

Capital Stock Conference
March 1997
Agenda Item V

MORTALITY AND SURVIVAL FUNCTIONS

Originally published as Chapter III from "Methods Used by OECD Countries to Measure Stocks of Fixed Capital", National Accounts: Sources and Methods No. 2. OECD, Paris 1993.

III. MORTALITY AND SURVIVAL FUNCTIONS

38. This chapter looks at the assumptions made about the distribution of retirements around the average service life. In the brief exposition of the PIM in Chapter I, it was noted that various assumptions can be made about the weights to be attached to each year's investment expenditures. Assigning a weight of unity during the average service life of the asset corresponds to what is described below as "simultaneous exit"; assets of a particular type all remain in the stock until they reach the average service life assumed for that asset, at which point they are all withdrawn. This is a convenient assumption for purposes of computation but it is not very realistic. Common sense suggests that, like people, some assets will be withdrawn before they reach the average length of life and that some will remain active for several years longer.

39. Below, there is first a discussion of the characteristics of the various functions available for simulating retirement patterns, and this is followed by a table summarising the information available on the mortality functions used by OECD countries.

40. Figure 1 shows typical mortality and survival functions underlying the various retirement patterns used by the OECD countries. The mortality functions (first column) show rates of retirement over the lifetimes of the longest-lived member of a group of assets of a particular type installed in a given year: they are essentially probability density functions with the area under each curve equal to unity. The survival functions (in the second column) show what proportion of the original members of the group of assets are still in service at each point during the lifetime of the longest-lived member of the group.

a) *Linear*

41. With a linear retirement pattern, assets are assumed to be discarded at the same rate each year from the time of installation until twice the average service life. The mortality function is a rectangle whose height - the rate of retirement - equals $1/2L$ where L is the average service life: the survival function shows that the surviving assets are reduced by a constant amount each year, equal to $50/L$ per cent of the original group of assets. No OECD countries presently use this retirement pattern for their regular capital stock statistics, although it has been used in Canada for "alternative capital stock" estimates designed to show the impact on the capital stock estimates of using different retirement patterns. See Koumanakos (1980).

b) *Delayed linear*

42. With a delayed linear retirement pattern, discards are assumed to occur at a constant rate over some period shorter than $2L$. Retirements start later and finish sooner than in the simple linear case. The United Kingdom and New Zealand use this retirement pattern for their capital stock estimates. In the case of the United Kingdom, it is assumed that all assets are retired over the period from 80 per cent to 120 per cent of their average service life. The rate of retirement in the mortality function is therefore equal to $1/L (1.2-0.8)$ or $250/L$ per cent per year during the period when the retirements are assumed to occur. For

the New Zealand estimates, four different types of delayed linear functions are used for different kinds of assets; retirements range from 4-25 per cent of the average to 4-82 per cent.

43. Until 1975, the United Kingdom estimates were based on the assumption that all assets of a given type were retired simultaneously when they reach the average service life for that group. This was found to produce a marked unevenness in the stock data for the early 1970s when the large amounts of plant and equipment installed in the 1940s for war-time production reached the end of their service lives. In the absence of any firm information about the actual retirement patterns, the delayed linear function was adopted because of its computational simplicity. It is interesting to note, however, that on the basis of a company survey, Barna (1961) concluded that a delayed linear function, with discards starting 3 to 5 years after installation, best fitted the mortality pattern of manufacturing equipment.

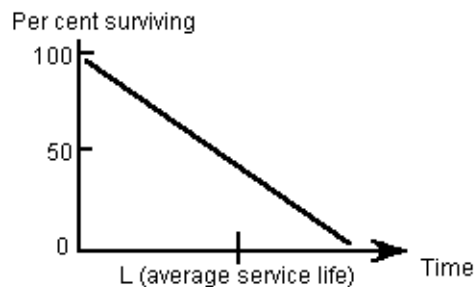
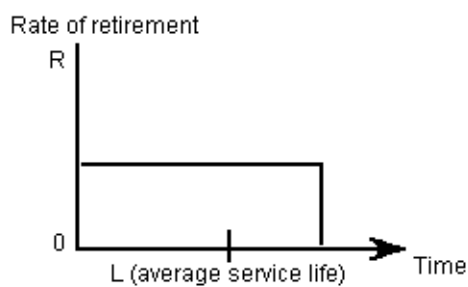
Figure 1

