Factor Models in Economics Research

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Economic Intuition

- A life cycle model of youth and adult decision making over horizon $\bar{T}$:
  - Agent maximizes

$$\int_0^{\bar{T}} \exp(-\rho t)U(c(t), \ell(t); \eta) \, dt$$

subject to dynamic constraints:

- $\bar{A}(t) = Y(t)h(t)\ell(t) - P(t)'c(t) + rA(t)$,
- $h(t) = \varphi(h(t), l(t), \tau)$,
- $Y(t) = R(h(t); \gamma)$,

and initial conditions $h(0), A(0)$. 
Latent dimensions might play a critical role:

preferences $\eta = \eta(\theta), \rho = \rho(\theta),$

human capital productivity $\tau = \tau(\theta),$

direct market productivity $\gamma = \gamma(\theta),$

$h(0) = h_0(\theta),$
$A(0) = A_0(\theta).$

Thus, these factors ($\theta$), unobserved heterogeneity, should explain a variety of outcomes.

Candidates for $\theta$?
Latent Skills and Outcomes:

The evidence demonstrates independent and important roles of latent cognitive and socio-emotional skills. They determine schooling attainment, labor market outcomes and social behavior.

Static Roy Model Framework

- Potential outcomes, two regimes (treatment and control)

\[
Y_1 = \mu_1(X) + U_1 \\
Y_0 = \mu_0(X) + U_0
\]

- Decision rule

\[
D = 1[\mu_D(Z) - V \geq 0]
\]

- Factor structure: \( U_1, U_0, V \) might information on unobserved dimensions. Thus,

\[
U_1 = \alpha_1 \theta + \epsilon_1 \\
U_0 = \alpha_0 \theta + \epsilon_0 \\
V = \alpha_V \theta + \epsilon_V
\]

where \( \epsilon_V \perp \perp \epsilon_1 \perp \perp \epsilon_0 \), and \( \theta \perp \perp (\epsilon_V, \epsilon_1, \epsilon_0) \)
Static Roy Model Framework

- Potential outcomes, two regimes (treatment and control)

\[
\begin{align*}
Y_1 &= \mu_1(X) + U_1 \\
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- Decision rule

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\[
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\]

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Roy Model Framework

The Roy Model provides a simple framework to analyze the effects of latent skills on outcomes. Factor models can be integrated into this economic setting.
Figure 2. Human Development at Each Stage (inputs/outputs)
Figure 2. Human Development at Each Stage (inputs/outputs)
Figure 2. Human Development at Each Stage (inputs/outputs)
Where do skills come from?

- $\theta_t$ is (latent) skill and $I_t$ represents investments. Thus,

\[
\begin{align*}
\theta_t &= g(\theta_{t-1}, I_{t-1}) + \nu_t \\
I_{t-1} &= \iota(\theta_{t-1}) \\
\theta_0 &= \text{initial condition}
\end{align*}
\]

- Do skills evolve over time? Sarzosa and Urzua (2013) for South Korea:

(a) Age 14
(b) Age 17
Implications?
Endowments, skills, labor market outcomes, and inequality

- Consider, for instance, earnings:

\[ W_t(i, j) = W \left[ \underbrace{H_0(i), S_t(i)}_{\text{Efforts/Circumstances}}, \underbrace{Z_t(j), T_t(i, j)}_{\text{Firms Match}} \right] \]

- Income inequality?

\[ \text{Gini}_t = G \left[ \left[ W_t(i, j) \right]_{i=1}^{N(j)} \right]_{j=1}^{J} \]

- Thus, the evolution of latent skills should shape the income distribution.
Implications?
Endowments, skills, labor market outcomes, and inequality

Consider, for instance, earnings:

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Thus, the evolution of latent skills should shape the income distribution.
Implications?

