RECENT DEVELOPMENTS IN INDEX NUMBER THEORY AND PRACTICE

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INTRODUCTION

When inflation was running at 10 or 20 per cent per year, as happened in many OECD countries in the **1970s** and **1980s,** the accuracy with which it could be measured was not particularly important. Policy objectives could be framed simply in terms of reducing the rate of inflation, perhaps with the eventual aim of eliminating it altogether. However, it is not so easy to determine when a precise rate of inflation, such as zero inflation, has actually been realised. Measured rates of inflation (and real growth) are based on index numbers, and it is worth recalling that there is a variety of different index numbers to choose from, all of which will be registering different rates of change at any given moment of time. Moreover, there is still no consensus among economists about what is the best type of index number from a conceptual or theoretical point of view. In addition, there are various ways in which biases can creep into the measures in practice.

These problems do not disappear when inflation is reduced. The achievement of price stability may be a superficially attractive objective, but economists do not intend this to be taken too literally. The efficient working of the price mechanism requires that relative prices are continually changing in response to changes in supply and demand. Indeed, unless relative prices are flexible, markets cannot be expected to operate efficiently. Thus, even when the general price level is stable, changes in relative prices require that some prices actually rise while others fall. Indeed, in such circumstances, if one were able to observe the price movements of all commodities, one would see an entire frequency distribution of positive and negative price changes centred about zero. The index number problem would still be present, therefore. It would consist essentially of deciding what is the most appropriate form of average to take of these positive and negative price changes. While there may be an instinctive tendency to opt for some kind of arithmetic average, this may be not so easy to justify on theoretical grounds. For example, it would not be unreasonable to define zero inflation as a situation in which positive and negative price changes occur with equal frequency, but in this situation most conventional index numbers would be likely to be registering positive rates of inflation (assuming that the frequency distribution of price changes is positively skewed, as seems likely to be the case in practice). Conversely, zero inflation as conventionally measured may well imply that negative price changes occur with greater frequency than positive changes.

Indeed, far from disappearing when there is no systematic tendency for prices to rise or fall, the index number problem is likely to assume a rather perverse form. Not merely will different index numbers, i.e. different forms of average, tend to lead to different results, some of them will register positive changes and others negative, so that different indices may be moving in different directions.

Index number problems, like the poor, may be conveniently forgotten, but they are always with us. The main purpose of this article is to bring users of economic data up-to-date about recent developments in index number theory and practice by providing a survey of the progress made in the last ten or fifteen years. The paper assumes a basic knowledge of the main index numbers and their properties – in particular, Laspeyres, Paasche and Fisher – but not much more. In fact, there have been a number of significant theoretical advances made in the last decade or *so* which tend to suggest that the index numbers in common use are mostly second best from a theoretical viewpoint. In particular, there is steadily increasing support for the use of chain indices whereas most of the indices actully used for economic analysis and policy making employ fixed weights. In this paper, therefore, particular attention is paid to the properties and behaviour of chain indices and also to the pros and cons of using chain indices more extensively in both national accounts and the macroeconomic models which depend upon national accounts data. The last section of the paper presents some new types of chain indices which have some attractive properties from both a theoretical and practical viewpoint and which throw new light on the properties, and biases, of conventional fixed weight indices.

I. RECENT DEVELOPMENTS IN INDEX NUMBER THEORY

There has been a recent resurgence of interest in index number theory following a number of important articles and books which appeared in the early **1970s:** for example, Griliches **(1971),** Pollak **(1971),** Christensen, Jorgenson and Lau **(1971),** Fisher and Shell **(1972),** and Samuelson and Swamy **(1974).** During the middle and late **1970s,** further important contributions were made by Diewert and by Eichorn and Voeller which are summarised in some detail in this section. At the same time, an almost new field of index number theory has been opened up by the development of multilateral international price and volume measures stemming from the pioneering work of Kravis, Heston and Summers on the U.N./World Bank International Comparison Project **(1982).** This paper will, however, confine itself to inter-temporal indices.

There are two fundamentally different approaches to index number theory. The first is the so-called "axiomatic" approach, which has been improved and refined by the work of Eichhorn and Voeller over the last decade or so (see 1983 for a summary account of this work). The axiomatic approach is one in which the theoretical foundations of index numbers are built on certain postulates, or axioms, which are meant to be so general that any index must satisfy them in practice. This approach stems from the work of Irving Fisher (1922) who required indices to satisfy certain conditions, or tests, if they were to be useful for economic analysis or policy making.

The second approach is the so-called "economic theoretic" approach, which seeks to define price or volume indices with reference to underlying utility or production functions. It stems from the early work of Konus on the "true cost of living" (1924) and is associated with a string of famous names in economics from Keynes and Frisch to Hicks and Samuelson.

A. The axiomatic approach to index number theory

The axiomatic approach starts from the actual prices and quantities observed in the two periods or situations being compared. The prices and quantities are treated as independent variables (in contrast to the economic theoretic approach, in which the quantities are deemed to be functions of the prices). Eichhorn and Voeller define an index as a function of the observed prices and quantities which satisfies four basic axioms: monotonicity, proportionality, price dimensionality and commensurability (see 1983, pp. 417-8). These axioms may be summarised as follows in the case of a price index. Monotonicity requires that a price index is increased whenever any of the prices in the current period are raised or any of the prices in the base period are lowered. Proportionality requires that when all prices in the current period are uniformly greater or lower than those in the base period by some fixed proportion, the index should equal that proportion. Price dimensionality requires that the same proportional change in the unit of currency in both periods (e.g. from pence to pounds) does not change the index, while commensurability requires that a change in the unit of quantity for any commodity in both periods (e.g. from kilos to tons) does not change the index.

These axioms are described as "basic properties which are desirable for every price index" and almost all the indices in common use satisfy them (1983, p. 418). Indices which have these properties automatically satisfy various tests of the type which Irving Fisher proposed, such as the identity test, the weak proportionality test and the mean value test.

The class of indices satisfying the above general criteria is extremely wide – in fact too wide. In order to narrow the field, Eichhorn and Voeller consider the implications of imposing further conditions, such as the time reversal test, the factor reversal test and the product test. The product test is the weak version of Fisher's famous factor reversal test. It states that the product of a price and a quantity index should equal the expenditure ratio where the price and quantity indices do not necessarily have to have the same form, but must both satisfy the four basic axioms for an index number. The product test is extremely important for economic analysis whenever time series data are available in current values. The product test requires that when the change in the current values is divided by a price index, we should come out with a recognisable and acceptable quantity index, even if it has a different form or properties from the price index. The combination of a Laspeyres quantity with Paasche price index provides the classic example of a pair of indices satisfying the product test even though neither index on its own satisfies the factor reversal test. This property of Laspeyres and Paasche indices is, of course, extensively exploited in economic statistics where one index (e.g. a GDP deflator) may be obtained ***indirectly*** by dividing a value ratio by another index (e.g. a Laspeyres quantity index).

