Mathematics for a new Taj Mahal

Secondary: (ages 11 – 14)	Mathematics

The Taj Mahal is one of the wonders of the world and most visitors cannot wait to get an iconic photo of the beautiful building. Look closely and you'll see a great example of line symmetry: one vertical line down the middle, and one along the waterline. In this task, students question how the Taj Mahal was constructed and use their knowledge of symmetry, lines, angles, geometrical shapes, and diagrams to make their own designs like the Taj Mahal.

NB: This lesson plan includes annexes with adaptations for remote learning and additional assessment possibilities

Time allocation	3-4 lesson periods
Subject content	Apply mathematics to everyday life Calculate area, length, and other geometrical properties Use parallel and perpendicular lines to reflect symmetry
Creativity and critical thinking	This unit has a **creativity** and **critical thinking** focus: • Generate and play with a range of strategies and unusual ideas to envision how to solve a real-life maths problem • Consider different perspectives and reflect on strengths and limitations of proposed solutions
Other skills	Collaboration, Communication, Persistence/Perseverance
Key words	geometry; trigonometry; arithmetic; design; symmetry; architecture; inquiry; India; ratios; angles; parallel; perpendicular

Products and processes to assess

Students discuss and create construction plans for a new Taj Mahal and use creative and critical thinking to identify ways to use mathematical principles to support this. They respond to mathematical challenges and produce written work responding to a series of questions about real-life applications of mathematics. At the highest levels of achievement, they consider several ways of formulating and answering problems, some of which are novel or original, and are able to explain and justify their positions.

Teaching and Learning plan

This plan suggests potential steps for implementing the activity. Teachers can introduce as many modifications as they see fit to adapt the activity to their teaching context.

Step	Duration	Teacher and student roles	Subject content	Creativity and critical thinking
1	Lesson period 1	The teacher introduces the lesson with a brief history of the Indian civilization and the various challenges historians, archaeologists, and others have faced to understand how the Taj Mahal was constructed. The teacher may decide at this point to ask the students to think of and list as many questions as they can about how the Taj Mahal was built. As appropriate, this can be followed with an initial introductory discussion of possible ways in which different branches of mathematics can help to plan the design and construction of buildings.	Relating mathematical concepts to real life: geometry, trigonometry, scale ratios, pulley systems etc., as appropriate to teaching context Concept of symmetry in mathematical terms	Building understanding of the context of the problem Posing questions to identify gaps in knowledge
2		The teacher sets students the challenge of creating a construction plan for a building like the Taj Mahal and makes them aware of the resources they will have for their construction, i.e.: * Workers have different geometrical shaped blocks. These are made of wood and can be attached to the other blocks by their edges. * Each block weights 10 to 15 kilograms. You will need to arrange the blocks taking into account their shapes and sizes in order to make a symmetrical building. * You can have as many workers as you need. The construction project employed around 20,000 artisans under the guidance of a board of architects led by Ustad Ahmad Lahauri. The domed marble tomb is part of an integrated complex consisting of gardens and two red-sandstone buildings surrounded by a crenelated wall on three sides.	Calculating area, length, and other geometrical properties Exploring symmetry and ratios	Generating questions and playing with unusual ideas to work out how to use mathematics to approach a real-life problem. Envisioning and planning how to meaningfully solve a mathematics problem Considering different perspectives and

