



# OECD Average and OECD Total

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## INTRODUCTION

In all PISA initial and thematic reports, the OECD gives the results for each country, but also two additional aggregated estimates: the OECD average and the OECD total.

The OECD average, sometimes also referred to as the country average, is the mean of the data values for all OECD countries for which data are available or can be estimated. The OECD average can be used to see how a country compares on a given indicator with a typical OECD country. The OECD average does not take into account the absolute size of the population in each country, *i.e.* each country contributes equally to the average. The contribution of the smallest OECD country, *i.e.* Luxembourg, is equivalent to the one of the largest country, *i.e.* the United States.

The OECD total considers all the OECD countries as a single entity, to which each country contributes proportionally to the number of 15-year-olds enrolled in its schools. It illustrates how a country compares with the OECD as a whole.

In PISA 2003, all OECD countries as well as several partner countries participated. It is possible, however, that for a particular cycle, data for one or several OECD countries may not be available for specific indicators. Researchers should, therefore, keep in mind that the terms OECD average and OECD total refer to the OECD countries included in the respective comparisons for each cycle and for a particular comparison.

For simple statistics such as a mean or a percentage, the OECD average and the OECD total statistics and their respective standard errors can be mathematically computed. If  $C$  OECD countries participated, then the OECD average mean and its respective sampling variance are equal to:

$$\hat{\mu} = \frac{\sum_{i=1}^C \hat{\mu}_i}{C} \quad \text{and} \quad \sigma^2_{(\hat{\mu})} = \frac{\sum_{i=1}^C \sigma^2_{(\hat{\mu}_i)}}{C^2}$$

The OECD total mean and its respective sampling variance are equal to:

$$\hat{\mu} = \frac{\sum_{i=1}^C w_i \hat{\mu}_i}{\sum_{i=1}^C w_i} \quad \text{and} \quad \sigma^2_{(\hat{\mu})} = \frac{\sum_{i=1}^C w_i^2 \sigma^2_{(\hat{\mu}_i)}}{\left[ \sum_{i=1}^C w_i \right]^2},$$

with  $w_i$  being the sum of the student final weights for a particular country.

While these formulae can be used for the computation of a mean or a percentage, they cannot be used for most other statistics. Such statistics can only be obtained directly from the data set.

## RECODING OF THE DATABASE FOR THE ESTIMATION OF THE OECD TOTAL AND OECD AVERAGE

As stated in Chapter 3, the sum of the student final weights per country is an estimate of the 15 year-old population in that country. Therefore, the OECD total statistic can easily be obtained by deleting the partner country data. Then the statistic is computed using the OECD variable as breakdown



variable (the OECD variable is a constant since the partner countries have been deleted from the file) instead of country (CNT). The standard error is obtained as usual by using the 80 replicates. Box 11.1 provides the SPSS® syntax for the computation of the mathematics performance per gender for the OECD total and Table 11.1 provides the results of the procedure.

**Box 11.1 ■ SPSS® syntax for the OECD total for the mathematics performance per gender**

```
get file 'c:\PISA\Data2003\INT_stui_2003.sav'.
compute OECD=0.
if
  (cnt='AUS' | cnt='AUT' | cnt='BEL' | cnt='CAN' | cnt='CZE' | cnt='DNK' |
   cnt='FIN' | cnt='FRA' | cnt='DEU' | cnt='GRC' | cnt='HUN' | cnt='ISL' |
   cnt='IRL' | cnt='ITA' | cnt='JPN' | cnt='KOR' | cnt='LUX' | cnt='MEX' |
   cnt='NZL' | cnt='NOR' | cnt='POL' | cnt='PRT' | cnt='ESP' | cnt='SWE' |
   cnt='CHE' | cnt='GBR' | cnt='USA' | cnt='NLD' | cnt='SVK' | cnt='TUR') OECD=1.
select if (OECD=1 & not missing(st03q01)).
save outfile='c:\pisa\Data2003\OECD.sav'.

* DEFINE MACRO.
include file 'c:\pisa\macros\mcr_SE_pv.sps'.

* CALL MACRO.
PV  nrep = 80/
   stat = mean/
   dep = math/
   grp = oecd st03q01/
   wgt = w_fstuwt/
   rwgt = w_fstr/
   cons = 0.05/
   infile = 'c:\pisa\Data2003\OECD.sav' /.
```

**Table 11.1 ■ Structure of the output data file from Box 11.1**

ST03Q01	STAT	SE
1	483.93	1.25
2	494.04	1.32

The OECD average requires an additional step. The student final weights need to be recoded, so that the sum of the student final weights per country is equal to a constant, *e.g.* 1000. This can easily be implemented with the procedure described in Box 11.2.<sup>1</sup> Table 11.2 presents the results of the procedure. The breakdown variables are OECD and ST03Q01 as in the example above for calculating OECD total and its standard error. The two weight arguments are different. The full student weight (W\_FSTUWT) is replaced by the senate weight (SENATE). The replicate weights have been transformed by the same linear transformation as the full student weight and are now called S\_FSTR1 to S\_FSTR80.

**Box 11.2 ■ SPSS® syntax for the OECD average for the mathematics performance per gender**

```

get file 'C:\PISA\Data2003\INT_stui_2003.sav'.
compute OECD=0.
If
  (cnt='AUS' | cnt='AUT' | cnt='BEL' | cnt='CAN' | cnt='CZE' | cnt='DNK' |
   cnt='FIN' | cnt='FRA' | cnt='DEU' | cnt='GRC' | cnt='HUN' | cnt='ISL' |
   cnt='IRL' | cnt='ITA' | cnt='JPN' | cnt='KOR' | cnt='LUX' | cnt='MEX' |
   cnt='NZL' | cnt='NOR' | cnt='POL' | cnt='PRT' | cnt='ESP' | cnt='SWE' |
   cnt='CHE' | cnt='GBR' | cnt='USA' | cnt='NLD' | cnt='SVK' | cnt='TUR') OECD=1.
select if OECD=1.
sort cases by cnt.
aggregate outfile='c:\temp\population.sav' /break=cnt /pop=sum(w_fstuw).
match files file=* /table='c:\temp\population.sav' /by cnt.

compute senate=(w_fstuw/pop)*1000.

define mcr ().
!do !r=1 !to 80.
compute !concat('s_fstr',!r)=(!concat('w_fstr',!r)/pop)*1000.
!doend.
!enddefine.
MCR.
save outfile='c:\pisa\Data2003\OECD.sav'.

weight by senate.
fre cnt.

*=====
* DEFINE MACRO.
include file 'c:\pisa\macros\mcr_SE_pv.sps'.

* CALL MACRO.
PV nrep = 80/
  stat = mean/
  dep = math/
  grp = oecd st03q01/
  wgt = senate/
  rwgt = s_fstr/
  cons = 0.05/
  infile = 'c:\pisa\Data2003\OECD.sav'/.

```

**Table 11.2 ■ Structure of the output data file from Box 11.2**

ST03Q01	STAT	SE
1	494.41	0.76
2	505.53	0.75

It is worth noting that the standard error is higher for the OECD total than it is for the OECD average. In the case of the OECD total, 40 per cent of the data come from just two countries (the United States and Japan), and these two countries do not have large sample sizes compared to the other OECD countries.



## DUPLICATION OF THE DATA FOR AVOIDING THREE RUNS OF THE PROCEDURE

If a researcher is interested in the country estimates as well as the OECD total and the OECD average, then three runs of the procedure are needed: one for the country estimates, one for the OECD total estimate and one for the OECD average estimate.

In order to avoid such repetitions, it is possible to duplicate three times the data for the OECD countries in such a way that the procedure directly provides the estimates for each country, as well as the OECD total and the OECD average estimates.

Box 11.3 presents the SPSS® syntax for the generation of such data sets. It consists of the following steps:

- A new categorical variable, denoted OECD and separating OECD and partner countries, is computed. A value of 1 for this variable designates OECD countries, whereas a value of 4 designates partner countries. A second alphanumerical variable, denoted COUNTRY and set as CNT, is created.
- OECD countries are selected and saved in TEMP2. The variable OECD is set as 2 and the COUNTRY variable is set as TOT.
- On the TEMP2 file, the sum of the student final weights per country is computed through the procedure described in Box 11.2. The final weights are transformed in such a way that the sum per country is equal to 1000. The same linear transformation is applied to the 80 replicates. The CNT variable is set as AVG and the OECD variable is set as 3. The new weight variables overwrite the old weight variables. These new data are saved into TEMP3.
- After sorting TEMP2 and TEMP3, the three temporary files are merged and saved in a final SPSS® data file.

SPSS macros presented in the previous chapters can be applied on this new data file. The breakdown variables are now OECD and COUNTRY instead of CNT. The output data file will consist of 43 rows. The first 30 rows will be the results of OECD countries. The next two rows will present the OECD total and the OECD average estimates. Finally, the 11 last rows will present the estimates for the partner countries.

## COMPARISONS BETWEEN OECD AVERAGE OR OECD TOTAL ESTIMATES AND A COUNTRY ESTIMATE

Only OECD countries that are fully adjudicated contribute to the OECD average and the OECD total estimates and their respective standard errors. Therefore, the expected value of the covariance between a country sampling variance and the OECD aggregate sampling variance will differ from 0 if the country's values are included in the OECD aggregate values, because the two are not independent. Indeed, if the sampling variance of one country increases, then the OECD aggregate sampling variance will also increase.

If a researcher wants to test the null hypothesis between an OECD country and the OECD aggregate estimate, then the covariance should be estimated, as explained in Chapter 10. Since the covariance is expected to be positive, then the correct standard error estimate should be smaller than the standard error obtained from the formulae.

**Box 11.3 ■ SPSS® syntax for the creation of a larger data set that will allow in one run the computation of the OECD total and the OECD average**

```
* COUNTRIES.
*-----
get file 'C:\PISA\Data2003\INT_stui_2003.sav'.
compute OECD=4.
if (cnt='AUS' | cnt='AUT' | cnt='BEL' | cnt='CAN' | cnt='CZE' | cnt='DNK' | cnt='FIN'
| cnt='FRA' | cnt='DEU' | cnt='GRC' | cnt='HUN' | cnt='ISL' | cnt='IRL' | cnt='ITA'
| cnt='JPN' | cnt='KOR' | cnt='LUX' | cnt='MEX' | cnt='NZL' | cnt='NOR' | cnt='POL'
| cnt='PRT' | cnt='ESP' | cnt='SWE' | cnt='CHE' | cnt='GBR' | cnt='USA' | cnt='NLD'
| cnt='SVK' | cnt='TUR') OECD=1.
save outfile='C:\TEMP\TEMP1.SAV'.

* OECD TOTAL.
*-----
get file 'C:\PISA\Data2003\INT_stui_2003.sav'.
select if
(cnt='AUS' | cnt='AUT' | cnt='BEL' | cnt='CAN' | cnt='CZE' | cnt='DNK' | cnt='FIN'
| cnt='FRA' | cnt='DEU' | cnt='GRC' | cnt='HUN' | cnt='ISL' | cnt='IRL' | cnt='ITA'
| cnt='JPN' | cnt='KOR' | cnt='LUX' | cnt='MEX' | cnt='NZL' | cnt='NOR' | cnt='POL'
| cnt='PRT' | cnt='ESP' | cnt='SWE' | cnt='CHE' | cnt='GBR' | cnt='USA' | cnt='NLD'
| cnt='SVK' | cnt='TUR').
compute OECD=2.
COMPUTE CNT='TOT'.
save outfile='C:\TEMP\TEMP2.SAV'.

* OECD AVERAGE.
*-----
get file 'C:\PISA\Data2003\INT_stui_2003.sav'.
select if
(cnt='AUS' | cnt='AUT' | cnt='BEL' | cnt='CAN' | cnt='CZE' | cnt='DNK' | cnt='FIN'
| cnt='FRA' | cnt='DEU' | cnt='GRC' | cnt='HUN' | cnt='ISL' | cnt='IRL' | cnt='ITA'
| cnt='JPN' | cnt='KOR' | cnt='LUX' | cnt='MEX' | cnt='NZL' | cnt='NOR' | cnt='POL'
| cnt='PRT' | cnt='ESP' | cnt='SWE' | cnt='CHE' | cnt='GBR' | cnt='USA' | cnt='NLD'
| cnt='SVK' | cnt='TUR').
compute OECD=3.
sort cases by cnt.
aggregate outfile='c:\temp\population.sav' /break=cnt /pop=sum(w_fstuwt).
match files file=* /table='c:\temp\population.sav' /by cnt.
compute w_fstuwt=(w_fstuwt/pop)*1000.
define mcr ().
!do !r=1 !to 80.
compute !concat('w_fstr',!r)=(!concat('w_fstr',!r)/pop)*1000.
!doend.
!enddefine.
mcr.
weight by w_fstuwt.
fre cnt.
COMPUTE CNT='AVG'.
save outfile='C:\TEMP\TEMP3.SAV'.

* COMBINE FILES.
*-----
add files file='C:\TEMP\TEMP1.SAV' /file='C:\TEMP\TEMP2.SAV' /file='C:\
TEMP\TEMP3.SAV'.
sort cases by oecd cnt.
FORMATS oecd (f1.0).
save outfile='c:\pisa\data2003\ALL3.sav'.
cros cnt by oecd.
```

...



```
* DEFINE MACRO.
include file 'c:\pisa\macros\mcr_SE_pv.sps'.

* CALL MACRO.
PV nrep = 80/
  stat = mean/
  dep = math/
  grp = oecd cnt/
  wgt = w_fstuwt/
  rwgt = w_fstr/
  cons = 0.05/
  infile = 'c:\pisa\data2003\ALL3.sav' /.
```

Since partner countries do not contribute at all to the OECD aggregate estimates, estimating the covariance is not necessary. The standard error on the difference can be directly obtained from the country standard error and the aggregate standard error.

Table 11.3 provides:

- The country mean performance in mathematics as well as the OECD average and the OECD total;
- The standard error on these mean estimates;
- The difference between the country and the OECD total;
- The standard error on this difference, using the formula provided in Chapter 10, *i.e.* without an estimation of the covariance;
- The standard error on this difference, using the replicate, *i.e.* with an estimation of the covariance;
- The difference between the country and the OECD average;
- The standard error on this difference, using the formula provided in Chapter 10, *i.e.* without an estimation of the covariance; and
- The standard error on this difference, using the replicate, *i.e.* with an estimation of the covariance.

The correct standard error estimates are in bold. The differences between the biased and unbiased estimates for OECD countries are not very large, except in the case of the United States and Germany for the OECD total.

The differences for partner countries are not very large either. As the expected covariance for partner countries are 0, both standard errors are on average unbiased. However, it is recommended to use the standard error directly obtained with the formula.

**Table 11.3 ■ Country mathematics performance means and their respective standard errors, country difference with OECD total and OECD average, their respective standard errors with or without estimation of the covariance**

	Country		OECD total			OECD average		
	<i>Mean</i>	<i>SE</i>	<i>DIF</i>	<i>SE without</i>	<i>SE with</i>	<i>DIF</i>	<i>SE without</i>	<i>SE with</i>
AUS	524.27	2.15	35.27	2.40	<b>2.12</b>	24.27	2.24	<b>2.03</b>
AUT	505.61	3.27	16.61	3.44	<b>3.49</b>	5.61	3.33	<b>3.27</b>
BEL	529.29	2.29	40.29	2.52	<b>2.42</b>	29.29	2.37	<b>2.23</b>
CAN	532.49	1.82	43.49	2.11	<b>2.08</b>	32.49	1.92	<b>1.96</b> ...



Table 11.3 (continued) ■ Country mathematics performance means and their respective standard errors, country difference with OECD total and OECD average, their respective standard errors with or without estimation of the covariance

	Country		OECD total			OECD average		
	Mean	SE	DIF	SE without	SE with	DIF	SE without	SE with
CHE	526.55	3.38	37.56	3.55	<b>3.48</b>	26.55	3.44	<b>3.38</b>
CZE	516.46	3.55	27.46	3.70	<b>3.90</b>	16.46	3.60	<b>3.51</b>
DEU	502.99	3.32	13.99	3.49	<b>3.42</b>	2.99	3.38	<b>3.30</b>
DNK	514.29	2.74	25.29	2.95	<b>2.99</b>	14.29	2.82	<b>2.67</b>
ESP	485.11	2.41	-3.89	2.64	<b>2.60</b>	-14.89	2.49	<b>2.47</b>
FIN	544.29	1.87	55.29	2.15	<b>2.07</b>	44.29	1.97	<b>1.91</b>
FRA	510.80	2.50	21.80	2.72	<b>2.45</b>	10.80	2.58	<b>2.46</b>
GBR	508.26	2.43	19.26	2.65	<b>2.41</b>	8.26	2.51	<b>2.39</b>
GRC	444.91	3.90	-44.09	4.05	<b>3.94</b>	-55.09	3.95	<b>3.81</b>
HUN	490.01	2.84	1.02	3.03	<b>3.20</b>	-9.99	2.91	<b>2.95</b>
IRL	502.84	2.45	13.84	2.67	<b>2.56</b>	2.84	2.53	<b>2.41</b>
ISL	515.11	1.42	26.11	1.78	<b>1.78</b>	15.11	1.56	<b>1.48</b>
ITA	465.66	3.08	-23.33	3.26	<b>3.11</b>	-34.34	3.14	<b>2.98</b>
JPN	534.14	4.02	45.14	4.16	<b>3.88</b>	34.14	4.06	<b>3.94</b>
KOR	542.23	3.24	53.23	3.41	<b>3.34</b>	42.23	3.30	<b>3.16</b>
LUX	493.21	0.97	4.21	1.45	<b>1.48</b>	-6.79	1.16	<b>1.20</b>
MEX	385.22	3.64	-103.78	3.80	<b>3.55</b>	-114.78	3.70	<b>3.64</b>
NLD	537.82	3.13	48.83	3.31	<b>3.19</b>	37.82	3.19	<b>3.10</b>
NOR	495.19	2.38	6.19	2.61	<b>2.69</b>	-4.81	2.46	<b>2.41</b>
NZL	523.49	2.26	34.49	2.50	<b>2.41</b>	23.49	2.34	<b>2.31</b>
POL	490.24	2.50	1.24	2.72	<b>2.82</b>	-9.76	2.58	<b>2.54</b>
PRT	466.02	3.40	-22.98	3.57	<b>3.30</b>	-33.98	3.46	<b>3.23</b>
SVK	498.18	3.35	9.19	3.51	<b>3.46</b>	-1.82	3.41	<b>3.31</b>
SWE	509.05	2.56	20.05	2.77	<b>2.48</b>	9.05	2.64	<b>2.40</b>
TUR	423.42	6.74	-65.58	6.82	<b>6.48</b>	-76.58	6.77	<b>6.46</b>
USA	482.88	2.95	-6.11	3.14	<b>2.38</b>	-17.12	3.02	<b>2.90</b>
TOT	489.00	1.07						
AVE	500.00	0.63						
BRA	356.02	4.83	-132.98	<b>4.95</b>	4.89	-143.98	<b>4.87</b>	4.77
HKG	550.38	4.54	61.39	<b>4.66</b>	4.80	50.38	<b>4.58</b>	4.68
IDN	360.16	3.91	-128.84	<b>4.05</b>	4.03	-139.84	<b>3.96</b>	3.88
LIE	535.80	4.12	46.80	<b>4.26</b>	4.16	35.80	<b>4.17</b>	4.13
LVA	483.37	3.69	-5.62	<b>3.84</b>	3.88	-16.62	<b>3.74</b>	3.76
MAC	527.27	2.89	38.27	<b>3.08</b>	3.13	27.27	<b>2.95</b>	2.85
RUS	468.41	4.20	-20.59	<b>4.33</b>	4.47	-31.59	<b>4.24</b>	4.33
THA	416.98	3.00	-72.02	<b>3.18</b>	3.38	-83.02	<b>3.06</b>	3.20
TUN	358.73	2.54	-130.26	<b>2.75</b>	2.55	-141.27	<b>2.61</b>	2.57
URY	422.20	3.29	-66.80	<b>3.46</b>	3.41	-77.80	<b>3.35</b>	3.30
YUG	436.87	3.75	-52.13	<b>3.90</b>	3.85	-63.13	<b>3.81</b>	3.78

## CONCLUSIONS

This chapter was devoted to the concept of the OECD total and the OECD average. For simple statistics such as a mean or a percentage, these aggregated estimates and their respective standard errors can directly be obtained from the country individual estimates.



In most cases, nevertheless, the estimate and its standard error can only be computed from the data base. SPSS<sup>®</sup> syntax was provided.

In order to avoid three runs for obtaining individual country estimates as well as OECD aggregate estimates, the SPSS<sup>®</sup> syntax for creating a larger data set was also provided.

Finally, following the issues raised in the previous chapter devoted to comparisons, any comparison that involves a particular country and an OECD aggregate estimate was discussed

### *Note*

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1. As an alternative, a country weight, CNTFAC1, can be also used.





# Trends

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## INTRODUCTION

Policy makers and researchers demand information on how indicators change over time. The longer the time period the more reliable the trend indicator. One example would be an analysis of the impact of reforms to the education system, where policy makers would seek to measure changes in the targeted area to gauge how effective their policies have been. In the early 1960s, for example, most of the OECD countries implemented education reforms to facilitate access to tertiary education, mainly through financial help. One indicator of the impact of these reforms would be to calculate the percentage of the population with a tertiary qualification for several different years to show how this has evolved. Computing this trend indicator is a straightforward statistical manipulation, since the measure (*i.e.* whether or not an individual has completed tertiary education) is quite objective and the information is in most cases available at the population level. Nevertheless, such measures can be slightly biased by, for example, differing levels of immigration over the time period, student exchange programmes, and so on.

Of course trends over time on one particular indicator need careful interpretation. Policy makers would also need to take into account changes to the economic context of the country, such as rising unemployment rates. For example, an increase in the percentage of tertiary graduates does not necessarily prove that the reform to the education system was effective. Further, when comparing trend indicators across countries it is important to consider how comparable the definition of the indicator is from country to country, for example tertiary education might mean something different in each country.

The PISA project offers a unique opportunity to extend the computation of trend indicators on educational outcomes by looking at student performance in reading, mathematical and scientific literacy.

For the trend measures to be reliable, the comparability of the target population, the data collection procedures, and the assessment framework need to be consistent over time. Being able to use the results from PISA as trend indicators is one of the major aims of the project.

PISA 2000 and PISA 2003 used the same assessment frameworks and the data collection procedures were essentially unchanged. In PISA 2000, the target population was defined as all 15-year-olds in grade 5 or higher grades. In PISA 2003, it was defined as all 15-year-olds in grade 7 or higher grades. In PISA 2000, only a very small percentage of 15-year-olds were attending grade 5 or grade 6 (Austria = 0.03%, Canada = 0.03%, Czech Republic = 0.06%, Germany = 0.02%, Hungary = 0.59%, Latvia = 0.27%, Portugal = 1.25% and Russia = 0.04%). Therefore, except for Portugal, the change in the target population should not significantly affect trend indicators.

