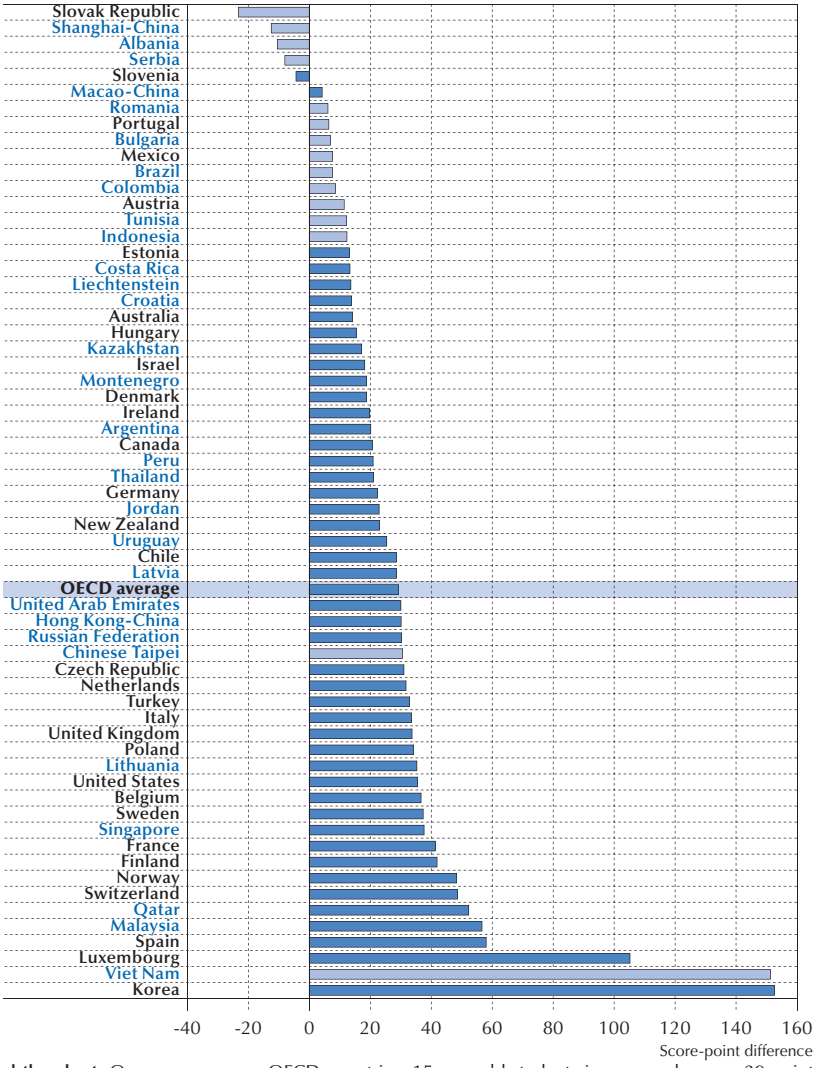
In order to determine how familiarity with mathematics is related to performance on demanding tasks, another analysis focuses on student performance on PISA tasks of various levels of difficulty or requiring different skills (Box 3.1 provides more details on analysis at the task level). Does greater exposure to and greater familiarity with mathematics mean that students will be equipped with all the skills they need to face complex mathematics problems?

Figure 3.12 shows that the greater students' familiarity with mathematics concepts, the more mathematics items (on the paper-based PISA assessment[7]) students answer correctly, on average across OECD countries. The association is stronger for more challenging tasks. For example, a one-unit increase in the *index of familiarity with mathematics* more than doubles (2.6 times) the likelihood that students answer correctly a difficult item like ARCHES Question 2 (difficulty of 785 on the PISA scale), but raises by only 1.5 times the likelihood of a correct response to an easy

■ Figure 3.11 ■

Difference in mathematics performance across grades related to exposure to pure mathematics

Score-point difference between grades in the same school associated with a one-unit difference in the index of exposure to pure mathematics



How to read the chart: On average across OECD countries, 15-year-old students in one grade score 29 points more in mathematics than students one grade below in the same school if the difference in the *index of exposure to pure mathematics* between the two grades is equal to one unit. The estimates can be interpreted as the effect of pure mathematics on perfomance after accounting for observable and unobservable differences across schools.

Notes: Only students in the modal grade and one grade below or above the modal grade are included in the analysis.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the score-point difference between grades.

Source: OECD, PISA 2012 Database, Table 3.11.

StatLink *http://dx.doi.org/10.1787/888933377391*

Box 3.1. **Analysis of performance in PISA at the task level**

One of the key strengths of PISA is that the assessment items (tasks) vary considerably in the type of response format, the contexts in which the problem sets are framed, and the type of content knowledge and cognitive processes that they aim to assess. The PISA mathematics tasks can be classified according to what is required of students, including their:

- knowledge of the mathematics **content areas** of *change and relationships*, *space and shape*, *quantity* and *uncertainty and data*. These overarching "big ideas" guide the conceptual understanding of traditional mathematics topics, such as algebra and functions, geometry and measurement.
- capacity to address problems framed in **contexts** dealing with personal life, the social environment in which students live, the world of work, or the use of mathematics in science and technology.
- capability to engage in the cognitive **processes** needed to cover the full cycle of mathematics modelling – formulate, employ and interpret.

PISA items are ranked according to their level of difficulty. The difficulty of the items is calculated after the test is conducted, using a scaling approach known as Item Response Theory, which estimates the difficulty of items and students' score on the test simultaneously. The lower the percentage of students who give the correct answer, the more difficult the item. For example, students with a score of 348 points have a 62% probability (this probability was chosen by the PISA consortium as part of the scoring design) of solving CHARTS Question 1 (see examples at the end of the chapter).[8] This item thus sits at 348 points on the PISA scale of difficulty. Analysing performance at the task level allows for identifying the relative strengths and weaknesses of countries or groups of students in particular areas and processes of mathematics. This section looks at how students perform across items at different levels of difficulty and focuses on four PISA tasks:

Description	Content	Process	Percentage of correct responses at the international level
CHARTS Question 1			
The students are shown a bar graph reporting the sales of CDs from four music bands. They need to identify and extract a data value from the chart to answer a simple question.	Uncertainty and data	Formulate	87%
DRIP RATE Question 1			
The task's stimulus explains a formula used by nurses to calculate the drip rate (in drops per minute) for infusions. The students need to interpret an equation linking four variables, and provide an explanation of the effect of a specified change to one variable on a second variable if all the other variables remain unchanged.	Change and relationships	Employ	22%

...

Description	Content	Process	Percentage of correct responses at the international level
REVOLVING DOOR Question 2			
The task describes a revolving door and presents diagrams that include information above the diameter and different positions of the door wings. The students are asked to calculate the maximum arc length that each door can have so that air never flows between entrance and exit. Students not only have to apply their knowledge of circle geometry (the formula of the circumference) but also engage in sophisticated reasoning to formulate a mathematical model. The task gives students no suggested approaches so that they have to invent their own strategies.	Space and shape	Formulate	3%
ARCHES Question 2			
The problem contains technical terms that students need to interpret in relation to a diagram. The students are asked to formulate a geometric model and apply their procedural knowledge of trigonometry or of the Pythagorean theorem to calculate a length.	Space and shape	Formulate	5%

item like CHARTS Question 1 (difficulty of 348). Students who solve ARCHES can interpret a text containing technical terms, and apply their procedural knowledge (trigonometry or Pythagorean theory) to calculate a length.

Knowing mathematics terminology, facts and procedures has a positive impact on overall performance and is even more valuable for solving more challenging problems. It may seem that if you want to improve students' ability to solve difficult mathematics problems, you may just extend the coverage of the mathematics curriculum and give students more time to practice their procedural skills. But that is only partly true. It takes more than content knowledge and practice to develop a good problem solver.

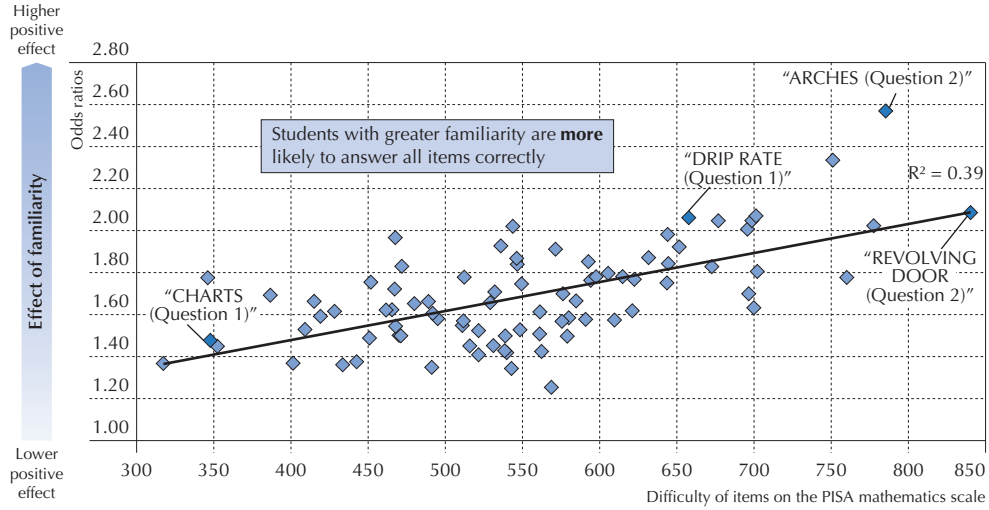
For example, compare the items DRIP RATE Question 1 and REVOLVING DOOR Question 2, both of which show strong associations between familiarity and correct answers. Both are difficult test questions (although not equally difficult) that require students to use their knowledge to solve new problems (see the full text of both tasks at the end of this chapter).

DRIP RATE Question 1 is a task at difficulty Level 5 that requires students to answer a question using a formula $\left(Drip\ rate = \frac{dv}{60n}\right)$ that is explicitly stated in the stimulus. REVOLVING DOOR Question 2 is the most difficult task in the assessment, lying at the upper end of Level 6. It asks students to engage in complex geometric reasoning and to perform calculations based on a formula that they should know but that is not recalled in the stimulus. The real problem of designing an efficient revolving door described in the item's stimulus needs to be translated

■ Figure 3.12 ■

Familiarity with mathematics and success on PISA items, by items' difficulty

OECD average (31 countries)



How to read the chart: Values greater than 1 on the vertical axis mean that a one-unit increase in the *index of familiarity with mathematics* increases the probability of answering a given question correctly.

Notes: The OECD countries included in the analysis are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, the Netherlands, New Zealand, Poland, Portugal, Slovenia, the Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The analysis only includes the items administered to those countries.

The difficulty of the item is set at a 0.62 threshold, meaning that a student who scores 600 in mathematics has a 62% chance of correctly answering an item with a difficulty level of 600.

Source: OECD, PISA 2012 Database, Table 3.12.

StatLink http://dx.doi.org/10.1787/888933377406

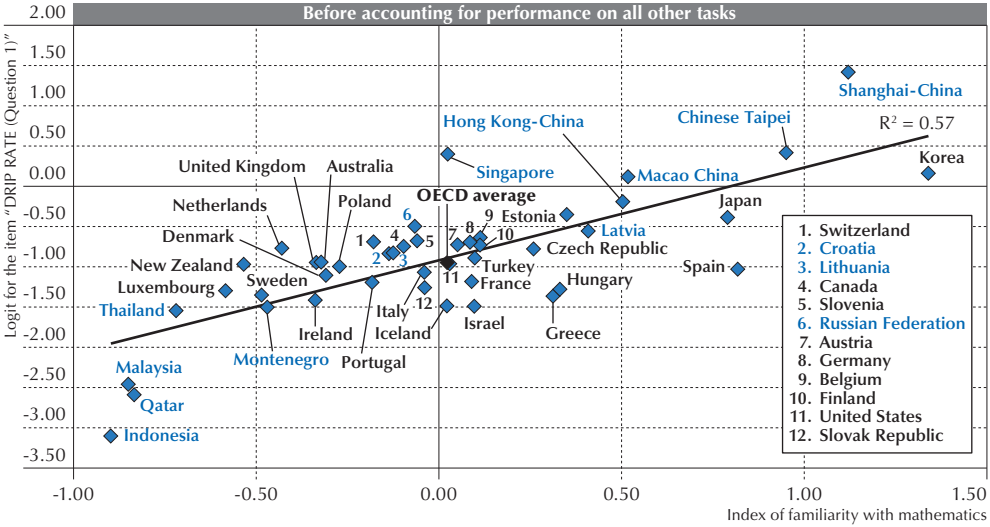
into geometrical terms and back again at multiple points. Only people who are highly skilled in mathematics have the capacity to model a real situation in a mathematical form.

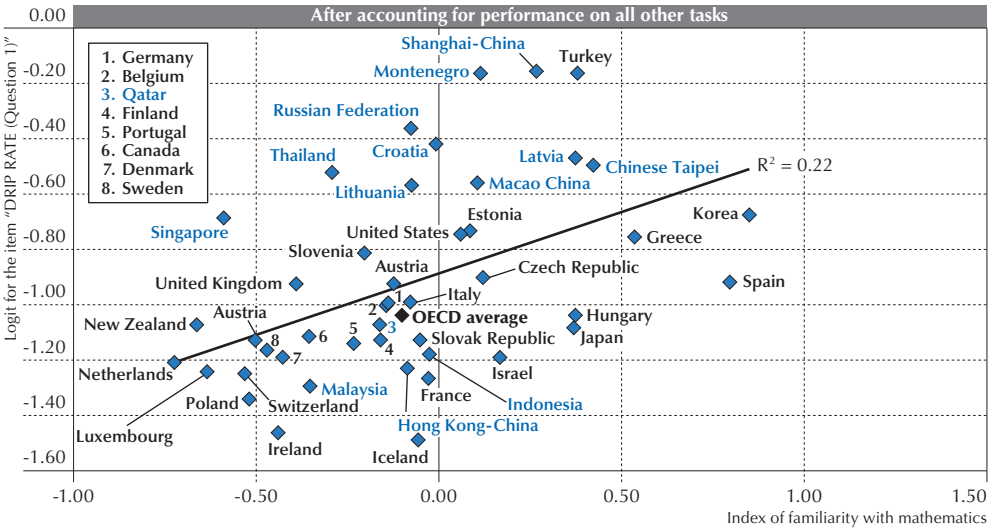
The top panel of Figure 3.13 shows that students with greater familiarity with mathematics are more likely to complete correctly a relatively difficult task, like DRIP RATE Question 1. Between-country differences in familiarity explain 57% of the variation in the correct response rate. This strong relationship between familiarity and performance on DRIP RATE might be due to a causal effect of familiarity on students' capacity to respond correctly, or to the fact that the countries where students perform better in PISA (because of the quality of the teachers, the motivations of students or parents, or other possible reasons) are also the countries with a more challenging mathematics curriculum. Looking at the relationship between solution rates to the DRIP RATE Question 1 task and familiarity *after accounting* for countries' performance on all the other mathematics tasks can help to clarify the direction of causality. Even after taking into account students' performance on all the other tasks in the PISA test, familiarity with mathematics is still positively associated with completing the task DRIP RATE correctly, and explains 22% of the system-level variation in the solution rate (bottom panel of Figure 3.13).

■ Figure 3.13 ■

Familiarity with mathematics and performance on a difficult mathematics task

Country average logit for the PISA Level 5 item "DRIP RATE (Question 1)"





Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
The OECD average is based on 31 countries with available data.
Source: OECD, PISA 2012 Database, Table 3.13.
StatLink http://dx.doi.org/10.1787/888933377412

This means that a higher familiarity with mathematics matters relatively more in explaining performance on a task that requires students to apply their procedural knowledge to a real setting than in other types of tasks.

Similarly, the top panel of Figure 3.14 shows that students in countries with greater familiarity with mathematics are more likely to answer correctly REVOLVING DOOR Question 2. But after accounting for performance on all the other tasks, a country's average level of familiarity with mathematics is not correlated with the percentage of students who answered correctly (bottom panel of Figure 3.14). After taking students' overall mathematics ability into account, greater familiarity is positively associated with the ability to answer the task DRIP RATE correctly but not the item REVOLVING DOOR Question 2, which requires students to engage in more advanced reasoning.

The different relationship between familiarity and performance in the two tasks, before and after accounting for students' overall performance, suggests that familiarity can improve performance in PISA, but only up to a point. The same analysis gives similar results when performed at the student level. After accounting for performance on all the other tasks, familiarity with mathematics is positively related to correct answers to DRIP RATE in 13 of 31 OECD countries and to REVOLVING DOOR in only in 4 of 31 OECD countries (Table 3.15). Frequent exposure to mathematics can make a difference to students trying to tackle problems like DRIP RATE, which states the main terms of the problem and requires students to apply procedures they learned at school. But familiarity with mathematics alone may not be sufficient for solving problems that require the ability to think and reason mathematically, like REVOLVING DOOR.

Developing competence and flexibility to solve demanding problems thus requires both a solid knowledge of mathematics content and extensive practice in searching for creative solutions to mathematics problems. Effective mathematics teachers cover the fundamental elements of the mathematics curriculum and still find the time to expose student to problems that promote conceptual understanding and activate students' cognitive abilities. Recognising mathematics problem solving as one of the ultimate goals of mathematics education, many countries are making specific efforts to develop higher-order thinking skills through the mathematics curriculum: see Box 3.2 for some examples.

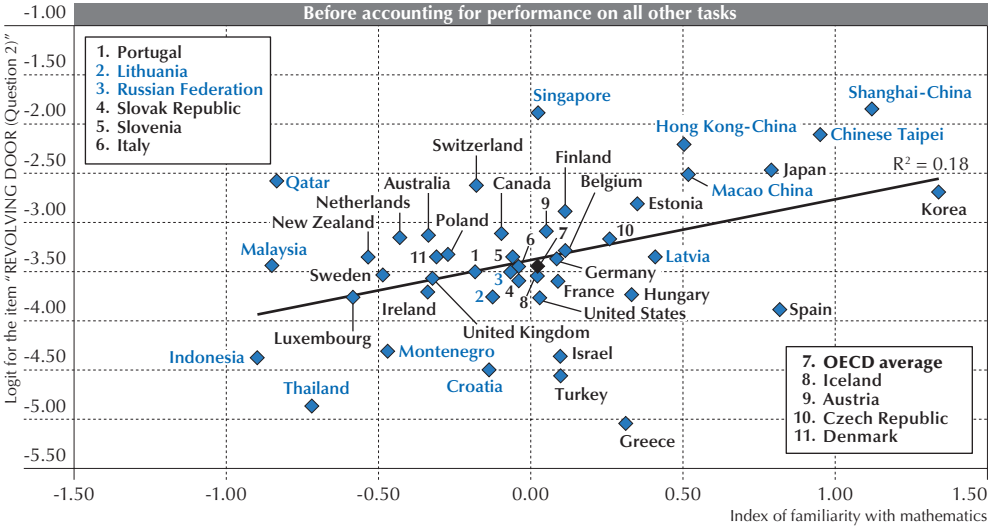
THE LINKS BETWEEN OPPORTUNITY TO LEARN, MATHEMATICS LITERACY AND SOCIO-ECONOMIC STATUS

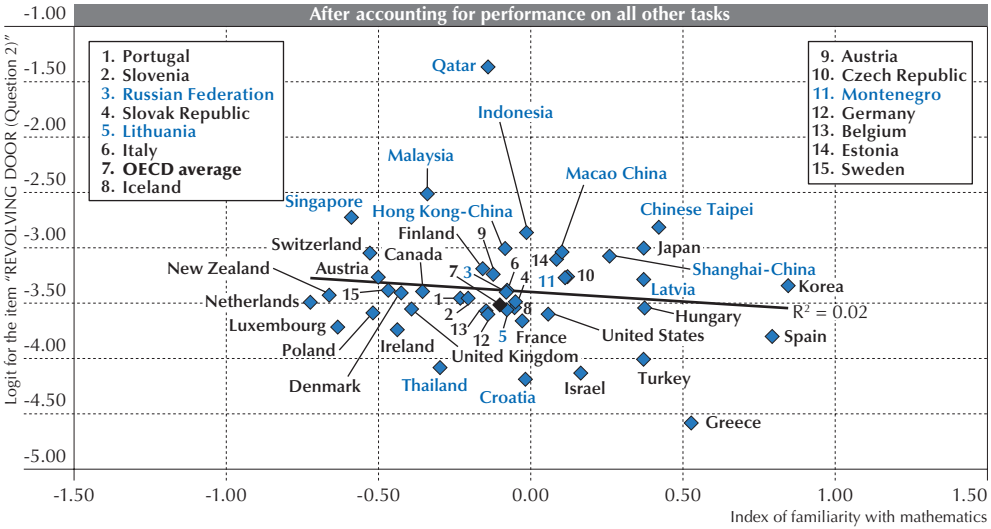
The analyses presented so far have shown a strong link between opportunity to learn and socio-economic status, and another strong link between opportunity to learn and performance in PISA. Putting the two stories together, how much of the performance gap related to socio-economic status can be explained by the frequency of exposure to mathematics? Figure 3.15 shows that around 19% of the performance difference between socio-economically advantaged and disadvantaged students can be attributed to differences in their familiarity with mathematics, on average across OECD countries (16% after taking into account other student and school characteristics). In Korea, the performance gap related to socio-economic status would be

■ Figure 3.14 ■

Familiarity with mathematics and performance on the most difficult mathematics task

Country average logit for the PISA Level 6 item "REVOLVING DOOR (Question 2)"





Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.