The product test is therefore a ***sine qua non*** for most economic statistics, but adding the product test to the list of four basic axioms does not narrow the class of possible indices very greatly. However, adding Fisher's circular test as well produces spectacular results as the ensuing set of possible indices is empty, as Eichhorn and Voeller prove. There is no possible index, i.e. an index satisfying the four basic axioms, which also satisfies both the product test and the circular test (1983, pp. **446-7).** This is an example of an "inconsistency theorem" or "non-existence theorem" to use the terminology of Eichorn and Voeller.

Fisher's circular test has always been controversial, and Fisher himself finally decided to abandon circularity, or transitivity as it is usually called today. First of all it is useful to clarify what is meant by transitivity. If ${}_{A}I_{B}$ represents an index B based on A, then transitivity requires that:

$$ {}_{A}I_{C} = {}_{A}I_{B} \cdot {}_{B}I_{C} \tag{1} $$

Transitivity implies that a direct comparison between A and **C** should yield the same result as an indirect comparison between A and ***C*** via ***5.*** When the indices which make up the individual links of a chain index are transitive, the ratio between the end points of the chain is the same as that yielded by a direct comparison between the two end points.

Under what conditions can transitivity be achieved? Some new insights have been gained in the last few years as a result of the work of Eichhorn and others in Germany. The following theorem has been independently derived by Hacker and Krtscha in **1979 (1979);** namely, that if a price index is to satisfy both the proportionality axiom and the circular test it must depend only on the prices and not on the quantities in successive periods, or locations. An example of such an index is a weighted geometric mean of the price relatives with ***fixed*** weights (which some authors call a Cobb-Douglas index). In this case, only the price movements from period to period are taken into account and the changes in quantities do not enter into the calculation of the index. Such an index can be described as a chain index in only a purely formal or trivial sense of the term. The whole rationale behind a temporal chain index is that the total change registered between the first and last periods should depend upon the path traced out by all prices and quantities in the intervening periods. This path reflects the gradual process by which economic units adjust to the changing economic environment in which they find themselves.

The theorem proved by Hacker and Krtscha shows that, if a chain index does indeed reflect this process of adjustment by utilizing information on ***both*** prices and quantities, the proportionality axiom has to be sacrificed. Suppose, for purposes of argument, that a chain price index is constructed whose links consist of Laspeyres price indices connecting successive time periods and that all prices eventually return to their initial values. The proportionality axiom requires that the index for the last period should be unity, but Hacker and Krtscha have proved that this cannot be the case. It has long been known that, ***in practice,*** a chain Laspeyres index does not return to unity in these circumstances, but without rigorous proof that this must be so. This phenomenon has been described as "drifting" and it shows that chain indices may be unsuitable whenever prices or quantities return approximately to their initial levels after some kind of disturbance, the classic example being regular seasonal fluctuations. Instead of cancelling out over a complete year, some components of the seasonal fluctuations are liable to get permanently incorporated into a monthly chain index causing the index to drift further and further away from a trend based on annual averages for the same series. Such behaviour is quite unacceptable in general. In a recent paper (1983, pp. 540-1), Szulc gives a simple numerical example to illustrate the fact that a chain Laspeyres does not return to unity when all prices return to their initial values, while an algebraic demonstration of why this is *so* is given later in this paper.

Given Eickorn and Voeller's non-existence theorem (which draws on the theorem of Hacker and Krtscha), it is natural to ask which of the conditions ought to be relaxed in order to enable an index to be defined. The four basic axioms of Eichorn and Voeller seem uncontroversial, while the product test is a ***sine qua*** non of economic statistics, as already tested. It appears, therefore, that transitivity may have to be abandoned.

At first sight, it is rather disconcerting to have to abandon transitivity. If lack of transitivity is interpreted as being symptomatic of some kind of logical inconsistency in the underlying system of measurement, it is disturbing to find that the Laspeyres index, which is probably the most widely used economic index, is intransitive (except in certain trivial special cases). However, lack of transitivity may not be so undesirable in practice. What it really implies is that it is necessary to choose ***either*** a chain index ***or*** a fixed weight index if results which are numerically inconsistent with each other are to be avoided. Not surprisingly, the choice of method involves a trade-off. It is fairly obvious that if a chain index is preferred, direct comparisons between pairs of years which are separated in time must be abandoned. It is not so widely appreciated, however, that if the alternative solution of fixed weight indices is adopted, direct comparisons between all pairs of years which do not include the base year, including comparisons between consecutive years, have to be abandoned and replaced by indirect comparisons – in effect, by chain indices.

Consider, for example, the usual situation in national accounts, where time series are compiled at the constant prices of some base year 0; in effect, this is equivalent to calculating a series of Laspeyres volume indices based on year *0.* As a result, the year to year movements are implicitly chain indices in which period t + 1 is linked to period t via period 0. The volume index between t + 1 and t is in fact given by:

$$\frac{\Sigma p_0 q_{t+1}}{\Sigma p_0 q_t} = \frac{\Sigma p_0 q_0}{\Sigma p_0 q_t} \cdot \frac{\Sigma p_0 q_{t+1}}{\Sigma p_0 q_0} \qquad (2)$$

$$= \underset{t \text{ volume } 0}{\text{Paasche}} \cdot \underset{0 \text{ volume } t+1}{\text{Laspeyres}}$$

Thus, the year to year change between t and t + 1 is measured by using period 0 as a link between them. The (backwards) Paasche volume index for period *0* based on period *t* is multiplied by the (forwards) Laspeyres volume index for t + 1 based on period 0. This is not exactly a simple measure conceptually, but the **loss** of simplicity in year-to-year measures is the price paid for the convenience of working with a fixed base year.

