Taxation, financial intermodality and the least taxed path for circulating income within a multinational enterprise.∗


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Abstract

When minimizing their overall tax liabilities, multinational enterprises exploit the various provisions of interjurisdictional tax arrangements, not hesitating to circulate flows indirectly and through various financial vehicles. This paper proposes to nest modelling such strategies into graph theory and network analysis. Such an exercise enables to compute strategy supported effective tax rates and to question the design of interjurisdictional tax arrangements.

Keywords : Taxation, Multinational firms, Application of graph and network theory.

JEL : H32, H73, C61

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Multinational firms frequently use complex paths when they circulate funds among affiliates located in different countries or jurisdictions and, very often, the choice of a path is primarily dictated by tax considerations.

So far, however, economic investigation has mainly focused on direct bilateral flows between one affiliate, the source of the income, say a subsidiary or a branch, and the parent company, possibly a resident taxpayer of another jurisdiction, computing for that purposes, effective (bilateral) tax rates.

The most popular such concept and statistics is that of marginal effective tax rate set forth by King and Fullerton (1984); it is an equilibrium concept in the classical sense, measuring, by a single statistics, the relative wedge at equilibrium, generated by the tax system under different assumptions regarding the source of funds, the type of assets or the status of the stockholder. Applications of King-Fullerton methodology are numerous, including Ocde (1991), Ruding (1992) and EU Commission (2001). Criticisms and extensions have been proposed a.o. by Alworth (1988, extension to MNE’s), Boadway and Bruce (1992), and Gérard (1993). The last paper, especially, shows that King-Fullerton marginal effective tax rate is just a specific case of a larger class of effective tax rates, that corresponding to a classical equilibrium. Another such specific case is the average effective tax rate put forward by Devereux and co-authors Chennels and Griffith, see a.o. Chennels and Griffith (1997) and Devereux and Griffith (1998, 2003).

In those contributions however, as mentioned before, as far as international taxation is concerned, the flow of income is deemed to circulate directly between the source country and the country of residence of the investor, thus involving only two jurisdictions, at most. This way of computing effective tax rates can be regarded as little compatible with actual tax behaviour of MNE’s, since it doesn’t pay enough attention to their tax strategies.

Unlike that, in this paper, we focus on complex paths and allow a flow of funds circulating between two jurisdictions to make strategic detours through one or more other entities and jurisdictions. We consider especially two interrelated tax strategies - treaty shopping and vehicle changing - and use an algorithm based on graph and network theory, to compute strategy supported effective tax rates.

Multinational firms use the former strategy - treaty shopping - when they try to use the provisions of international tax treaties to find out profitable detours. They use the latter - vehicle changing - when, at some point in the flow journey, thus when the flow goes through a given member entity of the
multinational firm, they change one type of financial flow into another one, like turning an interest into a dividend in a low tax jurisdiction to take profit of that feature. Such strategies are part of the daily business of tax managers of multinational enterprises, in short MNE’s. However they strategies do not exhaust the opportunities provided by the tax systems; as a matter of fact, multinational enterprises also use instruments like transfer prices, management fees and royalties to exploit the provisions and opportunities of the various domestic tax laws and interjurisdictional tax arrangements [see e.g. Bernard and Weiner (1990), Grubert (2002)].

To cope with that, this paper suggests an approach based on graph and network theory, especially on a modified version of a well known Dijkstra (1959) algorithm [for an introduction to graph and network theory, see e.g. Hillier and Liebermann (2001)] and reader accustomed with intermodal transportation will probably see something familiar in our approach. Therefore, in the sequel of this paper we use the words vehicle and mode equivalently, and we see vehicle changing as a form of intermodality. We consider the MNE search for the least taxed path over types of funds and detours through indirect paths, as a particular application of shortest path computation. However, as far as taxation is concerned, the problem may become somewhat more complicated. Indeed, standard shortest path algorithms require that the cost of going through an arc of the path - say to go from one jurisdiction to the next one - is fixed. In taxation however, double tax relief mechanisms and anti-abuse provisions violate that requirement and imply that the cost of going through an arc can be contingent to the history of the path or to part of that history.

Our motivation in conducting that research is first to get a better understanding of tax planning strategies conducted by MNE’s, second to propose a representation of those strategies capturing them in an extension of the now well known concept of effective tax rate, and third to pave the way for a better regulation of international flows of funds, especially through an improved design of tax treaties.

