

Please cite this paper as:

Antolin, P. (2007), "Longevity Risk and Private Pensions", *OECD Working Papers on Insurance and Private Pensions*, No. 3, OECD Publishing.  
[doi:10.1787/261260613084](https://doi.org/10.1787/261260613084)



OECD Working Papers on Insurance  
and Private Pensions No. 3

# Longevity Risk and Private Pensions

Pablo Antolin

JEL Classification: C15, C32, G23, J11, J26, J32



## **LONGEVITY RISK AND PRIVATE PENSIONS**

**P. Antolin**

**January 2007**

**OECD WORKING PAPER ON INSURANCE AND PRIVATE PENSIONS**

**No. 3**

---

Financial Affairs Division, Directorate for Financial and Enterprise Affairs  
Organisation for Economic Co-operation and Development  
2 Rue André Pascal, Paris 75116, France  
[www.oecd.org/daf/fin](http://www.oecd.org/daf/fin)

**ABSTRACT/RÉSUMÉ****Longevity risk and private pensions**

This paper examines how uncertainty regarding future mortality and life expectancy outcomes, *i.e.* longevity risk, affects employer-provided defined benefit (DB) private pension plans liabilities. The paper argues that to assess uncertainty and associated risks adequately, a stochastic approach to model mortality and life expectancy is preferable because it permits to attach probabilities to different forecasts. In this regard, the paper provides the results of estimating the Lee-Carter model for several OECD countries. Furthermore, it conveys the uncertainty surrounding future mortality and life expectancy outcomes by means of Monte-Carlo simulations of the Lee-Carter model.

In order to assess the impact of longevity risk on employer-provided DB pension plans, the paper examines the different approaches that private pension plans follow in practice when incorporating longevity risk in their actuarial calculations. Unfortunately, most pension funds do not fully account for future improvements in mortality and life expectancy. The paper then presents estimations of the range of increase in the net present value of annuity payments for a theoretical DB pension fund. Finally, the paper discusses several policy issues on how to deal with longevity risk emphasizing the need for a common approach.

*JEL codes:* J11, J26, J32, G23, C15, C32

*Keywords:* Demographic forecast; mortality and life expectancy; life tables; longevity risk, retirement; private pensions; defined-benefit pension plans; Lee-Carter models; Monte Carlo methods, histograms.

\*\*\*\*\*

**Risque de longévité et pensions privées**

Cet article examine l'impact de l'incertitude concernant l'évolution de la mortalité et de l'espérance de vie (risque de longévité) sur le passif des fonds de pensions privés à prestations définies. Cet article soutient l'argument que, pour évaluer de manière adéquate cette incertitude et les risques associés, il est préférable de recourir à une approche stochastique pour l'établissement de projections de mortalité et d'espérance de vie afin de pouvoir associer des probabilités à des prévisions différentes. A ce sujet, le présent article présente les résultats de l'application simulée du modèle Lee-Carter à plusieurs pays membres de l'OCDE. En outre, il illustre l'incertitude concernant l'évolution de la mortalité et de l'espérance de vie par le biais de simulations Monte Carlo du modèle Lee-Carter.

Afin d'évaluer l'impact du risque de longévité sur les fonds de pensions privés à prestations définies, cet article étudie les différentes manières dont les régimes de pension privés intègrent le risque de longévité dans leurs calculs actuariels. La plupart des fonds de pension ne reflètent malheureusement pas complètement une amélioration future de la mortalité et de l'espérance de vie. Le présent article présente ensuite une estimation de la possibilité d'accroissement de la valeur nette actuelle des rentes viagères pour un fonds de pension théorique. Enfin, il étudie plusieurs questions de politique liées à la gestion du risque de longévité en soulignant le besoin d'une approche commune.

*Classification JEL :* J11, J26, J32, G23, C15, C32

*Mots clés:* Prévisions démographiques; mortalité et l'espérance de vie; tableaux de survie; risque de longévité; pensions privées; fonds de pension à prestation définie; le modèle Lee-Carter; simulation Monte Carlo; histogrammes.

**Copyright OECD, 2007**

**Applications for permission to reproduce or translate all, or part of, this material should be made to: Head of Publications Service, OECD, 2 rue André-Pascal, 75775 Paris Cédex 16, France.**

## LONGEVITY RISK AND PRIVATE PENSIONS

P. Antolin<sup>1</sup>

### I. Introduction

1. The length of time people are expected to live in most OECD countries has increased by 25 to 30 years during the last century. These gains in life expectancy are good news. However, policy makers, insurance companies and private pension managers worry about the impact that these gains may have on retirement finances. As long as gains in life expectancy are foreseeable and they are taken into account when planning retirement, they would have a negligible effect on retirement finances. Unfortunately, improvements in mortality and life expectancy are uncertain. In this regard, longevity risk is associated with the risk that future mortality and life expectancy outcomes turn out different than expected.

2. As a result of this uncertainty surrounding future developments in mortality and life expectancy, individuals run the risk of outliving their resources and being forced to reduce their standard of living at old ages. Pension funds and life annuity providers (*e.g.* insurance companies), on the other hand, run the risk that the net present value of their annuity payments will turn out higher than expected, as they will have to pay out a periodic sum of income that will last for an uncertain life span. In this context, individuals bear the full extent of the longevity risk when this risk is ‘uncovered’. However, private pension funds and national governments providing defined retirement benefits, as well as financial institutions providing lifetime annuity payments face this longevity risk.

3. The main purpose of this paper is to disentangle how uncertainty regarding future mortality and life expectancy outcomes would affect employer-provided defined benefit (DB) private pension plans liabilities. In this regard, the paper first focuses on assessing the uncertainty surrounding future developments in mortality and life expectancy, *i.e.* longevity risk.<sup>2</sup> Secondly, it examines the impact that longevity risk could have on employer-provided DB pension plans.

4. In order to assess the uncertainty surrounding future mortality and life expectancy outcomes, Section 2 first examines the link between mortality and life expectancy, explaining how life tables are constructed from mortality data. Secondly, it provides an overview of the developments in mortality and

---

<sup>1</sup> The author would like to thank André Laboul, Fiona Stewart and Juan Yermo for comments on earlier drafts. He also thanks delegates to the Working Party on Private Pensions of the OECD Insurance and Private Pensions Committee for useful discussions, and participants at the OECD/IOPS Global Forum on Private Pensions held in Istanbul on November 7-8, 2006; the Chatham House Seminar on Redistributing the Risk: Public and Private Approaches to Retirement Provision held in London on October 9-10, 2006; the 5th Conference on Regulation and Supervision of Pension Funds held in Lisbon on the 22<sup>nd</sup> of June, 2006; the 2006 European Pensions and Investment Summit held in Montreux, Switzerland; as well as participants at ASSAL XVII Annual Meeting held in Lisbon in May 2006. The views expressed herein are those of the author and do not necessarily reflect those of the OECD or the governments of its Member countries. The author is sole responsible of any errors.

Contact information: Pablo Antolín, Private Pensions Unit, Financial Affairs Division, Directorate for Financial and Enterprise Affairs, Organization for Economic Co-operation and Development, 2, rue André Pascal, Paris 75116, France. E-mail: pablo.antolin@oecd.org

<sup>2</sup> Throughout this paper, point forecasts on mortality and life expectancy are not discussed because the aim of the paper is to provide ways of exploring and assessing uncertainty instead of providing another set of projections.

life expectancy over the past century. The improvements seen in mortality and life expectancy were unanticipated as the consistent underestimation of actual outcomes illustrates. Thirdly, Section 2 focuses on the main problem facing pension funds, that is, to forecast the future path of mortality and life expectancy to ascertain their future liabilities. In this context, after discussing the main arguments behind two divergent views as regards the outlook for human longevity, the paper discusses different approaches available to forecast or project mortality and life expectancy. It then argues that a stochastic approach to model mortality and life expectancy is preferable because it permits to attach probabilities to different forecasts and, as a result, uncertainty and risks can be gauged adequately. Consequently, the paper presents a stochastic approach to model uncertainty surrounding mortality and life expectancy. In this regard, it provides the results of estimating the Lee-Carter model for several OECD countries. However, as the goal is far from providing just another set of forecasts but to assess the uncertainty surrounding different mortality and life expectancy outcomes, Section 2 concludes with Monte-Carlo simulations of the Lee-Carter model. These randomly generated simulations facilitate the task of assessing the uncertainty surrounding those forecasts.

5. The second part of the paper focuses on the impact that longevity risk may have on employer-provided DB pension plans. Longevity risk affects the net liabilities of DB pension plans through their lifetime annuity payments as unexpected improvements in mortality and life expectancy increase the length of the payment period. Section 3 first examines the different approaches that private pension plans follow in practice when incorporating future improvements in mortality and life expectancy in their actuarial calculations. While some pension funds account for future improvements in mortality and life expectancy, but only partially, others use only the latest available life tables when evaluating their liabilities. Secondly, Section 3 assesses the importance of the impact of longevity risk on the liabilities of private pension plans. For this task, the paper presents estimations of the range of increase in the net present value of annuity payments for a theoretical DB pension fund. The results suggest that the younger is the membership structure of a pension fund, the more exposed to longevity risk the pension fund is. However, older pension funds have less room for manoeuvre to deal with the costs associated with the materialization of longevity risk.

6. Finally, Section 4 discusses several policy issues on how to deal with longevity risk, with a particular emphasis on indexing pension benefits to life expectancy. The first task would be to agree on a common stochastic methodology to assess future mortality and longevity outcomes. Governmental agencies are the best placed institutions to produce those forecasts. However, as the membership structure differs among pension funds, making assumptions regarding the overall population renders those forecasts less useful for particular pension funds. In this regard, pension funds are inclined to use different mortality tables according to socio-economic status. However, this remains controversial. Finally, changes in the regulatory framework requiring pension plans to fully account for future improvements in mortality and life expectancy may be required.

