Statistical properties of firm growth rates

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The “small shocks, large business cycle” (B. Bernanke, late 2000s chairman of the Fed) puzzle:

• The fluctuations of large economic systems should “average out” idiosyncratic shocks (CLT-style arguments).
• Aggregate volatility however remains very high (US GDP growth since 1950 ~3% ± 2.5%).

How can we try to explain this?

The goal of this talk: look at the empirical facts describing firms growth rates, using various data sources (French INSEE, CRSP, CompuStat, FactSet...).
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There is **no such thing as a representative firm**!

The fluctuations of aggregates of these quantities need not average out quickly! (X. Gabaix’ granularity)
Gibrat’s model

Take $S_t^i$, the size of firm $i$ at time $t$. Growth is modeled as

$$S_{t+1}^i = (1 + g_t^i) S_t^i$$

with

$$g_t^i \simeq \log(1 + g_t^i) = \log \left( \frac{S_{t+1}^i}{S_t^i} \right)$$

the firm’s growth-rate.

Under **mild assumptions** (bankruptcy probability, commercial exchange...) this model can reproduce **power-law distributed firm sizes**.
Understanding fluctuations

We rewrite things as

\[ g_t^i = \mathbb{E}(g_t^i) + \sigma_i \tilde{g}_t^i \]

Avg. growth

Fluctuations

\[ \sigma_i \simeq \text{std}(g_t^i) \]

Scale of the fluctuations (volatility)

\[ \tilde{g}_t^i = (g_t^i - \mathbb{E}(g_t^i))/\sigma_i \]

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Scale of the fluctuations (volatility)

“Randomness”

Known facts: \( \mathbb{E}(g_t^i) \) is large for small, young firms, but then is pretty much the growth of the whole economy.

We focus on the fluctuations.
The rescaled growth rates \( \tilde{g}_t^i = (g_t^i - \mathbb{E}(g_t^i))/\sigma_i \) have a **gaussian central region**, and their distribution seems to be **universal**.

Dynamically, they show persistent **trend following** (autocorrelation) up to about \(~10\) years.
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How to account for remaining volatility?

Hint: correlations and heterogeneity!
Firms' growth rates fluctuations $\sigma_i$ have very different values!

Empirically, they depend on:

- The size of the firm (bigger firms tend to fluctuate less) $\sigma \propto S^{-0.2}$,
- Its activity (sector),
- Its network properties (firms very dependent on a few, key suppliers may be very volatile!).

![Chart showing yearly growth-rate standard deviations for different sectors](chart.png)
Heterogeneous volatility

Aggregating (even gaussian) variables with heterogeneous scalings can produce **fat-tailed variables**.
Cross-firm correlations

Something still left to do: understand cross-firms correlations.

Intuitively: some firms depend heavily on others for their production.

Their growth rates may be correlated and thus one may see large avalanches of positive/negative fluctuations propagating along the supply-chain. (T. Dessertaine’s talk)
What to draw from this?

• Increasing amount of available data shows statistical regularities implicating common mechanisms that govern the life, growth and death of firms. (nearly all of our observations hold in US & French data!)

• This advocates in favor of empirically-based industrial dynamics, where data precedes and trumps theory.

• All in all, this would allow for a better understanding of how large macroeconomic fluctuations emerge from small idiosyncratic fluctuations.
Thank you!

Questions?