Other issues that need to be included in the interpretation of trend indicators are student and school participation rates and coverage indices. A higher or a lower school participation rate might explain partly observed differences.

Behind these preliminary precautions, the computation of trend indicators in PISA raises two statistical issues:

- PISA collects data on a sample and therefore any statistic has to be associated with a sampling error. The next section will discuss how to compute such sampling error on a trend indicator.
- Between 20 and 30 items per domain from the 2000 assessments were included in the 2003 assessment to ensure a psychometric link. These anchor items were used to scale the PISA 2000 and the PISA 2003 assessments on a common scale. As one can easily imagine, selecting other



anchor items would have returned slightly different results on the trend performance indicators. It follows that any comparison between two PISA cycles in the student performance will require the addition of another error component, *i.e.* the item sampling error.

### THE COMPUTATION OF THE STANDARD ERROR FOR TREND INDICATORS ON VARIABLES OTHER THAN PERFORMANCE

For any country, the PISA 2000 and the PISA 2003 samples are independent. Therefore, the standard error on any trend indicators not involving achievement variables can be computed as follows:

$$\sigma_{(\hat{\theta}_{2003} - \hat{\theta}_{2000})} = \sqrt{\sigma_{(\hat{\theta}_{2003})}^2 + \sigma_{(\hat{\theta}_{2000})}^2}, \text{ with } \theta \text{ representing any statistic.}$$

However, the computation of a difference between PISA 2000 and PISA 2003 and its standard error are relevant only if the two measures are identical. For instance, in the PISA 2000 and PISA 2003 databases, there are several indices derived from the student questionnaires with exactly the same variable names (for instance, HEDRES for home educational resources, BELONG for the student's sense of belonging to the school, and so on). The questions that were used to derive these indices have not changed, but as the scaling was done independently in 2000 and in 2003, there is no guarantee that the 2000 and the 2003 metrics are comparable. Further, these indices were standardised at the OECD level to get a mean of 0 and a standard deviation of 1. The 2000 standardisation differs from the 2003 one. It is therefore not recommended to compute trend indicators on questionnaire indices.

In the case of the PISA questionnaire indices, as the questions have not been modified, the underlying concepts are similar. Therefore, the correlation coefficients between these indices and the student performance can directly be compared. However, as the item parameters were estimated in 2003 without any link with the PISA 2000 data, the metric of the scales might be slightly different and an absolute increase in, for example, the sense of belonging might be simply a result of the scaling, or the standardisation, without any attitudinal change in the student. For the same reasons, regression coefficients for indices derived from student questionnaire data cannot be compared between 2000 and 2003.

The Highest International Social and Economic Index (denoted HISEI in the databases) satisfies all the conditions for the computation of trend indicators. Indeed, the questions were not changed and the transformation used on the ISCO categories in 2000 was implemented in 2003 without any modification.

Table 12.1 presents, per country, the mean estimate of HISEI and its standard error for PISA 2000 and PISA 2003, as well as the difference between the two estimates, the standard error of this difference and the standardised difference, *i.e.* the difference divided by its standard error.

For Germany (DEU), the means for HISEI in 2000 and in 2003 are respectively equal to 48.85 and 49.33. The difference between these two data collection is therefore equal to:

$$49.33 - 48.55 = 0.48.$$

The standard errors on these mean estimates are equal to 0.32 and 0.42. The standard error on the difference estimate is equal to:

$$\sigma_{(\hat{\theta}_{2003} - \hat{\theta}_{2000})} = \sqrt{\sigma_{(\hat{\theta}_{2003})}^2 + \sigma_{(\hat{\theta}_{2000})}^2} = \sqrt{(0.32)^2 + (0.42)^2} = 0.53$$



The standardised difference, *i.e.* the difference estimate divided by its standard error, is equal to:

$$\frac{0.48}{0.53} = 0.91$$

As the standardised difference is included in the interval  $[-1.96; 1.96]$ , the difference on the mean estimate for HISEI between 2000 and 2003 is not statistically different from 0 with a type I error of 0.05.

Table 12.1 shows that the difference is statistically different from 0 in 13 countries: Austria, Belgium, Brazil, the Czech Republic, Indonesia, Iceland, Korea, Liechtenstein, Luxembourg, Mexico, Thailand, the United Kingdom and the United States.

It would be unrealistic to consider these differences as simply a reflection of social and economic changes in these 13 countries. In a period of three years, some changes can occur, but these cannot explain by themselves the size of the observed increases or decreases.

**Table 12.1 ■ Trend indicators between PISA 2000 and PISA 2003 for HISEI per country**

	PISA 2000		PISA 2003		Difference		
	Mean	SE	Mean	SE	Estimate	SE	STD difference
AUS	52.25	(0.50)	52.59	(0.30)	0.34	(0.58)	0.59
AUT	49.72	(0.29)	47.06	(0.52)	-2.66	(0.59)	-4.49
BEL	48.95	(0.39)	50.59	(0.38)	1.65	(0.54)	3.05
BRA	43.93	(0.59)	40.12	(0.64)	-3.81	(0.87)	-4.39
CAN	52.83	(0.22)	52.58	(0.27)	-0.25	(0.35)	-0.73
CHE	49.21	(0.53)	49.30	(0.43)	0.09	(0.68)	0.13
CZE	48.31	(0.27)	50.05	(0.34)	1.74	(0.44)	3.98
DEU	48.85	(0.32)	49.33	(0.42)	0.48	(0.53)	0.91
DNK	49.73	(0.43)	49.26	(0.45)	-0.47	(0.63)	-0.75
ESP	44.99	(0.62)	44.29	(0.58)	-0.70	(0.85)	-0.83
FIN	50.00	(0.40)	50.23	(0.36)	0.23	(0.54)	0.42
FRA	48.27	(0.44)	48.66	(0.47)	0.39	(0.64)	0.61
GBR	51.26	(0.35)	49.65	(0.39)	-1.61	(0.52)	-3.07
GRC	47.76	(0.60)	46.94	(0.72)	-0.83	(0.93)	-0.88
HUN	49.53	(0.47)	48.58	(0.33)	-0.95	(0.57)	-1.65
IDN	36.38	(0.77)	33.65	(0.61)	-2.73	(0.98)	-2.77
IRL	48.43	(0.48)	48.34	(0.49)	-0.09	(0.69)	-0.13
ISL	52.73	(0.28)	53.72	(0.26)	0.99	(0.38)	2.62
ITA	47.08	(0.31)	46.83	(0.38)	-0.24	(0.49)	-0.50
JPN	50.54	(0.62)	49.98	(0.31)	-0.56	(0.69)	-0.80
KOR	42.80	(0.42)	46.32	(0.36)	3.52	(0.55)	6.36
LIE	47.46	(0.94)	50.73	(0.75)	3.27	(1.21)	2.71
LUX	44.79	(0.27)	48.17	(0.22)	3.38	(0.35)	9.76
LVA	50.15	(0.54)	50.28	(0.52)	0.13	(0.75)	0.18
MEX	42.48	(0.68)	40.12	(0.68)	-2.37	(0.96)	-2.46
NLD	50.85	(0.47)	51.26	(0.38)	0.42	(0.61)	0.68
NOR	53.91	(0.38)	54.63	(0.39)	0.72	(0.54)	1.33
NZL	52.20	(0.37)	51.46	(0.36)	-0.74	(0.51)	-1.45
POL	46.03	(0.47)	44.96	(0.34)	-1.07	(0.58)	-1.85
PRT	43.85	(0.60)	43.10	(0.54)	-0.75	(0.81)	-0.92
RUS	49.38	(0.45)	49.86	(0.38)	0.49	(0.59)	0.82
SWE	50.57	(0.39)	50.64	(0.38)	0.07	(0.55)	0.12
THA	33.02	(0.57)	36.01	(0.43)	2.99	(0.72)	4.18
USA	52.40	(0.79)	54.55	(0.37)	2.15	(0.87)	2.47



It is also possible that the quality of the samples might explain some of the differences. As a student's propensity to participate positively correlate with his or her academic records and as on average low achievers come from lower social background variables than high achievers, an increase or a decrease in the student participation rates might affect the HISEI mean.

A change in the percentage of missing data for the HISEI variable would be another explanation that can be easily verified. On average, students who do not provide their parents' jobs are lower achievers. Therefore, one should expect low social background characteristics, so that an increase of missing data could be associated with an increase of the HISEI mean and inversely.

Table 12.2 provides the percentages of missing data for the HISEI variables in PISA 2000 and PISA 2003 databases. These results do not really confirm the hypothesis. For instance, in the United States, the percentages of missing data were respectively about 14 per cent in 2000 and about 6 per cent in 2003 and the means of HISEI were respectively 52.40 and 54.55. In 9 out of the 13 countries where the HISEI means significantly differ, either an increase of the HISEI mean is associated with a decrease of the percentage of missing data or the inverse. In the three other countries, *i.e.* Belgium, the Czech Republic and Mexico, the relationship is consistent with the hypothesis.

Table 12.2 ■ Percentages of missing data for HISEI

	PISA 2000		PISA 2003		Difference		
	%	SE	%	SE	Estimate	SE	STD difference
AUS	4.15	(0.38)	7.91	(1.56)	3.76	(1.61)	2.33
AUT	2.06	(0.20)	3.62	(0.32)	1.56	(0.38)	4.13
BEL	5.02	(0.45)	6.11	(0.48)	1.09	(0.66)	1.66
BRA	7.90	(0.62)	8.75	(1.03)	0.86	(1.20)	0.71
CAN	3.00	(0.18)	12.34	(0.76)	9.34	(0.78)	11.93
CHE	3.36	(0.32)	3.06	(0.26)	-0.30	(0.41)	-0.72
CZE	1.90	(0.42)	5.65	(1.19)	3.75	(1.26)	2.97
DEU	3.05	(0.34)	9.92	(0.63)	6.87	(0.72)	9.55
DNK	7.12	(0.85)	2.73	(0.37)	-4.40	(0.92)	-4.76
ESP	4.48	(0.49)	3.70	(0.37)	-0.78	(0.62)	-1.27
FIN	1.96	(0.22)	1.44	(0.16)	-0.52	(0.27)	-1.92
FRA	6.23	(0.51)	4.61	(0.45)	-1.61	(0.68)	-2.37
GBR	5.15	(0.44)	7.23	(1.17)	2.07	(1.25)	1.66
GRC	4.04	(0.57)	5.81	(0.41)	1.78	(0.70)	2.53
HUN	3.02	(0.36)	5.39	(0.42)	2.37	(0.55)	4.31
IDN	6.99	(0.64)	8.67	(0.53)	1.67	(0.83)	2.03
IRL	3.23	(0.34)	4.32	(0.57)	1.09	(0.66)	1.65
ISL	2.19	(0.24)	2.30	(0.25)	0.11	(0.35)	0.31
ITA	2.73	(0.46)	2.47	(0.28)	-0.26	(0.54)	-0.48
JPN	62.52	(3.47)	11.25	(0.81)	-51.27	(3.56)	-14.41
KOR	7.34	(0.49)	2.36	(0.21)	-4.97	(0.54)	-9.29
LIE	5.49	(1.41)	3.02	(0.85)	-2.47	(1.64)	-1.50
LUX	9.55	(0.50)	3.62	(0.29)	-5.92	(0.58)	-10.27
LVA	5.02	(0.52)	3.34	(0.39)	-1.68	(0.66)	-2.56
MEX	8.51	(0.59)	5.07	(0.44)	-3.43	(0.74)	-4.65 ...



Table 12.2 (continued) ■ Percentages of missing data for HISEI

	PISA 2000		PISA 2003		Difference		
	%	SE	%	SE	Estimate	SE	STD difference
NLD	3.07	(0.65)	7.64	(1.34)	4.57	(1.49)	3.07
NOR	2.44	(0.31)	3.18	(0.39)	0.74	(0.50)	1.49
NZL	3.92	(0.39)	14.13	(0.43)	10.22	(0.58)	17.60
POL	6.90	(0.79)	2.33	(0.30)	-4.57	(0.85)	-5.39
PRT	3.72	(0.42)	2.76	(0.28)	-0.96	(0.50)	-1.90
RUS	3.16	(0.33)	2.14	(0.30)	-1.02	(0.45)	-2.27
SWE	2.48	(0.30)	2.63	(0.31)	0.15	(0.43)	0.35
THA	10.95	(1.38)	5.85	(0.64)	-5.09	(1.52)	-3.35
USA	14.58	(1.95)	5.88	(0.38)	-8.70	(1.99)	-4.38

This simple example shows that the interpretation of trend indicators is quite complex. The social and economic structure of a country should remain unchanged over a period of three years, so that no differences occur between two cycles. However, as shown, this difference appears significant in all 13 countries.

Changes in the school or student participation rates and in the distribution of missing data might sometimes explain these significant differences. It is therefore recommended to implement some verification before trying to interpret calculated differences as a real change in the population characteristics.

### THE COMPUTATION OF THE STANDARD ERROR FOR TREND INDICATORS ON PERFORMANCE VARIABLES

#### Anchoring of the PISA 2000 and PISA 2003 performance scales

The PISA 2000 database contains five plausible values for each of the following domains or subdomains:

- Mathematics
- Reading
  - Reading/retrieving information
  - Reading/interpreting
  - Reading/reflecting
- Science

The PISA 2003 database also contains five plausible values for each of the following domains or subdomains:

- Mathematics
  - Mathematics/space and shape
  - Mathematics/change and relationship
  - Mathematics/uncertainty
  - Mathematics/quantity



- Problem solving
- Reading
- Science

The psychometric procedures used to link the PISA 2000 and PISA 2003 performance scales are different for mathematics than they are for reading and science.

Reading was the major domain in 2000 and 28 of the 140 items developed for the 2000 assessment were used for the 2003 assessment. The 2003 data were therefore reported on the 2000 reading scale. The science assessment data of 2003 are also reported on the 2000 science scale as 25 of the 30 items developed for the 2000 assessment were used for the 2003 assessment.

Mathematics, as the major domain, was the subject of major development work for PISA 2003. Further, the mathematics assessment in 2000 only covered two of the four content areas (*space and shape* and *change and relationships*). Twenty items out of the 85 items used in 2003 assessment come from the 2000 assessment. Because of this broadening in the assessment, it was deemed inappropriate to report the PISA 2003 mathematics scores base on the scale for the PISA 2000 mathematics scores.

However, to provide countries with some trend indicators, the mathematics subscales *space and shape*, and *change and relationship* of the PISA 2000 were reported on the PISA 2003 scales.<sup>1</sup>

The steps for anchoring the PISA 2003 reading and science data on the 2000 scales are:

1. Calibration of the 2003 reading and science data to get the PISA 2003 item parameters, *i.e.* the relative difficulty of the item on the Rasch scale.
2. Based on these item parameters, generation of the plausible values for reading and science on the PISA 2003 data.
3. Based on the item parameters of step 1, but only on the link items, generation of plausible values for Reading and Science on the PISA 2000 data. By this time, two sets for plausible values are available for PISA 2000: the original set of plausible values included in the PISA 2000 database and the set of plausible values based on the PISA 2003 item parameters. Unfortunately, the mean and the standard deviation of the new set of plausible values will slightly differ from the PISA 2000 original plausible values. These differences reflect the changes in the difficulty of the link items between 2000 and 2003. As a reminder, the mean and the standard deviation for the OECD average were set respectively at 500 and 100 in 2000. Let us suppose that the new set of plausible values return a mean of 505 and a standard deviation of 110. The new set of plausible values for the PISA 2000 data has to be transformed so that their mean and standard deviation is respectively equal to 500 and 100.
4. This step consists of the computation of the linear transformation that will guarantee that the mean and the standard deviation of the new set of plausible values on the PISA 2000 data has a mean of 500 and a standard deviation of 100. This linear transformation can be written as

$$PV_{cal\_2000} = \alpha + \beta * PV_{cal\_2003} \text{ with } \beta = \frac{\sigma_{cal\_2000}}{\sigma_{cal\_2003}} \text{ and } \alpha = (\mu_{cal\_2000} - \beta * (\mu_{cal\_2003})).$$

In the example,  $\beta = 100 / 110 = 0.909$  and  $\alpha = (500 - (0.909 * 505)) = 40.955$ ;<sup>2</sup> and



5. This linear transformation is applied on the PISA 2003 plausible values. This linear transformation applied on the Reading or Science PISA 2003 plausible values guarantees that the student performance in 2003 is comparable to the student performance in 2000.

As stated earlier, with another set of link items, the linear transformation would have been different. As a consequence, there is an uncertainty in the transformation due to sampling of the link items, referred as the linking error.

The steps for anchoring the two mathematics PISA 2000 subscales on the PISA 2003 subscales are:

1. Calibration of the 2003 mathematics data to get the PISA 2003 item parameter;
2. Based on these item parameters, generation of the PISA 2003 plausible values; and
3. Based on the 2003 item parameters, generation of plausible values for the mathematics PISA 2000 data.

Similarly, the estimation of the trend would have been slightly different with another set of anchor items in reading and in science. It is therefore important to integrate this error component in the standard error of the trend indicator.

### **Inclusion of the linking error in the computation of the standard error**

For each link item, we have two item parameter estimates that are now on the same metric: the 2000 item parameter and the 2003 item parameter. Some of these link items show an increase of the relative difficulty, some show a decrease, but on average, the difference is equal to 0. This means that some items seem more difficult in 2003 than they were in 2000 or the inverse.

As the subset of link items can be considered as a simple random sample of an infinite population of link items, the linking error can be computed as:

$$\sigma_{(\text{Linking\_error})} = \sqrt{\frac{\sigma^2}{n}} \text{ where:}$$

$\sigma^2$  represents the variance of the item parameter differences, and  $n$  denotes the number of link items used.

If the item parameters from the 2003 calibration perfectly match the item parameters from the 2000 calibration, then the relative difficulty of the link items would not have changed. All the differences between the relative difficulty in 2000 and in 2003 would be equal to 0 and therefore, the linking error would be equal to 0.

As the differences in the item parameters increase, the variance of these differences will increase and consequently the linking error will increase. It makes sense for the uncertainty around the trend to be proportional to the changes in the item parameters.

Also, the uncertainty around the trend indicators is inversely proportional to the number of link items. From a theoretical point of view, only one item is needed to measure a trend, but with only one item, the uncertainty will be very large. If the number of link items increases, the uncertainty will decrease.



Table 12.3 provides the centred item parameters (*i.e.* item difficulty differences) for the reading link items for PISA 2000 and PISA 2003, as well as the difference between the two sets of estimates.

**Table 12.3 ■ Item parameter estimates in 2000 and 2003 for the reading link items**

Item Name	Centered <i>Delta</i> in 2003	Centered <i>Delta</i> in 2000	Difference
R055Q01	-1.28	-1.347	-0.072
R055Q02	0.63	0.526	-0.101
R055Q03	0.27	0.097	-0.175
R055Q05	-0.69	-0.847	-0.154
R067Q01	-2.08	-1.696	0.388
R067Q04	0.25	0.546	0.292
R067Q05	-0.18	0.212	0.394
R102Q04A	1.53	1.236	-0.290
R102Q05	0.87	0.935	0.067
R102Q07	-1.42	-1.536	-0.116
R104Q01	-1.47	-1.205	0.268
R104Q02	1.44	1.135	-0.306
R104Q05	2.17	1.905	-0.267
R111Q01	-0.19	-0.023	0.164
R111Q02B	1.54	1.395	-0.147
R111Q06B	0.89	0.838	-0.051
R219Q01T	-0.59	-0.520	0.069
R219Q01E	0.10	0.308	0.210
R219Q02	-1.13	-0.887	0.243
R220Q01	0.86	0.815	-0.041
R220Q02B	-0.14	-0.114	0.027
R220Q04	-0.10	0.193	0.297
R220Q05	-1.39	-1.569	-0.184
R220Q06	-0.34	-0.142	0.196
R227Q01	0.40	0.226	-0.170
R227Q02T	0.16	0.075	-0.086
R227Q03	0.46	0.325	-0.132
R227Q06	-0.56	-0.886	-0.327

The variance of the difference is equal to 0.047486. The link error is therefore equal to:

$$\sigma_{(Linking\_error)} = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{0.047486}{28}} = 0.041.$$

On the PISA reading scale with a mean of 500 and a standard deviation of 100, it corresponds to 3.75.

The linking errors between PISA 2000 and PISA 2003 are:

- Reading..... 3.75
- Science ..... 3.02
- Mathematics/space and shape ..... 6.01
- Mathematics/change and relationship..... 4.84

A common transformation has been estimated from the link items, and this transformation is applied to all participating countries. It follows that any uncertainty that is introduced through the linking



is common to all students and all countries. Thus, for example, suppose that the unknown linking error between PISA 2000 and PISA 2003 in reading resulted in an overestimation of student scores by two points on the PISA 2000 scale. It follows that every student's score would be overestimated by two score points. This overestimation would have effects on certain, but not all, summary statistics computed from the PISA 2003 data. For example, consider the following:

- Each country's mean would be overestimated by an amount equal to the link error, in our example this is two score points;
- The mean performance of any subgroup would be overestimated by an amount equal to the linking error, in our example this is two score points;
- The standard deviation of student scores would not be affected because the overestimation of each student by a common error does not change the standard deviation;
- The difference between the mean scores of two countries in PISA 2003 would not be influenced because the overestimation of each student by a common error would have distorted each country's mean by the same amount;
- The difference between the mean scores of two groups (*e.g.* males and females) in PISA 2003 would not be influenced, because the overestimation of each student by a common error would have distorted each group's mean by the same amount;
- The difference between the performance of a group of students (*e.g.* a country) between PISA 2000 and PISA 2003 would be influenced because each student's score in PISA 2003 would be influenced by the error; and
- A change in the difference between two groups from PISA 2000 to PISA 2003 would not be influenced. This is because neither of the components of this comparison, which are differences in scores in 2000 and 2003 respectively, is influenced by a common error that is added to all student scores in PISA 2003.

In general terms, the linking error need only be considered when comparisons are being made between PISA 2000 and PISA 2003 results, and then usually when group means are being compared.

The most obvious example of a situation where there is a need to use the linking error is in the comparison of the mean performance for a country between PISA 2000 and PISA 2003.

In PISA 2000, the mean in reading literacy for Germany is equal to 483.99 with a standard error of 2.47. In PISA 2003, the mean for Germany is equal to 491.36 and the standard error is equal to 3.39. The difference between 2000 and 2003 is therefore equal to  $491.36 - 483.99 = 7.37$ . The average performance of the German students has therefore increased by 7.37 scores on the PISA 2000 reading scale.