Thereafter, in section 2, we consider a two-jurisdiction world, thus limiting the investigation to direct flows, and we use that simple framework to set forth our formalisation. Then section 3 extends the investigation to more than two jurisdictions allowing for strategic detours. A critical discussion of the approach is proposed in section 3 as well as some conclusions and avenues for further investigation and application.
1 A 2-jurisdiction world

The income produced by an economic activity can be taxed in the jurisdiction where it is actually produced, or in the jurisdiction where the entity which receives that income has its residence. In the former case, the income is said taxed at source or according to the Source Principle, in the latter it is said taxed according to the Residence Principle [for a discussion see e.g. Mintz and Tulkens (1990) and Gérard (2002)]. It is up to the authority of a jurisdiction to decide for one principle or the other. However an income can then be taxed twice, according to the source principle in the jurisdiction where it is generated, and according to the residence principle when paid to a resident of another jurisdiction; then we speak of double taxation.

Also, although an income can circulate under various forms - dividend, interest, capital gain, profit - we first suppose here a single such mode or vehicle, say dividend. Then we relax that assumption.

Consider then a parent company located in jurisdiction $R$ with an active subsidiary located in jurisdiction $S$ and assume that income circulates as a dividend. The corporate tax rates are $\tau^R$ and $\tau^S$ respectively. Moreover, when an income flows across the border between the two jurisdictions a third tax is often due, called a withholding tax and noted $m^{SR}$; its value is determined either by jurisdiction $S$ unilaterally or by a tax treaty between the two jurisdictions. Therefore we could speak of multiple taxation rather of double taxation.

As a consequence one unit of income generated in $S$ becomes a net income $y^{SR}$ available for distribution in jurisdiction $R$, with

$$y^{SR} = (1 - \tau^R) (1 - m^{SR}) (1 - \tau^S)$$

an equation which illustrates what Feldstein and Hartman (1979) name full taxation after deduction.

Let us assume tentatively that such system is in operation.

Using a graph representation, we say that the flow from initial node $S$ to final node $R$ consists of three arcs. We term the first one the $SS$-intrajurisdictional arc; the tax cost factor associated to using that first arc is $1 - t^{SS}$ where $t^{SS} = \tau^S$. The second arc is the $SR$-interjurisdictional arc and the associated tax cost factor is $1 - t^{SR}$ with $t^{SR} = m^{SR}$. Finally the
last arc is called the \textit{RR-intra}jurisdictional arc and its associated tax cost factor is $1 - t^{SR}$ with $t^{RR} = \tau^R$.

![Diagram of Direct Flow]

**Figure 1 - Direct flow**

The value of $y^{SR}$ will depend on the tax rates observed in equation (1). Adopting a graph and network theory notation we can rewrite equation (1),

$$ y^{SR} = \sum_{(i,j) \in A} \ln \left(1 - t^{ij}\right) x^{ij} $$

where $A = \{(i, j)\}$ is the set of arcs, each arc being of origin $i$ and destination $j$, and $x^{ij}$ takes the value 1 if the arc is used, otherwise it is equal to 0. Moreover $x^{ij} = 1$ implies that there exists at least one $x^{jk} = 1$ except for $j = R$ and at least one $x^{hj} = 1$ except for $h = S$. If we further define $N$ the set of nodes - each arc goes from one node to another one -, then a graph can be characterised by a set of arcs and a set of nodes and noted $G(A, N)$.

Nevertheless, mechanisms of \textit{double taxation relief} have been set up in order to eliminate, or at least to alleviate, the effect of the series of levies on the same initial amount. They are termed \textit{unilateral} if one jurisdiction has decided, on its own, to grant a double taxation relief to its resident or non-resident taxpayers. They are called \textit{bilateral} if they are organized by an agreement between two jurisdictions, according e.g. to the tax treaty model

$\text{More formally, we require that,}$

$$ x_{ij} = \sum_{k \in N^+(j)} x_{jk} $$

$$ \sum_{j \in N^+(S)} x_{Sj} = 1, \sum_{j \in N^-(R)} x_{jR} = 1 $$

$$ x_{ij} \geq 0, x_{ij} \in \{0, 1\} $$

where $N^-(j) = \{i \in N \mid \exists (i, j) \in A\}$ and $N^+(j) = \{k \in N \mid \exists (j, k) \in A\}$. 

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set up by OECD\textsuperscript{2} or that suggested by UN. Finally they are *multilateral* if based on a multilateral arrangement among a set of countries, like the European Union Parent-Subsidiary Directive [see European Tax Handbook, 1999 or European Union “EC Parent-Subsidiary Directive of 23 July 1990” (90/435/EEC)].

