4

PISA and the U.S. Common Core State Standards for Mathematics

How does PISA relate to the education standards that apply within the United States? Most U.S. states have adopted the Common Core State Standards for Mathematics (CCSSM) as their state mathematics standard. A relevant question therefore is how performance measured by PISA relates to the CCSSM and whether faithful implementation of CCSSM is likely to improve the U.S. performance in the PISA test? This chapter provides an initial investigation into this.

Chapter 2 presented the United States' performance in PISA 2012 and described the factors associated with good performance and high levels of equity. Chapter 3 then analyzed the results at the item level in order to reveal the strengths and weaknesses in the mathematical competencies of 15-year old students in the United States. But how does PISA relate to the education standards that apply within the United States? Most U.S. states have adopted the Common Core State Standards for Mathematics (CCSSM) as their state mathematics standard (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010 – hereafter referred to as CCSSM, 2010). A relevant question therefore is how performance measured by PISA relates to the CCSSM – and would faithful implementation of CCSSM be likely to improve the United States performance in the PISA test? This chapter provides an initial investigation into this by seeking to understand, in mathematical terms, how CCSSM relates to the PISA measures and vice versa.

COMPARING THE PISA ASSESSMENT FRAMEWORK TO THE U.S. COMMON CORE STATE STANDARDS FOR MATHEMATICS

Assessment items can be thought of as samples from some larger domain. Every individual item in PISA is a particular, partial expression of a more general goal, idea, or set of goals and ideas outlined in the PISA framework. No finite set of PISA items quite expresses what is expressed in the PISA framework. Therefore, before relating specific PISA items to CCSSM, it is important to consider the relationship between the PISA framework and CCSSM.

Assessment frameworks and standards documents are designed to serve different purposes. State education standards such as CCSSM are intended to bring coherence to the academic functions of systems for education. If programs for curricula, assessment and instruction are all aligned with the standards, then to that extent they are also aligned with one another and jointly promote the state's vision for learning. Assessment frameworks such as those for PISA, on the other hand, are intended only to specify a measurement construct, not to ground all of the major academic functions of a school system.

Because assessment frameworks and state standards documents are designed to serve different purposes, it is a subtle undertaking to compare one to the other. Nevertheless, to a certain degree, comparisons can be made. That is because each type of document verges to some extent into the other's territory. Education standards aren't designed to specify an assessment program completely, but they do have extremely strong implications for what is assessed. And while assessment frameworks aren't designed to be a basis for an aligned curriculum or instruction, they do make implicit and/or explicit claims about what is valuable for students to learn. A fruitful comparison of the PISA framework and CCSSM can be made if the documents' different purposes are recalled as areas of overlap are explored. Important similarities, and important differences, will emerge from this comparison.

Structure of the Common Core State Standards for Mathematics

In order to understand the procedure used in this chapter for analyzing 2012 PISA items relative to the state standards, it is important to be aware of the high-level structure of the Common Core State Standards for Mathematics. CCSSM includes two distinct kinds of standards: practice standards and content standards. First there are eight Standards for Mathematical Practice, which are as follows:

- MP.1. Make sense of problems and persevere in solving them.
- MP.2. Reason abstractly and quantitatively.
- MP.3. Construct viable arguments and critique the reasoning of others.
- MP.4. Model with mathematics.
- MP.5. Use appropriate tools strategically.
- MP.6. Attend to precision.
- MP.7. Look for and make use of structure.
- MP.8. Look for and express regularity in repeated reasoning.

As described in CCSSM:

> The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual

inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (CCSSM, 2010:6).

As will be described in greater detail later in this chapter, MP.4 ("Model with mathematics") is the most relevant practice standard for a PISA analysis, though others are of some relevance too.

The second kind of standards appearing in CCSSM are the Standards for Mathematical Content. The content standards are presented in two sections, one for kindergarten to eighth grade (K–8) and one for high school. In *Grades K – 8* there are standards for each grade, each of which states an expectation of what students will understand or be able to do by the end of the grade. The content standards for each grade are grouped into clusters and the standards and clusters are in turn organized into domains, such as "Counting and Cardinality" or "Number and Operations – Fractions." Because of the importance of coherence in supporting mathematics achievement, the K – 8 content standards were designed not by a process of sorting topics into grades, but rather by a process of weaving together progressions across grades.

The *High School* content standards are presented in six mathematical categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

For five of these six categories (all except modeling), the content standards are presented using the domain/cluster/standard structure used for grades K–8. In the case of modeling, no dedicated standards are given, but an extended description of modeling is provided, including the modeling cycle discussed later in this chapter (CCSSM 2010, 72-73). In addition, content standards in the other five categories are marked with a star symbol to highlight them as opportunities for modeling.

A closer look at mathematics in the PISA framework

In the PISA framework, mathematical literacy consists of being able to:

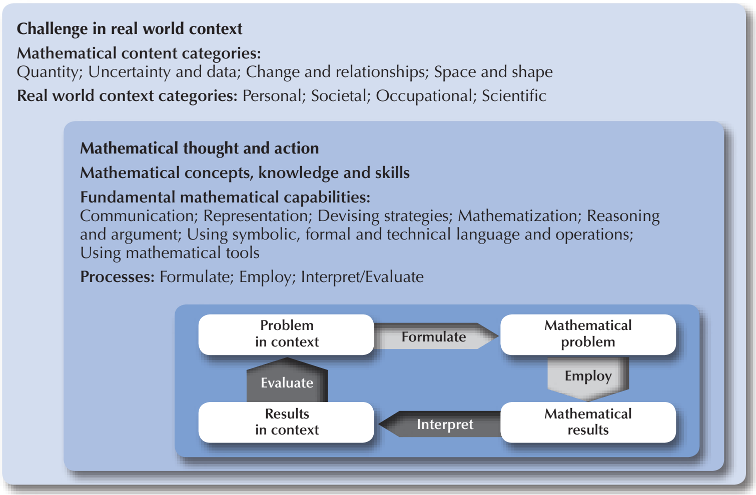
1. Formulate situations mathematically
2. Employ mathematical concepts, facts, procedures, and reasoning
3. Interpret, apply and evaluate mathematical outcomes

Formulating, employing, and interpreting have always been the basis for PISA's construct of mathematics. These three capacities are reported as subscales in PISA.

