Temporal Disaggregation Using Multivariate Structural Time Series Models

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Abstract

In this paper we provide a multivariate framework for temporal disaggregation of time series observed at a certain frequency into higher frequency data. The suggested method uses the seemingly unrelated time series equations model and is estimated by the Kalman filter. The methodology is flexible enough to allow for almost any kind of temporal disaggregation problem of both raw and seasonally adjusted time series. Comparisons with other temporal disaggregation methods proposed by the literature are presented using a wide OECD data set.

KEY WORDS: Temporal disaggregation; Multivariate structural time series models; Common structural components; Kalman filter

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A problem often faced by National Statistical Institutes (NSI's) and more generally by economic researchers is the interpolation or distribution of economic time series observed at low frequency into compatible higher frequency data. While interpolation refers to the estimation of missing observations of stock variables, a distribution (often called by some authors simply temporal disaggregation) problem occurs for flow and time averages of stock variables. In the distribution case, for example, the problem concerns the estimation of intraperiod values for a given time series subject to the constraint that their sums (or averages) equal the aggregates over the lower frequency.

The need for temporal disaggregation can stem from a number of reasons. For example NSI's, due to the high costs involved in collecting the statistical information needed for estimating national accounts, could decide to conduct large sample surveys only annually. Consequently, quarterly (or even monthly) national accounts could be obtained through an indirect approach, that is by using related quarterly (or monthly) time series as indicators of the short-term dynamics of the annual aggregates. As another example, econometric modelling often implies the use of a number of time series, some of which could be available only at lower frequencies, and therefore it could be convenient to disaggregate these data instead of estimating, with a significant loss of information, the complete model at the level of the lower frequencies. In what follows, the term temporal (or time) disaggregation is used for both interpolation and distribution cases.

Temporal disaggregation has been extensively considered in the econometric and statistical literature and numerous solutions have been proposed. Broadly speaking, two alternative approaches have been followed: 1) methods which do not involve the use of related series but rely upon purely mathematical criteria or time series models to derive a smooth path for the unobserved series; 2) methods which make use of the information obtained from related indicators observed at the desired higher frequency.

The first approach comprises the model-based methods (Stram and Wei 1986; Wei and Stram 1990) relying on the $ARIMA$ representation of the series to be disaggregated (e.g., see Eurostat 1999 for a survey and taxonomy of temporal disaggregation methods). The latter approach includes, amongst others, the adjustment procedure due to Denton (1971) and the methods proposed by Chow and Lin (1971), Fernández (1981) and Litterman (1983). The use of state space models for temporal disaggregation, originally developed by Harvey and Pierce (1984), has been further developed in the framework of structural time series models by Durbin and Quenneville (1997), Harvey (1989) and, in a recent application, by Harvey and Chung (2000). In this paper, the authors use seemingly unrelated structural time series (SUTSE) models to obtain timely estimates of the underlying change in unemployment when the target series is available in a time-aggregated form and a related time series, possibly defined at higher frequency, can be used to improve estimates of the rate of change of the target series itself.

In the spirit of the work by Harvey and Chung (2000), we further develop the use of SUTSE models for temporal disaggregation and compare the effectiveness of the SUTSE approach with other methods. Since there is usually no behavioral relationship between the series to be disaggregated and the set of related variables, we consider the SUTSE model as a more appropriate framework than the traditional univariate regression approach to represent and solve temporal disaggregation issues (see Harvey 1989, pp.463-464).

According to the categorization discussed above, the SUTSE approach is based on the use of related time series, but here the term 'related' assumes a different meaning. In fact, this approach implicitly assumes that the series to be disaggregated and the set of related series are affected by a similar environment. Consequently, they should move together and measure similar things, though none of them necessarily causes the other. Common components restrictions - such as common trends, common cycles and common seasonalities - may be tested and imposed quite naturally in such a context. This represents a further departure from traditional literature on time disaggregation. For example, both the $ARIMA(1,1,0)$ structure imposed on the residuals of the time-disaggregation models by Litterman (1983) and the $I(1)$ structure hypothesized by Fernández (1981) implicitly assume the lack of a co-integration relationship between the series to be disaggregated and the related series.

The SUTSE approach is flexible enough to allow for almost any kind of disaggregation problem (i.e. annual to quarterly, annual to monthly, quarterly to monthly, ...) and to handle interpolation, distribution and extrapolation of time series. Furthermore, it can be applied to both raw and seasonally adjusted data. Another advantage of the methodology suggested is that it can allow for simultaneous disaggregation and seasonal adjustment of the series, whereas NSI's usually undertake these two procedures separately (first disaggregation, and then seasonal adjustment of the estimated raw series).

In this paper temporal disaggregation is treated as a missing observation problem applied to variables which are defined over different timing intervals. In this particular context, the SUTSE model is extended so as to allow for different timing intervals of the relevant series. The SUTSE model is estimated by using the Kalman filter (KF) and then optimal estimates of missing observations are obtained by a smoothing algorithm.

The setup of the article is as follows. In the next Section we provide an introduction to SUTSE models and their statistical treatment. In Section 2 the state space form (SSF) for a general problem of time disaggregation is presented starting from the local linear trend (LLT) model. In Section 3 we compare in theoretical terms the SUTSE approach with the main competitors. Section 4 contains the results of empirical analyses using a wide OECD real data set. Conclusions and lines for future research are presented in Section 5.

1. SUTSE MODELS AND THEIR STATISTICAL TREATMENT

SUTSE models are widely treated in the literature and represent a multivariate generalization of structural time series models (see, e.g., Harvey 1989; Fernández and Harvey 1990; Harvey and Koopman 1997). Given a cross-section of time series $\mathbf{y}_t = (y_{1t}, \ldots, y_{Nt})'$, it is assumed that each y_{it}, $i = 1, 2, \ldots, N$ and $t = 1, 2, ..., n$, is not directly related with the others, although subject to similar influences. $\mathbf{y}_t$ is expressed in terms of additive N-dimensional unobserved components, e.g. level $\boldsymbol{\mu}_t$, slope $\boldsymbol{\beta}_t$, cycle $\boldsymbol{\psi}_t$, seasonality $\boldsymbol{\gamma}_t$ and irregular $\boldsymbol{\xi}_t$, which are allowed to be contemporaneously correlated.

SUTSE models allow for a wide range of formulations: here we start from the multivariate LLT model, where $\mathbf{y}_t$ consists of a stochastic trend plus a white noise:

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\xi}_t, \qquad \boldsymbol{\xi}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\xi), \tag{1}$$

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_t + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\eta), \tag{2}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\zeta}_t, \qquad \boldsymbol{\zeta}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\zeta), \tag{3}$$

where the $\boldsymbol{\Sigma}_h$'s, $h = \boldsymbol{\xi}$, $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$, are the covariance matrices of system disturbances, $\boldsymbol{\xi}_t$, $\boldsymbol{\eta}_t$ and $\boldsymbol{\zeta}_t$, mutually uncorrelated in all time periods.