The most popular systems of double taxation relief are the *crediting method* and the *exemption method*.

The crediting method allows the taxpayer in the residence jurisdiction to impute on its domestic tax liabilities, subject to the restrictions examined below, the tax paid abroad. The crediting method is termed *without deferral* or *direct* if the imputable tax is limited to the withholding tax, i.e. the tax levied when the flow crosses the border, \( m^{\text{SR}} \) above; it is called *with deferral* or *indirect* if it is extended to the upstream corporate tax paid abroad, \( \tau^S \). Credit is said *partial* if only a fraction \( \omega \) of the upstream tax is imputable, and *full* if \( w = 1 \), it is said *solely imputable* if it is upper limited to the domestic tax liabilities of the MNE, and *repayable* if it may generate a net refund from domestic tax authorities. Notice that in international taxation it is usually solely imputable.\textsuperscript{3}

The exemption method consists in deciding that an income is tax exempt, sometimes but up to a fraction \( \mu \), in one of the jurisdictions, most usually that of the residence of the taxpayer.

Finally, there are also anti-abuse provisions that currently tax the passive income or/and the income routed through low-tax jurisdictions; therefore parameter \( \mu \) and \( w \) can depend on the value of the corporate tax rate in jurisdiction \( S \).

\textsuperscript{2}Articles 23A and 23B of the OECD model tax treaty provide that, when the foreign-source income is taxed in the source jurisdiction, the destination jurisdiction has either to use the exemption method (article 23A) or the crediting method (article 23B) in order to prevent double taxation

\textsuperscript{3}As to the choice between a crediting method and an exemption method, international economic arguments rely on the merits of *tax neutrality* [Inland Revenue (1999)]. Economists speak of *Capital Export Neutrality* if the tax system provides no incentive for resident taxpayers in a given jurisdiction to invest at home rather than abroad or vice versa, and of *Capital Import Neutrality* when all agents investing in a given jurisdiction are subject to the same taxation by that jurisdiction on similar investments, whether they are or not residents of the jurisdiction. Gérard (2002) proposes a detailed analysis of the respective properties of the mechanisms.
1.1 The foreign tax crediting method

Tax cost factors $1 - t^{SS} = 1 - \tau^S$ and $1 - t^{SR} = 1 - m^{SR}$ are unaffected by the choice between foreign tax crediting and exemption. Unlike those parameters, $1 - t^{RR}$ does depend on that choice.

Under the crediting method, assuming that the credit is solely imputable,

$$t^{RR} = \max\{0, \tau^R \left(1 + \omega^{SR}\right) - \omega^{SR}\}$$

with

$$\omega^{SR} = \frac{w_m^{R, HR^{SR}}}{(1 - m^{SR})} + \frac{w_\tau^{R, \tau^{SR}}}{(1 - m^{SR})(1 - \tau^S)}$$

where $w_m^R$ and $w_\tau^R$ are the imputation rates with a subscript $m$ or $\tau$ indicating which tax is imputed and a superscript $R$ referring to the jurisdiction where that tax can be imputed.

Notice that if both imputation rates are equal to unity,

$$t^{RR} = \max\{0, \tau^R - \left[\tau^S + m^{SR} \left(1 - \tau^S\right)\right]\}$$

so that the residence principle applies if the second element of the bracket is positive - then $y^{SR} = 1 - \tau^R$ and the tax system is said capital export neutral - while the source principle applies if that element is non positive - then $y^{SR} = \left(1 - m^{SR}\right) \left(1 - \tau^S\right)$ and the system is capital import neutral. This point seems to corroborate Mintz and Tulkens (1996) who refer to and discuss Feldstein and Hartman (1979) and Musgrave and Musgrave (1984) on that issue.\(^4\)

