On the Dynamics of Wage Distributions and Unemployment Volatility
Labour Market Dynamics with Sequential Auctions and Heterogeneous Workers

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Motivation
Heterogeneous ability

- Explore the role of heterogeneous worker ability in a search-matching model of unemployment and wages with aggregate productivity shocks.
- Recent literature:
  - Moscarini and Postel-Vinay (2008): wage posting model, heterogeneous firms, random search
  - Menzio and Shi (2009): wage posting, heterogeneous matches, directed search
- Our model: sequential auctions, heterogeneous workers, random search
  - wage posting implies strategic interactions
  - directed search implies perfect sorting
Main results

• Solves Shimer’s unemployment volatility puzzle (Mortensen-Pissarides cannot amplify productivity shocks enough).
• Helps to understand business cycle fluctuations of wage distributions
Plan

1. Theory
2. Calibration/estimation
3. Results
Theoretical Setup
Aggregate shocks

- Time is discrete and indexed by $t \in \mathbb{N}$.
- The global state of the economy is described by a Markov chain $y_t \in \{y_1 < \ldots < y_N\}$ with transition probability matrix $\Pi = (\pi_{ij})$.
- Aggregate shocks accrue at the beginning of each period.
Aggregate shocks

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There are $M$ types of workers and $\ell_m$ workers of each type (with $\sum_{m=1}^{M} \ell_m = 1$).

Each type is characterized by a time-invariant ability $x_m$, $m = 1, \ldots, M$, with $x_m < x_{m+1}$.
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Matches

- Workers are paired with identical firms to form productive units.
- The per-period output of a job, if the worker is of ability $x_m$ and aggregate productivity is $y_i$, is $y_i(m) = x_m y_i$.
- Matches form and break at the beginning of each period, after the new aggregate state has been revealed.
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Unemployment and job destruction

• Let \( u_t(m) \) denote the proportion of unemployed in the population of workers of ability \( x_m \) at the end of period \( t - 1 \). Let \( u_t = \sum_{m=1}^{M} u_t(m)\ell_m \) denote the aggregate unemployment rate.

• I denote as \( S_i(m) \) the surplus of a match \((x_m, y_i)\), that is, the present value of the match minus the value of unemployment and minus the value of a vacancy (assumed to be nil).

• Only matches with \( S_i(m) > 0 \) are viable.

• At the beginning of period \( t \),
  • A fraction \( 1\{S_i(m) \leq 0\}[1 - u_t(m)]\ell_m \) of employees is endogenously laid off.
  • Another fraction \( \delta 1\{S_i(m) > 0\}[1 - u_t(m)]\ell_m \) is exogenously destroyed.
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Meetings

• For simplicity, we assume that workers meet employers at exogenous rates. It is easy to work out an extension of the model with a standard matching function if necessary.

• Let $\lambda_0$ and $\lambda_1$ denote the respective search intensities of unemployed and employed workers.

• At the beginning of period $t$,
  • A fraction $\lambda_0 1\{S_i(m) > 0\} u_t(m) \ell_m$ of unemployed workers meet an employer.
  • A fraction $\lambda_1 (1 - \delta) 1\{S_i(m) > 0\}[1 - u_t(m)] \ell_m$ of employed workers meet an alternative employer.
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Unemployment dynamics

• The law of motion of individual-specific unemployment rates is such that, at the end of period $t$,

$$u_{t+1}(m) = \begin{cases} 
1 & \text{if } S_i(m) \leq 0, \\
 u_t(m) + \delta(1 - u_t(m)) - \lambda_0 u_t(m) & \text{if } S_i(m) > 0.
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Rent sharing

• I use Postel-Vinay and Robin’s (2002) sequential auction model.

• Wage contracts are long term contracts that can be renegotiated by mutual agreement only.

• PVR add search frictions limited competition of employers for workers. In the basic version,
  • Employers have full monopsony power with respect to workers. Hence, unemployed workers are paid their reservation wage. Makes the value of unemployment easy to compute.
  • Yet, on-the-job search triggers Bertrand competition. Because firms are identical and there is no mobility cost, Bertrand competition transfers the whole surplus to the poached workers. Makes the surplus easy to compute.
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Renegotiation

- Let $i$ be a new aggregate state. With probability $\mathbb{1}\{S_i(m) > 0\}(1 - \delta)(1 - \lambda_1)$, the employee is not laid off and does not receive an outside offer.

- Still the wage can change if a productivity shock moves the current wage outside the bargaining set.

- We assume that the new wage contract is the closest point in the bargaining set (H&H, T&W, McLeod and Malcomson, 1993, Postel-Vinay and Turon, 2007).