The LLT model may take a variety of forms: in particular, when $\boldsymbol{\Sigma}_\zeta = \mathbf{0}$ the stochastic slope reduces to a fixed slope and the trend reduces to a multivariate random walk with drift

(RWD); when $\boldsymbol{\Sigma}_\eta = \mathbf{0}$, while $\boldsymbol{\Sigma}_\zeta$ remains positive semidefinite, a smooth trend or integrated random walk (IRW) is modelled; finally, $\boldsymbol{\Sigma}_\eta = \boldsymbol{\Sigma}_\zeta = \mathbf{0}$ implies a deterministic linear trend. Different forms also arise when restrictions on the covariance matrices $\boldsymbol{\Sigma}_h$'s are introduced. These may concern the rank of any of the $\boldsymbol{\Sigma}_h$'s, implying a common component restriction, and/or proportionality of the $\boldsymbol{\Sigma}_h$'s to each other, that is homogeneity.

The LLT collapses to the local level (LL) model when there is no slope component. Then, the system is defined by equation (1) and by a random walk (RW): $\boldsymbol{\mu}_{t+1} = \boldsymbol{\mu}_t + \boldsymbol{\eta}_t$. The restriction $\boldsymbol{\Sigma}_\eta = \mathbf{0}$ leads the RW to become a constant level. In both cases the LLT and the LL models can allow for more complicated expressions by introducing a cyclical and/or a seasonal component.

Equations (1)-(3) can be written more compactly in the following SSF:

$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}_t\boldsymbol{\alpha}_t + \mathbf{H}_t\boldsymbol{\varepsilon}_t, \ \boldsymbol{\alpha}_1 \sim N(\mathbf{0}, \mathbf{P}), \tag{4}$$

$$\mathbf{y}_t = \mathbf{Z}_t\boldsymbol{\alpha}_t + \mathbf{G}_t\boldsymbol{\varepsilon}_t, \tag{5}$$

where the system matrices $\mathbf{T}_t$, $\mathbf{H}_t$, $\mathbf{Z}_t$ and $\mathbf{G}_t$ can be time-varying to allow for missing observations. The state vector $\boldsymbol{\alpha}_t$ is such that $\boldsymbol{\alpha}_t = (\boldsymbol{\mu}_t', \boldsymbol{\beta}_t')'$, $\boldsymbol{\varepsilon}_t \sim NID(\mathbf{0}, \mathbf{I})$ and, dropping the subscripts, the system matrices are $\mathbf{T} = \begin{pmatrix} \mathbf{I}_N & \mathbf{I}_N \\ \mathbf{0} & \mathbf{I}_N \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} \boldsymbol{\Gamma}_\eta & \boldsymbol{\Gamma}_\zeta \\ \mathbf{0} & \boldsymbol{\Gamma}_\zeta \end{pmatrix}$, $\mathbf{Z} = [\mathbf{I}_N, \mathbf{0}]$ and $\mathbf{G} = [\mathbf{0}, \mathbf{0}, \boldsymbol{\Gamma}_\varepsilon]$, with the $\boldsymbol{\Gamma}_\mathbf{h}$'s lower triangular matrices such that $\boldsymbol{\Sigma}_\mathbf{h} = \boldsymbol{\Gamma}_\mathbf{h}\boldsymbol{\Gamma}_\mathbf{h}'$ and $\mathbf{h} = \boldsymbol{\eta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\xi}$.

SUTSE models are estimated in the time domain by using the KF. Once their SSF have been set up, the KF yields the one-step ahead prediction errors and the Gaussian log-likelihood function via the prediction error decomposition. The system matrices $\mathbf{T}_t$, $\mathbf{H}_t$, $\mathbf{Z}_t$, and $\mathbf{G}_t$ of the SSF (4)-(5) depend on a set of unknown parameters, denoted by $\boldsymbol{\varphi}$. Then, numerical optimization routines can be used to maximize the log-likelihood function with respect to $\boldsymbol{\varphi}$. Once $\boldsymbol{\varphi}$ has been estimated, the output of the KF may be used for different purposes, such as forecasting, diagnostic checking and smoothing. In particular, the backward recursions given by the smoothing algorithm yield optimal estimates of the unobserved components. The treatment of the diffuse initial conditions is approached efficiently by using the methods proposed by Koopman (1997) and Koopman and Durbin (1999).

2. TIME DISAGGREGATION WITH SUTSE MODELS

Both interpolation and distribution find an optimal and general solution in the KF framework where they are treated as missing observation problems. The Kalman filtering and smoothing (KFS) allows an adjustment in the dimension of the data and, in particular, in the system matrices $\mathbf{Z}_t$ and $\mathbf{G}_t$. Moreover, if for certain values of t no observations are available, the KFS can be simply run by skipping the updating equations, without affecting the validity of the prediction error decomposition.

While in the interpolation case the SSF introduced in Section 1 remains valid, in the distribution case the model and the observed timing intervals are different. By extending the discussion in Harvey (1989, p. 309) and Harvey and Chung (2000), we indicate with δ the model frequency and with $\delta_1^+, \ldots, \delta_N^+$ the frequencies at which the unobserved disaggregated flows $y_{1,t}, \ldots, y_{N,t}$ are respectively observed. Model and observed frequencies are such that their ratios, denoted $\delta_i = \delta/\delta_i^+$, are integers for each i. Then, the aggregates are such that:

$$y_{i,t}^{\dagger} = \sum_{r=0}^{\delta_i - 1} y_{i,t-r}, \quad t = \delta_i, 2\delta_i, \ldots. \tag{6}$$

2.1 Extending the LLT model

Let us suppose that $\mathbf{y}_t$ is generated by the LLT model (1)-(3), but that some elements of $\mathbf{y}_t$ are observed in temporally aggregated form. Following Harvey (1989) and Harvey and Chung (2000), let us define the cumulator as:

$$y^c_{i,t-\delta_i+r} = \sum_{j=1}^{r} y_{i,t-\delta_i+j}, \quad r = 1, \ldots, \delta_i, \quad i = 1, \ldots, N \tag{7}$$

so that $y^c_{i,t} = y^\dagger_{i,t}$ for $t = \delta_i, 2\delta_i, \ldots$. The cumulator can also be written as:

$$\begin{aligned} \mathbf{y}^c_t &= \mathbf{C}_t \mathbf{y}^c_{t-1} + \boldsymbol{\mu}_t + \boldsymbol{\xi}_t = \\ &= \mathbf{C}_t \mathbf{y}^c_{t-1} + \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t + \boldsymbol{\zeta}_t + \boldsymbol{\xi}_t, \end{aligned} \tag{8}$$

where $\mathbf{C}_t = diag\left(c_{1t}, c_{2t}, \ldots, c_{Nt}\right)$ and:

$$c_{it} = \begin{cases} 0 & t = 1, \delta_i + 1, 2\delta_i + 1, \ldots \\ 1 & \text{otherwise.} \end{cases} \tag{9}$$

Then, the state vector in equations (4)-(5) becomes $\boldsymbol{\alpha}_t = \left(\boldsymbol{\mu}'_t, \boldsymbol{\beta}'_t, \mathbf{y}^{c'}_t\right)'$ and the system matrices are given by $\mathbf{T}_t = \begin{pmatrix} \mathbf{I}_N & \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N & \mathbf{0} \\ \mathbf{I}_N & \mathbf{I}_N & \mathbf{C}_t \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} \boldsymbol{\Gamma}_\eta & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_\zeta & \mathbf{0} \\ \boldsymbol{\Gamma}_\eta & \boldsymbol{\Gamma}_\zeta & \boldsymbol{\Gamma}_\xi \end{pmatrix}$ and $\mathbf{Z} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I}_N \end{pmatrix}$, with $\mathbf{y}^c_0 \equiv \mathbf{0}$, and $\mathbf{G} = \mathbf{0}$. Note that, though the form above is not the most parsimonious, it allows the use of fast Kalman filtering and smoothing as suggested by Koopman and Durbin (2000). Further, the system matrix $\mathbf{T}$ is now time-varying as denoted by the subscript t.