The standard error on this difference, as mentioned here above, is influenced by the linking error. The standard error is therefore equal to:

$$SE = \sqrt{\sigma_{(\hat{\mu}_{2000})}^2 + \sigma_{(\hat{\mu}_{2003})}^2 + \sigma_{(linking\_error)}^2}$$

$$SE = \sqrt{(2.47)^2 + (3.39)^2 + (3.75)^2} = 5.63$$



As the standardised difference between PISA 2000 and PISA 2003, *i.e.* (7.37/5.63) is included in the interval [-1.96; 1.96], the null hypothesis of no difference is not rejected. In other words, Germany's performance in reading has not changed between 2000 and 2003.

Table 12.4 provides the estimates of the reading performance in Germany per gender in 2000 and 2003, with their respective standard errors, as well as the difference estimates and their respective standard errors.

**Table 12.4 ■ Mean performance in reading per gender for Germany**

		Performance in reading	Standard error
2003	Girls	512.93	3.91
	Boys	470.80	4.23
	Difference	42.13	4.62
2000	Girls	502.20	3.87
	Boys	467.55	3.17
	Difference	34.65	5.21

As the comparison for a particular country between 2000 and 2003 is affected by the linking error, the comparison for a particular subgroup between 2000 and 2003 is also affected by the linking error. Therefore, the standard error has to include the linking error.

The trend indicators for German boys and for German girls are, respectively, equal to:

$$Trends_{Girls} = 512.93 - 502.20 = 10.73$$

$$SE_{Girls} = \sqrt{(3.91)^2 + (3.87)^2 + (3.75)^2} = 6.66$$

$$Trends_{Boys} = 470.80 - 467.55 = 3.25$$

$$SE_{Boys} = \sqrt{(4.23)^2 + (3.17)^2 + (3.75)^2} = 6.48$$

Both differences are not statistically different from 0.

On the other hand, the gender difference in 2003 is not affected by the linking error. Indeed, both subgroup estimates will be underestimated or overestimated by the same amount and therefore the computation of the difference will neutralize this difference. Consequently, the trend indicator on the gender difference and its standard error will be equal to:

$$Trends_{Gender\_dif} = 42.13 - 34.65 = 7.43$$

$$SE_{Gender\_dif} = \sqrt{(4.62)^2 + (5.21)^2} = 6.96$$

This means that the change in gender difference in Germany for Reading between 2000 and 2003 was not statistically significant, even though it appears from Table 3 to have widened considerably.

In the PISA 2000 and PISA 2003 initial reports, student performance is also reported by proficiency levels (see Chapter 8). As the linking error affects the country mean estimate, the percentages of



students at each level will also be affected. However, an overestimation or an underestimation of the PISA 2003 results of  $X$  points on the PISA scale will have a different impact on the percentages of students at each proficiency level for each country. If the percentage is small, then the impact will be small. If the percentage is large, then the impact will be larger. It would have been too complex to provide for each country and for each proficiency level a linking error. It was therefore decided not to take into account the linking error for the comparison of percentages of students at each proficiency level between PISA 2000 and PISA 2003. This means that the standard errors on the difference between 2000 and 2003 are underestimated.

## CONCLUSIONS

This chapter was devoted to the computation of the standard error on trend indicators. The comparison of any variable other than performance variables is quite straightforward as the PISA 2000 and the PISA 2003 samples are independent. However, as stated previously, such comparisons are only relevant if the 2000 and the 2003 measures are comparable.

The comparison of performance mean estimates is more complex as it might require the inclusion of the linking error in the standard error depending on the statistic. For instance, Figure 2.6d in *Learning for Tomorrow's World – First Results from PISA 2003* (OECD, 2004a), presents the trends in mathematics/space and shape average performance between 2000 and 2003. The trend indicator has integrated the linking error in its standard error. Figure 2.6c in the same report presents the trends between 2000 and 2003 on the 5<sup>th</sup>, 10<sup>th</sup>, 25<sup>th</sup>, 75<sup>th</sup>, 90<sup>th</sup> and 95<sup>th</sup> percentiles and the linking error was not integrated into the standard error of the trends. Broadly speaking, the PISA 2003 initial report has integrated the linking error only in tables where the country mean performance is compared between 2000 and 2003.

Due to the growing interest in trend indicators and their political impacts, it is essential to interpret significant changes with caution. A significant change might simply be due to a difference in the school or student participation rate or in the pattern of missing data.

## Notes

1. The PISA 2000 database has been updated to integrate this new set of plausible values.
2. Actually, the linear transformation was applied on the plausible values before their transformation on the PISA scale with a mean of 500 and a standard deviation of 100. Further, different transformations were applied by gender (*i.e.* girls, boys and missing gender). The linear transformations per gender are: (i) girls:  $2000\_PVs = 0.0970 + (0.8739 * 2003\_PVs)$ , (ii) boys:  $2000\_PVs = 0.0204 + (0.8823 * 2003\_PVs)$ , (iii) missing gender:  $2000\_PVs = 0.0552 + (0.8830 * 2003\_PVs)$ . In science, the linear transformation is:  $2000\_PVs = -0.01552 + (1.0063 * 2003\_PVs)$ .



# Multilevel Analyses

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## INTRODUCTION

For the past two decades, education survey data have increasingly been analyzed with multilevel models. Indeed, since linear regression models fail to take into account the potential effects that may arise from the way in which students are assigned to schools or to classes within schools, they may give an incomplete or misleading representation of the efficiency of the education systems. In some countries, for instance, the socio-economic background of a student may partly determine the type of school that he or she attends and there may be little variation therefore in the socio-economic background of students within each school. In other countries or systems, schools may draw on students from a wide range of socio-economic backgrounds, but within the school, the socio-economic background of the student impacts the type of class he or she is allocated to and, as a result the within-school variance. A linear regression model that does not take into account the hierarchical structure of the data will not differentiate between these two systems.

The use of multilevel models (Goldstein, 1995), also called hierarchical linear models (Bryk and Raudenbush, 1992), acknowledges the fact that students are nested within classes and schools. The relative variation in the outcome measure – between students, within the same school and between schools – can therefore be evaluated.

## SIMPLE LINEAR REGRESSION

A linear equation can always be represented by a straight line. An equation with two variables will be represented in a two dimension space, and an equation with three variables, in a three dimensional space, and so on.

The following equation is graphically represented in Figure 13.1.

$$Y = 5 + 4X$$

Since all linear equations are represented by a straight line, it is only necessary to identify two points that belong to the line to be able to graph it. If  $X$  is equal to 1, then  $Y$  will be equal to 9. If  $X$  is equal to 10, then  $Y$  will be equal to 45. The straight line with the points (1, 9) and (10, 45) corresponds to the equation.

Figure 13.1 ■ Graphical representation of a linear equation

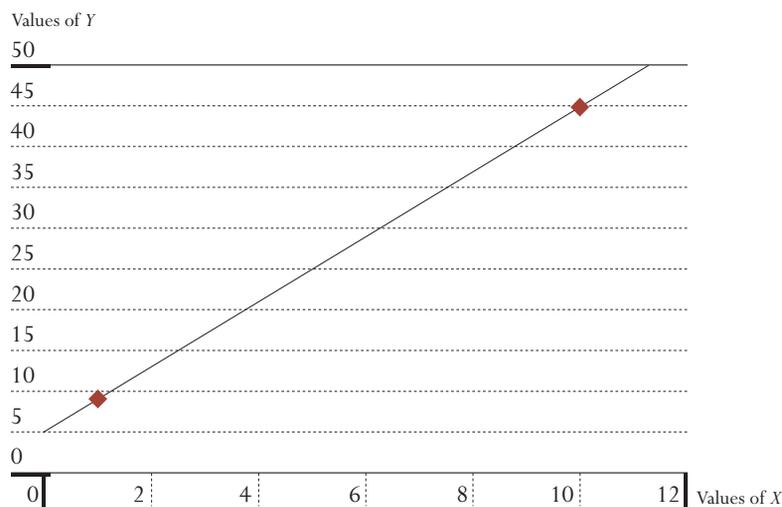




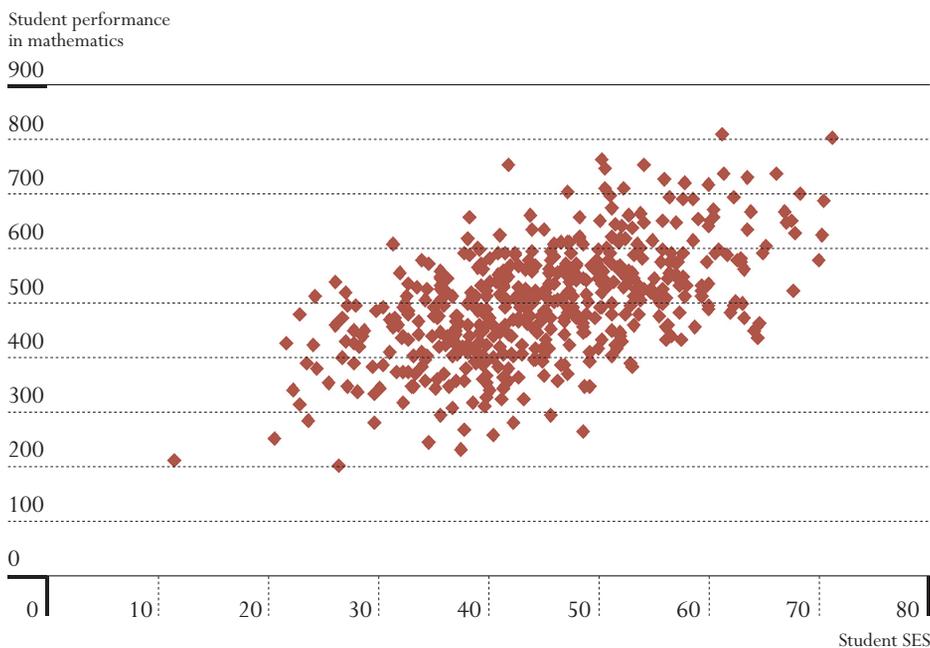
Figure 13.1 shows the graphical representation of the equation  $Y = 5 + 4X$ . As the figure shows, the line crosses the  $Y$  axis at 5. The point  $(0, 5)$  is called the intercept. It gives the value of  $Y$  when  $X$  is equal to 0. The  $X$  factor, or regression coefficient in statistical terms, gives the slope of the straight line. It tells us about the  $Y$  increase for an additional unit on the  $X$  axis. In the example considered, if  $X$  increases by one unit, then  $Y$  increases by four units.

The general expression of a linear equation with two variables is:

$$Y = a + bX, \text{ with } a \text{ the intercept and } b \text{ the regression coefficient.}$$

Although human processes can also be described with a similar approach, they are less deterministic. Let us graphically represent the relationship that might exist between the family socio-economic background (SES) of students and their academic performance at school.

**Figure 13.2 ■ Relationship between student socio-economic background and academic performance**



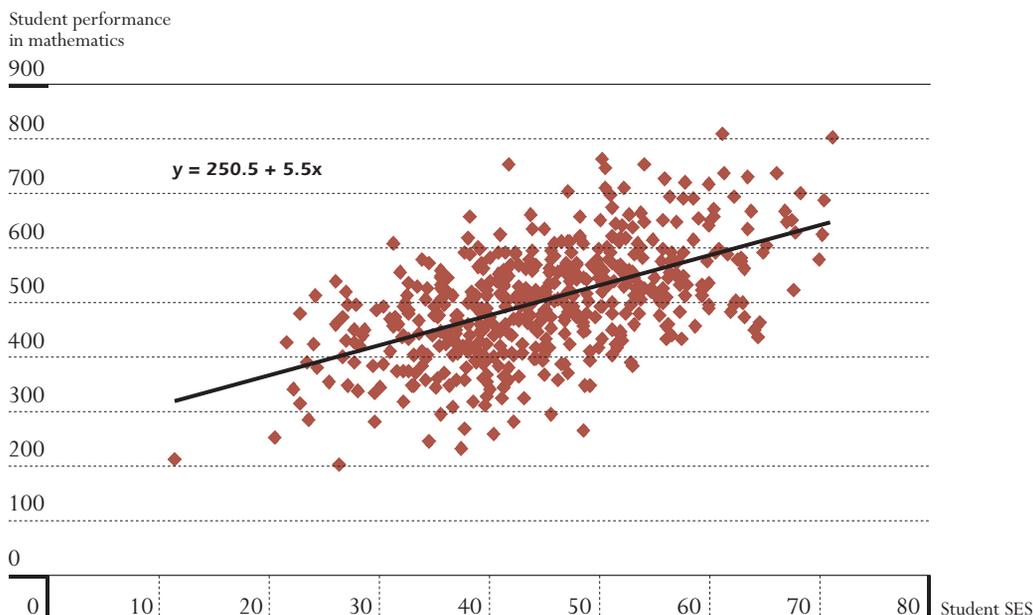
As Figure 13.2 shows, there is a positive relationship between the student's socio-economic background and the academic performance. Students from higher socio-economic backgrounds tend to perform better at school. However, unlike a linear equation, not all points are located on a straight line, meaning that students from a low socio-economic background may perform well academically, and that students from a high socio-economic background may perform poorly.

Statisticians use a linear regression analysis to quantify such relationships. The process in this particular example is similar to a linear equation with two variables. It will consist of computing an equation  $Y_i = \alpha + \beta X_i$ , with  $Y_i$  being the academic performance of student  $i$ , and  $X_i$  being his or her family socio-economic background. This equation can also be represented by a straight line denoted regression line.



The regression line in Figure 13.3 corresponds to the regression equation,  $Y_i = 250.5 + 5.5X_i$ . One measure of socio-economic status used for PISA 2000 and for PISA 2003 (Ganzeboom *et al.*, 1992) is the index of highest occupational status of parents called HISEI. This index ranges from 16 to 90 with an average of about 50 and a standard deviation of approximately 15. The performance in mathematics has an international mean of 500 and a standard deviation of 100. This equation shows that an increase of one unit on the HISEI scale is associated with an increase of 5.5 points on the PISA mathematics scale, on average.

**Figure 13.3 ■ Regression line of the socio-economic background on the student performance in mathematics**



This regression equation can also be used to predict the mathematics performance of a student if the socio-economic background is known. For instance, this regression equation will predict, for each student with a HISEI value of 60, a score of  $250.5 + (5.5 \times 60) = 580.5$ . In other words, any student with a HISEI of 60 will have a predicted score of 580.5. However, as shown by Figure 13.3, some of these students have a performance very close to this predicted score, usually denoted  $\hat{Y}_i$ , but the others either perform better, or at a lower level.

Before the computation of the regression equation, each student in the sample could be characterised by HISEI,  $X_i$ , and by performance in mathematics,  $Y_i$ . Now, each student can also be characterized by his or her predicted score,  $\hat{Y}_i$ , and by the difference between the observed score and predicted score ( $Y_i - \hat{Y}_i$ ), usually denoted as the residual (or  $\epsilon_i$ ).

**Table 13.1 ■ HISEI, mathematics performance, predicted score and residual**

Students	HISEI	Observed score	Predicted score	Residual
1	49	463	520	-57
2	53	384	542	-158
3	51	579	531	+48
4	42	404	481.5	-77.5
5	42	282	481.5	-199.5



The first student has an HISEI value of 49 and a mathematics performance of 463. Based on his or her socio-economic background, one would have predicted a score of 520. This student has, therefore, a lower performance than expected. The residual is equal to -57. On the other hand, the third student has a performance of 579 and an expected score of 531. This student performs better than expected.

Table 13.1 shows that the observed scores, the predicted scores and the residual scores present some variability on which variance coefficients can be computed. The regression equation and the regression line are constructed in a way that minimise the variance of the residual, denoted residual variance. This means that:

- The regression equation must include the point  $(\mu_x, \mu_y)$ ;
- The mean of the predicted score is equal to the mean of the observed score  $(\mu_{\hat{y}} = \mu_y)$ ; and
- The mean of the residual must be equal to 0.

Finally, a regression analysis can be extended to several explanatory variables. If  $k$  predictors are incorporated in the regression, then the equation will be written as:

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_k X_{ki}$$

For instance, the mathematics performance on the PISA test can be explained by the student family background, his or her gender, the time spent each week on homework, interest in mathematics and so on.

**Box 13.1 ■ Interpretation of a regression coefficient and an intercept**

A regression coefficient reflects the change of units on the Y axis (the dependent variable – in this particular case, the increase on the mathematics scale) per unit change of the X axis. The interpretation of a regression coefficient depends on the measurement unit of an independent variable. Therefore, the statistical effect of different independent variables can not be compared, unless these independent variables have the same measurement units.

To achieve this, independent variables can be standardised so that the measurement units become the standard deviation. If all variables have a standard deviation of 1, the regression coefficients of different variables can be directly compared. The regression coefficients will reflect the increase on the mathematic scale per standard deviation of the independent variables.

Suppose that two independent variables denoted X1 and X2 are used to explain the mathematical performance of students in two countries. The tables below provide the regression coefficients and the standard deviation of X1 and X2 before and after standardising the independent variables.

	Before standardisation				After standardisation			
	X1		X2		X1		X2	
	$\beta_1$	$\sigma_{(x_1)}$	$\beta_1$	$\sigma_{(x_2)}$	$\beta_1$	$\sigma_{(x_1)}$	$\beta_1$	$\sigma_{(x_2)}$
Country A	10	2	15	3	5	1	5	1
Country B	5	1	7.5	1.5	5	1	5	1

...



The results are quite different. Based on the regression coefficients after standardisation, it seems that the two independent variables have the same statistical effect on the mathematic performance in both countries. Assume that  $X_1$  represents the time spent at home for the homework. In country A, the increase of one hour spent on homework is associated with an increase of 10 points on the mathematic scale while in country B, an additional hour is associated with the increase of 5 points on the mathematic scale. While the standardisation of the variables allows comparisons, the interpretation of a particular regression coefficient becomes more complex as it does not refer anymore to the original scale.

Thus, there is no single algorithm to solve this problem. It depends on the nature of the independent variable and the purpose of the analyses.

The interpretation of the intercept is even more complex as it depends on the standard deviation and the mean of the independent variables. Let us suppose that HISEI is standardised to a mean of zero and a standard deviation of one. The regression coefficient would reflect the increase in mathematics per standard deviation on the socio-economic status scale. The intercept would therefore represent the performance of a student with a transformed HISEI score of 0. In a model with only standardised variables, it would reflect the performance of a hypothetical student who has average scores for all independent variables.

### **SIMPLE LINEAR VERSUS MULTILEVEL REGRESSION ANALYSES**

The previous simple linear regression has shown the relationship between socio-economic background and mathematics performance at the population level, *i.e.* the 15-year-olds attending an educational institution.

A relationship between the socio-economic background of the student and performance in mathematics does not necessarily imply that richer countries will have a higher performance average than developing countries. Further, the relationship observed at the student level across schools does not necessary imply that the same phenomenon will be identified within each school.

Multilevel regression analyses recognize that sampled units are nested within larger units. Instead of computing one regression equation on the whole dataset, the multilevel regression analysis will compute a regression equation per larger unit. In all education surveys, students are nested within schools. Therefore, a multilevel regression analysis will compute a regression equation per school.

Figure 13.4 shows four graphs that highlight the distinction between a linear regression and a multilevel linear regression model. These four graphs represent the relationship between student socio-economic backgrounds and mathematics performance estimates in different countries.

The thick black line represents the regression line when the hierarchical structure of the data is not taken into account. The thin red lines represent the relationship between these two variables within particular schools. For each school, there is a regression line (a red line in this particular example). The larger black point on the linear regression lines (black) represents the point with the mean of

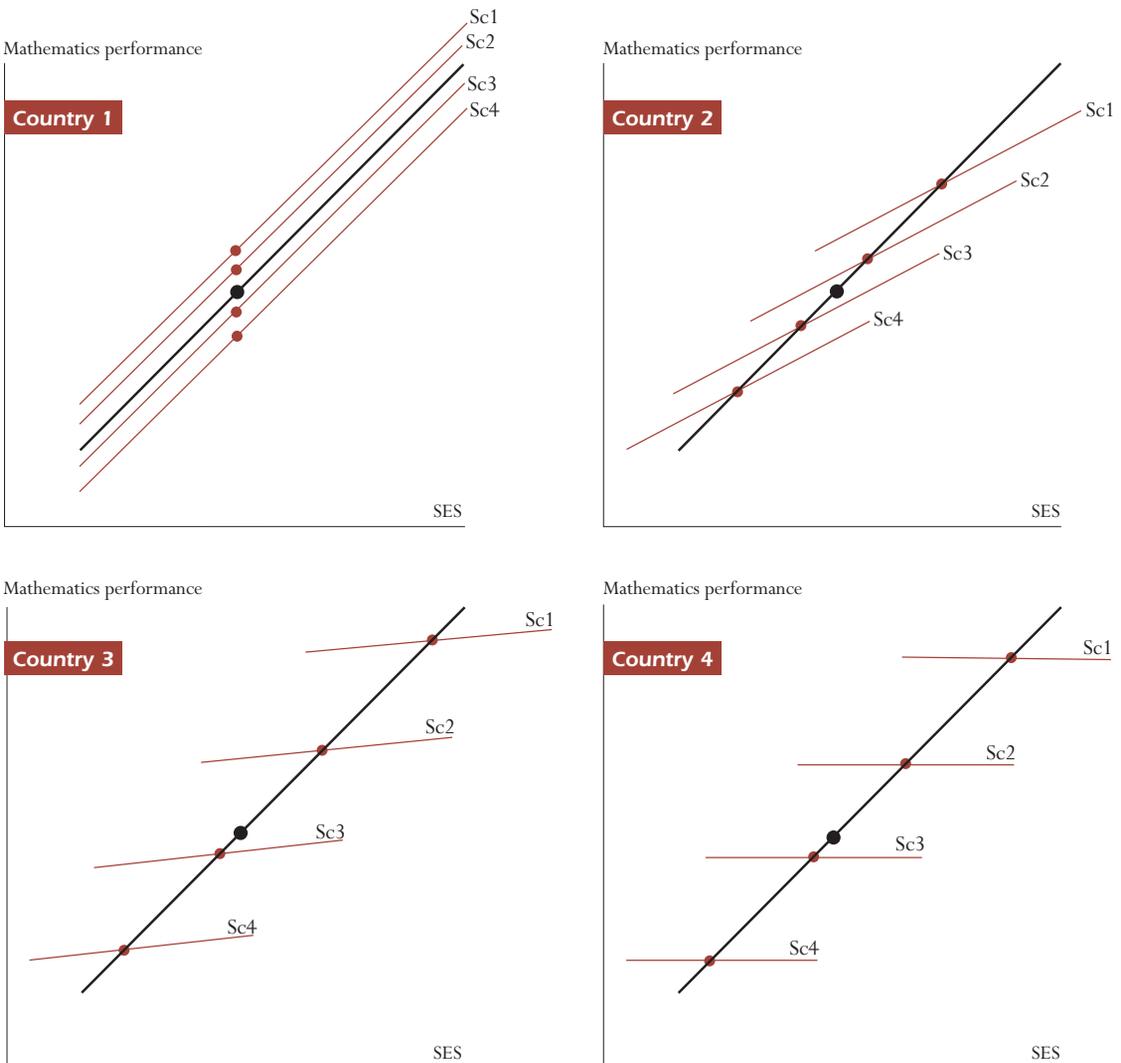


$X$  and  $Y$  as coordinates,  $(\mu_x, \mu_y)$ , and the red point on the multilevel regression lines represents the point with the school mean of  $X$  and  $Y$  as coordinates,  $(\mu_{xi}, \mu_{yi})$ .