The OECD average is based on 31 countries with available data.

Source: OECD, PISA 2012 Database, Table 3.14.

StatLink http://dx.doi.org/10.1787/888933377428

reduced by 29 points (34% of the total) if disadvantaged students had the same familiarity with mathematics as advantaged students have (Table 3.16).

Box 3.2. **Integrating higher-order thinking skills in the mathematics curriculum**

In addition to covering relevant mathematics contents, a number of countries have recently reformed their mathematics curricula with a view to fostering students' higher-order thinking skills and problem-solving ability. A few examples are reported below.

The national curriculum for mathematics in **England** published in 2013 aims to ensure that all pupils not only become fluent in the fundamentals of mathematics, but also acquire the skills to reason mathematically and to solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication (Department for Education, England, 2013). Also the new Curriculum for Excellence (CfE) in **Scotland** emphasises the development of higher-order skills, such as thinking about complex issues, problem solving, analysis and evaluation; creativity; and critical-thinking skills – making judgements and decisions, developing arguments and solving complex problems (Education Scoltland, 2011).

In **Korea**, teaching and learning problem solving was part of the curriculum since the 1980s. The 2007 revision aimed at further engaging students in mathematical processes such as mathematical reasoning, problem solving, and communication. As a part of the 2007 revision of the lower secondary school curriculum, problem solving was integrated into all mathematics areas (Kim et al., 2012).

Mathematical problem solving is the central priority of **Singapore**'s mathematics framework introduced in the 1990s. The primary aim of the mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving, which includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself (Ginsburg et al., 2005; Ministry of Education, Singapore 2012). In addition, the Teach Less, Learn More (TLLM) initiative launched in 2003 aimed at reducing the curriculum content taught via direct teaching and engage students in more thinking and problem-solving tasks (Berinderjeet et al., 2009).

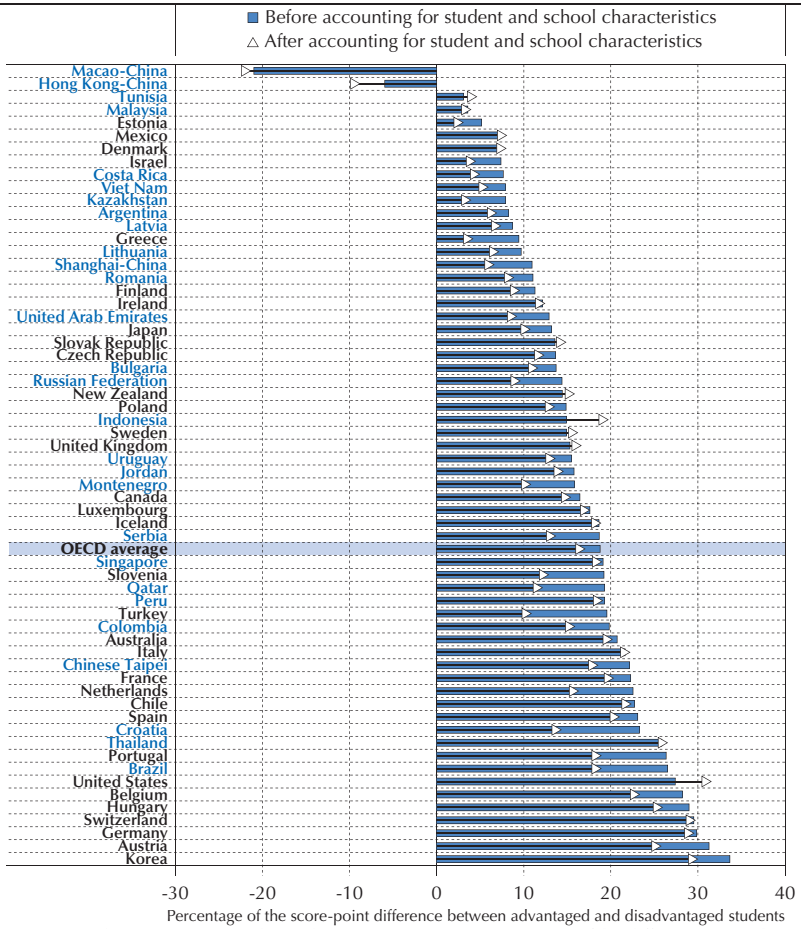
Again, looking at individual tasks in PISA can provide a more fine-grained picture of how opportunity to learn mediates the relationship between socio-economic status and mathematical literacy. Figure 3.16 shows that disadvantaged students lag behind other students across all items, but more so on the most difficult items. On average across OECD countries, a disadvantaged student can be expected to be 23% less likely to solve the easy item CHARTS Question 1 than the average student, but he or she is more than 70% less likely to solve REVOLVING DOOR Question 2.

Figure 3.16 also shows that when disadvantaged students' relative lack of familiarity with mathematics is taken into account, the performance gap related to socio-economic status narrows. But the effect varies, depending on the difficulty of the mathematics problem. For example, the effect is stronger on ARCHES Question 2, a task that mostly requires students to

■ Figure 3.15 ■

Differences in performance related to familiarity with mathematics, by socio-economic status

Percentage of the score-point difference between advantaged and disadvantaged students explained by different familiarity with mathematics



How to read the chart: The OECD average shows that across OECD countries, 19% of the difference in mathematics scores between advantaged and disadvantaged students is explained by disadvantaged students being less familiar with mathematics. This percentage decreases to 16% after accounting for student and school characteristics.

Notes: "Student and school characteristics" include: student's gender, mathematics learning time, whether student's country of birth is different from that in which the test was conducted, rural location of the school, private or public ownership of the school, academic or vocational track, school's selectivity, and indices of teacher support, use of cognitive-activation strategies and disciplinary climate.

Socio-economically advantaged students are defined as those students in the top quarter of the *PISA index of economic, social and cultural status* (ESCS). Disadvantaged students are students in the bottom quarter of ESCS.

In Hong Kong-China and Macao-China, the percentage is negative because disadvantaged students reported greater familiarity with mathematics than advantaged ones. In these economies, eliminating the difference in familiarity would increase the performance gap between advantaged/disadvantaged students.

Countries and economies are ranked in ascending order of the percentage of the performance gap between advantaged and disadvantaged students explained by familiarity with mathematics, before accounting for school characteristics.

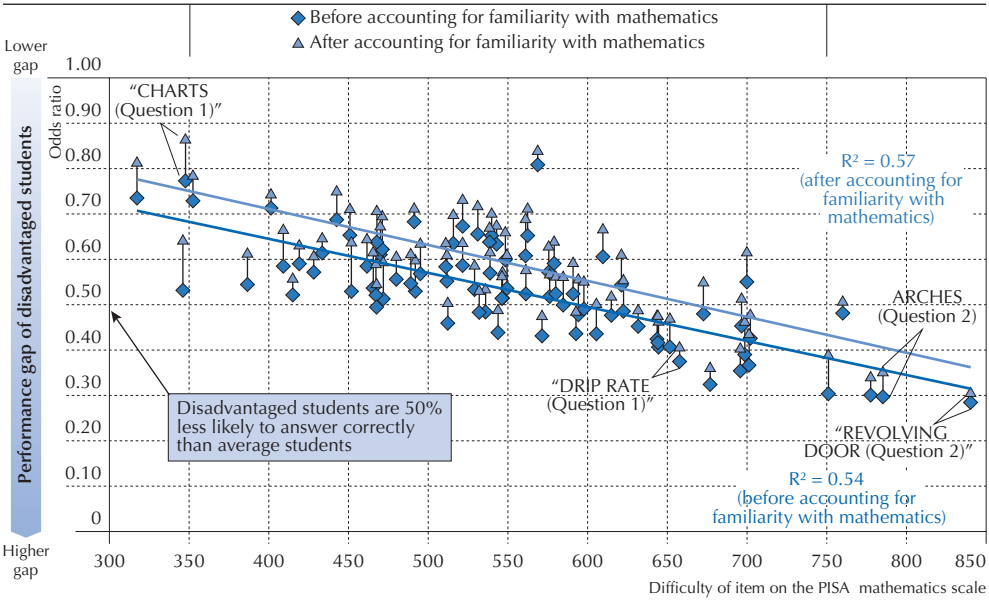
Source: OECD, PISA 2012 Database, Table 3.16.

StatLink http://dx.doi.org/10.1787/888933377436

■ Figure 3.16 ■

Socio-economic status and mathematics performance, by item difficulty

Change in the probability of answering an item correctly associated with socio-economic disadvantage



How to read the chart: A value of 1 on the vertical axis means that disadvantaged students have the same likelihood of answering the item correctly as the average student, while a value of 0.5 means they are 50% less likely to answer correctly. For each item, the distance between the triangle and the diamond reflects the effect of disadvantaged students' lesser familiarity with mathematics on the performance gap between disadvantaged and average students.

Notes: The OECD countries included in the analysis are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, the Netherlands, New Zealand, Poland, Portugal, Slovenia, the Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The analysis only includes the items administered to those countries.

The difficulty of the item is set at a 0.62 threshold, meaning that a student who scores 600 in PISA mathematics has a 62% chance of answering correctly an item with a difficulty level of 600.

Source: OECD, PISA 2012 Database, Table 3.17.

StatLink http://dx.doi.org/10.1787/888933377448

apply procedural knowledge, than on REVOLVING DOOR Question 2, a task that requires students to use of a broader set of mathematics skills.

Which mathematics skills do disadvantaged students have fewer opportunities to develop at school? The PISA mathematics framework defines a set of mathematics competencies that students need to have in order to make use of their knowledge in a variety of contexts (Box 3.3; OECD, 2013a). There is a substantial overlap among these competencies; indeed it is usually necessary to draw on several of them at once to solve a challenging problem.

Box 3.3. **Fundamental mathematical competencies**

The mathematics assessment framework describes a set of fundamental mathematical competencies needed by individuals to make use of their mathematics knowledge and skills (OECD, 2013a). These capabilities are derived from the mathematical competencies described in previous research (Niss and Højgaard, 2011). The framework for the 2012 PISA survey defines the following capabilities:

- **Communication:** This capability involves reading, decoding and interpreting statements, questions, tasks or objects enabling the individual to form a mental model of the situation, as an important step in understanding, clarifying and formulating a problem. Communication may also involve presenting and explaining one's mathematical work or reasoning.
- **Mathematising:** Mathematical literacy can involve transforming a problem defined in the real world to a strictly mathematical form (which can include structuring, conceptualising, making assumptions, and/or formulating a model), or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem.
- **Representation:** This can entail selecting, interpreting, translating between, and using a variety of representations to capture a situation, interact with a problem, or to present one's work. Representations may include graphs, tables, diagrams, pictures, equations, formulae and concrete materials.
- **Reasoning and argument:** This capability involves logically rooted thought processes that explore and link problem elements so as to make inferences from them, check a justification that is given or provide a justification of statements or solutions to problems.
- **Devising strategies for solving problems:** This involves a set of critical control processes that guide an individual to effectively recognise, formulate and solve problems. This skill is characterised as selecting or devising a plan or strategy to use mathematics to solve problems, as well as guiding its implementation.
- **Using symbolic, formal and technical language and operations:** This involves understanding, interpreting, manipulating, and making use of symbolic expressions within a mathematics context (including arithmetic expressions and operations), as well as understanding and using formal constructs based on definitions, rules and formal systems, and using algorithms with these entities.

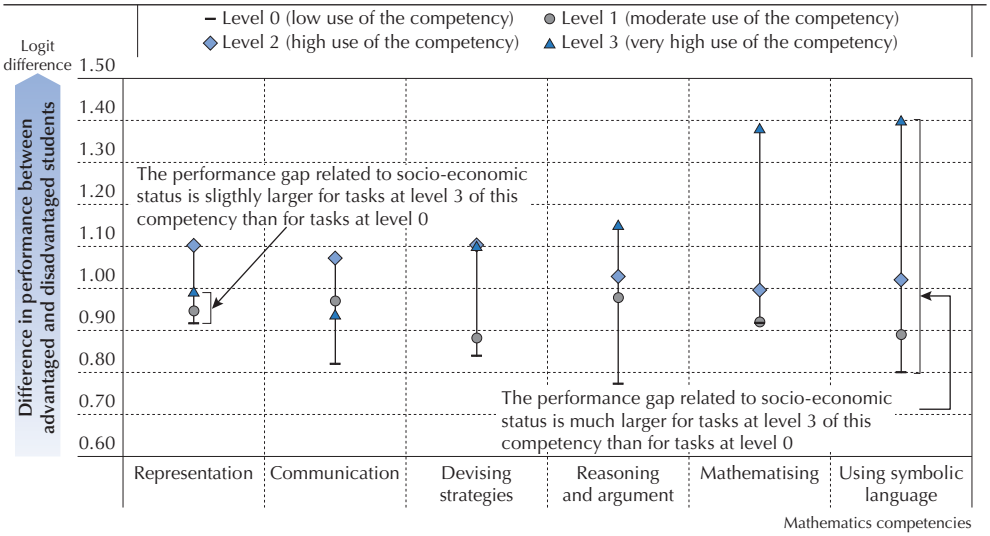
Experts involved in PISA implementation analysed PISA mathematics survey questions and judged the extent to which successfully answering those questions demanded the activation of six mathematical competencies mentioned in the PISA framework. The study involved the operational definition of these competencies and the description of four levels of each competency, recognising some degree of overlapping and interaction across competencies (Turner, 2012). An empirical validation of this classification found that the set of six competencies could predict more than 70% of the variability in item difficulty (Turner and Adams, 2012).

Mathematics experts who were involved in the development of PISA classified the mathematics items according to the type and level of competencies they require (Turner, 2012). Figure 3.17 shows that the performance gap between advantaged and disadvantaged students is significantly wider for those tasks that require greater use of two fundamental competencies: "using symbolic, formal and technical language and operations", and "mathematising", defined as the ability to construct a mathematical model from a real situation, finding a mathematical solution, and interpreting and validating the solution. As Chapter 5 will further discuss, these results suggest that, if the performance gap related to socio-economic status is to be fully closed, disadvantaged students would benefit not only from any policy that increases opportunities for them to develop technical and procedural mathematics skills, but also from more experience with mathematical modelling and using symbolic language.

■ Figure 3.17 ■

Socio-economic status and success on PISA mathematics tasks, by required mathematics competencies

Logit differences between advantaged and disadvantaged students according to the level of competency required by the task, OECD average

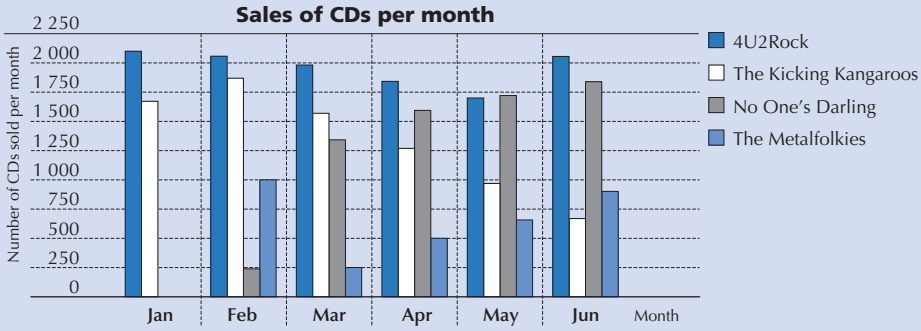


Notes: Socio-economically advantaged students are defined as those in the top quarter of the *PISA index of economic, social and cultural status* (ESCS). Disadvantaged students are those in the bottom quarter of ESCS.
The classification of PISA mathematics items by type and level of competency required is drawn from Turner (2012).
A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
All values are statistically significant.
Source: OECD, PISA 2012 Database, Table 3.18.
StatLink http://dx.doi.org/10.1787/888933377455

EXAMPLES OF PISA MATHEMATICS UNITS

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



CHARTS – *QUESTION 1*

How many CDs did the band *The Metalfolkies* sell in April?

A. 250

B. 500

C. 1 000

D. 1 270

Scoring

Description: Read a bar chart
Mathematical content area: Uncertainty and data
Context: Societal
Process: Interpret

Full Credit

B. 500

No Credit

Other responses.
Missing.

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \dfrac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

DRIP RATE – *QUESTION 1*

A nurse wants to double the time an infusion runs for.

Describe precisely how D changes if n is **doubled** but d and v do not change.

.....

.....

.....

Scoring

Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant
Mathematical content area: Change and relationships
Context: Occupational
Process: Employ

Full Credit

Explanation describes both the direction of the effect and its size.

- It halves
- It is half
- D will be 50% smaller
- D will be half as big

Partial Credit

A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.

- D gets smaller *[no size]*
- There's a 50% change *[no direction]*
- D gets bigger by 50% *[incorrect direction but correct size]*

No Credit

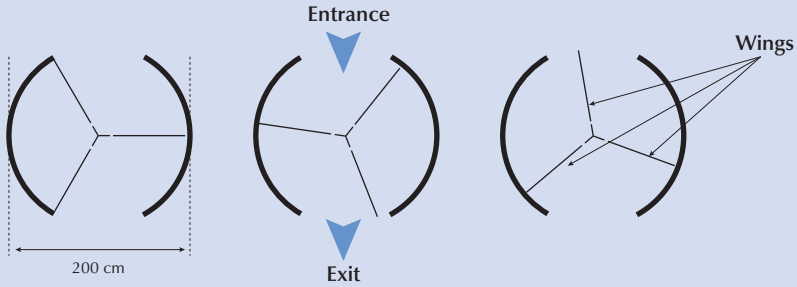
Other responses.

- D will also double *[both the size and direction are incorrect.]*

Missing.

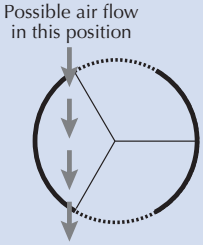
REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



REVOLVING DOOR – *QUESTION 2*

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .....cm

Scoring

Description: Interpret a geometrical model of a real life situation to calculate the length of an arc
Mathematical content area: Space and shape
Context: Scientific
Process: Formulate

Full Credit

Answers in the range from 103 to 105. *[Accept answers calculated as 1/6th of the circumference (100 π/3). Also accept an answer of 100 only if it is clear that this response resulted from using π = 3. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]*

No Credit

Other responses.

- 209 *[states the total size of the openings rather than the size of "each" opening]*

Missing.

Notes

1. This information is reported by representatives of countries/economies who responded to the TIMSS Eight Grade Curriculum Questionnaire. The full set of data is available at http://timss.bc.edu/timss2011/international-database.html.

2. The logit transformation of the percentage of students who responded correctly to the item takes into account the non-linear relationship between answering questions correctly and the items' difficulty (OECD, 2013c). A logit value of 0 means that 50% of respondents answered the question correctly; positive logits mean higher rates of correct answers and negative logits mean lower rates of correct responses.

3. Average difficulties by content area displayed in Figure 3.2 are based the following number of PISA test items: change and relationships: 8 items; quantity: 10 items; space and shape: 9 items; uncertainty and data: 7 items.

4. To understand how the results in Figure 3.5 are derived, consider the following two equations relating PISA average scores and learning time for students in each school and grade:

$$Score_{Mg} = \beta\ Hours_{Mg} + v_g + u_{Mg} \quad (1)$$

$$Score_{Rg} = \beta\ Hours_{Rg} + v_g + u_{Rg} \quad (2)$$

Where the subscripts *R* and *M* indicate students' averages in PISA reading and mathematics, respectively, the subscript *g* indicates the average for all students in the same grade within the same school, v_g represents characteristics of schools and grades that do not vary across subjects of instruction, and u is an error term.

Taking the difference of (1) and (2), gives:

$$\Delta Score_{Mg-Rg} = \hat{\beta}\ \Delta Hours_{Mg-Rg} + \Delta u_{Mg-Rg} \quad (3)$$

As can be seen, the first-difference (fixed-effect) regression in (3) estimates the relation between learning time and PISA scores ($\hat{\beta}$)accounting for subject-invariant differences across schools and grades (the term v_g gets cancelled out when differencing equations 1 and 2).

5. The *index of disciplinary climate* summarises students' reports on the frequency of noise, disorder and inactivity due to disciplinary issues in the classroom.

6. The results in Figure 3.11 are derived similarly to those shown in Figure 3.5 (see previous endnote). Consider the following two equations relating PISA average scores and exposure to pure mathematics for students in two contiguous grades:

$$Score_{0S} = \beta\ Exposure_{0S} + v_S + u_{0S} \quad (1)$$

$$Score_{1S} = \beta\ Exposure_{1S} + v_S + u_{1S} \quad (2)$$

Where the subscripts *0* and *1* indicate averages for students attending school *s* in grades *0* and *1*, respectively, the subscript v_S represents characteristics of schools that do not vary across grades, and u is an error term.

Taking the difference of (2) and (1), gives:

$$\Delta Score_{1-0S} = \hat{\beta}\ \Delta Exposure_{1-0S} + \Delta u_{1-0S} \quad (3)$$

As can be seen, the first-difference (fixed-effect) regression in (3) estimates the relation between exposure to pure mathematics and PISA scores ($\hat{\beta}$) accounting for grade-invariant differences across schools (the term v_S gets cancelled out when differencing equations 1 and 2).

7. The item-level analysis presented in this chapter is restricted to paper-based items because these are common to the largest number of countries.

8. The item, ARCHES Question 2, is not included in the examples at the end of this chapter because it is not a released item.