Endowments, skills, labor market outcomes, and inequality

- Pre-labor market conditions
  \[ H_0 \] (e.g., cognitive ability)

- Skills
  \[ S_t \] (e.g., functional literacy)

- Tasks
  \[ T_t \] (e.g., customer assistance)

Consider, for instance, earnings:

\[
W_t(i,j) = W \left[ \begin{array}{c} H_0(i), S_t(i) \\ \underline{\text{Efforts/Circumstances}} \\ Z_t(j), T_t(i,j) \\ \underline{\text{Firms Match}} \end{array} \right]
\]

- Income inequality?

\[
Gini_t = G \left[ \left[ W_t(i,j)^{N(j)} \right]_{i=1}^{J} \right]_{j=1}^{J}
\]

- Thus, the evolution of latent skills should shape the income distribution.
How to identify the distribution of unobserved factors?

- Strong functional form assumptions are usually imposed.
- Additional information might provide a flexible alternative.

Measurement system: Let $T$ denote a vector of test score (e.g. math score, IQ test)

$$T = \mu_T(X) + U_T = \mu_T(X) + \alpha_T \theta + \varepsilon_T$$

where $(X_T, \varepsilon_T) \perp \perp \theta$.

- We can link/anchor latent factors to, for example, test scores.
- The set up recognizes that $T$ is not a direct measure of $\theta$. 
How to identify the distribution of unobserved factors?

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- We can link/anchor latent factors to, for example, test scores.
- The set up recognizes that $T$ is not a direct measure of $\theta$. 
Identification

- Information on at least three test scores ($T_1$, $T_2$, $T_3$).
- For simplicity, $\theta$ is scalar and omit $X$. Thus,

$$T_1 = \alpha T_1 \theta + \epsilon_{T_1}$$
$$T_2 = \alpha T_2 \theta + \epsilon_{T_2}$$
$$T_3 = \alpha T_3 \theta + \epsilon_{T_3}$$

- We can compute RHS from $\text{Cov}(T_i, T_j) = \alpha_{T_i} \alpha_{T_j} \sigma^2_{\theta}$, and

$$\frac{\text{Cov}(T_1, T_2)}{\text{Cov}(T_2, T_3)} = \frac{\alpha_{T_1}}{\alpha_{T_3}} \quad \text{and} \quad \frac{\text{Cov}(T_1, T_2)}{\text{Cov}(T_1, T_3)} = \frac{\alpha_{T_2}}{\alpha_{T_3}}$$

- By normalizing $\alpha_{T_3} = 1$, we get $\alpha_{T_1}$ and $\alpha_{T_2}$. 
Finally, we can rewrite the system as:

\[ \frac{T_1}{\alpha_{T_1}} = \theta + \frac{\epsilon_{T_1}}{\alpha_{T_1}} = \theta + \epsilon'_{T_1} \]

\[ \frac{T_2}{\alpha_{T_2}} = \theta + \frac{\epsilon_{T_2}}{\alpha_{T_2}} = \theta + \epsilon'_{T_2} \]

and we can apply Kotlarski’s Theorem (Kotlarski, 1967) to identify

\[ f_{\epsilon_{T_1}} (\cdot), f_{\epsilon_{T_2}} (\cdot), f_{\theta} (\cdot) \]

We can identify the whole model applying this logic.
Estimation: MLE

- We observe $Y_j$, $T_j$, $D_j$ for $j = 1, ..., N$, with
  \[ Y_j = D_j Y_{1,j} + (1 - D_j) Y_{0,j} \]

- Conditional on unobserved abilities, $U_1, U_0, V$ and $U_T$ are mutually independent. Thus,
  \[
  \prod_{j=1}^{N} f(Y_j, T_j, D_j | X, X_T, Z) = \prod_{j=1}^{N} \int f(Y_j, T_j, D_j | X, X_T, Z, \theta) dF(\theta)
  \]
  where we can write
  \[ f(Y_j, T_j, D_j | X, X_T, Z, \theta) = f(Y_j, D_j | X, Z, \theta) f(T_j | X_T, \theta) \]
Estimation: MLE