Formulating, employing and interpreting are sequential steps in what the PISA framework refers to as the modeling cycle. See Figure 4.1 below, which is Figure 1.1 in the PISA framework (OECD 2013, p. 26).

■ Figure 4.1 ■

The modeling cycle in the PISA framework



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PISA items often concentrate on a single process category or step of its modeling cycle. The approximate target distribution of score points by process category is shown in Table 4.1 (see Table 1.1 of OECD 2013, p. 38).

■ Table 4.1. ■
Approximate distribution of PISA items across process categories

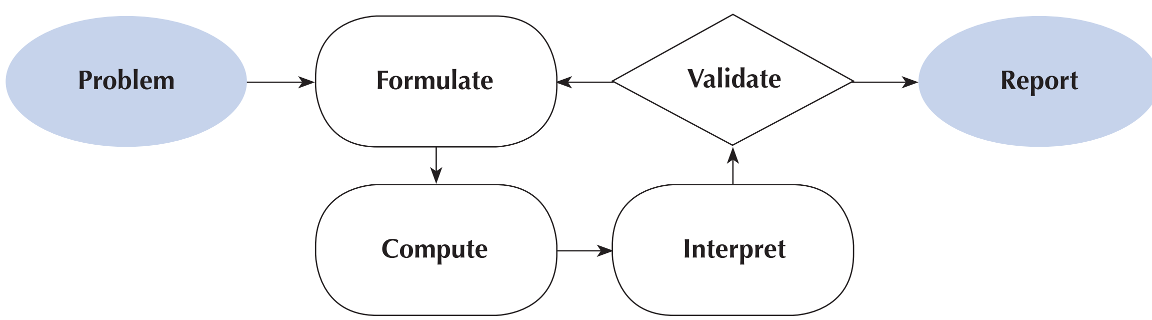
Process category	Percentage of score points
Formulating situations mathematically	Approximately 25
Employing mathematical concepts, facts, procedures and reasoning	Approximately 50
Interpreting, applying and evaluating mathematical outcomes	Approximately 25
TOTAL	**100**

Mathematical literacy in the Common Core State Standards for Mathematics

Mathematical literacy is important in CCSSM. First, from the earliest grades, the standards focus strongly on arithmetic in part because without arithmetic, a person will never be mathematically literate. Second, the phrase “real-world and mathematical problems” is a refrain in the standards (with over 20 occurrences), which points to the standards’ balanced attention to pure and applied mathematics. Writings on mathematical literacy appear in the bibliography to the standards. Also themes directly relevant to mathematical literacy appear in follow-up documents to CCSSM such as criteria for publishers produced by the CCSSM writing team.

But the most direct and visible correlate of PISA’s construct of mathematical literacy within CCSSM is the high school category of modeling. CCSSM presents a modeling cycle that closely resembles PISA’s own (Figure 4.2, from CCSSM 2010, p. 72).

■ Figure 4.2 ■
The Modeling cycle from CCSSM



Note that “Report” in CCSSM is partially related to “Evaluation” in PISA, and also partially related to “Communication”, which the PISA framework considers to be an underlying “fundamental mathematical capability”, but not explicitly a modeling step or a process category on a par with “Formulate”, “Employ” and “Interpret”. So there is some relationship between these. And yet, “Report” in CCSSM would appear straightforwardly to suggest that a student might actually be asked to produce a brief piece of writing as part of a modeling exercise and such a step can indeed be found in some modeling tasks present in the field.[1, 2] Reporting on this sort of scale is less prevalent in PISA.

■ Table 4.2. ■
Correspondences between the CCSSM Modeling cycle and the PISA Modeling cycle

CCSSM modeling step	Related PISA modeling step
Formulate	Formulate
Compute	Employ
Interpret	Interpret
Validate	Evaluate
Report	N/A

Following through the correspondences in Table 4.2, one can produce a rough map of PISA's approximate target distribution across different aspects of modeling in CCSSM terms (Table 4.3).

■ Table 4.3. ■

Rough map of approximate target distribution of PISA items in CCSSM Modeling terms

CCSSM modeling step	Percentage of related PISA score points
Formulate	Approximately 25
Compute	Approximately 50
Interpret	Approximately 25
Evaluate	Small
Report	Small
TOTAL	**100**

Despite the differences above, what is clear is that in high school, CCSSM – influenced substantially by PISA itself – invests significantly in mathematical literacy, and that this extensive shared territory makes comparisons between PISA and CCSSM both possible and valuable.

ANALYZING THE PISA 2012 MATHEMATICS ITEMS RELATIVE TO CCSSM

Design of the PISA assessment in mathematics

The following analysis is focused on the paper-and-pencil assessment of mathematics in PISA 2012; it does not discuss the computer-based mathematics assessment that PISA implemented in 2012 as a bridge to the 2015 assessment, which will be fully computer delivered.

> In PISA, students encounter the test not as a disconnected sequence of one-off questions, but rather as a series of "units". Each unit presents a description of a real-world context using stimuli such as text, tables, charts and figures. After reading the description of the situation, students answer anywhere from one to four questions about the situation. Thus, an "item" refers to one question in a unit.[3]

For the paper-and-pencil assessment, a total of 56 units containing a total of 110 mathematics items were implemented. However, two clusters of items were solely used in so called "easier" test booklets that were typically used by lower-performing countries. As the United Sates did not use these booklets, the items analyzed in this chapter are restricted to the 84 items that were in the booklets administered in the United States and most other countries.

Issues to consider when analyzing PISA tasks in CCSSM terms

Analyzing PISA tasks in CCSSM terms is a nontrivial exercise. One major source of difficulty is that mathematical literacy – in both PISA and CCSSM – can involve using elementary mathematics to solve sophisticated problems (Steen, 2007). Since the goal of PISA is to assess the cumulative yield of mathematics performance over the school career of 15-year-olds, rather than solely what students have learned at the age of 15 or at 10th grade level, the mathematical techniques needed for some PISA tasks might be first introduced in CCSSM during the middle grades, or perhaps even the elementary grades. Nevertheless, it would be wrong to call such a task an elementary-grade task according to CCSSM. That is because PISA tasks typically require problem-solving processes that are not expected in CCSSM at these grades, and that very few elementary school students could command in any case. A valid procedure of analysis ought to capture accurately the "lag" between content and process in tasks without misleadingly suggesting either that PISA is too easy or that CCSSM is too hard.