		* You have a plentiful supply of wooden blocks. You also have all the water, sand, and clay you need for making additional structures like fountains and a garden. Students organize themselves into teams of 4-5 and begin discussing the challenge. What do they need to work out in order to come up with a construction plan? (e.g. How will they know where to place materials and how will they transport them etc.) Teacher can visit each group to encourage students to be as imaginative in their thinking as possible, help them organize their ideas, and prompt them to consider the challenges listed in the appendix, as appropriate. Students then begin to work through the challenges, where necessary drawing on their knowledge of mathematics to generate possible solutions. Students should keep a record of their thinking, calculations, and diagrams used or produced at each step of the process	Using arithmetic to create work plans, estimates of the amount of materials needed, etc. Identifying relevant mathematical concepts and drawing on knowledge of mathematics to support planning process	challenges linked to the proposed solutions
3	Lesson period 2 (this may continue into a third period)	The teacher checks on progress of groups thus far and provides guidance to continue with their work. Students can be encouraged to assess and revise their thinking as appropriate and the teacher may help them become aware of any assumptions they are making (e.g. regarding materials, people, time, resources etc.). If the teacher has a particular related area of mathematical knowledge they want to reinforce, they may choose to present this to the class as a whole at this point. Students continue to work on their plans with their groups until they have finished, when they can be asked to present their work to the rest of the class if desired.	Opportunities to reinforce particular areas of mathematical knowledge as related to the local curriculum	Reflecting on steps taken so far, identifying and challenging own assumptions, and reviewing alternatives
4	Lesson period 3/4	The teacher provides challenge questions for students to answer independently about the activity, for example: 1.		

	a) What was the logic behind this building? b) What was the significance of this building? c) Why is symmetry used in the buildings? d) How did mathematics help you meet this challenge? 2. Which part of your work are you most proud of? Why? Which part of the exercise was the most difficult for your group? Why? After completing the activity, what questions do you still have? 3. Write an argumentative paragraph in response to the following question: *"Building on the exercise, discuss how and why symmetry can be used in the design of buildings and other objects"* Students can be asked to give examples which include references to actual measurements in their answer. A potential extension is also to ask students to discuss potential advantages of asymmetry relative to symmetry: for instance, is it more or less constraining than symmetry? 4. After looking at all the evidence we have studied in class overall what did you learn about Mughal architecture from our symmetrical building exercise? What did you learn about mathematics? What, if anything, are you still unsure or uncertain about? What was your personal contribution to the group's success? How did your group effectively work together? Students answer questions independently in writing and conclude the lesson by reporting back in small groups their findings and challenges.	Understanding the use of symmetry Summarizing how and why they used mathematics (arithmetic, geometry, trigonometry etc.) during the challenge Communicating mathematical knowledge gained and remaining challenges	Reflecting on steps taken to pose and solve a problem using mathematics Explaining strengths and acknowledging uncertainty and limits of proposed plans

Resources and examples for inspiration

Web and print

- Blueprints, history, and picture of the Taj Mahal: https://en.wikipedia.org/wiki/Taj_Mahal

Other

- Projector, poster board, markers/pencils, lined/graph paper, scale, measuring tape

Opportunities to adapt, extend, and enrich

- A more extended consideration of the Golden Ratio could be incorporated into this activity. See, for example, https://www.mathsisfun.com/numbers/golden-ratio.html
- Links could be made to history, geography, visual arts, physical science, and intercultural understanding.

Creativity and critical thinking rubric for maths

- Mapping of the different steps of the lesson plan against the OECD rubric to identify the creative and/or critical thinking skills the different parts of the lesson aim to develop

	CREATIVITY Coming up with new ideas and solutions	Steps	CRITICAL THINKING Questioning and evaluating ideas and solutions	Steps
INQUIRING	Make connections to other maths concepts or to ideas from other disciplines	1	Identify and question assumptions and generally accepted ways to pose or solve a maths problem	1,3
IMAGINING	Generate and play with several approaches to pose or solve a maths problem	2	Consider several perspectives on approaching a maths problem	2,3
DOING	Pose and envision how to solve meaningfully a maths problem in a personally novel way	2	Explain both strengths and limitations of different ways of posing or solving a math problem based on logical and possibly other criteria	4
REFLECTING	Reflect on steps taken to pose and solve a maths problem	3,4	Reflect on the chosen maths approach and solution relative to possible alternatives	3,4

Annex 1: Remote learning adaptations

This plan suggests potential steps for implementing the activity in online modes of delivery. Teachers can introduce modifications as they see fit to adapt the activity to their teaching context.