## **II. The uncertainty surrounding mortality and life expectancy**

### ***The link between mortality and life expectancy: life tables***

7. Life tables provide a summary description of mortality, survivorship and life expectancy for a specified population. They can contain data for every single year of age (complete life tables) or by 5- or 10-year intervals (abridged life tables). In its simplest form, a life table can be generated from a set of age-specific death rates (ASDR). ASDR are calculated as the ratio of the number of deaths during a year (from vital statistics) to the corresponding population size (from censuses and annual population size estimates). They are commonly expressed as per 1,000 habitants. Mortality rates on the other hand are the probability that an individual of a given exact age will die during the period in question (*i.e.* the probability of dying). In the case of annual probabilities, the denominator is the size of the generation who reach age  $n$  during the

year in question, and the numerator is the number of individuals from this generation who die between age  $n$  and age  $n+1$ .<sup>3</sup> The annual probability of dying by age differs from the annual death rate because the latter is the proportion of people of that age who die during the year, while the probability of dying is the proportion of people at that age dying during the age interval.<sup>4</sup>

8. Therefore, life tables provide a link between mortality and life expectancy. The final outcome of a life table is the mean number of years still to be lived by a person who has reached that exact age (*i.e.* the age-specific life expectancies), if subjected throughout the rest of his or her life to the current age-specific probabilities of dying. Table 1 is an example of a life table for males in France in 2003. It is constructed from the age-specific death rates, expressed in death rates per thousand. The first column reports different ages  $x$ . In the second column,  $m_x$  is the observed period age-specific death rates per capita (*i.e.* dividing ASDRs by thousand). The next column contains the age-specific probabilities of dying,  $q_x$ , computed as  $(2 \cdot m \cdot n) / (2 + m \cdot n)$  where  $n$  is the width of the age interval. In the case of the open-ended age interval 110+ the probability of dying is one. Column four shows the mean number of person-years lived in the interval by those dying in the interval,  $a_x$ .<sup>5</sup> People are assumed to die in the middle of the age-interval, however, at birth people are assumed to die at the beginning of the interval, while at ages 110+ people are assumed to die late in the interval.

9. The next columns compute the number of survivors at each age  $x$  of a hypothetical cohort of 100,000 individuals,  $l_x$ ; the number of deaths in the cohort between two consecutive ages,  $d_x$ ; the number of person-years lived by the cohort,  $L_x$ , and the total person-years remaining at each age,  $T_x$ .<sup>6</sup> Life expectancies at age  $x$ ,  $e_x$ , are computed by dividing  $T_x$  by  $l_x$ .

10. Therefore, given age-specific mortality rates, a life table provides the associated age-specific life expectancies (Table 1). Having a link between mortality and life expectancy, the next sub-section focuses on reviewing developments in both variables over the last hundred years in several OECD countries.

### [Table 1 Life Table, France 2003]

#### *The uncertainty surrounding mortality outcomes*

11. Mortality rates have declined steadily over the past century, which have translated into large increases in life expectancy at both birth and age 65 (Figure 1 and Table 2). These declines stem from substantial reductions in mortality rates at younger ages and, to some extent, improvements at old-ages. During the first part of the 20<sup>th</sup> century, the decline in mortality was mainly due to a reduction of infectious diseases affecting mainly young ages. During the last decades of the 20<sup>th</sup> century, the decline in mortality

<sup>3</sup> Some deaths occur during the year in question, while other deaths occur the following year.

<sup>4</sup> For example, a person reaching age 65 in 2000 who dies at age 65 but in 2001 will be counted when calculating the probability of dying at 65 in 2000, but it will not be counted when calculating the death rate at age 65 in 2000.

<sup>5</sup> This is a key variable. When using 1-year age groups it is assumed that people die in the middle of the age-interval (*i.e.* a value of 1/2), when using a 5-year age intervals you can assume also the middle of the interval (1/2) or, if data is available, use the single year age data to build the mean.

<sup>6</sup> The number of deaths,  $d_x$ , is computed by multiplying the number of survivors of the cohort by the probability of dying. The number of survivors,  $l_x$ , at age  $x+1$  is the difference between those surviving at  $x$  minus those dying at  $x$ . Computing the number of person-years lived,  $L_x$ , is a bit more tricky because we do not know when people dying in the age interval died, at the beginning, middle or the end. It is generally assumed in the middle. When using 1-year intervals this assumption is alright, but when using 5-years interval it may not be fully accurate. The formula is  $(n \cdot (1 - (d \cdot a)))$ . Finally,  $T_x$  is obtained by accumulating the  $L$  column backwards.

was due to reductions in deaths due to chronic diseases affecting primarily older ages.<sup>7</sup> This is confirmed when looking at the increases in life expectancy at birth and at age 65 during the 20<sup>th</sup> century (Table 2). Life expectancy at birth increased faster during the first-half of the 20<sup>th</sup> century while life expectancy at age 65 increased faster during the second-half, as comparing the top and bottom panels in Table 2 confirms. Employer-provided DB pension plans are mostly affected by changes in mortality and life expectancy at older ages. In this regard, it is important to highlight that for most OECD countries, more than half of the improvement in life expectancy since the 1960s is due to increases in life expectancy at age 65.

**[Figure 1. Life expectancy and mortality rates in selected OECD countries, 1950-2002]**

**[Table 2. Life Expectancy in selected OECD countries, 1900-2000]**

12. Past projections have consistently underestimated actual improvements in mortality rates and life expectancy. Improvements in mortality rates and life expectancy have increased the number of years that people spend in retirement, bringing in financial troubles for DB pension funds, individuals and social security systems. During the past decades governmental agencies, actuaries and academics have tried to project and forecast mortality rates and life expectancy to assess future liabilities. However, past projections have consistently underestimated improvements. Table 3 shows how life expectancy projections by international organisations (e.g. the UN and Eurostat) and actuaries have failed to account for actual improvements. A positive sign indicates that life expectancy at birth in 2003 has already bypassed the UN projected life expectancy for the average of the period 2000-2005 (first column) and the Eurostat projection for 2005.<sup>8</sup> In the same context, Figure 2 shows the United States Social Security Administration (SSA) projections of life expectancy consistently below actual outcomes.<sup>9</sup>

**[Table 3. Comparing past projections with realized gains in life expectancy]**

**[Figure 2. Life expectancy projections by the United States Social Security Administration and actual outcomes]**

13. Moreover, projections for the next 50 years incorporate a slower improvement in mortality and life expectancy than in the recent past. Future projections by international organisations and national statistical institutes assume that the projected gains in life expectancy at birth for the next 50 years will slow down by almost half from the gains experienced in the second half of the last century (Table 4). Unfortunately, it is impossible to assess the likelihood surrounding these projections because they are deterministic, and as such they do not incorporate a distribution function or probabilities to assess the likelihood of the range of possible outcomes.

**[Table 4. Comparing past with projected gains in life expectancy]**

14. Future increases in life expectancy will have to come mainly from further declines in mortality rates at old ages. There is a certain degree of uncertainty about the extent of future improvements in mortality rates and life expectancy. Nevertheless, as mortality rates at young and middle ages have reached very low levels, improvements would have to come from declines in mortality at old ages, that is, from

---

<sup>7</sup> This reduction was mainly due to reduced illnesses from cardio-vascular diseases.

<sup>8</sup> UN projections were produced in 1999 using data up to 1995, while Eurostat projections were produced in 2000 using data up to 1999.

<sup>9</sup> Siegel (2005) also reports that the projection by the United States Actuary's office have been consistently below actual values for most projection years (see Table 10 in his report).

increases in life expectancy at age 65 or more, and, in particular, at very old ages (85+). However, there are different views as regard the outlook for human longevity (Siegel, 2005).

15. There is a great debate on the extent of those increases in longevity. Essentially, there are two groups, those who argue that there are no limits to life expectancy (e.g. Oeppen and Vaupel, 2002) and those who are more conservative (e.g. Olshansky et al, 2005). The first group concludes from historical trends and age trajectories that no limits can be set to life expectancy. They argue that mortality is likely to level off after some (unspecified) threshold, and, as a result, longevity would be uncapped and would keep increasing in the next decades. However, this view remains controversial. The more conservative group argues that the epidemiological transition<sup>10</sup>, as well as the massive reductions in mortality rates required to produce even small increases in life expectancy, suggest that increases in life expectancy will slow down if not to stop (Olshansky et al, 2005). They believe that human live might have natural limits. Furthermore, the empirical evidence showing that survival probability curves have become increasingly rectangular or compressed (Kannisto, 2000) suggests that there are limits to life expectancy. Unfortunately, the compression of mortality or “rectangularization” theory is not conclusive (Siegel, 2005).

16. Therefore, there is a large degree of uncertainty surrounding future improvements in mortality and life expectancy, in particular at old ages. This uncertainty requires a different approach to model future improvements and, in particular, to assess the uncertainty surrounding these improvements. In this context, the next sub-section argues for using a stochastic instead of a deterministic approach to forecasts those improvements as it allows attaching probabilities to a full range of different forecasts and thus it allows assessing uncertainty and risks adequately.

#### *Approaches to forecast mortality and life expectancy*

17. There are several approaches available in the literature to project mortality rates (CMI, 2004, 2005a; Wong-Fupuy and Haberman, 2004). Public pension systems or private pension funds providing defined pension benefits need population projections to assess the number of people who will potentially be entitled to a pension. The main inputs necessary to produce population projections are assumptions regarding fertility, mortality and net migration flows. As the paper focuses on longevity risk and its impact on defined-benefit pension plans, the focus is on mortality and life expectancy projections. In this regard, there are several approaches to model mortality and life expectancy. There are process-based methods which use models based on the underlying biomedical processes; there are also explanatory-based approaches which employ a causal forecasting approach involving econometric relationships; and there are extrapolative methods which are based on projecting historical trends in mortality forward.