The simple linear regression analysis, graphically represented by the black lines, shows that the expected score of a student from a higher socio-economic background is considerably higher than the expected score of a student from a lower socio-economic background. The comparison between the four graphs shows the similarity of the relationship between the student's socio-economic background and the student performance at that level between countries. Based on simple linear regression analyses, one would conclude that the relationship between the socio-economic background and the student performance is identical in the different countries.

However, the multilevel regression analyses clearly distinguish the relationship between the two variables in the four countries.

Figure 13.4 ■ Linear regression analysis versus multilevel regression analysis





In country 1, the multilevel regression lines are similar and close to the simple linear regression line. This means that:

- Regarding the socio-economic background of the student ( $X$  axis):
  - The different schools are all attended by students coming from a wide range of socio-economic backgrounds. All the within-school regression lines cover the whole range of values on the  $X$  axis; and
  - The schools have the same socio-economic intake, *i.e.* the mean of the student socio-economic background. Indeed, the projections of the red dots on the  $X$  axis are very close to each other.
- Regarding the student performance in mathematics ( $Y$  axis):
  - In each school, there are low, medium and high achievers. All the within-school regression lines cover the  $Y$  axis; and
  - On average, the schools have a similar level of performance. Indeed, the projections of the red dots on the  $Y$  axis are very close to each other. It also means that the between-school variance is quite small.
- Regarding the relationship between the socio-economic background and mathematics performance:
  - In each school, there is a strong relationship between socio-economic background and achievement. Within all schools, low socio-economic background students perform well below high socio-economic background students. The slope of the within-school regression line indicates the strength of the relationship.

Each school in country 1 can therefore be considered as a simple random sample of the population and each school reflects the relationships that exist at the population level.

The opposite of country 1 is graphically represented by country 4. The multilevel regression lines differ considerably from the simple linear regression line. In that particular case, it means that:

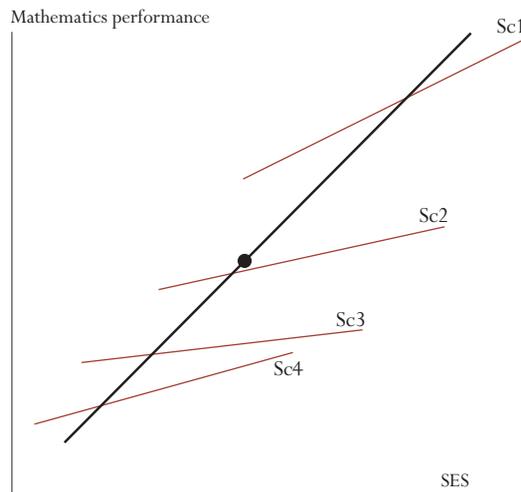
- Regarding the socio-economic background of the student ( $X$  axis):
  - The schools do not cover the range of socio-economic backgrounds that exist at the population level. School 1 is mainly attended by high socio-economic background students while school 4 is mainly attended by low socio-economic background students; and
  - The schools have therefore different socio-economic intakes as the projections of the red dots on the  $X$  axis would show. In other words, there is a significant socio-economic segregation at the school level.
- Regarding the student performance in mathematics ( $Y$  axis):
  - The schools do not cover the range of the student performance that exists at the population level. School 1 is mainly attended by high achievers and school 4 is mainly attended by low achievers; and
  - Schools largely differ by their average performance level, as the projections of the red dots on the  $Y$  axis would show. In country 4, the school performance variance is therefore very important.
- Regarding the relationship between the socio-economic background and mathematics performance:
  - In each school, there is no relationship between the socio-economic background and achievement.
  - Within a particular school, the socio-economic background of the student does not matter. What does matter is not the socio-economic background of the student but the school he or she will attend. But the socio-economic background of the student will determine the school he or she will attend.

Countries 2 and 3 present intermediate situations between these two extreme examples.

## FIXED EFFECT VERSUS RANDOM EFFECT

For the cases examined so far, the within-school regression lines were all parallel, but multilevel regression analyses also allow the regression slope to vary. In the former, the effect, *i.e.* the  $X$  effect, will be considered as fixed, while in the latter, the effect will be considered as random. Figure 13.5 represents a case with a random effect.

Figure 13.5 ■ A random multilevel model



Mathematically, in the case of one explanatory variable, the two models can be differentiated as follows:

$$Y_{ij} = \alpha_j + \beta X_{ij} + \varepsilon_{ij} \text{ for a fixed effect}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

and

$$Y_{ij} = \alpha_j + \beta_j X_{ij} + \varepsilon_{ij} \text{ for a random effect}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

$$\beta_j = \gamma_{10} + U_{1j}$$

The subscript  $i$  in the equations refers to the student<sup>1</sup> (also denoted level 1 in the multilevel model literature), and the subscript  $j$  refers to the school (or level 2). In an equation, the presence of the subscript  $j$  for a regression coefficient means that it can vary from one school to another.

The term  $\varepsilon_{ij}$  denotes the residual of the equation, *i.e.* the difference between the observed score  $Y_{ij}$  and the predicted score  $\hat{Y}_{ij}$ . This residual is normally distributed with a mean of 0 and a constant level 1 (*i.e.* the student level) variance, usually denoted  $\sigma^2$ .

As shown by these two equations, the intercept  $\alpha_j$  is always considered as a random effect. Considering the intercept as a fixed parameter would reduce the multilevel model to a linear regression analysis. The intercept  $\alpha_j$  can further be divided into a fixed part, *i.e.*  $\gamma_{00}$  denotes the overall intercept and is equal to the average of the school intercepts  $\alpha_j$ , and secondly into a random part, *i.e.*  $U_{0j}$ , denoting school departure from the overall intercept. This school departure  $U_{0j}$  is assumed to have a mean of 0 and a variance  $\tau_0^2$ .



The  $\beta$  coefficient in the first equation has no subscript  $j$ , meaning that the effect  $X$  cannot vary from one school to the other. The regression lines are therefore parallel and thus the  $X$  effect is considered as fixed. On the other hand, the  $\beta$  coefficient in the second equation has a subscript  $j$ , meaning that it can vary from one school to another. The regression lines are no longer parallel and thus the  $X$  effect is now considered as random. As previously, this regression coefficient  $\beta_j$  can be divided into a fixed part and a random part. The fixed part  $\gamma_{10}$  is called overall regression coefficient and corresponds to the mean of the regression coefficients  $\beta_j$ . The random part  $U_{1j}$  is the school departure from the overall regression coefficient. It has a mean of 0 and a variance denoted  $\tau_1^2$ .

Random effects and fixed effects can be combined in a single multilevel regression analysis. For instance, in the following equation, two student explanatory variables are introduced in the model, one is considered as fixed,  $X_1$ , and the other one as random,  $X_2$ .

$$Y_{ij} = \alpha_j + \beta_1 X_{1ij} + \beta_2 X_{2ij}$$

### SOME EXAMPLES WITH SPSS®

Usually, two types of indices are relevant in multilevel analyses: the regression coefficients and the decomposition of the variance into the different levels, *i.e.* the student level (or level 1) and school level (or level 2).

Multilevel regression analyses always report the residual variance at the different levels – the between-school variance and the within-school variance that are not explained by the predictors included in the model.

However, scientific reports usually show the explained variance. The conversion of the residual variance into percentages of explained variance just requires the comparison of the school and student variance coefficients with their respective residual variance coefficients.

#### Example 1

The decomposition of the total variance can be easily obtained with a multilevel regression model. Applying the following model

$$Y_{ij} = \alpha_j + \varepsilon_{ij}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

will provide unbiased estimates of the between-school variance and the within-school variance. As the regression model has no predictors, the school intercepts, *i.e.*  $\alpha_j$  will therefore be equal or close to the school means. The variance of  $U_{0j}$  will be equal to the between-school variance. As each student will be assigned his or her school mean as predicted score, the variance of  $\varepsilon_{ij}$  will be equal to the within-school variance.

SPSS® offers two procedures for multi-level analyses: VARCOMP enables researchers to do (multi-level) variance decomposition, MIXED is a procedure, which can be used for multi-level modelling. The SPSS® MIXED and VARCOMP procedures allow for modelling multilevel regression. However, both procedures require the standardisation of the weights, *i.e.* the sum of the weights is equal to the number of students in the dataset. If the BY statement is used, then the standardisation will be done by category of the breakdown variable.



Box 13.2 provides the SPSS® syntax for this standardisation, as well as a short checking procedure.<sup>2</sup> Please note that in order to write variance estimates into an output file, the VARCOMP procedures need to be used (see Box 13.3).

### Box 13.2 ■ Standardisation of the PISA 2003 final weights

```
GET FILE 'c:\PISA\Data2003\INT_stui_2003.sav'.
SORT CASES BY cnt schoolid stdstd.

*** COMPUTE STANDARDISED WEIGHT ***.

WEIGHT OFF.
AGGREGATE OUTFILE= 'c:\temp\templ.sav'
  /BREAK = cnt
  /popwgt=SUM(w_fstuwt)
  /smpsize=NU.
EXEC.

MATCH FILE FILE=*
  /TABLE = "c:\temp\templ.sav"
  /BY cnt.
EXEC.

COMPUTE std_wgt=(w_fstuwt/popwgt)*smpsize.
EXEC.

*** VERIFICATION ***.

WEIGHT OFF.
FREQ cnt.
WEIGHT BY std_wgt.
FREQ cnt.

WEIGHT OFF.

SAVE OUTFILE = 'c:\temp\INT_stui_2003.sav'.
```

Box 13.3 provides the SPSS® syntax for a multilevel regression model as well as the SPSS® syntax for the computation of the intra-class correlation.

### Box 13.3 ■ SPSS® syntax for a multilevel regression model – Example 1

```
*** VARCOMP ***.

WEIGHT OFF.
SPLIT FILE BY CNT.
VARCOMP
  pvlmath BY schoolid
  /RANDOM = schoolid
  /METHOD = ML
  /OUTFILE = VAREST("c:\temp\decompvar.sav")
  /REGWGT = std_wgt
  /INTERCEPT = INCLUDE .
SPLIT FILE OFF.

GET FILE "c:\temp\decompvar.sav".
COMPUTE rho=vc1/(vc1+vc2).
FREQ rho.
RENAME VARS (vc1 vc2 = intcept residual).
SAVE OUTFILE = "c:\temp\rho.sav"/ KEEP cnt intcept residual rho.
```



The `RANDOM` statement defines the second level of the analyses. The first statement following the name of the procedure (either `VARCOMP` or `MIXED`) specifies the model, including the dependent and then the predictor variables following the keyword `WITH`. The `VARCOMP` procedure also requires to define the second level in the model statement using the `BY` keyword (for example: `pv1math BY schoolid WITH hisei`). In this particular example, there is no predictor included in the model statement. Therefore the school and within-school residual variances will be equal to the school and within-school variance estimates. The `RANDOM` statement distinguishes between fixed and random predictors, as explained in the previous section. It should be noted that when using the `MIXED` procedure “intercept” always needs to be mentioned. The `REGWGT` statement should be followed by the normalised student-level weight. In order to get results by country the command should be preceded by the `SPLIT FILE BY` (grouping variable) statement and followed by `SPLIT FILE OFF`. The `VARCOMP PROCEDURE` allows to include an `OUTFILE` statement which allows to write the variance estimates into another SPSS® system file. The variance estimates will be saved in the file “*decompvar.sav*”. Further options for `OUTFILE` statement in `VARCOMP` are `COVB` (covariance matrix of variance estimates) and `CORB` (correlation matrix of variance estimates).

Table 13.2 provides the between-school and within-school variance estimates and the intra-class correlation. These variance estimates were saved in the file “*decompvar.sav*”. As shown in Box 13.3, the intra-class correlation<sup>5</sup> is equal to:

$$\rho = \frac{\sigma_{\text{between-school}}^2}{\sigma_{\text{between-school}}^2 + \sigma_{\text{within-school}}^2} = \frac{\tau_0^2}{\tau_0^2 + \sigma^2}$$

with  $\sigma_{\text{school}}^2$  or  $\tau_0^2$  the between-school variance and  $\sigma_{\text{within-school}}^2$  or  $\sigma^2$  the within-school variance. In Australia, the between-school variance is equal to 1919.11<sup>4</sup> and the within-school variance is equal to 7169.09. The intra-class correlation is therefore the percentage of the total variance that is accounted for by the school. It reflects how schools differ in their student average performance. In Australia, the intra-class correlation is therefore equal to  $1919.11 / (1919.11 + 7169.09) = 0.21$ . The estimate of the intra-class correlation ranges from 0.04 in Iceland to 0.63 in the Netherlands.

## Example 2

The following examples are based on the data of Luxembourg. The school sample size in Luxembourg, *i.e.* 29, will allow the presentation of the school parameter estimates. In Example 2, the socio-economic background of the student, denoted HISEI, is introduced as a fixed factor.

### Preparation of the data file

In the PISA databases, there are no missing data for the final weight and for the student performance estimate. However, there are missing values for variables that might be used as predictors in a multilevel regression model. These missing data generate two major issues:

- The sum of the weights will slightly differ from the number of cases that will be used by the regression models. Note that cases with missing values are automatically<sup>5</sup> dropped from any regression models.
- The school and student variances from different models cannot be compared as missing values are not always random. For instance, low socio-economic background students are usually less likely to provide answers about their mother’s and/or father’s occupations.



**Table 13.2 ■ Between- and within-school variance estimates and intra-class correlation**

Country	Between-school variance	Within-school variance	rho
AUS	1919.11	7169.09	0.21
AUT	5296.65	4299.71	0.55
BEL	7328.47	5738.33	0.56
BRA	4128.49	5173.60	0.44
CAN	1261.58	6250.12	0.17
CHE	3092.60	6198.65	0.33
CZE	4972.45	4557.50	0.52
DEU	6206.92	4498.70	0.58
DNK	1109.45	7357.14	0.13
ESP	1476.85	6081.74	0.20
FIN	336.24	6664.98	0.05
FRA	3822.62	4536.22	0.46
GBR	1881.09	6338.25	0.23
GRC	3387.52	5991.75	0.36
HKG	4675.30	5298.26	0.47
HUN	5688.56	4034.66	0.59
IDN	2769.48	3343.87	0.45
IRL	1246.70	6110.71	0.17
ISL	337.56	7849.99	0.04
ITA	4922.84	4426.67	0.53
JPN	5387.17	4668.82	0.54
KOR	3531.75	5011.56	0.41
LIE	3385.41	5154.08	0.40
LUX	2596.36	5806.97	0.31
LVA	1750.22	6156.52	0.22
MAC	1416.99	6449.96	0.18
MEX	2476.01	3916.46	0.39
NLD	5528.99	3326.09	0.62
NOR	599.49	7986.58	0.07
NZL	1740.61	7969.97	0.18
POL	1033.90	7151.46	0.13
PRT	2647.70	5151.93	0.34
RUS	2656.62	6021.44	0.31
SVK	3734.56	4873.69	0.43
SWE	986.03	8199.46	0.11
THA	2609.38	4387.08	0.37
TUN	2821.00	3825.36	0.42
TUR	6188.40	4891.13	0.56
URY	4457.08	5858.42	0.43
USA	2395.38	6731.45	0.26
YUG	2646.00	4661.59	0.36

To avoid these two problems, it is recommended to delete any cases with missing data for the different predictors that will be used in the regression models before the weight standardisation. As the next multilevel regression model examples will use two student level variables, *i.e.* HISEI for the student socio-economic background, and ST03Q01 for the student gender and two school level variables, *i.e.* the percentage of girls in the school, PCGIRLS, and the type of schools, SCHLTYPE, cases with missing data for at least one of these four variables will be deleted before the weight standardisation.



Box 13.4 presents the SPSS<sup>®</sup> syntax. It consists of:

- Merging the student data file and the school data file with the variables of interest;
- Deleting the cases with at least one missing data for the predictor; and
- Standardising the weight.

Before deletion of cases with missing values, there are 3 923 records in the Luxembourg database. After deletion, 3 782 are left. About 3.5 per cent of the cases are deleted. If too many cases are deleted, for instance, more than 10 per cent, then either the variables with too many missing values should be dropped from the analyses, or imputation methods should be used.

### Rerunning the empty multilevel model

After deletion of cases with missing values with the syntax in Box 13.4, the empty multilevel model, *i.e.* a multilevel regression model without any predictor in Box 13.3, is run to obtain the between-school and within-school variance estimates. The between-school and the within-school variance estimates, saved in the “*decompvar.sav*” file, are now respectively equal to 2 563.30 and 5 734.35 instead of 2 596.36 and 5 806.97.

#### Box 13.4 ■ SPSS<sup>®</sup> syntax for standardising PISA 2003 final weights with deletion of cases with missing values in Luxembourg

```
GET FILE "c:\PISA\Data2003\INT_stui_2003.sav".
SELECT IF (cnt = 'LUX').
EXEC.

SORT CASES BY cnt schoolid stdstd.

SAVE OUTFILE = "c:\temp\LUXstui2003.sav"
  /KEEP cnt schoolid stdstd w_fstuwt pvlmath hisei st03q01.

GET FILE "c:\PISA\Data2003\INT_schi_2003.sav".
SELECT IF (cnt = 'LUX').
EXEC.

SORT CASES BY cnt schoolid.

SAVE OUTFILE = "c:\temp\LUXsch2003.sav"
  /KEEP cnt schoolid schltype pcgirls.

MATCH FILE FILE = "c:\temp\LUXstui2003.sav"/
  /TABLE = "c:\temp\LUXsch2003.sav"
  /BY CNT SCHOOLID.
EXEC.

COUNT nbmiss = hisei st03q01 schltype (missing).

SELECT IF (nbmiss = 0).
EXEC.

*** COMPUTE STANDARDISED WEIGHT ***.

WEIGHT OFF.
AGGREGATE OUTFILE= 'c:\temp\templ.sav'
  /BREAK = cnt
  /popwgt=SUM(w_fstuwt)
  /smpsize=NU.
EXEC.

MATCH FILES FILE=*
  /TABLE = "c:\temp\templ.sav"
  /BY cnt.
EXEC.

COMPUTE std_wgt=(w_fstuwt/popwgt)*smpsize.
EXEC.
```



Box 13.4 (continued) ■ SPSS® syntax for standardising PISA 2003 final weights with deletion of cases with missing values in Luxembourg

```
*** VERIFICATION ***.

WEIGHT OFF.
FREQ cnt.
WEIGHT BY std_wgt.
FREQ cnt.
WEIGHT OFF.

SAVE OUTFILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath
      /FIXED = intercept
  / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept | SUBJECT(schoolid)
      / REGWGT=std_wgt
      /SAVE = FIXPRED PRED.
```

Estimating the “empty” model with the MIXED will have only one fixed parameter  $\gamma_{00}$ , which is 492.36 for the Luxembourg data.

SPSS® unfortunately does not have output files with the random parameters at the second level units. With an empty model, these random parameters would only include the school departure  $U_{0j}$ . Table 13.3 is a printout of a random parameter file which was computed using the SAS program. It contains:

- The breakdown variables used in the model, *i.e.* CNT;
- The effect, *i.e.* the intercept or as it will be shown later, the random predictor, the estimate;
- The class variable, *i.e.* the SCHOOLID;
- The estimate;
- The standard error on the estimate;
- The number of degrees of freedom (the number of students minus the number of schools);
- The *t* statistic; and
- The probability that the estimates differ from 0.

For instance, the departure of the school 00001 from the overall intercept 492.36 is only 0.71. This departure does not differ from 0, as shown by the *t* statistic and its associated probability value. In other words, the intercept of school 00001 is not significantly different from the overall intercept. On the other hand, the intercept of school 00002 is significantly higher than the overall intercept.

### Shrinkage factor

In the case of an empty model, one might consider that the sum of the overall intercept  $\gamma_{00}$  and a particular school departure  $U_{0j}$  should be perfectly equal to the school performance mean.

Multilevel models shrink the school departures. To illustrate this shrinkage process, suppose we have an educational system with 100 schools. Assume that the school performance means are perfectly identical. In other words, the between-school variance is equal to 0. If 20 students are tested within each school, it is expected that school mean estimates will slightly differ from the school means.



Table 13.3 ■ Printout of the random parameter file as computed with SAS®

CNT	Effect	SCHOOLid	Estimate	StdErrPred	DF	tValue	Probt
LUX	Intercept	00001	0.71	13.00	3753	0.05	0.96
LUX	Intercept	00002	66.39	11.63	3753	5.71	0.00
LUX	Intercept	00003	-23.71	11.03	3753	-2.15	0.03
LUX	Intercept	00004	-44.68	12.18	3753	-3.67	0.00
LUX	Intercept	00005	-8.56	10.68	3753	-0.80	0.42
LUX	Intercept	00006	61.90	11.34	3753	5.46	0.00
LUX	Intercept	00007	-68.69	12.39	3753	-5.54	0.00
LUX	Intercept	00008	61.14	11.62	3753	5.26	0.00
LUX	Intercept	00009	81.64	11.10	3753	7.36	0.00
LUX	Intercept	00010	-62.00	11.37	3753	-5.45	0.00
LUX	Intercept	00011	33.19	25.14	3753	1.32	0.19
LUX	Intercept	00012	-11.35	12.54	3753	-0.91	0.37
LUX	Intercept	00013	15.56	10.47	3753	1.49	0.14
LUX	Intercept	00014	8.01	11.25	3753	0.71	0.48
LUX	Intercept	00015	37.55	12.36	3753	3.04	0.00
LUX	Intercept	00016	-46.59	10.95	3753	-4.26	0.00
LUX	Intercept	00017	-33.61	10.98	3753	-3.06	0.00
LUX	Intercept	00018	-76.02	12.54	3753	-6.06	0.00
LUX	Intercept	00019	-70.43	12.96	3753	-5.43	0.00
LUX	Intercept	00020	57.54	11.17	3753	5.15	0.00
LUX	Intercept	00021	8.04	11.01	3753	0.73	0.47
LUX	Intercept	00022	-0.67	25.14	3753	-0.03	0.98
LUX	Intercept	00023	84.27	10.90	3753	7.73	0.00
LUX	Intercept	00024	29.88	11.12	3753	2.69	0.01
LUX	Intercept	00025	63.74	11.69	3753	5.45	0.00
LUX	Intercept	00026	-33.65	11.15	3753	-3.02	0.00
LUX	Intercept	00027	-8.29	11.53	3753	-0.72	0.47
LUX	Intercept	00028	-36.89	13.84	3753	-2.66	0.01
LUX	Intercept	00029	-84.43	10.96	3753	-7.71	0.00

Indeed, within particular schools, predominantly high achievers or low achievers may be sampled so that the school mean is respectively overestimated or underestimated. As the number of sampled students within schools increases, the difference between the school mean and its estimate is likely to decrease. Therefore, the shrinkage factor is inversely proportional to the number of sampled students within schools.

