Decomposing Changes in Aggregate Loan-to-Value Ratios

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Abstract: The Loan-to-Value ratio for a group of households can be defined as the total amount of outstanding mortgage loans divided by the total value of the housing stock. Compositional change of the household group can affect the Loan-to-Value ratio. In this paper we decompose a change in the overall ratio into three terms: the contribution of "continuing" households, which are present in each time period, and the contributions of "entering" and "exiting" households. Similar decompositions – in this case relating to firms – exist in the literature on productivity analysis. However, as far as we know, our decomposition has not been described before. We provide an empirical illustration for the Netherlands, based on microdata from the Dutch tax office.

Key words: decomposition, household dynamics, mortgage loans, property values.

JEL Classification: C43, G21, R21.

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1. Introduction

The financial crisis of 2008 underlined the need for accurate and timely information on the housing market. Statistical agencies around the world have indeed been improving and extending their housing market statistics. A major achievement in Europe was the setting up and harmonization of house price indexes, coordinated by Eurostat. In the Netherlands, users also asked for indicators of the financial risk faced by homeowners and mortgage lenders. One example of such indicators is the Loan-to-Value ratio for a group of households. This ratio is defined as the total amount of outstanding mortgage loans divided by the total value of the corresponding housing stock. Recently, Statistics Netherlands decided to publish Loan-to-Value ratios, based on data that stem from the Dutch tax office.

The Loan-to-Value (LTV) ratio is a key term in real estate financing, particularly in the case of mortgage loans.² It refers to the amount of money borrowed against the market value of the property. The property valuations are often done by appraisers. In several countries, loans with an LTV ratio exceeding 0.8 require mortgage insurance, as the risk of the borrower defaulting is deemed too great for the lender.³ The LTV ratio is also used by lenders to assess the risk involved when someone who owns a property for a number of years wants to refinance the property in order to take cash out. Due to repayments, the outstanding loan amount usually diminishes over time. But whether the owner's equity position in a property (1 minus the LTV ratio) increases, depends also on the value change of the property. As a result of the financial and economic crisis, many countries have witnessed a substantial drop in house prices, so most homeowners' LTV ratios have probably gone up.

As will be shown later on, the aggregate LTV ratio for a group of households is a weighted average of the LTV ratios pertaining to the constituent households. If all of

¹ Eurostat also funded the compilation of a *Handbook on Residential Property Price Indexes*. The latest draft of the handbook can be downloaded from www.epp.eurostat.ec.europa.eu.

² In Australia and New Zealand, the Loan-to-Value Ratio is usually abbreviated as LVR.

³ This is the case in e.g. the U.S. and Australia. The purchaser has to make a down payment of 20% or more to avoid paying mortgage insurance premiums. In the Netherlands, mortgage insurance is unusual. Many first-time buyers make no down payment at all. Their LTV ratios are sometimes well above 1 at the time of purchase because they borrow an amount that also covers additional costs (which is allowed in the Netherlands). Dutch banks charge higher interest rates when the LTV ratio exceeds 0.8, however, so there is actually an incentive for paying down.

the individual ratios increase, and if the weights do not change, the aggregate ratio also increases. However, a group of households for which statistical information is collected is typically dynamic; unlike a constant panel, its size and composition will change over time. In particular, apart from the "continuing" households, which are present in each time period, there may also be "entering" and "exiting" households. Such compositional changes can affect the overall LTV ratio. To give an example: households that recently purchased a house often have relatively high LTV ratios because they have not yet been able to repay a large part of the mortgage loan; when these households enter the group in question, they will put an upward pressure on the overall ratio. Moreover, the weights attached to the individual households need not be constant, which could also affect the overall ratio.

The purpose of this paper is to show how a change in the aggregate LTV ratio can be decomposed into three terms: the contribution due to a change in the LTV ratio of the continuing households plus the contributions of entering and exiting households. Similar decompositions – in this case relating to firms instead of households – can be found in the literature on productivity analysis. Aggregate productivity is defined as the ratio of aggregate output to aggregate input. Our decomposition is readily applicable to productivity but, as far as we know, has not been described before. In contrast to most other decompositions, we follow a *two-stage approach*. We first decompose the change in the overall ratio into contributions of the sub-sets of continuing, entering and exiting elements and then disaggregate the result into individual contributions, whereas others directly focus on the micro level.⁴

The remainder of the paper is organized as follows. In section 2 we outline the proposed decomposition in general terms (and speak of the *y*-to-*x* ratio). In section 3 we discuss two decompositions that have been suggested in the productivity measurement literature and argue why we prefer the proposal in section 2. Section 4 focuses on the LTV ratio and explains why an unweighted measure, or the median, is unsatisfactory. A number of problems related to household dynamics, such as the identification of the continuing households, are also addressed. Section 5 describes the data utilized, which stem from the Dutch tax office and cover 2006-2007, and presents the empirical results. Section 6 concludes.

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⁴ See Balk (2003) for an overview of single-stage decompositions of productivity change proposed in the literature. Balk and Hoogenboom-Spijker (2003) apply these decompositions to Dutch microdata.

2. A Two-Stage Decomposition

In this section we will derive a decomposition of the change (difference) in the ratio of two aggregate values. Let U^t denote a set of elements (persons, households, or firms) in period t, and suppose that data on two variables, x and y, is collected; x_i^t and y_i^t denote the observations for element i in period t. The aggregate values are $x^t = \sum_{i \in U^t} x_i^t$ and $y^t = \sum_{i \in U^t} y_i^t$, respectively. The y-to-x ratio is defined as

$$R^{t} = \frac{y^{t}}{x^{t}} = \sum_{i \in U^{t}} y_{i}^{t} \cdot \sum_{i \in U^{t}} x_{i}^{t} \cdot$$
 (1)

Notice that R^t can be written as a weighted average of individual or micro y-to-x ratios $R_i^t = y_i^t / x_i^t$:

$$R^{t} = \frac{\sum_{i \in U^{t}} x_{i}^{t} (y_{i}^{t} / x_{i}^{t})}{\sum_{i \in U^{t}} x_{i}^{t}} = \sum_{i \in U^{t}} s_{i}^{t} R_{i}^{t} , \qquad (2)$$

where $s_i^t = x_i^t / \sum_{i \in U^t} x_i^t$ denotes the share of element *i* in the aggregate x^t .