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4

Opportunity to Learn and Students' Attitudes Towards Mathematics

This chapter explores the relationship between opportunity to learn and students' attitudes towards mathematics, including their interest in mathematics, mathematics self-concept and anxiety towards mathematics. Teaching practices that have an impact on students' self-concept towards mathematics, peer effects and parents' influence on their child's attitudes towards mathematics are also examined.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

"I don't expect, and I don't want, all children to find mathematics an engrossing study, or one that they want to devote themselves to either in school or in their lives. Only a few will find mathematics seductive enough to sustain a long-term engagement. But I would hope that all children could experience at a few moments in their careers...the power and excitement of mathematics...so that at the end of their formal education they at least know what it is like and whether it is an activity that has a place in their future."
– David Wheeler (1975, quoted in Cross, 2004)

Positive feelings towards mathematics are closely linked to the ability to solve problems. In fact, it is highly unlikely that mathematical literacy can be acquired and put into practice by someone who does not have some degree of self-confidence, curiosity, interest in and desire to practice and understand mathematics (OECD, 2010; OECD, 2013b).

If not everyone is born to become a mathematician, everyone needs to be able to reason mathematically. Without the correct mindset, a primary school student who has had few opportunities to play with numbers at home might struggle to understand the meaning of arithmetic operations; without arithmetic skills, this same student will have a hard time making sense of algebra. Stimulating positive emotions during mathematics lessons is key to helping students of all ages understand the reasoning behind mathematics, encouraging them to appreciate the value of mathematics, and allowing them to decide whether mathematics should have a central place in their future studies and careers.

What the data tell us

- On average across OECD countries in 2012, 38% of students reported that they study mathematics because they enjoy it and 53% of students reported that they are interested in the mathematics they do at school. More girls (65%) than boys (54%) reported that they often worry about mathematics classes, and disadvantaged students were more likely than advantaged students to believe they are just not good at mathematics.
- Exposure to more complex mathematics concepts, as measured by the *index of familiarity with mathematics*, is associated with lower self-concept and higher anxiety among students in the bottom quarter of mathematics performance, and with higher self-concept/lower anxiety among students in the top quarter of mathematics performance, on average across OECD countries.
- Having hard-working friends can increase mathematics self-concept, but students can develop lower beliefs in their own ability when they compare themselves to higher-achieving peers.
- On average across OECD countries, high-performing students whose parents do not like mathematics are 73% more likely to feel helpless when they face a mathematics problem than high-performing children of parents who like mathematics.
- Students whose mathematics teachers differentiate tasks according to students' abilities and who encourage students to work in small groups have higher mathematics self-concept than students whose teachers do not engage in these practices.

What these results mean for policy

- Mathematics curricula, and the teachers who follow them, should strike a balance between making the material more challenging and aiming to bolster students' – especially low-performing students' – confidence and reduce their anxiety towards mathematics.
- School leaders and teachers should make a careful use of competition and rankings within the classroom, because students' mathematics self-concept is strongly influenced by how their familiarity with mathematics compares with that of their peers.
- Parents should be made aware of their role in transmitting mathematics anxiety to their children and should help motivate them.
- Specific communication training can help teachers to provide more effective feedback to the students who are least familiar with mathematics.

Previous PISA analyses have shown that opportunity to learn mathematics (OTL) is positively related not only to students' performance, but also to positive attitudes towards mathematics. For example, students' confidence in solving specific pure and applied mathematics problems is closely linked to their exposure to similar sets of problems in the classroom (OECD, 2013b). This chapter extends these analyses by focusing on the relationship between OTL and three aspects of students' attitudes towards mathematics (see Box 4.1 for the definition and measurement of these constructs):

- students' intrinsic and instrumental motivation to learn mathematics
- students' mathematics self-concept
- students' mathematics anxiety.

Figure 4.1 illustrates this chapter's analysis of the relationship among exposure to mathematics, performance in mathematics and students' attitudes towards mathematics. Opportunities to practice mathematics in class are positively associated with mathematics performance (Chapter 3); through this channel, they also strengthen students' interest in mathematics, support their self-confidence and reduce anxiety towards mathematics. The type and level of difficulty of mathematics tasks can also be directly associated with students' attitudes, although the direction of the relationship is unclear. On the one hand, more exposure to challenging mathematics problems can stimulate the interest and raise the self-concept of students who are well-trained in mathematics reasoning. On the other hand, exposure to complex mathematics problems can increase pressure on students who have gaps in their learning.

This chapter also looks at other mediators of the relationship between OTL and students' attitudes towards mathematics. The characteristics of students' peers, parents' attitudes towards mathematics, and the quality of teachers and their teaching practices influence how exposure to mathematics in class affects students' drive, self-beliefs and anxiety.

Box 4.1. **Students' attitudes towards mathematics analysed in this chapter**

The Student Questionnaire that was part of the PISA 2012 assessment included 67 questions that allowed for the construction of ten indices about students' attitudes towards mathematics. This chapter analyses the following indices and indicators:

1) Students' intrinsic and instrumental motivation to learn mathematics

- Intrinsic motivation to learn mathematics (or mathematics interest) measures students' drive to perform an activity purely for the joy gained from the activity itself. The *index of mathematics interest* is based on the degree to which students "strongly agree", "agree", "disagree" and "strongly disagree" with the statements: a) I enjoy reading about mathematics; b) I look forward to my mathematics lessons; c) I do mathematics because I enjoy it; d) I am interested in the things I learn in mathematics.
- Instrumental motivation to learn mathematics refers to students' drive to learn mathematics because they perceive it to be useful to their future studies and careers. The *index of instrumental motivation to learn mathematics* is based on the degree to which students "strongly agree", "agree", "disagree" and "strongly disagree" with the statements: a) Making an effort in mathematics is worth it because it will help me in the work that I want to do later on; b) Learning mathematics is worthwhile for me because it will improve my career; c) Mathematics is an important subject for me because I need it for what I want to study later on; d) I will learn many things in mathematics that will help me get a job.

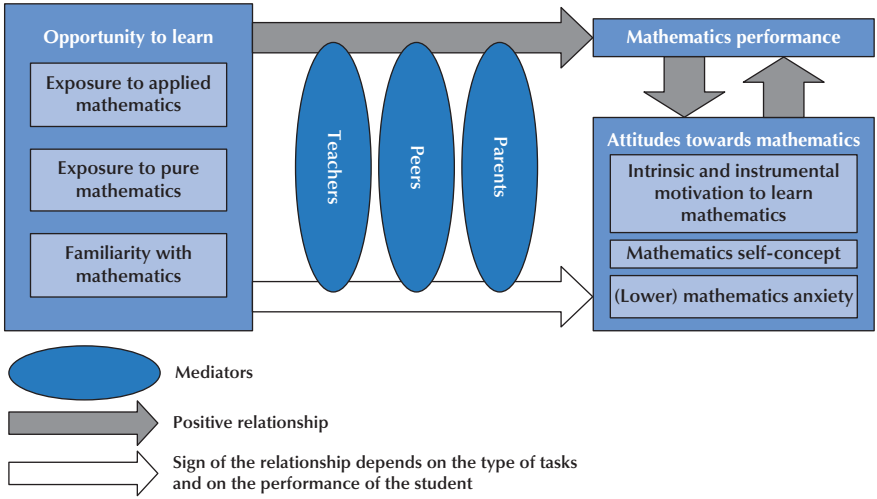
2) Mathematics self-concept measures students' beliefs in their own mathematics abilities. The *index of mathematics self-concept* is based on the degree to which students "strongly agree", "agree", "disagree" and "strongly disagree" with the statements: a) I am just not good at mathematics; b) I get good grades in mathematics; c) I learn mathematics quickly; d) I have always believed that mathematics is one of my best subjects; e) In my mathematics class, I understand even the most difficult work.

3) Mathematics anxiety measures the feelings of helplessness and stress that students can have when faced with a mathematics problem. The *index of mathematics anxiety* is based on the degree to which students "strongly agree", "agree", "disagree" and "strongly disagree" with the statements: a) I often worry that it will be difficult for me in mathematics classes; b) I get very tense when I have to do mathematics homework; c) I get very nervous doing mathematics problems; d) I feel helpless when doing a mathematics problem; e) I worry that I will get poor marks in mathematics.

Responses to all items were coded so that higher values correspond to a higher level of the construct. All indices were scaled so to have an OECD mean of zero and an OECD standard deviation of one.

■ Figure 4.1 ■

Direct and indirect relationship between opportunity to learn and attitudes towards mathematics



HOW STUDENTS' MOTIVATION AND SELF-BELIEFS VARY ACROSS COUNTRIES AND SUBGROUPS OF STUDENTS

All countries and economies are trying to find ways to spark students' interest in mathematics so that they are motivated to learn. Figure 4.2 shows that, on average across OECD countries in 2012, only 38% of students reported that they study mathematics because they enjoy it; in Austria, only 24% of students reported that they enjoy studying mathematics. By contrast, more than 70% of students in Indonesia and Thailand reported that they enjoy studying mathematics.

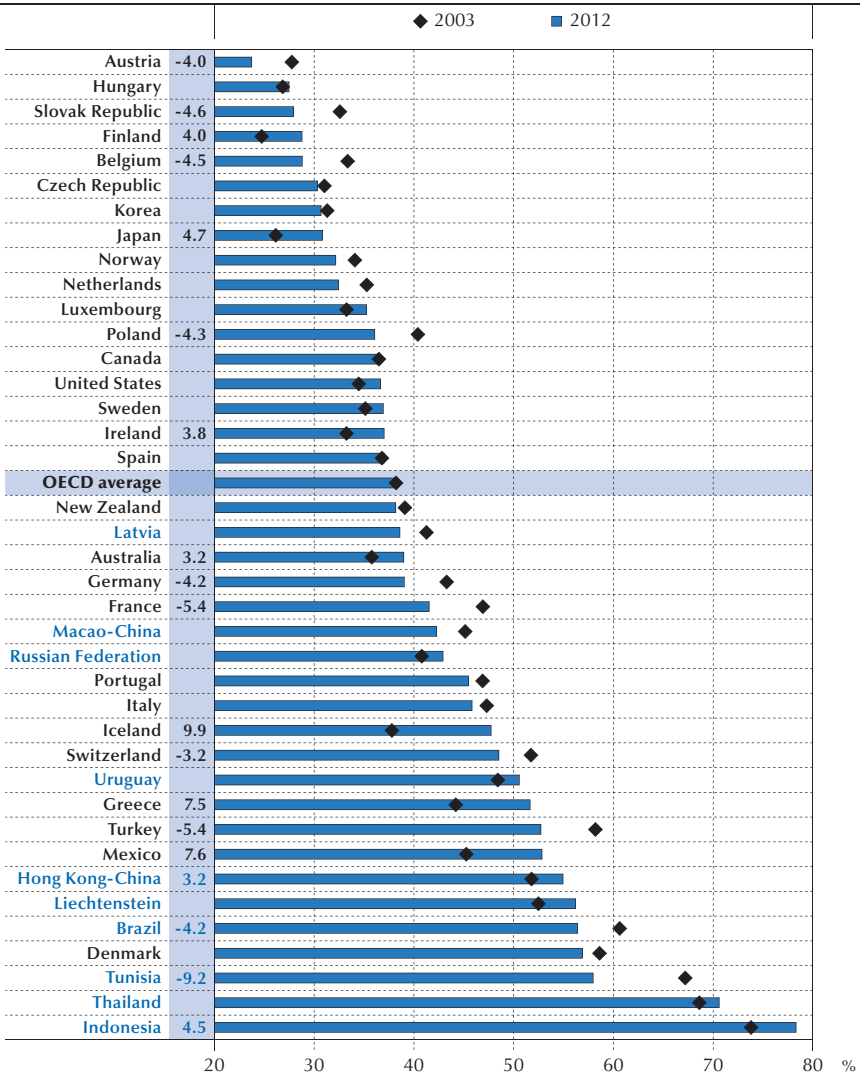
The percentage of students who reported that they enjoy mathematics increased by 4 percentage points or more between 2003 and 2012 in Finland, Greece, Iceland, Indonesia, Japan and Mexico. Students were more likely to report being interested in mathematics than enjoying mathematics when asked about the mathematics they do at school. On average across OECD countries in 2012, 53% of students reported that they are interested in the things they learn in mathematics, the same percentage observed in 2003 (Table 4.1). Between 2003 and 2012, the share of students who reported that they are interested in mathematics grew by 14 percentage points in Greece, but shrunk by 22 percentage points in the Slovak Republic.

How students feel about their mathematics ability (their mathematics self-concept) shapes their behaviour, especially when facing challenging problems (Bandura, 1977). PISA 2012 data show that, on average across OECD countries, 43% of students reported that they agree or strongly agree that they are not good at mathematics; 59% reported that they get good grades in mathematics; 37% reported that they understand even the most difficult work; 52% reported that they learn mathematics quickly; and 38% reported to have always believed that mathematics is one of their best subjects (Table 4.5a). These responses vary markedly across countries and

■ Figure 4.2 ■

Change between 2003 and 2012 in the percentage of students who enjoy mathematics

Percentage of students who reported that they "agree" or "strongly agree" with the statement "I do mathematics because I enjoy it"



Notes: Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown. The OECD average only accounts for countries that participated in both PISA 2003 and PISA 2012 assessments.
Only statistically significant differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the percentage of students in 2012 who agreed with the statement "I do mathematics because I enjoy it".
Source: OECD, PISA 2012 Database, Table 4.1.
StatLink http://dx.doi.org/10.1787/888933377468

economies. Over 60% of students in Albania, Argentina, Indonesia, Chinese Taipei and Thailand reported that they feel they are not good at mathematics, while these negative perceptions were shared by less than 30% of students in Denmark, Israel and Viet Nam.

Particular groups of students are more likely to develop negative beliefs about their mathematics capacities. Figure 4.3 shows that, in 2012, socio-economically disadvantaged students were much more likely than advantaged students to report that they are not good at mathematics. The socio-economic gap in this measure of mathematics self-concept is larger than 25 percentage points in Bulgaria, France, Greece, Liechtenstein, Portugal and Tunisia. The lower self-confidence in academic ability among disadvantaged students is clearly related to their poorer performance at school. But the concentration within a school of students with a history of negative experiences with mathematics can generate "contagion effects" that reinforce the negative relationship between economic disadvantage and self-concept.

Students who hold negative views about their academic abilities are more likely to suffer from mathematics anxiety. Feelings of anxiety can begin as early as elementary school, and are often prompted by social cues conveying the message that mathematics should be feared (Beilock and Willingham, 2014). The consequences of mathematics anxiety are mild to severe: from minor frustration to overwhelming physical reactions, such as pain (Ashcraft and Moore, 2009). Individuals with high mathematics anxiety perform worse because their worry over-rides their cognitive resources (Maloney and Beilock, 2012).

PISA data show clearly that mathematics anxiety is not limited to a minority of individuals nor to one country. Across OECD countries, around 59% of students often worry that it will be difficult for them in mathematics classes. Girls are more likely to report mathematics anxiety than are boys: on average across OECD countries in 2012, 65% of girls and 54% of boys reported feeling worried about their mathematics classes (Figure 4.4). In Denmark, Finland and Liechtenstein, the difference in the percentage of girls and boys who are anxious towards mathematics is larger than 20 percentage points.

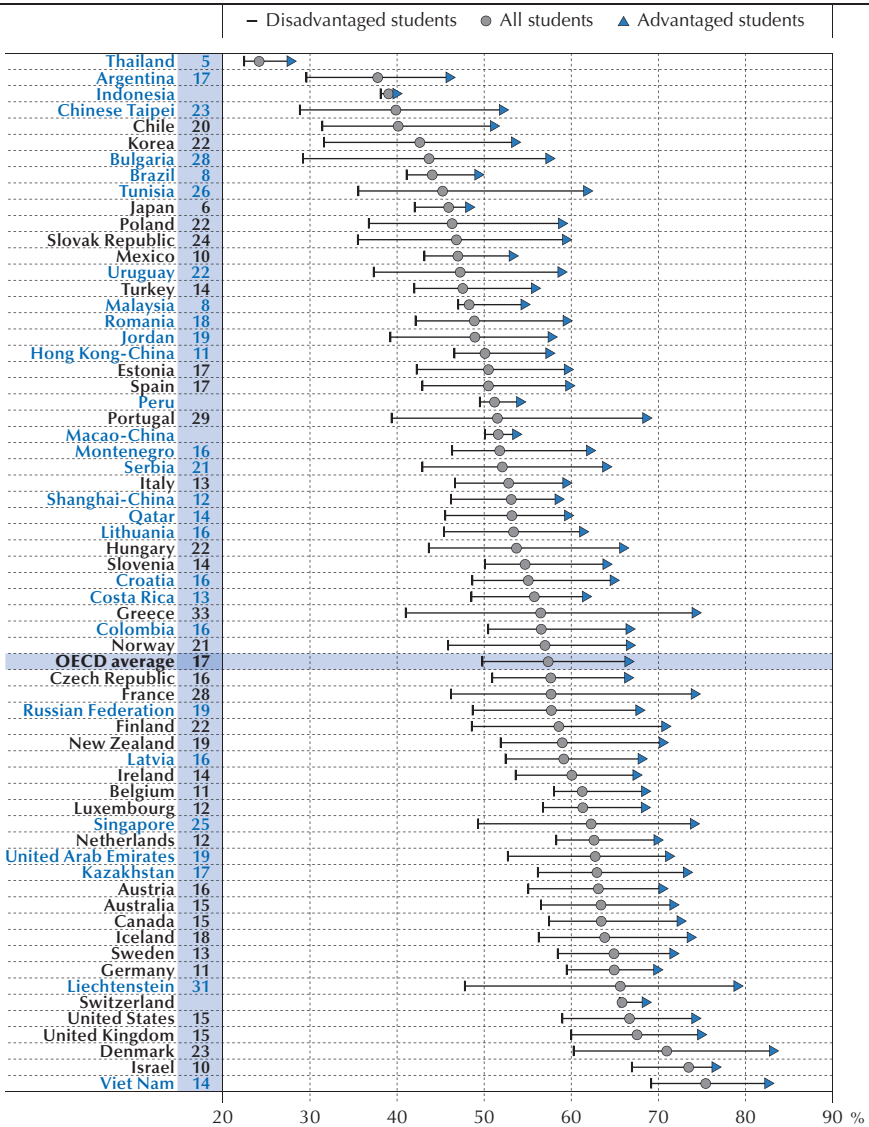
The fear of making mistakes often disrupts performance among gifted girls who "choke under pressure" (OECD, 2015a). The mathematics anxiety experienced by many girls and women has multiple roots. Lower expectations for girls and/or stereotypical thinking that labels mathematics as a more "masculine" subject contribute to mathematics anxiety. Indeed, they might be the greatest obstacles for talented girls and women to overcome on the way to high mathematics achievement (Lavy and Sand, 2015).

The importance of attitudes, beliefs and feelings about mathematics goes beyond the immediate learning context. Students' education pathways and careers might depend on the confidence they have in their abilities to solve mathematics tasks (Hackett and Betz, 1995). In 2012, girls and socio-economically disadvantaged students were less likely than the average student to report that they intended to take additional mathematics courses after the end of compulsory schooling (Figure 4.5).

■ Figure 4.3 ■

Mathematics self-concept, by students' socio-economic status

Percentage of students who reported that they "disagree" or "strongly disagree" with the statement "I am just not good at mathematics"



Notes: Disadvantaged students are defined as those students in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS). Advantaged students are students in the top quarter of ESCS.

Only statistically significant percentage-point differences between advantaged and disadvantaged students are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of all students who disagreed with the statement "I'm just not good at mathematics".

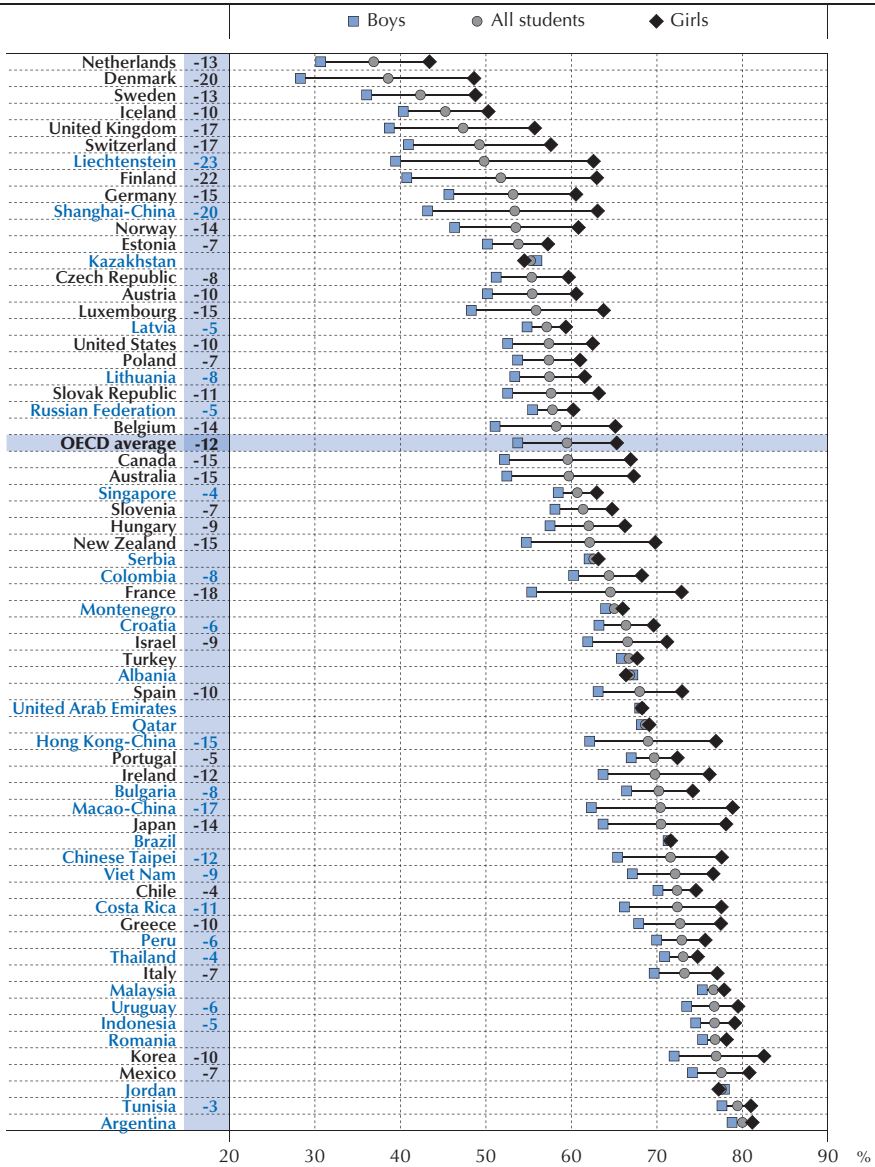
Source: OECD, PISA 2012 Database, Table 4.2.