Example Let us consider a quarterly disaggregation of an annual variable y_{1t} through a related quarterly time series y_{2t}. The relevant indices of the SSF (4)-(5) are $N = 2$, $\delta = 4$, δ^+_1=1 and δ^+_2= 4. Moreover $c_1 = (0, 1, 1, 1, 0, 1, 1, 1, ...)$, $c_2 = (0, 0, 0, 0, 0, 0, 0, 0, ...)$,and the system matrices $\mathbf{T}_t$, $\mathbf{H}$, and $\mathbf{Z}$ can be easily obtained.

2.2 Cyclical components

In structural time series models the cyclical component is a linear function of sines and cosines. When a distribution problem occurs, the treatment of this component is similar to the treatment of the level. In a cycle plus noise model the state vector is defined as $\boldsymbol{\alpha}_t = \left(\overline{\boldsymbol{\psi}}'_t, \overline{\boldsymbol{\psi}}^{*\prime}_t, \mathbf{y}^{c\prime}_t\right)'$, and the system matrices become $\mathbf{T}_t = \begin{pmatrix} \mathbf{C}_{\boldsymbol{\Psi}} & \mathbf{S}_{\boldsymbol{\Psi}} & \mathbf{0} \\ -\mathbf{S}_{\boldsymbol{\Psi}} & \mathbf{C}_{\boldsymbol{\Psi}} & \mathbf{0} \\ \mathbf{C}_{\boldsymbol{\Psi}} & \mathbf{S}_{\boldsymbol{\Psi}} & \mathbf{C}_t \end{pmatrix}$ and $\mathbf{H}= \begin{pmatrix} \boldsymbol{\Gamma}_{\boldsymbol{\kappa}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_{\boldsymbol{\kappa}} & \mathbf{0} \\ \boldsymbol{\Gamma}_{\boldsymbol{\kappa}} & \mathbf{0} & \boldsymbol{\Gamma}_{\xi} \end{pmatrix}$, where $\mathbf{C}_{\boldsymbol{\Psi}} = \rho \cos\lambda \cdot \mathbf{I}_N$, $\mathbf{S}_{\boldsymbol{\Psi}} = \rho \sin\lambda \cdot \mathbf{I}_N$, the frequency of the cyclical component is $0 \leq \lambda \leq \pi$ and the damping factor is $0 < \rho \leq 1$. The matrix $\boldsymbol{\Gamma}_{\boldsymbol{\kappa}}$ is obtained by Cholesky decomposition of the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\kappa}}$ for the cyclical disturbances $\boldsymbol{\kappa}_t$.

2.3 Common Components

Though SUTSE models do not necessarily require common components, common factor restrictions on level, slope, irregular and cycle may be introduced by imposing rank restrictions on the covariance matrices of disturbances driving the components of interest. In this respect, of particular convenience is a factorization of the covariance matrix $\boldsymbol{\Sigma}_{\mathbf{h}}$ such as $\boldsymbol{\Sigma}_{\mathbf{h}} = \boldsymbol{\Theta}_{\mathbf{h}}\mathbf{D}^2_{\mathbf{h}}\boldsymbol{\Theta}'_{\mathbf{h}}$, $\mathbf{h} = \boldsymbol{\eta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\xi}$, $\boldsymbol{\kappa}$, where $\boldsymbol{\Theta}_{\mathbf{h}}$ is a lower triangular matrix with 1's along the principal diagonal and with $\mathbf{D}_{\mathbf{h}}$ diagonal (note that $\boldsymbol{\Gamma}_{\mathbf{h}} = \boldsymbol{\Theta}_{\mathbf{h}}\mathbf{D}_{\mathbf{h}}$). Then, a model with r common restrictions for the $\mathbf{h}$-th component, $1 < r \leq N$, reduces the dimension of $\mathbf{D}_{\mathbf{h}}$ to $N - r$ and $\boldsymbol{\Theta}_{\mathbf{h}}$ to a full rank $(N \times (N - r))$ matrix. The limit case given by $r = N$ lets the $\mathbf{h}$-th component be deterministic since $\boldsymbol{\Sigma}_{\mathbf{h}} = \mathbf{0}$.

2.4 Seasonal Component

The LLT model in (1) can be extended to deal with seasonal time series. Here we consider the dummy seasonal (DS) model in which the multivariate seasonal component $\boldsymbol{\gamma}_t$

is such that:

$$S(L)\boldsymbol{\gamma}_t = \boldsymbol{\omega}_t, \quad \boldsymbol{\omega}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\omega), \tag{10}$$

where $S(L) = 1 + L + ... + L^{\delta-1}$, with L the lag operator and $\boldsymbol{\omega}_t$ the vector of disturbances driving the seasonal pattern.

Hotta and Vasconcellos (1999) discuss the aggregation problem of the DS model in univariate time series. When a flow variable is aggregated across time, the form (10) does not change except the case where δ is a multiple of the aggregation period. In this case seasonality is unobserved and it is confused with the irregular component.

In a multivariate context, we consider the case in which seasonality is observed only for some elements of $\mathbf{y}_t^\dagger$. Then, the limited seasonal information is to be distributed among all the elements of $\mathbf{y}_t^\dagger$ by restricting in some way model (10). A first solution is the common seasonal model: if r is the number of elements of $\mathbf{y}_t^\dagger$ for which seasonality is not observed, both $\boldsymbol{\gamma}_t$ and $\boldsymbol{\omega}_t$ will become $((N-r)\times 1)$ vectors. Thus, for a simple seasonal plus irregular model, $\mathbf{y}_t^\dagger$ is represented by:

$$\mathbf{y}_t^\dagger = \boldsymbol{\Theta}_\omega \boldsymbol{\gamma}_t + \overline{\boldsymbol{\gamma}}_t + \boldsymbol{\xi}_t, \tag{11}$$

where $\boldsymbol{\Theta}_\omega$ follows the definition of the previous Section and $\overline{\boldsymbol{\gamma}}_t$ is a $(N \times 1)$ vector of fixed seasonal effects.