1.2 The exemption method

Under the exemption method, income is not subject to the corporate tax either in the $S$ or the $R$ jurisdiction, except possibly for a small part $\mu$ of

\[^4\]Let us add that excess credit can often be deducted against the tax base. Formally if only a fraction $\gamma$ of the tax credit can be effectively used so that $\gamma \left[w_m^R(1 - \tau^R) m^{SR} (1 - \tau^S) + w_\tau^R(1 - \tau^R) \tau^S\right] = \tau^R(1 - \gamma)(1 - m^{SR})$ then, the parent company can deduct the excess credit against its corporate tax base, getting a tax rebate which amounts to $\tau^R \left(1 - \gamma\right) \left[w_m^R(1 - \tau^R) m^{SR} (1 - \tau^S) + w_\tau^R(1 - \tau^R) \tau^S\right]$ Of course that rebate is smaller than the advantage provided by repayability of the excess credit since $\tau^R < 1$.\]
it (5 percent according to EU Parent-Subsidiary Directive of July 23, 1990). Usually, the exempting jurisdiction is $R$ and we should speak more precisely of inflow exemption. Should the exempting jurisdiction be $S$, we would speak of outflow exemption. Assume the former, so that $\mu = \mu^R$ and allow that parameter to be contingent to the origin of the flow, so that it is noted $\mu^{SR}$. Then,

$$t^{RR} = \mu^{SR} \tau^R$$

If $\mu^{SR}$ vanishes, like in France, so does $t^{RR}$ too, and the exemption method is an application of the source principle, producing a capital import neutral international tax system as long as $m^{SR}$ does not depend on $S$, and generating no tax externality against destination jurisdiction $R$.

### 1.3 Synthesis and extension to more than a single financial mode

The net income in jurisdiction $R$ generated by a pretax income equal to unity in jurisdiction $S$ can be written,

$$y^{SR} = 1 - T$$

where $T$ stands for the effective (bilateral) tax rate with,

$$T = \tau^S + m^{SR}(1 - \tau^S) + t^{RR} (1 - m^{SR}) (1 - \tau^S)$$

where

$$t^{RR} = \theta \max \{ 0, \tau^R (1 + \omega^{SR}) - \omega^{SR} \} + (1 - \theta) \mu^R \tau^R$$

with $\theta = 1$ under the crediting method and $\theta = 0$ otherwise.

If various financial modes are permitted, $y^{SR}$ also depends on the vehicle $f$ chosen by the MNE for circulating the income. Then the tax cost factors have to be adapted accordingly, e.g. to reflect that interest uses to be deductible against the tax base in the paying jurisdiction. Seeking for the least taxed path then consists in selecting $f$ such that it

$$\min_f \sum_{(i,j) \in A} |\ln (1 - t_{ij}^f)| x_{ij}^f$$

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The value of \( f \) is selected by the MNE at any intrajurisdictional arc but the ultimate one, where that choice is then irrelevant; thus here only in \( S \). Now each variable has also to be characterised by a subscript \( f \) indicating the vehicle used, dividend \((f = 1)\), interest \((f = 2)\), capital gain \((f = 3)\) or profit \((f = 4)\).

2 Least taxed path in a \( n > 2 \) - jurisdiction world

Suppose now that the world consists of at least three jurisdictions. To circulate a flow from affiliate \( S \) to parent \( R \) the MNE has now to choose between the direct path of the previous section and a detour through entity located in \( L \). Also it has to decide of the financial vehicle to be used for leaving \( S \) and, in case of detour through \( L \), of the possible change of mode at intrajurisdictional arc \( L L \).

In the case illustrated by Figure 2, it may be that the best strategy is to leave \( S \) with an interest - then \( 1 - t^{SS}_2 = 1 \) since interest is deductible against the tax base in \( S \) - , then to go to \( L \), which implies \( 1 - t^{SL}_2 = 1 - m^{SL}_2 \). In \( L \), the interest is taxed at rate \( t^{LL}_2 = \tau^L \) before the proceed to be paid out as a dividend; that detour can seem profitable when \( \tau^L \) is low. The dividend is subject to a withholding tax when leaving \( L \) so that \( 1 - t^{LR}_1 = 1 - m^{LR}_1 \) and finally it is taxed in \( R \) at rate \( t^{RR}_1 \). Then, equation (9) is rewritten,

\[
t^{RR}_f = \theta \max \{ 0, \tau^R (1 + \omega^k_{j}) - \omega^k_{f} \} + (1 - \theta) \mu^k_{j} \tau^R (11)
\]

with \( k = S, L \).

As an example imagine that jurisdiction applies the exemption method for foreign dividends except for dividends coming from a tax heaven, and assume that jurisdiction \( L \) is precisely a tax heaven from \( R \) viewpoint; then it may be that \( \mu^{SR}_1 = 0 \) but \( \mu^{LR}_1 = 1 \).