The situation is worse for the price measures derived indirectly from such volume measures, the so-called price "deflators". By dividing the change in values at current prices between t and t + 1 by the expression given in (2) above, the following expression is obtained:

$$\text{Price deflator} = \frac{\Sigma p_0 q_t}{\Sigma p_t q_t} \cdot \frac{\Sigma p_{t+1} q_{t+1}}{\Sigma p_0 q_{t+1}} \qquad (3)$$

$$= \underset{t \text{ price } 0}{\text{Laspeyres}} \cdot \underset{0 \text{ price } t+1}{\text{Paasche}}$$

that is, by the (backwards) Laspeyres price index for period 0 based on period t multiplied by the (forwards) Paasche price index for *t + 1* based on 0. This expression does not even satisfy Eichorn and Voeller's proportionality axiom, because if $p_{it+1} = \lambda p_{it}$ for every commodity *i,* the "price deflator" given by (3) is not equal to λ (except in the trivial special case in which the quantity weights are identical in *t* and t + 1). Thus, the GDP and other price deflators widely used to measure year to year rates of inflation by economic analysts and policy makers do not satisfy Eichorn and Voeller's four basic, and apparently innocuous, axioms: they do not even qualify as price indices on the axiomatic approach.

On the axiomatic approach, no index number emerges as front runner for all purposes, and at this point it is convenient to turn to consider the alternative approach based on utility or production functions.

B. The economic theoretic approach to index numbers

The discussion of the previous section focused on the properties of index numbers in relation to the actual price and quantity observations in the situations being compared by postulating that indices must behave in certain ways in certain circumstances, e.g. when all prices change by the same proportion. The economic theoretic approach is quite different. In particular, the prices and the quantities are not treated as separate independent variables because the quantities are assumed to be functions of the prices. Thus, the information on which an economic theoretic index is based does not consist of two price vectors and quantity vectors, but rather two vectors of prices plus a functional relationship connecting the quantities to the prices in both the situations being compared. As the parameters of this underlying function are generally unknown and incapable of being estimated in most real world situations, it follows that economic theoretic indices, although precisely defined, cannot be calculated in practice (except in special circumstances).

There are two main kinds of functions relating quantities to prices, namely utility functions and production functions. In order to have a more neutral terminology, Diewert has proposed the use of the term "aggregator function" to cover both (1981, p. 163). As the present discussion is intended to be as brief as possible, however, it is convenient to focus on price indices and utility functions to the neglect of quantity indices and production functions. However, most of the conclusions reached apply, ***mutatis mutandis,*** to quantity indices and production functions.

The classic example of an economic theoretic index is the cost of living index which, following Pollak, may be defined as "the ratio of the minimum expenditures required to attain a particular indifference curve under two price regimes" (1971, p. 94). It should be noted that the index depends not only on the two sets of prices but also on a specific indifference map or preference ordering ***and*** the choice of a base indifference curve.

The problem is to make inferences about such an index from actual observations on prices and quantities. The main results are well known and will be only briefly summarised here. Consider two situations A and B. Assuming the actual situation in A represents the utility maximising situation of a rational consumer, then the Laspeyres index provides an upper bound to the theoretic index based on A. Similarly, the Paasche index provides a lower bound to the theoretic index based on B. In general, there are two theoretic indices depending on whether the index takes situation A or situation B as its reference point, because A and B lie on different indifference curves.

The reasoning behind these conclusions is simple. Suppose that the money income of a consumer is increased by the same proportion as the Laspeyres index based on his actual expenditures in the first situation, or situation A. It follows, by definition, that he could buy the same basket of goods in B as in A. However, assuming that the relative prices of commodities change between A and B, it also follows that the consumer has the opportunity to increase his utility by changing his pattern of consumption between A and B to take advantage of the changes in relative prices. Thus, by adjusting his pattern of consumption he must be better off in practice. The Laspeyres index therefore provides an upper bound to the "true" index as defined by Pollak. Similar reasoning establishes that the Paasche index provides a lower limit to the "true" index based on B.

it should be noticed that a theoretic index does not have to be defined with reference to the preferences of the individual whose expenditures are actually under observation. One could, for example, define a true cost of living index between the United Kingdom and Germany with reference to the tastes and income level of a typical French consumer, or a typical European consumer. It can be argued, for example, that when making multilateral price comparisons among a group of countries, the theoretically most appropriate indices are those which are actually based on the preferences and income of an average consumer for the group as a whole. Moreover, if the multilateral indices relate to the preference ordering of a single individual they are more likely to be transitive. Indeed, if the preferences are homothetic (i.e. if each indifference curve is a uniform enlargement, or contraction, of each other so that all indifference curves have the same "shape"), the indices will actually be transitive, as Samuelson and Swamy have pointed out (1974). The same point has also been noted by Diewert (1983, Theorem 1, p. 169). Thus, from the viewpoint of the European Economic Community, the theoretically most appropriate index between the United Kingdom and Germany is not necessarily one which involves either a typical U.K. consumer or a typical German consumer, but a typical Community consumer. In general, however, the literature has focused on the case in which the tastes and income are those of the individual, or individuals, whose expenditures are actually observed in one or other of the two situations being compared.

The next question is to see what assumptions are required in order to be able to make more precise inferences about the underlying theoretic index, or indices, from actual price and quantity data. Among the more important results to be established more than half a century ago is the fact that the theoretic index is invariant to the level of utility, or base indifference curve, when the preferences are homothetic. This implies, for example, that when preferences are homothetic the Laspeypes and Paasche indices provide upper and lower bounds to the same underlying theoretic index, in which case there is a strong temptation to take some kind of average of the two as the best estimate of the theoretic index. Fisher's Ideal Index is, of course, the most famous example of such an index, but Samuelson and Swamy have proved that any symmetric mean of the Laspeyres and Paasche index will approximate the theoretic index up to the third order in accuracy (1974, p. 582).

It is generally agreed, of course, that it is not plausible to assume that preferences are homothetic in the real world. For example, Pollak remarks that "these results are important not because we believe that peoples' indifference maps are homothetic but because we believe they are not." (1971, p. 114). Thus, in general, cost of living indices do vary significantly with income, or at least may be presumed to do so. Moreover, Samuelson and Swamy also warn against the "common fallacy" of assuming that, if the Laspeyres and Paasche indices are not too far apart, they must provide bounds for the theoretic index by giving an example where the theoretic index lies outside the interval spanned by the Laspeyres and Paasche indices (1974, p. 585). On the other hand, there is more justification for assuming that production functions may be homothetic so that the conclusions reached in the homothetic case may be more relevant to production indices than price indices.