A second complication arises from the fact that the Standards for Mathematical Practice are not a taxonomy. Traditional procedures for coding content should not be applied to the Standards for Mathematical Practice. A mathematical task, or a mathematical behavior, might easily exhibit/evoke/resonate with several practice standards at once – or none. Thus, in a collection of *N* items, there is no reason why the sum of the weights across the eight practice standards must add up to *N*. Practice standards cannot be thought of even approximately as drawers in a filing cabinet.

Because mathematical literacy in PISA relates so closely to modeling in CCSSM, one expects that every PISA task will involve practice standard MP.4. MP.4 tells a large part of the story of how PISA and CCSSM relate through the lens of the practice standards, though the other standards for mathematical practice are of some relevance also.

Data generated for each analyzed task

In order to address the complications described above and analyze the PISA tasks relative to CCSSM in the most appropriate way, an analysis procedure to generate the following data for each of the 84 PISA 2012 mathematics items that were administered in the United States:

1. Progression co-ordinate
2. Modeling attributes (includes the use of technology and tools)
3. Modeling intensity level

These terms are defined next and illustrated using publicly released items from the PISA 2012 assessment.

Progression co-ordinate

Viewed from a high enough level, the CCSSM content standards could be described as a sequence of time-ordered sets of content goals.

The first set in the sequence is the kindergarten content standards; the last set is the high school content standards.

If one considers that the CCSSM document *D* consists of, or defines, a sequence of time-ordered sets of content goals D_1, …, $D_{n,}$ one refers to the subscript *k* in D_k as the "progression co-ordinate". Progression co-ordinates for CCSSM are defined in Table 4.4.

■ Table 4.4. ■
Definition of progression co-ordinates for CCSSM

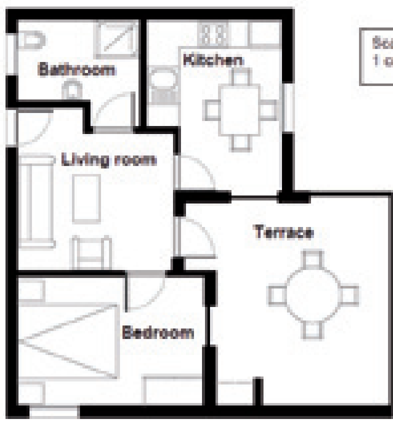
Progression co-ordinate	Content goal set
K	D_k
0	Standards for Mathematical Content—Kindergarten
1	Standards for Mathematical Content—Grade 1
2	Standards for Mathematical Content— Grade 2
3	Standards for Mathematical Content— Grade 3
4	Standards for Mathematical Content— Grade 4
5	Standards for Mathematical Content— Grade 5
6	Standards for Mathematical Content— Grade 6
7	Standards for Mathematical Content— Grade 7
8	Standards for Mathematical Content— Grade 8
9	Standards for Mathematical Content—High School

Mathematical tasks, such as PISA items, can be assigned a progression co-ordinate by reference to a theoretically optimal user of content at that grade level, i.e. one who can reliably complete all tasks up to that level. Thus, just because a task has progression co-ordinate *k*, this does not mean that CCSSM sets an expectation that all students at the corresponding grade level be able to reliably complete the task. The most one can say in general is that a task with progression co-ordinate *k* is at or beyond the corresponding grade level.

As an illustration of the analysis of progression co-ordinate, we consider the following PISA 2012 released mathematics item (Figure 4.3), one of the 84 items analyzed in this chapter.

■ Figure 4.3. ■
Apartment purchase (Item PM00FQ01)

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



Scale:
1 cm represents 1 m

Question 1: APARTMENT PURCHASE

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

Homever, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the **four** lengths that are needed to estimate the total floor area of the apartment.

This item was assigned the progression co-ordinate $k = 4$ in light of content standard 4.MD.7d. This standard reads as follows (emphasis added):

> Recognize area as additive. **Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems** (CCSSM 2010, p. 25)

The processes required for success in the "Apartment purchase" task depend on, but are not reducible to, the knowledge and skills that are articulated in content standard 4.MD.7d. Therefore, just because the "Apartment purchase" task has content progression $k = 4$, this does not mean that the PISA test includes fourth-grade tasks, nor does it mean that CCSSM requires fourth graders to solve tasks appropriate for 15-year-olds.

Modeling attributes

Each PISA item was located along five continua that reflect various attributes of modeling tasks (Box 4.1). In each of the following continua, the left-hand side is "less about modeling" while the right-hand side is "more about modeling.[4]"For each of the five attributes, the task was located along the continuum using heuristic values 0, 1, 2.

Box 4.1 **Attributes of modeling tasks**

A_1. Well posedness
Well posed → element(s) of intentional ambiguity → freedom to specify and simplify the problem.
Model is judged correct/incorrect → model is judged useful/not useful, a good start or not.

A_2. Authenticity
Using specified math the real goal → life and realism the real goal, with math content a means not an end.
Cooked-up data/facts → actual or realistic data/facts.
Problem posed with words → problem posed with words and artifacts.
Units abstracted away → units or dimensional thinking prominent in problem and solution.

A_3. Scaffolding
Routine task → non-routine task.
Fully scaffolded → partly scaffolded → unscaffolded.
Key variables are declared or evident → key variables not declared/not evident.
Easy to identify the math to use → careful thought about quantities and their relationships required in order to identify the math to use.

A_4. Coverage of the Modeling cycle
Less coverage → more coverage (formulate, compute, interpret, critique/validate, improve, report).

A_5. Technology and tools
No technology or tools required → technology or tools required → required without prompting.

Note that "Technology and tools" includes technology such as spreadsheets, calculators, graphing software, and so on, as well as tools such as measurement scales or even remembered formulas.

As an illustration of the analysis of modeling attributes, consider the following publicly released PISA 2012 mathematics item, again one of the 84 items analyzed in this chapter. Attributes of the item are shown in Table 4.5.

■ Figure 4.4. ■

MP3 Players (Item PM904Q02)

Translation Note: The use of zeds is important to the unit, so please do not adapt "zed" into an existing currency.

Question 2: MP3 PLAYERS

Olivia added the prices for the MP3 player, the headphones and the speakers on her calculator.

The answer she got was 248.