In case of full but non repayable imputation in \( R \), as long as the second element in the bracket is positive, it is contingent to the respective values of \( k^{kk} \) and \( m^{kR} \). Indeed,

\[
\omega^k_{f} = \frac{w^{kR}_{j} m^{kR}_{f}}{(1 - m^{kR}_{f})} + \frac{w^{kR}_{f} \tau^k}{(1 - m^{kR}_{f}) (1 - \tau^k)} (12)
\]
Generalising backward, we have that

\[ t_{ij}^{jf} = \theta \max \left\{ 0, \tau_i^j \left( 1 + \omega_{kj}^{jf} \right) - \omega_{kj}^{jf} \right\} \]

\[ + (1 - \theta) \mu_{kj}^{jf} \tau_i^j \]

(13)

with

\[ \omega_{kj}^{jf} = \frac{w_{kj}^{m_i} m_{kj}^{j}}{(1 - m_{kj}^{j})} + \frac{w_{kj}^{m_i} \tau_j^k}{(1 - m_{kj}^{j}) (1 - \tau_j^k)} \]

(14)

To find out the sequence of \( x_{ij}^{jf} \) which solves the problem of the least taxed path, we use the modified Dijkstra algorithm presented thereafter.

### 2.1 A modified Dijkstra algorithm

The algorithm set up and used in this paper is an extension of the well known Dijkstra (1959) algorithm, described in Appendix B. Although Dijkstra algorithm is based on a single fixed cost associated to each arc and a single distance value associated to each node, ours allows for relating variable costs, possibly contingent to the history of the path, to each of the arcs of the graph and therefore, relating a table values to each node.
Our extension\footnote{A suggestion of Pierre Semal was seminal for that part of the work.} of Dijkstra’s algorithm is formally presented in Appendix A; we believe that the reader will gain a better insight of our approach following the application thereafter.

## 2.2 Application

The following is an application with vehicle changing. Suppose three member states of the European Union and the conditions fulfilled for the application of the EU Parent-Subsidiary directive. As a consequence, withholding tax rates are zero on dividends and so are the cost of going through such inter-jurisdictional arcs. Moreover withholding rates on interests, if non-zero, as small enough to be actually imputed on the corporate tax liabilities of the firm which receives the interests, so that they can be set to zero. Corporate tax rates amounts to 0.4017 in Belgium, 0.30 in Denmark and 0.35 in The Netherlands; interest income are fully taxed and interest payments fully deductible against the corporate tax base in the three jurisdictions. However, while intercorporate dividends are fully exempt in Denmark and The Netherlands (\(\mu = 0\)), they are subject to corporate tax on 5 percent of their value in Belgium (\(\mu = 0.05\)).

Let us first consider direct flows under the form of a dividend and an interest respectively. The value of the corresponding final income \(y^{SR}\) have been reported on tables 2 and 3. Effective bilateral tax rates are \(1 - y^{SR}\).

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<th>To</th>
<th>B</th>
<th>Dk</th>
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<td>From</td>
<td></td>
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<tr>
<td>B</td>
<td>0.58628</td>
<td>0.5983</td>
<td>0.5983</td>
</tr>
<tr>
<td>Dk</td>
<td>0.68594</td>
<td>0.7000</td>
<td>0.7000</td>
</tr>
<tr>
<td>Nl</td>
<td>0.63694</td>
<td>0.6500</td>
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Table 1: Direct flows, dividends
Table 2: Direct flows, interests

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improvement is possible with respect to the figures of table 2. Similarly, no detour is paying as long as dividends are concerned. Channelling through Denmark or The Netherlands doesn’t affect the value of the flow while going via Belgium decreases that value.

If changing the financial vehicle in the intermediate jurisdiction is permitted, then some gain can be possible. Let us start with an interest and turn it into a dividend in the intermediate jurisdiction. We see - on Table 4, which provides the $y_{SR}^L$, $L$ being Denmark - that a detour through that country is profitable; this is due to the fact that the corporate tax rate is lower in that country. Unlike that, starting with a dividend and turning it into an interest in the intermediate jurisdiction is not profitable: dividend will be taxed at source, at best exempted in the intermediate jurisdiction where turned into an interest, and then taxed again as interest income in the destination jurisdiction.

