Anomalous Market Impact and Agent Based Models

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Price Impact

What is price impact?

► Price impact = correlation between an arriving order and the subsequent price change
► Buy/sell trades push the price up/down – on average
► This is highly relevant:
  > Allows information to be included in prices
  > Induces extra execution costs
    – large but often overlooked
  > Makes marked-to-market valuation over-optimistic
  > Can lead to crashes
    – the impact of a trade can trigger other trades
Intuition: price impact is inversely related to market liquidity

Liquidity? What liquidity?

- Immediate liquidity at any given moment is small, and affected by tick size, priority rules, fees, market makers, HFT, etc.
  - For a liquid small tick stock the instantaneous volume at best is approx. $10^{-5}$ of market cap., while the total daily traded volume is 500 times larger
- Most of the available volume is “latent”, and only progressively gets revealed during the day
- Large trades are sliced/diced and executed incrementally (target VWAP)
  - What is the (average) impact of a “metaorder” of size $Q$?
Impact of Metaorders

A universal empirical result?
Independently but consistently reported by many groups since the mid-eighties (Loeb 83, BARRA 95, Almgren 05, Engle, Kissel, JPM, DB, LH, CFM)

A metaorder of size Q has a sqrt price impact:

\[ I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}} \]

where:
- \( Q \) is the volume of the metaorder
- \( \sigma_T \) is the volatility of the market
- \( V_T \) is the total volume traded in the market (\( Y \) of order 1)

Important note:
- Impact is usually small compared to vol itself
- Requires a lot of averaging to be seen
Impact of Metaorders

A universal empirical result? (CFM)

\[ I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}} \]
Remarkable stability of results:

- Style of trading, strategies, markets, period (1980 – 2016), tick sizes, treatment of data etc.
- Hints that microstructure and HFT effects are not relevant, only “macro-liquidity”
- Impact is, to a first approximation, independent on the time to complete the metaorder (!), only depends on Q
- Impact is non-linear even for $Q << V_T$

A genuine “physical law” of financial markets?

Understanding why is important both conceptually and for applications

\[ I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}} \]
A theoretical model for latent liquidity

An “Agent Based” numerical model and a stylized theory

► People randomly send buy/sell orders
► These orders are “eaten” by transactions
► Realistic statistics for order flow
► No “fundamental” price, no fancy behavioral assumptions
  > Only random walks and random flows
  > The probability to find an unexecuted order close to the price is linearly small
  > Hence: $Q = \Delta p^2$
  > A universal theory for a universal law

\[
\rho_{st}(u) = \rho_\infty \left(1 - e^{-u/u*}\right); \quad u^* = \sqrt{\frac{\sigma^2}{2v}}
\]

\[
\frac{\partial \rho_{A,B}}{\partial t} = \frac{\sigma^2}{2} \frac{\partial^2 \rho_{A,B}}{\partial u^2} + \lambda - v \rho_{A,B}
\]
Direct comparison with Bitcoin

\[
\frac{\partial \rho_{A,B}}{\partial t} = \frac{\sigma^2}{2} \frac{\partial^2 \rho_{A,B}}{\partial u^2} + \lambda - \nu \rho_{A,B}
\]

J. Donier (2015)
Results of the model

Results: exact square root impact, decay of impact

\[ \frac{\partial \rho_{A,B}}{\partial t} = \frac{\sigma^2}{2} \frac{\partial^2 \rho_{A,B}}{\partial u^2} + \lambda - \nu \rho_{A,B} \]
So what?

1: The true cost of trading

Naïve answer: the bid-ask spread (sensitive to microstructure, etc.)

True for small trades, but as size grows, impact costs quickly dominate (although often disregarded)

Orders of magnitude: for $Q=2\%$ of daily volume and $2\%$ vol:

\[ \text{Cost} = 1 \text{ bp} + 0.5 \times 2\% \times \sqrt{1\%} = 1 + 14 \text{ bp} \]

Impact is dominated by “true liquidity” and very little by microstructure

- Complaints about HFT have misplaced focus: impact is unavoidable and much larger than spreads. Dark pools cannot alleviate impact
- Irreducible impact—induced « haircut » to marked-to-market valuation: a long stock position on $1\%$ of market cap must be marked down by $\sim 1\%$!
So what?

2: Intrinsic Market Fragility

Broader Consequences for Market stability/fragility

► Liquidity at the best price is vanishingly small (it is eaten by the diffusive motion of prices)

► This imposes a splitting up of metaorders...

► ...and leads to an anomalously large impact for small trades

► Liquidity fluctuations are bound to play a crucial role: Micro-crises and jumps in prices without news, as indeed seen empirically ever since markets exist

► Beware marked-to-market valuations impact-induced spirals (e.g. the « Quant Crunch »)