When $\overline{\boldsymbol{\gamma}}_t$ is set to zero, model (11) leads to the stronger restriction of *similar seasonals.* In other words, the seasonal pattern is proportional among the elements of $\mathbf{y}_t^\dagger$. Finally, seasonality is *identical* when $r = N-1$ and $\boldsymbol{\Theta}_\omega$ is further restricted to a N vector of ones.

Example Let us consider a seasonal plus irregular model with annual, quarterly and monthly time series, given respectively by y_{1t}, y_{2t} and y_{3t}. The relevant model indices are $N = 3$, $\delta = 12$, $\delta_1^+ = 1$, $\delta_2^+ = 4$ and $\delta_3^+ = 12$. Since seasonality is fully observed for the first series only, one disturbance drives the stochastic seasonal component. Then, imposing similar seasonals the state vector is $\boldsymbol{\alpha}_t = \left(\gamma_t, \gamma_{t-1}, ..., \gamma_{t-10}, \mathbf{y}_t^{c\prime}\right)'$ and the system matrices $\mathbf{T}_t$ and $\mathbf{H}$ are $\mathbf{T}_t = \begin{pmatrix} \mathbf{T}_\gamma & \mathbf{0} \\ \mathbf{Z}_\gamma & \mathbf{C}_t \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} \mathbf{H}_\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_\xi \end{pmatrix}$ and $\mathbf{T}_\gamma = \begin{pmatrix} & -1_{11}' \\ \mathbf{I}_{10} & \mathbf{0} \end{pmatrix}$, with $\mathbf{H}_\gamma = diag(\sigma_\omega, 0_{10})$, $\mathbf{Z}_\gamma = \mathbf{\Theta}_{\boldsymbol{\omega}} \otimes e_{11}$ and $\mathbf{\Theta}_{\boldsymbol{\omega}} = (1, \theta_{\omega_2}, \theta_{\omega_3})'$; 1_m and 0_m denote $m-$vectors of ones and zeros respectively, and e_m is the first row of the identity matrix $\mathbf{I}_m$.

3. COMPARISON WITH OTHER METHODS

The main methods for temporal disaggregation proposed by the literature hypothesize a simple linear univariate relationship between the unknown low-frequency variable y_{1t} and the high-frequency related time series y_{2t}. The most important difference among the various methods lies in the structure imposed on the residuals of the hypothesized econometric relationship. Starting from the high frequency variables regression, $y_{1t} = \alpha + \beta y_{2t} + u_t$, the key issue - after temporal aggregation and estimation of the model with observed series - is the identification of the covariance matrix of the disturbances of the high frequency model from the estimated covariance matrix of the low-frequency model. This model is derived from the available low-frequency data, possibly by imposing an $ARIMA$ structure on the data generating process of $u_t \mid y_{2t}$. In particular, it is assumed to be an $AR(1)$ process by Chow and Lin (1971), an $I(1)$ process by Fernández (1981), and an $ARIMA(1,1,0)$ process by Litterman (1983). Though the proposal of Stram and Wei (1986) encompasses the other models, as it considers a general $ARIMA(p,d,q)$ structure of the data generation process of the aggregated series and, if an indicator is available, an $ARIMA(p,d,q)$ model for the residuals, NSI's often limit their consideration to the Chow and Lin's family of procedures because of their computational simplicity (see Bloem, Dippelsman and Mæhke 2001). The quadratic minimization approach suggested by Denton (1971), strongly favoured by Bloem, Dippelsman and Mæhke (2001), can be easily seen as a special case of the least squares approach of Chow and Lin (1971) (see Fernández 1981).

Implicit assumptions of the univariate approaches are the weak exogeneity of y_{2t} and the existence of a behavioural relation between y_{1t} and y_{2t}. As stated by Harvey (1989), none of these assumptions are necessarily fulfilled in current practices.

Then, the question naturally arises as to which circumstances a multivariate approach is *equivalent* to the univariate approaches utilised for temporal disaggregation so far. In order to theoretically compare the various methods, it is important to note that the reduced form of the LLT model is multivariate $IMA(2,2)$. Therefore, comparable models can only be obtained if we impose restrictions on its form, and in particular on the MA component.

Let us assume that $N = 2$ and consider the following factorization of the covariance matrix $\mathbf{\Sigma}_h$'s for the h-th component, $h = \boldsymbol{\xi}$, $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$:

$$\mathbf{\Sigma}_h = \begin{pmatrix} \boldsymbol{\sigma}_{1h}^2 & \rho_h \boldsymbol{\sigma}_{1h} \boldsymbol{\sigma}_{2h} \\ \rho_h \boldsymbol{\sigma}_{1h} \boldsymbol{\sigma}_{2h} & \boldsymbol{\sigma}_{2h}^2 \end{pmatrix}. \tag{12}$$

Starting from the multivariate LL model, whose reduced form is multivariate $IMA(1,1)$, Harvey (1989, 1996) has shown the conditions to obtain fully efficient estimates of the parameters of interest from the univariate model, namely the hypotheses for y_{2t} to be weakly exogenous. However, these conditions lead to residuals which are either $IMA(1,1)$ or white noise. The first case arises if the following results hold: (a) if $\mathbf{\Sigma}_{\boldsymbol{\xi}}$ and $\mathbf{\Sigma}_{\boldsymbol{\eta}}$ are positive definite and the system is homogeneous, $\mathbf{\Sigma}_{\boldsymbol{\eta}} = q_\eta \mathbf{\Sigma}_{\boldsymbol{\xi}}$, with known homogeneity coefficient; (b) if $\sigma_{2\xi} = 0$; (c) if $\sigma_{2\eta} = 0$; (d) if $\theta_\eta = \theta_\xi$ in the factorization of the covariance matrix discussed in Section 2.3. The second case occurs if $\boldsymbol{\Sigma}_\eta = \mathbf{0}$, or if $\sigma_{2\xi} = 0$ with the further restriction that $\rho_h = 1$, namely common levels.

The condition that must be satisfied to obtain a single equation form equal to the model proposed by Fernández (1981) with $IMA(1,0)$ disturbances is, in some respects, more stringent, as we need $\boldsymbol{\Sigma}_\xi = \mathbf{0}$ (the two series are equal to their trend components). The same result is obtained by making the same assumption but starting from the LLT model with fixed slope. In this case, the final univariate model contains a time trend as an additional regressor. Further, weak exogeneity is obtained only if the slope is known and identical for the two series.

The model for temporal disaggregation proposed by Chow and Lin (1971) is obtained from a modified LL model where the level component is $AR(1)$ with autoregressive coefficient equal for both series, namely $\boldsymbol{\mu}_t = \phi \boldsymbol{\mu}_{t-1} + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\eta)$, with ϕ scalar and known. The conclusions here are the same as those discussed before for the model proposed by Fernández (1981). Anyway, in this situation weak exogeneity can also be obtained by assuming the homogeneity restriction with q_ξ known. However, in the majority of situations q_ξ is unknown, then y_{2t} is no longer weakly exogenous and there is a loss of information in neglecting the equation for y_{2t}. In these cases, the single equation estimator is only asymptotically efficient. Similar conclusions can be obtained from a modified LL model with common levels and common $AR(1)$ residuals. Because y_{1t} and y_{2t} are co-integrated, it follows from the results in Stock (1987) that the obtained estimates are (super-)consistent and efficient in large samples even if y_{2t} and the residuals themselves are correlated by construction. Note that the hypothesis of common levels actually reduces the integration order of the residual component, so that it is no longer an $I(1)$ process.