The shrinkage factor<sup>6</sup> is equal to:

$$\frac{n_j \sigma_{\text{between-school}}^2}{n_j \sigma_{\text{between-school}}^2 + \sigma_{\text{within-school}}^2},$$

with  $n_j$  being the number of students in school  $j$  in the sample (Goldstein, 1997).



Table 13.4 presents, for each school, the average performance in mathematics, the number of students used in the multilevel regression model, the departure from the overall intercept estimated by the empty multilevel regression model, as presented in Table 13.3 and the sum of the overall intercept  $\gamma_{00}$  and the school departure  $U_{0j}$ . Box 13.5 shows how to compute Table 13.4 using SPSS® (results in output file “schoolmeans.sav”).

**Box 13.5 ■ SPSS® syntax for a multilevel regression model – Example 2 (1)**

```

GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath
    /FIXED = intercept
    / PRINT = G SOLUTION
    / METHOD = ML
    / RANDOM = intercept | SUBJECT(schoolid)
    / REGWGT=std_wgt
    /SAVE = FIXPRED PRED.
compute schresid = PRED_1 - FXPRED_1.
exec.

weight by std_wgt.
aggregate outfile = "c:\temp\schoolmeans.sav" /
    break = schoolid /
    schmn dep fxpred pred = mean (pvlmath, schresid, FXPRED_1, PRED_1 )/
    nstud=nu.
exec.

```

**Table 13.4 ■ School performance in mathematics, number of students per school and corrected mean**

School	School mean	Number of students	Departure $U_{0j}$	$\gamma_{00} + U_{0j}$
00001	493.1	67	0.7	493.1
00002	560.0	120	66.4	558.8
00003	468.3	179	-23.7	468.6
00004	446.6	94	-44.7	447.7
00005	483.7	233	-8.6	483.8
00006	555.2	146	61.9	554.3
00007	421.8	83	-68.7	423.7
00008	554.6	116	61.1	553.5
00009	575.1	167	81.6	574.0
00010	429.4	131	-62.0	430.4
00011	535.2	8	33.2	525.6
00012	480.7	78	-11.3	481.0
00013	508.0	289	15.6	507.9
00014	500.5	150	8.0	500.4
00015	530.9	87	37.6	529.9
00016	445.2	184	-46.6	445.8
00017	458.3	183	-33.6	458.8
00018	414.2	73	-76.0	416.3
00019	419.6	66	-70.4	421.9
00020	550.7	162	57.5	549.9
00021	500.5	174	8.0	500.4
00022	491.5	8	-0.7	491.7
00023	577.6	185	84.3	576.6
00024	522.7	169	29.9	522.2
00025	557.3	117	63.7	556.1
00026	458.2	151	-33.7	458.7
00027	483.9	126	-8.3	484.1
00028	453.9	53	-36.9	455.5
00029	406.9	183	-84.4	407.9



As shown, the difference between the school performance mean and the sum  $\gamma_{00} + U_{0j}$  is:

- Proportional to the school departure, *i.e.* the shrinkage factor mainly affects low and high performing schools; and
- Inversely proportional to the number of observed students in the school.

**Introduction of HISEI as a fixed effect**

With the introduction of the student level variable HISEI as a fixed effect, the equation can be written as:

$$Y_{ij} = \alpha_j + \beta_1(HISEI)_{ij} + \varepsilon_{ij}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

The SPSS® syntax for this model is presented in Box 13.6 and parts of the SPSS® output is presented in Box 13.7.

**Box 13.6 ■ SPSS® syntax for a multilevel regression model – Example 2 (2)**

```
GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath WITH hisei
      /FIXED = intercept hisei
      / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept | SUBJECT(schoolid)
      / REGWGT=std_wgt.
```

**Box 13.7 ■ SPSS® output – Example 2**

**Estimates of Covariance Parameters<sup>a, b</sup>**

Parameter	Estimate	Std. Error
Residual	5551.5060563	128.1500411
Intercept [subject = SCHOOLID] Variance	1950.3945680	530.9974935

**Estimates of Fixed Effects<sup>a, b</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	446.7649734	9.2614577	43.341	48.239	.000	428.0917086	465.4382383
HISEI	.9479007	.0823676	3780.841	11.508	.000	.7864114	1.1093899

a. Dependent variable: plausible value in math.  
b. Residual is weighted by std\_wgt.

Only one change has been introduced in comparison with the syntax presented in Box 13.5. The name HISEI has been added to the model statement.



The overall intercept  $\gamma_{00}$  is now equal to 446.76 and the within-school regression coefficient  $\beta_1$  is equal to 0.9479. This means that, within a particular school, an increase of 1 unit on the HISEI scale will be associated with an increase of 0.9479 on the mathematics scale. By comparison, the linear regression coefficient of HISEI on the mathematics performance is equal to 2.05. The relationship between SES and student achievement in Luxembourg education system seems to be similar to the one in the hypothetical examples for country 2 or as country 3 in Figure 13.4.

The between-school and within-school residual variable estimates, respectively denoted  $\tau_0^2, \sigma^2$  are equal to 1 949.09 and 5 551.53.

The percentage of variance explained by the HISEI variable can be computed as:

$$1 - \frac{1949.09}{2563.07} = 0.24 \text{ at the school level and}$$

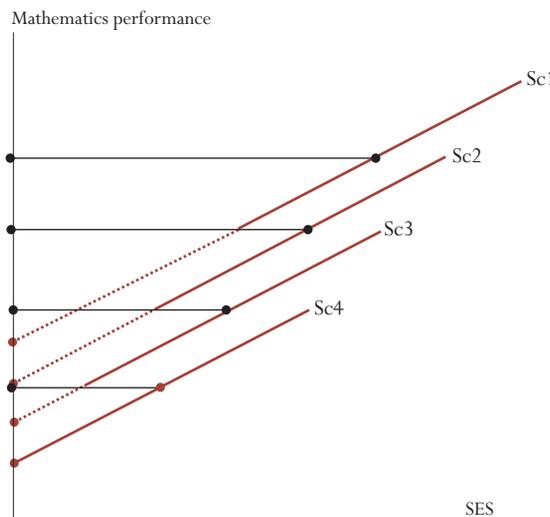
$$1 - \frac{5551.53}{5734.39} = 0.03 \text{ at the student level.}$$

How can a student level variable explain about 24 per cent of the between-school variance and only 3 per cent of the within-school variance? This mainly reflects the school socio-economic background segregation. Some of the Luxembourg schools are mainly attended by students with high socio-economic backgrounds while some other schools are mainly attended by students with low socio-economic backgrounds.

Figure 13.6 provides a graphical explanation of this phenomenon. The between-school variance in any case can be graphically represented by the variability of the school intercepts on the  $Y$  axis.

Note that the between-school variance can be obtained by an empty multilevel regression model. In that particular case, the intercept is close to the orthogonal projection of the school performance average on the  $Y$  axis, as shown by the black line in Figure 13.6. As explained in the previous section, the difference between the school mean and the intercept results from the application of the shrinkage factor.

Figure 13.6 ■ Graphical representation of the between-school variance reduction





The between-school residual variance can be obtained by the extension of the regression line on the Y axis, as shown by the red discontinuous line in Figure 13.6. As one can see, the range of the black intercepts is larger than the range of the red intercepts.

Broadly speaking, a student level variable will have an impact on the between-school variance if:

- Schools differ in the mean and the range of students in regard with that variable (see countries 2, 3 and 4) in Figure 13.4; and
- The within-school regression coefficient of that variable differs from 0. The case of country 4 in Figure 13.4 illustrates a case where using the HISEI variable at the student level in the model will not reduce the between-school variance. On the other hand, the introduction of the school socio-economic intake, *i.e.* the school HISEI mean, will have in country 4 a substantial impact on the between-school variance.

**Example 3**

Example 3 is similar to Example 2, except that HISEI is now considered as a random effect. The SPSS® syntax is presented in Box 13.8. The equation can therefore be written as:

$$Y_{ij} = \alpha_j + \beta_{1j}(HISEI)_{ij} + \varepsilon_{ij}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

**Box 13.8 ■ SPSS® syntax for a multilevel regression model – Example 3**

```
GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath WITH hisei
      /FIXED = intercept hisei
      / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept hisei | SUBJECT(schoolid)
      / REGWGT=std_wgt.
```

The variable HISEI has been added to the random statement.

The fixed parameter file contains the overall intercept  $\gamma_{00}$  and HISEI overall regression coefficient  $\gamma_{10}$ . Like the school intercepts which are divided into two parts, an overall intercept and a school departure, the within-school regression coefficient is divided into two parts: an overall regression coefficient (the fixed part, denoted  $\gamma_{10}$ ) and a school regression coefficient departure (the random part, denoted  $U_{1j}$ ).

The overall intercept and regression coefficient are presented in Table 13.5. The overall intercept is equal to 449.59 and the overall HISEI regression coefficient is equal to 0.89. As shown by the *t* statistic and its associated probability, both parameters are significantly different from 0.

**Table 13.5 ■ Fixed parameter output**

CNT	Effect	Estimate	StdErrPred	tValue	Probt
LUX	Intercept	449.59	9.69	46.39	0.00
LUX	HISEI	0.89	0.11	8.17	0.00



The random parameter file lists the school departures:

- $U_{0j}$  from the intercept  $\gamma_{00}$ , *i.e.* 449.59 ; and
- $U_{1j}$  from HISEI regression coefficient  $\gamma_{10}$ , *i.e.* 0.89.

As HISEI is now considered as a random effect, it is meaningless to interpret the school departure from the overall intercept. Table 13.6 presents the school departure from the overall HISEI regression coefficient for the first 13 schools.<sup>7</sup>

**Table 13.6 ■ Random parameter output computed with SAS®**

CNT	Effect	SCHOOL	Estimate	StdErrPred	DF	tValue	Probt
LUX	HISEI	00001	0.22	0.31	3724	0.71	0.48
LUX	HISEI	00002	0.04	0.26	3724	0.15	0.88
LUX	HISEI	00003	0.29	0.26	3724	1.13	0.26
LUX	HISEI	00004	-0.51	0.29	3724	-1.75	0.08
LUX	HISEI	00005	-0.08	0.25	3724	-0.31	0.76
LUX	HISEI	00006	0.07	0.28	3724	0.26	0.79
LUX	HISEI	00007	-0.04	0.29	3724	-0.13	0.90
LUX	HISEI	00008	-0.13	0.27	3724	-0.49	0.62
LUX	HISEI	00009	-0.29	0.25	3724	-1.19	0.23
LUX	HISEI	00010	-0.17	0.26	3724	-0.65	0.52
LUX	HISEI	00011	0.07	0.34	3724	0.19	0.85
LUX	HISEI	00012	-0.04	0.28	3724	-0.14	0.89
LUX	HISEI	00013	0.82	0.22	3724	3.66	0.00

The HISEI regression coefficient for school 00001 is equal to  $0.89+0.22=1.11$ , but it cannot be considered as significantly different from the overall intercept. Out of the 13 schools presented in Table 13.6, only school 00013 presents a regression coefficient that significantly differs from the overall coefficient, as shown by the  $t$  statistics of the probability. The HISEI regression coefficient is equal to  $0.89+0.82=1.71$  and as shown by the  $t$  statistic or the probability, this within-school regression coefficient is significantly different from the overall regression coefficient.

SPSS now provides three variance estimates:

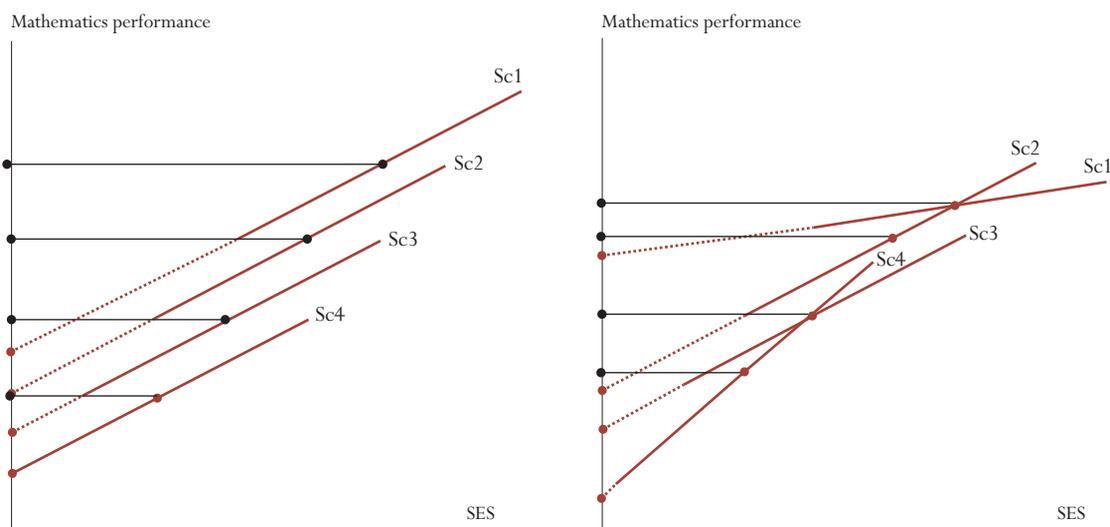
- The between-school residual variance  $\tau_0^2$ , *i.e.* 2 147.64;
- The within-school residual variance  $\sigma^2$ , *i.e.* 5 509.34; and
- The variance of HISEI regression coefficients  $\tau_1^2$ , *i.e.* 0.1275. This is also the variability of the regression coefficient departure.

In comparison with Example 2, the between-school residual variance has slightly increased and the within-school residual variance has slightly decreased. The reduction of the within-school variance is not surprising as the random effect can only better fit the data.

Figure 13.7 helps to understand the increase of the between-school residual variance. The regression coefficient for school 00001 (Sc1) is slightly less steep so that the extension of the regression line will be higher than previously on the  $Y$  axis. Further, the regression coefficient is slightly steeper for school 00004 (Sc4), so that the extension of the regression line will be a bit lower on the  $Y$  axis.



Figure 13.7 ■ Reduction of the between-school residual variance for a fixed and a random model



**Example 4**

In Example 4, the student gender, denoted ST03Q01 in the PISA database, is added as a fixed factor to the previous model. The equation can be written as:

$$Y_{ij} = \alpha_j + \beta_{1j} (HISEI)_{ij} + \beta_2 (ST03Q01)_{ij}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

Box 13.9 presents the SPSS® syntax.

**Box 13.9 ■ SPSS® syntax for a multilevel regression model - Example 4**

```
GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath WITH hisei st03q01
      /FIXED = intercept hisei st03q01
      / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept hisei | SUBJECT(schoolid)
      / REGWGT=std_wgt.
```

The fixed parameters are respectively equal to 419.68 for the overall intercept, 0.86 for the overall HISEI regression coefficient and 20.7927 for the overall gender coefficient.

The between-school residual variance  $\tau_0^2$  is equal to 2 167.41 and the within-school residual variance  $\sigma^2$  is equal to 5 415.34. Finally, the variance of the school HISEI regression coefficient  $\tau_1^2$  is equal to 0.1313.

This model explains  $1 - \frac{2167.41}{2563.07} = 15.3$  per cent of the between-school variance and  $1 - \frac{5415.34}{5734.39} = 5.6$  per cent of the within-school variance.

The gender regression coefficient of 20.8 reflects the expected gender difference within any school, after controlling for HISEI.



**Box 13.9 ■ Interpretation of the within-school regression coefficient**

The expected within-school gender difference can greatly differ from the overall gender difference, especially in a highly tracked system. It appears that girls are more likely to attend an academic track while boys are more likely to attend a vocational track. The linear regression coefficient of gender on the student performance does not take into account this differential attendance. If the different tracks are organised by different schools, as in Germany for instance, a multilevel regression model will take this differential attendance into account, so that the gender multilevel regression coefficient will substantially be different from the linear regression coefficient. The table below provides the linear and multilevel regression coefficients for gender on the German PISA 2003 data.

At the population level, boys outperform girls by 8.9 in mathematics while girls outperform boys by 42.1 in reading. But within a particular school, the expected differences in mathematics and in reading are respectively equal to 30.7 and -19.3.

**Gender differences in Germany**

	Mathematics	Reading
Simple linear regression coefficient	8.9	-42.1
Multilevel regression coefficient	30.7	-19.3

Gender can also be considered as a random factor. The equation can therefore be written as:

$$Y_{ij} = \alpha_j + \beta_{1j}(HISEI)_{ij} + \beta_{2j}(ST03Q01)_{ij}$$

$$\alpha_j = \gamma_{00} + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

$$\beta_{2j} = \gamma_{20} + U_{2j}$$

Box 13.11 presents the variance estimate of the random parameters as well as the regression coefficient estimates of the fixed parts of the model.

**Box 13.11 ■ SPSS® output – Example 4**

**Estimates of Fixed Effects<sup>a, b</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.
Intercept	419.3613485	10.0172881	43.829	41.864	.000
HISEI	.8606827	.1097952	38.918	7.839	.000
ST03Q01	21.0238222	3.1530028	31.424	6.668	.000

**Estimates of Covariance Parameters<sup>a, b</sup>**

Parameter		Estimate	Std. Error
Residual		5400.8730832	125.3934221
Intercept [subject = SCHOOLID]	Variance	1904.7788816	613.6594064
HISEI [subject = SCHOOLID]	Variance	.1348145	.0714020
ST03Q01 [subject = SCHOOLID]	Variance	70.5719428	56.7028751

a. Dependent variable: plausible value in mathematics.

b. Residual is weighted by std\_wgt.



As shown in Box 13.11, the variability of  $U_{2j}$ , i.e. the school departure on the gender regression coefficient is quite large. This indicates that the gender differences vary from one school to another.

**Example 5**

The last equation in Example 4 was  $Y_{ij} = \alpha_j + \beta_{1j} (HISEI)_{ij} + \beta_{2j} (ST03Q01)_{ij}$ . This equation mainly models the student performance variability within schools by introducing student level predictors. However, due to the segregation effect, these student level predictors can explain some of the between-school variance.

It is also possible to introduce a predictor school level variable. Suppose that one is interested in the effect of the school type on the school mean performance. The equation can be written as:

$$Y = \alpha_j + \beta_{1j}(HISEI)_{ij} + \beta_{2j}(ST03Q01)_{ij} + \varepsilon_{ij}$$

$$\alpha_j = \gamma_{00} + \gamma_{01}(SCHLTYPE)_j + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + U_{1j}$$

$$\beta_{2j} = \gamma_{20} + U_{2j}$$

In other words, as the school type variable is identical for all students within a particular school, this variable will only have an effect on the school intercepts. Given the socio-economic background and the gender composition of the schools, does the school type explain why some schools perform better than expected and why some schools perform at a lower level than expected?

The SPSS® syntax is presented in Box 13.12.

**Box 13.12 ■ SPSS® syntax for a multilevel regression model - Example 5 (1)**

```
GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath WITH hisei st03q01 schltype
      /FIXED = intercept hisei st03q01 schltype
      / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept hisei | SUBJECT(schoolid)
      / REGWGT=std_wgt.
```

Table 13.7 presents the results for the fixed parameters.

**Table 13.7 ■ Fixed parameters – Example 5**

CNT	Effect	Estimate	StdErrPred	DF	tValue	Probt
LUX	Intercept	320.47	66.69	27	4.81	0.00
LUX	HISEI	0.86	0.11	28	7.84	0.00
LUX	ST03Q01	20.69	2.59	3723	7.98	0.00
LUX	SCHLTYPE	35.14	23.36	3723	1.50	0.13

As shown by Table 13.7, the school type variable is not significant. In other words, it cannot be stated that government-dependent private schools differ from public schools once the student socio-economic background and the student gender is controlled.



**Example 6**

The model can finally be extended by trying to understand why the school HISEI and ST03Q01 regression coefficients vary. Two hypotheses to test are:

- The HISEI regression coefficients differ between public school and private government: dependent schools, and
- The ST03Q01 regression coefficients is related to the percentage of boys and girls in the school.

The equation can be written as:

$$Y_{ij} = \alpha_j + \beta_{1j}(HISEI)_{ij} + \beta_{2j}(ST03Q01)_{ij} + \epsilon_{ij}$$

$$\alpha_j = \gamma_{00} + \gamma_{01}(SCHLTYPE)_j + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SCHLTYPE)_j + U_{1j}$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}(PCGIRLS)_j + U_{2j}$$

Box 13.13 presents the SPSS® syntax for running this model. Testing whether the HISEI regression coefficients differ according to the school type is similar to testing the interaction between the school type and the HISEI regression coefficients. Therefore, in SPSS, the term “hisei\*schltype” has to be added in the FIXED statement, as well as “st03q01\*pcgirls”. Please note that the model statement following the procedure name lists “schltype” and “pcgirls” without the interaction terms.

**Box 13.13 ■ SPSS® syntax for a multilevel regression model - Example 5 (2)**

```
GET FILE = "c:\temp\LUX2003.sav".
weight off.
MIXED pvlmath WITH hisei st03q01 schltype pcgirls
      /FIXED = intercept hisei st03q01 schltype hisei*schltype
      st03q01*pcgirls
      / PRINT = G SOLUTION
      / METHOD = ML
      / RANDOM = intercept hisei st03q01| SUBJECT(schoolid)
      / REGWGT=std_wgt.
```

**Box 13.14 ■ SPSS® output – Example 5**

Estimates of Fixed Effects <sup>a, b</sup>					
Parameter	Estimate	Std. Error	df	t	Sig.
Intercept	291.4022663	71.2240565	39.885	4.091	.000
HISEI	1.8607710	.9019687	51.684	2.063	.044
ST03Q01	19.8852339	11.7315332	50.628	1.695	.096
SCHLTYPE	45.1271348	24.8610903	39.203	1.815	.077
HISEI * SCHLTYPE	-.3504893	.3137546	50.175	-1.117	.269
ST03Q01 * PCGIRLS	2.6190012	24.8681643	49.929	.105	.917

Estimates of Covariance Parameters <sup>a, b</sup>			
Parameter		Estimate	Std. Error
Residual		5397.4444256	125.3180631
Intercept [subject = SCHOOLID]	Variance	1757.3290219	578.2565538
HISEI [subject = SCHOOLID]	Variance	.1426724	.0735291
ST03Q01 [subject = SCHOOLID]	Variance	71.0821703	56.3930823

<sup>a</sup>. Dependent variable: plausible value in mathematics.  
<sup>b</sup>. Residual is weighted by std\_wgt.



Table 13.8 presents the fixed parameters in relation to the equation. As shown in the SPSS® output in Box 13.14, the random HISEI regression coefficient is not significantly associated with the school type.