Olivia's answer is incorrect. She made one of the following errors. Which error did she make?

A. She added one of the prices in twice.
B. She forgot to include one of the three prices.
C. She left off the last digit in one of the prices.
D. She subtracted one of the prices instead of adding it.

■ Table 4.5. ■

Modeling attributes of the "MP3 Players" (Item PM904Q02)

Attribute	Relevant continua	Value assigned (0, 1, 2)	Comment
A_1. Well-posedness	Well posed → element(s) of intentional ambiguity → freedom to specify and simplify the problem Model is judged correct/incorrect → model is judged useful/not useful, a good start or not	0	The task is completely well posed. There are no meaningful choices to be made by the student in representing the situation mathematically. There is an unambiguous notion of correctness for the task. The mathematical work will not be seen as part of any iterative process of refinement.
A_2. Authenticity[5]	Using specified math the real goal → life and realism the real goal, with math content a means not an end Cooked-up data/facts → actual or realistic data/facts Problem posed with words → problem posed with words and artifacts Units abstracted away → units or dimensional thinking prominent in problem and solution	1	It is clear that life is the real goal of the task. The problem is posed with words and artifacts. However, the numbers in the problem are clearly devised in such a way as to foster particular strategies. The quantities in the problem are dimensioned (units of zeds), but units are not prominent in the problem or solution.

Attribute	Relevant continua	Value Assigned (0, 1, 2)	Comment
A_3. Scaffolding	Routine task → non-routine task Fully scaffolded → partly scaffolded → unscaffolded Key variables are declared or evident → key variables not declared/not evident Easy to identify the math to use → careful thought about quantities and their relationships required in order to identify the math to use	1	The task is not entirely routine. But the multiple-choice format implies a certain amount of scaffolding. The universe of possible errors is presented and constricted to a small set, rather than having likely errors defined by the student.
A_4. Coverage of the Modeling cycle	Less coverage → more coverage (formulate, compute, interpret, critique/validate, improve, report)	0	The answer choices can be seen as different potential models for what went wrong in Olivia's calculation. But the fact that the models appear as multiple-choice options weakens this sense, and it would be a subtle reading of the task in any case. A student would not likely understand his or her work on the task in these terms.
A_5. Technology and tools	No technology or tools required → technology or tools required → required without prompting	1	A calculator can be helpful in completing the task; its use is not prompted.

As this example shows, critical judgment is involved in assigning a number 0, 1, or 2 to each attribute, which required some subjective judgment. Therefore, the specific ratings given should not be over-interpreted.

Modeling level

Each PISA item was also assigned an overall modeling level (Box 4.2):[6]

Box 4.2 **Modeling levels**

Level 0 Pure mathematics, no context

Proof … fluency … procedure … concept … single or multi-step

Level 1 Modeling/application[7]

1.1 One-step problem using D_k content … problem with "thin context"[8]
1.2 Multi-step problem; lower values along most or all of the modeling dimensions $A_1 – A_5$

Level 2 Modeling

2.1 Well-posed multi-step problem; higher values along at least some of the modeling dimensions $A_1 – A_5$
2.2 Less than well-posed problems with model formulation; computation; and interpretation of results
2.3 Least well-posed problems and/or four or more steps of the modeling cycle

For example, the "Apartment purchases" task and the "MP3 players" items were both assigned Level 1.2.

The term "multi-step" should be interpreted judiciously. The "steps" in a "multi-step" item might not take the form of a sequence of discrete calculations; in some cases, the "steps" might instead take the form of multiple nodes in a web of reasoning. Conversely, consider a problem about substituting a value into a complicated equation. Arriving at the final numerical answer might involve several sub-calculations, but such a problem could well be thought of as "single-step" anyway.

Modeling levels were not designed to "spread out" the PISA tasks. They were designed to reflect the full spectrum of tasks that one might see in a faithful implementation of CCSSM.

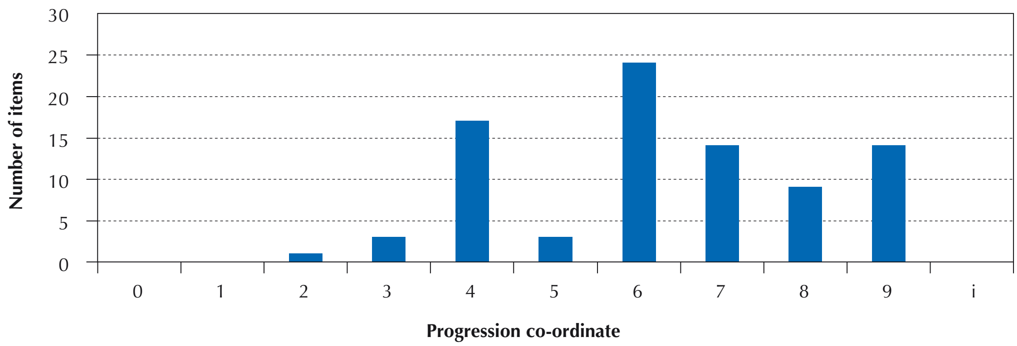
RESULTS OF ANALYZING PISA 2012 ITEMS RELATIVE TO THE U.S. COMMON CORE STATE STANDARDS FOR MATHEMATICS

Progression co-ordinate

Mathematical literacy has been described as involving the application of mathematics to solve sophisticated problems (Steen, 2007). Given PISA's intent to assess the cumulative yield of mathematics performance up to the age of 15 years, rather than solely to assess the mathematical content acquired at the age of 15 years, then one would expect PISA items to vary in their progression co-ordinates. Figure 4.5 shows the relative frequency of the 84 items administered in the United States by their progression co-ordinates.

■ Figure 4.5. ■

Distribution of PISA items by progression co-ordinate



The maximum of the distribution is $k = 6$. This is consistent with PISA's aims, reflecting the fact that at grade 6, the progressions in CCSSM lead to the development of new and powerful tools such as ratio, rate, and percent; new ideas such as center and variation; and the culmination of decimal computation.

Fewer items' progression co-ordinates fall at $k = 8$; when they do, they are more often related to the CCSSM Functions domain than to the Expressions and Equations domain.