If the starting point is the modified LLT model with autoregressive slopes and $\mathbf{\Sigma}_{\boldsymbol{\xi}} = \mathbf{0}$, weak exogeneity is obtained if the model is trend homogeneous, $\mathbf{\Sigma}_{\boldsymbol{\zeta}} = q_{\zeta}\mathbf{\Sigma}_{\boldsymbol{\eta}}$, with q_{η} known. Furthermore, as stated before, unless the autoregressive parameters are equal in the two equations and known, there is a loss of efficiency in parameters estimates if these are based on single equation estimation. In other words, we do not have weak exogeneity. Anyway, the resulting residuals of the single equation for y_{1t} are $ARIMA(1,1,1)$: then, the model adds to the residual of the model proposed by Litterman (1983) a moving average component, and can be judged as more close to the wider assumptions of the approach by Stram and Wei (1986) and Wei and Stram (1990).

It should be noted that the assumptions of common levels and slopes, even when $\mathbf{\Sigma}_{\boldsymbol{\xi}} = \mathbf{0}$, do not lead us to obtain weak exogeneity for y_{2t} in the equation of interest. Therefore, according to what stated for the model by Fernández (1981), also the model favoured by Litterman (1983) is in conflict with the likely property in this context of co-integration between y_{1t} and y_{2t}.

4. RESULTS OF THE EMPIRICAL ANALYSES

In this Section we compare the results obtained by applying the structural approach with those obtained from the main methods for temporal disaggregation proposed so far by the literature and discussed in Section 3. The results of the structural approach have been generated using the Ox program, version 3.0 (see Doornik 2001), and the SsfPack package (see Koopman, Shephard and Doornik 2002), while for the other methods we have benefited from the program Ecotrim developed by Eurostat (see Eurostat 1999).

4.1 The data set used for comparisons

The comparisons are carried out by considering a wide data set drawn from OECD (2002). The time series chosen refer to the twelve biggest OECD countries in terms of Gross Domestic Product (GDP) at current prices in 2001. Eight sets of data are used: 1) Industrial production index and deliveries in manufacturing; 2) GDP and industrial production index; 3) Consumer and producer price indices; 4) Private consumption and GDP; 5) GDP deflator and consumer price index; 6) Broad and narrow money supply; 7) Short-term and long-term interest rates; 8) Imports evaluated on a f.o.b. (free on board) and c.i.f. (costs, insurances and freights) basis. In total, we have 96 cases considered (8 data sets times 12 countries). Whenever available in the OECD data set, unadjusted data have been preferred to the corresponding seasonally adjusted series. The eight sets of data have been chosen in order to cover almost all the various situations which typically occur in practice. In particular, in case 6) we have an interpolation problem (stock variables, last observation of the reference period as benchmark), while in the other cases we face a distribution problem (index series in all cases but 8), where a distribution of flow variables is considered). As regards frequencies, the following cases are considered: a disaggregation problem from quarterly to monthly observations, $\delta_i = 3$, in cases 1), 3) and 6); from annual to quarterly in cases 2), 3), 4) and 8); and from annual to monthly in case 7). Following the terminology used for the univariate framework, the related series is the second series indicated in every set of data, whilst the first series is the 'dependent' variable.

Once the results of the disaggregations have been obtained for all the methods, they can be compared with the actual data using standard statistics, such as root mean squared or mean absolute errors. In what follows, we report the results obtained in terms of root mean squared percentage errors (RMSPE) only to save space. The results obtained with other criteria, available upon request, do not change the ordering of the results and the conclusions reported below.

Though the related series have been chosen on an *ad hoc* basis so as to be the closest approximation to the dependent variable in the data set, nonetheless a variety of situations arise in our experiments. In some cases, the related series are very close to the target variable, both as concerns their meaning and time-series properties. This is the case of experiment 8), where the c.i.f and f.o.b evaluations of imports differ only for insurance and freight charges. In other cases, though the related series is part of the dependent series, the two variables can substantially differ as a consequence, for example, of the increasing importance of new forms of payment, such as in case 6), or different short and long-term elasticities to economic conditions and expectations, such as in case 7). In some experiments the variables are casually linked (cases 1), 3) and 4)), while in others they are only subject to a similar economic environment and simply measure similar things (cases 2) and 5)). The heterogeneity in the data set used is further increased by countries' specificities.

4.2 Testing in the SUTSE model

One of the main advantages of the proposed approach is that it does not impose any a priori structure on the data. The final model depends on the stochastic behavior of the data and on their representation in terms of univariate/multivariate structural time series model. Unobserved components may assume a variety of forms and common component restrictions may be imposed, whenever appropriate, in a very straightforward and natural way. In this respect, an important question to be addressed regards the form of the trend component and the determination of whether common factor restrictions can be imposed. However, though common factors are desirable in our context, SUTSE models only impose them as special cases and the SUTSE approach can give gains from using related indicators even if there are significant differences in the time behaviour of the related and the target series, for example a tendency of one series in the system to drift apart (see Harvey and Chung, 2000).This aspect will be further clarified below.

Given that most of the time-series in our data set show a clear upward trend, our preferred approach consists in starting from the general multivariate LLT model, augmented to include seasonal and cyclical components whenever appropriate.

Concerning the seasonal component, two situations can arise: a) the component is observed only for the high frequency series; b) the component is observed with different frequencies on both series. In the first case, our choice has been either to use, whenever feasible, the information drawn from the estimate of the factor loading of the level, or to impose identical seasonality when the levels of the series are compatible. In the latter case, one can also consider the information on the seasonal factor loading estimated from the low-frequency model.

Once a first estimation is run, some of the variances of the system in the general LLT model can be fixed by the system estimation itself because approximately equal to zero.

The literature has recently proposed a number of tests for the form of the trend component and for the existence of common factors (see Nyblom and Harvey 2000, 2001). However, in general it is not possible to represent the observed aggregated series by a fully orthogonal structural model (see Harvey 1989; Hotta and Vasconcellos 1999). This could invalidate the use of the non parametric versions of these tests. One possible solution, adopted in the disaggregations discussed below, consists in using parametric versions of the tests based on the innovations obtained from the model estimated at the lower frequency the series are observed (see, e.g., Harvey 2001; Nyblom and Harvey 2000, 2001). These tests start from estimating the nuisance parameters of the unrestricted model, then follow in running the Kalman filter and smoother with the appropriate variances set to zero to extract the innovations or 'smoothing errors', and to construct tests which have the same asymptotic properties and more reliable size in small samples (see Nyblom and Harvey 2001).

