Scenarii for endogenous liquidity crises

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An example of liquidity crisis: the flash crash

Figure: Price of the SPMini future contract during the flash crash, 6th of May 2010
The study of Joulin et al.\textsuperscript{1} shows that anomalous price movements ($> 4\sigma$) on the scale of one minute:

- Only 5\% are news related (i.e. exogenous).
- 95\% are not news related (i.e. endogenous).

Necessity to have a feedback in the price formation mechanism in order to take into account the endogeneity!

Example of feedback in a physical system

Figure: Flock of birds: collective behaviour emerging from a zero-intelligence feedback.
Definition of the limit order book

Continuous double auction markets - the limit order book

Bid (buy orders)
- Limit orders (rate $\lambda^f_t$)
- Market orders (rate $\lambda^m_t$)

Mid-price $p_t$

Ask (sell orders)
- Cancellations (rate $\lambda^c_t$)

Volume

Spread $S_t$

Price p/share

Liquidity: Number of orders in the order book.
Rate: Number of orders per unit time.
Limit order: Buy or sell the item at its specified price.
Market order: Buy or sell the item immediately, at the current best price.
Figure: Example of a real order book.
Necessity to put some feedback of past price on the price formation process. Possible mechanism of feedback:

\[ \lambda_t^c \propto \lambda_0^c + \alpha_K \left( \int_0^t \sqrt{2t} e^{-\beta(t-s)} dp_s \right)^2 \]

Feedback on past price changes on cancelations:
We would like to motivate this choice of feedback by looking at the data.

We aim to fit the following rate of events:

$$\lambda_t \propto \text{base rate} + \text{past trend contribution} + \text{square past trend contribution}$$  \hspace{1cm} (1)

**Take home message**

The **past square trend** diminishes the **future liquidity** equally at the bid and at the ask. This effect is mainly in the **cancelations**.

The **past trend** has a slight effect on the bid-ask imbalance (*i.e.* difference of liquidity between the bid and ask).
## Numerical simulations

- **N** size of the system *i.e.* the number of available price levels.
- **T** time of simulation.
- **$\alpha_K$ feedback intensity:**
  \[
  \lambda_t^c \propto \alpha_0^c + \alpha_K \left( \int_0^t \sqrt{2\beta} e^{-\beta(t-s)} dp_s \right)^2.
  \]
- **$1/\beta$:** time scale of the feedback.

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**Video of an order book**

We measure the time $\tau_c$ of first liquidity crisis: first time when one side of the order book empties.
Figure: Snapshots of the order book. The left figure is taken at the beginning of the simulation and the right figure is taken during a period of high volatility. We can see that the liquidity almost dries out.
Figure: Stability map: Crisis probability $\mathbb{P}[\tau_c \leq T]$ for $T = 200$, $N = 280$, $\lambda_0^\ell = 10$, $\lambda_0^G = 1$ and $\lambda_0^m = 20$. The blue region corresponds to a stable order book, whereas the red region corresponds to liquidity crises.

Numerical simulations show that we have an exact phase transition. For an infinite order book ($N = T = +\infty$), if $\alpha_K < \alpha^*$ there is no crisis and if $\alpha_K > \alpha^*$ there are crises. $\alpha^*$ is the critical point.
To make this scenario consistent, we have two possibilities:

- Real financial markets would have to sit below, but very close to the critical point and his critical point is attractive (see Self-Organized Criticality).

- The feedback parameters $\alpha_K$ is time dependent and occasionally visit the unstable phase.
Key ingredient: **spread opening events trigger more spread opening events.**

Model definition:

- Number of spread opening events before $t$: $S_t^+$
- Rate of spread closing event: $\lambda_0^-$
- Rate of spread opening event: $\lambda_t^+ = \lambda_0^+ + \epsilon X_t^2 = \text{base rate} + \text{feedback}$
- Number of spread opening events on time scale $1/\beta$: $X_t := \int_0^t \beta e^{-\beta(t-s)} dS_s^+$.

Let's call $\tau_c$ the time of liquidity crisis.

- If $\epsilon > 0$ there is always a liquidity crises.
- If $\epsilon > 0$ is small enough, before the liquidity crisis the spread seems stable: it is **metastable**.
- $\log \mathbb{E}[\tau_c] \approx \frac{1}{\beta \epsilon} \log \frac{1}{\epsilon \lambda_0^+}$ for $\epsilon > 0$ small enough.
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Figure: Set of parameters: $\lambda_0^+ = 1$, $\lambda_0^- = 0.5$ and $\epsilon = 0.2$. (a) Survival function (sf) of the time of metastability which is found to be exponential. (b) Evolution of the average metastability time with $\epsilon$. The dotted red curve is the continuous time prediction. The plain red curve is obtained by multiplying the term in the exponential by a empirical factor 2.5. (c) Typical metastable trajectory.
Physical interpretation of metastability: the "particule" tries to minimize $V$. When it is trapped in $X_{eq}$, it needs a fluctuation large enough to cross the barrier $V(X^*) - V(X_{eq})$ and go beyond $X^*$.
Conclusions
On empirical data, the past square trend diminishes the future liquidity.
Two scenarii of liquidity crises: one from a phase transition, the other from a metastable description.

Outline
We could also design an empirical test that could help discriminating between the second order phase transition and activation scenarii.
We also would design a complete protocol to estimate the price feedback and use it to predict liquidity crises.

Thank you!