- Essentially, we constrain wage rigidity in a way that is consistent with what we believe are the main lessons of the literature on self-enforced wage contracts and limited commitment (forgetting yet about risk aversion).
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The value of unemployment

- Let $U_i(m)$ denote the present value of remaining unemployed for the rest of period $t$ for a worker of type $m$ if the economy is in state $y_i$.
- An unemployed worker receives a flow-payment $z_i(m)$ for the period.
- The value of unemployment solves the following linear Bellman equation:

$$U_i(m) = z_i(m) + \frac{1}{1 + r} \sum_j \pi_{ij} U_j(m).$$

- At the beginning of the next period, the state of the economy changes to $y_j$ with probability $\pi_{ij}$.
- The continuation value is $U_j(m)$ whether the worker exits unemployment or not as she gets paid her reservation wage anyway.
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The match surplus

- The surplus of a match \((x_m, y_i)\) solves the following (nearly linear) Bellman equation:

\[
S_i(m) = y_i(m) - z_i(m) + \frac{1 - \delta}{1 + r} \sum_j \pi_{ij} S_j(m)^+, 
\]

where we denote \(x^+ = \max(x, 0)\).

- After a productivity shock from \(i\) to \(j\) the continuation surplus is zero if the match is destroyed.
- If the worker is poached, Bertrand competition transfers the whole surplus to the worker whether s/he moves or not.

- This nearly-linear system of equations can be numerically solved by value function iteration.
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The worker surplus

- Let \( W_i(w, m) \) denote the present value of a wage \( w \) in state \( i \) to a worker of type \( m \).

- The worker surplus, \( W_i(w, m) - U_i(m) \), satisfies the following Bellman equation:

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W_i(w, m) - U_i(m) = w - z_i(m) \\
+ \frac{1 - \delta}{1 + r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) > 0\} \left[ \lambda_1 S_j(m) \\
+ (1 - \lambda_1)(W_j^*(w, m) - U_j(m)) \right]
\]

where, after renegotiation, \( W_j^*(w, m) - U_j(m) = \)

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\begin{cases} 
0 & \text{if } W_j(w, m) - U_j(m) \leq 0 \\
S_j(m) & \text{if } W_j(w, m) - U_j(m) \geq S_j(m) \\
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For all aggregate states $y_i$ and all worker types $x_m$, there are only two possible (re)negotiated wages:

- **Starting wages**: Either the worker was offered a job while unemployed, and he can only claim $w_i(m)$ such that $W_i(w_i(m), m) = U_i(m)$ (his reservation wage).

- **Promotion wages**: Or he was already employed and he benefits from a wage rise to $\bar{w}_i(m)$ such that $W_i(\bar{w}_i(m), m) - U_i(m) = S_i(m)$ (the employer’s reservation value).
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Wage distributions

- The support of the wage distribution is the union of all sets $\Omega_m = \{w_i(m), \bar{w}_i(m), \forall i\}$.
- Let $g_t(w, m)$ denote the measure of workers of ability $m$ employed at wage $w \in \Omega_m$ at the end of period $t - 1$.
- The law of motion for $g_t(w, m)$ is obtained by equating inflows and outflows as for unemployment rates.
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Calibration and Results
Data

- Unemployment/employment series from BLS (overall and by unemployment duration)
- Productivity per person and real hourly compensation (renormalised per person) from BLS
- JOLTS
- CPS
Unemployment transitions (CPS - BLS)

- Estimate unemployment exit rate as
  \[ f = 1 - \frac{F.U - F.U5}{U}, \]
  where \( U5 \) is the number of unemployed with less than 5 weeks unemployment.

- Estimate job destruction rate as
  \[ s = \frac{F.U5}{E}. \]
Time aggregation

- Iterate law of motion of unemployment.
- For example,

\[
\begin{align*}
    u_{t+2} &= s_{t+1}(1 - u_{t+1}) + (1 - f_{t+1})u_{t+1} \\
    &= s_{t+1} + [1 - s_{t+1} - f_{t+1}]u_{t+1} \\
    &= s_{t+1} + (1 - s_{t+1} - f_{t+1})[s_t + (1 - s_t - f_t)u_t] \\
    &= s_{t+1} + \underbrace{(1 - s_{t+1} - f_{t+1})s_t}_{\bar{s}} + \underbrace{(1 - s_{t+1} - f_{t+1})(1 - s_t + f_t)u_t}_{\bar{f}}.
\end{align*}
\]

- Keep only first month of each quarter
Employment transitions (JOLTS)

- From $H$ (monthly hires), $S$ (separations) and $Q$ (quits), estimate

$$f_{JOLTS} = (H - Q)/U,$$
$$s_{JOLTS} = (S - Q)/E,$$
$$q_{JOLTS} = Q/E.$$
Unemployment exit rate