In what follows we will distinguish two time periods, the base period t = 0 and the comparison period t = 1. Using expression (2), the change (difference) in R between period 0 and period 1 can be written as

$$R^{1} - R^{0} = \frac{y^{1}}{x^{1}} - \frac{y^{2}}{x^{2}} = \sum_{i \in U^{1}} s_{i}^{1} R_{i}^{1} - \sum_{i \in U^{0}} s_{i}^{0} R_{i}^{0} .$$

$$(3)$$

Let C denote the set of continuing elements, which are present in both time periods. The set of exiting elements, which are only present in period 0, is denoted by X; the set of entering elements, which are only present in period 1, is denoted by N. Thus, we have $U^0 = C \cup X$ and $U^1 = C \cup N$, and (3) can be decomposed as

$$R^{1} - R^{0} = \sum_{i \in C} s_{i}^{1} R_{i}^{1} - \sum_{i \in C} s_{i}^{0} R_{i}^{0} + \sum_{i \in N} s_{i}^{1} R_{i}^{1} - \sum_{i \in V} s_{i}^{0} R_{i}^{0} .$$

$$(4)$$

Another way of writing (4) is

$$R^{1} - R^{0} = s_{C}^{1} \sum_{i \in C} s_{Ci}^{1} R_{i}^{1} - s_{C}^{0} \sum_{i \in C} s_{Ci}^{0} R_{i}^{0} + s_{N}^{1} \sum_{i \in N} s_{Ni}^{1} R_{i}^{1} - s_{X}^{0} \sum_{i \in X} s_{Xi}^{0} R_{i}^{0} ,$$

$$(5)$$

⁵ We will use the term time *period* throughout the paper even though the variables can relate to points in time rather than discrete periods.

in which $s_{Ci}^1 = x_i^1 / \sum_{i \in C} x_i^1$, $s_{Ci}^0 = x_i^0 / \sum_{i \in C} x_i^0$, $s_{Ni}^1 = x_i^1 / \sum_{i \in N} x_i^1$ and $s_{Xi}^0 = x_i^0 / \sum_{i \in X} x_i^0$ are the x shares of the individual elements with respect to the sets to which they belong, and with $s_C^1 = \sum_{i \in C} x_i^1 / \sum_{i \in U^1} x_i^1$, $s_C^0 = \sum_{i \in C} x_i^0 / \sum_{i \in U^0} x_i^0$, $s_N^1 = \sum_{i \in N} x_i^1 / \sum_{i \in U^1} x_i^1$ and $s_X^0 = \sum_{i \in X} x_i^0 / \sum_{i \in U^0} x_i^0$ denoting the x shares of the sub-sets C, N and X. Similar to (1) and (2), we write the y-to-x values for C, N and X as

$$R_C^0 = \sum_{i \in C} s_{Ci}^0 R_i^0 \; ; \; R_C^1 = \sum_{i \in C} s_{Ci}^1 R_i^1 \; ; \; R_N^1 = \sum_{i \in N} s_{Ni}^1 R_i^1 \; ; \; R_X^0 = \sum_{i \in X} s_{Xi}^0 R_i^0 \; . \tag{6}$$

Substitution of (6) into (5) yields

$$R^{1} - R^{0} = s_{C}^{1} R_{C}^{1} - s_{C}^{0} R_{C}^{0} + s_{N}^{1} R_{N}^{1} - s_{N}^{0} R_{N}^{0}.$$

$$(7)$$

Equation (7) is a convenient starting point for decomposing the change in the aggregate y-to-x ratio, $R^1 - R^0$, into the contributions of the sub-sets C, N and X. We first rewrite (7) as

$$R^{1} - R^{0} = s_{C}^{1} R_{C}^{1} - s_{C}^{0} R_{C}^{0} + (s_{C}^{0} R_{C}^{1} - s_{C}^{0} R_{C}^{1}) + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}$$

$$= s_{C}^{0} (R_{C}^{1} - R_{C}^{0}) + (s_{C}^{1} - s_{C}^{0}) R_{C}^{1} + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}.$$
(8)

Alternatively, (7) can be rewritten as

$$R^{1} - R^{0} = s_{C}^{1} R_{C}^{1} - s_{C}^{0} R_{C}^{0} + (s_{C}^{1} R_{C}^{0} - s_{C}^{1} R_{C}^{0}) + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}$$

$$= s_{C}^{1} (R_{C}^{1} - R_{C}^{0}) + (s_{C}^{1} - s_{C}^{0}) R_{C}^{0} + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}.$$

$$(9)$$

Decompositions (8) and (9) are equally valid. To avoid making an arbitrary choice and to treat (8) and (9) in a symmetric manner, we will take the mean. This leads to

$$R^{1} - R^{0} = \left[\left(s_{C}^{0} + s_{C}^{1} \right) / 2 \right] \left(R_{C}^{1} - R_{C}^{0} \right) + \left(s_{C}^{1} - s_{C}^{0} \right) \left[\left(R_{C}^{0} + R_{C}^{1} \right) / 2 \right] + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}$$

$$= s_{C}^{01} \left(R_{C}^{1} - R_{C}^{0} \right) + \left(s_{X}^{0} - s_{N}^{1} \right) R_{C}^{01} + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}$$

$$= s_{C}^{01} \left(R_{C}^{1} - R_{C}^{0} \right) + s_{N}^{1} \left(R_{N}^{1} - R_{C}^{01} \right) - s_{N}^{0} \left(R_{N}^{0} - R_{C}^{01} \right), \tag{10}$$

where we defined $s_C^{01} = (s_C^0 + s_C^1)/2$ and $R_C^{01} = (R_C^0 + R_C^1)/2$ in the second expression and used $s_C^0 = 1 - s_X^0$ and $s_C^1 = 1 - s_X^1$.

The first term in the last expression of (10) measures the contribution of the set of continuing elements by multiplying the change in the *y*-to-*x* ratio for set C, $R_C^1 - R_C^0$, by its average *x* share s_C^{01} in the two periods. The second term measures the contribution of the set of entering elements. Note that if R_N^1 is greater than the average ratio for the

continuing elements, R_C^{01} , then the set of entering elements will raise the overall ratio. The set of exiting elements will lower the overall ratio if R_X^0 is above R_C^{01} , as shown by the third term.

For some purposes it could be useful to express the change in the aggregate y-to-x ratio in terms of the contributions of the *individual elements* rather than the sets C, N and X. Using (6), the change in the aggregate ratio for the set of continuing elements can be decomposed as

$$R_{C}^{1} - R_{C}^{0} = \sum_{i \in C} s_{Ci}^{1} R_{i}^{1} - \sum_{i \in C} s_{Ci}^{0} R_{i}^{0} = \sum_{i \in C} s_{Ci}^{0} (R_{i}^{1} - R_{i}^{0}) + \sum_{i \in C} R_{i}^{1} (s_{Ci}^{1} - s_{Ci}^{0})$$

$$= \sum_{i \in C} [s_{Ci}^{0} (R_{i}^{1} - R_{i}^{0}) + R_{i}^{1} (s_{Ci}^{1} - s_{Ci}^{0})], \qquad (11)$$

or alternatively as

$$R_C^1 - R_C^0 = \sum_{i \in C} [s_{Ci}^1(R_i^1 - R_i^0) + R_i^0(s_{Ci}^1 - s_{Ci}^0)].$$
 (12)

Again, for reasons of symmetry we prefer taking the mean of (11) and (12):

$$R_C^1 - R_C^0 = \sum_{i \in C} \left[s_{Ci}^{01} (R_i^1 - R_i^0) + R_i^{01} (s_{Ci}^1 - s_{Ci}^0) \right], \tag{13}$$

where $s_{Ci}^{01} = (s_{Ci}^0 + s_{Ci}^1)/2$ and $R_i^{01} = (R_i^0 + R_i^1)/2$ for $i \in C$. Equation (13) shows that the change in the aggregate *y*-to-*x* ratio for the continuing elements depends not only on (the weighted average of) the changes in the micro ratios but also on the changes in the micro *x* shares. Put differently, due to a shift towards elements with lower *x* shares, the aggregate *y*-to-*x* ratio can decrease even when all micro ratios increase.