18. Extrapolative models are the type of models most used by actuaries and official agencies. These models express age-specific mortality as a function of calendar time using past data and as such, they can be deterministic or stochastic. Deterministic models forecast by directly extending past trends and, as a result, they do not come with standard errors or forecast probabilities. Stochastic models, on the other hand, forecast using probability distributions. They fit a statistical model to the historical data and then project into the future. As an outcome of the forecast process, forecast values have probabilities attached that allow assessing the likelihood that an outcome will occur. Among the extrapolative stochastic methods the literature distinguishes between (1) models based on the interdependent projection of age-specific

---

<sup>10</sup> Gains in longevity during the last century were mainly at young ages and were based on successes dealing with infectious and parasitic diseases. These are externally caused and relatively easy to treat with vaccines and immunization. However, future gains should focus on old ages where there is a predominance of degenerative diseases of later life, such as cancer, cardiovascular diseases and diabetes. These are chronic and progressive, and more difficult to treat.



mortality or hazard rates (including graduation models, CMI)<sup>11</sup>; (2) models using standard time series procedures like the Lee-Carter method (Lee and Carter, 1992) where a log linear trend for age specific mortality rates is often assumed for the time-dependent component; and (3) models using econometric modeling (e.g. Spline models).

19. However, governmental agencies tend to extrapolate historical trends in a deterministic manner, while actuaries use several smoothing approaches, generally parametric approaches (e.g. Gompertz model). Governmental agencies project mortality rates by using past trends and expert opinion. Parametric smoothing models are the most familiar to actuaries since they have been used for mortality graduations.<sup>12</sup> Neither of these approaches provides therefore forecast probabilities. Both Eurostat and the United States Census Bureau population projections use a deterministic approach.<sup>13</sup> They use historical trends in age-specific mortality rates (ASMR), generally the last 15 years, and assume that they will continue in the future, however, weighted by some expert assessment of the causes of death (European Commission, Eurostat, 2005; Hollman et al., 2000). They estimate the values of the ASMR in an intermediate year (e.g. 2018) and at the end target year (e.g. 2050). This is done by applying the improvement rate to the average mortality rate in the last 3 to 5 available years.<sup>14</sup> Finally, ASMRs for each intermediate year are calculated by an interpolation method based on fitting third degree curves. They extrapolate the intermediate years by assuming a parametric function such as the logistic or the Gompertz function.

20. Additionally, governmental agencies and actuaries use different populations when projecting mortality and life expectancy. Governmental agencies produce mortality tables and project life expectancy for the population of their respective countries as a whole. However, private pension plans use their own actuarial mortality tables, because mortality rates of pension funds' participants can differ substantially from those of the overall population. It is a well know fact that mortality rates are lower, and life expectancy higher, for women, high educated and high income people (Goldman, 1991; and Drever et al., 1996). However, using life tables differentiating by socio-economic groups could give raise to another different set of problems (see Section 4 below). In addition, in some countries, private pension funds use mortality tables from other countries (e.g. from the United Kingdom) as their data records do not go far back enough.

21. Finally, due to the lack of enough data, estimating and forecasting mortality rates and life expectancy for the very old (those aged 85 or more) is challenging. Data at very old ages are not very accurate because of small sample problems. Only a few countries have official population statistics which are sufficiently accurate to produce reliable estimates of death rates at higher ages. It is commonly accepted that between ages 30 and 85, age-specific death rates tend to rise roughly at a fixed rate of increase.<sup>15</sup> This rate of increase tend to fall for ages above 85, and even possibly, at the more extreme ages, to become zero or negative, although one cannot be certain of the latter because of the sparseness of the data above age 100 (Robine and Vaupel, 2002; Wilmoth, 1998). This paper uses data form the Human

---

<sup>11</sup> Among these are parametric models which fit a specific distribution (e.g. logistic, Gompertz) to the historical data and forecast given the statistical characteristics of the parametric distribution. Some commentators consider them neither deterministic nor stochastic.

<sup>12</sup> Examples are the Gompertz model and its many generalizations (e.g. Gompertz-Makeham family) which have been used for recent CMIB graduations.

<sup>13</sup> The United States SSA projects mortality using causes of death (Siegel, 2005). This is also a deterministic approach and it incorporates expert opinion.

<sup>14</sup> Always distinguishing by age and gender.

<sup>15</sup> Roughly in accordance with the Gompertz curve.

Mortality Database ([www.mortality.org](http://www.mortality.org)) and takes at face value the age-specific death rates at very old age provided.<sup>16</sup>

### *Measuring uncertainty surrounding mortality and longevity outcomes*

22. The uncertainty surrounding future mortality and life expectancy outcomes can be gauged using a stochastic approach because it attaches probabilities to different outcomes, permitting therefore to assess uncertainty and risks adequately. Future developments in mortality rates and life expectancy are uncertain, but some paths or trajectories are more likely than others. Hence, attempts to forecast mortality and life expectancy should include a range of possible outcomes, and probabilities attached to that range. Together, these elements constitute the ‘prediction interval’ for the mortality and life expectancy variables concerned. There is a clear trade-off between greater certainty (higher odds) and better precision (narrower intervals). This subsection presents the results of examining the uncertainty surrounding forecasts of mortality and life expectancy using the Lee-Carter stochastic methodology (Lee and Carter, 1992) by means of Monte-Carlo simulations.<sup>17,18</sup>

23. Forecasts of mortality rates and life expectancy using the Lee-Carter stochastic approach were produced for 6 OECD countries.<sup>19</sup> The Lee-Carter model suggests a log-bilinear model in the variables  $x$  (age) and  $t$  (time) for estimating the force of mortality at age  $x$  in year  $t$ ,  $m(x,t)$ :

$$\ln(m(x,t)) = a(x) + b(x) \cdot k(t) + \varepsilon(x,t) \quad \text{or} \quad m(x,t) = e^{a(x)+b(x) \cdot k(t)+\varepsilon(x,t)} \quad [1]$$

24. The  $a(x)$  coefficients describe the average level of the  $\ln(m(x,t))$  surface over time. Therefore,  $\exp(a(x))$  is the general shape of mortality at age  $x$ . The  $b(x)$  coefficients capture the sensitivity of the logarithm of the force of mortality at age  $x$  to variations in  $k(t)$ , where  $k(t)$  is a time-varying index of the level of mortality that describes the change in overall mortality over time. That is, as  $k(t)$  falls mortality falls and vice-versa. Moreover, if  $k(t)$  decreases linearly, then the cause of mortality or mortality rate,  $m(x,t)$ , decreases exponentially at each age and at a rate that depends on  $b(x)$ .<sup>20</sup> For ages with high values of the coefficient  $b(x)$ , mortality rates would change faster. If the coefficients  $b(x)$  were to be equal at all ages, then mortality rates would change at the same rate at all ages. Finally, the error term  $\varepsilon(x,t)$  reflects particular age-specific historical influences not captured in the model. Lee and Carter (1992) assume that the error term is normally distributed with mean zero and variance  $\sigma^2$ , *i.e.*  $\varepsilon(x,t) \sim N(0, \sigma^2)$ .

<sup>16</sup> The Human Mortality Database (HMD) was created to provide detailed mortality and population data to researchers, students, journalists, policy analysts, and others interested in the history of human longevity. The project began as an outgrowth of earlier projects in the Department of Demography at the University of California, Berkeley, USA, and at the Max Planck Institute for Demographic Research in Rostock, Germany. The database contains detailed data for a collection of 26 countries.

<sup>17</sup> The Lee-Carter and the P-Spline model (Currier *et al.*, 2004) are the two approaches recommended by the Mortality Committee of the English actuarial profession (CMI, 2005a and 2006). Stata programmes to estimate mortality using the Lee-Carter have been developed in house, while Stata programmes to estimate mortality using S-splines are being considered.

<sup>18</sup> Monte Carlo simulations are the result of repeating the estimated Lee-Carter model several thousand times using random number generators for the error terms.

<sup>19</sup> These include France, the Netherlands, Spain, Sweden, the United Kingdom, and the United States. Results are available upon request.

<sup>20</sup> The coefficients  $b(x)$  can be negative, in particular for old ages, reflecting an increase in the likelihood of dying at very high ages. For negative values of  $b(x)$ ,  $m(x,t)$  increases exponentially as  $k(t)$  decreases linearly.

25. The model cannot be identified (i.e. estimated uniquely) because  $b(x)$  and  $k(t)$  appear through their product. Therefore, it is often assumed that  $\sum_x k(t) = 0$  and  $\sum_x b(x) = 1$  to ensure identifiability of the model. Using maximum likelihood techniques and the Singular Value Decomposition (SVD) method (see Lee & Carter, 1992),  $a(x)$ ,  $b(x)$  and  $k(t)$  are estimated.<sup>21</sup> In order to obtain forecasts of mortality rates, Lee and Carter (1992) propose to use a time series model for  $k(t)$ . The standard choice is an autoregressive model, AR(1), for  $k(t)$ .

$$k(t) = \alpha + \beta \cdot k(t-1) + e(t) \quad [2]$$

26. The error term is distributed  $N(0,1)$ . During the estimation procedure, dummy variables were used to control for extreme events, for example the flu epidemic in 1918 or the two world wars. Having fitted the model for the time-varying index,  $k(t)$  is projected forward. Using these forecasts of  $k(t)$  and the previously estimated values for  $a(x)$  and  $b(x)$ , one obtains age-specific mortality rates for future years. Finally, using the methodology to produce life tables described above, age-specific life expectancies for future years were obtained.<sup>22</sup>

27. However, the purpose of this paper is far from producing another set of projections but to assess the uncertainty surrounding future mortality and life expectancy prospects. With the aim of providing the most useful information to policy makers and pension funds, the paper explores therefore ways of conveying the degree of uncertainty surrounding those forecasts. In this regard, the stochastic method used to obtain forecasts of mortality rates and life expectancy permits to attach probabilities to these forecasts and thus assess uncertainty around any possible outcome, such as the mean or the central forecast. In this context, this paper conveys the uncertainty surrounding future mortality and life expectancy outcomes (i.e. longevity risk) by means of frequency distributions and cumulated probabilities generated from 10,000 Monte-Carlo simulations of the Lee-Carter model of mortality.<sup>23,24</sup>

28. The likelihood that life expectancy in 2050 will turn out lower or higher than a determined forecast value (e.g. the central forecast) measures the risk or uncertainty surrounding that forecast. Figure 3 shows the histogram and the cumulative probability of 10,000 simulations for life expectancy at age 65 in 2050 using the Lee-Carter model. The cumulative probability function provides the probability or likelihood that life expectancy will be lower or equal than the corresponding forecast value. In this framework, Figure 3 shows that the central forecast lies above the median life expectancy for all countries, that is, the likelihood that life expectancy will turn out higher than the central forecasts is lower than 50 percent. In particular, this likelihood varies from 62 percent in France to 82 percent in the United

<sup>21</sup> During the estimation procedure  $k(t)$  is first estimated to minimize errors in the log of death rates rather than the death rates themselves. Therefore, in a second step  $k(t)$  is re-estimated taking the estimates of  $a(x)$  and  $b(x)$  from the first step as given. The new values of  $k(t)$  are found by an iterative search such that for each year, given the actual population age distribution, the implied number of deaths will equal the actual number of deaths.