There is also a noticeable spike in the distribution at grade 4. One of the main factors leading to this peak is that grade 4 is when CCSSM first expects students to be using all four basic operations to solve multi-step problems with whole numbers, including interpreting remainders (4.OA.3). In light of what PISA tasks are aiming to do, the peak in this distribution at grade 4 reveals the fact that the skills referenced in 4.OA.3 already represent quite a powerful body of mathematics. Standard 4.OA.3 deserves to be thought of right alongside 7.EE.3 as a "pinnacle". (Indeed, 4.OA.3 might be thought of as a "whole-number version" of 7.EE.3, which is concerned with solving multi-step real-life and mathematical problems posed with positive and negative rational numbers.)

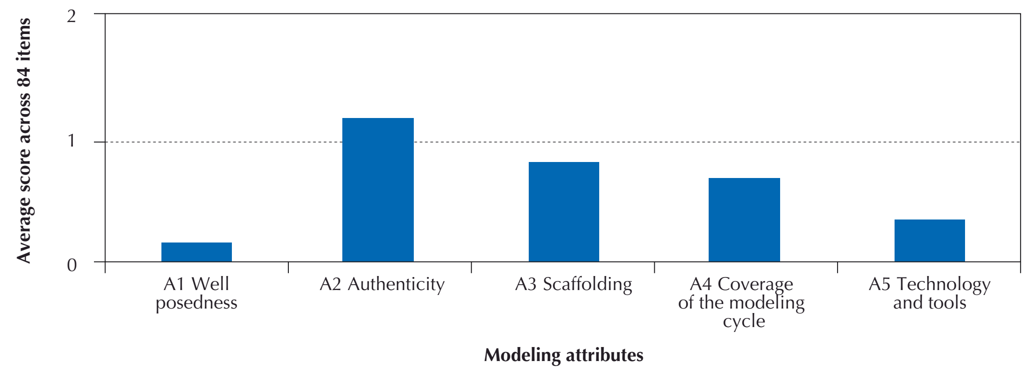
A secondary factor contributing to the spike at grade 4 is that the numerical values in PISA items may be simple enough to foster mental computations. (All other things being equal, the two calculations 120/6 and 139.13/7.662 would receive progression co-ordinates of 4 and 6, respectively.)

Steen's observation (Steen, 2007) about humble mathematics being put to sophisticated uses is often thought of specifically in relation to arithmetic, algebra and geometry. But it is equally true when applied to statistics. Only four of the PISA 2012 items with progression co-ordinates of 9 were referable to the high school Statistics and Probability category in CCSSM – about the same number of items at that level referable to the high school Algebra category (three items) or the high school Functions category (four items). This too is consistent with PISA's purposes. Statistics, as it is used in work and life, relies little on advanced techniques of the kind first introduced in high school. It relies, rather, on fertile uses of the fundamental notions of center, variation, and distribution that are first introduced in the middle grades in CCSSM and that continue to be relevant throughout high school, college, work and life.

Modeling attributes

Figure 4.6 shows the average score in each modeling attribute. That is, for each modeling attribute, the 0-1-2 scores for that attribute were summed over all 84 items and then divided by 84. A brief discussion of each attribute follows the figure.

■ Figure 4.6. ■
Average score by modeling attribute



Attribute A_1 *(Well posedness).* Problems that are anything less than perfectly posed have always been rare on standardized tests – in fact, usually such problems are completely absent by design. Reasons for this include psychometric difficulties as well as the difficulty of explaining the ins and outs of modeling to a public unused to seeing such work on tests. Against this backdrop, the 18 PISA problems from 2012 that scored above zero on this attribute represent a qualitatively greater proportion than one would typically find in the item bank for a U.S. state mathematics test from before CCSSM. This reflects PISA's intent to assess the capacity of students to identify problems even in ambiguous contexts, as they would be in real-world situations. At the same time, it should be noted that none of those 18 items scored a 2 on this attribute, and many are more well-posed than some prototypical modeling tasks one finds in the field.[9]

Attribute A_2 *(Authenticity).* PISA 2012 items had relatively high authenticity, as is clear enough from a brief scan of the released items. PISA items are often posed using interesting artifacts from real life, they often involve units and dimensional thinking, and they often put mathematics in service of the phenomenon.

Attribute A_3 *(Scaffolding).* PISA includes a mixture of completely unscaffolded items, partly scaffolded items, and highly scaffolded items. In some cases, a multiple-choice answer format effectively serves intentionally or unintentionally to scaffold an item. Psychometric considerations and the cost of hand scoring have a strong effect on the degree of scaffolding in tasks.

Attribute A_4 *(Coverage of the modeling cycle).* The average score for this attribute was relatively low. This was to be expected, as the design of PISA is such that individual items tend to focus on a single step of the cycle.

Attribute A_5 *(Technology and tools).* Many PISA items that involve computations do not actually require a calculator, as the chosen numbers are amenable to mental arithmetic, or the rubric may allow an approach that involves rounding to convenient numbers. Where technology and tools are in play, the technology involved is generally only a four-function calculator. Of course, it is not be feasible to include spreadsheets and other tools central to working quantitatively in the paper-and-pencil test. Tasks in the computer assessment component of PISA will lend themselves to a higher core on attribute A_5.

Modeling intensity

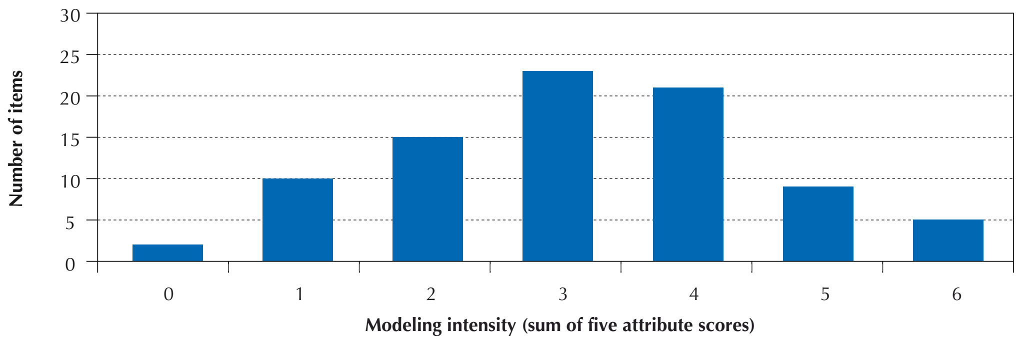
A heuristic measure of *modeling intensity* can be defined by summing the 0-1-2 attribute scores across all five modeling attributes. If this is done, the 2012 PISA items exhibit the distribution shown in Figure 4.7, which covers data for the 84 items administered in the United States.