Important new results and insights have been gained in the last decade as a result of the work of W.E. Diewert (1983 and 1981, for example). First, it is useful to recall the result (apparently established for the first time in 1925 by Buscheguennce) that, when the utility function can be represented by a homogeneous quadratic function, Fisher's Ideal Index is exact (1925); that is to say, it coincides with the underlying theoretic index as defined above. While this is an interesting result, it does not provide a justification for the general use of the Ideal Index because its validity depends on the assumption that the underlying utility function assumes one particular (and not very plausible) form out of an infinity of possible functional forms. Exact indices can be deduced for some other underlying utility functions, but each one constitutes a special case which does not provide a satisfactory basis for generalisation.

In the last decade, however, Diewert has made important theoretical advances by introducing the notion of "superlative" index numbers. The idea is not to search for more and more special cases, but to provide a genuine basis for generalisation by working with "flexible" functional forms. A flexible functional form is one "which is capable of providing a second order differential approximation to an arbitrary twice continuously differentiable linearly homogeneous aggregator function" (1981, p. 185). The class of linearly homogeneous aggregator functions covers a wide range of possible utility (or production) functions, and Diewert's objective is to identify specific functions which will provide close local approximations to any individual function in the class without having to know, or estimate, the parameters of the latter. Once a flexible functional form has been identified, it is possible to specify the theoretic index associated with it which in turn must provide a close approximation to a range of other possible theoretic indices. Diewert describes an index as "superlative" if it is exact for some flexible functional form.

Diewert has pointed out that the homogeneous quadratic function used by Buscheguennce is, in fact, an example of a flexible functional form (1981, p. 185). Thus, he concludes that Fisher's Ideal Index is not merely exact in one special case but may be presumed to provide a close approximation to a range of other theoretic indices. It is thus not merely ideal but superlative.

However, the homogeneous quadratic function is not the only case of a flexible functional form. Another is provided by the homogeneous translog function introduced by Christensen, Jorgenson and Lau in the context of productivity measurement (1971). The homogeneous translog function also provides a second-order approximation to an arbitrarily twice-continuously differentiable linearly homogeneous function. The theoretic index corresponding to the translog function is the so-called Tornqvist index (which was also considered as a possible index by Fisher, incidentally). The Tornqvist (or translog) price index, T, is defined as follows:

$$P_T = \prod_i \left(\frac{p_{it}}{p_{io}}\right)^{\frac{1}{2}(s_{io} + s_{it})} \quad (4)$$

where s_{it} and s_{i0} denote the shares of total expenditure accounted for by commodity i in periods t and 0 respectively. This index is an example of what might be called a "symmetric" index: namely, one which attaches equal importance to prices and quantities in both situations. Fisher's index is another example of a symmetric index.

Two examples of flexible functional forms have been given so far and they both turn out to be special cases of a more general flexible functional form, namely a quadratic mean of order r aggregator function defined as:

$$M_r = \left(\sum_i \sum_j a_{ij} p_i^{\frac{r}{2}} p_j^{\frac{r}{2}}\right)^{\frac{1}{r}} \quad (r \neq 0) \quad (5)$$

when $r = 2$ we obtain the homogeneous quadratic function used by Buscheguennce, while the homogeneous translog aggregator function is the limiting case of the function as r tends to zero (1981, p. 189). Thus, there is a whole family of possible flexible functional forms depending on the values assigned to r. Corresponding to these there is a family of price index numbers defined as:

$$P_r = \left\{\sum_i s_{i0} \left(\frac{p_{it}}{p_{i0}}\right)^{\frac{r}{2}}\right\}^{\frac{1}{r}} \left\{\sum_j s_{jt} \left(\frac{p_{jt}}{p_{j0}}\right)^{\frac{-r}{2}}\right\}^{\frac{-1}{r}} \quad (6)$$

When $r = 2$, Fisher's Ideal Index is obtained.

Given the multiplicity of possible superlative indices corresponding to different values of r, it must be inevitably asked which index formula should be used in practice. The answer suggested by Diewert is that it does not appear to matter very much as they will all give the same answer to a very high degree of approximation, at least for "normal" time series data (**1981,** p. **189;** and **1983,** p. **186).** Certainly, the two specific superlative indices considered above, namely the Fisher and Tornqvist (or translog) indices will tend to give similar results in practice because both are essentially averages of price relatives which treat the quantities, or expenditures, in the two situations symmetrically.

There is, however, another factor to be taken into consideration because the flexible functional forms in (5) above all imply homothetic preferences, except for the limiting case of the translog function. Moreover, it is important to remember that the assumption of homotheticity, at least for preferences, is generally agreed to be unrealistic. Thus, although Fisher's Ideal Index may also be superlative, its theoretical validity still rests on a restrictive and unrealistic assumption. For this reason, Diewert's final choice seems to be the Tornqvist index **(1983,** p. **187)** although he continues to be attracted by the Fisher index for other reasons. However, these conclusions are somewhat paradoxical because, although the Tornqvist or translog index does not require homothetic preferences whereas the Fisher index does, both are symmetric indices which seem likely to yield similar results in practice, as just noted. The empirical results obtained by Hansen and Lucas **(1984,** pp. **32-3),** provide support for the view that the two indices tend to behave in a very similar fashion.

Diewert also argues strongly for the use of chain indices on the grounds that the shorter the time period, the closer together the Laspeyres, Paasche and all the superlative indices will tend to be, and hence the closer they will tend to approximate to the underlying theoretic index.

In general, it may be concluded from this very brief survey of work on economic theoretic indices that there has been a resurgence of interest in this topic in the last decade, starting with the major paper by Samuelson and Swamy in the **1974** ***American Economic Review.*** The impetus has been sustained through a sequence of important contributions from Diewert and others, and possibly also as a result of the very high rates of inflation in the **1970s.** In addition, there has been an explosion of work on international purchasing power parities which has thrown up some traditional index number problems in a more stark form. One conclusion which seems to emerge from this work, especially that of Diewert, is a reaffirmation of the desirable properties of certain symmetric indices, notably Fisher's Ideal Index and the less familiar Tornqvist Index, both of which are "superlative". The theoretical advance lies not so much in the introduction of new index numbers to match new aggregator functions but in the realisation that certain kinds of functions are flexible, i.e. provide good approximations to a class of other functions. The stumbling block is that many of the specific conclusions or firm results continue to depend on the unrealistic assumption of homothetic aggregator functions, although the introduction of the translog aggregator function constitutes a step forward away from homotheticity. Samuelson and Swamy may be given the last word on this subject:

> "Empirical experience is abundant that the Santa Claus hypothesis of homotheticity in tastes and in technical change is quite unrealistic. Therefore, we must not be bemused by the undoubted elegances and richness of the homothetic theory. Nor should we shoot the honest theorist who points out to us the unavoidable truth that in nonhomothetic cases of realistic life, one must not expect to be able to make the naïve measurements that untutored common sense always longs for ..." (1974, p. 592).