<sup>22</sup> Stata programmes to estimate the Lee-Carter model and produce forecast for the period 2005-2100 are available upon request, as well as Stata programmes generating life tables from age-specific mortality rates.

<sup>23</sup> Another way of presenting the uncertainty surrounding forecasts would be to present the point forecast and the standard deviation. It could be presented using a fan graph as the inflation report of the Bank of England does (King, 2006).

<sup>24</sup> Monte-Carlo is a technique that involves using random numbers. In particular, it produces simulations of the Lee-Carter model by using random number generators for the random terms in the Lee-Carter equations (1) and (2). The Stata programmes, available upon request, assume that the mean and the variance of  $\varepsilon(x,t)$  and  $e(t)$  are those obtained from the errors of fitting the Lee-Carter model to the historical data.

States.<sup>25,26</sup> In other words, there is a 12 to 38 percent chance that life expectancy will turn out higher than the central forecasts, depending on the country examined. In addition, the probability of any deviation regarding life expectancy is also easy to determine. There is a 5 percent likelihood that life expectancy at 65 in 2050 will be one year higher than the central forecast for each country.<sup>27</sup>

**[Figure 3. Histogram and Cumulative probability of life expectancy at birth and at age 65 in 2050, selected OECD countries]**

29. Consequently, the risk or uncertainty surrounding the forecasts is large, but the magnitude of the likely deviation is relatively small.<sup>28</sup> Thanks to using a stochastic approach to model mortality and life expectancy, the uncertainty surrounding life expectancy can be measured by attaching probabilities to a range of future mortality and life expectancy outcomes. The next step is therefore to evaluate how this uncertainty affects pension fund liabilities. In this regards, the next section calculates the increase in the net present value of annuity payments as mortality and life expectancy changes.

**III. The impact of longevity risk on defined-benefit private pension plans.**

30. This section examines the impact of longevity risk on employer-provided DB private pension plans. The previous section showed that using a stochastic approach to forecasts mortality and life expectancy permits to attach probabilities to a range of different forecasts and thus assess the uncertainty surrounding future mortality and life expectancy outcomes. However, private pension funds are concerned with the effect of this uncertainty on their pension liabilities. As the main impact of this longevity risk on net pension liabilities is through their guarantee annuity payments, this section evaluates the changes in the net present value of annuity payments as mortality and life expectancy evolves. These changes are evaluated for pension fund members at different ages, and for pension funds with different age-membership structure.

***How does longevity risk affect DB private pension plans?***

31. The main impact of longevity risk on the net pension liabilities of employer-provided DB private pension plans is through their annuity payments. An annuity is an agreement for one person or organisation to pay another (the annuitant) a stream or series of payments (annuity payments). Annuities are intended to provide the annuitant with a steady stream of income over a number of years, which can start immediately or in the future. The capital and investment proceeds are generally tax-deferred. There are many categories of annuities. They can be classified in several ways, for example: (1) according to the underlying investment into fixed or variable; (2) according to the primary purpose, i.e. accumulation or pay-out, into deferred or immediate; (3) according to the nature of pay-out commitment into fixed period, fixed amount or lifetime; and (4) according to the premium payment arrangement into single or flexible premium. In a fixed annuity, the insurance company or pension fund guarantees the principal and a minimum rate of interests, while in a variable annuity the annuity payment depends on the investment performance of the

---

<sup>25</sup> The distribution function is more skew to the right in some countries like the United States. This is most likely due to larger variance of errors when fitting the Lee-Carter model.

<sup>26</sup> That is,  $0.62 \leq \Pr(\text{LEx} \leq \text{LEx}^c) \leq 0.82$  depending on the country. LEx is life expectancy and  $\text{LEx}^c$  is the central forecast.

<sup>27</sup>  $\Pr(\text{LEx} \leq \text{LEx}^c + 1) = 0.95$ .

<sup>28</sup> This deviation depends on the variance used in the random number generators to produce Monte Carlo simulations. This exercise uses the variance of the fitted errors, which depend on whether dummies are used or not in the estimation of the Lee-Carter model to control for extreme events like the flu epidemic or the two world wars.

underlying portfolio. An immediate annuity is designed to pay an income one-time or an income stream immediately after the immediate annuity is bought, while in a deferred annuity, the annuitant receives the payment(s) at a later time. Fixed period annuities pay an income for a specified period of time (e.g. 10 years), while lifetime annuities provide income for the remaining life of the annuitant. A single premium annuity is an annuity funded by a single payment, while a flexible premium is an annuity intended to be funded by a series of payment. Flexible annuities are only deferred.

32. As employer-provided DB private pensions guarantee a fixed future stream of payments at retirement to their members for the rest of their life, the analysis throughout focuses on the impact of longevity risk on fixed, deferred, lifetime and flexible premium annuities. Longevity risk would have its larger effect on annuities that are fixed, deferred and for the lifetime of the annuitant once retirement age is reached. The impact of longevity risk on fixed period annuities, on the other hand, is less clear-cut. Moreover, the magnitude of the impact of longevity risk on annuity payments would depend not only on the type of annuity guarantees but on how pension funds account for improvements in mortality and life expectancy when calculating the net present value of annuity payments.

### ***How private pension funds account for future improvements in mortality and/or life expectancy?***

33. Unfortunately, pension funds do not seem to fully account for future improvements in mortality and life expectancy. Recent studies, in particular, the research by the Actuarial Profession and Cass Business School (2005), found that current practice varies considerably across the EU. Pension funds in some countries incorporate an allowance for expected future improvements in mortality, while others use tables that relate to mortality observed over a period in the past, without allowing for the fact that life expectancy may continue to increase (Belgium, Denmark, Norway, Sweden, and Switzerland). Of those countries incorporating an allowance for future improvements in mortality, Austria, France, Germany (for only 25 years and using 1996 as the base year), Ireland (improvements incorporated only until 2010), Italy, the Netherlands, Spain, and the United Kingdom use forecasts; while Canada, Finland, and the United States, despite of having mortality tables with built in mechanisms to take into account future changes in mortality, generally do not use them.

34. Furthermore, there is not a consistent or standard methodology to incorporate future improvements in mortality and life expectancy. In this regard, there is a big problem with tracking longevity risk because the lack of a standard methodology makes mortality calculations arbitrary and difficult to compare across pension funds and, let alone, countries.<sup>29</sup>

35. Moreover, in most countries pension funds are restricted as regard the demographic assumptions they can incorporate in their assessments. The study by the Groupe Consultatif Actuariel Europeen on actuarial methods and assumptions used in the valuation of retirement benefits in the EU and other European countries, suggests that only in a few countries (Ireland, Italy, the Netherlands, Portugal and the UK) demographic assumptions are chosen by the actuary without any direct restrictions from the supervisory or taxation authorities.<sup>30</sup> Rigid regulation regarding demographic assumptions can be inadequate when restricting pension funds from using alternative mortality tables that incorporate improvements in mortality and life expectancy, as they may not result in a better assessment of longevity risk.

---

<sup>29</sup> The Cass Business School (2005) reports the large variety of approaches used by those pension funds adjusting partially their liability calculations for mortality and life expectancy improvements.

<sup>30</sup> In addition, the study by the Group Consultatif Actuariel Europeen notes that only in Ireland, Cyprus, France, Italy and the United Kingdom an allowance for improvements in mortality is part of the demographic assumptions.

36. Therefore, as a result, the impact of longevity risk is compounded. The impact of the uncertainty surrounding future improvements in mortality and life expectancy (*i.e.* longevity risk) on employer-provided DB private pension plans is compounded as few actuaries and pension schemes account for future improvements in mortality and life expectancy, and those that account for improvements generally do it only partially. In addition, the base tables used for demographic assumptions, even if adjustments for future improvements in mortality are included, are almost 10 years old, from the early to mid-1990s. Furthermore, the lack of standard methods to forecast mortality and life expectancy, and the fact that these methods are generally far from being fully stochastic complicate any comparative analysis and make the task of examining the impact of longevity risk on pension fund liabilities fuzzier.

### ***The impact of longevity risk on net pension liabilities***

37. This sub-section focuses on the impact of unexpected improvements in mortality and life expectancy on the net present value of annuity payments of an employer-provided DB fund.<sup>31</sup> Firstly, it presents the results of calculating the increase in the net present value in 2005 of annuity payments due to a pension fund member at different ages. The pension benefit is a fixed amount of 10,000€ at 2005 values and it is paid over the lifetime of a member after reaching retirement age at 65. Secondly, this sub-section discusses the results of calculating the increase in the net present value in 2005 of annuity payments of a hypothetical DB pension fund given different age-membership structures. For simplicity, the pension fund is assumed to be closed to new members.<sup>32</sup>

38. Results show that the gap in the net present value of annuity payments between taking into account mortality and life expectancy improvements or not, is inversely related with the age of pension fund members. Table 5 reports the increase in the net present value of annuity payments for several ages. This increase is the result of comparing the net present value of annuity payments when using the latest available mortality tables and when using mortality tables that account for improvements in mortality and life expectancy. In this particular case, the calculations reported assume that life expectancy at birth and at age 65 increases by 1.2 and 0.8 years per decade, respectively. As a result of taking into account these improvements in mortality and life expectancy, benefit payments to a 25 year old member in 2005 increase by almost ¼ with respect to the case when no account for improvements is taken. This increase drops to 3.3 percent for a 65 year old member. This inverse relationship stems from the fact that the exposure of the pension fund to improvements in life expectancy is larger the younger the individual is today.

