Betting on Death and Capital Markets in Retirement:

A Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans

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Pension and Long-Run Investment
Paris
Three Uncertainties in Retirement: A Financial Perspective

Investment Returns?

Bequest?

Investment horizon?

Source: Die Zeit
"Rente“=Retirement / „Ziel“=Goal
The Benchmark Life Annuity

• Characteristics
  ➢ Constant (real) annuity payments until death
  ➢ Pooling of longevity risk
  ➢ Offered by commercial insurance companies
  ➢ No bequest potential, low flexibility

• History
  ➢ In 1308 the convent of St. Denis sold the archbishop of Cologne a life annuity paying 400 Livres p.a. for a single premium of 2,400.
  ➢ Important financial instrument during the middle ages

• Present Relevance
  ➢ Thin private annuity markets around the world
  ➢ Also countries with substantial DC-pension plans
Life Annuity Benefits: Using German data

<table>
<thead>
<tr>
<th>Mortality Table</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retirement Age</td>
<td>(Real) Life Annuity € p.a. for € 100 premium</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>5.82</td>
<td>5.02</td>
</tr>
</tbody>
</table>

Immediate Annual Life-long Real Annuity Benefits per EUR 100 Single Premium: Total Expense Loadings 2.785%; (Real) Discount Factor 1.5%; German annuitant mortality tables

➤ Mortality “drag” at the cost of no bequest potential
Phased Withdrawal Plans

• Retirement assets invested in Individual Pension Account
  ➢ Asset Allocation ?

• Retiree consumes from the IPA periodically
  ➢ Withdrawal Rule ?

• Advantages compared to Life Annuity
  ➢ High flexibility, liquidity
  ➢ Bequest potential
  ➢ Higher benefits

• Risks of Phased Withdrawal Plans
  ➢ Lower benefits than Life Annuity  ➔ Consumption Shortfall
  ➢ Longevity risk (No risk pooling)  ➔ “Betting on Death”
  ➢ Capital market risk  ➔ “Betting on Capital Markets”
Phased Withdrawal Plans

**Types of Withdrawal Plans**

- **Fixed Withdrawals**
  - constant
  - increasing
  - decreasing
  Amount in EURO

- **Asset Allocation**
  - Stocks
  - Bonds
  - Cash
  - Mixed

- **Variable Withdrawals**
  - constant
  - increasing
  - decreasing
  Benefit-to-wealth ratio
Phased Withdrawal Rules

- **Fixed Benefit Rule:** $10,000 p.a. (or same benefits as life annuity)

- **Fixed Percentage Rule:**
  - Constant benefit to wealth ratio, e.g. 5% p.a. of remaining wealth

- **1/T Rule:** Withdrawal fraction set to maximum plan duration $T$ (= 110)
  - Increasing benefit-to-wealth ratio

- **1/E[$T(x)$] Rule:** Withdrawal fraction is updated by retiree’s remaining life expectancy (according to a mortality table)
Research Approach

• Evaluate these different strategies against life annuity benchmark
• Stochastic Model (mortality / investments)
• Possible objective functions
    – Only look at shortfall probability
    – Only examine withdrawal plans with fixed benefits
  ➢ Specific utility functions (Blake, Campbell/Viciera)
    – Must assume exact risk preferences, but…
Our Contributions

➔ Using risk value models:
  ➢ Our risk measure incorporates both probability and size of loss
  ➢ Compare fixed with different variable withdrawal rules
  ➢ Optimize asset allocation
  ➢ Optimize design parameters of variable payment schedule
  ➢ Study portfolios of withdrawal plans and annuities
Withdrawal plans: Risk-Minimizing Investment Allocation

- Objective function:

\[ EPV_{\text{Shortfall}} = \sum_{t=1}^{T} t p_x \frac{E[\max(z - B_t, 0)]}{(1 + r)^t} \]

- This risk measure accounts for:
  - Mortality risk
  - Time preferences
  - Risk preferences for investment uncertainty

- Vary investment mix and withdrawal fraction to minimize Expected PV of Shortfall
Optimized Withdrawal Rules in Risk-Return Context

✓ EPV_Benefits reflects expected present value of benefit payments conditional on survival:

\[
EPV_{\text{Benefits}} = \sum_{t=1}^{T} \frac{t \cdot p_x \cdot E(B_t)}{(1+r)^t}
\]