- We observe $Y_j$, $T_j$, $D_j$ for $j = 1, \ldots, N$, with

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- Conditional on unobserved abilities, $U_1, U_0, V$ and $U_T$ are mutually independent. Thus,

  $$\prod_{j=1}^N f(Y_j, T_j, D_j|X, X_T, Z) = \prod_{j=1}^N \int f(Y_j, T_j, D_j|X, X_T, Z, \theta) dF(\theta)$$

  where we can write

  $$f(Y_j, T_j, D_j|X, X_T, Z, \theta) = f(Y_j, D_j|X, Z, \theta)f(T_j|X_T, \theta)$$
Three Economic Applications

1. Multidimensional ability.
2. Social interactions.
3. Dynamic effects of Training.
The multidimensionality of skills, ability and knowledge must be at the “center stage of the theoretical and empirical research on child development, educational attainment and labor market careers”. (Altonji, 2010)

Unlike standard constructs, it reduces the probability of attending a four-year college, while presenting positive reward on the labor market.

For individuals with very high levels of mechanical ability but low levels of cognitive and socio-emotional ability, not going to college is associated with higher expected hourly wage.
Basic Idea and Approach

- USA & NLSY79. Armed Services Vocational Aptitude Battery (ASVAB) and the Armed Forces Qualification Test (AFQT). ASVAB: arithmetic reasoning, word knowledge, paragraph comprehension, mathematics knowledge, numerical operations, coding speed, general science, auto and shop information, electronics information, and mechanical comprehension.

Mechanical comprehension section

Ability to solve simple mechanics problems and understand basic mechanical principles

- Roy Model, three correlated factors, college and earnings, MCMC.
Roy Framework

\[ Y = \begin{cases} 
Y(0) = \mu_0 + \varepsilon_0 & \text{if } D = 0 \\
Y(1) = \mu_1 + \varepsilon_1 & \text{if } D = 1 
\end{cases} \]

\[ D = 1 \{ Z \gamma + \varepsilon_D \geq 0 \} \]

- Not assuming normality on the unobserved components \( \varepsilon_0, \varepsilon_1, \varepsilon_D \), imposing instead the following structure:

\[ \varepsilon_0 = \lambda_0^C \theta_C + \lambda_0^M \theta_M + \lambda_0^S \theta_S + e_0 \]
\[ \varepsilon_1 = \lambda_1^C \theta_C + \lambda_1^M \theta_M + \lambda_1^S \theta_S + e_1 \]
\[ \varepsilon_D = \lambda_D^C \theta_C + \lambda_D^M \theta_M + \lambda_D^S \theta_S + e_D \]

- Assuming

\[ e_0 \perp e_1 \perp e_D \text{ and } (\theta_C, \theta_M, \theta_S) \perp (e_0, e_1, e_D) \]
Latent Factors: Flexible Distributions

\[ \theta_{c,i} \sim \sum_{k=1}^{K} p_k N \left( \mu^k_c, (\sigma^k_c)^2 \right) \]

\[ \theta_{s,i} \sim \sum_{l=1}^{L} p_l S \left( \mu^l_s, (\sigma^l_s)^2 \right) \]

\[ \theta_{m,i} = \alpha_1 \theta_{c,i} + \alpha_2 \theta_{2,i} \]

with

\[ \theta_{2,i} \sim \sum_{j=1}^{J} p_j N \left( \mu^j_2, (\sigma^j_2)^2 \right) \]

\[ \sum_{k=1}^{K} p_k = \sum_{j=1}^{J} p_j = \sum_{l=1}^{L} p_l = 1 \]

In this case, we use mixtures of 2 normals so \( K = J = L = 2 \) and

\[ E[\theta_c] = E[\theta_m] = E[\theta_s] = 0 \]

Finally, \((\theta_c, \theta_s) \perp \theta_2 \) and \((\theta_c, \theta_m) \perp \theta_s \).
Results
Variance Decomposition