Step	Duration	Teacher and student roles	Subject content	Creativity and critical thinking	Adaptations and considerations for online modes of delivery
1	Lesson period 1	The teacher introduces the lesson with a brief history of the Indian civilization and the various challenges historians, archaeologists, and others have faced to understand how the Taj Mahal was constructed. The teacher may decide at this point to ask the students to think of and list as many questions as they can about how the Taj Mahal was built. As appropriate, this can be followed with an initial introductory discussion of possible ways in which different branches of mathematics can help to plan the design and construction of buildings.	Relating mathematical concepts to real life: geometry, trigonometry, scale ratios, pulley systems etc., as appropriate to teaching context Concept of symmetry in mathematical terms	Building understanding of the context of the problem Posing questions to identify gaps in knowledge	NB: Many of these adaptations rely on students and teachers having access to internet-connected devices. In contexts, where this is not the case, more use could be made of phone calls, WhatsApp type applications, and paper worksheets. The introduction could be a recorded video that students watch asynchronously or a live whole-class video conferencing call can be held to begin the unit. Question generation can be done independently or during video conferencing breakout rooms. It may be helpful to use a collaborative document or whiteboard to collect and share all student questions. Group discussions can take place in breakout rooms, in independently scheduled group calls with the teacher joining at set times, through a written discussion board or WhatsApp group.

2	The teacher sets students the challenge of creating a construction plan for a building like the Taj Mahal and makes them aware of the resources they will have for their construction, i.e.: * Workers have different geometrical shaped blocks. These are made of wood and can be attached to the other blocks by their edges. * Each block weights 10 to 15 kilograms. You will need to arrange the blocks taking into account their shapes and sizes in order to make a symmetrical building. * You can have as many workers as you need. The construction project employed around 20,000 artisans under the guidance of a board of architects led by Ustad Ahmad Lahauri. The domed marble tomb is part of an integrated complex consisting of gardens and two red-sandstone buildings surrounded by a crenelated wall on three sides. * You have a plentiful supply of wooden blocks. You also have all the water, sand, and clay you need for making additional structures like fountains and a garden. Students organize themselves into teams of 4-5 and begin discussing the challenge. What do they need to work out in order to come up with a construction plan? (e.g. How will they know where to place materials and how will they transport them etc.) Teacher can visit each group to encourage students to be as imaginative in their thinking as possible, help them organize their ideas, and prompt them to consider the challenges listed in the appendix, as appropriate.	Calculating area, length, and other geometrical properties Exploring symmetry and ratios Using arithmetic to create work plans, estimates of the amount of materials needed, etc. Identifying relevant mathematical concepts and drawing on	Generating questions and playing with unusual ideas to work out how to use mathematics to approach a real-life problem. Envisioning and planning how to meaningfully solve a mathematics problem Considering different perspectives and challenges linked to the proposed solutions	The teacher can assign groups a set way of communicating (e.g. regular group calls, a discussion board, text messaging, or breakout rooms during whole-class sessions). A schedule can be made for when the teacher will join small group calls or breakout rooms to give feedback and check progress. Students can also use online tools such as Google Sketch or Tinkercad to work collaboratively. Once they have understood the task, groups can begin to work independently with the teacher joining for some of each group discussion. Clear guidelines will need to be set for when they should finish the task, how they should behave in small group discussions, and when and how they can contact the teacher if they are stuck. Students can be asked to keep a record of their work, for example, by using an online collaborative document (e.g. google sheets), a digital portolio (e.g. Open School eportfolio, Sesame, or Seesaw) or using an online whiteboard (e.g. Miro). These can be viewed in real time by the teacher as an additional method to assess progress.