StatLink http://dx.doi.org/10.1787/888933377470

■ Figure 4.4 ■

Mathematics anxiety, by gender

Percentage of students who reported that they "agree" or "strongly agree" with the statement "I often worry that it will be difficult for me in mathematics classes"



Note: Only statistically significant percentage-point differences between boys and girls are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of all students who agreed with the statement "I often worry that it will be difficult for me in mathematics classes".

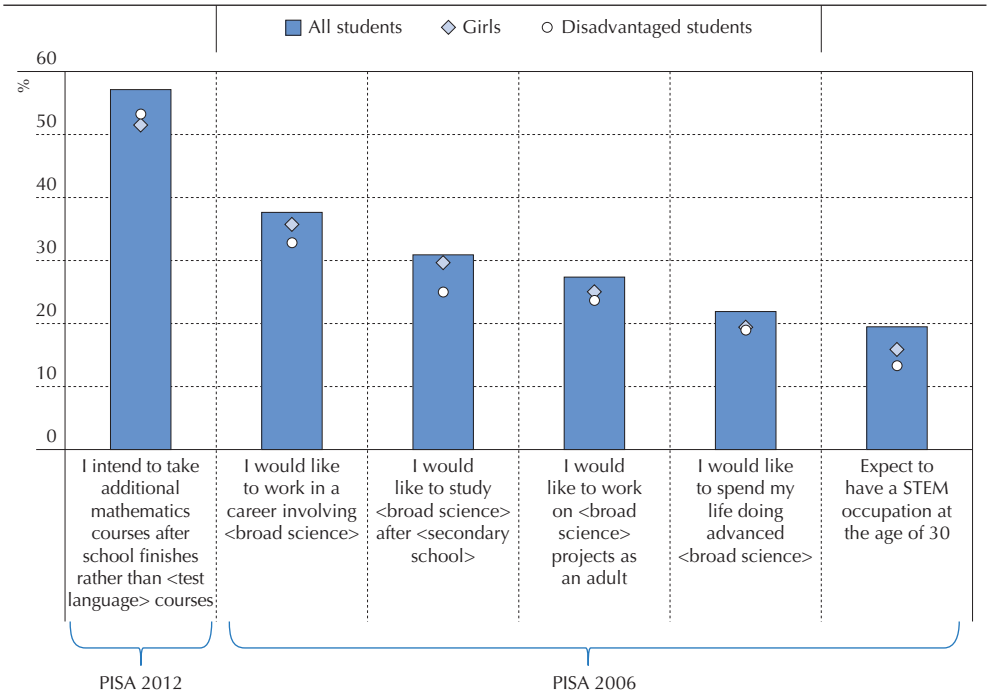
Source: OECD, PISA 2012 Database, Table 4.3.

StatLink http://dx.doi.org/10.1787/888933377487

■ Figure 4.5 ■

Students' intentions, motivation and expected careers in STEM fields, by gender and socio-economic status

Percentage of students who reported that they "agree" or "strongly agree" with the following statements, OECD average



Notes: Disadvantaged students are defined as those students in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS).
STEM refers to science, technology, engineering and mathematics.
Source: OECD, PISA 2006 and 2012 Databases,Tables 4.4a, b and c.
StatLink http://dx.doi.org/10.1787/888933377493

Data on career expectations in PISA 2006 also show that the students more at risk of developing negative attitudes towards mathematics at school – namely girls and disadvantaged students – are also less likely to expect to pursue a scientific career. For example, on average across OECD countries, 19% of students expected to work as a STEM (science, technology, engineering and mathematics) professional (Table 4.4c). Only 13% of disadvantaged students expected to work in one of these fields. These students might back away from the race to mathematics-intensive jobs at the starting line. Similarly, persistent gender-biased expectations about mathematics ability perpetuate the under-representation of women in science and engineering professions (OECD, 2015a).

RELATIONSHIPS BETWEEN OPPORTUNITY TO LEARN AND ATTITUDES TOWARDS MATHEMATICS

Exposure to pure mathematics, familiarity with mathematics and students' attitudes towards mathematics

Exposure to relatively complex mathematics topics may increase the self-concept of students who are relatively well-prepared and ready to be challenged, but it may undermine the self-confidence of students who do not feel up to the task. Recognising the role of attitudes in mathematics performance, some countries and economies explicitly include activities to help students develop positive attitudes towards mathematics in their mathematics curricula (see Box 4.2).

Figure 4.6 looks at how exposure to pure mathematics tasks (linear and quadratic equations) is associated with different measures of mathematics self-concept. On average across OECD countries, students who are more exposed to pure mathematics (by one additional unit on the *index of exposure to pure mathematics*) are over 30% more likely to disagree that they are just

Box 4.2. **Developing positive mathematics attitudes as a curriculum objective**

Acknowledging the relationship between attitudes towards mathematics and exposure to mathematics, a number of countries and economies have included the development of positive attitudes towards mathematics as one of the goals of their mathematics curricula. For instance, the new **Australian** mathematics curriculum "encourages teachers to help students become *self-motivated, confident* learners [italics ours]" (ACARA, 2016).

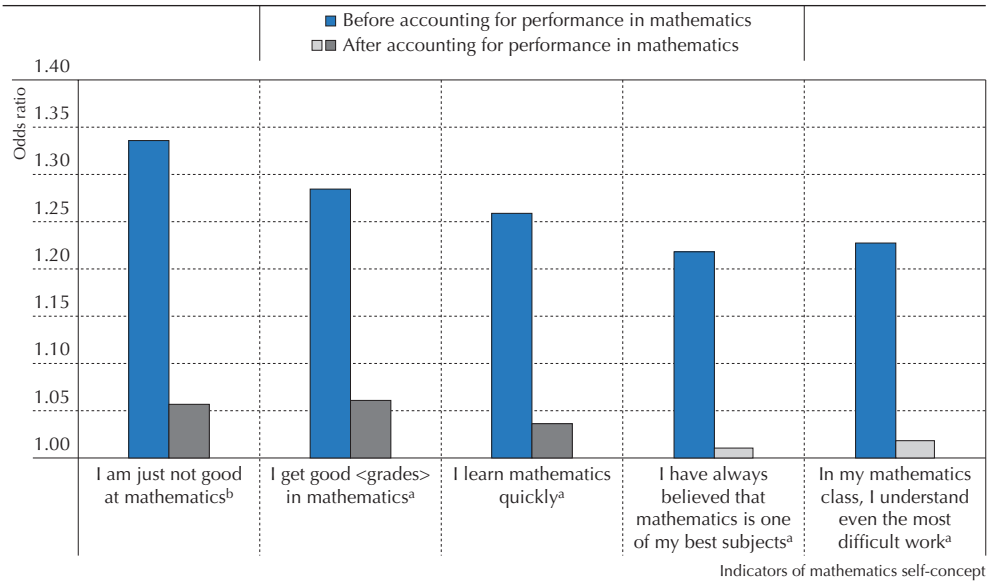
The 2007 and 2011 revisions of the mathematics curriculum in **Korea** introduced a number of new objectives, including the need to develop positive attitudes towards mathematics among students. This idea stemmed from the recognition that previous Korean curricula had fostered the cognitive aspects of mathematics teaching, while attitudes towards mathematics were considered as secondary, albeit instrumental, for developing students' cognitive abilities. Results from the Trends in International Mathematics and Science Study (TIMSS) and PISA confirmed this, as they showed that Korean students consistently displayed high performance in mathematics and problem solving, but had low interest and self-confidence in mathematics (Li and Lappan, 2014). As a part of the 2011 revision of the mathematics curriculum, some content was eliminated or rearranged to significantly reduce students' study load, creating some time for creative and self-directed activities to foster interest in and motivation to learn mathematics (Lew et al., 2012).

Attitudes towards mathematics are one of the five key elements of the mathematics framework that is at the heart of **Singapore**'s mathematics curriculum (Ministry of Education of Singapore, 2012). The framework considers that mathematics education should enable students to develop positive attitudes towards mathematics, including beliefs about the usefulness of mathematics, interest and enjoyment in learning mathematics, confidence in using mathematics and perseverance in solving problems. Similarly, one of the goals of the mathematics curriculum in lower secondary schools in **Hong Kong-China** is to develop positive attitudes towards mathematics (Mullis et al., 2012).

■ Figure 4.6 ■

Exposure to pure mathematics and students' self-concept

Change in the likelihood of students reporting to "agree"/"strongly agree" (a) or "disagree"/"strongly disagree" (b) with the following statements on mathematics self-concept associated with a one-unit increase in the index of exposure to pure mathematics



How to read the chart: An odds ratio of 1.26 corresponding to the statement "I learn mathematics quickly" means that a student who is one standard deviation more exposed to pure mathematics is 26% more likely to agree or strongly agree that he/she learns mathematics quickly compared with a student who is less exposed to pure mathematics.
Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Statistically significant odds ratios are marked in a darker tone. All values before accounting for performance in mathematics are statistically significant.
Source: OECD, PISA 2012 Database, Table 4.5b.
StatLink http://dx.doi.org/10.1787/888933377507

not good at mathematics and they are more than 20% more likely to agree that they get good grades in mathematics, that they learn mathematics quickly, that they have always believed that mathematics is one of their best subjects, and that they understand even the most difficult work, compared to students who are less exposed to pure mathematics.

However, once students' mathematics performance is taken into account, the relationship between exposure to mathematics tasks, such as equations, and self-concept becomes weaker. Students of similar ability who are more exposed to pure mathematics are 6% more likely to disagree that they are not good at mathematics, 6% more likely to agree that they get good grades in mathematics, 4% more likely to agree that they learn mathematics quickly and just as likely to agree with the other statements as students who are less exposed (Figure 4.6). After accounting for performance in mathematics, exposure to pure mathematics is not associated with any of the statements about mathematics self-concept in almost half of the countries and economies (Table 4.5b).

This finding suggests that mathematics performance explains the positive relationship between exposure to mathematics and mathematics self-beliefs (see Figure 4.1). High-performing students may be more likely to be exposed to pure mathematics and to express positive feelings towards mathematics. After accounting for mathematics ability, an increase in exposure to pure mathematics corresponds to an increase in mathematics self-concept by more than 0.2 units (equivalent to 20% of a standard deviation for the OECD average) in Kazakhstan, Jordan and Tunisia, but it corresponds to a decrease in mathematics self-concept in Belgium, Denmark, Macao-China, the Netherlands and Switzerland (Table 4.6).

The strong mediating role of mathematics performance is even more apparent when looking at the relationship between self-concept and the *index of familiarity with mathematics*. Figure 4.7 shows that, on average across OECD countries, a one-unit increase in the *index of familiarity with mathematics* corresponds to an increase in the *index of mathematics self-concept* by 10% of a standard deviation before taking mathematics performance into account; but it corresponds to a decrease in that index, by 10% of a standard deviation, when comparing students of similar ability. In Austria, Germany and Liechtenstein, a one-unit increase in the *index of familiarity with mathematics* corresponds to a decrease in the *index of mathematics self-concept* by more than 20% of a standard deviation, after taking mathematics performance into account. As the *index of familiarity with mathematics* is measured on relatively difficult concepts for 15-year-old students, like vectors, complex numbers and congruent figures, it appears that, in most countries, exposure to advanced topics often challenges students' beliefs in their own mathematics abilities.

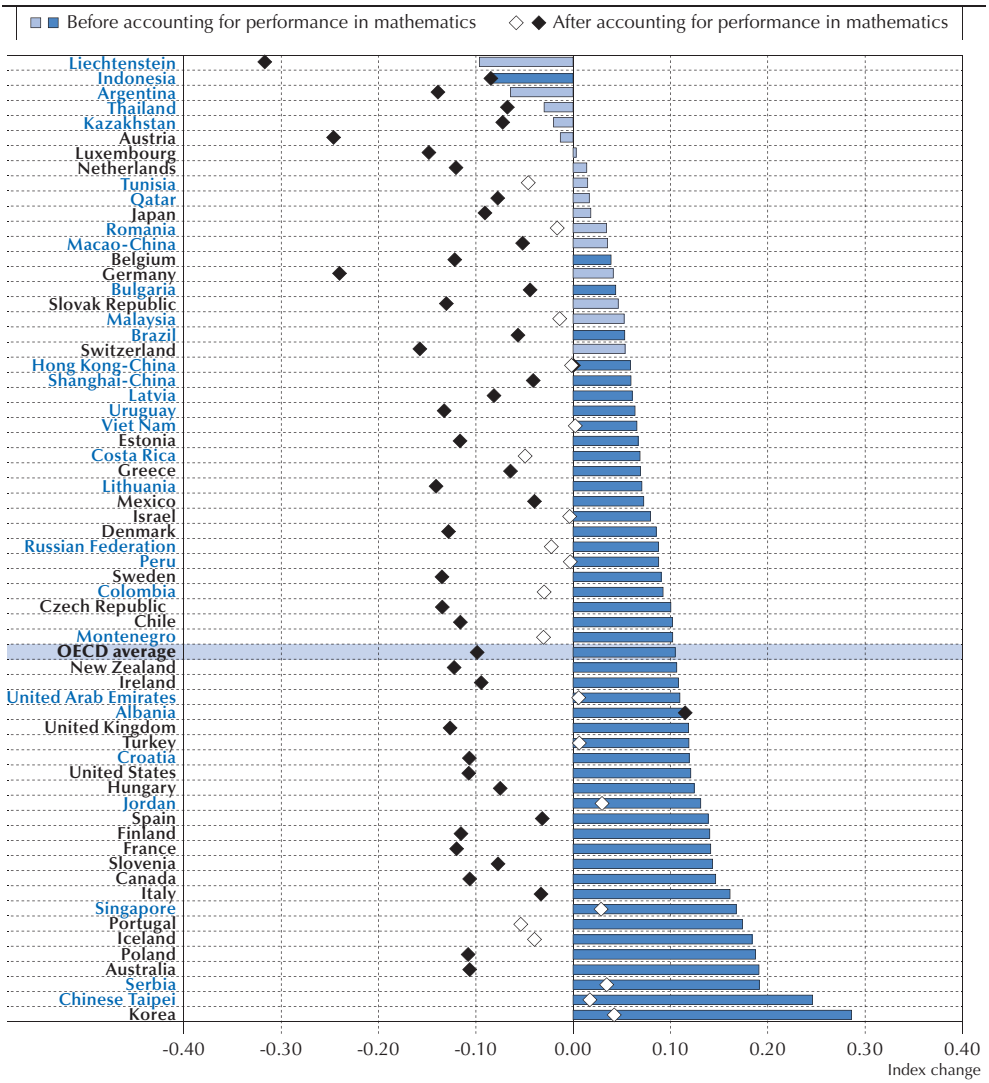
Exposure to relatively formal mathematics, as measured by the *index of exposure to pure mathematics* may also be associated with more anxiety towards mathematics among students who are not sufficiently prepared to learn. On average across OECD countries, students who are more exposed to pure mathematics are 14% more likely to say that they worry that they will get poor grades in mathematics, 8% more likely to worry that it will be difficult for them in mathematics classes, and 4% more likely to report that they get very nervous doing mathematics problems (but 2% less likely to report that they get very tense when they have to do mathematics homework), compared with students of similar mathematics ability who are less exposed to these tasks (Table 4.8b).

Moreover, Figure 4.8 shows that, on average across OECD countries, greater exposure to complex concepts, as measured by the *index of familiarity with mathematics*, is associated with higher anxiety among students in the bottom quarter of the mathematics performance distribution and with lower mathematics anxiety among students in the top quarter of the distribution. The increase in mathematics anxiety among low-performing students associated with a one-unit increase in the *index of familiarity with mathematics* is larger than 10% of a standard deviation in Australia, Austria, Canada, the Czech Republic, France and Chinese Taipei. A similar increase in familiarity with mathematics is associated with a decrease in anxiety by more than 10% of a standard deviation among high-performing students in Albania, Colombia, Hong Kong-China, Hungary, Malaysia, Peru, Romania, Serbia and Slovenia. Similar patterns are found when looking at the effect of exposure to pure mathematics on anxiety along the performance distribution. In France, both high-performing and low-performing students reported

■ Figure 4.7 ■

Relationship between familiarity with mathematics and students' self-concept

Change in the index of mathematics self-concept *associated with a one-unit increase in the* index of familiarity with mathematics



Notes: The *index of mathematics self-concept* is based on the degree to which students agree with the statements: I'm just not good in mathematics; I get good grades in mathematics; I learn mathematics quickly; I have always believed that mathematics is one of my best subjects and In my mathematics class, I understand even the most difficult work.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the relationship between familiarity with mathematics and students' self-concept before accounting for performance in mathematics.

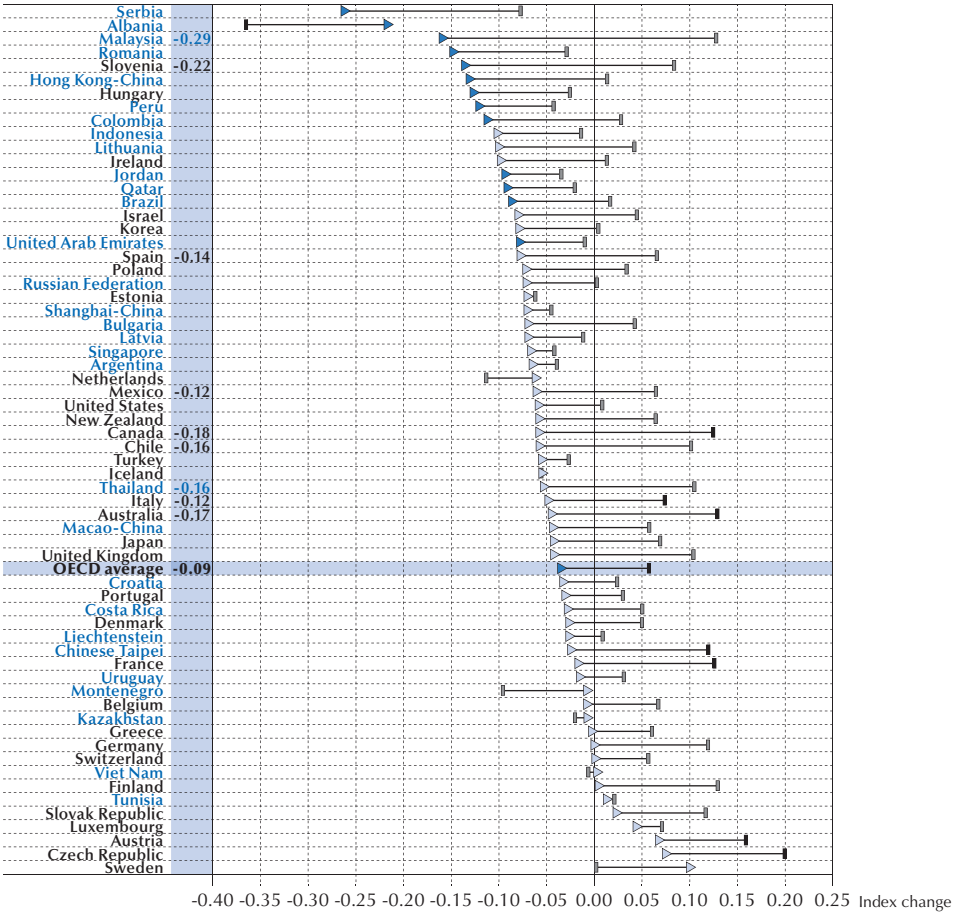
Source: OECD, PISA 2012 Database, Table 4.6.

StatLink http://dx.doi.org/10.1787/888933377511

■ Figure 4.8 ■

Familiarity with mathematics and mathematics anxiety, by students' performance in mathematics

Change in the index of mathematics anxiety *associated with a one-unit increase in the* index of familiarity with mathematics, *among students in the top and bottom 25% of the mathematics performance distribution*



Notes: The *index of mathematics anxiety* is based on the degree to which students agree with the statements: I often worry that it will be difficult for me in mathematics classes; I get very tense when I have to do mathematics homework; I get very nervous doing mathematics problems; I feel helpless when doing a mathematics problem; and I worry that I will get poor marks in mathematics.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).