After controlling for the latent variables, we are able to explain between 34 and 65 percent of the total variance, except for the Rotter Scale.
Joint Distribution Cognitive and Mechanical Ability

\[ \sigma_{\theta^M} = 0.58, \ \sigma_{\theta^c} = 0.73, \ \sigma_{\theta^s} = 0.89, \ \text{COV}(\theta^c, \theta^m) = 0.21, \ \rho_{\theta^c \theta^m} = 0.52 \]
Distribution Measurements vs Estimated Cognitive Ability: Marginal CDF
Same sorting by Schooling

\[ \sigma_{\theta^c} = 0.73, \quad \text{COV}(\theta^c, \theta^m) = 0.21, \quad \rho_{\theta^c \theta^m} = 0.52 \]
Distribution Measurements vs Estimated Mechanical Ability: Marginal CDF

Distributions and Sorting Differ Between Measurements and Estimated Mechanical ability

\[
\sigma_{\theta^M} = 0.58, \quad COV(\theta^C, \theta^M) = 0.21, \quad \rho_{\theta^C \theta^M} = 0.52
\]
Schooling Choices
Effect of 1 SD of each ability: Mechanical Predicts Lower Schooling

<table>
<thead>
<tr>
<th></th>
<th>Cognitive</th>
<th>Mechanical</th>
<th>Socio-emotional</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Decision</td>
<td>0.229***</td>
<td>-0.095***</td>
<td>0.024***</td>
</tr>
</tbody>
</table>

Note: Standard errors in parenthesis. College Decision equation includes family background controls, cohort dummies and geographical controls for region and urban residence at the age of 14. For hourly wages we control for cohort dummies as well as geographical controls for region and urban residence at age 25.
Hourly Wages
Effect of 1 SD of each ability: Mechanical has Positive Returns

Table: College Attendance and Hourly Wages: Estimated Marginal Effects

<table>
<thead>
<tr>
<th></th>
<th>Cognitive</th>
<th>Mechanical</th>
<th>Socio-emotional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log hourly wages</td>
<td>0.098***</td>
<td>0.014***</td>
<td>0.041***</td>
</tr>
<tr>
<td>w0</td>
<td>0.047***</td>
<td>0.044***</td>
<td>0.033***</td>
</tr>
<tr>
<td>w1</td>
<td>0.108***</td>
<td>-0.031***</td>
<td>0.047***</td>
</tr>
</tbody>
</table>

Note: Standard errors in parenthesis. College Decision equation includes family background controls, cohort dummies and geographical controls for region and urban residence at the age of 14. For hourly wages we control for cohort dummies as well as geographical controls for region and urban residence at age 25.
Conclusions

- We analyze the importance of mechanical, cognitive and socio-emotional ability.
- We show that like standard measures of ability:
  - Mechanical ability is positively rewarded by the labor market,
  - But that in contrast to these other measures, it predicts the choice of low levels of schooling: reduces the likelihood of attending college.
- For individuals with very high levels of mechanical ability but low levels of standard ability (cognitive and socio-emotional), not going to college is associated with the highest expected hourly wage, despite the high returns associated with college.
Three Economic Applications

1. Multidimensional ability.
2. Social interactions: Bullying.
3. Dynamic effects of Training.
Sarzosa & Urzua (2019): Bullying

60,000 children miss school every day in the US because of fear of being bullied. Bully victims are between 2 to 9 times more likely to consider suicide than non-victims in the US. In the UK at least half of suicides among young people are related to bullying.

Examines longitudinal data on teenagers to assess the effects of skills on several outcomes in the context of bullying. Unobserved cognitive and non-cognitive ability as drivers of bullying.

Bulling victims are more likely to be depressed, feel sick, have mental health issues and feel stressed that non victims. Bullies are more likely to be depressed, smoke, and feel stressed that non bullies. Non-cognitive skills significantly reduce the chances of being bullied during high school.
Basic Idea and Approach

- S. Korea & Junior High School Panel (JHSP) of the Korean Youth Panel Survey (KYP). Cognitive (CS) and Non-Cognitive (NCS) measures, self-reported bullying, the proportion of peers that report being bullies in the class and the proportion of peers in the classroom that come from a violent family.