Substituting expressions (13), $R_N^1 = \sum_{i \in N} s_{Ni}^1 R_i^1$ and $R_X^0 = \sum_{i \in X} s_{Xi}^0 R_i^0$ into (10) and using $s_N^1 s_{Ni}^1 = s_i^1$ and $s_X^0 s_{Xi}^0 = s_i^0$ yields

$$R^{1} - R^{0} = \sum_{i \in C} s_{C}^{01} [s_{Ci}^{01} (R_{i}^{1} - R_{i}^{0}) + R_{i}^{01} (s_{Ci}^{1} - s_{Ci}^{0})] + \sum_{i \in N} s_{i}^{1} (R_{i}^{1} - R_{C}^{01}) - \sum_{i \in V} s_{i}^{0} (R_{i}^{0} - R_{C}^{01}).$$
(14)

The contribution of $i \in C$, for example, equals $s_C^{01}[s_{Ci}^{01}(R_i^1 - R_i^0) + R_i^{01}(s_{Ci}^1 - s_{Ci}^0)]$. Note that we follow a *two-stage approach*: in the first stage we distinguish between the sets (sub-aggregates) of continuing, entering and exiting elements and in the second stage we step down to the micro level.

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⁶ In a productivity context, this paradox was noted by Fox (2002). Prior to that, Fox (1999) described his first paradox: "Using a standard definition of cost efficiency, a multi-product firm may be more efficient in producing each product than any other firm, yet it may not be the most efficient firm overall."

A number of alternative decompositions have been proposed in the literature on productivity growth. In section 3 below, we will discuss two of them and argue why we prefer decompositions (10) and (14).

3. Alternative Decompositions

One alternative approach could be the following. The starting point is again equation (4) but now we focus directly on the micro level, i.e., on the contributions of the *individual* continuing, entering and exiting households rather than the sub-aggregates C, N and X. As noted by Balk (2003), an arbitrary scalar a (a scaling factor) can be incorporated into (4):

$$R^{1} - R^{0} = \sum_{i \in C} s_{i}^{1}(R_{i}^{1} - a) - \sum_{i \in C} s_{i}^{0}(R_{i}^{0} - a) + \sum_{i \in N} s_{i}^{1}(R_{i}^{1} - a) - \sum_{i \in N} s_{i}^{0}(R_{i}^{0} - a),$$

$$(15)$$

because $\sum_{i \in C} s_i^0 + \sum_{i \in X} s_i^0 = \sum_{i \in C} s_i^1 + \sum_{i \in N} s_i^1 = 1$. Equation (15) can be written as

$$R^{1} - R^{0} = \sum_{i \in C} s_{i}^{1} R_{i}^{1} - \sum_{i \in C} s_{i}^{0} R_{i}^{0} - a \sum_{i \in C} (s_{i}^{1} - s_{i}^{0}) + \sum_{i \in N} s_{i}^{1} (R_{i}^{1} - a) - \sum_{i \in X} s_{i}^{0} (R_{i}^{0} - a).$$
 (16)

Two possible ways of writing the right-hand side of (16) are

$$\sum_{i \in C} s_i^0(R_i^1 - R_i^0) + \sum_{i \in C} R_i^1(s_i^1 - s_i^0) - a \sum_{i \in C} (s_i^1 - s_i^0) + \sum_{i \in N} s_i^1(R_i^1 - a) - \sum_{i \in X} s_i^0(R_i^0 - a);$$
 (17)

$$\sum_{i \in C} s_i^1(R_i^1 - R_i^0) + \sum_{i \in C} R_i^0(s_i^1 - s_i^0) - a \sum_{i \in C} (s_i^1 - s_i^0) + \sum_{i \in N} s_i^1(R_i^1 - a) - \sum_{i \in X} s_i^0(R_i^0 - a).$$
 (18)

Taking the average of (17) and (18) and some rearranging yields

$$R^{1} - R^{0} = \sum_{i \in C} s_{i}^{01} (R_{i}^{1} - R_{i}^{0}) + \sum_{i \in C} (R_{i}^{01} - a)(s_{i}^{1} - s_{i}^{0}) + \sum_{i \in N} s_{i}^{1} (R_{i}^{1} - a) - \sum_{i \in X} s_{i}^{0} (R_{i}^{0} - a)$$

$$= \sum_{i \in C} \left[s_{i}^{01} (R_{i}^{1} - R_{i}^{0}) + (R_{i}^{01} - a)(s_{i}^{1} - s_{i}^{0}) \right] + \sum_{i \in N} s_{i}^{1} (R_{i}^{1} - a) - \sum_{i \in X} s_{i}^{0} (R_{i}^{0} - a), \quad (19)$$

where $R_i^{01} = (R_i^0 + R_i^1)/2$, as before, and $s_i^{01} = (s_i^0 + s_i^1)/2$.

What value for a should be chosen? Balk (2003) calls $(R^0 + R^1)/2$, the average aggregate level of R (in his case productivity), "a rather natural choice". But this choice would make the contribution of the continuing elements dependent on R_N^1 and R_X^0 . In our opinion a more suitable choice for a is $R_C^{01} = (R_C^0 + R_C^1)/2$, the average value for the set of continuing elements. Decomposition (19) then becomes

$$R^{1} - R^{0} = \sum_{i \in C} \left[s_{i}^{01} (R_{i}^{1} - R_{i}^{0}) + (R_{i}^{01} - R_{C}^{01}) (s_{i}^{1} - s_{i}^{0}) \right]$$

$$+ \sum_{i \in N} s_{i}^{1} (R_{i}^{1} - R_{C}^{01}) - \sum_{i \in V} s_{i}^{0} (R_{i}^{0} - R_{C}^{01}).$$

$$(20)$$

If $R_i^{01} = R_C^{01}$ for all $i \in C$, the contribution due to share changes disappears. It is easily demonstrated that this also holds for our decomposition (14). The second and third term in (20), the aggregate contributions of the entries and exits, are equal to those in (14). Hence, the first term in (20), the aggregate contribution of the continuing elements, is equal to the first term in (14). However, the contribution of an individual element $i \in C$ in (20), $s_i^{01}(R_i^1 - R_i^0) + (R_i^{01} - R_C^{01})(s_i^1 - s_i^0)$, is different from that in (14), which was $s_C^{01}[s_C^{01}(R_i^1 - R_i^0) + R_i^{01}(s_C^1 - s_C^0)]$.

There are two reasons why we prefer (14) over (20). If R_i^{01} is the same for all $i \in C$, then in decomposition (14) the total contribution due to share changes vanishes. This nice property does not hold for decomposition (20), except when $R_i^{01} = R_C^{01}$ for all $i \in C$. More importantly, measuring the contribution of the *set* of continuing elements requires calculating its aggregate y-to-x ratio in a similar way as the overall ratio. Thus, taking equation (7) as the starting point, and following a two-stage approach, makes a lot of sense.