### **[Table 5. The increase in the NPV of annuity payment, 2005]**

39. Therefore, pension funds with an older age-membership structure will experience a smaller impact from longevity risk on their liabilities. However, they may have less room for manoeuvre to correct for changes in longevity risk. The age composition of the pension fund members is quite important to determine the overall impact of unexpected improvements in mortality and life expectancy. The right hand side panel of Table 5 shows the increase in the net present value of annuity payments for a “theoretical pension fund” according to its membership structure. This increase is smaller as the membership ages. Table 5 indicates that an un-expected improvement in life expectancy at birth of one year per decade could

<sup>31</sup> In this sub-section, the paper does not provide a whole range of possible outcomes. The next step in the current project is to provide this range using Monte Carlo simulations and thus be able to assess uncertainty. In this case, random number generators can also be incorporated in interest rates, discount rates and wage assumptions, allowing therefore assessing the whole range of risks affecting annuity payments.

<sup>32</sup> Additionally, wages are assumed to grow at 3.5% nominally in line with productivity growth of 1.75% and inflation of 1.75%. The discount rate is set at 3.5% offsetting therefore any impact due to differences between discounting and the growth of payments. Hence, the changes in annuity payments are only due to changes in life expectancy.

increase pension fund liabilities by as much as 10 percent.<sup>33</sup> Taking into account that funding regulations of pension funds suggest that a deviation in liability calculations of more than 5% is over the acceptable margins of risk, the impact of longevity risk needs to be reckon with.<sup>34</sup> Furthermore, following the results in Section 2, there is a 10 to 30 percent chance that the net present value of annuity payment increases by as much as 10 percent.<sup>35</sup>

40. Additionally, the impact of longevity risk on pension fund liabilities is reinforced by reductions in interest rates. Pension funds have recently experienced sharp increases in their liabilities as interest rates fell. Lower interest rates result in lower discount rates, giving a relatively higher weight to the future. As longevity risk is back-loaded, reductions in interest rates increase the impact of longevity risk on the net present value of annuity payments. In this regard, Table 6 compares the change in annuity payments stemming from different interest rates (*i.e.* discount rates) with the change due to improvements in life expectancy. A two percentage point reduction in interest rates increases annuity payments by around 18-20 percentage points, while an improvement in life expectancy of 1.2 years per decade increases annuity payments by only 2.4 percent for a 65 year old individual, and almost 20 percent for a 25 year old individual. The combination of both effects can lead to an increase in the net present value of annuity payments of as much as 213 percent. Hence, life insurance companies and pension funds are strongly affected by the interest rate-longevity correlation risk.<sup>36</sup>

**[Table 6. Impact of longevity improvements and changes in interest rates on annuity payments]**

#### IV. Policy issues

41. The paper have shown that longevity risk, defined as the uncertainty surrounding future developments in mortality and life expectancy, has a non-negligible impact on the liabilities of employer-provided DB pension plans through their lifetime annuity payments as they depend on the length of time people is expected to live. Section 3 provided a measure of this impact on the net present value of annuity payments for a “theoretical pension fund”. It showed that the magnitude of this impact depends on the age structure of the pension fund membership. As a result, pension funds with a younger membership structure will experience a larger impact from longevity risk because the pension fund is exposed for longer to uncertain improvements in mortality and life expectancy.

42. Unfortunately, the impact of longevity risk is compounded as few pension plans account for future changes in mortality and life expectancy, and those funds that do, they account for improvements only partially. Adding to the problem, pension funds use mortality tables that are almost a decade old in

<sup>33</sup> It is important to recall here that the difference between the projections of life expectancy for 2050 prepared by national statistical institutes and the simple extrapolation of past trends (Table 4) is one year per decade.

<sup>34</sup> A recent study by Cass Business School (2005) compares mortality assumptions used in corporate pension liability calculations across EU countries, Canada and the United States. Considering a pension scheme with an actuarial deficit £200m (*e.g.* with assets of £800m and liabilities of £1000m), calculated using UK’s mortality assumptions, they find an increase in the net liabilities of £63m when using French mortality assumptions, but a reduction in net liabilities of £131m (*i.e.* net liabilities of £69m) when using Dutch mortality assumptions.

<sup>35</sup> Section 2 showed that there is a 10 to 30 percent uncertainty surrounding the likelihood that life expectancy at 65 in 2050 would be one year higher than the central forecasts.

<sup>36</sup> The ECB (2005) reports that a 10% improvement in longevity leads to an increase by 5.4% of the net present value of the immediate annuity – an immediate annuity being a regular income payable throughout life, which is usually secured in exchange for a lump sum – to meet an annual payment of 10,000 euro over 25 years, based on a 3% interest rate. With interest rates equal to 5% and 10% respectively, this figure would fall to 4.2% and 2.1%.

most cases. Moreover, the task of assessing the best way to account for improvements in mortality and life expectancy is complicated by the lack of a common methodology to account for longevity risk.

43. In this respect, there is a clear advantage from using a common methodology to forecasts mortality rates and life expectancy. In this regard, this paper have argued for using a stochastic model as it permits to attach probabilities and thus assess the degree of uncertainty surrounding future mortality and life expectancy outcomes.<sup>37</sup> Unfortunately, many small and medium size pension funds may not have the financial resources or the technical capability to produce forecasts using a common methodology. Insofar as governmental agencies (e.g. national statistical institutes) have the resources and technical capabilities, they could produce them. However, assumptions regarding the overall populations rather than the specific membership populations of private pension plans may not be of much use to them. Governmental agencies could produce forecasts for the entire population and for different subgroups according to gender, age, income and educational level. Hence, different pension funds could use the corresponding sub-population that matches its current membership structure more closely.

44. However, using mortality tables differentiating according to socio-economic status and gender have its own problems as it could give raise to problems of discrimination. Arguments in favour of differentiating tables include that using an average life expectancy index penalises people with higher life expectancy (e.g. women, high educated and high income people) favouring people with lower life expectancy (e.g. men, low educated and low income people). Moreover, private pension plans need to hedge against their own longevity risk, *i.e.* the risk attached to their own membership structure, instead of an average longevity risk.

45. Furthermore, there may be a need for a change in the regulatory framework requiring pension funds to fully account for future improvements in mortality and life expectancy as well as guiding pension funds regarding the type of approaches best suited to forecasts those improvements and assess its associated uncertainty.

46. Finally, in addition to incorporating improvements in mortality with the help of a common methodology and the use of average or differentiated mortality tables, the impact of longevity risk on employer-provided DB plans can be partly offset by indexing pension benefits to life expectancy.<sup>38</sup> However, indexing benefits to life expectancy shifts part of the longevity risk back to individuals, removing one of the main incentives individuals have to acquire annuities. In this regard, differentiating between individual and aggregate or cohort longevity risk can be of help. Individual risk is associated to each individual and it can be easily offset by pooling risks. Therefore, it would be more efficiently undertaken if assumed by pension funds, as they are best placed to pool individual specific risks. The aggregate or cohort risk, on the other hand, is more difficult to address or hedge against. Therefore, this risk is more open to be shared by pension funds and individuals by indexing benefits to cohort longevity changes.<sup>39</sup>

---

<sup>37</sup> CMI (2005a, 2006) and Lee (1998) also argue for using a stochastic approach to forecast mortality and life expectancy.

<sup>38</sup> Yet, indexing also brings up the previous discussion related to using average or differentiated mortality tables.

<sup>39</sup> Antolin and Blommestein (2006) deal with this issue in the context of whether government should issue longevity bonds to hedge against longevity risk. OECD (2005) provides further discussions on the issue of longevity bonds and government involvement. This issue could be addressed further at the following Working Party in Debt Management (WPDM) and the Financial Markets Committee (CMF).



## REFERENCES

- Antolin, P. and H. Blommestein (2006), "Governments and the market for longevity-index bonds". Paper prepared for the forthcoming 26<sup>th</sup> SUERF Colloquium, 12-14 October, Lisbon.
- Continuous Mortality Investigation, CMI, (2004), "Projecting future mortality: A discussion paper", Mortality Committee, Working Paper 3.
- Continuous Mortality Investigation, CMI, (2005a), "Projecting future mortality: Towards a proposal for a stochastic methodology", Mortality Committee, Working Paper 15.
- Continuous Mortality Investigation, CMI, (2005b), "The Graduation of CMI 1999-2002 Mortality Experience: Proposed Annuitant and Pensioner Graduations", Mortality Committee, Working Paper 16.
- Continuous Mortality Investigation, CMI, (2006), "Stochastic projection methodologies: Further progress and P-spline model features, example results and implications", Mortality Committee, Working Paper 20.
- Drever, F.M. Whitehead, and M. Roden (1996), "Current Patterns and Trends in Male Mortality by Social Class (Based on Occupation)" *Population Trends* 86:15-20.
- European Central Bank, ECB (2005), "Longevity risk, Interest rates and Insurance Companies and Pension funds" *Financial Stability Review*: December 2005.
- European Commission, Eurostat (2005), "EUROPOP2004: Methodology for drafting mortality assumptions".
- Friedberg, L. and A. Webb (2005), "Life is cheap: using mortality bonds to hedge aggregate mortality risk" *Center for Retirement Research Working Paper 2005-13*.
- Goldman, N. (2001). Social inequalities in health: disentangling the underlying mechanisms. In M. Weinstein, A. Hermalin, & M. Stoto (Eds.), *Population Health and Aging: Strengthening the Dialogue Between Epidemiology and Demography* (pp. 118-139): Annals of the New York Academy of Sciences.
- Groupe Consultatif Actuariel Europeen (2001), "Actuarial Methods and Assumptions used in the Valuation of Retirement Benefits in the EU and other European Countries." Working Paper, Edited by David Collinson.
- Hollmann, F.W., T.J. Mulder, and J.E. Kallan (2000), "Methodology and Assumptions for the Population Projections of the United States: 1999-2100", *Population Division Working Paper No. 38*.
- Kannisto, V. (2000), "Measuring the Compression of Mortality", *Demographic Research*, 3.