- Roy Model, two factors, bullying and different outcomes, MLE.
Setting

- Structural model implemented is a set of measurement systems that are linked by a factor structure

$$Y_1 = X_Y \beta^Y_1 + \alpha^Y_{1,C} \theta^C + \alpha^Y_{1,N} \theta^N + e^Y_1 \quad \text{if} \ D = 1$$

$$Y_0 = X_Y \beta^Y_0 + \alpha^Y_{0,C} \theta^C + \alpha^Y_{0,N} \theta^N + e^Y_0 \quad \text{if} \ D = 0$$

$$D = 1 \left[ X_D \beta^D + \alpha^D_{Y,D} \theta^C + \alpha^D_{Y,D,N} \theta^N + e^D > 0 \right]$$

$$T = X_T \beta^T + \alpha^T_C \theta^C + \alpha^T_N \theta^N + e^T$$

- $D$ denotes bullying (at age 15), potential outcomes (at age 18) include depression, life satisfaction, sickness, smoking, mental health. The model is estimated using MLE.
Results
Decomposing Variances of Non-Cognitive Measures

Locus of Control, Responsibility, Self-esteem

Unobservables, Latent Endowment, Observables
## Determinants of Bullying

**Table:** Non-Cognitive and Cognitive Skills at age 14 and the Probability of Being Bullied at age 15 ($\tau_1$)

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
<th>(3)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>Std. Err</td>
<td>Coeff.</td>
<td>Std. Err</td>
<td>Coeff.</td>
<td>Std. Err</td>
</tr>
<tr>
<td>Age in Months</td>
<td>0.0034</td>
<td>0.010</td>
<td>0.0021</td>
<td>0.010</td>
<td>0.0009</td>
<td>0.010</td>
</tr>
<tr>
<td>Male</td>
<td>0.3234***</td>
<td>0.068</td>
<td>0.2900***</td>
<td>0.069</td>
<td>0.3076***</td>
<td>0.068</td>
</tr>
<tr>
<td>Youngsiblings</td>
<td>-0.1153*</td>
<td>0.067</td>
<td>-0.1117*</td>
<td>0.067</td>
<td>-0.1159*</td>
<td>0.067</td>
</tr>
<tr>
<td>% Peer Bullies</td>
<td>-3.8408**</td>
<td>1.711</td>
<td></td>
<td></td>
<td>4.8220**</td>
<td>2.147</td>
</tr>
<tr>
<td>% Peer Vnt Fam</td>
<td>0.8107**</td>
<td>0.340</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Peer Vnt Fam$^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Cogn. Skills</td>
<td>-0.2810**</td>
<td>0.132</td>
<td>-0.2986**</td>
<td>0.133</td>
<td>-0.3004**</td>
<td>0.133</td>
</tr>
<tr>
<td>Cognitive Skills</td>
<td>0.0759</td>
<td>0.058</td>
<td>0.0784</td>
<td>0.059</td>
<td>0.0804</td>
<td>0.059</td>
</tr>
<tr>
<td>Observations</td>
<td>2,690</td>
<td></td>
<td>2,690</td>
<td></td>
<td>2,690</td>
<td></td>
</tr>
</tbody>
</table>
Outcomes: $D = 0$ no bullying, $D = 1$ bullying

Table: Outcome Equations (age 18, $\tau_2$) by Bullying Status $D$ (age 15, $\tau_1$)