It is worthwhile mentioning that we made an implicit assumption in deriving equation (10) from (7). Similar to what was done above, we could include an arbitrary scalar $a \neq 0$ in (7), because $s_C^0 + s_X^0 = s_C^1 + s_N^1 = 1$, yielding

$$R^{1} - R^{0} = s_{C}^{1}(R_{C}^{1} - a) - s_{C}^{0}(R_{C}^{0} - a) + s_{N}^{1}(R_{N}^{1} - a) - s_{X}^{0}(R_{X}^{0} - a)$$

$$= s_{C}^{1}R_{C}^{1} - s_{C}^{0}R_{C}^{0} - a(s_{C}^{1} - s_{C}^{0}) + s_{N}^{1}(R_{N}^{1} - a) - s_{X}^{0}(R_{X}^{0} - a).$$
(21)

As before, there are a number of ways to proceed. Equation (21) can be written as

$$R^{1} - R^{0} = s_{C}^{0}(R_{C}^{1} - R_{C}^{0}) + R_{C}^{1}(s_{C}^{1} - s_{C}^{0}) - a(s_{C}^{1} - s_{C}^{0}) + s_{N}^{1}(R_{N}^{1} - a) - s_{X}^{0}(R_{X}^{0} - a),$$
or, alternatively, as

$$R^{1} - R^{0} = s_{C}^{1}(R_{C}^{1} - R_{C}^{0}) + R_{C}^{0}(s_{C}^{1} - s_{C}^{0}) - a(s_{C}^{1} - s_{C}^{0}) + s_{N}^{1}(R_{N}^{1} - a) - s_{X}^{0}(R_{X}^{0} - a).$$
 (23)

Taking the mean of (22) and (23) yields

$$R^{1} - R^{0} = s_{C}^{01}(R_{C}^{1} - R_{C}^{0}) + R_{C}^{01}(s_{C}^{1} - s_{C}^{0}) - a(s_{C}^{1} - s_{C}^{0}) + s_{N}^{1}(R_{N}^{1} - a) - s_{N}^{0}(R_{N}^{0} - a)$$

$$= s_{C}^{01}(R_{C}^{1} - R_{C}^{0}) + (R_{C}^{01} - a)(s_{C}^{1} - s_{C}^{0}) + s_{N}^{1}(R_{N}^{1} - a) - s_{N}^{0}(R_{N}^{0} - a).$$
(24)

The second term in the last expression of (24) measures the contribution of a change in x shares of the set C of continuing elements to the change in the overall y-to-x ratio. We prefer choosing $a = R_C^{01}$ in order to get rid of this nuisance term. But then of course (24) simplifies to our decomposition (10).

Diewert and Fox (2010) propose another decomposition. Starting from equation (7), which we repeat for convenience, they proceed as follows:

$$R^{1} - R^{0} = s_{C}^{1} R_{C}^{1} - s_{C}^{0} R_{C}^{0} + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0} = (1 - s_{N}^{1}) R_{C}^{1} - (1 - s_{X}^{0}) R_{C}^{0} + s_{N}^{1} R_{N}^{1} - s_{X}^{0} R_{X}^{0}$$
$$= (R_{C}^{1} - R_{C}^{0}) + s_{N}^{1} (R_{N}^{1} - R_{C}^{1}) - s_{X}^{0} (R_{X}^{0} - R_{C}^{0}).$$
(25)

Equation (25) differs in two important respects from our proposal (10). First, in (25) the contribution of the set of continuing elements is independent of its relative size, which seems counterintuitive, whereas in (10) the contribution depends on its average x share. Second, in equation (25) the contributions of the entering and exiting elements depend on the difference between their y-to-x ratios and those of the continuing elements in the respective periods, while in (10) R_N^1 and R_X^0 are evaluated against the "fixed" average R_C^{01} . Our approach has the advantage that the contributions of the entering and exiting elements cancel each other out when the two sets have identical y-to-x ratios and their relative importance in terms of x is the same.

Diewert and Fox (2010) also apply the "Bennet (1920) type decomposition" (13) to decompose the y-to-x change of the set of continuing elements. They follow a similar two-stage approach as we do, but their first-stage decomposition (25) differs from our decomposition (10). They make a final adjustment to ensure that the result is invariant to changes in the units of measurement by dividing both sides of (25) by the base period value R^0 . In the case of Loan-to-Value ratios, this adjustment is less relevant as y and x will typically be expressed in the same currency unit, e.g., in euros or in thousands of euros.

In section 4 we will turn to Loan-to-Value ratios. As will become clear, one of the biggest problems is to find a definition of the household that enables us to identify continuing households.

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⁷ Haltiwanger (2000) argues that if the productivity levels of entering and exiting firms are the same, then the sum of the contributions of entering and exiting firms should be zero. This does not hold for equation (10) unless the x shares are also identical. In Appendix 1 we show how decomposition (10) relates to an *index* of productivity change when the x shares are equal.

4. Loan-to-Value Ratios

The measurement of the Loan-to-Value (LTV) ratio and the practical implementation of the decomposition into contributions of the continuing, entering and exiting households raise a number of questions, such as

- What is the appropriate target population?
- What exactly do we mean by "loan" and "value"?
- Why are the unweighted LTV ratio and the median ratio unsuitable?
- How should the continuing, entering and exiting households be defined?
- Is the necessary data available?

Before going into these questions, we take a brief look at two definitions of the household. The System of National Accounts 2008 and the U.S. Census Bureau define the household differently:

"A household is a group of persons who share the same living accommodation, who pool some, or all, of their income and wealth and who consume certain types of goods and services collectively, mainly housing and food."

(Eurostat et al., 2009)

"A household includes all the people who occupy a housing unit as their usual place of residence." (U.S. Census Bureau, Glossary for the 2010 Census, http://factfinder2.census.gov)

Sharing a housing unit or living accommodation, such as a house or an apartment, plays a crucial role in both definitions, but the SNA places a few additional restrictions. For our purpose, the Census Bureau's definition will suffice. According to them, "a housing unit is owner-occupied if the owner or co-owner lives in the unit even if it is mortgaged or not fully paid for".

LTV ratios are obviously only relevant for homeowners, not for renters, so the answer to the first question seems quite straightforward: the target population consists of households living in owner-occupied housing units. In addition, it could be useful to exclude households without a mortgage loan; they paid off their debt and there is no risk involved any more, in any case no risk of default. User needs should also be taken into account, especially when it comes to distinguishing sub-populations. Users may require LTV ratios for different income groups, households in different regions of the country, etc.

⁸ Statistics Netherlands indeed intends to publish LTV ratios both for all owner-occupiers and for owner-occupiers with a mortgage loan.

The owner has usually taken out a mortgage loan to finance the purchase of the property. Some people also own a second house, for instance a vacation home, which may be mortgaged as well. In this paper, we exclude the latter and look at primary (first) homes only, including properties that have more than one mortgage lien. Note that we exclude non-mortgage loans. The value of the house should reflect the market value. Property valuations are typically determined by appraisers. In some countries, property assessments are needed for tax purposes and are available for the entire housing stock. If the properties are re-assessed regularly, for example on an annual basis, then these official valuations could be used to compute LTV ratios.