Table 3: Possible indirect flows, interests turned into dividends

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<tbody>
<tr>
<td>From</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.6859*(Dk)</td>
<td>0.7000(No)</td>
<td>0.7000* (Dk)</td>
</tr>
<tr>
<td>Dk</td>
<td>0.6859*(Dk)</td>
<td>0.7000(No)</td>
<td>0.7000* (Dk)</td>
</tr>
<tr>
<td>Nl</td>
<td>0.6859*(Dk)</td>
<td>0.7000(No)</td>
<td>0.7000* (Dk)</td>
</tr>
</tbody>
</table>

In terms of effective tax rate, the detour through Denmark generates a strategy supported effective tax rate equal to $1 - y_{SR}^L$ smaller, or not larger, than that corresponding to a direct flow, $1 - y_{SR}$.

As an example consider the case of a flow between The Netherlands and Belgium; direct flows generates effective tax rates equal to $1 - 0.63694 = 12$
0.363 06 and $1 - 0.5983 = 0.401 7$ for a dividend and an interest respectively, while the strategy supported rate amounts to $1 - 0.6859 = 0.314 1$.

Now let us see how that result can be obtained using our algorithm. For the ease of the exposition we first suppose a flow to Belgium leaving The Netherlands as an interest. The cost factor associated to an arc is

$$c_{ij}^d = |\ln (1 - r_{ij}^d)|$$ (15)

Initially - Figure 3a - the table associated to each node indicates a distance from the origin $d = \infty$, except that associated to the first node, which is indicated $d = 0$. Since interests are deductible against the tax basis in The Netherlands, the cost of moving from $N_1$ to $N_2$ is zero implying that in the table associated to node $N_2$, the distance value from $N_1$ is no longer $d = \infty$, but $d = 0$, the sum of the distance value associated to $N_1$ and the cost of going from $N_1$ to $N_2$ - Figure 3b.

From $N_2$ we can move at a same 0 cost to both node $D_1$ (the entry of Denmark) and $B_1$ (the entry of Belgium) using interjurisdictional nodes. Those moves implies that 0 is substituted for $\infty$ in the tables associated to those nodes Figure 3c.
The cost of using intrajurisdictional arc $D_1D_2$ is either 0 if the income enters and leaves the Danish affiliate as an interest, since incoming interests are taxed but interest outflow is deductible against the corporate tax basis, or $0.357 = |\ln (1 - 0.30)|$ if incoming interests are turned into dividends; therefore the table associated to node $D_2$ will now contain two distance values contingent to the financial mode used, 0 (int) and $0.357$ (div) - Figure 3d. Similarly we can go from $N_2$ to $B_1$ using the corresponding interjurisdictional arc and changing the distance at node from $\infty$ to 0 - also in Figure 3d.

On Figure 3e we move from $D_2$ to $B_1$ producing a new table associated to that latter node.

Finally we reach the intrajurisdictional Belgian arc. Then, dividends are taxed at $0.05 \cdot 0.4017$ and interests are at full corporate tax rate $0.4017$ so that the costs respectively are $0.02 = |\ln [1 - 0.05 \cdot 0.4017]|$ and $0.514 = \infty$.
In the last table - Figure 3f - we observe that the minimum distance is 0.377 so that the detour through Denmark with change of financial mode in that country is profitable.

It turns out that the largest final income is \( \exp(-0.377) = 0.686 \) so that the strategy supported effective tax rate on a flow of interest from The Netherlands to Belgium in a world including also Denmark is \( 1 - 0.686 = 0.314 \).

The same exercise can be conducted starting with a dividend.

3 Conclusion

Combining a standard technique of international public finance - computation of effective tax rates - with a useful tool of graph and network analysis - Dijkstra’s algorithm to compute a shortest path - we have set up and applied a technique allowing us to compute a strategy supported effective tax rate, by which is meant an effective tax rate which incorporates the possibility of a profitable detour through one or several third jurisdictions with possibly the change of financial vehicle or mode, whenever the flow is channelled through an intrajurisdictional arc of the detour. Doing so we have also proposed a modified version of the Dijkstra algorithm which permits to have several costs associated to an arc, and, as a consequence, a table of informations, including distances from the origin, rather than just a single valued such distance.

Such way to capture the effective tax rate actually paid by a multinational firm seems to us more realistic than simply measuring the levy on a bilateral flow. From a fiscal policy point of view, the approach conducted here enables to question the design of international tax arrangements. Tentatively formulated, the question is: \textit{is an international tax system where such detours are profitable an efficient device?}
That approach has however some limitations. One is limitation to such modes of circulating flows as dividends, interests, capital gains or profits, ignoring other tax shifting methods like the use of transfer pricing and intangibles. Another one\(^6\), only relevant when a tax system like that in the US is in operation, is related to the fact that our tool seems to imply that excess credit are computed per country; however we think that such limitation can be eliminated.