In the LL model, a test for the null hypothesis that $\mathbf{\Sigma}_{\eta} = \mathbf{0}$ against the homogeneous alternative $\mathbf{\Sigma}_{\eta} = q\mathbf{\Sigma}_{\xi}$, $q > 0$, is given by $\eta(r, N) = tr\left(\mathbf{S}^{-1}\mathbf{C}\right)$, with $\mathbf{C} = n^{-2}\sum_{i=1}^{n}\left(\sum_{t=1}^{i}\boldsymbol{\nu}_t\right)\left(\sum_{t=1}^{i}\boldsymbol{\nu}_t\right)'$, $\mathbf{S} = n^{-1}\sum_{t=1}^{n}\boldsymbol{\nu}_t\boldsymbol{\nu}_t'$ and $\boldsymbol{\nu}_t$ the Kalman filter estimated residuals. In the LLT with constant slope, the test above is indicated with $\eta'(r, N)$, with different rejection regions from $\eta(r, N)$. In the IRW model, the test for $\mathbf{\Sigma}_{\zeta} = \mathbf{0}$ against the homogeneous alternative $\mathbf{\Sigma}_{\zeta} = q\mathbf{\Sigma}_{\xi}$ is given by $\zeta = tr\left(\mathbf{S}^{-1}\mathbf{T}\right)$, where $\mathbf{T} = n^{-2}\sum_{i=1}^{n}\left[\sum_{s=1}^{i}\sum_{r=1}^{s}\boldsymbol{\nu}_t\right]\left[\sum_{s=1}^{i}\sum_{r=1}^{s}\boldsymbol{\nu}_t'\right]$ (see Nyblom and Harvey 2001). Finally, the test against a stochastic slope with $\mathbf{\Sigma}_{\eta} > \mathbf{0}$ is again given by $\eta(r, N)$.

Furthermore, we might test for a specified number r of common levels/slopes, that is we might test for the rank of the relevant covariance disturbance. The test is constructed on the sum of the $N - r$ smallest eigenvalues of the matrices constructed on the innovations.

4.3 Results of comparison

As an illustrative example of the proposed disaggregation method, we consider the quarterly GDP and the industrial production index for the US. The series are seasonally adjusted and cover the sample period 1960.q1-2002.q1 ($n = 169$). Data on GDP are firstly annually aggregated and then quarterly disaggregated using a multivariate LLT model. Given the factorization (12), we have obtained the following estimates:

$$\begin{array}{lll} \widetilde{\sigma}_{1\eta} = 26.1 & \widetilde{\sigma}_{2\eta} = 11.2 & \widetilde{\rho}_{\eta} = 0.292 \\ \widetilde{\sigma}_{1\zeta} = 22.3 & \widetilde{\sigma}_{2\zeta} = 13.3 & \widetilde{\rho}_{\zeta} = 0.997 \\ \widetilde{\sigma}_{1\xi} = 0.252, & & \end{array}$$

with the irregular component of the second series $\widetilde{\sigma}_{2\xi}$ equal to zero, and consequently $\widetilde{\rho}_{\xi} = 0$. Concerning GDP, the diagnostics based on innovations are $Q(6) = 2.75$ and $NORM = 0.72$, while for the production index are $Q(13) = 37.45$ and $NORM = 43.69$, where $Q(P)$ is the Box-Ljung statistic based on the P autocorrelations and $NORM$ is the Bowman-Shenton normality test statistic based on skewness and kurtosis of residuals. If the model is correctly specified, the Box-Ljung statistics are asymptotically distributed with $P - 1$ degrees of freedom, whereas $NORM$ is distributed as a χ^2 with 2 degrees of freedom. There is no evidence of serial correlation and non normality in the GDP residuals, while this is the case for the production index, possibly because of large changes in the series not completely reflected in GDP data. This is a feature our structural model shares with the other competitors. For example, the second best solution in our experiments is the Litterman's model (see Table 1), where we have a Ljung Box Q-statistic of the residuals in the annual relation at lag 7 equal to 34.94. Recently, the literature has shown the existence of different forms of non linearities in the US industrial production index, due for example to outliers, asymmetries in business cycle movements and time irreversibility. When seasonal unadjusted data are used, as in the example considered below, serial correlation and non normalities can also emerge as a result of either of seasonal heteroscedasticity and trends, or different seasonal behaviour between the target and the related series. Structural models can be amended in a number of ways to obtain better fits in these circumstances (see Proietti 1999, and for the seasonal case, Proietti 1998). The use of logarithms or other transformations of the original series could add to the fit of the structural model, but can create relevant problems, particularly for temporal disaggregations of flow series. We do not address these relevant issues in the present context. The important point to make here is that the structural approach still works properly even though the fit we obtain is not entirely satisfactory.

The results of the quarterly disaggregation of GDP have been compared with the actual quarterly data to obtain a RMSPE equal to 0.355. The estimates indicate the existence of a common slope ($\eta\,(1,2)$ is equal to 0.075, well below the 5% critical value of 0.218) and an irregular component approximately equal to zero. Imposing such restrictions does not change final results and the diagnostics reported above. The gains from using an indicator series in the structural approach are substantial, as clearly emerges from comparing the results here obtained with those of the univariate model, for which the RMSPE is equal to 0.465 (see Table 1). As noted by Harvey and Chung (2000), this may be due to the fact that, though the slopes of the two series are highly correlated, the two levels show a tendency to drift apart, possibly as a consequence of the decreasing contribution of the industrial sector to total GDP. This evidence shows that a high correlation between the disturbances in the two series - and hence the existence of common factors - not necessarily leads to big increases in the accuracy of final estimates.

The LLT model could also be estimated with a cyclical component. In this respect, an interesting question concerns whether its inclusion adds something to the accuracy of the model. An alternative specification, in line with the findings in Harvey and Jeager (1993) for US quarterly GNP, is the IRW model plus a cyclical component. The results are:

$$\begin{array}{lll} \widetilde{\sigma}_{1\zeta} = 14.3 & \widetilde{\sigma}_{2\zeta} = 8.0 & \widetilde{\rho}_{\zeta} = 0.973 \\ \widetilde{\sigma}_{1\kappa} = 20.9 & \widetilde{\sigma}_{2\kappa} = 11.3 & \widetilde{\rho}_{\kappa} = 0.739, \end{array}$$

with $\widetilde{\sigma}_{1\xi} = \widetilde{\sigma}_{1\xi} = 0$ and the cyclical period equal to 12.5 quarters. The diagnostics are $Q(6) = 2.13$ and $NORM = 0.18$ for GDP and $Q(13) = 32.44$ and $NORM = 29.11$ for the index of production, somewhat below those of the complete system. The RMSPE of the disaggregation model is equal to 0.351, a result very close to that obtained from the LLT model. The ζ-test, equal to 28.5, clearly rejects the null of a deterministic slope. Similar results are obtained for Spain, where the inclusion of the cyclical component reduces the RMSPE with respect to the LLT model with common slope of only 0.5%.