As shown by equation (2), the aggregate LTV ratio is a weighted average of the LTV ratios pertaining to the individual households, where the weights are equal to the properties' value shares. We could alternatively compute the unweighted average of the individual ratios. However, this measure is not representative of the group as a whole. Some households are "more important" than others, and the property value reflects the *economic importance*. The weighted average has a straightforward interpretation: it says what percentage of the value of the owner-occupied housing stock is mortgaged. This is not to say that the unweighted average has no meaning. Like the median, it is a measure of central tendency – a location measure – of the distribution of the individual ratios. This distribution can be of importance to users, in particular for determining how many households have an LTV ratio that exceeds some threshold value.

The biggest conceptual and practical problem with decomposing a change in the LTV ratio is identifying the continuing households. Once this is done, the entering and exiting households are implicitly identified, given the target population. The difficulty in establishing whether households stay "the same" is an old problem that has frustrated longitudinal household analyses for a long time. Broadly speaking, there are two issues involved which are often interrelated: a changing household composition and residential mobility. As long as the composition of the household remains unchanged and none of the members moves house, we can safely speak of the same household. Complications can arise if these requirements do not hold.

⁹ Households that actually defaulted and became renters will be classified as exiting households because they are no longer part of the target population even if they still have a liability (remaining debt) after the house has been sold.

¹⁰ Many household characteristics do change, of course. For example, the age of all the members definitely changes over time!

Suppose that the household moves house while its composition does not change. If it moves to a rental home, then the household exits the target population. But what if it buys another place? According to the definitions above, the household continuous to exist. On the other hand, the home and the location are different, and a new mortgage loan might replace the former one. For our purpose it could be more appropriate to view this household as both exiting and (provided that the new location fits into the target population) entering rather than continuing. Another problem area is the splitting up of households, say because of divorce. At least one of the household members is likely to move out and start a new household or move in with someone else. Assuming the other members stay in the home, should we consider them as the same household as before but with a different composition or as a new household? And, for our purpose, should the choice depend on whether the conditions of the mortgage loan change?

In section 5 we present empirical evidence for the Netherlands and describe how households are identified and followed over time. We are unable to offer a real solution to the problems mentioned above. Our identification method is a pragmatic one, driven by data availability. Admittedly, this will make the interpretation of the decomposition a little unclear. There are some other issues which should also be taken into account when interpreting our results.

5. Data and Results for the Netherlands

Our data set is based on income tax files and contains microdata on household wealth – financial capital and liabilities – in December 2006, 2007 and 2008. It basically covers all private households in the Netherlands, from which we selected the owner-occupiers. Outstanding mortgages on the primary (first) dwelling are distinguished as a separate category. The officially appraised values of the properties are also included in the data set. ¹¹ Dutch properties are re-assessed annually, and we have valuations at our disposal for each year. However, the appraisals relate to January and hence lag behind almost a

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¹¹ In the Netherlands, the valuations are needed for both income tax and local property tax purposes. They are determined by private companies specialized in mass appraisals, but the municipalities are ultimately responsible for the results. Statistics Netherlands' house price index is based on the Sale Price Appraisal Ratio (SPAR) method which combines selling prices and official valuations. For more information, see de Vries *et al.* (2009).

full year. We used the overall house price index to update the appraisals to December. This equal re-scaling of individual LTV ratios is a potential source of error because in reality, different market segments exhibit different price trends. The results also depend on the accuracy of the appraisal data. At the micro level, these data are quite noisy and do not always reflect the "true" market values. On the other hand, we expect the quality of the data to be good enough to compute LTV ratios at the aggregate level.

Besides data quality issues, certain types of mortgages hamper the interpretation of the LTV ratio as a measure of financial risk. Tax treatment of debt-financed owner-occupied housing in the Netherlands is very favorable. Mortgage interest payments are fully deductible at the marginal tax rate, so Dutch households have strong incentives to maintain high levels of mortgage debt. Due to deregulation and increased competition among credit providers, new financial products were introduced in the 1990s. Mortgage financing combined with a capital insurance policy in particular became popular. With this type of mortgage, principal repayments are paid into an insurance policy rather than deducted from the outstanding mortgage. The borrower can maximize mortgage interest deductions by not paying off the debt while at the same time accumulating capital in the insurance policy to pay off the debt when the mortgage term expires. Consequently, the estimated LTV ratio will overstate the risk as compared to a situation without this type of mortgages.

Everyone in the Netherlands has a personal "citizen service number", which is included in the original data files. This number is used by many Dutch organizations. To avoid statistical disclosure, Statistics Netherlands replaced it by a randomly chosen identification number. A *reference person* has been assigned to each household in the data set, typically the person, or one of the people, in whose name the home is owned or rented. All other household members (all of the people that occupy the same housing unit) receive the reference person's identification number, and so this number identifies the household. We will use it to track households over time. In other words, we assume that a household stays "the same", and will be classified as continuing, as long as the identification number exists. In general the number is left unchanged when the reference person moves house or when the composition of the household changes. As mentioned in section 4, this may not be the preferred way to identify the (continuing) households, but at this stage it is the best we can do.

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¹² This is in agreement with the U.S. Census Bureau's definition of the *householder*.

Table 1 lists some descriptive statistics for our microdata set. The total number of households with an owned dwelling increased from 3.8 million in 2006 to 4.0 million in 2008.¹³ This increase was mainly due to a shift from rental to owner-occupied homes; the share of owner-occupiers increased from 53.7% to 55.2%. The average value of the outstanding mortgage loan (per household, including households without a mortgage loan) amounted to €134,000 in 2006 and increased by 5.3% and 3.6% in 2007 and 2008, respectively. In each year the average value of the house was slightly more than twice the average amount of the loan; rounded to two decimal places, the aggregate LTV ratio was 0.47 in 2006 and 2007 and 0.48 in 2008.

Table 1. Descriptive statistics (owner-occupiers)*

	`				
	2006	2007		2008	
Number of households	3,761,400	3,854,500		4,034,400	
Disposable income (mean)	€38,100	€41,500	(+8.8%)	€42,100	(+1.5%)
Mortgage loan (mean)	€134,000	€141,000	(+5.3%)	€146,100	(+3.6%)
Appraised value (mean)	€285,200	€301,600	(+5.8%)	€307,200	(+1.9%)
LTV ratio	0.46979	0.46757		0.47550	

^{*} Changes with respect to the preceding year in parentheses.

Purely as background information, Table 1 also provides the average disposable household income for all owner-occupiers on an annual basis. Disposable income grew by 8.8% in 2007 and 1.5% in 2008 with respect to the preceding year. Dividing average disposable income by the average value of the mortgage loan gives an indication of the aggregate Loan-to-Income (LTI) ratio. ¹⁴ The LTI ratio was 3.51, 3.40 and 3.47 in 2006, 2007 and 2008. The decompositions discussed in sections 2 and 3 can also be applied to LTI ratios. We have chosen not to do this as disposable income in our data set does not exactly meet Statistics Netherlands' definition and, in this respect, has been measured only crudely.