Statistically significant differences between students in the top and bottom quarter of performance in mathematics are shown next to the country/economy name.

Only countries and economies with available data are shown.

Countries and economies are ranked in ascending order of the index change for the students in the top quarter of performance in mathematics.

Source: OECD, PISA 2012 Database, Table 4.10b.

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greater anxiety when they are more exposed to pure mathematics tasks, such as solving linear and quadratic equations (Table 4.10b).

All in all, the analysis of the relationship between opportunities to learn mathematics, mathematics performance and attitudes towards mathematics suggests that exposure to challenging mathematics content can help improve performance (Chapter 3). At the same time, such exposure could backfire if students, particularly low-achieving students, develop negative attitudes and self-beliefs towards mathematics as a result. Chapter 5 discusses the implications of these results for policy, and suggests that mathematics curricula, and the teachers who follow those curricula, should strike a balance between making mathematics lessons challenging and aiming to bolster students' self-confidence and reduce their anxiety towards mathematics.

Exposure to applied mathematics and students' attitudes towards mathematics

By contrast, exposing students to problems that ask them to apply mathematics in a real context seems to be a way to expand their opportunity to learn while, at the same time, reinforcing their self-beliefs. On average across OECD countries, students who reported that they are frequently exposed to contextualised mathematics problems (such as interpreting a trend in a chart; see Chapter 1) tend to have higher values on the *index of mathematics self-concept* by 10% of a standard deviation compared with students who are less frequently exposed to these problems, after accounting for their mathematics performance (Figure 4.9). The increase in self-concept associated with being frequently exposed to contextualised mathematics problems is larger than 25% of a standard deviation in Austria, Bulgaria, Croatia, Luxembourg and Montenegro (Table 4.7a).

Students' practice with relatively simple applied mathematics tasks, as measured by the *index of exposure to applied mathematics*, seems to have a different effect on mathematics anxiety than exposure to more abstract content, once students' performance is taken into account. On average across OECD countries, more frequent exposure to the relatively difficult mathematics concepts captured by the *index of familiarity with mathematics* and the *index of exposure to pure mathematics* increases students' anxiety, after accounting for mathematics performance (Table 4.9). By contrast, the association between more frequent exposure to applied mathematics and mathematics anxiety is not statistically significant, after accounting for mathematics performance.

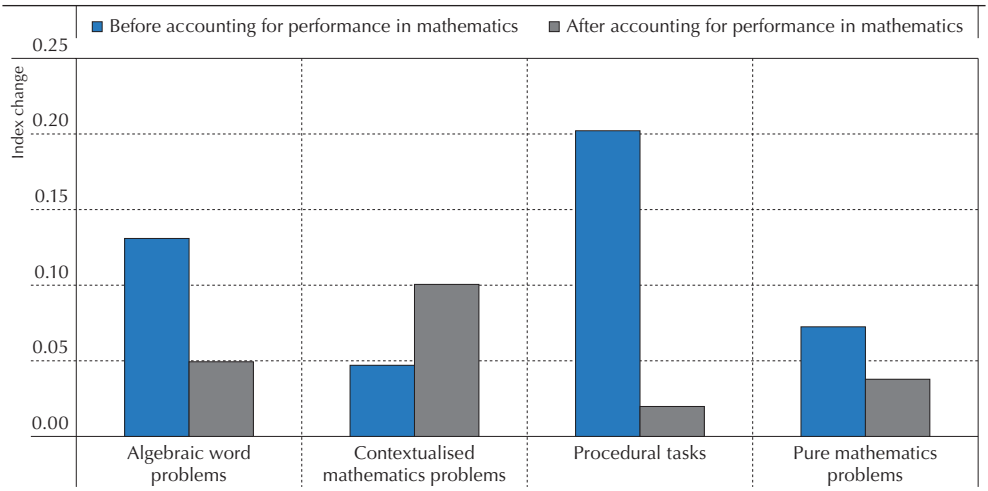
Mathematics assessments and anxiety

Anxiety can be related not only to exposure to mathematics during lessons but also to mathematics assessments (Reys et al., 2014). Mathematics anxiety causes greater deterioration in performance when mathematics knowledge and practice is tested under timed, high-stakes conditions (Ashcraft and Moore, 2009). On average across OECD countries, students tend to show higher values on the *index of mathematics anxiety* when they are more frequently exposed to pure mathematics problems during school tests (Table 4.7d; see Chapter 1 for examples of the various tasks).

■ Figure 4.9 ■

Relationship between exposure to mathematics tasks during mathematics lessons and students' self-concept

Change in the index of mathematics self-concept *associated with frequent exposure to mathematics tasks during during lessons, OECD average*



Notes: The *index of mathematics self-concept* is based on the degree to which students agree with the statements: I'm just not good in mathematics; I get good grades in mathematics; I learn mathematics quickly; I have always believed that mathematics is one of my best subjects and In my mathematics class, I understand even the most difficult work.
See Chapter 1 for examples of the various tasks.
All values are statistically significant.
Source: OECD, PISA 2012 Database, Table 4.7a.
StatLink *http://dx.doi.org/10.1787/888933377533*

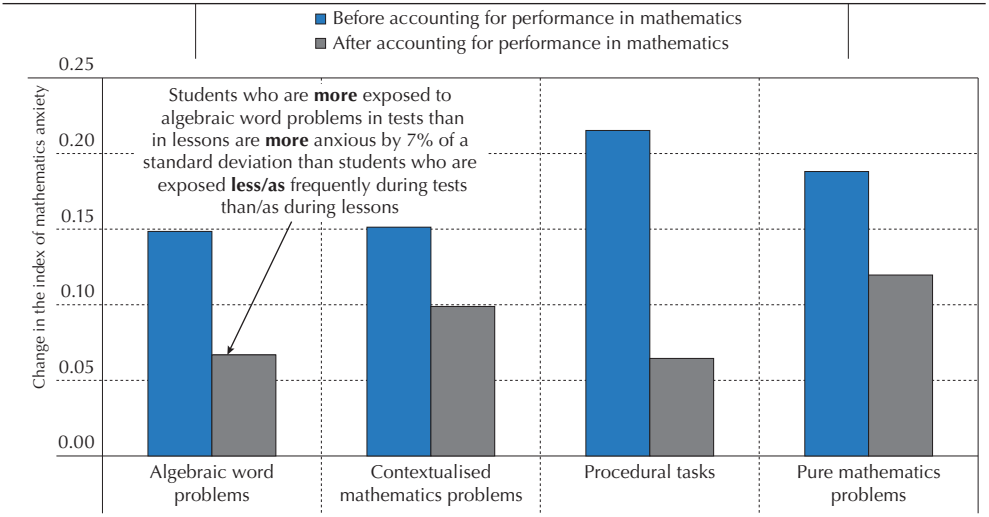
Students are more likely to feel anxious if they are assessed on a topic that they have not practiced very often during classroom lessons. About 10% of students, on average across OECD countries, reported that they are more frequently exposed to algebraic word problems in tests than in lessons; 17% of students in the United Kingdom reported that they are more frequently exposed to pure mathematic problems in tests than they are in mathematics classes (Table 4.7e).

Figure 4.10 shows that, on average across OECD countries, students who are exposed more frequently to pure and contextualised mathematics tasks in tests than in lessons feel more anxious by at least 10% of a standard deviation than students who are exposed less or equally frequently during tests than during lessons, after accounting for their mathematics performance. Students in Austria who are exposed more frequently to pure and contextualised mathematics tasks in tests than in lessons feel more anxious by more than 30% of a standard deviation than students who are exposed less or equally frequently during tests than during lessons, after accounting for their mathematics performance (Table 4.7f). More frequent use of contextualised problems in tests than in lessons is associated with a larger increase in mathematics anxiety than more frequent use of procedural tasks in tests than in lessons, possibly because contextualised problems are more unpredictable than procedural problems.

■ Figure 4.10 ■

Mathematics anxiety and the mismatch between what is taught and what is tested

Change in the index of mathematics anxiety *associated with more frequent exposure to mathematics tasks during tests than during lessons, OECD average*



How to read the chart: This figure compares students who are exposed more frequently to mathematics tasks in tests than in lessons to students who are exposed to mathematics tasks less/as frequently in tests than/as in lessons.
Notes: The *index of mathematics anxiety* is based on the degree to which students agree with the statements: I often worry that it will be difficult for me in mathematics classes; I get very tense when I have to do mathematics homework; I get very nervous doing mathematics problems; I feel helpless when doing a mathematics problem; and I worry that I will get poor marks in mathematics.
All values are statistically significant.
Source: OECD, PISA 2012 Database, Table 4.7f.
StatLink *http://dx.doi.org/10.1787/888933377548*

MEDIATING FACTORS BETWEEN EXPOSURE TO MATHEMATICS AND ATTITUDES TOWARDS MATHEMATICS

Students learn mathematics in the context of their classroom and families, and the relationship between exposure to mathematics and self-beliefs is likely to be mediated by students' peers, parents and teachers. This section considers how peers, parents and teachers can influence the relationship between exposure to mathematics and students' intrinsic and instrumental motivation to learn mathematics, as well as their self-concept and anxiety towards mathematics.

The benefits and possible shortcomings of hard-working and well-prepared peers

Peer quality and behaviour are important determinants of student outcomes (Sacerdote, 2001; Jencks and Mayer, 1990). Peer effects are central to many important policy issues in education, including the impact of ability tracking between and within schools, "mainstreaming" special education students, and programmes for racial and economic desegregation. The effect of grouping students in classrooms by ability, for example, affects students' achievement not only by influencing

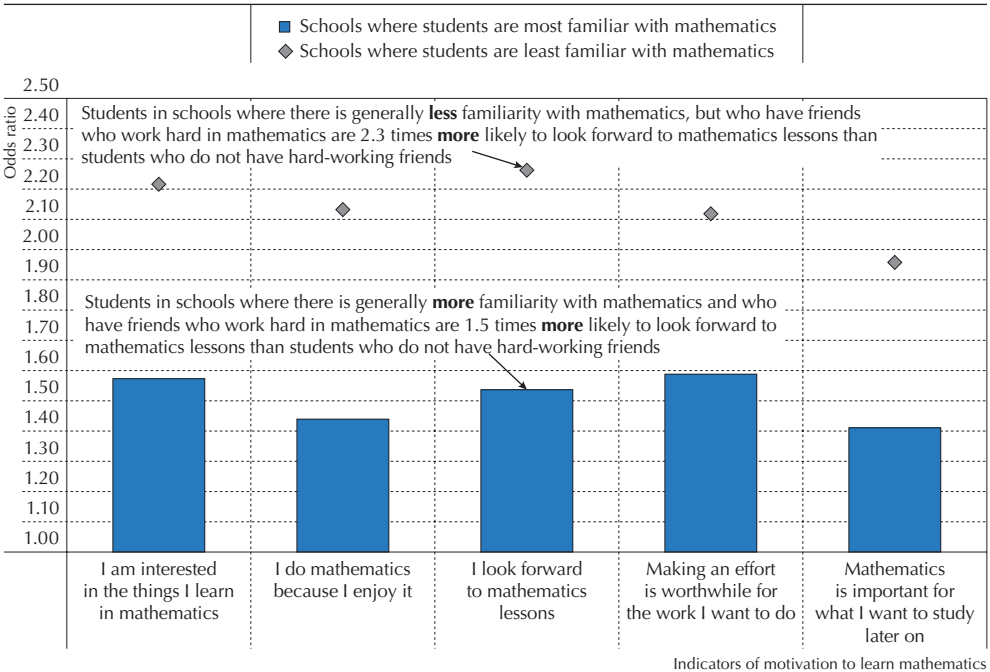
the instruction they are exposed to, but also by limiting their classroom interactions to those with only high-achieving peers (for those in higher tracks) or to those with only low-achieving peers (for those in lower tracks [Angrist and Lang, 2004; Cooley, 2007; Fryer and Torelli, 2010]).

The characteristics of students' peers are clearly related to the academic credentials of the schools they attend. PISA 2012 data show that 56% of students in schools where the students' level of familiarity with mathematics is above the country average and 49% of the students in schools where the students' level of familiarity with mathematics is below the country average reported having friends who work hard on mathematics, on average across OECD countries (Table 4.12a). In both types of schools, students of similar ability with hard-working friends have higher intrinsic and instrumental motivation to learn mathematics (Figure 4.11).

■ Figure 4.11 ■

Motivation to learn mathematics and peers' attitudes, by schools' level of familiarity with mathematics

Change in the likelihood of students reporting that they "agree" or "strongly agree" with the following statements on the motivation to learn mathematics associated with having friends who work hard in mathematics, OECD average



How to read the chart: An odds ratio of 2.22 corresponding to the statement "I am interested in the things I learn in mathematics" means that a student who has friends who work hard in mathematics is 2.22 times more likely to agree or strongly agree with the statement than a student who does not have friends who work hard in mathematics.
Notes: Schools with less (more) familiarity with mathematics are defined as those schools where the average familiarity with mathematics is significantly lower (higher) than the country average familiarity with mathematics.
All odds ratios are statistically significant.
Source: OECD, PISA 2012 Database, Table 4.11.
StatLink http://dx.doi.org/10.1787/888933377553

This relationship is not causal, as hard-working and motivated students are likely to self-select into groups of hard-working friends. But it is interesting to observe that the positive relationship between a student's interest in mathematics and the attitudes towards mathematics of his or her peers are stronger in the schools that teach relatively less advanced mathematics, even after taking mathematics performance into account. The data thus suggest that students in schools that are not academically challenging would benefit even more than students in academically rigorous schools from having a network of friends who like mathematics. A possible explanation is that students in academically rigorous or socio-economically advantaged schools have more access to other resources (e.g. parental involvement) that are substitutes for peers who are interested in mathematics.

To the extent that attending a school with highly motivated peers creates incentives for students to work on mathematics, disadvantaged students can be expected to benefit from attending advantaged schools. However, desegregation policies that allow disadvantaged students to attend high-achieving schools do not always work as expected. As a result of a background of disadvantage and low achievement, a disadvantaged student attending an advantaged school may suffer from social isolation or even discrimination if he or she is not well-prepared to be among a minority in the school (Montt, 2012). The disadvantaged student may also suffer from an "invidious comparison" with his/her higher-achieving peers, leading to lower self-concept and achievement (Hoxby and Weingarth, 2005).

Figure 4.12 shows that students' mathematics self-concept is associated with the students' relative position in the school. In almost all countries and economies, students who reported less familiarity with mathematics than the average student in the school have lower mathematics self-concept. Particularly in Korea and Chinese Taipei, students who reported a level of familiarity with mathematics below the average student in their school feel like "the small frogs of the pond" (Marsh and Hau, 2003), meaning that their self-concept is undermined by social comparisons with peers who have a greater familiarity with mathematics.

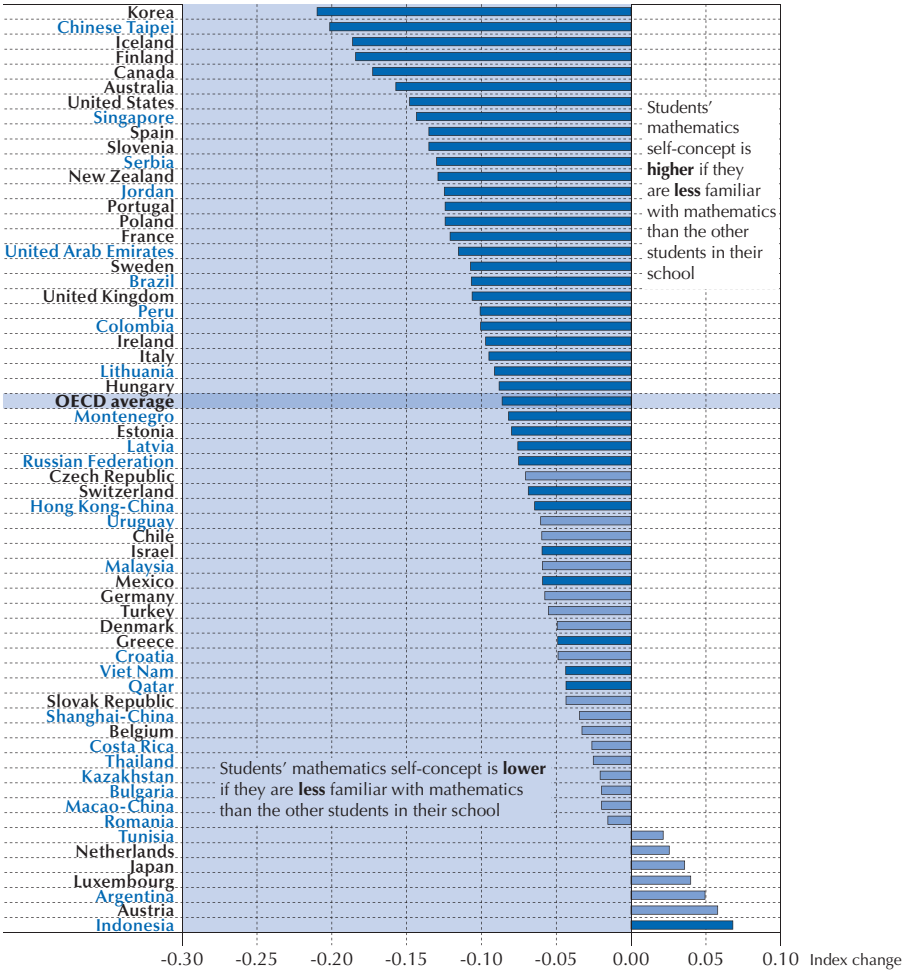
The research on the effects of school composition has often failed to integrate the impact of students' relative social and academic standing on the psychological and competitive atmosphere in the school. This is problematic because this atmosphere affects students' achievement, well-being, beliefs and expectations about their future studies and jobs. The organisation of school systems and teaching practices can have an effect on whether an academically or socio-economically disadvantaged student is pushed ahead through positive "contagion" from his or her higher-achieving peers, or pushed back by humiliating social comparisons.

For example, both positive and negative peer effects tend to be weaker in stratified school systems because these systems institutionalise the norms and information students consider when forming their expectations (Montt, 2012). A closer look at the real-life experiences of poor and minority students in high-achieving schools can help to identify the psychological, pedagogical and instructional challenges of "detracking" and integration across achievement levels (Rosenbaum, 1999).

■ Figure 4.12 ■

Mathematics self-concept and relative familiarity with mathematics compared with schoolmates

Change in the index of self-concept *associated with a one-unit difference between the school's average familiarity and the student's familiarity with mathematics*



Notes: The *index of mathematics self-concept* is based on the degree to which students agree with the statements: I'm just not good in mathematics; I get good grades in mathematics; I learn mathematics quickly; I have always believed that mathematics is one of my best subjects and In my mathematics class, I understand even the most difficult work.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).

The results are based on a multi-level model and account for the gender and socio-economic status of the student (at the student level) and for the percentage of girls, the socio-economic profile of the school and the average level of familiarity with mathematics in the school (at the school level).

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the index change in familiarity with mathematics associated with larger differences between the school's and the student's familiarity.

Source: OECD, PISA 2012 Database, Table 4.12b.

StatLink http://dx.doi.org/10.1787/888933377566

Parental involvement and children's mathematics anxiety

Parents are their children's first and longest-serving teachers (Maloney et al., 2015). Many of the basic ideas that support formal mathematics later in life are constructed during the first years of a child's life, through interaction with the adults in their surrounding environment. Over the course of these interactions, parents' attitudes towards mathematics are often passed on to their child. What makes mathematics different from other subjects is that it is considered socially acceptable to say "I am not a maths person" or "I never liked maths". Unlike other weaknesses that are hidden, mathematical illiteracy is flaunted by many people (Paulos, 2011). When a parent does that in front of a child, he or she is suggesting that mathematics is not important or that only some people need to understand mathematics.

PISA data show that parents participate actively in their child's learning. On average across the 11 countries and economies where the PISA Parent Questionnaire was distributed, 81% of parents reported discussing how the child is doing at school at least once a week; 78% of parents reported eating the main meal with their child almost every day; 65% of parents reported spending time just talking to their child at least once a week; 19% of parents reported obtaining mathematics materials for their child at least once a week; and 32% of parents reported discussing with their child how mathematics can be applied to everyday life at least once a week (OECD, 2013b: Table III.6.1a).

On average, these activities are positively associated with students' mathematics performance and with other indicators of students' attitudes towards learning, such as the likelihood of arriving on time for classes (OECD, 2013b). However, research shows that the effects of parental involvement on a child's achievement vary according to parents' behaviour, the child's grade level, and the racial, ethnic and socio-economic background of the family (Robinson and Harris, 2014).