References


\(^6\)That second limitation, only valid when a tax system like that in operation in US, has been pointed out to us by Alan Auerbach.


Appendix A: A modified Dijkstra algorithm

Diverse algorithms exist to solve the kind of problem we investigate in this paper but Dijkstra’s algorithm seems to be the most appropriated. Nevertheless, Dijkstra is limited when facing the peculiarities of this tax optimization problem.

The modified algorithm will work at the path’s level, guiding each path from one node to another. Moreover, some different transfers reaching the same node could have the same characteristics but different costs, then they will be compared in order to retain the cheapest one. Two paths may have the same properties (i) if the type of income are identical, (ii) if the income is channeled (or not) through a jurisdiction subject to a more favorable tax regime and (iii) if the preceding tax costs are identical. Each new path created from a node to the next one will be added to the paths group of the next node if there is no identical transfer already existing at this next node. At the end of the graph (at the last node) a final comparison is still conducted, from the cheapest transfer to the most expensive one. The outcome of the algorithm is then a ranking of paths characterised by cost, type and net income at destination.

Let us now introduce the technical features of the algorithm before turning to an application.

Configuration of a node

Each node - $N_K$ - of the graph $G(A,N)$ is represented by a table - $\text{Table}_{N_K}$ - with some rows - $(\text{Row}_{N_{K,1}} , \text{Row}_{N_{K,2}} , .. \text{Row}_{N_{K,P}})$ - filled at some iteration by a repatriation of income. If a new transfer has the possibility to reach the node - $N_K$ - , it is then included in a new row - $\text{Row}_{N_{K,P+1}}$ - of the table - $\text{Table}_{N_K}$ - when no identical path already exists at this node.

The size of the table depends on the number of repatriations of the income (with different characteristics) which can reach the node.

Progressively, all the paths included in the table $\text{Table}_{N_K}$ for the node $N_K$ go to the next node $N_{K+1}$ - i.e. the $\text{Table}_{N_{K+1}}$ - for $(N_K, N_{K+1}) \in A$, the set of arcs of the graph.

The algorithm chooses the cheapest path from all the tables of the graph at every iteration, guiding it from the corresponding node to the next one.

$\text{Table}_{N_K}$ for the node $N_K$ :

$\{\text{Row}_{N_{K,1}} \text{ type “Row”}\},$
$\{\text{Row}_{N_{K,2}} \text{ type “Row”}\},...$
$\{\text{Row}_{N_{K,P}} \text{ type “Row”}\}$
So, a row represents a path, with its characteristics, having reached the corresponding node at some iteration. It contains the history of the transfer, its value in terms of net income, the type of the income repatriated, a conditional information and some other factors, e.g. it may consist of a series of numbered fields like, where,

(1) : the name of all the previous nodes reached, from the first one to the corresponding node - $N_K$ -, on this path
(2) : the preceding tax costs previously paid on this transfer.
(3) : the net income amount, brought to this node, through this path, at this iteration.
(4) : the type of the income repatriated here.
(5) : the answer to the question: is this repatriation of income going through a jurisdiction subject to a privileged tax regime ?
(6) : the answer to the question: is the row definitive?
(7) : the name of the node.

Finally a row is called definitive if the corresponding path has already been checked, more exactly if it was used for going from the node - $N_K$ - to the next.

The algorithm

First, let us introduce the following notations,

$Set_{N_K} = \text{set of rows, from the } N_K \text{ node, already considered as "definitive"}$

$= \{ Row_{N_K1} , Row_{N_K2} , \ldots , Row_{N_KP_K} \}$

$Set'_{N_K} = \text{set of rows, from the } N_K \text{ node, not already checked}$

$= \{ Row_{N_KP_{K+1}} , Row_{N_KP_{K+2}} , \ldots , Row_{N_KQ_K} \}$

$N = \text{set of all the nodes of the graph}$

$Set = \{ Set_{N_K} \mid \forall N_K \in N \} \quad Set' = \{ Set'_{N_K} \mid \forall N_K \in N \}$

$ROW = \text{set of all the rows for all the nodes}$

$= \{ Row_{N_KR_K} \in Set_{N_K} \cup Set'_{N_K} \mid \forall N_K \in N \}$

$A = \text{set of all the arcs of the graph}$

$d = \text{distance function.}$

Note that the distance function corresponds to the absolute value of the Napierian logarithm of the net income amount, brought to this node, through this path, at this iteration.