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¹³ These figures differ somewhat from the official figures as published on Statistics Netherlands' online database "Statline". One of the reasons is that we identified owner-occupiers as households that have a positive value for owned dwelling in our data set. The official figures on the other hand rely on combined (and cleaned) microdata from a number of sources and on different rules for identifying owner-occupier households.

¹⁴ The LTI ratio is another famous indicator of financial risk in the context of owner-occupied housing. Again, we could alternatively restrict the population to owner-occupier households with a mortgage loan.

Figures 1a, 1b and 1c depict the (smoothed) relative frequency distribution of the individual LTV ratios in 2006, 2007 and 2008. These graphs illustrate the advantage of working with integral microdata, when we do not have to worry about sampling issues.

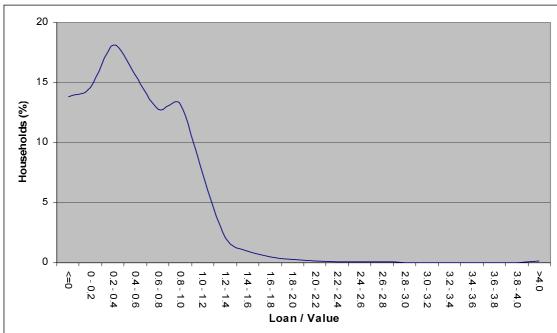
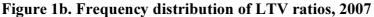
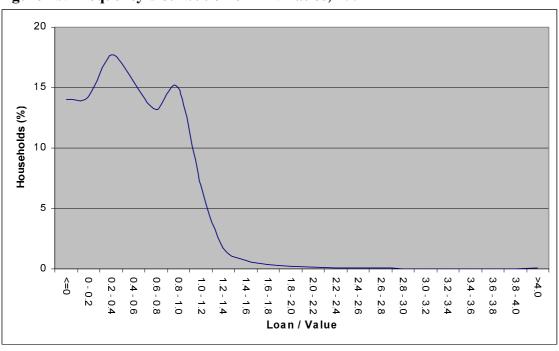


Figure 1a. Frequency distribution of LTV ratios, 2006





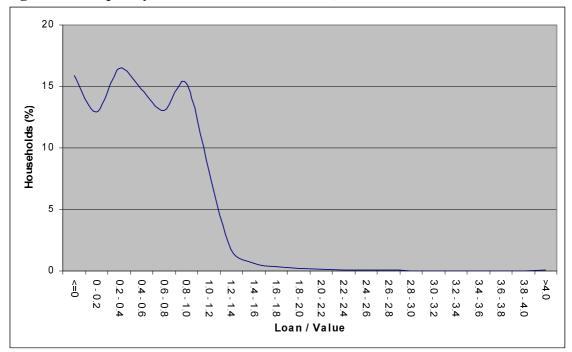


Figure 1c. Frequency distribution of LTV ratios, 2008

A couple of points are worth noting. Approximately 15% of the owner-occupier households do not have a mortgage on their house. Most likely, many of them belong to the elderly, who have paid off their debt. Some 12% have an LTV ratio of greater than 1. Part of this group probably consists of first-time buyers, most of whom are relatively young, who did not make a down payment. Another part of this group may consist of households with the type of mortgage described above, where they accumulate capital in an insurance policy and pay off the debt when the mortgage term expires. The latter households, as well as first-time buyers who did make a down payment or paid off a small part of their debt, might also explain the remarkable bimodality of the frequency distributions, i.e., they could be the cause of the second mode (peak) at an LTV ratio of 0.8-1.0 (including 1). The first mode is at an LTV ratio of 0.2-0.4, and becomes less important in the course of time.

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¹⁵ A negligible fraction of these households has a negative loan, which is difficult to understand. It does remind us of the fact that, while sampling errorr is absent, non-sampling error can be important. For the compilation of official statistics, some cleaning of the microdata will clearly be needed. As mentioned previously, we have not thoroughly checked and cleaned the data.

¹⁶ The frequency distributions of mortgage loan and house value are shown in Appendix 2. The shape of the distribution of house value is much more stable over time than that of outstanding mortgage loan.

Table 2 displays the aggregate data to be used as input to decompositions (10) and (25). Note that we do not present any results for second-stage decompositions into the contributions of individual households since this does not seem particularly relevant in the context of LTVs, in contrast to, for example, productivity measurement. Three pairs of years (or rather, December months) will be compared: 2006 with 2007, 2007 with 2008, and 2006 with 2008. The notation in Table 2 corresponds to the notation in sections 2 and 3. For example, R^0 and R^1 denote the overall LTV ratio in the first and second year of each bilateral comparison. The values are of course identical to those in the last row of Table 1.

Table 2. Aggregate data for decompositions (10) and (25)

	00 0	• • • • • • • • • • • • • • • • • • • •	
	2006-2007	2007-2008	2006-2008
R°	0.46979	0.46757	0.46979
$R_{\scriptscriptstyle C}^{\scriptscriptstyle 0}$	0.46908	0.46799	0.46979
$R_{\scriptscriptstyle X}^{\scriptscriptstyle 0}$	0.48389	0.45754	0.46982
$R^{\scriptscriptstyle 1}$	0.46757	0.47550	0.47550
$R_{\scriptscriptstyle C}^{\scriptscriptstyle 1}$	0.45382	0.47094	0.45871
$R_{\scriptscriptstyle N}^{\scriptscriptstyle 1}$	0.67386	0.52883	0.60080
$R_{\scriptscriptstyle C}^{\scriptscriptstyle 01}$	0.46145	0.46947	0.46425
$oldsymbol{S}_{X}^{0}$	0.04782	0.04054	0.06769
$oldsymbol{\mathcal{S}}_N^{1}$	0.06247	0.07877	0.11814
$oldsymbol{\mathcal{S}}_{C}^{01}$	0.94485	0.94034	0.90708

Looking at Table 2, two interesting observations can be made. First, the LTV ratio of the entering households, R_N^1 , is higher than that of the continuing households in period 1, R_C^1 . This is in accordance with a priori expectations. However, the values of 0.67, 0.53 and 0.60 are much lower than we would have expected. As mentioned earlier, it is commonly known that many first-time buyers in the Netherlands do not make a down payment and have an LTV ratio of roughly 1. Thus, our finding suggests that the sets of entering households also include households that are not first-time buyers.¹⁷ This has to do with how we identified continuing, hence entering (and exiting) households.

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¹⁷ In 2007 and 2008, we record 7% and 8% entering households with respect to the preceding year. In 2008, we record 13% entering households with respect to 2006. Indeed, there cannot be that many first-time buyers. A puzzling finding is that the number of entering households between 2006 and 2008 (548 thousand) is a lot smaller than the sum of the entering households between 2006 and 2007 (283 thousand) and between 2007 and 2008 (351 thousand).