II. CHAIN INDICES

In the recent work on both the axiomatic and the economic theoretic approaches to index numbers, there seems to be an increasing preoccupation with chain indices. However, surprisingly little is known about the property and behaviour of chain indices except in one or two special cases of which the most familiar is that in which prices return to their initial values.

At the very least, it is necessary to know more about how the behaviour of chain Laspeyres and Paasche indices compares with that of their corresponding direct counterparts. It is sometimes assumed that chaining tends to reduce the index number spread between Laspeyres and Paasche indices, and there are case studies to support this view. However, other studies suggest that chaining does not necessarily reduce the index number spread and may possibly even increase it: i.e. the chain Laspeyres index may exceed the direct Laspeyres, while the chain Paasche may be below the direct Paasche. Moreover, expert opinion has been divided on this point. The most cautious conclusion is that of R.G.D. Allen who argued that "there is no reason to expect that a chain Laspeyres index drifts above the direct Laspeyres index nor, equally, that it tends to—correct for any propensity for the direct index to run high. Empirical evidence is needed..." (1975, p. 188). There is little doubt that this uncertainty, indeed confusion, about the properties of chain indices tends to discourage their use.

As chain indices depend on the path followed by prices and quantities over the period covered, it should be obvious at the outset that no progress can be made on this issue unless something is known, or some assumption is made, about the way in which prices and quantities move over the period in question. In a recent paper (1983), Bodhan Szulc has addressed this question directly by considering two polar cases. The first case is that in which prices and quantities tend to keep moving smoothly in the same general direction throughout the period covered. In other words, the pattern of relative prices at the beginning of the period is gradually transformed into the pattern prevailing at the end of the period without much disturbance along the way. In this case, Szulc has shown that the chain Laspeyres will lie below the direct Laspeyres, and the chain Paasche above the direct Paasche, so that chaining will tend to reduce the index number spread between Laspeyres and Paasche, possibly quite significantly. The contrary case, of course, has to be that in which relative prices and quantities tend to oscillate. In other words, some commodities become relatively cheaper at first only to become relatively dearer again towards the end of the period. Szulc describes relative prices as 'bouncing" (1983, p. 548) in this case, relative price changes being first positive, then negative (or vice versa). When "bouncing" is predominant Szulc shows that the chain Laspeyres tends to exceed the direct Laspeyres, while the chain Paasche tends to be lower than the direct Paasche: i.e. the index number spread is increased by chaining. If some prices move smoothly while others bounce, the chain indices may not differ significantly from their direct counterparts. Szulc's proofs rely on the standard assumption that movements in relative prices and relative quantities are negatively correlated, i.e. that we are observing the responses of price takers to movements in the relative prices with which they are confronted. It is this phenomenon which is, of course, responsible for the spread between the Laspeyres and Paasche indices in the first place.

Against this background it is interesting to come back to the hypothetical case in which the prices in some later period are proportional to those in some earlier period, *i.e.* the pattern of relative prices, after some disturbance, returns to its initial state. For example, prices of seasonal products may behave in approximately this way over a period of twelve months. Whatever price changes occur initially must be subsequently reversed if relative prices are to return to their initial state so that "bouncing" must be prevalent and the chain Laspeyres should exceed the direct Laspeyres according to Szulc. Repetitions of this kind of cycle over subsequent periods will cause the chain Laspeyres to 'drift" further and further away from the direct Laspeyres.

Because of the attention which this case has attracted, and because of the importance of the proportionality axiom in the axiomatic approach to index number theory, it is worth examining it more closely. Consider a comparison between two non-consecutive periods of time 0 and t in which the price of every commodity in period t is a multiple of that in period 0, i.e. $p_{it} = \lambda p_{io}$. The direct Laspeyres and Paasche price indices for t based on 0 must be λ ; indeed, because of the proportionality axiom, every acceptable price index ought to equal λ according to Eichorn and Voeller. Now introduce a link between periods 0 and t at period k where $0 < k < t$. The chain Laspeyres price index connecting 0 and t using period k as the link is:

$$\frac{\Sigma p_k q_0}{\Sigma p_0 q_0} \cdot \frac{\Sigma p_t q_k}{\Sigma p_k q_k} = \lambda \left\{ \frac{\Sigma p_k q_0}{\Sigma p_0 q_0} \bigg/ \frac{\Sigma p_k q_k}{\Sigma p_0 q_k} \right\} \tag{7}$$

given that p_{it} = A p_{i0}.

The expression in brackets is the ratio of the Laspeyres to the Paasche price index for period k based on period 0. Thus, the ratio of the chain Laspeyres index to the direct Laspeyres index for period t based 0 is equal the ratio of the Laspeyres and Paasche indices for the link period k based 0. As the ratio of a Laspeyres to a Paasche index increases with the covariance between the price and quantity relatives, as Bortkiewicz (1925) first showed, it follows that the discrepancy between the chain and the direct Laspeyres indices connecting 0 and t increases with the covariance between the price and quantity relatives between 0 and k, the link period. This covariance reflects the extent to which the pattern of relative prices in the link period k differs from that in periods 0 and t. In other words, the discrepancy between the chain and the direct Laspeyres indices increases to the extent that passing from 0 to t via k involves a detour (in an economic sense).

Of course, the moral to be drawn from this example is that a chain index should not be used in these circumstances. When two situations are very similar, or identical, from an economic point of view they should be compared directly, and not indirectly, by means of a link through another situation which differs significantly from both of them. There is no sense, for example, in using India as the link in a chain comparison between France and Germany, or for that matter **1980** as the link between **1987** and **1988.**

Thus, it is already possible to draw some tentative conclusions about when chain indices should, and should not, be used:

i) A chain index should ***not*** be used when the two situations being compared are similar from an economic point of view and the chaining involves linking through a situation which is dissimilar to both. Similarity in this context refers essentially to the extent to which the patterns of relative prices in the two situations resemble each other.

ii) Conversely, a chain index should be used when the two situations being compared are very dissimilar to each other and when the linking can be realised by passing through an intermediate point which lies between them. The ideal intermediate situation would be one in which the pattern of relative prices in the link situation could be roughly approximated by some kind of average of the relative prices in the two situations being compared. In these circumstances, the index number spread between Laspeyres and Paasche will tend to be significantly reduced by chaining – a point to be elaborated further below.