✓ EPV_Bequest measures expected present value of inheritance the retiree passes to heirs in the event of death:

\[
EPV_{\text{Bequest}} = \sum_{t=1}^{T} \frac{t-1 \cdot p_x \cdot q_{x+t} \cdot E(V_t)}{(1+r)^t}
\]
## Optimization Results:
**“Stand Alone Withdrawal Rules”**

### Results for Male (Retirement Age 65):

**Benchmark Real Life Annuity €5.82 p.a./ €100**

<table>
<thead>
<tr>
<th>Strategy</th>
<th>EPV Shortfall</th>
<th>EPV Benefits</th>
<th>EPV Bequest</th>
<th>Investment Weights (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>Real Annuity €5.82</td>
<td>0</td>
<td>97.29</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Fixed Benefit = €5.82</td>
<td>3.58</td>
<td>93.41</td>
<td>53.19</td>
<td>30</td>
</tr>
<tr>
<td>Fixed Pct. = 5.82%</td>
<td>12.58</td>
<td>92.53</td>
<td>66.06</td>
<td>50</td>
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<tr>
<td>1/T Rule Age 110</td>
<td>34.95</td>
<td>82.68</td>
<td>134.41</td>
<td>50</td>
</tr>
<tr>
<td>1/E(T) Rule</td>
<td>8.27</td>
<td>103.08</td>
<td>39.80</td>
<td>20</td>
</tr>
</tbody>
</table>

**Benefits from Withdrawal Plan**:

65

age
### Results for Male (Retirement Age 65 Switching Age 75):
*Benchmark Real Life Annuity €5.82 p.a./€100*

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<thead>
<tr>
<th>Strategy</th>
<th>EPV Shortfall</th>
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<td></td>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>Real Annuity €5.82</td>
<td>0</td>
<td>97.3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Fixed Benefit until 85</td>
<td>2.8</td>
<td>103.4</td>
<td>33.5</td>
<td>15</td>
</tr>
<tr>
<td>Fixed Pct. Opt ω=7.4%</td>
<td>7.4</td>
<td>108.8</td>
<td>32.3</td>
<td>25</td>
</tr>
<tr>
<td>1/T Rule Opt Age 88</td>
<td>9.5</td>
<td>108.3</td>
<td>35.1</td>
<td>20</td>
</tr>
<tr>
<td>1/E(T) Rule</td>
<td>5.4</td>
<td>104.1</td>
<td>31.2</td>
<td>15</td>
</tr>
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Portfolio of Phased Withdrawal Plan and Deferred Life Annuity starting at Age 85

Results for Male (Retirement Age 65 Switching Age 75):
Benchmark Real Life Annuity €5.82 p.a./€100

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<th>Investment Weights (in %)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>Real Annuity 5.828</td>
<td>0</td>
<td>99.0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>Fixed Payment until 85</td>
<td>5.3</td>
<td>100.0</td>
<td>34.4</td>
<td>79</td>
</tr>
<tr>
<td>Fixed Perc. opt. 9.1%</td>
<td>13.4</td>
<td>110.1</td>
<td>33.7</td>
<td>50</td>
</tr>
<tr>
<td>1/T-Rule (T=84)</td>
<td>10.0</td>
<td>110.2</td>
<td>21.2</td>
<td>68</td>
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<tr>
<td>1/E(T)-Rule</td>
<td>14.6</td>
<td>111.9</td>
<td>37.7</td>
<td></td>
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</table>
Conclusions

• Phased withdrawal plans offer many advantages: flexibility, bequests, and possibly higher consumption than life annuities.

• Yet a phased withdrawal plan also requires that attention be devoted to asset allocation and withdrawal rules.

• To minimize the shortfall-risk of consuming less than a real annuity benchmark, retirees should invest their assets more in fixed income than in equities.

• For a fixed withdrawal rule compared to no annuity:
  – Mandatory deferred annuitization and/or a switching rule can enhance expected payouts & cut expected shortfall risk
  – But at cost of reduced bequests.
“The secret to living well is to die without a cent in your pocket”
“But I seem to have miscalculated”

Source: Financial Times