Second, for each of the three comparisons, the (period 1) share of entering households in the value of the housing stock exceeds the (period 0) share of exiting households.¹⁸ We therefore cannot assume constancy over time of the value shares for the continuing households, as is done in the Appendix.

Table 3 contains the results for our preferred decomposition (10) of the change (Δ) in the aggregate LTV ratio pertaining to the three bilateral comparisons. Given the extremely small changes, the results as such are not overly exciting. However, they do illustrate the importance of distinguishing between the sub-groups, in particular between continuing and entering households. For example, the change in the LTV ratio for 2006-2007 (-0.00223) is the net result of a negative contribution of the continuing households (-0.01443), an almost offsetting positive effect of the entering households (+0.01327), and a very small negative contribution of the exiting households (-0.00107). Notice that the contributions of each sub-group for the comparisons 2006-2207 and 2007-2008 do not add up to the corresponding contributions for the comparison 2006-2008. This is a feature of the method used.

Table 3. Decomposition (10)

	Δ LTV ratio	Continuing	Entering	Exiting
2006-2007	-0.00223	-0.01443	+0.01327	-0.00107
2007-2008	+0.00794	+0.00278	+0.00468	+0.00048
2006-2008	+0.00571	-0.01005	+0.01614	-0.00038

Finally, Table 4 presents the results for the decomposition proposed by Diewert and Fox (2010), given by equation (25). We were a bit critical of their approach, among other things because we feel it overstates the contribution of the continuing households. They measured this contribution simply by the change in the *y*-to-*x* ratio (in this case the LTV ratio), whereas in our decomposition (10) the change is multiplied by the average *x* share (appraised value share) of the continuing households. For example, for the 2006-2008 comparison we multiplied the value of -0.01108 in Table 4 by the value for s_C^{01} in Table 2, 0.90708, to obtain our estimate of -0.01005 in Table 3.

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¹⁸ The value shares for the entering households are roughly the same as the population shares mentioned in footnote 17. The estimated value of the Dutch owner-occupied housing stock amounts to 504 billion euros, 543 billion euros, and 589 billion euros in 2006, 2007, and 2008.

Table 4. Decomposition (25)

	Δ LTV ratio	Continuing	Entering	Exiting
2006-2007	-0.00223	-0.01526	+0.01374	-0.00071
2007-2008	+0.00794	+0.00295	+0.00456	+0.00043
2006-2008	+0.00571	-0.01108	+0.01679	-0.00000

6. Conclusions

Statistics Netherlands decided to publish LTV ratios for all owner-occupier households, and for a number of sub-populations. The data source will be the *Income Panel Survey*, which is a sample of approximately 70,000 households drawn from the integral tax data. The survey data are cleaned and augmented to meet Statistics Netherlands' concepts and definitions, such as the definition of disposable income. There are plans to change over to the full data set to compile income and wealth statistics and hence to compute LTV ratios.

In this paper we anticipated these developments and already exploited the full tax data set (but we did not do much data cleaning). This was necessary to identify the continuing, entering and exiting households and compute their effects on the change in the overall LTV ratio. Since the LTV ratio hardly changed during 2006-2007, the results may seem somewhat disappointing. Yet they do show the usefulness of distinguishing between continuing and "unmatched" (entering and exiting) households. Moreover, in the long run, and particularly for small groups of households, changes in the LTV ratios could be much bigger, which would strengthen the case for a decomposition analysis in order to better understand the sources of these changes.

We proposed a two-stage approach. In the first stage the change in the *y*-to-*x* ratio is decomposed into the contributions of the sub-sets of continuing, entering, and exiting households, and in the second stage the contribution of the sub-set of continuing households can be further decomposed into the contributions of (changes in the *y*-to-*x* ratios of) the individual households. In this paper we did not apply the second stage as this is not very useful for LTV ratios, given the huge number of households involved. It is relevant for productivity growth though, and it could be worthwhile to try and apply our first and second stage decompositions to productivity microdata.

Like in other countries, house prices in the Netherlands went down after 2008. For continuing households in particular, this implied an increase in risk, and extending our decomposition analysis to more recent years might give additional insight. Followup research should include taking a closer look at how the continuing households have been identified. If the identification procedure can be improved, then we may be able to distinguish first-time buyers as a separate group, which could be of importance to banks and other mortgage lenders and may also have policy implications.

It would be helpful if we knew more about the characteristics of the households and the owned properties. This is indeed what Statistics Netherlands is currently aiming at. Various administrative data sets, containing data that are relevant for housing market analyses, are linked at the household level. 19 Unfortunately, information on the type of mortgage is unavailable, so we cannot quantify the impact of mortgages where principal repayments are paid into an insurance policy rather than deducted from the outstanding mortgage.

Appendix 1: Decomposition (10) under Constant x Shares

In this Appendix we dwell a little bit upon our first-stage decomposition (10) in case the x share of the set C of continuing elements is constant. With $s_C^1 = s_C^0$, the x shares of the sets N and X of entering and exiting elements are $s_N^1 = s_X^0 = 1 - s_C^0$. Decomposition (10) then reduces to

$$R^{1} - R^{0} = s_{C}^{0}(R_{C}^{1} - R_{C}^{0}) + (1 - s_{C}^{0})(R_{N}^{1} - R_{C}^{01}) - (1 - s_{C}^{0})(R_{X}^{0} - R_{C}^{01}).$$
(A.1)

If we "consolidate" the contributions of N and X, given by the second and third term in (A.1), we have

$$R^{1} - R^{0} = s_{C}^{0} (R_{C}^{1} - R_{C}^{0}) + (1 - s_{C}^{0})(R_{N}^{1} - R_{X}^{0}).$$
(A.2)

Although unnecessary for applications such as Loan-to-Value ratios, in general it is recommendable to make (A.2) invariant to changes in the units of measurement. For instance, we could divide both sides of (A.2) by $R^0 \neq 0$, as suggested by Diewert and Fox (2010). This yields

¹⁹ The linked microdata sets will be used by Statistics Netherlands to compile official statistics and will also be available to researchers at universities and other institutions.

$$\frac{R^{1}}{R^{0}} - 1 = \frac{s_{C}^{0}}{R^{0}} (R_{C}^{1} - R_{C}^{0}) + \frac{1 - s_{C}^{0}}{R^{0}} (R_{N}^{1} - R_{X}^{0}).$$
(A.3)

Provided that $R_C^0 \neq 0$ and $R_X^0 \neq 0$, (A.3) can be written as

$$\frac{R^{1}}{R^{0}} - 1 = \frac{s_{C}^{0} R_{C}^{0}}{R^{0}} \left[\frac{R_{C}^{1}}{R_{C}^{0}} - 1 \right] + \frac{(1 - s_{C}^{0}) R_{X}^{0}}{R^{0}} \left[\frac{R_{N}^{1}}{R_{X}^{0}} - 1 \right]
= \frac{y_{C}^{0}}{y^{0}} \left[\frac{R_{C}^{1}}{R_{C}^{0}} - 1 \right] + \left[1 - \frac{y_{C}^{0}}{y^{0}} \right] \left[\frac{R_{N}^{1}}{R_{X}^{0}} - 1 \right] = \widetilde{s}_{C}^{0} \left[\frac{R_{C}^{1}}{R_{C}^{0}} - 1 \right] + (1 - \widetilde{s}_{C}^{0}) \left[\frac{R_{N}^{1}}{R_{X}^{0}} - 1 \right], \tag{A.4}$$

where $\tilde{s}_C^0 = y_C^0 / y^0$. Decomposition (A.4) says that when the *x* shares of the sets *N* and *X* are equal, their combined contribution to the *percentage change* in the overall *y*-to-*x* ratio is zero if $R_N^1 = R_X^0$.