The highest attribute sum for any of the 84 items was 6. Items with an intensity of 6 tended to be the "richest" of the modeling exercises – and also among the most difficult, as will be discussed further below.

No pronounced relationship was found between tasks' progression co-ordinates and their modeling intensities.

■ Figure 4.7. ■

Distribution of PISA items by modeling intensity

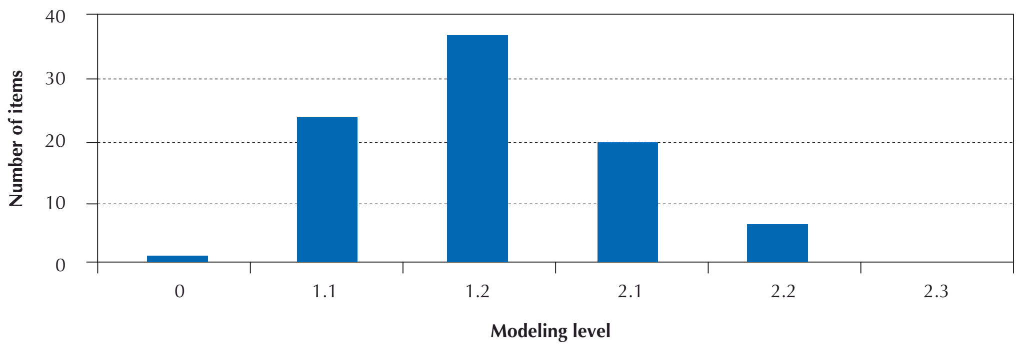


Modeling Levels

Most of the PISA items cluster at level 1.2 and its two neighbor levels, as shown in Figure 4.8, which reports data only for the 84 items administered in the United States.

■ Figure 4.8. ■

Distribution of PISA items by modeling level



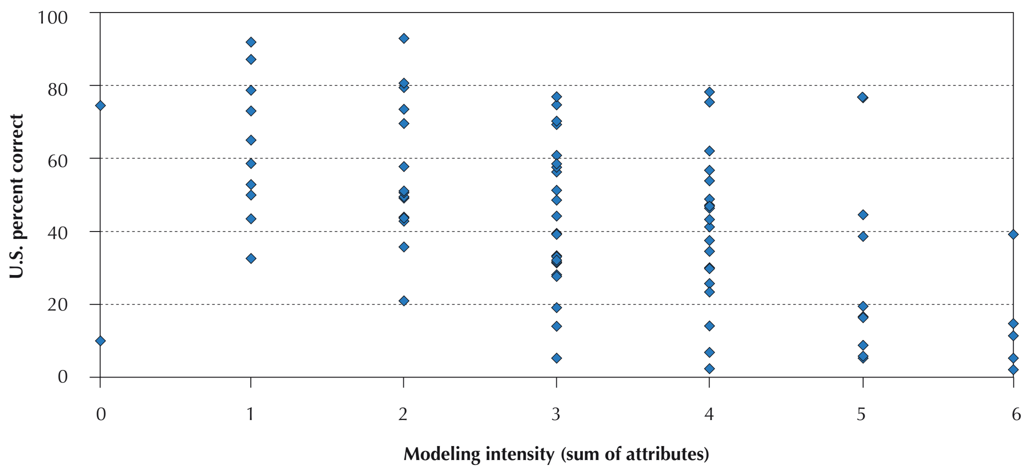
This distribution likely represents a higher general level of modeling than has been present in U.S. mathematics tests prior to CCSSM. For a number of reasons, including the nature of the construct being measured, items on United States state mathematics tests prior to CCSSM would likely cluster at lower levels (namely 0, 1.1, and a smaller amount of 1.2, with 2.1 also present in some cases).

Table 4.6 provides the full detail of the classification of the 84 PISA 2012 mathematics items against the dimensions of the CCSSM.

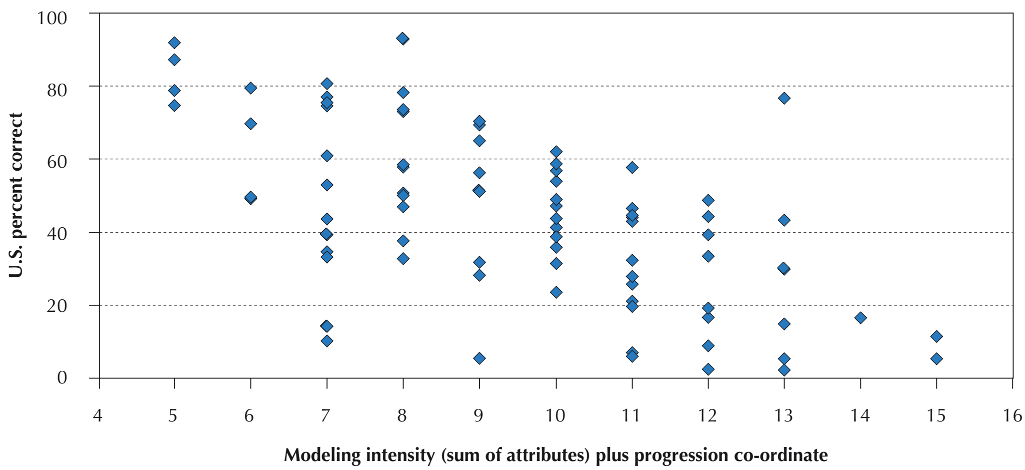
Relationship to student performance

It is interesting to see how students in the United States performed according to modeling intensities (sum of attribute scores). Figure 4.9 clearly shows a relationship between modeling intensity and performance (percent correct on the item). Additionally, Figure 4.10 shows the comparison between performance and "content plus practices" by simply adding the progression co-ordinate to the modeling intensity. Again a distinct relationship is evident, with R^2 values of around 0.3.