- King, M. (2004), "What Fates Imposes: Facing up to Uncertainty", *The Eight British Academy Annual Lecture*.
- Lee, R. and Carter (1992), "Modeling and forecasting U.S. mortality", *Journal of the American Statistical Association* 87(14), 659-671
- Lee, R. (1998), "Probabilistic Approaches to Population Forecasting", *Population and Development Review*, 24.
- Lee, R. and S. Tuljapurkar (2001), "Population forecasting for fiscal planning: issues and innovations" in *Demographic Change and Fiscal Policy*, A. Auerbach and R. Lee (eds), chapter 2. Cambridge University Press, UK.
- OECD (2005), "Ageing and Pension System Reform: Implications for financial markets and economic policies", *Financial Market Trends*, Vol. 2005(1).
- Oeppen, J. and J.W. Vaupel (2002), "Enhanced: Broken Limits to Life Expectancy" *Science*, vol. 296, 1029-1031.
- Olshansky, S.J., D. Passaro, R. Hershow, J. Layden, BA. Carnes, J. Brody, L. Hayflick, RN. Butler, DB Allison, DS. Ludwig, DS. (2005), "A Possible Decline in Life Expectancy in the United States in the 21st Century" *New England Journal of Medicine* 352:1103-1110
- Siegel, J. (2005), "The Great Debate on the Outlook for Human Longevity: Exposition and Evaluation of Two Diverging Views", *Society of Actuaries*.
- The Actuarial Profession and Cass Business School (2005), "Mortality research project - Mortality assumptions used in the calculation of company pension liabilities in the EU".
- Wong-Fupuy, C. and S. Haberman (2004), "Projecting Mortality Trends: Recent Developments in the United Kingdom and the United States", *North American Actuarial Journal*, 8 (2).

## TABLES AND FIGURES

Table 1. Life table, France 2003

Age	mx	qx	ax	lx	dx	Lx	Tx	ex
0	0.004	0.00394	0.06	100,000	394	99,630	7,957,453	79.57
1	0.000	0.00038	0.50	99,606	38	99,587	7,857,823	78.89
20	0.001	0.00066	0.50	99,187	66	99,154	5,968,537	60.17
30	0.001	0.00080	0.50	98,530	78	98,491	4,979,901	50.54
40	0.002	0.00170	0.50	97,441	165	97,358	3,999,349	41.04
50	0.004	0.00440	0.50	94,781	417	94,573	3,036,208	32.03
60	0.008	0.00794	0.50	89,416	710	89,062	2,112,789	23.63
70	0.017	0.01673	0.50	80,010	1,339	79,341	1,260,628	15.76
80	0.048	0.04669	0.50	60,702	2,834	59,285	545,424	8.99
90	0.170	0.15647	0.50	24,949	3,904	22,997	106,062	4.25
100	0.447	0.36562	0.50	1,520	556	1,242	3,069	2.02
109	0.751	0.54616	0.50	6	3	5	8	1.31
110+	0.778	1.00000	1.29	3	3	4	4	1.29

Source: Human Mortality Database (<http://www.mortality.org/index.html>).

Notes: 1. Selected ages from table period 1x1 (age by year).

mx is the per-capita annual death rate at age x.

qx is the probability of dying at age x.

ax is the mean number of person-years lived in the interval by those dying in the interval. It indicates when in the interval people dies (e.g. beginning, middle, end).

lx is the number of survivors at age x of a hypothetical cohort of 100,000 individuals.

dx is the number of deaths in the cohort between two consecutive ages.

Lx is the number of person-years lived at age x.

Tx is the total person-years remaining at each age x.

ex is life expectancy at age x.

**Table 2. Life Expectancy, 1900-2000**

Increase in number of years per decade

	at birth	at 65
<i>20th Century</i>		
Canada (1921-2002)	2.8	0.7
France (1900-2003)	3.3	0.9
The Netherlands (1900-2003)	2.9	0.6
Italy (1900-2002)	3.8	0.9
Spain (1908-2003)	3.9	0.9
Sweden (1900-2004)	2.7	0.6
United Kingdom (1900-2003)	3.1	0.7
United States (1933-2002)	2.3	0.8
<i>1960-2000</i>		
<i>EU15 Average<sup>1</sup></i>	2.0	
<i>OECD Average<sup>1</sup></i>	2.2	
Canada	1.9	1.0
France	2.2	1.3
The Netherlands	1.3	0.7
Germany	2.0	
Italy	2.6	1.2
Spain	2.5	1.1
Sweden	1.7	1.0
United Kingdom	1.7	0.9
United States	1.8	0.9

Source: Human Mortality Database and OECD Health Data 2005.

1. Unweighted average.

**Table 3. Comparing realized gains in life expectancy at birth with past projections (years)<sup>1</sup>**

	UN	Eurostat
OECD Average	0.8	
EU15 Average	0.7	0.4
Canada	0.2	
France	0.6	-0.3
Germany	0.6	0.3
Italy	1.1	0.7
Japan	1.5	
Mexico	1.9	
United Kingdom	0.5	-0.1
United States	-0.2	

Source: UN (1999), Eurostat (2000), OECD 2005 Health Data.

(1) A positive sign means that life expectancy in 2003 has already by passed projected life expectancy for the average 2000-2005 (UN) and 2005 (Eurostat).

**Table 4. Comparing past with projected gains in life expectancy at birth**

	In number of years per decade		Difference (B)-(A)
	(A) average gains 1960-2000	(B) projected gains 2000-2050 <sup>1</sup>	
<i>EU15 Average</i>	2.0	1.2	-0.8
<i>OECD Average</i>	2.2	1.2	-0.9
Canada	2.0	0.9	-1.1
France	2.2	1.8	-0.4
Germany	2.0	1.2	-0.8
Italy	2.4	1.8	-0.6
Mexico	4.1	1.2	-2.9
United Kingdom	1.8	1.6	-0.2
United States	1.7	1.4	-0.3

Source: OECD/DELSA Population database, OECD Health Data and Eurostat EUROPOP2004.

**Table 5 The increase in the net present value of annuity payments<sup>1</sup>**  
(percentage increase)

Age in 2005					Hypothetical pension fund		
25	40	55	65	70	(1)	(2)	(3)
23.6	15.3	7.3	3.3	2.4	10.4	9.6	8.2

Source: OECD calculations.

Notes: 1. Increase resulting from comparing the net present value at 2005 of annuity payments from 2005 'till 2090 when life expectancy at birth improves by 1.2 years per decade and life expectancy at 65 by 0.8 years per decade, with the NPV at 2005 of annuity payments when the latest available mortality tables (2005) are used without allowing for improvements in mortality.

(1) Membership structure in 2005: 65% aged 25-49; 20% aged 50-59; 10% aged 60-69; and 5% aged 70 or more.

(2) Membership structure in 2005: 60% aged 25-49; 20% aged 50-59; 15% aged 60-69; and 5% aged 70 or more.

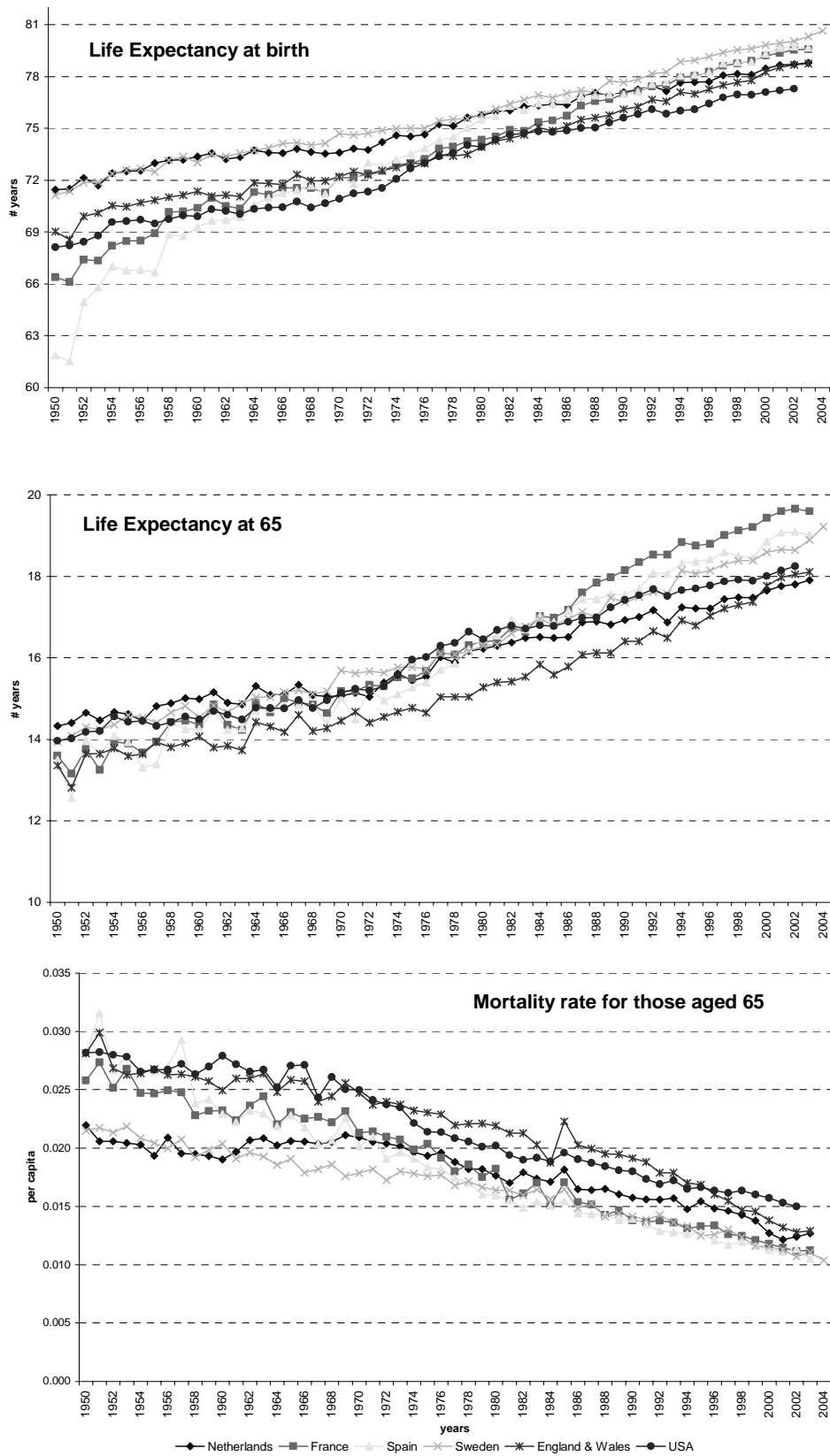
(3) Membership structure in 2005: 50% aged 25-49; 20% aged 50-59; 20% aged 60-69; and 10% aged 70 or more.

