Climate dynamics and dynamical system theory

Linear Single Degree of Freedom Oscillator

\[ f(t) \]

\[ x(t) \]

\[ m \]

\[ k \]

\[ c \]

\[ m = \text{mass} \]

\[ c = \text{damping} \]

\[ k = \text{stiffness} \]

\[ \ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = f(t) \]

\[ \omega_n^2 = \frac{k}{m} \quad \text{Natural frequency of the undamped system} \]

\[ 2\zeta \omega_n = \frac{c}{m} \quad \text{Damping factor} \]

Climate system as Forced Nonlinear Dampened System (thus leading to chaos)

\[ \ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x(x + ax^3) = f(t) \]

Problem: model formulation \(\Rightarrow\) to specify equation (●) for the considered system (e.g. the precipitation-runoff transformation)

Water resources system model as forced nonlinear dampened system

\[ V, \text{snow and ice storage} \Rightarrow \text{dampening;} \quad T \text{ and } P \Rightarrow \text{forcings} \]

\[ V = V(p,Q) \]
\[ Q = Q(T,V) \]

[Perona & Burlando, *Climate Dyn.*, submitted 2006]

\[ \frac{dV}{dt} = p - Q \]
\[ \frac{dQ}{dt} = c_1 V + c_2 VQ + c_3 VP + c_4 VT + c_5 P + c_6 VT^2 + c_7 Q \]

Transforming the equation system into a 2nd order ODE

\[ \ddot{V} - \left( c_7 + c_2 V \right) \dot{V} + \left[ c_1 + \left( c_2 + c_3 \right) p + c_4 T + c_6 T^2 \right] V = \dot{p} - \left( c_5 + c_7 \right) p \]

\[ \ddot{x} + \zeta(x) \dot{x} + \omega_n^2(t) x = f(t) \]

The \( c_1, \ldots, c_7 \) parameters are estimated using the trajectory method applied to a training set.
An example of application to the water resources of Aosta valley
Model performance

Reconstruction of the training set

[Perona & Burlando, AGU Hydrology, Days, 2006]
Model response to forcing

Recovering a stable dynamics

[Perona & Burlando, Climate Dyn., subm. 2006]
Model response to climate forcing

Hypothetical and IPCC scenarios forcing

(a) Q [10^6 m^3/day] vs Days

(b) V [10^6 m^3] vs Days

(c) Q [10^6 m^3/day] vs Julian day

(d) Q [10^6 m^3/day] vs V [10^6 m^3]

[Perona & Burlando, Climate Dyn., subm. 2006]
Model response to long-term forcing

Overlap of "Milankovitch type" of forcing with climate forcing ($P_{w} +25\%$, $P_{s} -25\%$, $T_{s} +4\,^\circ C$)

$T$ [1 yr]

$V$ [$10^6 m^3$]

$Q$ [$10^6 m^3$/day]

$t$ [days]

100 yrs
Model response to long-term forcing

Overlap of "Milankovitch type" of forcing with climate forcing ($P_w +25\%, P_s -25\%, T_s +4^\circ C$)

- $T$ 1 yr
- $V [10^6 m^3]$  :
- $Q$
- $V [day]$  :
- $t [days]$ 100 yrs
Concluding remarks

- Adaptation requires a sufficient knowledge and a limited uncertainty: these are, unfortunately, both lacking to the extent necessary to be convincing and put pressure on decision makers.

- Large scale/global adaptation should be replaced by prevention, although the climate system inertia may require as well some adaptation strategy.

- Local scale adaptation can take advantage of downscaled scenarios, which simulate the variability and the non-stationary character of the impact as opposed to (large scale) time slice steady state scenarios.

- Understanding whether the anthropogenic forcings may lead the climate system (or any of its subsystems) to new “stable” configurations may help identifying adaptation strategies.