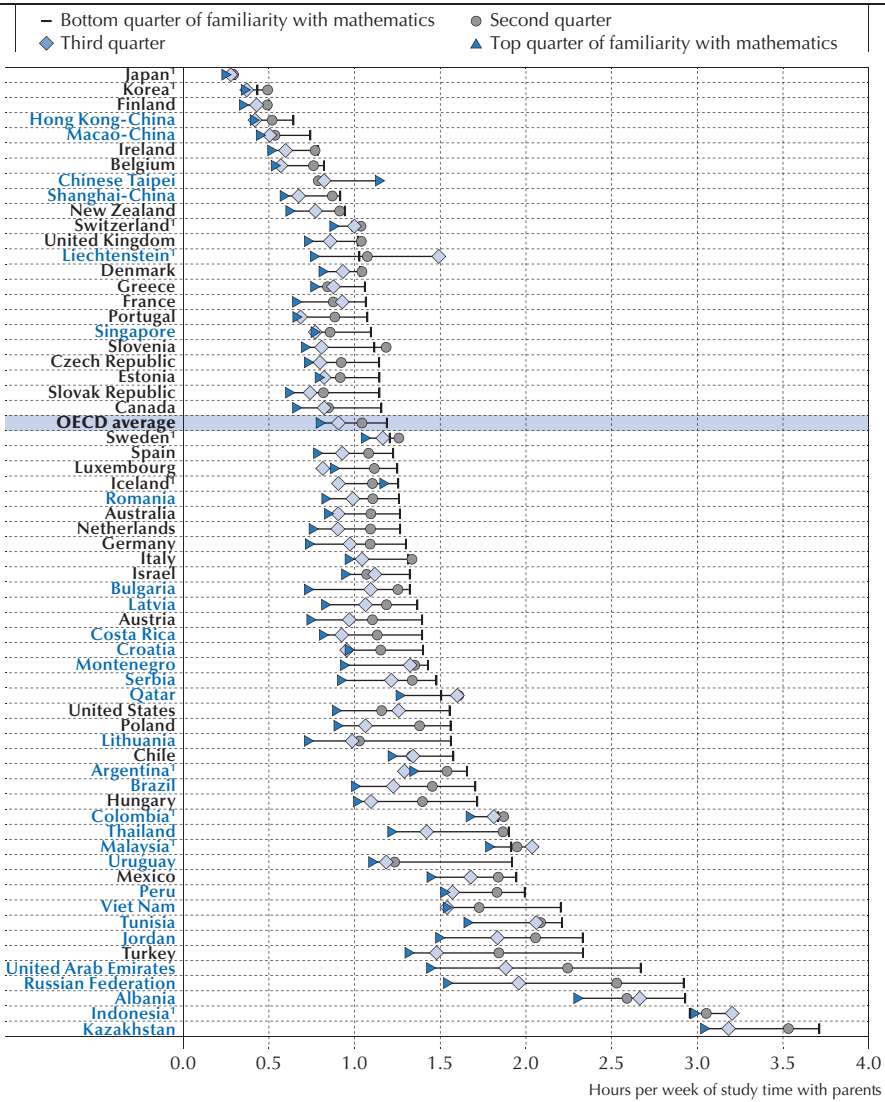
The major vehicle through which parents help their children with school subjects is homework. On average across OECD countries, 44% of 15-year-old students reported spending some time each week studying with a parent (Table 4.13). Studying with parents is much less common in East-Asian countries and economies. Less than 30% of students in Hong Kong-China, Japan, Korea, Macao-China, Shanghai-China, Singapore and Chinese Taipei study with their parents.

Parents are more likely to intervene when their children struggle with mathematics. Figure 4.13 shows that, on average, students in the lower quarter of the *index of familiarity with mathematics* spend 1 hour and 11 minutes studying with their parents, while students in the top quarter spend less than 1 hour.[1] The difference in study time with parents between students who are less familiar with mathematics and students who are more familiar with mathematics is larger than 1 hour per week in the Russian Federation, Turkey and the United Arab Emirates.

■ Figure 4.13 ■

After-school study time with parents, by students' level of familiarity with mathematics

Average number of hours per week that students spend studying with a parent or other family member



1. The difference between students in the top and in the bottom quarters of the *index of familiarity with mathematics* is not statistically significant.

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Countries and economies are ranked in ascending order of study time for students in the bottom quarter of familiarity with mathematics.

Source: OECD,PISA 2012 Database, Table 4.13.

StatLink http://dx.doi.org/10.1787/888933377572

What happens when a parent who dislikes mathematics, or is anxious about mathematics, tries to help his or her child with homework? Recent research has shown that when parents who are anxious towards mathematics frequently help their children with mathematics homework, their help can backfire, leading to greater mathematics anxiety and less mathematics learning for their children (Maloney et al., 2015). In expressing their own dislike of and confusion about mathematics, these parents may be inadvertently transferring their own attitudes to their children.

Mathematics anxiety is even evident among "smart" students. On average across OECD countries, 13% of the students in the top quarter of the mathematics performance distribution reported feeling helpless when doing mathematics problems (Table 4.14a). This lack of self-confidence is more common among top-performing girls (16%) than among top-performing boys (10%).

Parents who send positive messages about mathematics can help dispel the anxiety that distorts students' perceptions of their problem-solving abilities. Figure 4.14 shows that, on average across OECD countries, high-performing students who reported that their parents do not like mathematics are 73% more likely to feel helpless when they are doing mathematics problems than the high-performing children of parents who like mathematics. High-performing students in Croatia, Finland, Hungary and the Netherlands whose parents dislike mathematics are more than 2.5 times more likely to feel helpless when doing mathematics than high-performing students whose parents like mathematics.

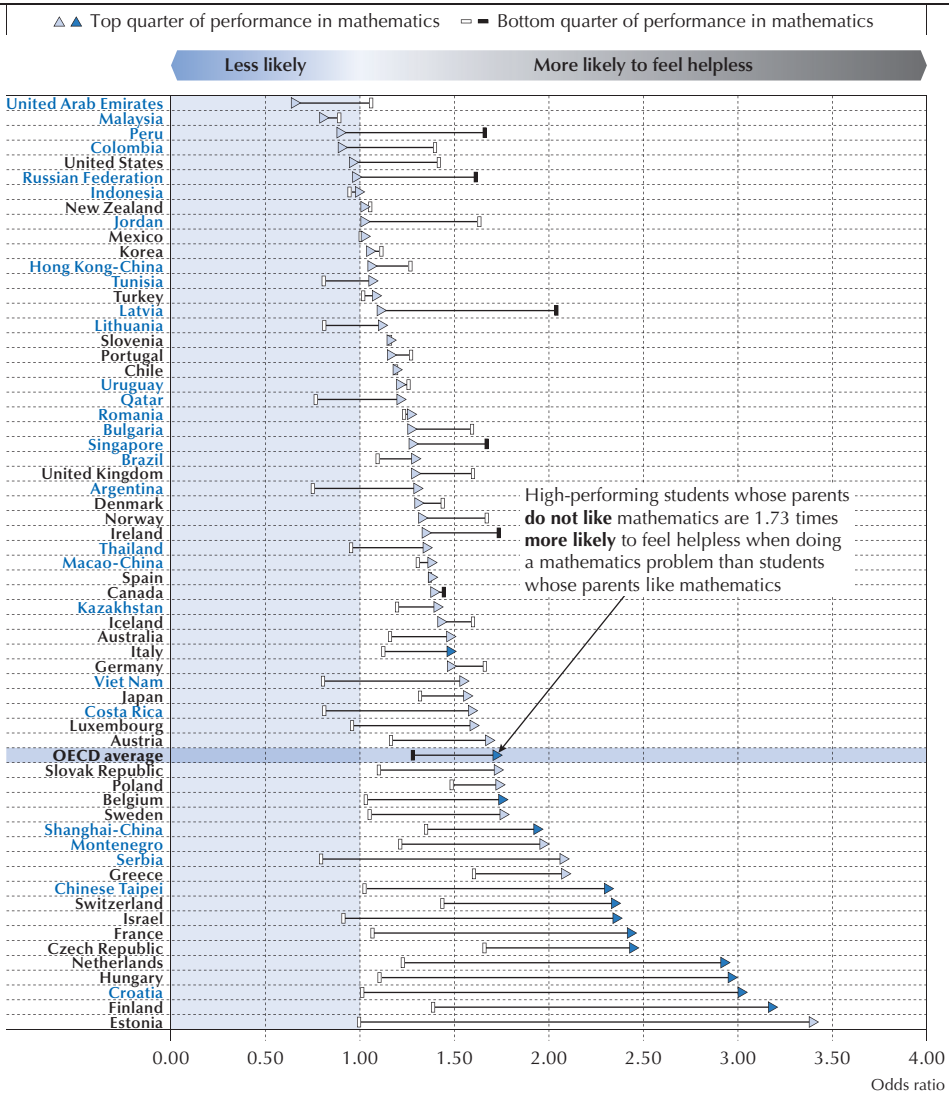
This finding confirms results from experimental research showing that mathematics success can be inherited, not only through genetics but also because children often believe that they cannot do much better than their parents. For example, an experiment examined students' concern about their relative performance when completing a mathematics task on a computer (Jury, Smeding and Darnon, 2015). In the experiment, researchers noted how often students' eyes moved towards an arrow on the screen showing how well they were doing compared with other students. Students whose parents did not complete higher education were more worried about others' performance than students from more-educated families. This concern explained their underperformance: when the arrow was not shown on the screen, there was no performance difference between the two groups of students. Students from less-educated families, or whose parents have a poorer record in mathematics, might thus be more likely to believe in the received wisdom that they cannot succeed, and are more sensitive to signals that might confirm this failure.

One essential ingredient for children's success is for parents to communicate that solving mathematics problems can be a pleasant and rewarding activity. This is a message that all parents should be sending early in their child's lives and that needs to be reinforced over time (Robinson and Harris, 2014).

■ Figure 4.14 ■

Parents' attitudes towards mathematics and students' anxiety, by performance in mathematics

Change in the probability that students feel helpless when doing mathematics problems associated with reporting that their parents do not like mathematics



How to read the chart: An odds ratio of 2 means that students whose parents do not like mathematics are twice as likely as students whose parents like mathematics to feel helpless when doing a mathematics problem.

Notes: The results take into account students' gender and socio-economic status.

Statistically significant odds ratios are marked in a darker tone.

Countries and economies are ranked in ascending order of the odds ratio for students in the top quarter of performance in mathematics.

Source: OECD, PISA 2012 Database, Table 4.14b.

StatLink http://dx.doi.org/10.1787/888933377584

Teachers' practices and attitudes towards mathematics

Teachers' communication about mathematics and students' anxiety

Together with parents, teachers are major role models for students. The way teachers communicate and structure their teaching practices is likely to affect students' attitudes towards mathematics. Traditional practices in mathematics classes, such as imposed authority, public exposure and timed deadlines, may cause great anxiety in many students (Curtain-Phillips, 1999). Consequently, teaching methods that use competition judiciously, communicate clearly, de-emphasise speed tests or drills, and reduce the pressure from major tests and examinations could alleviate students' mathematics anxiety (Beilock and Willingham, 2014; Furner and Berman, 2003; Maloney and Beilock, 2012; Rossnan, 2006).

In particular, communication about learning objectives and feedback on students' performance can be ways of reducing students' mathematics anxiety. A number of teachers' communication strategies, such as telling students what they have to learn, what is expected of them in a test, quiz or assignment, and how well they are doing in mathematics class, are related to less mathematics anxiety among students (Table 4.15).

However, such communication does not work in the same way (or is not expressed in the same way) for all students. Figure 4.15 shows that the communication practices listed above reduce anxiety among students who are more familiar with mathematics; but giving students feedback on their strengths and weaknesses in mathematics, and telling them what they need to do to become better in mathematics actually increase anxiety among students who are less familiar with mathematics. In interpreting the results, one has to keep in mind that this relationship may be mediated by the quality of the teachers: good teachers can provide greater exposure to mathematics content and communicate well with their students at the same time.

Teaching practices that build mathematics self-concept

Classroom practices that help students focus their attention and engage in the mathematics task at hand may also help reduce the incidence of poor performance that results from mathematics anxiety and low mathematics self-concept. A number of practices, such as differentiating tasks based on students' abilities, splitting students into small groups, helping students understand, and using cognitive-activation strategies, are associated with higher mathematics self-concept (Table 4.16). While the use of these teaching practices is associated with higher self-concept across students, regardless of how familiar they are with mathematics, the effect is not necessarily of the same magnitude for all students.

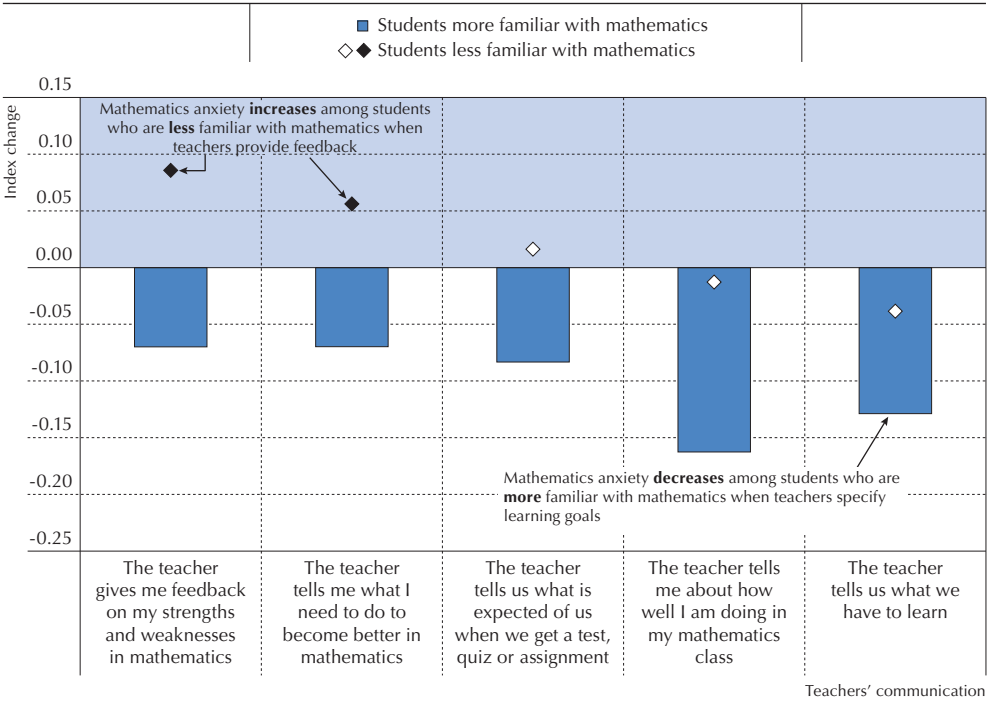
The use of teaching practices that differentiate tasks according to students' ability and encourage students to work in small groups is associated with a greater increase in self-concept among students who are less familiar with mathematics than among students who are more familiar with mathematics (Figure 4.16 and Table 4.16). Students who are less familiar with mathematics are likely to feel less competition with peers when they work in small groups and when they receive relatively more individualised teaching (Marsh, 1993; Pajares and Schunk, 2001). Giving students extra help, taking the time to explain the subject until students understand, and using

cognitive-activation strategies, such as assigning problems that require students to think for an extended time, seem to benefit all students. Students who are more familiar with mathematics benefit the most from these practices, possibly because students who do more advanced mathematics also tend to be paired with more effective teachers.

■ Figure 4.15 ■

Teaching practices and students' mathematics anxiety, by students' level of familiarity with mathematics

Change in the index of mathematics anxiety *associated with having mathematics teachers who provide feedback or specify learning goals in every or most lessons, OECD average*



Notes: The *index of mathematics anxiety* is based on the degree to which students agree with the statements: I often worry that it will be difficult for me in mathematics classes; I get very tense when I have to do mathematics homework; I get very nervous doing mathematics problems; I feel helpless when doing a mathematics problem; and I worry that I will get poor marks in mathematics.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Students with less (more) familiarity with mathematics are students in the bottom (top) quarter of the distribution of familiarity.

The results take into account students' gender and socio-economic status.

The OECD average for students who are more/less familiar with mathematics is calculated only for countries with a valid index change across both categories and across countries with available data.

Statistically significant values are marked in a darker tone. All values for students who are more familiar with mathematics are statistically significant.

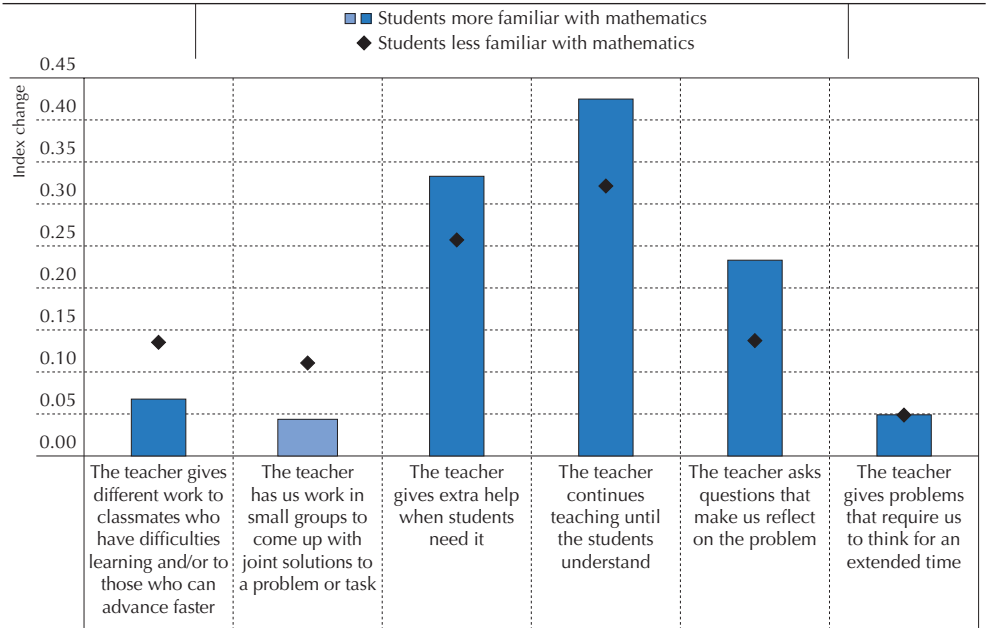
Source: OECD PISA 2012 Database, Table 4.15.

StatLink http://dx.doi.org/10.1787/888933377594

■ Figure 4.16 ■

Teaching practices and students' mathematics self-concept, by students' level of familiarity with mathematics

Change in the index of mathematics self-concept *associated with having mathematics teachers who provide feedback or specify learning goals in every or most lessons, OECD average*



Notes: The *index of mathematics self-concept* is based on the degree to which students agree with the statements: I'm just not good in mathematics; I get good grades in mathematics; I learn mathematics quickly; I have always believed that mathematics is one of my best subjects and In my mathematics class, I understand even the most difficult work.
The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Students with less (more) familiarity with mathematics are students in the bottom (top) quarter of the distribution of familiarity with mathematics.
The results take into account students' gender and socio-economic status.
The OECD average for students who are more/less familiar with mathematics is calculated only for countries with a valid index change across both categories and across countries with available data.
Statistically significant values are marked in a darker tone. All values for students who are less familiar with mathematics are statistically significant.
Source: OECD PISA 2012 Database, Table 4.16.
StatLink http://dx.doi.org/10.1787/888933377601

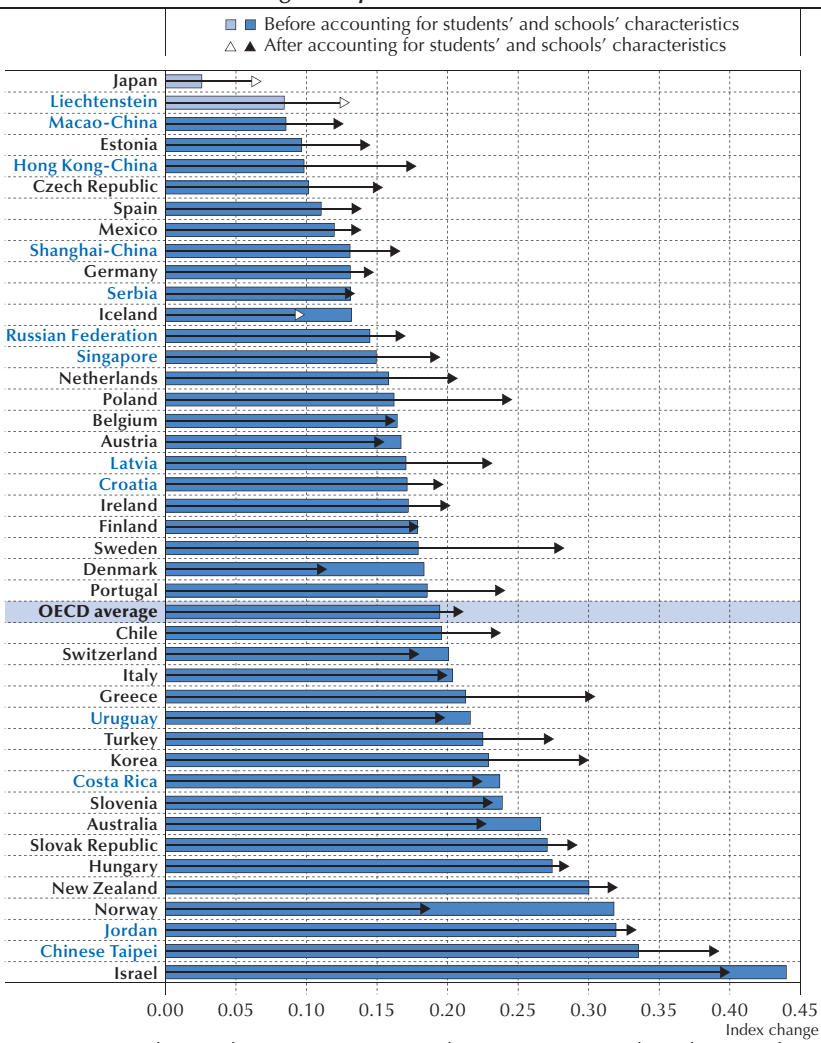
Innovative teaching instruments that foster motivation to learn mathematics

Using innovative teaching instruments can also help to engage students and spark an interest in mathematics. In particular, using technology in teaching mathematics can encourage students to become active participants during class. Figure 4.17 shows that using a computer during mathematics lessons is associated with an increase in the *index of intrinsic motivation for mathematics* (mathematics interest) corresponding to 19% of a standard deviation, on average across OECD countries. Even after taking into account students' and schools' characteristics,

■ Figure 4.17 ■

Students' interest in mathematics and their use of computers in mathematics lessons

Change in the index of intrinsic motivation *for mathematics associated with using a computer in mathematics class*



Notes: Intrinsic motivation to learn mathematics (or interest in mathematics) measures students' drive to perform an activity purely for the joy gained from the activity itself. The *index of intrinsic motivation for mathematics* is based on the degree to which students agree or disagree with the statements: I enjoy reading about mathematics; I look forward to my mathematics lessons; I do mathematics because I enjoy it; I am interested in the things I learn in mathematics.