**Table 13.8 ■ Fixed parameter estimates – Example 6**

Effect	Coefficient estimate	Coefficient
Intercept	291.40	$\gamma_{00}$
HISEI	1.86	$\gamma_{10}$
ST03Q01	19.88	$\gamma_{20}$
SCHLTYPE	45.12	$\gamma_{01}$
HISEI*SCHLTYPE	-0.35	$\gamma_{11}$
ST03Q01*PCGIRLS	2.62	$\gamma_{21}$

**Table 13.9 ■ Random parameter variance estimates - Example 6**

Effect	Variance estimate	Coefficient
Intercept	1757.37	$U_{0j}$
HISEI	0.1427	$U_{1j}$
ST03Q01	71.0154	$U_{2j}$
Residual	5397.46	$\epsilon_{ij}$

As shown by the reported probability, both null hypotheses have to be accepted, *i.e.* the school type is not associated with the HISEI slopes and the within-school gender difference is not associated with the percentage of girls in the school.

### LIMITATIONS OF THE MULTILEVEL MODEL IN THE PISA CONTEXT

This section aims to alert potential PISA data users of the limitations or the dangers of such models in the PISA context.

Such models are designed to decompose the student variance into:

- The between-school variance,
- The within-school variance, and
- The within class variance.

As PISA draws, per participating school, a random sample of an age population across grades and across classes, it allows the decomposition of the variance into two levels: a between-school variance and a within-school variance. Further, the overall variance is expected to be larger with an age sample than with a grade sample, unless the age population is attending a single grade, as in Iceland or Japan.

To allow meaningful international comparisons, these types of indicators require a common definition for a school and for a class. While there are no major issues on what a student is, there are from one country to another important differences between what a school is and what a class is.

International surveys in education are primarily interested in the student sample and therefore one might consider the school sample as a necessary step to draw an efficient sample of students that



minimizes the cost of testing. In this context, the definition of what a school is or what a class is, does not present any major issues. However, the increasing importance and popularity of multilevel analyses calls for more attention on these definition issues.

PISA 2000 and PISA 2003 do not give a detailed definition of a school. The emphasis in the sampling procedures was on developing a list of units that would guarantee full coverage of the enrolled 15-year-old population and that would additionally give acceptable response rates. Once a school was selected, it also had to be practical to sample 35 students or so from that school to assess them. Thus, the school frame was constructed with issues of student coverage and practical implementation of PISA administration in mind, rather than analytic considerations. Therefore, in the PISA databases, it is possible that the school identification represents different educational institutions that may not be comparable without any restriction. For instance, in some PISA countries, schools are defined as administrative units that may consist of several buildings not necessarily located close together. Other countries used the building as the school sampling unit and finally, a few countries defined a school as a track within a particular building. It is likely that the larger these aggregates are, the smaller the differences between these aggregates will be and the larger the differences within these aggregates will be. In this context, one would expect to observe high intra-class correlations in these countries and a nonsignificant within-school regression coefficient for the student socio-economic background (Kirsch *et al.*, 2002).

Besides this problem of an international definition of a school, data users have to be aware of the following issues:

- The choice of a school definition in a particular country may be dictated by the availability of the data. Indeed, the national centres have to include a measure of size of the 15-year-old population in the school sample frame (see Chapter 2). This information may be available at the administrative unit level, but not at the building level. In federal countries that count several educational systems, the available data might differ from one system to the other, so that the concept of a school might differ even within a particular country.
- For practical or operational reasons, the concept of schools might differ between two PISA data collections. For instance, some countries used the administrative units in the PISA 2000 school sample frame and the building units in the PISA 2003 school sample frame. Such changes were implemented to increase the school participation rate. These conceptual changes will influence the results of any variance decomposition and might also affect the outcomes of multilevel models. Moving from an administrative definition to a building definition will increase the intra-class correlation and should decrease the slope of the within-school regression coefficient. If such changes occur in a country, it is strongly advised not to compute any trends on variance decomposition or multilevel regressions.

As this example shows, multilevel analyses and variance decomposition analyses need to be interpreted in the light of:

- The structure of the educational systems; and
- The school definition used in the school sample frame.

Under the limitations provided in this section, multilevel regression analyses are certainly suitable and appropriate to describe how students are assigned to schools and what the major criteria are for such assignment. However, 10 or even 20 student and school variables will never be able to



model the complexity of an educational system. Further, PISA is measuring a cumulative process of about ten years of schooling. What we are doing today can certainly not explain what we are today. Consequently, the pedagogical practices and the school environment in which 15-year-olds are currently learning is unable to fully explain how these students perform today. In this context, policy recommendations should be made and interpreted with caution.

## CONCLUSIONS

This chapter firstly describes the concept of multilevel analyses and how to perform such models with SPSS®. It starts with the simplest model, denoted the empty model, and then progressively adds complexity by adding variables. Finally, in the PISA context, important methodological issues that limit the international comparability of the results have been discussed.

### Notes

---

1. For consistency with the literature on multilevel regression, subscripts  $i$  and  $j$  have been inverted in comparison with Chapter 2.
2. Multiplying the full student weight `W_FSTUWT` with the variable `CNTFAC2` produces the same weights (`COMPUTE std_wgt = w_fstuw*cntfac2`) as the syntax in Box 13.2. But the resulting standardised weights should only be used for multi-level models based on variables without any missing values. When estimating multi-level models including variables with missing values different standardised weight should be computed after deleting all cases with missing values.
3. See also Table 4.4 in Chapter 4.
4. Note that the results obtained from the SPSS® syntaxes in this chapter sometimes may differ from the examples in the text of this chapter, which were estimated using the SAS® program. These minor deviations are due to differences in the algorithms used by different statistical software packages.
5. A correlation matrix computed with the pairwise deletion option can however be used as input for a linear regression analysis.
6. This shrinkage factor has to be associated to the expected school mean square in an ANOVA model. Indeed,
$$E(MS_{school}) = n_j \sigma_{\text{between-school}}^2 + \sigma_{\text{within-school}}^2.$$
7. Note that this output file is not available with SPSS®; the results are presented here for the purpose of illustration and were computed with SAS®.



# Other Statistical Issues

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## INTRODUCTION

The PISA 2000 and the PISA 2003 initial reports included descriptions of the relationship between questionnaire indices and student performance by dividing the questionnaire indices into quarters and then reporting the mean achievement per quarter. The PISA reports also include the statistical concepts of relative risk and attributable risk. This chapter is devoted to these two specific issues.

## ANALYSES BY QUARTERS

As described in Chapter 4, the indices derived from the questionnaire data were generated with the Rasch model and students' estimates were reported with the WLEs. As previously mentioned, a WLE individual's estimate is a discontinuous variable.

Table 14.1 presents the distribution of the questionnaire index *interest in and enjoyment of mathematics* from the German PISA 2003 data set. This table clearly shows the discontinuous character of the variable.

To divide a questionnaire index into quarters, the 25<sup>th</sup>, 50<sup>th</sup>, and 75<sup>th</sup> percentiles have to be computed. These percentiles are respectively -0.6369, 0.029, and 0.973 for the index of *interest in and enjoyment of mathematics* for Germany.

There are two possible recoding procedures: lower versus equal or greater and equal or lower versus greater.

The SPSS<sup>®</sup> syntax is presented in Box 14.1.

### Box 14.1 ■ Two SPSS<sup>®</sup> syntax procedures for the recoding into quarters

```
get file 'C:\PISA\Data2003\int_stui_2003.sav'.
Select if cnt='DEU'.
Weight by w_fstuw.
FREQ VARIABLES=intmat /PERCENTILES= 25 50 75.

if (intmat < -0.6369) q1=1.
if (intmat >= -0.6369 and intmat < 0.029) q1=2.
if (intmat >= 0.029 and intmat < 0.973) q1=3.
if (intmat >= 0.973) q1=4.

if (intmat <= -0.6369) q2=1.
if (intmat > -0.6369 and intmat <= 0.029) q2=2.
if (intmat > 0.029 and intmat <= 0.973) q2=3.
if (intmat > 0.973) q2=4.
freq q1 q2.
```

Depending on the procedure adopted, the percentages of students in the bottom quarter, second quarter, third quarter, and top quarter are respectively equal to 24.88, 21.39, 27.80 and 25.93 or 34.53, 21.60, 25.33 and 18.54.

Neither of these two procedures generate quarters that precisely include 25 per cent of the students. Since the percentages of students in each quarter can vary among countries, no international comparisons can be performed.



**Table 14.1 ■ Distribution of the questionnaire index interest in and enjoyment of mathematics for Germany**

WLE	Percentage	Cumulative percentage	WLE	Percentage	Cumulative percentage
-1.783	10.20	10.20	0.477	0.10	64.30
-1.733	0.02	10.23	0.643	0.10	64.40
-1.700	0.02	10.25	0.643	9.53	73.93
-1.469	0.02	10.27	0.869	0.03	73.96
-1.258	7.53	17.80	0.912	0.04	74.00
-1.179	0.02	17.82	0.925	0.05	74.05
-1.147	0.02	17.85	0.940	0.02	74.07
-1.077	0.03	17.88	0.973	7.39	81.46
-0.971	0.08	17.95	1.044	0.03	81.49
-0.929	6.77	24.73	1.146	0.03	81.52
-0.739	0.15	24.88	1.299	5.27	86.79
-0.637	9.66	34.53	1.338	0.02	86.81
-0.619	0.13	34.66	1.346	0.04	86.85
-0.370	0.02	34.68	1.464	0.02	86.87
-0.335	0.07	34.74	1.568	0.04	86.91
-0.319	11.37	46.11	1.587	4.58	91.49
-0.250	0.01	46.13	1.702	0.01	91.51
-0.160	0.10	46.22	1.761	0.02	91.53
-0.045	0.05	46.27	1.792	0.04	91.57
0.029	9.86	56.13	1.817	0.05	91.62
0.057	0.04	56.17	1.827	0.03	91.64
0.068	0.08	56.25	1.891	4.72	96.37
0.132	0.07	56.32	2.091	0.04	96.41
0.229	0.06	56.39	2.119	0.02	96.43
0.300	0.02	56.41	2.161	0.07	96.50
0.345	7.75	64.15	2.335	0.04	96.54
0.448	0.02	64.17	2.373	3.46	100.00
0.462	0.02	64.20			

It was therefore necessary to distribute the students with a WLE equal to one of the 3 percentiles into the two respective adjacent quarters. For instance, 7.39 per cent of the students get a score equal to percentile 75. As 74.07 per cent of the students get a lower score, it is necessary to sample 0.93 per cent of the students with a score equal to percentile 75 and allocate them to the third quarter. The remaining 6.46 per cent will be allocated to the fourth quarter.

This random subsampling process is implemented by adding a small random variable to the questionnaire index. That random noise will generate more categories and therefore the three new percentiles will be able to divide the index variable into quarters that exactly include 25 per cent of



the students. Box 14.2 presents the SPSS<sup>®</sup> syntax for the addition of a random variable, as well as the computation of the percentiles and the recoding into quarters.

**Box 14.2 ■ SPSS<sup>®</sup> syntax for the questionnaire indices recoding into quarters**

```
get file 'C:\PISA\Data2003\int_stui_2003.sav'.
Weight by w_fstuw.
Select if cnt='DEU'.
Set seed=1.
compute newindex=intmat+rv.normal(0,.01).

FREQ VARIABLES=newindex /FORMAT=NOTABLE
  /PERCENTILES= 25 50 75.

if (newindex < -0.655) quart =1.
if (newindex >= -0.655 and newindex < 0.0263) quart =2.
if (newindex >= 0.0263 and newindex < 0.9622) quart =3.
if (newindex >= 0.9622) quart =4.

freq quart.
```

The outcomes of the FREQUENCY procedure will demonstrate that 25 per cent are allocated to each quarter.

This random allocation of some parts of the population to one of the four quarters adds an error component to the standard error. Indeed, in our example, the composition of the 0.93 per cent of the students allocated to the third quarter and the composition of the remaining 6.46 per cent allocated to the fourth quarter might differ between two runs of the procedure (unless the seed is set to a certain integer, like in Box 14.2).

To account for this new error component, the statistical approach adopted for the analyses of plausible values can be implemented. It will therefore consist of:

- Computing for each student a set of five plausible quarters;
- Per plausible quarter, computing the required statistic and its respective sampling variance by using the final and 80 replicate weights;
- Averaging the five estimates and their respective sampling variances;
- Computing the imputation variance; and
- Combining the sampling variance and the imputation variance to obtain the final error variance.

If the dependent variable is a set of plausible values, the procedure described in Chapter 6 will be used, except that each plausible value will be analyzed with a different plausible quarter. Box 14.3 presents the SPSS<sup>®</sup> syntax for the computation of the average mathematics performance per quarter of any questionnaire derived index.

The different steps of this procedure are:

1. From the initial questionnaire index, five new variables are created by adding a random number;
2. For each new variable, the 25<sup>th</sup>, 50<sup>th</sup> and 75<sup>th</sup> percentiles are computed and then imported in the temporary data file;



**Box 14.3 ■ SPSS® syntax for the computation of the average mathematics performance per quarter of a questionnaire derived index**

```

get file 'C:\PISA\Data2003\int_stui_2003.sav'.
Select if cnt='DEU'.
Save outfile='c:\pisa\data2003\DEU.sav'.

* DEFINE MACRO.
Include file 'C:\PISA\macros\mcr_SE_PV_WLEQRT.sps'.

* CALL MACRO.
PVWLEQRT      nrep = 80/
               stat = mean/
               pv = math/
               wle = intmat/
               grp = cnt/
               wgt = w_fstuwt/
               rwgt = w_fstr/
               cons = 0.05/
               infile = 'c:\pisa\data2003\DEU.sav' /.

* CHECK QUARTERS.
get file='C:\temp\quarters.sav'.
weight by w_fstuwt.
FREQUENCIES
  VARIABLES= intmat1 intmat2 intmat3 intmat4 intmat5 /FORMAT=NOTABLE
  /PERCENTILES= 25 50 75 /ORDER= ANALYSIS .
MEANS intmat1 by quart1 /CELLS MIN MAX COUNT NPCT.
MEANS intmat2 by quart2 /CELLS MIN MAX COUNT NPCT.
MEANS intmat3 by quart3 /CELLS MIN MAX COUNT NPCT.
MEANS intmat4 by quart4 /CELLS MIN MAX COUNT NPCT.
MEANS intmat5 by quart5 /CELLS MIN MAX COUNT NPCT.
weight off.

```

3. The five new variables are compared with their respective percentiles and the quarter allocations are saved in five categorical variables (these 10 new variables are saved in the temporary data file 'C:\temp\quarters.sav');
4. The statistic is computed for each plausible value of mathematics performance by one of the five new categorical variables; and
5. The final estimate and the final standard error are computed.

The results from these steps are presented in Table 14.2.

**Table 14.2 ■ Mean scores and standard errors on the mathematics scale for each quarter of index of interest in and enjoyment of mathematics**

CNT	quart (INTMAT)	stat (MATH)	SE
DEU	1	493	4.90
DEU	2	511	4.01
DEU	3	520	4.67
DEU	4	524	4.69



## THE CONCEPTS OF RELATIVE RISK AND ATTRIBUTABLE RISK

### Relative risk

The notion of relative risk is a measure of association between an antecedent factor and an outcome factor (Cornfield, 1951). The relative risk is simply the ratio of two risks, *i.e.* the risk of observing the outcome when the antecedent is present, and the risk of observing the outcome when the antecedent is not present. Table 14.3 presents the notation that will be used.

Table 14.3 ■ Labels used in a two-way table

		Outcome measure		
		Yes	No	Total
Antecedent measure	Yes	$P_{11}$	$P_{12}$	$P_{1.}$
	No	$P_{21}$	$P_{22}$	$P_{2.}$
	Total	$P_{.1}$	$P_{.2}$	$P_{..}$

$p_{..}$  is equal to  $\frac{n_{..}}{n}$ , with  $n_{..}$  the total number of students and  $p_{..}$  is therefore equal to 1,  $p_{.i}$ ,  $p_{.j}$  respectively represent the marginal probabilities for each row and for each column. The marginal probabilities are equal to the marginal frequencies divided by the total number of students. Finally, the  $p_{ij}$  values represent the probabilities for each cell and are equal to the number of observations in a particular cell divided by the total number of observations.

In this document, the conventions for the two-way table will be the following:

- The rows represent the antecedent factor with:
  - The first row for having the antecedent; and
  - The second row for “not having the antecedent”.
- The columns represent the outcome with:
  - The first column for having the outcome; and
  - The second column for not having the outcome.

In these conditions, the relative risk is equal to:

$$RR = \frac{(p_{11} / p_{1.})}{(p_{21} / p_{2.})}$$

Let us suppose that a psychologist wants to analyse the risk of a student repeating a grade if the parents recently divorced. The psychologist draws a simple random sample of students of grade 10. In this particular example, the outcome variable is present if the child is repeating grade 10 and the antecedent factor is considered present if the student’s parents divorced in the past two years. The results he obtained are presented in Table 14.4 and in Table 14.5.



Table 14.4 ■ Distribution of 100 students by marital status of the parents and grade repetition

	Repeat the grade	Not repeat the grade	Total
Parents divorced	10	10	20
Parents not divorced	5	75	80
Total	15	85	100

Table 14.5 ■ Probabilities by marital status of the parents and grade repetition

	Repeat the grade	Not repeat the grade	Total
Parents divorced	0.10	0.10	0.20
Parents not divorced	0.05	0.75	0.80
Total	0.15	0.85	1.00

The relative risk is therefore equal to:

$$RR = \frac{(p_{11}/p_{1.})}{(p_{21}/p_{2.})} = \frac{(0.10/0.20)}{(0.05/0.80)} = \frac{0.5}{0.0625} = 8$$

This means that the probability of repeating grade 10 is eight times larger if the parents recently divorced than if they had not recently divorced.

### Attributable risk

The attributable risk is equal to:

$$AR = \frac{(p_{11}p_{22}) - (p_{12}p_{21})}{(p_{.1}p_{2.})}$$

In the previous example, the attributable risk is equal to:

$$AR = \frac{(p_{11}p_{22}) - (p_{12}p_{21})}{(p_{.1}p_{2.})} = \frac{(0.10 \times 0.75) - (0.10 \times 0.05)}{(0.15 \times 0.80)} = 0.583$$

The attributable risk is interpreted as follows. If the risk factor could be eliminated, then the rate of occurrence of the outcome characteristic in the population would be reduced by this coefficient. With the next version of the formula, the meaning of the attributable risk, *i.e.* a reduction of the outcome if the risk factor disappears, is more obvious.

$$AR = \frac{(p_{.1}) - (p_{21}/p_{2.})}{(p_{.1})}$$

The expression  $p_{.1}$  represents the proportion of children in the whole sample with the outcome. The expression  $(p_{21}/p_{2.})$  represents the proportion of children who are not at risk, but nevertheless suffer from the outcome. The difference of these two proportions provides the absolute reduction if the risk was eliminated. Dividing this difference by the first expression transforms this absolute reduction into a relative reduction or a reduction expressed as a percentage.



These two formulae give the same coefficient:

$$AR = \frac{(p_{.1}) - (p_{21} / p_{2.})}{(p_{.1})} = \frac{(0.15) - (0.05 / 0.80)}{(0.15)} = 0.583$$

To express this result as a percentage, the coefficient needs to be multiplied by 100.

### INSTABILITY OF THE RELATIVE AND ATTRIBUTABLE RISKS

The relative risk and the attributable risk were developed for dichotomous variables. More and more often, these two coefficients are extended and are used with continuous variables. To apply the coefficients to continuous variables, a cutpoint for each variable needs to be set and the continuous variables need to be dichotomised.

It is important to recognise that when applied to dichotomised variables, the computed values of the relative risk and the attributable risk will depend on the value of the chosen cutpoint.

To demonstrate the influence of the cutpoint on the relative and attributable risks, two random variables were generated with a correlation of 0.30. These two variables were then transformed into dichotomous variables by using respectively the 10<sup>th</sup>, 15<sup>th</sup>, 20<sup>th</sup>, 25<sup>th</sup> and 30<sup>th</sup> percentiles as cutpoints. Table 14.6 presents the relative risk and the attributable risk for a range of choices for the cutpoints.

Table 14.6 ■ Relative risk and attributable risk for different cutpoints

Percentile	Relative risk	Attributable risk
10	2.64	0.13
15	2.32	0.16
20	1.90	0.15
25	1.73	0.15
30	1.63	0.15

Table 14.6 shows that the relative risk and, to a lesser extent, the attributable risk coefficients are dependent on the setting of the cutpoints, and therefore the interpretation of the value needs to be made in the light of this observation.

Such a comparison of the relative and attributable risks was computed for the PISA data to identify the changes depending on the cutpoint location. The antecedent factor was the mother's educational level and the outcome variable was reading achievement. Low reading achievement (having the outcome) was successively defined within countries as being below the 10<sup>th</sup>, 15<sup>th</sup>, 20<sup>th</sup>, 25<sup>th</sup>, 30<sup>th</sup> and 35<sup>th</sup> percentiles.

The relative risks for these different cutpoints are respectively on average (across OECD countries) equal to 2.20, 1.92, 1.75, 1.62, 1.53, and 1.46. The attributable risks are equal to 0.25, 0.21, 0.19, 0.17, 0.15, and 0.14, respectively.

Nevertheless, the correlations between the different relative risks and attributable risks are rather high, as shown in Table 14.7



**Table 14.7 ■ Correlation between relative risks and attributable risks at the 10<sup>th</sup> percentile with the 15<sup>th</sup>, 20<sup>th</sup>, 25<sup>th</sup> 30<sup>th</sup> and 35<sup>th</sup> percentiles**

	RR	AR
P15	0.96	0.98
P20	0.93	0.97
P25	0.92	0.96
P30	0.90	0.94
P35	0.87	0.92

In PISA, it was decided to use the 25<sup>th</sup> percentile as the cutpoint for continuous variables when calculating relative and attributable risks.

### COMPUTATION OF THE RELATIVE RISK AND ATTRIBUTABLE RISK

Depending on the variables involved in the computation of the relative risk and attributable risk, the procedure might differ. Indeed, these two statistical concepts require as input two dichotomous variables, such as gender (ST03Q01).

However, most of the variables in the PISA data bases are not dichotomous; they are categorical or continuous variables.

The recoding of a categorical into a dichotomous variable does not raise special issues. From a theoretical point of view, the purpose of the comparison needs to be decided upon, and the recoding will follow. For instance, in PISA 2000, the education levels of the parents are reported by using the ISCED classification (OECD, 1999b). If the contrast is on the distinction between tertiary versus non tertiary education, then the categorical variable can be recoded into a dichotomous variable. Students whose parents do not have a tertiary qualification will be considered at risk.

Numerical variables also have to be recoded into dichotomous variables. As stated earlier, the OECD has decided to divide numerical variables based on the 25<sup>th</sup> percentile.