		Students then begin to work through the challenges, where necessary drawing on their knowledge of mathematics to generate possible solutions. Students should keep a record of their thinking, calculations, and diagrams used or produced at each step of the process	knowledge of mathematics to support planning process		
3	Lesson period 2 (this may continue into a third period)	The teacher checks on progress of groups thus far and provides guidance to continue with their work. Students can be encouraged to assess and revise their thinking as appropriate and the teacher may help them become aware of any assumptions they are making (e.g. regarding materials, people, time, resources etc.). If the teacher has a particular related area of mathematical knowledge they want to reinforce, they may choose to present this to the class as a whole at this point. Students continue to work on their plans with their groups until they have finished, when they can be asked to present their work to the rest of the class if desired.	Opportunities to reinforce particular areas of mathematical knowledge as related to the local curriculum	Reflecting on steps taken so far, identifying and challenging own assumptions, and reviewing alternatives	This can take place in a whole-class video conferencing call before students return to their group calls/breakout rooms/discussion forums If students are asked to present, they can do this in a whole-class video conferencing session by sharing their screen – or they can be asked to pre-record their presentation and share a link, along with sharing permissions for any video or audio clips they have integrated into the presentation. Other students can then review these presentations asynchronously in between live sessions.
4	Lesson period 3/4	The teacher provides challenge questions for students to answer independently about the activity, for example: 1. a) What was the logic behind this building? b) What was the significance of this building? c) Why is symmetry used in the buildings? d) How did mathematics help you meet this challenge? 2. Which part of your work are you most proud of? Why? Which part of the exercise was the most difficult for your group? Why? After completing the activity, what questions do you still have?	Understanding the use of symmetry Summarizing how and why they used mathematics (arithmetic, geometry, trigonometry	Reflecting on steps taken to pose and solve a problem using mathematics Explaining strengths and acknowledging uncertainty and limits of proposed plans	This can be independent written work that has to be submitted to the teacher by an agreed date in an agreed format (e.g. over email, WhatsApp or on the learning platform). OR these can be oral discussions that take place in small group calls or breakout rooms. If students are asked to create written work on collaborative documents, these can easily be shared with other groups so that students can see similarities and

	3. Write an argumentative paragraph in response to the following question: *"Building on the exercise, discuss how and why symmetry can be used in the design of buildings and other objects"* Students can be asked to give examples which include references to actual measurements in their answer. A potential extension is also to ask students to discuss potential advantages of asymmetry relative to symmetry: for instance, is it more or less constraining than symmetry? 4. After looking at all the evidence we have studied in class overall what did you learn about Mughal architecture from our symmetrical building exercise? What did you learn about mathematics? What, if anything, are you still unsure or uncertain about? What was your personal contribution to the group's success? How did your group effectively work together? Students answer questions independently in writing and conclude the lesson by reporting back in small groups their findings and challenges.	etc.) during the challenge Communicating mathematical knowledge gained and remaining challenges		divergences in how the task has been approached by others.

Annex 2: Additional assessment activities

There are many ways this activity can be assessed in addition to attention to the processes and products already outlined. Here are just some possible activities, many of which can be used for either formative or summative assessment and which aim to assess both subject knowledge and creative and/or critical thought.

	DETAILS	POSSIBLE CRITERIA	POSSIBLE ONLINE MODES OF DELIVERY
HOMEWORK	Students could be asked to find another example of a famous building that uses symmetry and explain what this brings to the building by making comparisons to the Taj Mahal. Which is the better example of symmetry and why? Would everybody agree or is there different perspectives on this?	• Ability to identify and explain symmetry • Ability to appraise examples of symmetry • Ability to consider and review different perspectives on symmetry	Independent, asynchronous homework submitted to the teacher in an agreed timeframe and format
PORTFOLIO/ PROJECT	Students could be asked to record their activities, calculations, and reflections in a portfolio. The exact content of the portfolio could be designed by the students or assigned by the teacher and may also include self-assessment according to the rubrics	• Effort put in to creating a high quality output • Correct and relevant mathematical equations and knowledge • Ability to generate ideas for interesting and related portfolio entries • Ability to appraise own work and identify and explain critical and creative thinking	Digital portfolios can be used for this. There are many available online Or students could simply create and submit a PowerPoint/link or scanned copy of work done on paper, which documents their project. It is useful to provide teacher feedback during the process of development
EXAM/QUIZ	Students could be asked under timed conditions to “explain at least 3 different ways they used mathematics in this task and give some examples of the calculations made for each” and then “explain three ways we used creative/critical thinking in this task and give an example of what I mean for each”	• Correct and relevant mathematical equations • Ability to identify and evidence creative and critical thinking • Ability to generate ideas for ways in which mathematics can/did help meet an architectural challenge	In a live session or as a take-home exam