These conclusions can be reinforced by introducing an alternative line of reasoning. In the real world, the main problem confronting index number compilers is not the choice between the various index number formulae but the practical problem of how to deal with the fact that many commodities are to be found in only one of the two situations being compared. Although the quantity vectors are fully determined for both situations (consisting of elements which are either positive or zero) the price vectors are incomplete as prices are missing for some commodities in one or other of the two situations. It is not possible, therefore, to calculate price relatives for such commodities. Moreover, it is quite impractical to envisage estimating shadow prices for them on a large scale, especially since the products affected tend to be atypical. In a time series context, they tend to be old products which disappear as a result of obsolescence or the exhaustion of supplies or new products which appear as a result of technological progress.

Index number theory is embarassingly reticent on this subject because many of the theorems about index numbers rest on a tacit, and quite unrealistic, assumption that price and quantity data are available for all commodities in both situations. In other words, most of the theory refers to vectors of prices and quantities whose elements are assumed to be strictly positive in both situations. The consequences of dropping this assumption are far-reaching and can be illustrated by a simple example. Consider two situations A and B such that the prices of those commodities which are actually found in both, situations are identical, but some additional commodities are to be found in B which are not present in A. First, it may be noted that an axiom such as the proportionality axiom ceases to have much relevance in such a situation because it is impossible, by assumption, for all the prices in A to be proportional to those in B when some prices do not exist in A. In practice, any price index calculated between A and B will tend to be confined to those commodities found in both situations and such an index must be unity under the conditions postulated above. However, the fact that the set of consumption possibilities is larger in B than in A implies that the economic theoretic index for B based on A must be less than unity. To the extent that consumers actually avail themselves of the extra commodities present in B but not in A, it follows that they must be better off in B than A with an equal money income in both situations. Thus, the index compiled on the basis of commodities available in both situations will tend to overstate the underlying theoretic index for B based on A. In general, it may be concluded that in the real world, price indices which are inevitably restricted to commodities found in both situations will fail to capture the improvement of welfare associated with an enlargement of the set of consumption possibilities. The benefits brought by the introduction of new goods are not generally taken into account in price indices in the period in which the goods first make their appearance.

For this reason, many index number compilers and other authorities are convinced that actual price indices tend to have some upward bias because they do not capture all the benefits accruing from technological progress and the introduction of new goods or (to use a different terminology to express the same idea) improved "qualities" or "models" of existing goods.

Of course, the argument works in the opposite direction in the case of disappearing products which reduce the consumption possibility sets of consumers. Moreover, products do not always disappear because they have become obsolete: they may also disappear because the supply of some input becomes exhausted. It is a matter of judgement, or opinion, whether the effects of new products outweigh those of disappearing products, but the majority view seems to be that the enlargement of choice due to new products is the predominant factor so that, on balance, existing price indices seem likely to have an upward bias compared with the corresponding economic theoretic indices.

The fact that some products gradually disappear over time to be replaced by others is directly relevant to the question of whether or not to use a chain index. When the set of products is continually changing, it follows that the number of commodities whose prices can be compared directly tends to diminish the further apart the periods become. In other words, the proportion of the total values of the expenditures in the two periods which can actually be covered by direct price comparisons steadily diminishes the further apart they are. If, therefore, we insist on a direct comparison between two periods which are far apart – separated by, say, a quarter or half a century – we must accept the fact that it is likely that price relatives can be compiled for only quite a small proportion of the total values of the expenditures in both periods. In these circumstances, the coverage of the price relatives may become so poor as to bring into question the validity, or usefulness, of a direct comparison between the two periods (quite apart from the fact that the index number spread between the Laspeyres and Paasche indices for the commodities actually included in the price index will also tend to be very large).

On the other hand, if a chain index is employed, the amount of price information which can be exploited is greatly increased. When comparisons are made between consecutive time periods the overlap between the sets of commodities found in both periods is likely to be greatest. In general, it is clear that when calculating each individual link in a chain index it will be possible to use almost all of the price information available in both periods, and the problem created by new and disappearing products will tend to be minimised. This applies to each and every link in the chain, including the first and last links, so that the amount of price information in the first and last periods which is actually utilised will tend to be far greater than if a direct comparison is attempted between them.

The conclusion reached earlier with respect to the behaviour of chained Laspeyres or Paasche indices was that chaining is desirable when the patterns of relative prices in the two periods are very different from each other and when the prices and quantities move over time in a fairly smooth manner without too many fluctuations in relative prices or quantities along the way. In these circumstances, chaining will tend greatly to reduce the index number spread between Laspeyres and

Paasche indices as compared with their direct counterparts. Of course, the circumstances in which patterns of relative prices tend to differ greatly are when they are far apart in time. In these circumstances, however, not only will patterns of relative prices for those commodities available in both periods tend to diverge, the coverage of the price relatives will also tend to be very poor, and possibly insufficient, because very many of the products can be found in only one or other of the two periods. The previous argument in favour of chaining is thus reinforced by the fact that chaining permits a much greater amount of price information to be utilized. When each individual link in the chain connects consecutive time periods, most goods will be found in both periods so that the coverage of the price relatives will always tend to be very high.

The general conclusion to be drawn from this discussion is that when there is a conflict between a direct and an indirect comparison, the indirect comparison is often to be preferred. It follows that a direct comparison should not be elevated to the yardstick, or criterion, by which to evaluate alternative measures. The direct comparison is not always the best even though there is a tendency to assume this, at least implicitly, in some of the literature on index numbers.

A. Smooth chain indices

In this section a new type of index will be elaborated. As already explained, Szulc has recently shown that if the paths followed by prices and quantities are fairly smooth, a chain Laspeyres index will tend to lie below its direct counterpart, and ***vice versa*** for Paasche indices, thereby reducing the index number spread between them. It is therefore interesting to investigate the limiting case in which the paths followed by individual prices and quantities are as smooth as possible; namely, when each individual price and quantity rises, or falls, exponentially over the period covered.

Consider two time periods, 0 and t, which are not consecutive. For each commodity, the rates of change r_i and s_i can be calculated which satisfy the equations:

$$p_{it} = p_{i0}(1 + r_i)^t \qquad (7)$$

and

$$q_{it} = q_{i0}(1 + s_i)^t \qquad (8)$$

By using these equations to interpolate between 0 and t, the entire set of prices and quantities can be calculated thereby enabling various kinds of chain indices to be calculated. Such indices will be called "smooth chain indices" as all prices and quantities follow smooth growth paths.