■ Figure 4.9. ■
U.S. percent correct decreases with increasing modeling intensity



■ Figure 4.10. ■
U.S. percent correct decreases with increasing sum of modeling intensity and progression co-ordinate



FINAL OBSERVATIONS

The analysis in this chapter has shown some commonality between the PISA framework and items and the CCSSM. It classifies PISA 2012 items against the CCSSM progression according to where they sit in the progression of standards up to high school level, the degree to which they represent attributes of modeling and their modeling level. This has served to show a degree of commonality between the PISA and CCSSM constructs. This chapter began by asking whether faithfully implementing CCSSM would improve PISA results. The analysis suggests that this is intuitively plausible. The prominence of modeling in the U.S. high school standards has already influenced developers of large-scale assessment in the United States It will take time for excellent task designs to emerge and become widespread. But it seems safe to say that over the coming years, the high school curriculum in the United States will attend to modeling to a greater degree than has happened in the past. If more students work on more and better modeling tasks than happens today, then one could reasonably expect PISA performance to improve.

It may be that U.S. students seldom work on well-crafted tasks that situate algebra, proportional relationships and rational numbers within authentic contexts. More generally, one wonders if the application problems that most students have access to today are the worst of all worlds: fake applications that strive to make the mathematics curriculum more palatable, yet do no justice either to modeling or to the pure mathematics involved.

In effect, it could be argued that when it comes to authentic modeling, there has been too little “opportunity to learn”. If, as a result of quality implementation of CCSSM, the U.S. curriculum begins presenting high school students with better modeling problems – and if assessments included them as well – then it would be reasonable to believe, in a way that goes beyond simple correlations, that PISA scores would improve. That would be important not because of any drive for higher test scores *per se*, but rather because it would begin to signal that the United States is becoming a country whose citizens make frequent and productive use of mathematics in their work and life. What is certain, from an examination of the 2012 PISA 2012 items and our performance on them, is that the United States is not that country today.

■ Table 4.6 ■

Mapping of PISA items to aspects of the Common Core State Standards for Mathematics

Name	Item code	Progression co-ordinate k	A1 Well posedness	A2 Authenticity	A3 Scaffolding	A4 Coverage of the modeling cycle	A5 Technology and tools	Level of modeling intensity	Attribute Sum
Apartment Purchase	PM00FQ01	4	0	2	2	0	0	1.2	4
An Advertising Column	PM00GQ01	9	0	2	2	1	1	2.1	6
Wheelchair Basketball	PM00KQ02	7	0	1	1	1	1	1.2	4
A View with a Room	PM033Q01	7	0	1	0	0	0	1.2	1
Bricks	PM034Q01	7	0	0	1	0	0	1.2	1
Population Pyramids	PM155Q01	6	0	2	1	0	0	1.1	3
Population Pyramids	PM155Q02	6	0	2	1	0	1	1.2	4
Population Pyramids	PM155Q03	7	1	2	2	0	1	2.1	6
Population Pyramids	PM155Q04	6	0	2	2	0	0	2.1	4
Containers	PM192Q01	8	0	1	0	1	0	1.2	2
Pipelines	PM273Q01	4	0	0	1	0	1	1.2	2
Map	PM305Q01	7	0	2	0	0	2	1.2	4
Running Tracks	PM406Q01	7	1	1	1	1	1	2.1	5
Running Tracks	PM406Q02	7	1	1	1	1	1	2.1	5
Lotteries	PM408Q01	9	0	2	0	1	0	2.1	3
Diving	PM411Q01	6	0	2	1	0	1	1.2	4
Diving	PM411Q02	6	0	2	1	1	0	1.2	4
Transport	PM420Q01	6	1	1	0	0	0	1.1	2
Tossing Coins	PM423Q01	7	0	0	0	0	0	1.1	0
Braille	PM442Q02	3	1	2	0	1	0	1.2	4
The Thermometer Cricket	PM446Q01	2	0	2	0	1	0	1.1	3
The Thermometer Cricket	PM446Q02	8	0	2	1	1	1	2.1	5
Tile Arrangement	PM447Q01	8	0	0	1	0	0	1.1	1
The Third Side	PM462Q01	7	0	0	0	0	0	0.0	0
The Fence	PM464Q01	4	0	1	2	0	0	1.2	3
Running Time	PM474Q01	4	0	1	0	0	1	1.2	2
Cash Withdrawal	PM496Q01	4	0	1	0	1	0	1.2	2
Cash Withdrawal	PM496Q02	4	0	1	1	1	0	1.2	3
Telephone Rates	PM559Q01	6	0	1	2	1	0	2.1	4
Chair Lift	PM564Q01	6	1	1	1	1	0	2.2	4
Chair Lift	PM564Q02	6	1	1	2	1	1	2.2	6
Stop the Car	PM571Q01	9	0	1	0	1	0	1.2	2
Number Check	PM603Q01	4	0	1	1	1	0	1.2	3
Number Check	PM603Q02	4	0	1	1	1	1	1.2	4
Computer Game	PM800Q01	4	0	0	1	0	1	1.2	2
Labels	PM803Q01	7	1	1	1	1	0	2.2	4
Carbon Dioxide	PM828Q01	9	1	2	1	0	0	2.2	4
Carbon Dioxide	PM828Q02	9	0	1	0	0	0	1.1	1
Carbon Dioxide	PM828Q03	9	0	1	0	0	1	1.1	2
Drip Rate	PM903Q01	9	0	1	1	1	0	1.1	3
Drip Rate	PM903Q03	9	0	1	1	1	1	1.2	4
Tennis Balls	PM905Q01	4	0	2	1	1	0	1.2	4
Tennis Balls	PM905Q02	5	0	1	2	1	1	1.2	5
Crazy Ants	PM906Q01	6	0	1	0	0	1	1.1	2
Crazy Ants	PM906Q02	9	0	0	0	1	1	1.2	2
Speeding Fines	PM909Q01	6	0	1	0	1	0	1.1	2
Speeding Fines	PM909Q02	6	0	1	1	1	0	1.2	3
Speeding Fines	PM909Q03	6	0	1	1	1	0	1.2	3
Carbon Tax	PM915Q01	6	0	1	0	0	0	1.2	1
Carbon Tax	PM915Q02	6	0	0	0	1	0	1.2	1
Charts	PM918Q01	4	0	1	0	0	0	1.1	1
Charts	PM918Q02	4	0	1	1	1	0	1.2	3
Charts	PM918Q05	8	1	1	1	1	1	2.2	5
Z's Fan Merchandise	PM919Q01	4	0	1	0	0	0	1.2	1
Z's Fan Merchandise	PM919Q02	4	0	1	1	1	0	1.2	3
Sailing Ships	PM923Q01	7	0	1	0	0	0	1.1	1
Sailing Ships	PM923Q03	8	0	0	0	1	1	1.2	2
Sailing Ships	PM923Q04	9	0	2	2	1	1	2.1	6