CPS, flows  JOLTS
Job destruction rate

CPS, flows  JOLTS
Parameters

- Markov chain for $y_t = y_i$ for $i = 1, ..., N = 50$. Discretization of an AR(1) process for log-wages.
- Rates $\lambda_0, \lambda_1, \delta$ and $\tau$ (the probability of changing employer if poached).
- Ability: 500 Chebishev nodes in $[x, x + 1]$, where $x$ must be estimated.
- The distribution of individual ability is beta-distributed:
  \[ \ell_m = \text{betacdf}(x_m - x, \eta, \mu) \]
- The opportunity cost of employment (leisure cost) is specified as:
  \[ z_i(m) = z_0 + \alpha [y_i(m) - z_0] \]
- I set the unit of time equal to a quarter.
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- The opportunity cost of employment (leisure cost) is specified as:
  $$z_i(m) = z_0 + \alpha [y_i(m) - z_0].$$
- I set the unit of time equal to a quarter.
Parameters

- Markov chain for \( y_t = y_i \) for \( i = 1, \ldots, N = 50 \). Discretization of an AR(1) process for log-wages.
- Rates \( \lambda_0, \lambda_1, \delta \) and \( \tau \) (the probability of changing employer if poached).
- Ability: 500 Chebishev nodes in \([x, x+1]\), where \( x \) must be estimated.
- The distribution of individual ability is beta-distributed:
  \[
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  \]
- The opportunity cost of employment (leisure cost) is specified as:
  \[
  z_i(m) = z_0 + \alpha [y_i(m) - z_0] .
  \]
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  \[
  z_i(m) = z_0 + \alpha [y_i(m) - z_0].
  \]
- I set the unit of time equal to a quarter.
I set $\tau = 0.5$ (some evidence from Jolivet et al.)

From JOLTS data I estimate transition probabilities $f_{1t}$ (job-to-job mobility) and $s_t$ (job destruction). The model predicts a job-to-job mobility rate $f_{1t}$ such that $f_{1t} = \tau \lambda_1 (1 - s_t)$. This implies a rate of on-the-job offer arrival of $\lambda_1 = 0.12 \lambda_0$. 
Theoretical turnover rates

- **Exit rate from unemployment:**
  \[ f_{0t} = \lambda_0 \sum_m 1\{S_i(m) > 0\} u_t(m) \ell_m / u_t; \]

- **Quit rate (job-to-job mobility):**
  \[ f_{1t} = \tau \lambda_1 (1 - \delta) \sum_m 1\{S_i(m) > 0\} [1 - u_t(m)] \ell_m / (1 - u_t); \]

- **Lay-off rate:**
  \[ s_t = \delta + (1 - \delta) \sum_m 1\{S_i(m) \leq 0\} (1 - u_t(m)) \ell_m / (1 - u_t). \]
Estimation

- Parameter $\alpha$ is not identified from employment and turnover data. It can be chosen arbitrarily to be estimated later using wages.
- The remaining parameters $(\lambda_0, \delta, x, \eta, \mu, z_0)$ are estimated by simulating very long series of $T = 10,000$ observations so as to match the following moments:
  - The mean productivity is 1, the standard deviation of log productivity is equal to 0.0223 and its autocorrelation is 0.91;
  - The mean unemployment rate is 5.8%, the standard deviation of log unemployment is 0.214 and its kurtosis is 2.52;
  - The mean exit rate from unemployment is 78.5%.
- I estimate $z_0 = 0.77$, $\sigma = 0.023$, $\rho = 0.94$, $\lambda_0 = 0.99$, $s = 0.042$, $x = 0.73$, $\eta = 2.00$ and $\mu = 5.56$.
- Then, $\alpha = 0.64$ is found to match the elasticity of aggregate wages optimally.
### Actual moments

<table>
<thead>
<tr>
<th></th>
<th>productivity</th>
<th>unempl. rate</th>
<th>unempl. exit rate</th>
<th>job destruction rate</th>
<th>wage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td></td>
<td><strong>0.058</strong></td>
<td><strong>0.785</strong></td>
<td><strong>0.0470</strong></td>
<td></td>
</tr>
<tr>
<td><strong>std</strong></td>
<td><strong>0.0226</strong></td>
<td><strong>0.214</strong></td>
<td><strong>0.0784</strong></td>
<td><strong>0.163</strong></td>
<td><strong>0.0202</strong></td>
</tr>
<tr>
<td><strong>skewness</strong></td>
<td>-0.19</td>
<td>0.12</td>
<td>-0.96</td>
<td>-0.06</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>kurtosis</strong></td>
<td>3.06</td>
<td><strong>2.52</strong></td>
<td>5.05</td>
<td>2.69</td>
<td>3.06</td>
</tr>
<tr>
<td><strong>autocorrelation</strong></td>
<td>0.91</td>
<td>0.95</td>
<td>0.94</td>
<td>0.91</td>
<td>0.96</td>
</tr>
<tr>
<td><strong>corr with prod</strong></td>
<td>1</td>
<td>-0.51</td>
<td>0.43</td>
<td>-0.54</td>
<td>0.74</td>
</tr>
<tr>
<td><strong>corr with unempl</strong></td>
<td>-0.51</td>
<td>1</td>
<td>-0.91</td>
<td>0.97</td>
<td>-0.46</td>
</tr>
<tr>
<td><strong>reg on prod</strong></td>
<td>1</td>
<td><strong>-4.79</strong></td>
<td>1.49</td>
<td>-3.88</td>
<td><strong>0.659</strong></td>
</tr>
<tr>
<td><strong>reg on unempl</strong></td>
<td>1</td>
<td>-0.33</td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Fitted moments