In order to understand the properties of these indices it is convenient to start with the example of a chain Laspeyres index with only a single link; i.e. let $t = 2$. In this case, the smooth chain Laspeyres index takes the following form:

$$\frac{\Sigma\left\{\left(\frac{p_2}{p_0}\right)^{\frac{1}{2}}(p_0\,q_0)\right\}}{\Sigma\,(p_0\,q_0)} \qquad \frac{\Sigma\left\{\left(\frac{p_2}{p_0}\right)^{\frac{1}{2}}(p_0\,q_0)^{\frac{1}{2}}(p_2\,q_2)^{\frac{1}{2}}\right\}}{\Sigma\left\{(p_0\,q_0)^{\frac{1}{2}}\,(p_2\,q_2)^{\frac{1}{2}}\right\}} \tag{9}$$

For simplicity, further assume that all commodities have unit elasticities so that the pattern of expenditure does not vary between O and t and simplify the notation by writing:

$$x_i = \frac{p_{i2}}{p_{i0}} \quad \text{and} \quad w_i = (p_{i0}) \Big/ \Sigma\,(p_{i0}\,q_{i0}) \tag{10}$$

The chain Laspeyres index is then simply:

$$\left\{\Sigma\,(x_i)^{\frac{1}{2}}\,w_i\right\}^2$$

and the question is to compare the value of this index with the direct Laspeyres index, namely $\Sigma\,(x_i w_i)$.

Without loss of generality, we may also switch from weighted to unweighted indices bearing in mind that the unweighted indices may contain repeated observations of the same value. The question therefore reduces to comparing:

$$\left\{\Sigma\,\left(\frac{x_i}{n}\right)^{\frac{1}{2}}\right\}^2 \quad \text{with} \quad \Sigma\,\frac{x_i}{n} \tag{11}$$

and some simple algebra shows that the first expression must be less than the second (assuming the x's are not all equal to each other) the difference between them increasing with the variance of the x's. Thus, the smooth chain Laspeyres index must be less than it direct counterpart.

It is useful to give a simple numerical example at this point to illustrate the orders of magnitude involved. The data in the following table are chosen to have unit elasticities:

p_0	40	Pt	q_t
1	4	2	2
2	4	3	8/3
3	4	3	4

The direct Laspeyres price index for *t* on O is **133.22,** the Paasche is **124.14** and the Fisher **128.65.** Assuming $t = 2$ and introducing a link in period **1,** the value of each of the two link indices is **114.4** and their product, the smooth chain Laspeyres index, is equal to **130.86.** Thus, by introducing only a single link, the Laspeyres index falls from **133.33** to **130.86.**

It is clear, however, that the introduction of more links would reduce the resulting smooth chain Laspeyres even further. For example, if $t = 4$ the formula:

$$\left\{ \Sigma \left(\frac{x_i}{n} \right)^{\frac{1}{2}} \right\}^2 \quad \text{is replaced by} \quad \left\{ \Sigma \left(\frac{x_i}{n} \right)^{\frac{1}{4}} \right\}$$

and the arnooth chain Laspeyres falls to **129.66.** In the limit, the value of:

$$\left\{ \Sigma \left(\frac{x_i}{n} \right)^{\frac{1}{t}} \right\}$$

as $t \rightarrow \infty$ in this example is equal to **128.48** (just less than the direct Fisher).

Dropping the assumption of unit elasticities does not affect the general conclusions. First, it is necessary to give the general formula for smooth chain Laspeyres price indices, namely:

$$\prod_{k=0}^{k=t-1} \left[\frac{\Sigma \left\{ \left(\frac{p_t}{p_0} \right)^{\frac{1}{t}} (p_0\, q_0)^{\frac{t-k}{t}} \; (p_t\, q_t)^{\frac{k}{t}} \right\}}{\Sigma \left\{ (p_0\, q_0)^{\frac{t-k}{t}} \; (p_t\, q_t)^{\frac{k}{t}} \right\}} \right] \qquad (12) \quad \text{assuming } t \text{ links}$$

Although the individual price relatives remain constant from link to link, the expenditure weights gradually shift from those observed in period 0 to those in period *t.* Thus, the values of successive links in a smooth chain Laspeyres index are not constant: when demands are mostly inelastic so that an increase in relative price is associated with an increase in expenditure share, the values of successive link indices tend to increase in moving from 0 to *t;* conversely, when demands are predominantly elastic, the link indices tend to fall. It should also be noted that the greater the eiasticities, the stronger the negative correlation will tend to be between the price and quantity relatives and hence the greater the index number spread between the direct Laspeyres and Paasche indices for period *t.* It is in these circumstances that the smooth chain indices will tend to yield significantly different results from their direct counterparts.

Smooth chain Paasche indices behave in the opposite way to smooth chain Laspeyres indices. The smooth chain Paasche must always lie above its direct counterpart, so that the index number spread between smooth chain haspeyres and Paasche indices will generally be much smaller than that between their direct counterparts. Moreover, it may be inferred that as the number of links is increased the Laspeyres and Paasche indices tend to converge on each other. In the limit when all prices and quantities vary continuously and exponentially and the linking is done continuously, the smooth chain Laspeyres and Paasche indices coincide with each other and we are left with a single index, namely the Divisia index, and the conventional index number problem disappears. Moreover, in contrast to actual Divisia indices, which cannot usually be calculated because they depend on the unknown time paths of individual prices and quantities, the numerical value of a smooth Divisia index not only can be calculated but calculated simply from the price and quantity information for periods *0* and *t*. It is an operational index.

The value of the smooth Divisia in the numerical example above is 128.48 as compared with 128.65 for the Fisher and 128.49 for the Tornqvist. As the smooth Divisia treats the prices and quantities in periods *0* and *t* symmetrically, it is not surprising that its value turns out to be extremely close to other symmetric indices such as the Fisher and Tornqvist indices, which are also superlative indices in Diewert's sense.

In general, it may be concluded from this section, and the previous work of Szulc, that chaining will tend to minimise the traditional index number problem by greatly reducing the spread between alternative formulae, providedthat changes in prices and quantities proceed in a fairly smoth or regular fashion and that the chaining is done frequently.

Another way of looking at this is to note that when individual prices and quantities continue to change in a systematic manner, the cumulative effect of the changes over several periods is to shift the vectors of relative prices and quantities further and further away from their initial states. In these conditions, direct comparisons with the base period become progressively more difficult to execute and to interpret, whereas chaining between consecutive periods yields results which are more firmly based statistically and much less sensitive to choice of index number formula. On the other hand, if there were to be no trends underlying movements in prices and quantities, the differences between the vectors of prices and quantities for periods which are far apart would tend to be no greater than for periods which are close together and there would be no point in chaining. In practice, however, prices and quantities do tend to change in a systematic way as a result of permanent changes in demand and supply conditions so that chaining is generally to be preferred.