As a second example, we consider the quarterly aggregated industrial production index and monthly revenues for Germany in the sample 1991.m1-2002.m3 ($n = 135$). As the series are seasonally unadjusted, we start from the LLT model plus a seasonal component. The results are:

$$\begin{array}{lll} \widetilde{\sigma}_{1\eta} = 1.111 & \widetilde{\sigma}_{2\eta} = 1.538 & \widetilde{\rho}_{\eta} = 1.000 \\ \widetilde{\sigma}_{1\zeta} = 0.069 & \widetilde{\sigma}_{2\zeta} = 0.078 & \widetilde{\rho}_{\zeta} = 1.000 \\ \widetilde{\sigma}_{1\omega} = 0.319 & \widetilde{\sigma}_{2\omega} = 0.319 & \widetilde{\rho}_{\omega} = 1.000 \\ \widetilde{\sigma}_{1\xi} = 2.532 & \widetilde{\sigma}_{2\xi} = 4.286 & \widetilde{\rho}_{\xi} = -1.000. \end{array}$$

The diagnostics for the production index are $Q(6) = 14.35$ and $NORM = 14.79$, for deliveries we obtained $Q(11) = 43.15$ and $NORM = 0.69$. The RMSPE is equal to 2.328. Common restrictions on level and slope are naturally obtained by numerical optimization, in line with the results given by the test $\eta\,(1,2)$ on the slope component, equal to 0.046.

Considering the general results in terms of RMSPE, presented in Table 1 for the 96 examples, they indicate that the multivariate structural approach is likely to be more accurate than the other methods in virtually all the distributions of time series. Table 2 shows that the SUTSE approach has a percentage of success over the competitors which varies from about 78% (against the univariate structural model) to about 92% (against the Chow-Lin's approach). For all the experiments, the average gain in terms of RMSPE varies from about

14% in the case of the Litterman's procedure, to some 60% for the Denton's model. Among the methods using related time series, the approach by Litterman seems to be, on average, the second-best solution for time disaggregation issues. This is probably due to the fact that this approach is the closest to the SUTSE approach because, on the one hand, it requires less restrictions on the form of the underlying multivariate system, as noted before, and on the other hand it is more able to resemble the time behaviour of highly non-stationary time series. Almost at the same level, the approaches by Denton (1971) and Chow and Lin (1971) have quite unsatisfactory performances with respect to the other methods: their probabilities of success, even against methods which do not use related time series, is seldom above 0.4.

When no indicator is available, the best solution seems again to be offered by the use of a structural approach, with a gain in terms of accuracy over the approaches by Denton and Stram-Wei of about 15%. Another interesting result emerges from our experiments. *Coeteris paribus*, the use of a related series can have an impact on accuracy of final estimates which greatly depends on the method used for time disaggregation. In fact, while the gains obtained passing from the univariate structural model to the SUTSE approach are substantial (about 35%, with an increase of the probability of success of 22%), for the Denton's approach the use of a related time series seems to even worsen the final outcomes (a reduction of 3% in accuracy, with a probability of success of about 33%). Therefore, what this limited experiment seems to indicate is that the choice of the method for time disaggregation can be even more relevant than the use of a good reference series, even if the use of this series can add substantially in terms of accuracy when an appropriate framework for time disaggregation is chosen. Furthermore, as noted before, the gains from using the SUTSE over the univariate structural approach can be low when the series show similar behaviour. This is what emerges from cases C and E in our data set where, on average, the target and the related series are characterised by similar patterns. Here the gains in terms of RMSPE from using an indicator series is of 2.4% (against an average of 35.1% in the whole data set) and the probability of success reduces to 62.5% from an average of 78.1%.

Table 3 shows, for each experiment, the sample, the final model estimated and the results obtained from the parametric tests discussed above. It can be noted that in 58 over 96 cases the LBI tests confirm the appropriateness of the model finally chosen. The models contain stochastic levels without common factor restrictions in the majority of cases (75 over 96), and in about 50% of these models the slopes of both the indicator and the target series are deterministic. In fifteen cases we obtained a complete LLT model without restrictions, whilst in eleven cases the more appropriate model includes both common levels and slopes. The other preferred models account for the other twelve cases. A cycle is added in only two final models and the gains obtained are rather weak, as discussed before. In the cases where a seasonal component is needed, the proportional form has always been imposed. Regarding the monthly disaggregation of prices, we have found that in most experiments the seasonal component in both series is well described by a deterministic dummy component.

5. SUMMARY AND LINES FOR FUTURE RESEARCH

This paper describes a framework for time disaggregation of time series based on structural multivariate models. The approach here suggested has the advantage over the main competitors that it does not impose any particular structure on the data, and it is flexible enough to fit almost any temporal disaggregation problem. Furthermore, it easily allows for common factor restrictions among components, a quite natural circumstance which is not fully taken into account by other methods. We have shown that only under very particular conditions, usually not met in practice, the SUTSE approach reduces to the univariate methods mostly used in current practice by NSI's. The results of our experiments, which use a large number of time series covering almost all the various situations typically occurring in real life, show that the gains from using the multivariate/univariate structural approach can be substantial.

A feasible line for future research is the use of a logarithmic transformation for the variables characterised by increasing trends and variances. That could have some impact on the model chosen on the basis of the parametric tests used in this paper, and could improve diagnostics. Though modelling the series in logs creates no difficulties for stock variables, for flow series no simple solution exists at the moment simply because, for the aggregated variable, the logarithm of the sum is not equal to the sum of the logarithms. As suggested in Harvey and Pierce (1984), one way to proceed would be to assume that the logarithms of the variables are normally distributed, then to modify accordingly the state-space representation of the model, and finally to use the extended Kalman filter to obtain an approximation to the likelihood function computed by the prediction error decomposition.

Model selection strategy when observations are missing is another important issue to be further investigated. In this paper we have followed a 'reduction strategy' based on the results of parametric tests available in the literature. Advances in this field will certainly add to the choice of the final model for time disaggregation.

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Table 1. Results of time disaggregations: root mean square percentage errors (RMSPE)