"Students' and schools' characteristics" include: student's gender, socio-economic status and mathematics performance, and the school's socio-economic profile.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the change in the index of intrinsic motivation for mathematics *before accounting for students' characteristics.*

Source: OECD, PISA 2012 Database Table 4.17.

StatLink http://dx.doi.org/10.1787/888933377617

students' use of a computer in class is related to an increase in interest in mathematics of at least 30% of a standard deviation in Greece, Israel, Jordan, New Zealand and Chinese Taipei.

Dynamic graphical, numerical and visual technological applications (e.g. the interactive whiteboard) can provide new opportunities for teachers and students to interact with, represent and explore mathematics concepts. However, teachers need to be well-trained and practice extensively with these tools if they are to be effective (OECD, 2015b).

The importance of choosing well-framed and engaging problems

Mathematics tasks are central to students' learning because "tasks convey messages about what mathematics is and what doing mathematics entails" (National Council of Teachers of Mathematics, 1991: p. 24). The mathematics problems to which many students are exposed are often nothing more than routine exercises organised to provide practice on a particular mathematics technique that, usually, has just been demonstrated to the student (Schoenfeld,1992). The traditional mathematics class follows a linear structure: a task is used to introduce a technique; the technique is illustrated; more tasks are provided so that the student may practice the illustrated skills. Rather than being engaged in "real" problem solving, students only work on the tasks that have been set before them. This kind of routine, which assigns a passive role to students, might not stimulate interest and engagement.

Solving problems and making up new ones is the essence of mathematics (Boaler, 2015) and is what makes mathematics so intriguing to proficient practitioners. Presenting a problem and developing the skills needed to solve that problem can be more motivational for students than teaching students how to apply a procedure without a context. Teaching through real problems allows students to see a reason for learning a specific topic or concept, and thus become more deeply involved in learning it. Working with open-ended and modelling tasks, in particular, provides students with opportunities not just to apply mathematics but also to learn new mathematics concepts and practice their computational skills (Henningsen and Stein, 1997).

Teachers can more easily stimulate the interest of their students by posing problems that relate conceptual knowledge with a practical application that students find familiar. Research shows that students perform better when they are familiar with the context used to present a particular problem set – for example, when the context refers to an experience they personally lived through (Chiesi, Spilich and Voss, 1979; Alexander and Judy 1988; Alexander, Kulikowich and Schulze, 1994). Using practical applications should thus be encouraged as long as students are helped to transfer what they learn on the tasks to other contexts (see Box 1.3 in Chapter 1).

The structure of PISA assessments allows researchers to investigate how well students do in problems that are contextualised in different ways, as test items are cast in four different contexts: personal, occupational, societal and scientific. Problems classified in the personal context category focus on activities carried out by students, their families and their peers; problems in the occupational context are set in the world of work; problems in the societal context focus on the students' community, whether local, national or global; and problems classified in the scientific category relate to the application of mathematics to the natural world and to issues and topics related to science and technology (OECD, 2013a).

Figure 4.18 shows that socio-economically disadvantaged students perform relatively better on items framed in a personal context than on problems framed in the other contexts. On average across OECD countries, disadvantaged students were 39% less likely than other students to answer correctly test questions framed in a personal context, 45% less likely to answer correctly questions in a societal context, and about 50% less likely to answer correctly questions with an occupational or scientific context, after accounting for other characteristics of the tasks, such as their difficulty. A careful selection of the context of applied problem thus matters. Collaborative efforts involving mathematics researchers, teachers and students should develop and share contextualised tasks that are both challenging and engaging, particularly for those students who have low familiarity with mathematics.

DEVELOPING KNOWLEDGE OF AND ENGAGEMENT WITH MATHEMATICS AT THE SAME TIME

A coherent curriculum paired with well-structured instruction materials is a prerequisite for developing conceptual knowledge of mathematics. In turn, conceptual knowledge is important for mathematics problem solving (Chapter 3). However, simply assigning challenging mathematics tasks to students will not automatically engage them. Students cannot be expected to develop the capacity to think, reason and solve problems mathematically if teachers do not use the kinds of cognitive-activation teaching strategies that have been proven to be the most effective for student learning (Henningsen and Stein, 1997).

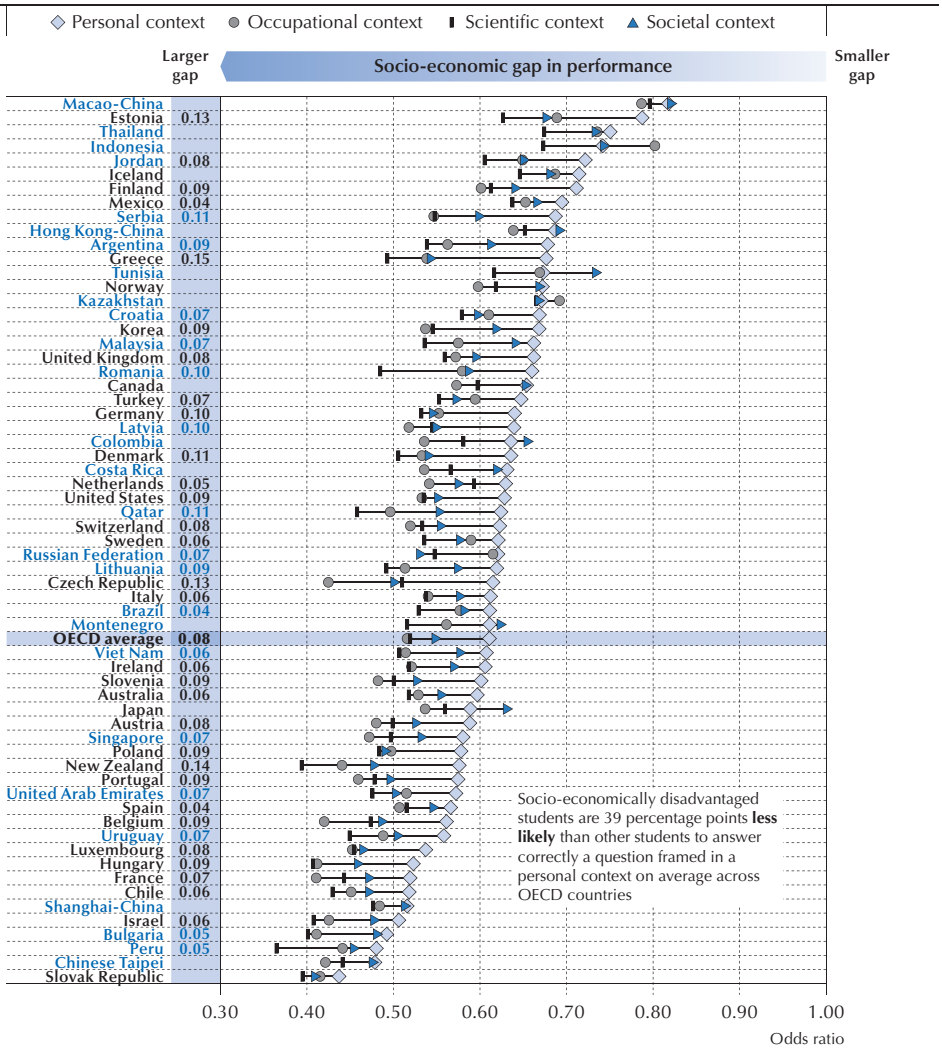
Figure 4.19 shows that students' interest in mathematics can complement exposure to content in developing mathematical literacy. On average across OECD countries, the relationship between exposure to pure mathematics and performance in PISA is stronger – by five score points – among students who are interested in what they study in class than among students who are not interested. In Hungary and Slovenia, the change in performance associated with more exposure to pure mathematics is more than 15 score points greater among students who reported that they are interested in what they learn at school than among students who reported that they are not interested.

Mathematics is more than a static, structured system of facts, procedures and concepts (Henningsen and Stein, 1997); but large numbers of mathematics students only see the facts and do not get a sense of the questions behind the answers (Boaler, 2015). Moreover, many students have problems keeping up with fast-paced mathematics lessons, and thus develop ever-widening gaps in their knowledge and understanding, and lose some self-belief. It is possible, although not easy, to make mathematics concepts more engaging and more finely tuned to the capacities of the weakest students in the class without compromising the integrity of the activity (Houssart, 2004). Everyone involved in mathematics education – teachers, school leaders, teacher educators, researchers, parents, specialist support services, school boards and policy makers, as well as students themselves – has a role to play in changing the way mathematics is taught so that it becomes more intriguing and engaging for all students (Anthony and Walshaw, 2009).

■ Figure 4.18 ■

Performance gap between disadvantaged and other students, by context of task

Change in the probability of answering correctly questions framed in different contexts associated with being in the bottom quarter of socio-economic status



How to read the chart: An odds ratio of 0.61 associated with questions framed in a personal context means that socio-economically disadvantaged students are 39 percentage points ([1-0.61]*100) less likely than other students to answer correctly a problem framed in a personal context.

Notes: Disadvantaged students are defined as those students in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS).

Odds ratios for all item contexts are statistically significant. Statistically significant differences between odds ratios referring to personal context items and odds ratios for all the other items are shown next to the country/economy name.

Countries and economies are ranked in descending order of the odds ratios for items framed in a personal context.

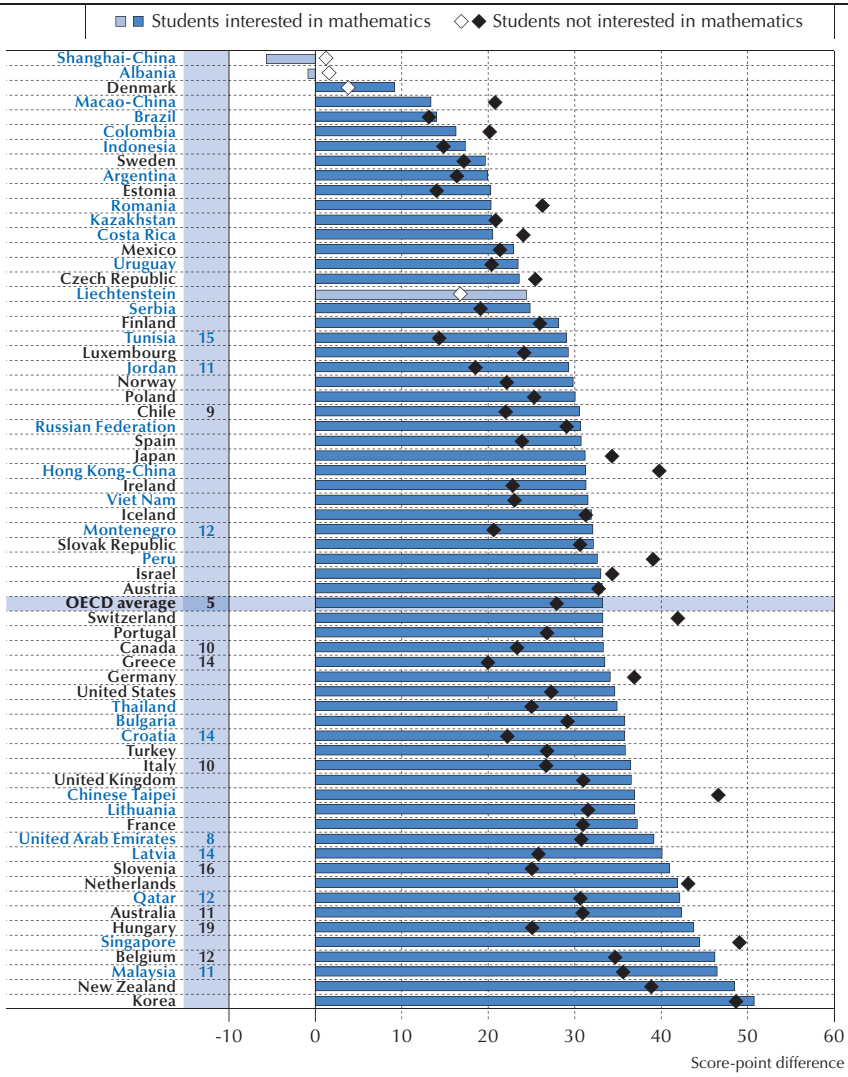
Source: OECD, PISA 2012 Database Table 4.18.

StatLink http://dx.doi.org/10.1787/888933377621

■ Figure 4.19 ■

Relationship between exposure to pure mathematics and mathematics performance, by students' interest in mathematics

Change in mathematics performance associated with a one-unit change in the index of exposure to pure mathematics, *by students' interest in mathematics*



Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Statistically significant differences between students who are interested and not interested in mathematics are shown next to the country/economy name.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the change in mathematics performance among students interested in mathematics.
Source: OECD, PISA 2012 Database Table 4.19.
StatLink http://dx.doi.org/10.1787/888933377639

Note

1. This result is robust to accounting for students' performance in mathematics.

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5

A Policy Strategy to Widen Opportunities to Learn Mathematics

Only a minority of the 15-year-old students in most countries understand and know well the core mathematics concepts in the curriculum. This chapter discusses a policy strategy to give all students similar opportunities to learn mathematics. Policy makers, curriculum designers, teachers and parents have an important role to play in the implementation of this strategy.

More than ever before, today's students need to understand mathematical ideas, compute fluently, engage in logical reasoning and communicate using mathematics. All these skills are central to a young person's preparedness to tackle problems that arise at work and in life beyond the classroom. But the reality is that many students are not reaching baseline levels of proficiency in mathematics (OECD, 2014; OECD, 2016). Large numbers of students also lack self-confidence in the subject, do not enjoy it and are unlikely to continue studying it voluntarily (Chapter 4).

How can these patterns be changed? This report shows that one possible way forward is to ensure that more students spend more "engaged" time learning mathematics concepts and practising complex mathematics tasks. The opportunity to learn mathematics content – the time students spend learning mathematics topics and practising mathematics tasks at school – can accurately predict mathematics literacy (Chapter 3). In the vast majority of countries, a substantial share of the performance disparities in PISA between socio-economically advantaged and disadvantaged students can be attributed to differences in these students' familiarity with mathematics concepts (Figure 3.16). Widening access to mathematics content can thus raise the average levels of achievement and, at the same time, reduce inequalities. While acquiring deep conceptual knowledge and procedural fluency, students should also have opportunities to practice their reasoning and modelling capabilities, and to develop positive attitudes and self-beliefs towards mathematics.

A policy strategy centred on giving all students similar opportunities to learn mathematics can reduce the number of students who lack the knowledge and understanding of mathematics expected of a 15-year-old. It can also encourage teachers to create and use engaging material to develop students' interest in mathematics. Ultimately, it can result in greater equity throughout the school system and thus in greater social mobility. Table 5.1 presents six policy recommendations that are part of this strategy.

DEVELOP COHERENT STANDARDS, FRAMEWORKS AND INSTRUCTION MATERIAL FOR ALL STUDENTS

Chapter 1 shows that only a minority of the 15-year-old students in most countries reported that they understand and know well the core mathematics concepts in the curriculum. For example, on average across OECD countries, less than 30% of students reported that they know well and understand the concept of arithmetic mean. Some students were never taught the topics, and many more were exposed to the topics so superficially that they only remember bits and pieces. Closing the gap between the expectations and the reality of mathematics learning is possible if education systems set the right objectives, and have the means to transform these objectives into everyday teaching and learning.

A first important step is to develop curriculum standards and frameworks that clearly define the mathematics content to be covered at each cycle of schooling and the targets to be reached by the end of compulsory schooling. Standards represent potential opportunities to learn (Schmidt and Burroughs, 2013) and define what students are expected to understand and be able to do. A framework is a more detailed list of the content and performance standards, by

grade level, that guides the development of the curriculum and the selection of instructional materials.

■ Table 5.1 ■

Policy recommendations to widen opportunities to learn

Policy recommendation	Goal	Who should be involved
Develop coherent standards, frameworks and instruction material for all students	▪ Reduce the number of students who have only superficial knowledge of core mathematics ideas ▪ Set high expectations for all students ▪ Set accountability targets while preserving teachers' autonomy	▪ Education policy makers, curriculum designers, teachers and those involved in the development of textbooks, assessments and teaching material
Help students acquire mathematics skills beyond content knowledge	▪ Strengthen the connections between school mathematics and the mathematics skills students will need for their future ▪ Reduce inequalities in quantitative skills ▪ Make school mathematics more engaging for all students	▪ Curriculum designers, teachers and those involved in the development of textbooks, assessments and teaching material
Reduce the impact of tracking on equity in exposure to mathematics	▪ Reduce the impact of socio-economic status on students' opportunities to learn ▪ Ensure that students in vocational tracks are exposed to a coherent curriculum	▪ Education policy makers
Learn how to handle heterogeneity in the classroom	▪ Give socio-economically disadvantaged students the same opportunities to learn as advantaged students ▪ Provide high-performing students with more challenging material	▪ Education policy makers and teachers
Support positive attitudes towards mathematics through innovations in curriculum and teaching	▪ Improve students' self-concept ▪ Reduce students' anxiety ▪ Foster motivation to learn mathematics, also after school hours	▪ Education policy makers, curriculum designers, teachers and parents
Monitor and analyse opportunities to learn	▪ Better understand the obstacles teachers face in covering the curriculum ▪ Better understand the obstacles students face in learning specific material ▪ Identify the impact of curriculum changes on students' acquisition of knowledge and skills	▪ Education policy makers, in collaboration with the research community

Content standards and frameworks should set the stage for the development of a core mathematics curriculum that provides all students, no matter their ability or socio-economic status, with the mathematics foundation for quantitative literacy and for study at a higher level. Giving all students equal opportunities to learn depends on how the curriculum is implemented (which will

be discussed later); but applying the same standards to all students can be a first step to break the pattern of differentiation that is based on the false assumption that only some students will need a strong background in mathematics for their future. For example, policy efforts to standardise secondary school curricula in Scotland in the 1980s improved average test scores and reduced socio-economic inequalities in education (Gamoran, 1996). Standards do not set upper bounds in content coverage and expected performance, so schools and teachers can adapt them to challenge high-performing students.

The standards should cover the full cycle of education to allow for the widest possible range of students to participate in a rigorous course of study from the outset. No standard can fully reflect the variety of abilities and needs among students in any given classroom. However, standards can provide clear signposts along the way to higher education and career readiness for all students, and can help identify early on those students who need special support.

What content should be specified in the standards and frameworks? International comparisons find that the same set of core mathematics ideas is covered in the curricula of most PISA countries and economies (Table 1.3). This suggests that some consensus about what mathematics content is important may already have been reached (Mullis et al., 2012). What varies greatly across countries is how this core content is organised in the frameworks.

The curriculum can provide more productive exposure to mathematics content across all students if it is coherent, focused and not repetitive. Coherence can be achieved by grouping the concepts into units that, in their logical sequence, reflect the hierarchical nature of mathematics and make it easier for students to build on the skills they acquired (Schmidt et al., 2002).

When students understand the connections between the topics, they stop seeing mathematics as a laundry-list of formulas to memorise, and start to make sense of what they learn. A sequential or spiral curriculum framework is particularly useful for students who may not have been exposed to certain content. Before starting a new unit, teachers can easily identify what prior knowledge is needed, whether their students have had the opportunity to acquire this knowledge, and whether it is useful to review or incorporate previously studied topics in the new unit.

Coherence across topics and a strong focus on key mathematics ideas matter more than the quantity of topics covered. Tightening the focus of the curriculum by addressing fewer topics each year can allow for much greater depth of learning. For example, in Singapore the mathematics framework covers a relatively small number of topics in depth, following a spiral organisation in which topics introduced in one grade are covered in later grades, but at a more advanced level. Students are expected to have mastered prior content, not repeat it (Ginsburg et al., 2005). Korea cut as much as 20% of the learning content in the 2011 revision of the curriculum in order to let teachers cover mathematics topics in greater depth (Li and Lappan, 2013).

Curriculum frameworks that allocate sufficient time to a set of key topics can also leave more scope for teachers to choose how to cover the material and to adapt their teaching to their specific classroom context. Standards are meant to support teachers in their choices, and not to unnecessarily limit their autonomy. Teachers still have to use their professional judgement,

creativity and autonomy individually and together with other teachers to find the best ways to help their students to learn. For example, teachers in Singapore are not bound by the sequence of topics as long as they can ensure that the inherent hierarchy and linkages of mathematics are maintained (Ginsburg et al., 2005).