**Table 6. Impact of longevity improvements and changes in interest rates on annuity payments**  
percentage change in the net present value of annuity payments, 2005-2090

Improvements in life expectancy	Interest rates		
	3.5	4.5	5.5
No improvements, latest available mortality table used (2005)			
individual aged 65 in 2005	118.6	108.6	100.0
individual aged 25 in 2005	254.6	158.9	100.0
Life expectancy improves by 1.2 years per decade			
individual aged 65 in 2005	122.3	111.6	102.4
individual aged 25 in 2005	312.7	192.6	119.8

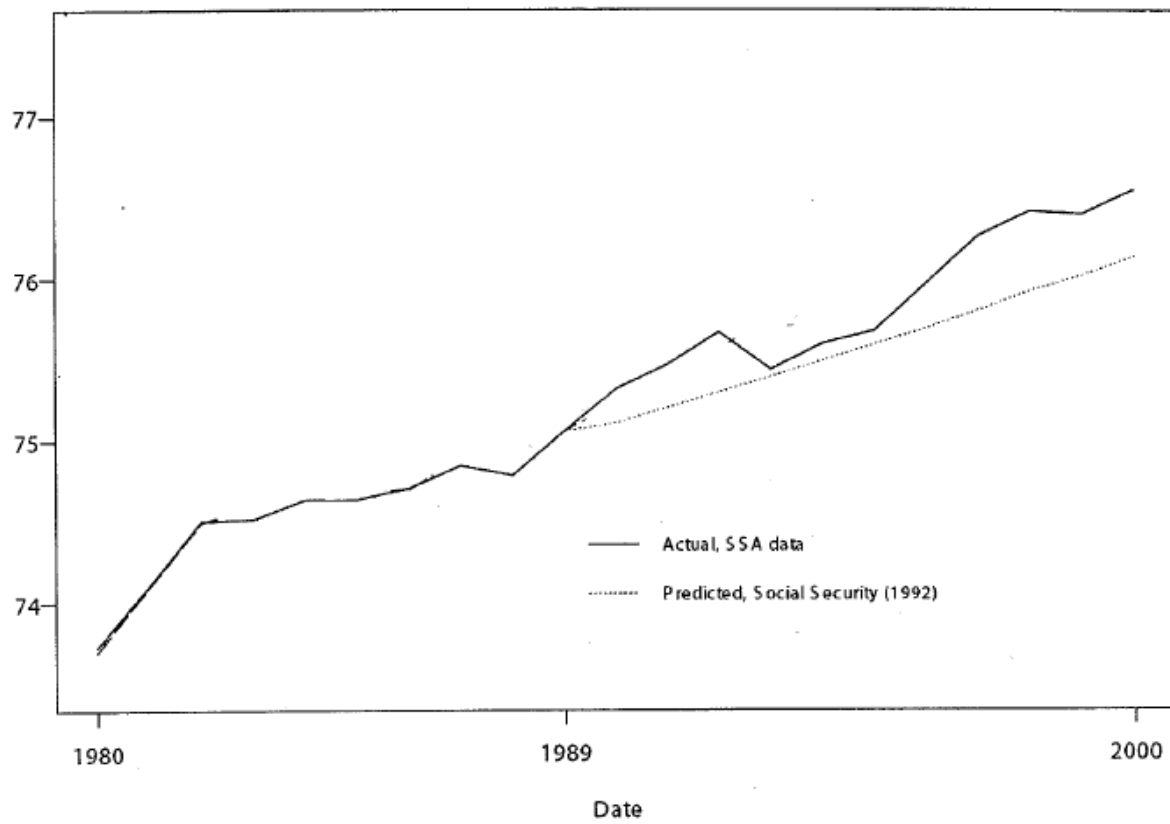
Source: OECD.

Figure 1. Life Expectancy and mortality rates in selected OECD countries, 1950-2003