It is convenient to express (A.4) in terms of index numbers:

$$\frac{R^1}{R^0} = \widetilde{s}_C^0 \frac{R_C^1}{R_C^0} + (1 - \widetilde{s}_C^0) \frac{R_N^1}{R_N^0}. \tag{A.5}$$

Equation (A.5) is the "ratio counterpart" of (A.2). The overall R (y-to-x) index uses the base period y shares, \tilde{s}_C^0 and $1 - \tilde{s}_C^0$, to aggregate the R index of the continuing elements and the "R-index" of the entering and exiting elements.

Equation (A.5) holds for $s_C^1 = s_C^0$. An unconditional equivalent of (A.5) is

$$\frac{R^1}{R^0} = \widetilde{s}_C^0 \frac{s_C^1}{s_C^0} \frac{R_C^1}{R_C^0} + (1 - \widetilde{s}_C^0) \frac{1 - s_C^1}{1 - s_C^0} \frac{R_N^1}{R_X^0}, \tag{A.6}$$

which is easily checked by substituting the various entities. In (A.6), the R index of the continuing elements and the "R-index" of the entering and exiting elements are adjusted for changes in the x shares. It follows that

$$\frac{R^{1}}{R^{0}} - 1 = \widetilde{s}_{C}^{0} \left[\frac{s_{C}^{1}}{s_{C}^{0}} \frac{R_{C}^{1}}{R_{C}^{0}} - 1 \right] + (1 - \widetilde{s}_{C}^{0}) \left[\frac{1 - s_{C}^{1}}{1 - s_{C}^{0}} \frac{R_{N}^{1}}{R_{N}^{0}} - 1 \right]. \tag{A.7}$$

(A.7) decomposes the percentage change in the overall *y*-to-*x* ratio into the contributions of the set of continuing elements and the set of "unmatched" elements.

Although they do provide additional insight, decompositions (A.6) and (A.7) are not really helpful because the changes in the indexes are not separated from the changes in the x shares.

Appendix 2: Frequency distributions

Figure 2a. Frequency distribution of outstanding mortgage loan, 2006

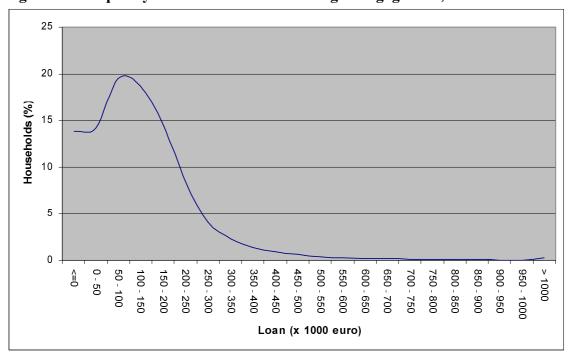


Figure 2b. Frequency distribution of outstanding mortgage loan, 2007

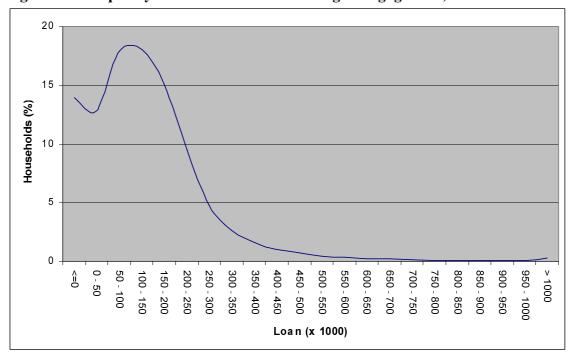


Figure 2c. Frequency distribution of outstanding mortgage loan, 2008

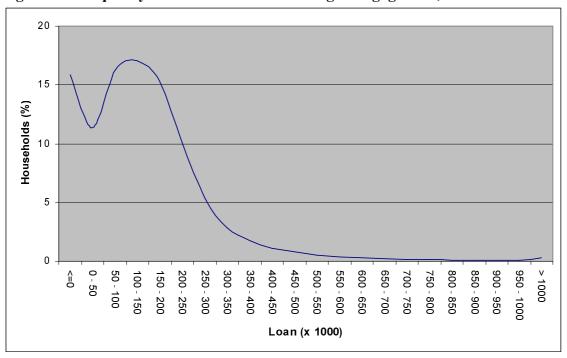


Figure 3a. Frequency distribution of appraised house value, 2006

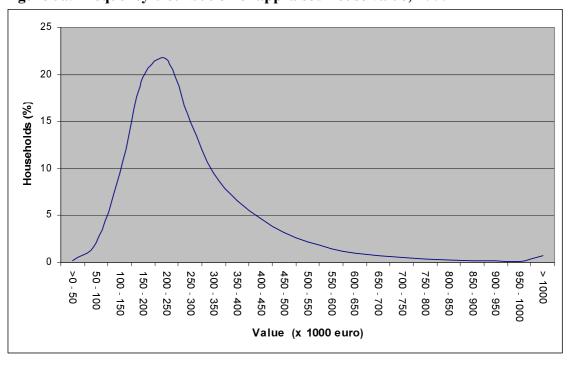


Figure 3b. Frequency distribution of appraised house value, 2007

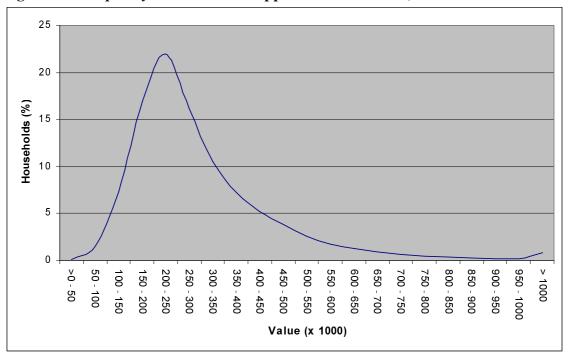
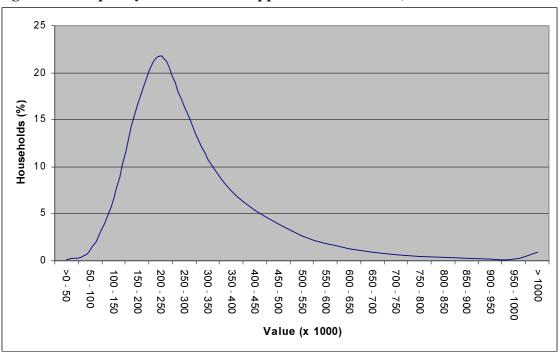


Figure 3c. Frequency distribution of appraised house value, 2008



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