B. The actual usage of chain indices

Given the advantages of chain indices, especially for the measurement of short-term movements, it is somewhat surprising to find that they are still quite rare in official statistics. For example, according to a recent survey by the International Labour Organisation, (ILO, **1987)** only eight out of **161** Consumer Price Indices in respect of **149** countries were chain indices. By far the most common method used is a time series of Laspeyres indices on a fixed base (which, in some cases, may be twenty or more years out of date). Among OECD countries, the most common situation is a Laspeyres index based on **1980,** although four OECD countries – France, Norway, Sweden and the United Kingdom – do actually use a chain index with annual links.

There are several reasons why chain indices are not used more extensively in practice. One reason may be that there is still insufficient understanding about their properties and behaviour. A much more important reason, however, is that chain indices are more costly because they require information on both prices and quantities in all periods, whereas a fixed weight Laspeyres price index requires information on quantities, or expenditures, in the base period only. Another relevant factor is that collecting information about prices and quantities is also more time consuming and could considerably delay the publication of the index. For this reason, the quantity or expenditure weights which are used by those countries which actually calculate a chain consumer price index are generally estimates based on the year $t - 7$, or $t - 2$, or perhaps some average of the two, rather than the inevitably unknown weights for the current year t.

It should also be noted that many countries whose indices are nominally described as a fixed weight Laspeyres indices do utilize a form of chaining surreptitiously. Within a given expenditure category, it is standard practice, in most OECD countries at least, to replace some items by others for pricing purposes as certain products, or brands, disappear from the market and are replaced by others. In other words, the price movements for two different products may be spliced, or linked, together to give one continuous series over a longer period, without changing the weight of the expenditure category within which the spliced series falls.

Thus, some of the practical advantages of chaining are secured by this procedure as the set of goods and services actually being priced is gradually modified over time in response to changing market conditions. However, because the weights assigned to each expenditure category remain fixed, even at the most detailed level, the resulting indices continue to be described as Laspeyres indices with fixed weights even though the items whose price movements are weighted together to arrive at the overall index do not remain fixed over the life of the index.

In the case of national accounts data, the typical situation in OECD countries is to have fixed weight Laspeyres volume indices, using base years which tend to vary from country to country, the associated derived, or implicit, price indices or deflators being Paasche type indices. The attraction of Laspeyres volume indices in national accounts is that they can be transformed into time series of values expressed at the constant prices of the base year. Such data are very convenient for econometric modelling or other analytical work because they obey the same accounting rules as current price data and can therefore be manipulated algebraically to obtain derived series, in particular residuals such as value added at constant prices.

Chain indices, however, cannot be transformed in the same way because they are not "additively consistent". They cannot be converted into time series at constant prices because, by definition, the prices which enter into an annual chain volume index are changing from year to year. If the chain indices referring to a set of interrelated aggregates in an accounting framework are mechanically converted into value terms by multiplying them by the values of those same aggregates in the base year, it will be found that the converted figures do not add up. They do not respect the rules of arithmetic or accounting constraints. The fact that chain indices are not additively consistent is a distinct disadvantage when working with a set of interdependent variables within an overall accounting framework or macroeconomic model, and this is another reason why fixed weight indices remain popular. However, as stated earlier in this paper, there is a price to be paid for the convenience of working with long time series at constant prices. The year-to-year changes in such series, which are often the main focus of interest, are not ideal measures of year to year movements since they implicitly involve a roundabout link via the base year. Moreover, the year-to-year changes in the associated price deflators do not even deserve to be described as index numbers on the axiomatic approach.

SUMMARY AND CONCLUSIONS

Measures of inflation and economic growth are based on index numbers. As there are several alternative index number formulae in common use, each yielding a different numerical result, there is always a range of possible rates of inflation or growth to choose from. There can be no unique measure which is demonstrably superior to all others in all circumstances so that not too much significance should be attached to the precise value of some particular index unless the properties, and limitations, of that index are well understood. Similarly, not too much attention should be paid to small differences in rates of inflation or growth between different countries as these may well be attributable. at least in part, to differences in methods of measurement. Insofar as policy concerns are directed towards the rate at which inflation or growth is accelerating or decelerating within the same country, the situation is better because differences in the rates of change of a given index between successive periods of time may be measured reasonably reliably even if the index itself is biased, provided the bias is consistent over time. The problem is to establish when a particular absolute rate of change, such as zero inflation, has been realised.

The purpose of this article has been to take a fresh look at index number theory and practice in the light of various advances which have been made in the last ten or fifteen years. The results are not very reassuring because the indices in common use emerge as rather obsolete and second best from a theoretical viewpoint.

Two main conclusions may be drawn from the recent theoretical literature on index numbers. The first is that indices which utilise the price and quantity information in both periods ***symmetrically*** are generally to be preferred. Among such indices, the Fisher and Tornqvist indices may be singled out because the use of these indices can be theoretically justified in terms of underlying aggregator functions in a wide range of circumstances. In this context, the main advance made in recent years is the introduction by Diewert of the concept of a "superlative index" which is an exact index based on an underlying aggregator function which has a flexible form and therefore capable of approximating closely to a wide range of other possible underlying functions. Both the Fisher and Tornqvist indices are superlative.

The second conclusion to be drawn is that chain indices are generally to be preferred to fixed weight indices. One advantage of chaining is that it will tend to reduce the index number spread between the values recorded by different indices, and in particular between the Laspeyres and Paasche formulae, provided that the evolution of individual prices and quantities over time is not too erratic or subject to fluctuation. In other words, chaining will tend to minimise the traditional index number problem by making the choice of formula less important numerically so that the use of indices which are second best from a theoretical viewpoint becomes less objectionable. Another reason for preferring chain indices is that they make it possible to exploit the available price and quantity to the fullest extent because the overlap between the set of commodities available tends to be greatest for consecutive periods. Probably the most important practical problems confronting index number compilers are how to treat changes in the quality of goods and services over time and how to capture the benefits accruing from the introduction of new goods, and these problems can be minimised by chaining.

In practice, most of the price and volume indices found in economic statistics and used for purposes of economic analysis and policy making are neither symmetric indices nor chain indices. ***Caveat*** user.

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