In the PISA 2000 and PISA 2003 databases, all numerical variables, except the performance scales, are discontinuous variables. To ensure that the 25<sup>th</sup> percentile will divide the variables into two categories that will include, respectively, 25 and 75 per cent, a random variable has to be added to the initial variable, as described in the section devoted to the analyses per quarter. Five relative risk and/or five attributable risk estimates are computed and then combined.

Finally, if plausible values are involved as outcome measures, then five estimates will also be computed and then combined. However, it is not necessary to add a random variable to the initial variable as it constitutes a continuous variable.

### CONCLUSIONS

This chapter was devoted to some statistical issues related to the way the OECD reported the PISA 2000 and PISA 2003 results in the initial reports, in particular questionnaire indices by quarters and the relative and attributable risks.





# SPSS<sup>®</sup> Macros

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## INTRODUCTION

This chapter presents the SPSS® syntax of the macros used in the previous chapters. These macros are also included in the CD provided with the manual.

Twelve macros were described. These are summarized in Table 15.1. The file names are in red and the macro names as well as their arguments are in black. All the macros have five common arguments:

- NREP =
- WGT =
- RWGT =
- CONS =
- INFILE =

The other arguments are specific to a particular macro. These specific arguments were largely explained in the previous chapters.

## STRUCTURE OF THE SPSS® MACROS

All SPSS® macros have the same structure.

- The first step consists of:
  - Reading in the INFILE data file and dropping all variables that are not necessary for the analysis.
- The second step is the iterative part of the macro:
  - The SPSS® procedure for computing the estimate is repeated 81 times or 405 times, depending on the procedure; and
  - At each run, the results are stored in a temporary file. The number of the replicate and the number of the plausible values if needed are then combined in one file.
- The final step is devoted to the computation of the final statistic and its respective standard error, *i.e.*:
  - The squared differences between the final estimate and the 80 replicates are computed;
  - The sum of the squared difference is computed and divided by 20; and
  - Final estimates, the sampling variance estimates and, in the case of the plausible values, the measurement variance are computed.

The SPSS® syntax is presented hereafter.



Table 15.1 ■ Synthesis of the twelve SPSS® macros

Requested statistics	Without plausible values	With plausible values
Mean, sd, sum, pgt, plt, pin, pout, fgt, flt, fin, fout	<p><b>mcr_SE_univ.sps (ch. 6)</b></p> <pre>univar nrep = /       stat = /       dep = /       grp = /       wgt = /       rwgt = /       cons = /       infile = ' ' / .</pre>	<p><b>mcr_SE_pv.sps (ch. 7)</b></p> <pre>PV nrep = /    stat = /    dep = /    grp = /    wgt = /    rwgt = /    cons = /    infile = ' ' / .</pre>
Percentage	<p><b>mcr_SE_GrpPct.sps (ch. 6)</b></p> <pre>GRPPCT nrep = /        within = /        grp = /        wgt = /        rwgt = /        cons = /        infile = ' ' / .</pre>	<p><b>mcr_SE_PctLev.sps (ch. 8)</b></p> <pre>PCTLEV nrep = /        within = /        grp = /        wgt = /        rwgt = /        cons = /        infile = ' ' / .</pre>
Regression coefficients	<p><b>mcr_SE_reg.sps (ch. 6)</b></p> <pre>REGnoPV nrep = /         ind = /         dep = /         grp = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre>	<p><b>mcr_SE_reg_PV.sps (ch. 7)</b></p> <pre>REG_PV nrep = /        ind = /        dep = /        grp = /        wgt = /        rwgt = /        cons = /        infile = ' ' / .</pre>
Correlation coefficients	<p><b>mcr_SE_cor.sps (ch. 6)</b></p> <pre>CORnoPV nrep = /         var1 = /         var2 = /         grp = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre>	<p><b>mcr_SE_cor_1PV.sps (ch. 7)</b></p> <pre>COR_1PV nrep = /         nopv = /         pv = /         grp = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre> <p><b>mcr_SE_cor_2PV.sps (ch. 7)</b></p> <pre>COR_2PV nrep = /         pv1 = /         pv2 = /         grp = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre>
Differences in mean, sd, sum, pgt, plt, pin, pout, fgt, flt, fin, fout	<p><b>mcr_SE_dif.sps (ch. 10)</b></p> <pre>difNOpv nrep = /         dep = /         stat = /         within = /         compare = /         categ = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre>	<p><b>mcr_SE_dif_PV.sps (ch. 10)</b></p> <pre>dif_pv nrep = /         dep = /         stat = /         within = /         compare = /         categ = /         wgt = /         rwgt = /         cons = /         infile = ' ' / .</pre>
PV mean, sd, sum, pgt, plt, pin, pout, fgt, flt, fin, fout within quarters of WLE indices		<p><b>mcr_SE_PV_WLEQRT.sps (ch. 14)</b></p> <pre>PVWLEQRT nrep = /          stat = /          pv = /          wle = /          grp = /          wgt = /          rwgt = /          cons = /          infile = ' ' / .</pre>



## Box 15.1 ■ SPSS® syntax of mcr\_SE\_univ.sps.

```
define univar (nrep = !charend('/') /
               stat = !charend('/') /
               dep = !charend('/') /
               grp = !charend('/') /
               wgt = !charend('/') /
               rwgt = !charend('/') /
               cons = !charend('/') /
               infile = !charend('/')).

get file !infile /keep !grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!dep).

*** COMPUTE ESTIMATE ***.

weight by !wgt.
aggregate outfile = !quote(!concat('c:\temp\',!stat,'_all.sav'))
/break=!grp /stat=!stat(!dep).

* REPLICATES.

!do !i= 1 !to !nrep.
weight by !concat(!rwgt,!i).
aggregate outfile = !quote(!concat('C:\temp\',!stat,'_',!dep,!i,'.sav'))
/break=!grp /statr=!stat(!dep).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!stat,'_',!dep,'1.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('C:\temp\',!stat,'_',!dep,!e,'.
sav')).
!Doend.

sort cases by !grp.
match files file=* /table=!quote(!concat('c:\temp\',!stat,'_all.sav')) /
by !grp.
exec.

*** COMPUTE SAMPLING VARIANCE (U) ***.

compute var=(statr-stat)**2.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=* / break=!grp/ stat= mean(stat)/ var=sum(var).

compute var=!cons*var.

*** COMPUTE STANDARD ERROR ***.

compute se=sqrt(var).
exec.

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp stat se.

!enddefine.
```



## Box 15.2 ■ SPSS® syntax of mcr\_SE\_pv.sps.

```
define PV (nrep = !charend('/') /
          stat = !charend('/') /
          dep = !charend('/') /
          grp = !charend('/') /
          wgt = !charend('/') /
          rwgt = !charend('/') /
          cons = !charend('/') /
          infile = !charend('/')).

get file !infile /keep !grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!concat(pv,1,!dep)
!concat(pv,2,!dep) !concat(pv,3,!dep) !concat(pv,4,!dep)
!concat(pv,5,!dep) .

*** COMPUTE STATISTIC ***.

weight by !wgt.
erase file='c:\temp\all.sav'.
aggregate outfile = 'c:\temp\all.sav' /break=!grp /
  stat1 stat2 stat3 stat4 stat5=!stat(!concat(pv1,!dep)
!concat(pv2,!dep) !concat(pv3,!dep) !concat(pv4,!dep) !concat(pv5,!dep)).

* REPLICATES.

!do !i= 1 !to !nrep.
weight by !concat(!rwgt,!i).
erase file=!quote(!concat('c:\temp\' ,!dep,!i,'.sav')).
aggregate outfile = !quote(!concat('c:\temp\' ,!dep,!i,'.sav'))/break=!grp /
  statr1 statr2 statr3 statr4 statr5=!stat(!concat(pv1,!dep)
!concat(pv2,!dep) !concat(pv3,!dep) !concat(pv4,!dep) !concat(pv5,!dep)).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('c:\temp\' ,!dep,'1','.sav')).
cache.

!Do !e = 2 !to !nrep.
add files/file=*/file=!quote(!concat('c:\temp\' ,!dep,!e,'.sav')).
!Doend.

sort cases by !grp.

match files file=*/table= 'c:\temp\all.sav'/by !grp.
exec.
```



## Box 15.2 (continued) ■ SPSS® syntax of mcr\_SE\_pv.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
    b=stat1 to stat5/
    c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
    break=!grp/
    stat1 to stat5= mean(stat1 to stat5)/
    var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp stat SE.

!enddefine.
```



## Box 15.3 ■ SPSS® syntax of mcr\_SE\_GrpPct.sps.

```
define GRPPCT (nrep = !charend('/') /
              within = !charend('/') /
              grp = !charend('/') /
              wgt = !charend('/') /
              rwgt = !charend('/') /
              cons = !charend('/') /
              infile = !charend('/')).

get file !infile /keep !within !grp !wgt !concat(!rwgt,1) to
!concat(!rwgt,!nrep)).
sort cases by !grp.
save outfile='c:\temp\sort.sav'.

*** COMPUTE ESTIMATE ***.

weight by !wgt.
aggregate outfile='c:\temp\temp1.sav' /break=!within /N_all=n.
aggregate outfile=* /break=!within !grp /N_grp=n.
exe.
match files file=* /table='c:\temp\temp1.sav' /by !within.
compute stat=100*(n_grp/n_all).
save outfile=!quote(!concat('c:\temp\',!grp,'.sav')).

* REPLICATES.

!do !i= 1 !to !nrep.
get file 'c:\temp\sort.sav'.
weight by !concat(!rwgt,!i).
aggregate outfile='c:\temp\temp2.sav' /break=!within /N_all=n.
aggregate outfile=* /break=!within !grp /N_grp=n.
exe.
match files file=* /table='c:\temp\temp2.sav' /by !within.
compute statR=100*(n_grp/n_all).
save outfile=!quote(!concat('c:\temp\',!grp,'_',!i,'.sav')).
erase file='c:\temp\temp2.sav'.
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('c:\temp\',!grp,'_1.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('c:\temp\',!grp,'_',!e,'.sav')).
!Doend.

sort cases by !within !grp.

match files file=* /table=!quote(!concat('c:\temp\',!grp,'.sav')) /by
!within !grp.
exec.
```



## Box 15.3 (continued) ■ SPSS® syntax of mcr\_SE\_GrpPct.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.  
  
compute var=(statr-stat)**2.  
exec.  
  
save outfile = 'c:\temp\regmod.sav'.  
  
aggregate outfile=*/  
  break=!within !grp/  
  stat= mean(stat)/  
  var=sum(var).  
  
compute var=!cons*var.  
  
  
  
*** COMPUTE STANDARD ERROR ***.  
  
compute SE=sqrt(var).  
exec.  
  
formats stat (f8.3)/ SE (f10.6).  
list cases/var= !within !grp stat SE.  
  
!enddefine.
```



## Box 15.4 ■ SPSS® syntax of mcr\_SE\_PctLev.sps.

```
define PCTLEV (nrep = !charend('/')/
              within = !charend('/') /
              grp = !charend('/') /
              wgt = !charend('/') /
              rwgt = !charend('/') /
              cons = !charend('/')/
              infile = !charend('/')).

*** COMPUTE ESTIMATE ***.

get file !infile /keep !within !concat(!grp,'1') !concat(!grp,'2')
!concat(!grp,'3') !concat(!grp,'4')
!concat(!grp,'5') !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)).
weight by !wgt.
aggregate outfile='c:\temp\temp10.sav' /break=!within /N_all=n.

!do !j=1 !to 5.
get file !infile /keep !within !concat(!grp,'1') !concat(!grp,'2')
!concat(!grp,'3') !concat(!grp,'4')
!concat(!grp,'5') !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)).
weight by !wgt.
aggregate outfile=* /break=!within !concat(!grp,!j) /N_grp=n.
exe.
match files file=* /table='c:\temp\temp10.sav' /by !within.
compute !concat('stat',!j)=100*(n_grp/n_all).
rename var (!concat(!grp,!j)=!grp).
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')) /keep=!within !grp
!concat('stat',!j).
!doend.

match files file='c:\temp\temp1.sav'
          /file='c:\temp\temp2.sav'
          /file='c:\temp\temp3.sav'
          /file='c:\temp\temp4.sav'
          /file='c:\temp\temp5.sav'
          /by !within !grp.
save outfile='c:\temp\all.sav' .
```



## Box 15.4 (continued – 1) ■ SPSS® syntax of mcr\_SE\_PctLev.sps.

```
* REPLICATES.

!do !i= 1 !to !nrep.
get file !infile /keep !within !concat(!grp,'1') !concat(!grp,'2')
!concat(!grp,'3') !concat(!grp,'4')
!concat(!grp,'5') !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)).
weight by !concat(!rwgt,!i).
erase file='c:\temp\temp20.sav'.
aggregate outfile='c:\temp\temp20.sav' /break=!within /N_all=n.

!do !j=1 !to 5.
get file !infile /keep !within !concat(!grp,'1') !concat(!grp,'2')
!concat(!grp,'3') !concat(!grp,'4')
!concat(!grp,'5') !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)).
weight by !concat(!rwgt,!i).

aggregate outfile=* /break=!within !concat(!grp,!j) /N_grp=n.
exe.
match files file=* /table='c:\temp\temp20.sav' /by !within.
compute !concat('statR',!j)=100*(n_grp/n_all).
rename var (!concat(!grp,!j)=!grp).
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')) /keep=!within !grp
!concat('statR',!j).
!doend.

match files file='c:\temp\temp1.sav'
/file='c:\temp\temp2.sav'
/file='c:\temp\temp3.sav'
/file='c:\temp\temp4.sav'
/file='c:\temp\temp5.sav'
/by !within !grp.
save outfile=!quote(!concat('c:\temp\',!grp,'_',!i,'.sav')).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('c:\temp\',!grp,'_1.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('c:\temp\',!grp,'_',!e,'.sav')).
!Doend.

sort cases by !within !grp.

match files file=* /table='c:\temp\all.sav' /by !within !grp.
exec.
```



## Box 15.4 (continued – 2) ■ SPSS® syntax of mcr\_SE\_PctLev.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
  b=stat1 to stat5/
  c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
  break=!within !grp/
  stat1 to stat5= mean(stat1 to stat5)/
  var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).
list cases/var= !within !grp stat SE.

!enddefine.
```



## Box 15.5 ■ SPSS® syntax of mcr\_SE\_reg.sps.

```
define REGnoPV (nrep = !charend('/') /
                ind = !charend('/') /
                dep = !charend('/') /
                grp = !charend('/') /
                wgt = !charend('/') /
                rwgt = !charend('/') /
                cons = !charend('/') /
                infile = !charend('/')).

get file !infile /keep !grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!dep !ind).
sort cases by !grp.
save outfile='c:\temp\sort.sav'.

*** COMPUTE ESTIMATE ***.

split file by !grp.

weight by !wgt.

REGRESSION
  /DEPENDENT !dep
  /METHOD=ENTER !ind
  /OUTFILE=COVB('C:\temp\coef.sav') .
get file='C:\temp\coef.sav'.
select if (rowtype_='EST').
rename var(CONST =b0).
VARSTOCASES /MAKE stat FROM b0 !ind
  /INDEX = ind(stat) /KEEP = !grp /NULL = KEEP.

erase file='c:\temp\all.sav'.
sort cases by !grp ind.
save outfile='c:\temp\all.sav'.

* REPLICATES.

!do !i=1 !to !nrep.
get file='c:\temp\sort.sav'.

*sort cases by !grp.
split file by !grp.

weight by !concat(!rwgt,!i).

REGRESSION
  /DEPENDENT !dep
  /METHOD=ENTER !ind
  /OUTFILE=COVB('C:\temp\coef.sav') .
get file='C:\temp\coef.sav'.
select if (rowtype_='EST').
rename var(CONST =b0).
VARSTOCASES /MAKE statR FROM b0 !ind
  /INDEX = ind(statR) /KEEP = !grp /NULL = KEEP.

save outfile=!quote(!concat('C:\temp\',!dep,!i,'.sav'))

!doend.
```



## Box 15.5 (continued) ■ SPSS® syntax of mcr\_SE\_reg.sps.

```
*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!dep,'1.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('C:\temp\',!dep,!e,'.sav')).
!Doend.

sort cases by !grp ind.

match files file=* /table='c:\temp\all.sav' /by !grp ind.
exec.

*** COMPUTE SAMPLING VARIANCE (U) ***.

compute var=(statr-stat)**2.
exec.

save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
  break=!grp IND/
  stat= mean(stat)/
  var=sum(var).

compute var=!cons*var.

*** COMPUTE STANDARD ERROR ***.

compute se=sqrt(var).
exec.

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp ind stat se.

!enddefine.
```



## Box 15.6 ■ SPSS® syntax of mcr\_SE\_reg\_PV.sps.

```
define REG_PV (nrep = !charend('/') /
              ind = !charend('/') /
              dep = !charend('/') /
              grp = !charend('/') /
              wgt = !charend('/') /
              rwgt = !charend('/') /
              cons = !charend('/') /
              infile = !charend('/')).

get file !infile /keep=!grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!IND !concat('pv1',!dep)
!concat('pv2',!dep) !concat('pv3',!dep) !concat('pv4',!dep)
!concat('pv',5,!dep) .
sort cases by !grp.
erase file 'c:\temp\sort.sav'.
save outfile='c:\temp\sort.sav'.

*** COMPUTE REGRESSION COEFFICIENTS ***.

!do !j=1 !to 5.

get file 'c:\temp\sort.sav'.

split file by !grp.

weight by !wgt.

REGRESSION
  /DEPENDENT !concat('pv',!j,!dep)
  /METHOD=ENTER !ind
  /OUTFILE=COVB(!quote(!concat('C:\temp\coef',!j,'.sav'))) .
get file=!quote(!concat('C:\temp\coef',!j,'.sav')).

select if (rowtype_='EST').
rename var(CONST=b0).
VARSTOCASES /MAKE !concat('stat',!j) FROM b0 !ind
  /INDEX = ind(!concat('stat',!j)) /KEEP = !grp /NULL = KEEP.
sort cases by !grp IND.
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')).
!doend.

match files file='c:\temp\temp1.sav'
  /file='c:\temp\temp2.sav'
  /file='c:\temp\temp3.sav'
  /file='c:\temp\temp4.sav'
  /file='c:\temp\temp5.sav'
  /by !GRP ind.
erase file='c:\temp\all.sav'.
save outfile='c:\temp\all.sav'.
```



## Box 15.6 (continued – 1) ■ SPSS® syntax of mcr\_SE\_reg\_PV.sps.

```
* REPLICATES.

!do !i=1 !to !nrep.
!do !j=1 !to 5.

get file 'c:\temp\sort.sav'.

split file by !grp.

weight by !concat(!rwgt,!i).

REGRESSION
  /DEPENDENT !concat('pv',!j,!dep)
  /METHOD=ENTER !ind
  /OUTFILE=COVB(!quote(!concat('C:\temp\coef',!j,'.sav'))) .
get file=!quote(!concat('C:\temp\coef',!j,'.sav')).
select if (rowtype_='EST').
rename var(CONST_=b0).
VARSTOCASES /MAKE !concat('statR',!j) FROM b0 !ind
  /INDEX = ind(!concat('statR',!j)) /KEEP = !grp /NULL = KEEP.
sort cases by !grp IND.
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')).
!doend.

match files file='c:\temp\temp1.sav'
  /file='c:\temp\temp2.sav'
  /file='c:\temp\temp3.sav'
  /file='c:\temp\temp4.sav'
  /file='c:\temp\temp5.sav'
  /by !GRP ind.

erase file=!quote(!concat('C:\temp\',!dep,!i,'.sav')).
save outfile=!quote(!concat('C:\temp\',!dep,!i,'.sav')).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!dep,'1','.sav')).
cache.

!Do !e = 2 !to !nrep.
add files/file=*/file=!quote(!concat('C:\temp\',!dep,!e,'.sav')).
!Doend.

SORT CASES BY !grp IND.
match files file=*/table= 'c:\temp\all.sav'/by !grp IND.
exec.
```



## Box 15.6 (continued – 2) ■ SPSS® syntax of mcr\_SE\_reg\_PV.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
    b=stat1 to stat5/
    c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
    break=!grp IND/
    stat1 to stat5= mean(stat1 to stat5)/
    var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp IND stat SE.

!enddefine.
```



## Box 15.7 ■ SPSS® syntax of mcr\_SE\_cor.sps.

```
define CORnoPV (nrep = !charend('/') /
                var1 = !charend('/') /
                var2 = !charend('/') /
                grp = !charend('/') /
                wgt = !charend('/') /
                rwgt = !charend('/') /
                cons = !charend('/') /
                infile = !charend('/')).

get file !infile /keep !grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!var2 !var1.
select if (not missing(!var1) & not missing (!var2)).
sort cases by !grp.
save outfile='c:\temp\sort.sav' .

*** COMPUTE ESTIMATE ***.

split file by !grp.

weight by !wgt.
descr !var1 !var2 / stat=mean stddev /save.
compute Y=!concat('Z',!var1)*!concat('Z',!var2).
split file off.
aggregate outfile=* /break=!grp /sumY=sum(Y) /n=n.
compute stat=sumY/n.

erase file='c:\temp\all.sav' .
save outfile='c:\temp\all.sav' /keep=!grp stat.

* REPLICATES.

!do !i=1 !to !nrep.
get file='c:\temp\sort.sav' .

split file by !grp.

weight by !concat(!rwgt,!i).
descr !var1 !var2 / stat=mean stddev /save.
compute Y=!concat('Z',!var1)*!concat('Z',!var2).
split file off.
aggregate outfile=* /break=!grp /sumY=sum(Y) /n=n.
compute statR=sumY/n.

erase file=!quote(!concat('C:\temp\',!var2,!i,'.sav')).
save outfile=!quote(!concat('C:\temp\',!var2,!i,'.sav')) /keep=!grp statR.
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!var2,'1.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('C:\temp\',!var2,!e,'.sav')).
!Doend.

sort cases by !grp.
match files file=* /table='c:\temp\all.sav' /by !grp.
exec.
```



## Box 15.7 (continued) ■ SPSS® syntax of mcr\_SE\_cor.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.  
  
compute var=(statr-stat)**2.  
exec.  
  
save outfile = 'c:\temp\regmod.sav'.  
  
aggregate outfile=*/  
    break=!grp/  
    stat= mean(stat)/  
    var=sum(var).  
  
compute var=!cons*var.  
  
*** COMPUTE STANDARD ERROR ***.  
  
compute SE=sqrt(var).  
exec.  
  
formats stat (f8.3)/ SE (f10.6).  
list cases/var= !grp stat SE.  
  
!enddefine.
```



## Box 15.8 ■ SPSS® syntax of mcr\_SE\_cor\_1PV.sps.