TYPICAL MORTALITY AND SURVIVAL FUNCTIONS

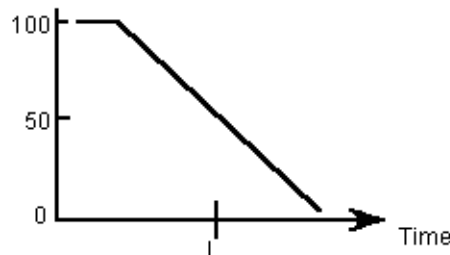
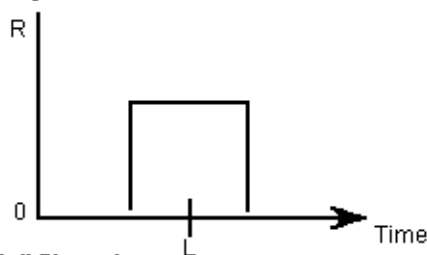
MORTALITY FUNCTIONS

SURVIVAL FUNCTIONS

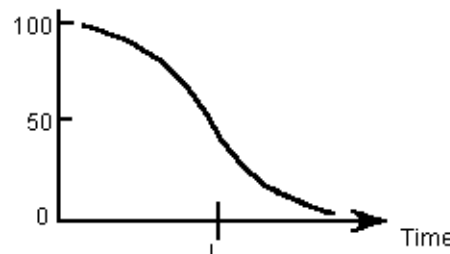
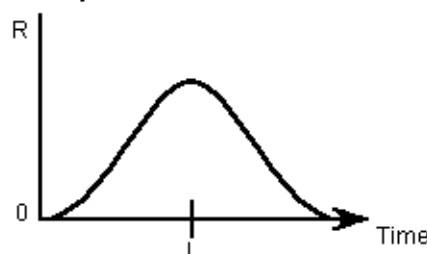
1. Linear



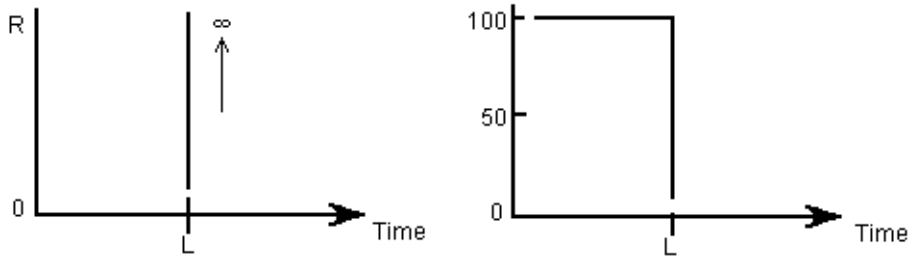
2. Delayed linear



3. Bell Shaped



4. Simultaneous exit



c) *Bell-shaped*

44. Most countries use some kind of bell-shaped mortality function whose parameters allow for various degrees of skewness and peakedness. The functions used are described as log-normal (France), gamma (Germany), logistic (Austria), Weibull (Finland) and Winfrey (United States, Australia and Sweden). In addition, Canada uses a quadratic approximation to the normal distribution, and a truncated bell-shaped function (also based on the quadratic) for its "alternative capital stock" estimates. See Koumanakos (1980). In most cases symmetrical functions are used, but Germany uses left-skewed gamma functions, and Sweden and Australia use right-skewed as well as symmetrical Winfrey functions.

45. These various functions have presumably been selected because they appear to fit whatever information is available in the countries concerned on actual retirements of various kinds of assets. However, empirical evidence is scarce and it seems likely that in most countries the selection of a particular function is based on the retirement patterns of only a few types of assets - notably transport equipment. The main exceptions are the so-called Winfrey functions which were developed at the Engineering School of the Iowa State College during the 1930s. Data were assembled on the mortality patterns of 176 types of assets, from which 18 standard mortality functions were derived - six being left-skewed, seven symmetrical, and five right-skewed. These are denoted by L, S and R respectively, with a subscript 0 through 6 moving from flatter to more peaked curves. The Winfrey S_1 function, for example, gives a flattish symmetrical curve with retirements spread over the period ± 95 per cent of the average service life; the Winfrey S_4 function is again symmetrical, but it is more peaked with retirements occurring around ± 70 per cent of the average service life.

Table 6. **Mortality functions used by OECD Countries for capital stock estimates**