<table>
<thead>
<tr>
<th>Bullied</th>
<th>(1) Depression $D = 0$</th>
<th>(1) Depression $D = 1$</th>
<th>(2) Drink $D = 0$</th>
<th>(2) Drink $D = 1$</th>
<th>(3) Smoke $D = 0$</th>
<th>(3) Smoke $D = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Cogn</td>
<td>-0.425*** (0.070)</td>
<td>-0.729*** (0.192)</td>
<td>-0.108*** (0.038)</td>
<td>-0.219* (0.118)</td>
<td>-0.093*** (0.024)</td>
<td>-0.159* (0.084)</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.108*** (0.033)</td>
<td>0.107 (0.089)</td>
<td>-0.005 (0.018)</td>
<td>-0.042 (0.052)</td>
<td>-0.034*** (0.012)</td>
<td>-0.138*** (0.037)</td>
</tr>
<tr>
<td>Obs</td>
<td>2,395</td>
<td>2,690</td>
<td>2,690</td>
<td>2,690</td>
<td>2,690</td>
<td>2,690</td>
</tr>
<tr>
<td>(4) Life Satisfaction</td>
<td>(4) Life Satisfaction</td>
<td>(4) Life Satisfaction</td>
<td>(5) Sick</td>
<td>(5) Sick</td>
<td>(6) Mental Health</td>
<td>(6) Mental Health</td>
</tr>
<tr>
<td>$D = 0$</td>
<td>$D = 1$</td>
<td>$D = 0$</td>
<td>$D = 1$</td>
<td>$D = 0$</td>
<td>$D = 1$</td>
<td></td>
</tr>
<tr>
<td>Non-Cogn</td>
<td>0.235*** (0.035)</td>
<td>0.477*** (0.098)</td>
<td>-0.043** (0.017)</td>
<td>-0.090 (0.068)</td>
<td>-0.022* (0.013)</td>
<td>-0.099* (0.059)</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.007 (0.017)</td>
<td>0.046 (0.044)</td>
<td>-0.001 (0.008)</td>
<td>0.011 (0.031)</td>
<td>0.008 (0.006)</td>
<td>0.016 (0.026)</td>
</tr>
<tr>
<td>Obs</td>
<td>2,690</td>
<td>2,514</td>
<td>2,514</td>
<td>2,514</td>
<td>2,514</td>
<td>2,514</td>
</tr>
</tbody>
</table>
What is the effect of Bullying (ATE, TT)?

- \( \text{ATE} = E[Y_1 - Y_0] \) and \( \text{TT} = E[Y_1 - Y_0 | D = 1] \)
- We can estimate these as a function of latent factors.

<table>
<thead>
<tr>
<th>Table: Treatment Effects: Outcomes at Age 18 (( \tau_2 )) of Being Bullied at Age 15 (( \tau_1 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Depression</strong></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td><strong>ATE</strong></td>
</tr>
<tr>
<td>(0.0755)</td>
</tr>
<tr>
<td><strong>TTE</strong></td>
</tr>
<tr>
<td>(0.0590)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Depression</strong></th>
<th><strong>Smoking</strong></th>
<th><strong>Drinking</strong></th>
<th><strong>Sick</strong></th>
<th><strong>Mental Hlth</strong></th>
<th><strong>Life Satisfact</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>inCollege</strong></td>
<td>0.1939**</td>
<td>0.1508**</td>
<td>0.0513</td>
<td>0.0242</td>
<td>0.1467*</td>
</tr>
<tr>
<td>(0.0924)</td>
<td>(0.0752)</td>
<td>(0.0818)</td>
<td>(0.0783)</td>
<td>(0.0814)</td>
<td></td>
</tr>
<tr>
<td><strong>TTE</strong></td>
<td>0.2735***</td>
<td>0.1450**</td>
<td>0.1422**</td>
<td>0.0825</td>
<td>0.2191***</td>
</tr>
<tr>
<td>(0.0725)</td>
<td>(0.0684)</td>
<td>(0.0635)</td>
<td>(0.0646)</td>
<td>(0.0642)</td>
<td></td>
</tr>
</tbody>
</table>

*Str:* Friends, Str:* Parent, Str:* School, Str:* Poverty, Str:* Total

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Factor Models

PIAAC Methodological Seminar June 14
ATE on Smoking as a function of $\theta$

Figure: ATE on Smoking
Three Economic Applications

1. Multidimensional ability.
2. Social interactions: Bullying.
3. Dynamic effects of Training.
Rodriguez, Saltiel & Urzua (2019): Dynamic Treatment Effects

Technological progress +
Mismatch in the labor market
=
Complexity and uncertainty in the demand for skills.