```
define COR_1PV (nrep = !charend('/') /
               nopv = !charend('/') /
               pv = !charend('/') /
               grp = !charend('/') /
               wgt = !charend('/') /
               rwgt = !charend('/') /
               cons = !charend('/') /
               infile = !charend('/')).

*** COMPUTE REGRESSION COEFFICIENTS ***.

get file !infile /keep=!grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!concat('pv1',!pv)
  !concat('pv2',!pv) !concat('pv3',!pv) !concat('pv4',!pv)
!concat('pv5',!pv) !nopv.

rename var (!concat('pv1',!pv)
  !concat('pv2',!pv) !concat('pv3',!pv) !concat('pv4',!pv) !concat('pv5',!
pv)=!concat('p1',!pv)
  !concat('p2',!pv) !concat('p3',!pv) !concat('p4',!pv)
!concat('p5',!pv)).

select if (not missing(!noPV)).
sort cases by !grp.
save outfile='c:\temp\sort.sav'.

!do !j=1 !to 5.
get file='c:\temp\sort.sav'.
split file by !grp.

split file layered by !grp.

!let !x=!nopv.
!let !y=!concat('p',!j,!pv).

weight by !wgt.
descr !x !y /stat=mean stddev /save.
compute P=!concat('Z',!x)*!concat('Z',!y).
split file off.

aggregate outfile=* /break=!grp /sumP=sum(P) /n=n.
compute !concat('stat',!j)=sumP/n.

save outfile=!quote(!concat('c:\temp\temp',!J,'.sav')) /keep=!grp
!concat('stat',!j).
!doend.

match files file='c:\temp\temp1.sav'
  /file='c:\temp\temp2.sav'
  /file='c:\temp\temp3.sav'
  /file='c:\temp\temp4.sav'
  /file='c:\temp\temp5.sav'
  /by !GRP.
erase file='c:\temp\all.sav'.
save outfile='c:\temp\all.sav'.
```



## Box 15.8 (continued – 1) ■ SPSS® syntax of mcr\_SE\_cor\_1PV.sps.

```
* REPLICATES.

!do !i=1 !to !nrep.
!do !j=1 !to 5.
get file='c:\temp\sort.sav' .

split file by !grp.

weight by !concat(!rwgt,!i) .
split file layered by !grp.

!let !x=!nopv.
!let !y=!concat('p',!j,!pv) .

descr !x !y /stat=mean stddev /save.
compute P=!concat('Z',!x)*!concat('Z',!y) .
split file off.

aggregate outfile=* /break=!grp /sumP=sum(P) /n=n.
compute !concat('statR',!j)=sumP/n.

save outfile=!quote(!concat('c:\temp\temp',!J,'.sav')) /keep=!grp
!concat('statR',!j) .
!doend.

match files file='c:\temp\temp1.sav'
           /file='c:\temp\temp2.sav'
           /file='c:\temp\temp3.sav'
           /file='c:\temp\temp4.sav'
           /file='c:\temp\temp5.sav'
           /by !GRP.

erase file=!quote(!concat('C:\temp\',!pv,!i,'.sav')) .
save outfile=!quote(!concat('C:\temp\',!pv,!i,'.sav')) .
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!pv,'1','.sav')) .
cache.

!Do !e = 2 !to !nrep.
add files/file=*/file=!quote(!concat('C:\temp\',!pv,!e,'.sav')) .
!Doend.

sort cases by !grp.
match files file=*/table= 'c:\temp\all.sav'/by !grp.
exec.
```



## Box 15.8 (continued – 2) ■ SPSS® syntax of mcr\_SE\_cor\_1PV.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
  b=stat1 to stat5/
  c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
  break=!grp/
  stat1 to stat5= mean(stat1 to stat5)/
  var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp stat SE.

!enddefine.
```



## Box 15.9 ■ SPSS® syntax of mcr\_SE\_cor\_2PV.sps.

```
define COR_2PV (nrep = !charend('/')/
                pv1 = !charend('/') /
                pv2 = !charend('/') /
                grp = !charend('/') /
                wgt = !charend('/') /
                rwgt = !charend('/') /
                cons = !charend('/')/
                infile = !charend('/')).

*** COMPUTE REGRESSION COEFFICIENTS ***.

get file !infile /keep=!grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!concat('pv1',!pv2)
!concat('pv2',!pv2) !concat('pv3',!pv2) !concat('pv4',!pv2)
!concat('pv',5,!pv2)
!concat('pv1',!pv1) !concat('pv2',!pv1) !concat('pv3',!pv1)
!concat('pv4',!pv1) !concat('pv',5,!pv1).
rename var (!concat('pv1',!pv1)
!concat('pv2',!pv1) !concat('pv3',!pv1) !concat('pv4',!pv1) !concat('pv'
,5,!pv1)=!concat('p1',!pv1)
!concat('p2',!pv1) !concat('p3',!pv1) !concat('p4',!pv1)
!concat('p',5,!pv1)).
rename var (!concat('pv1',!pv2)
!concat('pv2',!pv2) !concat('pv3',!pv2) !concat('pv4',!pv2) !concat('pv'
,5,!pv2)=!concat('p1',!pv2)
!concat('p2',!pv2) !concat('p3',!pv2) !concat('p4',!pv2)
!concat('p',5,!pv2)).
sort cases by !grp.
save outfile='c:\temp\sort.sav' .

!do !j=1 !to 5.
get file='c:\temp\sort.sav' .

split file by !grp.

weight by !wgt.
split file layered by !grp.

!let !x=!concat('p',!j,!pv1).
!let !y=!concat('p',!j,!pv2).

descr !x !y /stat=mean stddev /save.
compute P=!concat('Z',!x)*!concat('Z',!y).
split file off.

aggregate outfile=* /break=!grp /sumP=sum(P) /n=n.
compute !concat('stat',!j)=sumP/n.

save outfile=!quote(!concat('c:\temp\temp',!J,'.sav')) /keep=!grp
!concat('stat',!j).
!doend.
```



## Box 15.9 (continued – 1) ■ SPSS® syntax of mcr\_SE\_cor\_2PV.sps.

```
match files file='c:\temp\temp1.sav'
           /file='c:\temp\temp2.sav'
           /file='c:\temp\temp3.sav'
           /file='c:\temp\temp4.sav'
           /file='c:\temp\temp5.sav'
           /by !GRP.
erase file='c:\temp\all.sav' .
save outfile='c:\temp\all.sav' .

* REPLICATES.

!do !i=1 !to !nrep.
!do !j=1 !to 5.

get file='c:\temp\sort.sav' .
split file layered by !grp.

weight by !concat(!rwgt,!i) .
split file by !grp.

!let !x=!concat('p',!j,!pv1) .
!let !y=!concat('p',!j,!pv2) .

descr !x !y /stat=mean stddev /save.
compute P=!concat('Z',!x)*!concat('Z',!y) .
split file off.

aggregate outfile=* /break=!grp /sumP=sum(P) /n=n.
compute !concat('statR',!j)=sumP/n.

save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')) /keep=!grp
!concat('statR',!j) .
!doend.

match files file='c:\temp\temp1.sav'
           /file='c:\temp\temp2.sav'
           /file='c:\temp\temp3.sav'
           /file='c:\temp\temp4.sav'
           /file='c:\temp\temp5.sav'
           /by !GRP.

erase file=!quote(!concat('C:\temp\',!pv2,!i,'.sav')) .
save outfile=!quote(!concat('C:\temp\',!pv2,!i,'.sav')) .
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('C:\temp\',!pv2,'1','.sav')) .
cache.

!Do !e = 2 !to !nrep.
add files/file=*/file=!quote(!concat('C:\temp\',!pv2,!e,'.sav')) .
!Doend.

sort cases by !grp.
match files file=*/table= 'c:\temp\all.sav' /by !grp.
exec.
```



## Box 15.9 (continued – 2) ■ SPSS® syntax of mcr\_SE\_cor\_2PV.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
    b=stat1 to stat5/
    c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
    break=!grp/
    stat1 to stat5= mean(stat1 to stat5)/
    var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp stat SE.

!enddefine.
```



## Box 15.10 ■ SPSS® syntax of mcr\_SE\_dif.sps.

```
define difNOpv (nrep = !charend('\') /
               dep = !charend('\') /
               stat = !charend ('\') /
               within = !charend('\') /
               compare = !charend('\') /
               categ = !charend ('\') /
               wgt = !charend('\') /
               rwgt = !charend('\') /
               cons = !charend('\') /
               infile = !charend('\')).

get file !infile /keep !dep !within !compare !wgt !concat(!rwgt,1) to
!concat(!rwgt,!nrep) .

*** COMPUTE ESTIMATE ***.

weight by !wgt.
aggregate outfile = *
  /break=!within !compare /stat=!stat(!dep).
exe.
casestovars /id=!within /index=!compare .

!let !n=!length(!categ).
!let !m=!LENGTH(!substr(!blanks(!n),2)).

!let !var=" ".
!do !a=1 !to !m.
!let !s=!LENGTH(!concat(!blanks(!a),' ')).
!do !b=!s !to !n.

!let !d=!substr(!categ,!a,1).
!let !e=!substr(!categ,!b,1).
compute !concat('stat',!d,'_',!e)=
  !concat('stat.',!d)-!concat('stat.',!e).
!let !var=!concat(!var,'stat',!d,'_',!e,' ').
!doend.
!doend.

VARSTOCASES /MAKE stat FROM !var
  /INDEX = contrast(stat) /KEEP = !within /NULL = DROP.

save outile='c:\temp\all.sav'.
```



## Box 15.10 (continued – 1) ■ SPSS® syntax of mcr\_SE\_dif.sps.

```
* REPLICATES.

!do !i= 1 !to !nrep.
get file !infile /keep !within !compare !wgt !concat(!rwgt,1) to
!concat(!rwgt,!nrep) !dep).
weight by !concat(!rwgt,!i).

aggregate outfile = *
  /break=!within !compare /stat=!stat(!dep).
exe.
casestovars /id=!within /index=!compare .

!let !n=!length(!categ).
!let !m=!LENGTH(!substr(!blanks(!n),2)).

!let !var=" ".
!do !a=1 !to !m.
!let !s=!LENGTH(!concat(!blanks(!a),' ')).
!do !b=!s !to !n.

!let !d=!substr(!categ,!a,1).
!let !e=!substr(!categ,!b,1).
compute !concat('stat',!d,'_',!e)=
  !concat('stat.',!d)-!concat('stat.',!e).
!let !var=!concat(!var,'stat',!d,'_',!e,' ').
!doend.
!doend.

VARSTOCASES /MAKE statR FROM !var
  /INDEX = contrast(statR) /KEEP = !within /NULL = DROP.

save outfile=!quote(!concat('c:\temp\',!dep,!i,'.sav')).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('c:\temp\',!dep,'1','.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('c:\temp\',!dep,!e,'.sav')).
!Doend.

sort cases by !within CONTRAST.
match files file=* /table='c:\temp\all.sav' /by !within CONTRAST.
exec.

*** COMPUTE SAMPLING VARIANCE (U) ***.

compute var=(statr-stat)**2.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=* / break=!within contrast/ stat= mean(stat)/
var=sum(var).

compute var=!cons*var.
```

Box 15.10 (continued – 2) ■ SPSS® syntax of `mcr_SE_dif.sps`.

```
*** COMPUTE STANDARD ERROR ***.  
  
compute se=sqrt(var).  
exec.  
  
formats stat (f8.3)/ SE (f10.6).  
list cases/var= !within contrast stat se.  
  
string contr (a3).  
compute contr=substr(contrast,5,3).  
casestovars /id=!within /index=contr /groupby=index /drop=contrast var.  
  
!enddefine.
```



## Box 15.11 ■ SPSS® syntax of mcr\_SE\_dif\_PV.sps.

```

define dif_pv (nrep = !charend('\') /
              dep = !charend('\') /
              stat = !charend ('\') /
              within = !charend('\') /
              compare = !charend('\') /
              categ = !charend ('\') /
              wgt = !charend('\') /
              rwgt = !charend('\') /
              cons = !charend('\') /
              infile = !charend('\')).

*** COMPUTE ESTIMATE ***.

!do !j=1 !to 5.
get file !infile /keep !concat('pv1',!dep)
  !concat('pv2',!dep) !concat('pv3',!dep) !concat('pv4',!dep)
!concat('pv',5,!dep)
  !within !compare !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep) .
weight by !wgt.
aggregate outfile = *
  /break=!within !compare /stat=!stat(!concat('pv',!j,!dep)).
exe.
casestovars /id=!within /index=!compare .

!let !n=!length(!categ).
!let !m=!LENGTH(!substr(!blanks(!n),2)).

!let !var=" ".
!do !a=1 !to !m.
!let !s=!LENGTH(!concat(!blanks(!a),' ')).
!do !b=!s !to !n.

!let !d=!substr(!categ,!a,1).
!let !e=!substr(!categ,!b,1).
compute !concat('stat',!d,'_',!e)=
  !concat('stat.',!d) - !concat('stat.',!e).
!let !var=!concat(!var,'stat',!d,'_',!e,' ').
!doend.
!doend.

VARSTOCASES /MAKE !concat('stat',!j) FROM !var
  /INDEX = contrast(!concat('stat',!j)) /KEEP = !within /NULL = DROP.
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')).
!doend.

match files file='c:\temp\temp1.sav'
  /file='c:\temp\temp2.sav'
  /file='c:\temp\temp3.sav'
  /file='c:\temp\temp4.sav'
  /file='c:\temp\temp5.sav'
  /by !within contrast.
sort cases by !within CONTRAST.
save outile='c:\temp\all.sav' .

```



## Box 15.11 (continued – 1) ■ SPSS® syntax of mcr\_SE\_dif\_PV.sps.

```
* REPLICATES.

!do !i= 1 !to !nrep.
!do !j=1 !to 5.
get file !infile /keep !concat('pv1',!dep)
    !concat('pv2',!dep) !concat('pv3',!dep) !concat('pv4',!dep)
!concat('pv',5,!dep)
    !within !compare !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep) .
weight by !concat(!rwgt,!i).
aggregate outfile = *
    /break=!within !compare /stat=!stat(!concat('pv',!j,!dep)).
exe.
casetovars /id=!within /index=!compare .

!let !n=!length(!categ).
!let !m=!LENGTH(!substr(!blanks(!n),2)).

!let !var=" ".
!do !a=1 !to !m.
!let !s=!LENGTH(!concat(!blanks(!a),' ')).
!do !b=!s !to !n.

!let !d=!substr(!categ,!a,1).
!let !e=!substr(!categ,!b,1).
compute !concat('stat',!d,'_',!e)=
    !concat('stat.',!d)-!concat('stat.',!e).
!let !var=!concat(!var,'stat',!d,'_',!e,' ').
!doend.
!doend.

VARSTOCASES /MAKE !concat('statR',!j) FROM !var
    /INDEX = contrast(!concat('statR',!j)) /KEEP = !within /NULL = DROP.
save outfile=!quote(!concat('c:\temp\temp',!j,'.sav')).
!doend.

match files file='c:\temp\temp1.sav'
    /file='c:\temp\temp2.sav'
    /file='c:\temp\temp3.sav'
    /file='c:\temp\temp4.sav'
    /file='c:\temp\temp5.sav'
    /by !within contrast.

save outfile=!quote(!concat('C:\temp\',!dep,!i,'.sav')).
!doend.

*** COMBINE RESULTS ***.

get file =!quote(!concat('c:\temp\',!dep,'1','.sav')).

!Do !e = 2 !to !nrep.
add files file=* /file=!quote(!concat('c:\temp\',!dep,!e,'.sav')).
!Doend.

sort cases by !within CONTRAST.
match files file=* /table='c:\temp\all.sav' /by !within CONTRAST.
exec.
```



## Box 15.11 (continued – 2) ■ SPSS® syntax of mcr\_SE\_dif\_PV.sps.

```
*** COMPUTE SAMPLING VARIANCE (U) ***.

do repeat a=stat1 to stat5/
    b=stat1 to stat5/
    c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'c:\temp\regmod.sav'.

aggregate outfile=*/
    break=!within contrast/
    stat1 to stat5= mean(stat1 to stat5)/
    var1 to var5 = sum(var1 to var5).

do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.

compute pv_var=mean(var1 to var5).

*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.

compute stat=mean(stat1 to stat5).

do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.

compute pvmerr=.25*(sum(pvar1 to pvar5)).

*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.

compute SE=sqrt(pv_var+1.2*pvmerr).

formats stat (f8.3)/ SE (f10.6).

string contr (a3).
compute contr=substr(contrast,5,3).
list cases/var= !within contr stat SE.
save outfile='c:\temp\temp.sav' .
get file='c:\temp\temp.sav'/keep=!within contr stat se.
casestovars /id=!within /index=contr /groupby=index.

!enddefine.
```



## Box 15.12 ■ SPSS® syntax of mcr\_SE\_PV\_WLEQRT.sps.

```
define PVWLEQRT (nrep = !charend('/')
                /stat = !charend('/')
                /pv = !charend('/')
                /WLE = !charend('/')
                /grp = !charend('/')
                /wgt = !charend('/')
                /rwgt = !charend('/')
                /cons = !charend('/')
                /infile = !charend('/')).

get file=!infile /keep=!grp !wgt !concat(!rwgt,1) to !concat(!rwgt,!nrep)
!concat(pv,1,!pv)
!concat(pv,2,!pv) !concat(pv,3,!pv) !concat(pv,4,!pv) !concat(pv,5,!pv)
!wle.

select if not missing (!wle).
means !wle /cell=mean min max.

* COMPUTE CUMULATIVE COUNT WITHIN COUNTRIES.
*-----
sort cases by cnt.
autorecode cnt /into cnt# /print.

!do !s=1 !to 5.
set seed = !s.
!let !v=!concat(!wle,!s).
!let !c=!concat('cumfreq',!s).

compute !v=!wle+rv.normal(0,.01).
sort cases by cnt# !v.

do if ($casenum=1 or lag(cnt#) <> cnt#).
compute !c=w_fstuwt.
else if (cnt#=lag(cnt#)).
compute !c=w_fstuwt + lag(!c).
end if.
sort cases by cnt.
!doend.
save outfile='C:\temp\temp.sav'.

* DEFINE CUTSCORES.
*-----
weight by w_fstuwt.
aggregate outfile=* /break=cnt /total=max(cumfreq1).
compute cut25=total/(100/25).
compute cut50=total/(100/50).
compute cut75=total/(100/75).

match files file='C:\temp\temp.sav'
/table=*
/by cnt.
exe.
```



## Box 15.12 (continued – 1) ■ SPSS® syntax of mcr\_SE\_PV\_WLEQRT.sps.

```
* CREATE PERCENTILE GROUPS.

do repeat c=cumfreq1 cumfreq2 cumfreq3 cumfreq4 cumfreq5 /q=quart1 quart2
quart3 quart4 quart5.
if (c<cut25) q=1.
if (c>=cut25 & c<cut50) q=2.
if (c>=cut50 & c<cut75) q=3.
if (c>=cut75) q=4.
formats q (f1.0).
end repeat.
save outfile='C:\temp\quarters.sav' .

*** COMPUTE STATISTIC ***.
!do !j=1 !to 5.
get file='C:\temp\quarters.sav' /keep !grp !wgt !concat(!rwgt,1) to
!concat(!rwgt,!nrep) !concat(pv,1,!pv)
!concat(pv,2,!pv) !concat(pv,3,!pv) !concat(pv,4,!pv) !concat(pv,5,!pv)
quart1 quart2 quart3 quart4 quart5.
weight by !wgt.
rename var (!concat('quart',!j)=quart).
erase file=!quote(!concat('C:\temp\temp',!j,'.sav')).
aggregate outfile = !quote(!concat('C:\temp\temp',!j,'.sav')) /break=!grp
quart
!concat('stat',!j)=!stat(!concat('pv',!j,!pv)).
!doend.

match files file='C:\temp\temp1.sav'
/file='C:\temp\temp2.sav'
/file='C:\temp\temp3.sav'
/file='C:\temp\temp4.sav'
/file='C:\temp\temp5.sav'
/by !grp quart.
erase file='C:\temp\all.sav' .
save outfile='C:\temp\all.sav' .

* REPLICATES.

!do !i= 1 !to !nrep.
!do !j=1 !to 5.
get file='C:\temp\quarters.sav' /keep !grp !wgt !concat(!rwgt,1) to
!concat(!rwgt,!nrep) !concat(pv,1,!pv)
!concat(pv,2,!pv) !concat(pv,3,!pv) !concat(pv,4,!pv) !concat(pv,5,!pv)
quart1 quart2 quart3 quart4 quart5.
weight by !concat(!rwgt,!i).
rename var (!concat('quart',!j)=quart).
sort cases by !grp quart.
erase file=!quote(!concat('C:\temp\temp',!j,'.sav')).
aggregate outfile = !quote(!concat('C:\temp\temp',!j,'.sav')) /break=!grp
quart
!concat('statR',!j)=!stat(!concat('pv',!j,!pv)).
!doend.
```



## Box 15.12 (continued – 2) ■ SPSS® syntax of mcr\_SE\_PV\_WLEQRT.sps.

```
match files file='C:\temp\temp1.sav'
           /file='C:\temp\temp2.sav'
           /file='C:\temp\temp3.sav'
           /file='C:\temp\temp4.sav'
           /file='C:\temp\temp5.sav'
           /by !grp quart.
erase file=!quote(!concat('C:\temp\',!pv,!i,'.sav')).
save outfile = !quote(!concat('C:\temp\',!pv,!i,'.sav')).
!doend.
*** COMBINE RESULTS ***.
get file =!quote(!concat('C:\temp\',!pv,'1','.sav')).
cache.
!Do !e = 2 !to !nrep.
add files/file=*/file=!quote(!concat('C:\temp\',!pv,!e,'.sav')).
!Doend.
sort cases by !grp quart.
match files file=*/table= 'C:\temp\all.sav'/by !grp quart.
exec.
*** COMPUTE SAMPLING VARIANCE (U) ***.
do repeat a=stat1 to stat5/
          b=stat1 to stat5/
          c=var1 to var5.
compute c=(a-b)**2.
end repeat.
save outfile = 'C:\temp\regmod.sav'.
aggregate outfile=*/
           break=!grp quart/
           stat1 to stat5= mean(stat1 to stat5)/
           var1 to var5 = sum(var1 to var5).
do repeat a=var1 to var5.
compute a=!cons*a.
end repeat.
compute pv_var=mean(var1 to var5).
*** CALCULATING MEASUREMENT VARIANCE (Bm) ***.
compute stat=mean(stat1 to stat5).
do repeat a=stat1 to stat5/b=pvar1 to pvar5.
compute b=(a-stat)**2.
end repeat.
compute pvmerr=.25*(sum(pvar1 to pvar5)).
*** COMPUTE STANDARD ERROR [V = U + (1+1/M)Bm] ***.
compute SE=sqrt(pv_var+1.2*pvmerr).
formats stat (f8.3)/ SE (f10.6).
list cases/var= !grp quart stat SE.
!enddefine.
```