	Additionally or alternatively, students could be set a mini timed challenge in which they are presented with an architectural drawing for a new building and they have to generate mathematic calculations to help inform the design of at least three aspects of the building.		
CREATIVE OUTPUT	Students could be asked to write a 'why mathematics is useful in the real world' manifesto, giving examples from the activity to show how mathematics can be used to inform everyday and professional activities. What assumptions do you think people have about mathematics and how can you come up with creative content to challenge those assumptions?	• Correct and relevant mathematical equations, reasoning, and examples • Ability to generate original ideas and product original content to explain how maths can be used in everyday life • Ability to identify and challenge assumptions about mathematics	Independently, offline with regular check-ins from the teacher to provide formative feedback as appropriate. Submitted to the teacher through an agreed format
PERFORMANCE/ PRESENTATION	Students could be asked to give a presentation on how they used mathematics to help them design a new Taj Mahal. They can also be asked to reflect on the strengths and limitations of their work and how they used critical and creative thought.	• Correct and relevant mathematical equations, reasoning, and examples • Presenting style and skills • Ability to appraise own work and identify and explain critical and creative thinking	This can take place in a live video conference session or students can pre-record and submit a presentation/link. Note that video clips can be too heavy for some email systems. If the student emails a link to the presentation, they may need to ensure permissions are shared for each embedded video or audio clip individually in addition to presentation itself.

Annex 3

POSSIBLE CHALLENGES FOR STUDENTS TO CONSIDER:

1. It's easy to use the wooden blocks to construct a building, but how can you align the blocks to make a symmetrical shape?

2. Design a simple, labour efficient way to move blocks from the store to the construction site. What materials do you need to move each block? How many blocks will you need on each side of the symmetrical plane? How long will it take?

3. To ensure that the building gets off to a good start you need to make sure the 65 hectare (250m x 250m) base is perfectly smooth and level with no bumps, undulations, or hollows more than .3 of a metre above or below a perfectly level surface. How can you make sure the site is this level? Devise a survey and inspection system to achieve this.

4. You need to make sure the initial floor plan of the building is correct. Devise a survey and inspection system to achieve this.

Note: you cannot use a protractor since it's not accurate enough.

Hint: use geometry on a very large scale!

5. The building has to be right angled with no deviation anywhere from bottom to top. Devise a system to make sure every outer block is at exactly the correct angle. For this problem you do have a protractor. It's easy to get the first blocks into place, but it becomes much harder to move the blocks up for the top half of the structure. Devise a system to move blocks up to the top half of the structure in a labour efficient manner. You need to raise them 60 to 140m (the block at the very top is 140m above ground level).

Notes:

a) You can use pulleys since the early Indians did have them.

b) You cannot build a straight ramp up the side of the building.

7. Work out the logistics of the building.

a) List all the jobs directly necessary in building such a building.

b) Estimate how many people you need doing each job each day (on average) in order to cut out, move, raise, and correctly position the blocks. i.e. How many blocks need to be used?

c) What materials, tools, and supplies do you need to do the job? How much of each item do you need?

d) List the support materials and all the workers will need. i.e How much food, what sort of pottery, what materials must be made by someone else for the workers' use. Prepare an estimate of everything and every job needed to work on the structure.

e) Create an organizational structure showing who will oversee what, and who will report to whom.