The effectiveness of standards to promote innovative patterns of instruction is clearly related to the development of tools and training, including teachers' guides, textbooks and pre-service education that prepare teachers to teach the curriculum and provide opportunities for professional development. Textbooks, in particular, can disrupt the coherence of the standards if they introduce concepts too early or too late, if they superficially cover the same concepts year after year, or fail to show students explicitly how the new material connects to previous concepts. More efforts at the national and international levels should thus be devoted to defining and regularly updating the core criteria that mathematics textbooks should meet to be eligible for use in the classroom.

HELP STUDENTS ACQUIRE MATHEMATICS SKILLS BEYOND CONTENT KNOWLEDGE

The mastery of core concepts and procedures is a necessary component of mathematics learning and is needed to understand and solve novel problems. However, knowledge of concepts or formulas and procedural fluency alone are hardly sufficient for solving complex problems (Chapter 3); several other skills are central to mathematics proficiency. These include the ability to use a wide range of mathematics strategies; the ability to reason using mathematical ideas and to communicate one's reasoning effectively; the ability to use the knowledge and time at one's disposal efficiently; and the disposition to see mathematics as sensible, useful and worthwhile, coupled with a belief in one's own efficacy (Schoenfeld, 2006; National Research Council, 2001).

Content, representations, tasks and teaching materials should be chosen and organised within and across grades to support the integrated and balanced development of all these abilities. Mathematics frameworks should thus be explicit about how concepts and skills are interwoven within individual units, over a year, and during the course of education.

Outside of their coursework, many students have difficulty doing what may be considered elementary mathematics for their level of attainment. For example, only 22% of the students tested in PISA 2012 were able to interpret a simple equation and explain the effect of a change to one variable on a second variable (the DRIP RATE problem, see Chapter 3). This probably happens because too many students spend too much time routinely and mechanically solving well-defined tasks that are very close to the ones they have been taught. These tasks do not involve exploration, conjecturing and thinking…in other words, they do not provide opportunities for deep learning. Similarly, simple applied tasks that are commonly used at school, such as "calculate how many square metres of tiles you need to cover a floor" (see Chapter 1), are routine mathematics tasks "dressed up" in the words of everyday life, and do not require any deep thinking and modelling skill (Echazarra et al., 2016). Chapter 3 shows that students' exposure to this type of applied task has only a modest relationship with students' capacity to solve PISA problems (Figure 3.8b).

Problem solving, as a method of teaching mathematics, can be used to introduce core mathematics concepts through lessons involving exploration and discovery (Stein et al., 2008). For example, the PISA 2012 REVOLVING DOOR item (see Chapter 3) describes a revolving door through diagrams. Students are asked to calculate the maximum arc length that each door can have so that air never flows between entrance and exit. Solving in class a problem like REVOLVING DOOR can help students consolidate their understanding of circle geometry and, at the same time, show them how they can test multiple heuristic strategies, such as dividing the circled space of the revolving door into six equal parts or reworking the diagram to consider extreme cases. Whatever strategy students choose to use, in order to succeed they need to monitor how well they are progressing, and persevere or change direction accordingly. Knowledge, effective reasoning, strategy formulation, self-regulation and perseverance can all be developed while working on problems of this type.

A greater emphasis on problem solving does not make traditional topics obsolete or irrelevant. The core mathematics topics in school curricula (fractions, functions, vectors…) are basic to all mathematics activities, including modelling and application. Whether one can solve mathematics problems depends in fundamental ways on the mathematics one knows. For example, students who do not know the formula for the circumference (π times diameter) would have a very difficult time solving complex, contextualised problems like REVOLVING DOOR or ARCHES (Chapter 3).

But introducing problem-solving strategies – such as teaching students how to question, make connections and predictions, conceptualise, and model complex problems – requires time and probably some adjustment to content coverage. Chapter 2 shows that cognitive-activation teaching strategies, such as giving problems with no immediate solution, might be associated with less content coverage in disadvantaged learning environments (Figure 2.23b).

When teachers develop a routine that allows students to discover and work with mathematics, teachers also need to consider how students with different skills approach complex problems. Chapter 3 shows that disadvantaged students perform more poorly than other students in those tasks requiring modelling skills (Figure 3.17). Problem solving, modelling and application make mathematics lessons more demanding, both for teachers and for students. Weaker students – and particularly those from a disadvantaged background – are less confident and tend to prefer more external direction (Lubienski and Stilwell, 1998). These students might need additional support in, for example, identifying the intended mathematical ideas embedded in contextualised problems, or describing those ideas to the rest of the class. That said, mathematics teachers should not be discouraged from integrating problem solving in their instruction when teaching weaker classes. Students with less familiarity with mathematics can still participate if the teacher builds a supportive relationship with students, conducts individualised tutoring sessions, builds on what students know, preserves equity among students in the classroom and makes explicit the desired classroom norms (Lester, 2007; Boaler, 2002; Lubienski, 2002). Formal and informal teacher networks can be useful platforms for sharing experiences and ideas.

Restructured textbooks, teaching materials and dedicated training can help minimise the time needed to incorporate these teaching practices into an already full schedule. For example, the New Zealand government's website on mathematics education (http://nzmaths.co.nz/)

provides teaching materials on problem solving, such as sample problems that fit into a given curriculum unit. The Mathematics Assessment Project, a collaboration between the University of California at Berkeley and Nottingham University, developed a series of "formative-assessment lessons" whose purpose is to help teachers improve students' ability to apply their knowledge to non-routine problems. The lessons describe common patterns of student responses to the tasks and ways to deal with them; they also contain activities that help teachers assess students' understanding and build on it.[1]

Mathematics teachers would also find it easier to integrate problem solving in instruction if the assessment system reflected the value of this approach. In reality, few curricula make the ability to model and apply mathematics knowledge the object of systematic assessment and testing (Rosli et al., 2013). More efforts need to be invested in constructing, using and sharing new ways to assess mathematics problem-solving abilities.

REDUCE THE IMPACT OF TRACKING ON EQUITY IN EXPOSURE TO MATHEMATICS

Access to mathematics is unequally distributed across individuals, schools and school systems, and familiarity with mathematics is strongly related to students' socio-economic status (Chapter 2). Moreover, the organisation of most education systems tends to reinforce socio-economic inequalities in access to mathematics (Chapter 2). Selecting student into more homogenous groups through grade repetition, between-school tracking, academic admission requirements and school transfers is associated not only with a more unequal distribution of achievement, but also with more unequal access to mathematics content, which is the basis for improving mathematics literacy.

Students who are enrolled in vocational tracks are much more likely to come from disadvantaged families and to have low levels of familiarity with mathematics (Figure 2.16). The concentration of disadvantaged students in less challenging tracks strengthens the link between socio-economic status and opportunities to learn, not only because students in vocational programmes are unlikely to receive the same exposure to mathematics as students in academic tracks, but also because students' outcomes and attitudes towards mathematics are affected by their peers (Chapter 4; Field, Kuczera and Pont, 2007).

Moreover, the starting age of between-school tracking at the system level is strongly related to equity in opportunities to learn mathematics (Figure 2.15). In systems that start sorting students into different tracks as early as 10 or 11, the relationship between students' socio-economic status and their access to mathematics at age 15 is much stronger than in systems where students are first tracked at age 15 or 16.

Postponing the age at which students are first sorted into tracks can be difficult, given the costs involved in such a substantial reform, possible effects on drop-out rates, and the reluctance among teachers who may have to adjust their teaching methods to cater to a more heterogeneous group of students. But several countries have been successful in delaying the age at first tracking, and there is some evidence that this policy has reduced the gap in education and labour market outcomes related to socio-economic status (Meghir and Palme, 2005; Hanushek

and Woessmann, 2006). In Sweden and Finland, which reformed their education systems between the 1950s and the 1970s, a later age at tracking reduced inequalities in the labour market later on (Meghir and Palme, 2005; Pekkarinen, Uusitalo and Kerr, 2009; Pekkala Kerr, Pekkarinen and Uusitalo, 2013). More recently, Germany reformed its education structure to reduce the influence of socio-economic status on student achievement. Some regions delayed the assignment of students to different tracks until they were 12 rather than 10 years old; some regions chose to combine tracks (going from a three- to a two-track lower secondary system); and some regions increased their system's flexibility by allowing students in any of the three types of lower secondary school to go to any type of upper secondary school (OECD, 2011a). In 1999, Poland reformed the structure of its education system, deferring tracking in secondary education, embracing deep curriculum reform, and giving more autonomy to schools. Research has shown that the deferral of tracking contributed to the substantial improvement in international assessments (OECD, 2011b).

It may not be necessary to eliminate early tracking as long as the education system provides students with equal opportunities to learn. In the Czech Republic and Singapore, for example, where students are first tracked at age 11 or 12, students' and schools' socio-economic profile explains less than 10% of the variation in familiarity with mathematics, similar to the OECD average (Figure 2.15). In other words, students in vocational tracks should be exposed to the same core curriculum and to the same quality of mathematics teaching as other students, and the tracking system should be flexible enough to allow students to change tracks when and if they are ready to do so. Moving towards equivalence in mathematics instruction between pathways would ensure that students can choose their preferred course of study and be confident they will acquire the core skills they need for their adult life.

LEARN HOW TO HANDLE HETEROGENEITY IN THE CLASSROOM

Given the difficulty of delaying early tracking, some countries have replaced between-school tracking with ability grouping within schools or within classrooms. Selection for within-school grouping may be better informed, since students' abilities are more easily observed in individual schools; but within-school or within-class ability grouping reduces opportunities to learn for disadvantaged students just as tracking does. In fact, the relationship between socio-economic status and mathematics achievement is not necessarily weaker in systems that use ability grouping compared with systems that use between-school tracking (Chmielewski, 2014). The negative impact of early tracking, streaming and grouping by ability on equity of education outcomes can be mitigated by limiting the number of subjects or the duration of ability grouping and by increasing opportunities to change tracks or classrooms (OECD, 2012).

The real alternative to streaming and ability grouping is heterogeneous classes. Teaching these classes can be challenging, and education authorities may have to provide additional assistance, such as more personalised tutoring and/or more innovative teaching practices, to the students who would otherwise be placed in "low" tracks. Schools with mixed-ability classes must also avoid lowering academic standards and must provide their high-achievers with challenging material (Gamoran, 1996; Gamoran, 2002).

Teach heterogeneous classes effectively

It is easier and more efficient for teachers to deal with a smaller range of abilities than to provide instruction that is sufficiently broad to address the needs of all students or to "teach to the middle" (Darrow, 2003; Evertson, Sanford and Emmer, 1981). According to PISA results, many teachers believe that heterogeneous classes hinder learning, especially in socio-economically disadvantaged schools (Figure 2.17). Clearly, teachers need to adapt their teaching strategies depending not only on the average ability of the class but also on the degree of ability heterogeneity. For instance, Chapter 2 shows that some teaching practices, such as cognitive-activation strategies, are related in different ways to familiarity with mathematics and performance in mathematics across schools with different socio-economic profiles (Figure 2.23a and 2.23b).

Nevertheless, there are ways to help teachers work with heterogeneous classes other than by sorting students. As was mentioned above, curricula can be organised in a spiral, so that they cover key ideas several times, in order to provide students with multiple opportunities to learn important concepts at varying levels of complexity. In addition, classes can be made smaller as a way to make it less difficult to teach heterogeneous groups (Finland did this in the 1980s when it discontinued ability grouping [Kupari, 2008]).

Moreover, specific teaching practices, such as using curricula and pedagogies with multiple points of entry that are challenging, relevant and engaging, can be adopted in the context of whole-class instruction. Various student-oriented practices also appear to be successful in heterogeneous class environments, such as having students work in small (heterogeneous) groups, co-operative learning, keeping students actively involved and giving students control over their learning (Freedman, Delp and Crawford, 2005; Rubin, 2006). In particular, flexible grouping can be frequently reconfigured based on content, project and ongoing evaluation, as a way to nurture the idea that ability is not fixed, and to reduce the segregation that often comes with more rigid forms of grouping (Tomlinson, 2001). Co-operative learning strategies are also used in heterogeneous classes (Rothenberg, Mcdermott and Martin, 1998). Success for All, for example, is a programme for primary schools in the United States and the United Kingdom that combines co-operative learning with small ability groups that are frequently reorganised to reflect student progress (Loveless, 2016).

Given the crucial role of teachers in implementing curricula in challenging contexts, teachers who shift from teaching classes grouped by ability to teaching heterogeneous classes, as well as teachers addressing students in multicultural and diverse environments, should receive support and training on how to teach such classes (OECD, 2010; Rubin and Noguera, 2004).

Offer greater and individualised support to struggling students

Struggling students should receive instruction adapted to their ability and needs. For many low-performing students, this may include simply *more time* to learn. Engaged learning time is a key aspect of opportunity to learn, in addition to the curriculum content to which students are exposed. Longer instruction time is related to better performance – except when that learning time is wasted in a disruptive disciplinary climate (Chapter 3). The so-called "No Excuses" charter

schools in the United States provide an example of how combining dramatically increased instructional time, strict behaviour norms and a strong student work ethic has improved student achievement in low-income, minority and disadvantaged schools (Angrist et al., 2010; Thernstrom and Thernstrom, 2004).

In addition to more instruction, many disadvantaged students also need more tailored approaches and individualised support. In the context of the US debate on "de-tracking" (intended as the shift from ability grouping to mixed-ability, heterogeneous classes), targeted academic support is considered a key practice to be embedded in the regular organisation of the school year or day. Support classes can help struggling students catch up on skills and concepts they may have missed in the classroom, and can support them in completing their daily work, without preventing them from being exposed to a more academically demanding curriculum (Rubin, 2006). High-dose, targeted tutoring is one of several practices that have been shown to improve achievement in low-performing US charter schools (Dobbie and Fryer, 2013; Fryer, 2011). Of course, the effectiveness of this remedial support depends on the quality of its implementation, and particularly on teachers' preparedness.

Finland, for example, provides dedicated support to students in need. Half of children with special education needs are mainstreamed, rather than being enrolled in special schools. These students are helped by "special teachers" assigned to each school, based on the idea that if schools focus on early diagnosis and intervention, most students can be helped to achieve success in regular classrooms. These specially trained teachers work closely with the class teachers to identify students in need of extra help and to work individually or in small groups with these students so that they get the support they need to keep up with their classmates (OECD, 2011a).

Low-performing students in Singapore are given extra help from well-trained teachers and follow an alternative mathematics framework that covers all the mathematics topics in the regular framework, but at a slower pace and with more repetition (Ginsburg et al., 2005).

SUPPORT POSITIVE ATTITUDES TOWARDS MATHEMATICS THROUGH INNOVATIONS IN CURRICULA AND TEACHING

The types of mathematics that students are exposed to are related not only to their achievement but also to their attitudes and self-beliefs about mathematics (Chapter 4). Exposure to relatively complex mathematics topics may improve the attitudes and self-beliefs of students who are relatively well-prepared and ready to be challenged, but it may undermine the self-beliefs of students who do not feel up to the tasks. In Belgium, Denmark, Macao-China, the Netherlands and Switzerland, students of similar ability who are more exposed to pure mathematic tasks tend to display lower mathematics self-concept than students who are less exposed (Table 4.6). On average across OECD countries, greater exposure to complex concepts, as measured by familiarity with mathematics, is associated with less mathematics anxiety among high-performing students but with greater anxiety among low-performing students (Figure 4.8).

Given the link between attitudes towards mathematics and performance (OECD, 2013), in designing or revising their mathematics curricula, countries should find ways of improving performance and problem-solving skills without undermining the self-confidence or raising

the anxiety of low-performing students. Box 4.2 offers some examples from Australia, Hong Kong-China, Korea and Singapore of how the development of positive attitudes towards mathematics became an aim of the mathematics curriculum, and of how some of these countries and economies have reduced curriculum content to give more time to engaging activities that would improve students' motivation.

Teaching practices can play a big role in influencing students' attitudes towards learning. Chapter 4 discusses how various teaching practices – including feedback and communication, cognitive-activation strategies and student-oriented practices – are associated with students' attitudes and self-beliefs towards mathematics, even though the extent of the relationship varies according to students' familiarity with mathematics. Reinforcing basic numerical and spatial skills, reducing time pressure during tests, and bolstering teachers' ability and confidence to teach mathematics can also be ways of reducing students' mathematics anxiety (Maloney and Beilock, 2012; Beilock and Willingham, 2014).

However, it is also important to take into account that teaching practices and curriculum coverage may interact in affecting students' self-beliefs, especially among students who have had fewer opportunities to learn. Teaching practices aimed to reinforce students' self-beliefs and improve their attitudes towards learning may need to be adapted to students' readiness to learn the mathematics specified in the curriculum.

Teachers can also help to improve students' attitudes towards mathematics by engaging parents. Chapter 4 has also shown that parents are likely to transmit their mathematics anxiety to their children, especially through their involvement in homework. Teachers can make parents aware of their influence and of the importance of communicating positive messages about mathematics, and can suggest alternative ways of supporting their children in learning mathematics (such as external tutors or after-school classes).

MONITOR AND ANALYSE OPPORTUNITIES TO LEARN

The content of instruction, as defined in the curriculum, plays a fundamental role in students' mathematics achievement (Chapter 3; Gamoran, 2007; Schmidt et al., 2015). However, some would argue that what matters more is how a curriculum is implemented rather than the curriculum itself (Chapter 1). In that case, data collection and analysis must try to determine how well teachers cover the curriculum and whether the implemented curriculum adheres to the objectives set in the standards.

Measuring opportunities to learn involves more than a simple time metric. Data need to be collected all along the progression from what educators and policy makers specify in the standards, through the content to which students are exposed, to the ideas and practices that students understand (Floden, 2002). An analysis of data at each step of the progression can help clarify how much time teachers spend teaching each topic in the curriculum, how many items in a national assessment are devoted to the topic, and the degree to which students engage in the corresponding instructional activities.

Monitoring opportunities to learn thus requires combining multiple data-collection instruments and analyses. A core set of information should be collected directly from teachers, using either surveys questions that ask teachers about the content covered in their classes (such as those used in the Trends in International Mathematics and Science Study [TIMSS]), or teacher logs completed over the course of the school year. After comparing multiple indicators, Gamoran et al. (1997) concluded that the measure of OTL that more strongly correlates with students' achievement is an index that combines teacher-reported information about the proportion of class time spent on topics with information on the extent to which the instruction engaged students' problem-solving abilities. Collecting and processing log data and questionnaires can be made less cumbersome and costly by adapting digital approaches from other types of data collections, such as time-use studies.

Students' self-reported knowledge and exposure to mathematics tasks, as collected in PISA (see Chapter 1), provides another piece of information about what students actually get out of mathematics instruction. Collating the information on the intended curriculum (from official documents), on the implemented curriculum (from teachers' reports and textbooks), and on what students know and can do in class (from students' self-reports) can give a detailed picture of any missing steps in the progression from curriculum intentions to student outcomes.

All these data are of little value if they are not properly used to guide changes in the curriculum. In some countries and economies, including France, Hong Kong-China, the Netherlands, Singapore, the United Kingdom and the United States, national centres conduct multi-year research and curriculum-development programmes in school mathematics. These institutes and universities should engage in rigorous statistical comparisons of student performance under "traditional" and "experimental" curricula (where performance is evaluated on the basis of carefully designed assessments of different mathematics skills). These evaluations help identify what innovative elements in the curriculum should be strengthened. Ideally, results disaggregated by characteristics of the school (such as its socio-economic profile) could provide evidence of the effects of the curriculum and the kinds of support structures that are helpful in different implementation contexts (Schoenfeld, 2006).

Students' learning depends not only on the content covered by the teacher but also on other dimensions of the learning environment, such as an orderly classroom climate and the pedagogical choices teachers make. For example, Chapter 3 shows that long instruction time is related to high performance only in classrooms with a good disciplinary climate (Figure 3.6). Video studies can provide data directly from the classroom about how teachers structure and manage their classes, what types of support and student orientation they adopt, and whether they use teaching methods and tasks that challenge students' cognitive abilities, self-beliefs and attitudes (Tomáš and Seidel, 2013). Video studies can also showcase how students acquire mathematics knowledge, skills and understanding, and thus inform the design and update of mathematics standards. The Teaching and Leaning International Survey (TALIS) study is piloting an international video study of teaching practices. The objective of the study is to provide insights into effective teaching practices using classroom observations from countries with different teaching cultures. The pilot will also eventually produce a global video library showcasing a variety of teaching practices in a range of educational settings in participating countries.[2]

Notes

1. The formative assessment tasks and more information on the Mathematics Assessment Project are available at: map.mathshell.org.

2. More information on the TALIS international video study is available at: https://www.oecd.org/edu/school/TALIS-2018-video-study-brochure-ENG.pdf.

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PISA

Equations and Inequalities

MAKING MATHEMATICS ACCESSIBLE TO ALL

More than ever, students need to engage with mathematics concepts, think quantitatively and analytically, and communicate using mathematics. All these skills are central to a young person's preparedness to tackle problems that arise at work and in life beyond the classroom. But the reality is that many students are not familiar with basic mathematics concepts and, at school, only practice routine tasks that do not improve their ability to think quantitatively and solve real-life, complex problems.

How can we break this pattern? This report, based on results from PISA 2012, shows that one way forward is to ensure that all students spend more "engaged" time learning core mathematics concepts and solving challenging mathematics tasks. The opportunity to learn mathematics content – the time students spend learning mathematics topics and practising maths tasks at school – can accurately predict mathematics literacy. Differences in students' familiarity with mathematics concepts explain a substantial share of performance disparities in PISA between socio-economically advantaged and disadvantaged students. Widening access to mathematics content can raise average levels of achievement and, at the same time, reduce inequalities in education and in society at large.

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