Country	Type of mortality functions used
Canada	<i>Simultaneous exit.</i> Alternative estimates have also been made using linear, exponential and bell-shaped mortality functions. See Koumanakos (1980).
United States	<i>Bell-shaped.</i> Symmetrical Winfrey functions are used. For residential buildings, discards are spread over the period $\pm 95\%$ of average service life: For all other assets discards are spread over the period $\pm 55\%$ of the average service life. See <i>Fixed Reproducible Tangible Wealth in the United States, 1925-85</i> (1987).
Japan	<i>Simultaneous exit.</i>
Australia	<i>Bell-shaped.</i> In general, retirement distributions are based on symmetrical Winfrey S3 functions. Exceptions are made for alterations and additions and for real estate transfer expenses, both of which use a Winfrey SO function. See <i>Estimates of Depreciation and Capital Stock, Australia</i> (1985).
New Zealand	Delayed linear. Four different retirement patterns are used. For buildings with average service lives of 32 years retirements are spread evenly over the period $\pm 25\%$ of the average: for buildings with average lives of 55 years, $\pm 28\%$; for equipment with average lives of 17 years, $\pm 76\%$; for equipment with average lives of 27 years, $\pm 82\%$. See Campbell (1977).
Austria	<i>Bell-shaped.</i> Symmetrical quasi-logistic functions are used. See Kirner (1968).
Belgium	<i>Bell-shaped.</i> Quasi-logistic functions are used. For all vehicles, machinery and equipment and for private sector non-residential building and construction, symmetrical functions are used. For government and for all residential buildings, the distributions are right-skewed. See de Biolley and Gilot (1987).
Finland	<i>Bell-shaped.</i> Symmetrical Weibull functions are used. See <i>Kansantalouder tilinpito</i> (1980).
France	<i>Bell-shaped.</i> Asymmetrical log-normal functions are used. See Mairesse (1972).
Germany	a) Statistisches Bundesamt <i>Bell-shaped.</i> The official capital stock series compiled by the Statistisches Bundesamt use left-skewed gamma functions. See Lützel (1977). It has recently become apparent that in some industries fixed assets were being prematurely retired from the capital stock. In the case of mining and railroad and water transport, this was taken into consideration by temporarily reducing the average length of life. For oil refining, iron and steel and shipbuilding, special retirements were recorded since the late seventies and or early eighties. See Schmidt (1986). b) DIW <i>Bell-Shaped.</i> The Deutsches Institut für Wirtschaftsforschung (DIW) uses a symmetrical quasi-logistic function for estimates of capital stock by industry sector. See Kimer (1968).
Norway	<i>Simultaneous exit.</i>

Sweden	<i>Bell-shaped.</i> Winfrey functions are used. For most types of buildings and for transport equipment, right-skewed distributions are used of types R2, R3 and R4. For engineering construction and most other types of machinery and equipment, symmetrical distributions of types S1, S2 and S3 are used. See Tengblad (1976)
United Kingdom	Delayed linear. Retirements are spread evenly over the period $\pm 20\%$ of the average service lives. See Griffin (1975).

46. While Winfrey curves are empirically well-based, it is not obvious that functions found to fit the mortality patterns of fixed assets in use in the United States during the 1920s are still appropriate for the 1980s. Nearly a third of the 176 asset groups analysed by Winfrey consisted of railway equipment and structures, and the list covered relatively few manufacturing assets.

d) *Simultaneous exit*

47. The simultaneous exit mortality function is used for capital stock estimates in Canada, Japan and Norway. As already noted it was also used in the United Kingdom until 1975.

48. This function assumes that all assets are retired from the capital stock at the moment when they reach the average service life for the type of asset concerned. The survival function therefore shows that all assets of a given type and vintage remain in the stock until time L , at which point they are all retired together. This retirement pattern is sometimes referred to as "sudden exit" but this term is ambiguous. Whatever mortality pattern is used, individual assets are always retired suddenly: the distinguishing feature of this mortality function is that all assets of a given type and vintage are retired **simultaneously**.

49. Simultaneous exit may be regarded as a limiting case of a bell-shaped function where the peak approaches infinity and the variance approaches zero. The infinitely thin space inside the infinitely high vertical line sums to unity, as do the areas under the other mortality functions in Figure 1.

50. The assumption that all assets of a given vintage disappear simultaneously from the capital stock is clearly unrealistic. While it may be that the large majority of retirements occur around the average service life, it is clear that some assets are used with different intensities by different producers. It may therefore be assumed that simultaneous exit is used for much the same kind of reasons as the delayed linear function - computational simplicity and lack of information on actual retirement patterns.

51. It was noted above that the United Kingdom's capital stock estimates were based on the simultaneous exit function until implausible irregularities appeared in the series due to the (simultaneous) retirement of large quantities of war-time assets. The risk of this "echo-effect" happening depends to a large extent on the degree of detail in the breakdown of capital assets. For example, if it is assumed that all machinery in a particular industry has the same average service life, it is much more likely that the assumption of simultaneous exit will cause irregularities in the stock series than if 10 different types of machinery are distinguished for the industry group, each type having its own average service

life.

52. Table 6 summarises the information available to the OECD Secretariat about the kinds of mortality functions used in Member countries.

* Originally published as Chapter III from "Methods Used by OECD Countries to Measure Stocks of Fixed Capital", National Accounts: Sources and Methods No. 2. OECD, Paris 1993.