<table>
<thead>
<tr>
<th></th>
<th>productivity</th>
<th>unempl. rate</th>
<th>unempl. exit rate</th>
<th>job destruction rate</th>
<th>wage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean</strong></td>
<td>1</td>
<td>0.057</td>
<td>0.753</td>
<td>0.0435</td>
<td>0.932</td>
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<tr>
<td><strong>std</strong></td>
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<td>0.0153</td>
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<td><strong>skewness</strong></td>
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<td>-0.96</td>
<td>2.27</td>
<td>0.085</td>
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<tr>
<td><strong>kurtosis</strong></td>
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<td>3.20</td>
<td>4.30</td>
<td>10.81</td>
<td>3.81</td>
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<tr>
<td><strong>autocorrelation</strong></td>
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<td>0.94</td>
<td>0.95</td>
<td>0.02</td>
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<tr>
<td><strong>corr with prod</strong></td>
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<td>0.93</td>
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<tr>
<td><strong>corr with unempl</strong></td>
<td>-0.97</td>
<td>1</td>
<td>-0.98</td>
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<td>-0.92</td>
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<tr>
<td><strong>reg on prod</strong></td>
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<td>-9.47</td>
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<td>0.659</td>
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<td><strong>reg on unempl</strong></td>
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<td>-0.99</td>
<td>0.0023</td>
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</tbody>
</table>
Opportunity cost of employment

• I estimate $\alpha = 0.5$, $z_0 = 0.6115$.
• The mean leisure cost $z_t(m) = z_0 + \alpha [y_t(m) - z_0]$ averaged over worker types and time is 0.80, somewhere between Hagedorn and Manovskii (2008), 0.95, and Hall and Milgrom (2008), 0.70.
Worker heterogeneity

 Aggregate shock
 Match productivity
 viability threshold
 aggregate shock
 productivity by type
The amplifying Effect of Heterogeneity

Nonlinearity of the response function

unemployment rate
aggregate shock
Fit of unemployment series

Overall $R^2 = 28\%$; on 1948-1990: 45\%
Fit of unemployment exit rates

Overall $R^2 = 19\%$; on 1948-1990: 37%
Fit of job destruction rates

Overall $R^2 = 2.6\%$; on 1948-1990: 3.7\%
Fit of wages

Overall $R^2 = 44\%$; on 1948-1990: 56\%
Dynamics of wage inequality
## Fit of wage inequality indices

<table>
<thead>
<tr>
<th></th>
<th>Actual (males)</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elasticity</td>
<td>Volatility</td>
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<tr>
<td>P90</td>
<td>0.35</td>
<td>0.012</td>
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<tr>
<td>P80</td>
<td>0.33</td>
<td>0.011</td>
</tr>
<tr>
<td>P70</td>
<td>0.31</td>
<td>0.012</td>
</tr>
<tr>
<td>P60</td>
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<td>P50</td>
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<td>P40</td>
<td>0.50</td>
<td>0.018</td>
</tr>
<tr>
<td>P30</td>
<td>0.62</td>
<td>0.020</td>
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<tr>
<td>P20</td>
<td>0.69</td>
<td>0.025</td>
</tr>
<tr>
<td>P10</td>
<td>0.92</td>
<td>0.033</td>
</tr>
</tbody>
</table>
Decomposition of wage inequality

\[
\text{Var } w_{it} = \underbrace{\text{Var } \mathbb{E}(w_{it} | z_{it})}_{\text{between}} + \underbrace{\mathbb{E}\text{Var } (w_{it} | z_{it})}_{\text{within}}
\]
Conclusion

• We have proposed a simple dynamic search-matching model with cross-sectional wage dispersion and worker heterogeneous abilities.
• Fits unemployment volatility and the volatility of wages.
• The model is extremely simple to simulate.
• This is due to two very strong assumptions: firms have full monopsony power and they are identical.
• Next step: introduce a matching function and cross-country comparison