Now we can briefly describe the algorithm.
BEGIN :  
1/ Set = \{the first row of the first node = Row_{11}\}  
2/ Set’ = ROW\{Row_{11}\}  
3/ \(d(\text{Row}_{NK_{R_{K}}}) = \infty\) \((\forall \text{Row}_{NK_{R_{K}}} \in \text{Set’})\)  
4/ \(d(\text{Row}_{11}) = 0\)  

LOOP :  
1/ Choose \(\text{Row}_{NK_{R_{K}}}\) (the cheapest row)  
\[= \min\{d(\text{Row}_{NK_{R_{K}}}) : \text{Row}_{NK_{R_{K}}} \in \text{Set}\}\]  
2/ \(\text{Row}_{NK_{R_{K}}}\) is definitive then Set = Set \(\cup\) \{\(\text{Row}_{NK_{R_{K}}}\)\}  
\(\text{and Set’} = \text{Set’} \setminus \{\text{Row}_{NK_{R_{K}}}\}\)  
3/ \(\text{Row}_{NK_{R_{K}}}\) is analyzed : \(\forall (N_{K_{1}}, N_{K_{2}}) \in A =>\)  
4/ If it exists a row with the same type in \(N_{K_{2}}\)  
\(\rightarrow \text{YES}\)  
5/ Suppose that it is \(\text{Row}_{NK_{R_{K}}}\) :  
If \(d(\text{Row}_{NK_{R_{K}}}) > d(\text{Row}_{NK_{R_{K}}} + C_{NK_{1},NK_{2}})\)  
then \(d(\text{Row}_{NK_{R_{K}}}) = d(\text{Row}_{NK_{R_{K}}}) + C_{NK_{1},NK_{2}}\)  
and the predecessor of \(\text{Row}_{NK_{R_{K}}\text{}} = \text{Row}_{NK_{R_{K}}}\)  
\(\rightarrow \text{NO}\)  
5/ add a row in \(N_{K_{2}}\) : \(\text{Row}_{NK_{R_{K}}}\) with \(d(\text{Row}_{NK_{R_{K}}}) = d(\text{Row}_{NK_{R_{K}}} + C_{NK_{1},NK_{2}})\)  
and the predecessor of \(\text{Row}_{NK_{R_{K}}}\) = \(\text{Row}_{NK_{R_{K}}}\)  

END :  
Repeat the loop until \(\text{Set’}\) is empty  

The first row of the first node, which represents the entry point of the graph, is built with no previous characteristics but an income of 1, a given type of income, and a name. The value of a transfer until this node - i.e. its distance - is equal to zero and all the other distances are equal to infinity for all the other rows of all the nodes of the graph.

Let’s consider the cheapest path of the graph. The corresponding row is considered as definitive. Guiding this transfer from its corresponding node to any next node, it is compared with all the rows of this next node - i.e. with all the paths which have already reached the next node -. If a row with the same characteristics already exists on the next node, the comparison between the two paths will retain the cheapest. If it doesn’t, the cheapest path is simply added to the paths group of the next node. And so on, the progression is repeated.

The search for the least taxed path is completed when all the rows of all the nodes of the graph are definitive.
Appendix B: Dijkstra’s algorithm

If

- $G(N, A)$ is an oriented graph with $N$ the set of nodes and $A$ the set of arcs of the graph,
- $S$ is a set of nodes,
- $\bar{S}$ is the complementary of $S$,
- $d(i)$ is a distance function, i.e. the total distance between the first node of the graph (node “1”) and the node $i$,
- $c(i, j)$ is a cost function, i.e. the distance between node $i$ and node $j$, ($c(i, j) \geq 0$);

Then

1) Begin
2) $S = \bar{S} = N \setminus S$
3) $d(i) \leq \infty$, for each $i$ in $\bar{S}$
4) $d(1) \leq 0$ and the predecessor of node $i$ is the empty set
5) While $\bar{S}$ is not empty do
6) Begin
7) Let $i$, the node in $\bar{S}$ with the minimum distance value
8) $S \leq S \cup \{i\}$, $\bar{S} \leq \bar{S} \setminus \{i\}$
9) For each arc $(i, j)$ of the graph do
10) If $d(j) > d(i) + c(i, j)$ then
11) $d(j) < d(i) + c(i, j)$ and the predecessor of node $j$ is node $i$
12) end
13) end

At the end, $d(\text{\textquotedblleft the last node of the graph\textquotedblright})$ is the minimum distance between this node and the first one.