Workers invest in their human capital multiple times in their careers. Thus, what is the impact of human capital investments (training) on earnings in a dynamic world?

Effects of training differ by past training choices, dynamic substitutability, policy-relevant treatment effects differ by compliers type. Policy-makers should consider impact on labor market trajectories and how are these affected by training policies dinamically.
Basic Idea and Approach

- Chile & Administrative Information. Training during 1998-2010, unemployment Insurance database (workers’ monthly earnings in the quarter following a training period), college-entry standardized exam (test scores).

- Roy Model, dynamic decisions, one factor, MLE.
Decision Tree

First Year Training

Second Year Training

$D_1 = 1$

$Y_1(1)$

$D_1(h_2 = 1) = 1 \Rightarrow Y_2(1, 1)$

$D_1(h_2 = 1) = 0 \Rightarrow Y_2(1, 0)$

$D_1 = 1$

$Y_1(0)$

$D_1(h_2 = 0) = 1 \Rightarrow Y_2(0, 1)$

$D_1(h_2 = 0) = 0 \Rightarrow Y_2(0, 0)$
Dynamic-discrete choice model of training choices and earnings

- Individual chooses training for many periods, evaluating benefits and costs. Period-$t$ earnings depend on current and past training choices. Unobserved ability.

- Setting:
  - $h_t$: state of training decision in previous periods ($t-1, t-2,...$).
  - $D_t(h_t) \in \{0,1\}$: training choice, given history $h_t$.
  - $Y_t(h_t,j)$: outcomes in period $t$ given a training history $h_t$ and current decision $j \in \{0,1\}$.

- Formally:
  - Choices: $D_t(h_t) = \mathbf{1}[\mu(I(h_t)) + \theta \lambda(I(h_t)) + \epsilon_t(h_t) \geq 0]$.
  - Earnings: $Y_t(h_t,j) = \delta^Y(h_t,j) + \lambda^Y(h_t,j) \theta + \epsilon^Y_t(h_t,j)$.
  - $\theta$ is unobserved heterogeneity known only to the agent.
Results
Distribution of Unobserved Ability by Training History

- Always-trained (1,1)
- Trained Once (1,0)
- Trained Once (0,1)
- Never-trained (0,0)
Dynamic Treatment Effects

Let \( \tilde{Y}_1(j) \) be the present value of (observed) earnings associated with choosing training option \( j \) in period \( t = 1 \). In two periods:

\[
\tilde{Y}_1(1) \equiv Y_1(1) + \rho \left( D_2(j) Y_{i2}(j = 1, 1) + (1 - D_2(j)) Y_{i2}(j = 1, 0) \right)
\]

Dynamic Treatment Effects (DATE): \( \tilde{Y}(1) - \tilde{Y}(0) \).

- DATE can be decomposed as:

\[
\text{DATE} = \left( Y_{i1}(1) - Y_{i1}(0) \right) + \rho \left[ Y_{i2}(1, 0) - Y_{i2}(0, 0) \right] + \rho \left[ D_{i2}(1)(Y_{i2}(1, 1) - Y_{i2}(1, 0)) - D_{i2}(0)(Y_{i2}(0, 1) - Y_{i2}(0, 0)) \right]
\]

Direct effect (short-term)
Direct effect (medium-term)
Continuation value
Dynamic Treatment Effects: Direct Effects and Continuation Value

![Graph showing the relationship between deciles of ability and change in earnings.](image-url)

- Continuation value
- Direct effect (short-term)
- Direct effect (medium-term)
- ATE (DE+CV)
Conclusions

Large-scale, government-subsided training program to show novel evidence on dynamic treatment effects of job training.

Effects of training differ by past training choices, dynamic substitutability, policy-relevant treatment effects differ by compliers type.

Factor model allows the estimation of the impact on labor market trajectories and how are these affected by training policies dynamically.