Name	Item code	Progression co-ordinate k	A1 Well posedness	A2 Authenticity	A3 Scaffolding	A4 Coverage of the modeling cycle	A5 Technology and tools	Level of modeling intensity	Attribute Sum
Sauce	PM924Q02	6	0	1	1	1	0	1.2	3
Arches	PM943Q01	7	0	0	1	1	0	1.1	2
Arches	PM943Q02	8	0	1	2	1	0	2.1	4
Roof Truss Design	PM949Q01	8	0	1	1	1	0	1.1	3
Roof Truss Design	PM949Q02	8	0	1	1	1	0	1.1	3
Roof Truss Design	PM949Q03	7	0	1	1	1	0	1.1	3
Flu Test	PM953Q02	9	0	2	0	1	0	2.1	3
Flu Test	PM953Q03	9	0	1	1	1	0	1.2	3
Flu Test	PM953Q04	9	1	1	1	1	1	2.1	5
Medicine Doses	PM954Q01	6	0	1	1	1	0	1.1	3
Medicine Doses	PM954Q02	6	0	1	1	1	0	2.1	3
Medicine Doses	PM954Q04	6	0	1	1	1	1	2.1	4
Migration	PM955Q01	6	0	2	0	0	0	1.1	2
Migration	PM955Q02	6	1	2	1	1	0	1.1	5
Migration	PM955Q03	6	0	2	1	1	1	1.1	5
Employment Data	PM982Q01	4	0	1	0	0	0	1.1	1
Employment Data	PM982Q02	4	0	1	0	1	1	1.1	3
Employment Data	PM982Q03	5	0	1	0	1	1	1.2	3
Employment Data	PM982Q04	9	1	1	1	1	0	2.2	4
Spacers	PM992Q01	3	0	2	1	1	0	2.1	4
Spacers	PM992Q02	3	0	2	1	1	0	2.1	4
Spacers	PM992Q03	6	0	1	1	1	0	2.1	3
Revolving Door	PM995Q01	4	0	1	2	1	0	2.1	4
Revolving Door	PM995Q02	7	0	2	2	1	1	2.1	6
Revolving Door	PM995Q03	6	0	2	2	1	0	2.1	5
Bike Rental	PM998Q02	5	0	1	0	1	0	1.1	2
Bike Rental	PM998Q04	8	0	1	1	1	0	1.2	3
London Eye	PM934Q01	7	0	1	0	1	0	1.2	2
London Eye	PM934Q02	7	0	0	1	1	0	1.2	2
Seats in a Theatre	PM936Q01	8	0	1	0	1	0	1.1	2
Seats in a Theatre	PM936Q02	8	0	1	1	1	0	1.1	3
Racing	PM939Q01	9	0	0	1	1	0	1.2	2
Racing	PM939Q02	9	0	0	1	1	0	1.2	2
Climbing Mt. Fuji	PM942Q01	6	1	1	1	1	1	1.2	5
Climbing Mt. Fuji	PM942Q02	7	0	2	2	1	1	2.1	6
Climbing Mt. Fuji	PM942Q03	6	0	1	1	1	1	2.1	4
Part-Time Work	PM948Q01	2	0	2	1	0	0	2.1	3
Part-Time Work	PM948Q02	6	1	1	0	1	0	1.1	3
Part-Time Work	PM948Q03	7	0	2	2	1	1	2.1	6
Helen the Cyclist	PM957Q01	6	0	0	1	1	0	1.1	2
Helen the Cyclist	PM957Q02	9	0	1	0	1	0	1.2	2
Helen the Cyclist	PM957Q03	9	0	1	1	1	0	1.2	3
Chocolate	PM961Q02	6	1	1	2	1	1	2.2	6
Chocolate	PM961Q03	9	0	2	1	1	0	2.1	4
Chocolate	PM961Q05	6	1	1	1	2	1	2.2	6
Wooden Train Set	PM967Q01	5	0	2	2	1	0	1.1	5
Wooden Train Set	PM967Q03	5	0	1	1	1	0	1.1	3
Which Car?	PM985Q01	4	0	1	1	1	0	1.2	3
Which Car?	PM985Q02	5	0	1	0	0	0	1.1	1
Which Car?	PM985Q03	6	1	1	0	0	1	1.1	3
Garage	PM991Q01	i	0	0	0	0	0	1.2	0
Garage	PM991Q02	9	0	1	2	1	1	2.1	5

Notes

1. See for example *http://www.comap.com/product/textbooks/*, and the "Karnataka" task (in original form due to COMAP) on slide 21 of *http://www.parcconline.org/sites/parcc/files/GA_CCSS.pdf*

2. See for example "The Taxicab Problem" on p.74 of the Smarter Balanced content specifications (in original form due to the Shell Centre), *www.smarterbalanced.org/wordpress/wp-content/uploads/2011/12/Math-Content-Specifications.pdf*

3. Although PISA items are grouped into units, items are treated as independent in the PISA scoring model. Thus, for example, units are designed so that the answer to a later item in the unit does not depend upon having answered an earlier item in the unit correctly.

4. No value judgments are being made about the relative worth of tasks from either end of the continua. Neither extreme suffices in itself to meet CCSSM as written.

5. Of course, there is such a thing as a *mathematically* authentic task as well. Using the word to describe modeling here is not intended to compete with any of the word'*s other uses.*

6. Again, no value judgments are being made about the relative worth of tasks at each level or sub-level. Neither extreme suffices to meet CCSSM as written.

7. Note, all Level 1 problems are well-posed.

8. For an example of "thin context", see Illustrative Mathematics (undated) *http://www.illustrativemathematics.org/illustrations/436*

9. See for example *www.mathmodels.org/problems/probview.php?probnum=20053*

References

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OECD (2013), *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*, PISA, OECD Publishing, Paris.
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Steen (2007), *Moving Beyond Standards and Tests* p.4: ["High schools focus on elementary applications of advanced mathematics whereas most people really make more use of sophisticated applications of elementary mathematics. … Many who master high school mathematics cannot think clearly about percentages or ratios."]
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