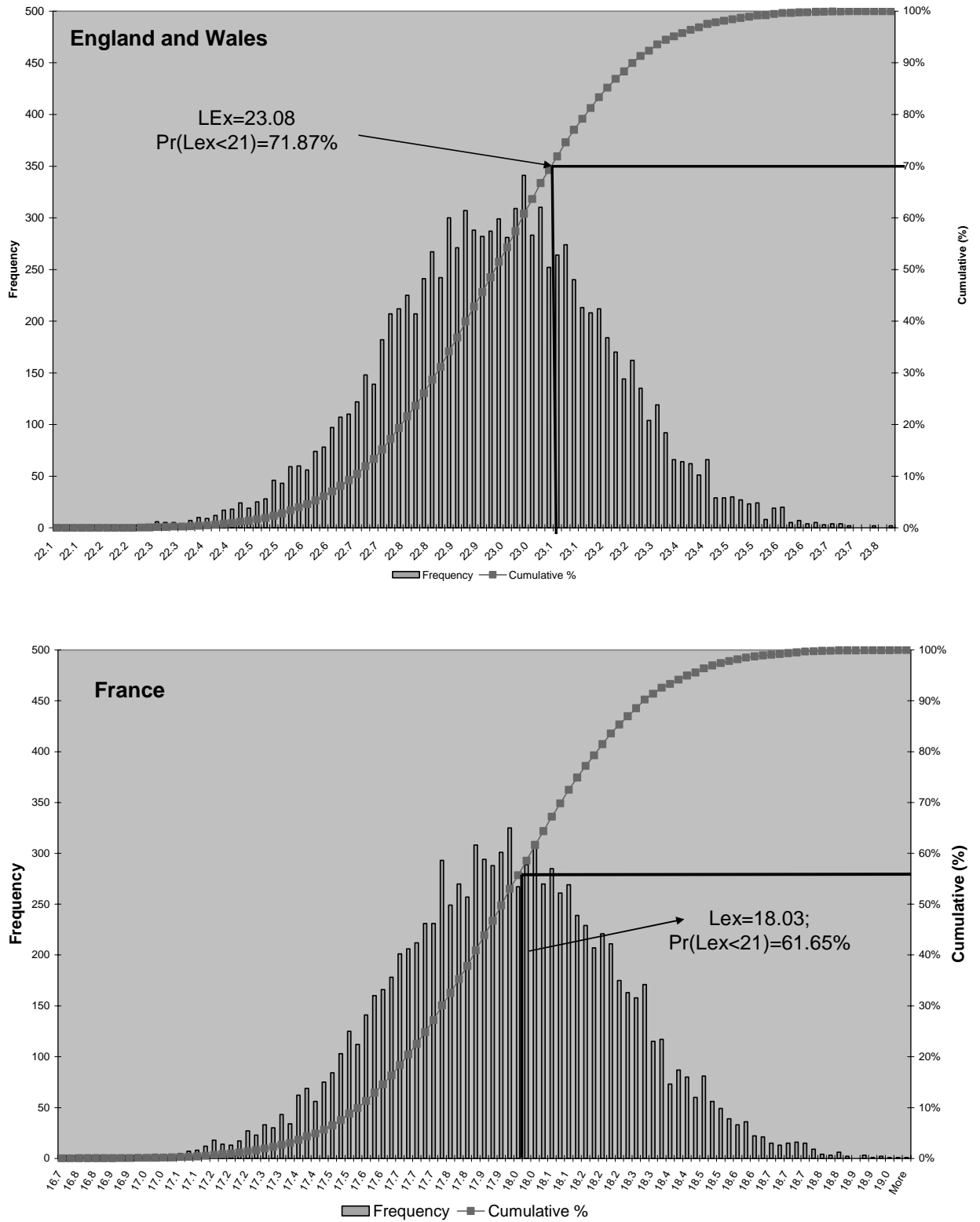


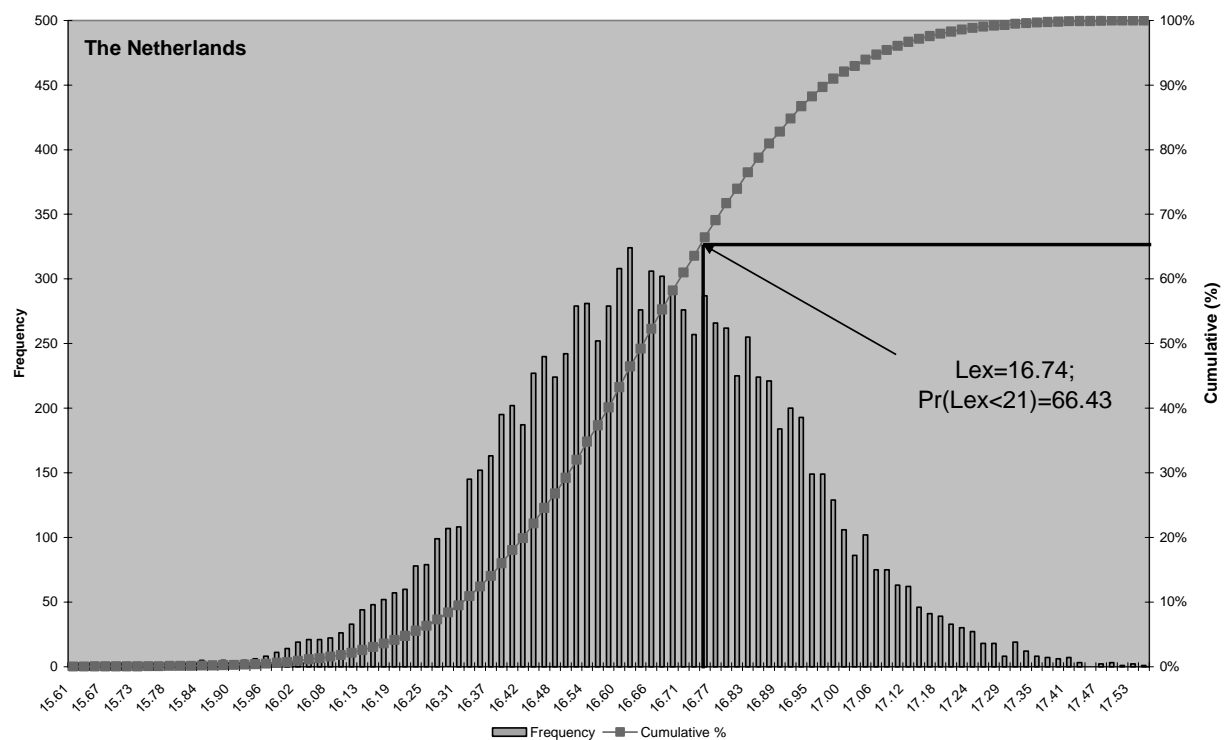
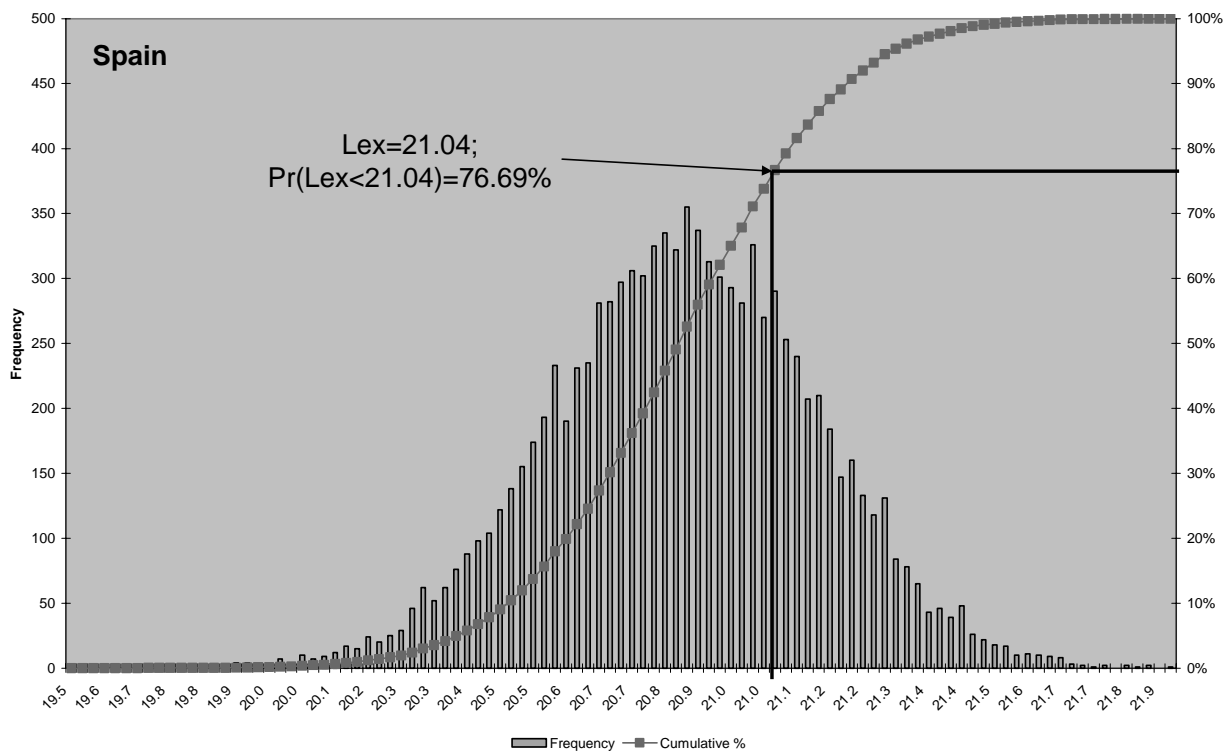
**Figure 2. Life expectancy projections by the United States Social Security Administration and actual outcomes**

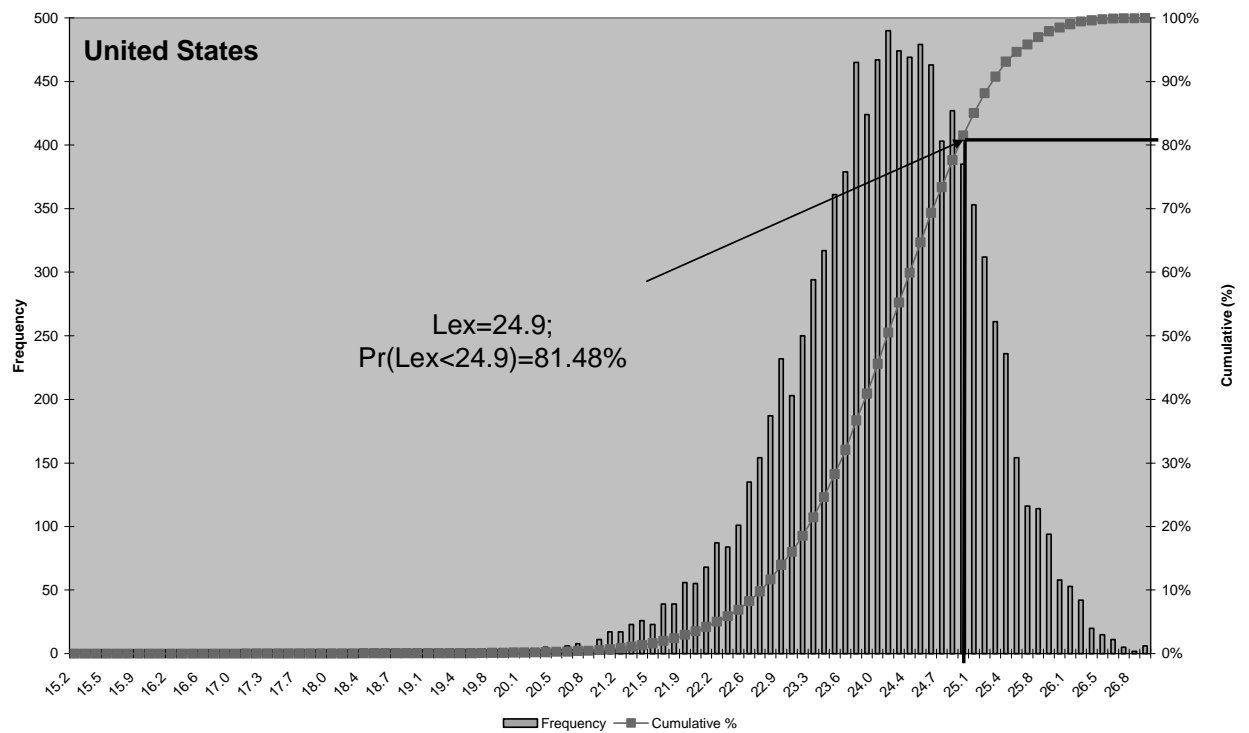
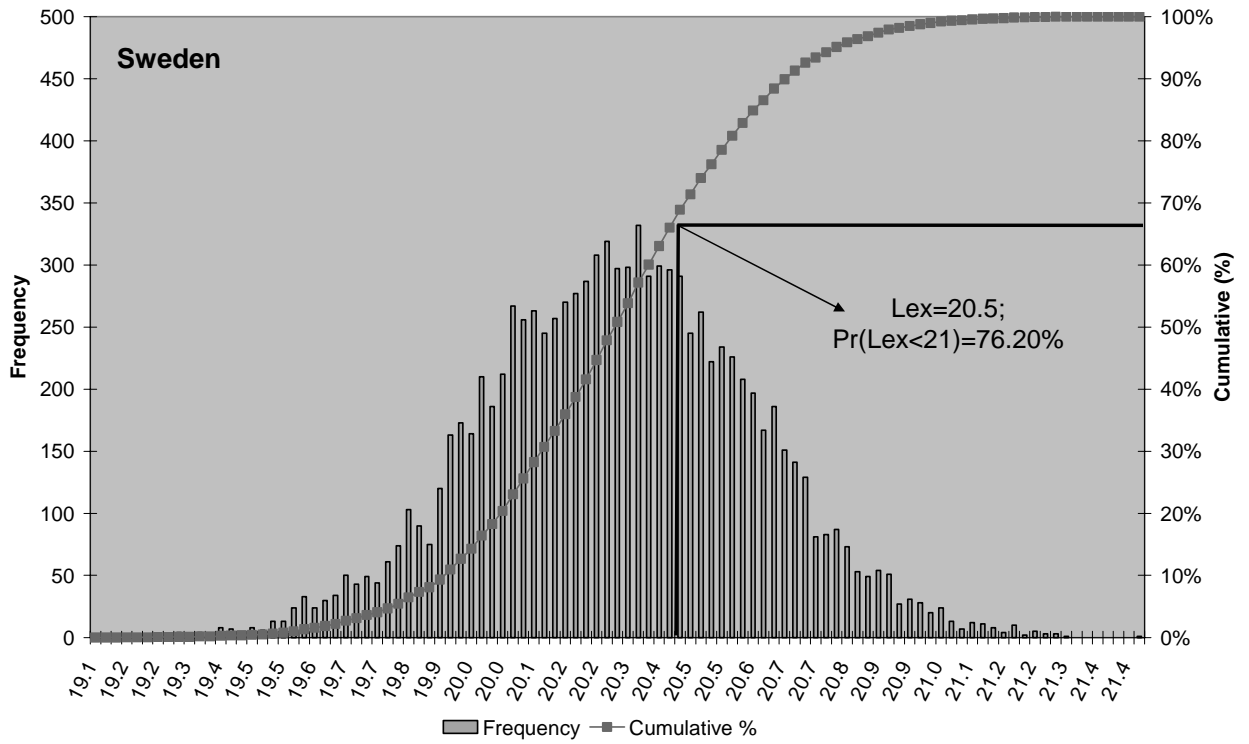


Source: Lee and Miller (2000); Lee and Tuljapurkar (2001).

**Figure 3. Life Expectancy at age 65 in 2050**  
10,000 Monte-Carlo simulations







Notes: Lex stands for life expectancy at age 65 in 2050.