COUNTRY	Can	Mex	Usa	Aus*	Jap	Kor	Fra	Deu	Ita	Nld	Esp	Gbr
	A. Quarterly industrial production - Monthly deliveries											
SUTSE	3.017	2.643	0.919	4.008	3.189	3.379	11.118	2.328	6.017	5.901	14.573	6.052
Chow-Lin	3.284	2.855	0.877	3.878	4.673	4.426	10.075	3.206	6.535	6.324	14.794	6.988
Fernández	3.381	2.851	0.887	3.010	4.773	3.517	9.992	7.831	6.814	6.008	14.798	6.540
Litterman	3.456	2.857	1.031	6.415	4.951	4.098	10.970	11.593	6.711	6.510	15.066	6.824
Denton	3.325	4.728	2.899	5.271	19.222	13.098	12.363	3.350	6.998	7.089	14.901	11.380
Structural	5.389	2.990	2.077	4.146	4.395	3.375	8.906	5.264	23.276	4.349	14.939	5.534
Denton	5.579	2.990	2.078	4.170	4.468	3.389	8.906	5.265	23.276	4.350	14.936	5.534
Stram-Wei	5.549	3.009	2.073	4.159	4.459	3.460	9.178	5.332	23.947	5.856	15.289	5.487
	B. Annual GDP - Quarterly industrial production											
SUTSE	0.202	2.244	0.351	0.513	0.484	0.561	0.221	0.347	0.484	0.509	0.456	0.474
Chow-Lin	0.213	2.287	0.751	1.214	0.648	0.683	0.670	0.431	0.604	1.856	0.942	1.486
Fernández	0.214	2.249	0.452	0.855	0.559	0.606	0.367	0.367	0.604	0.793	0.502	0.599
Litterman	0.213	2.245	0.358	0.613	0.495	0.633	0.256	0.369	0.504	0.509	0.454	0.457
Denton	0.205	2.329	0.718	0.984	1.031	0.877	0.536	0.745	1.208	1.069	0.716	0.882
Structural	0.394	2.403	0.465	0.623	0.480	0.785	0.255	0.438	0.511	0.535	0.480	0.574
Denton	0.419	2.404	0.473	0.626	0.493	0.885	0.257	0.417	0.521	0.545	0.488	0.598
Stram-Wei	0.447	2.408	0.496	0.623	0.491	0.800	0.265	0.420	0.552	0.544	0.481	0.585
	C. Quarterly consumer prices - Monthly producer prices											
SUTSE	0.209	3.115	0.161	0.460	0.323	0.229	0.191	0.161	0.112	0.253	0.519	0.349
Chow-Lin	0.317	5.279	0.341	0.671	0.347	0.296	0.443	0.199	0.271	0.278	0.798	0.396
Fernández	0.236	3.499	0.200	0.610	0.335	0.236	0.240	0.163	0.170	0.265	0.594	0.379
Litterman	0.215	3.310	0.158	0.469	0.334	0.241	0.190	0.161	0.114	0.279	0.478	0.395
Denton	0.300	3.146	0.317	0.593	0.355	0.266	0.349	0.177	0.196	0.310	0.543	0.370
Structural	0.205	2.439	0.161	0.467	0.336	0.284	0.192	0.165	0.112	0.269	0.482	0.300
Denton	0.205	2.425	0.162	0.467	0.336	0.287	0.188	0.162	0.127	0.276	0.454	0.311
Stram-Wei	0.204	2.398	0.163	0.486	0.339	0.287	0.187	0.162	0.114	0.273	0.461	0.309
	D. Annual private consumption expenditures- Quarterly GDP											
SUTSE	0.345	2.552	0.314	0.494	0.579	1.243	0.309	0.567	0.495	0.500	0.528	0.553
Chow-Lin	0.393	2.812	0.465	0.869	0.479	1.105	0.328	0.587	0.639	0.582	0.572	0.653
Fernández	0.372	2.863	0.434	0.777	0.510	1.173	0.309	0.533	0.572	0.597	0.586	0.617
Litterman	0.384	2.889	0.333	0.518	0.635	1.314	0.317	0.594	0.441	0.530	0.647	0.554
Denton	0.401	2.756	0.442	0.901	0.472	1.202	0.295	0.556	0.563	0.635	0.603	0.586
Structural	0.372	2.750	0.355	0.498	0.765	0.977	0.383	0.575	0.309	0.506	0.403	0.673
Denton	0.378	2.780	0.358	0.523	0.764	1.056	0.411	0.574	0.314	0.525	0.411	0.690
Stram-Wei	0.379	2.770	0.360	0.515	0.766	0.987	0.389	0.599	0.311	0.504	0.412	0.678

NOTE: Legenda - Can=Canada, Mex=Mexico, USA=The United States of America, Aus=Australia, Jap=Japan, Kor=North Korea, Fra=France, Deu=Germany, Ita=Italy, Nld=Netherlands, Esp=Spain, Gbr=Great Britain. *In cases A and C annual-quarterly exercise instead of quarterly-monthly.

Table 1 (follows). Results of time disaggregations: RMSPE

COUNTRY	Can	Mex	Usa	Aus	Jap	Kor	Fra	Deu	Ita	Nld	Esp	Gbr
E. Annual GDP deflator - Quarterly consumer prices												
SUTSE	0.310	6.321	0.170	0.901	0.217	0.850	0.434	0.261	0.585	0.371	0.333	0.492
Chow-Lin	0.330	11.474	0.211	0.999	0.224	0.856	0.450	0.340	0.621	0.379	0.440	0.759
Fernández	0.308	6.716	0.205	0.976	0.250	0.854	0.455	0.364	0.624	0.384	0.409	0.743
Litterman	0.266	7.004	0.178	0.930	0.248	0.841	0.437	0.341	0.604	0.383	0.396	0.532
Denton	0.325	6.619	0.233	1.001	0.272	0.846	0.457	0.389	0.604	0.395	0.384	0.747
Structural	0.306	6.022	0.177	0.907	0.219	1.041	0.419	0.202	0.730	0.375	0.296	0.497
Denton	0.372	6.173	0.177	0.914	0.222	0.995	0.421	0.310	0.767	0.387	0.312	0.510
Stram-Wei	0.349	6.071	0.178	0.904	0.214	0.974	0.430	0.374	0.753	0.379	0.270	0.511
F. Quarterly broad money supply - Monthly narrow money supply												
SUTSE	0.237	1.864	0.155	0.373	0.473	3.250	0.384	0.673	0.437	0.458	2.758	0.599
Chow-Lin	1.170	37.861	1.833	1.290	1.660	14.879	1.624	3.493	3.340	1.735	4.994	0.993
Fernández	0.690	4.940	0.596	0.860	1.068	5.029	0.705	1.961	2.233	1.111	3.490	0.870
Litterman	0.296	2.042	0.304	0.508	1.107	3.591	0.416	2.363	3.128	0.560	3.407	0.660
Denton	2.698	9.386	1.884	2.661	4.278	6.858	3.142	3.536	4.092	2.334	4.635	1.926
Structural	0.462	2.255	0.383	0.493	1.019	3.457	0.384	1.306	2.243	0.585	2.628	0.647
Denton	1.864	6.525	1.452	2.082	1.858	5.150	1.410	2.932	2.973	1.610	2.622	1.603
Stram-Wei	1.879	6.525	1.459	2.107	1.882	5.214	1.421	3.009	2.820	1.632	2.705	1.603
G. Annual short-term interest rates - Monthly long-term interest rates												
SUTSE	3.590	8.908	4.010	5.088	9.305	6.210	3.470	3.971	7.157	3.583	4.210	5.030
Chow-Lin	3.764	9.281	4.258	5.837	9.585	5.899	4.258	4.593	7.204	3.980	4.340	5.500
Fernández	3.616	8.958	4.013	5.081	10.113	6.004	3.478	4.306	7.157	3.652	4.210	5.436
Litterman	3.441	10.010	3.748	4.780	11.098	6.642	3.366	3.883	7.590	3.393	4.026	5.724
Denton	8.464	9.399	8.952	8.266	10.264	8.563	7.142	4.115	7.648	3.805	4.727	5.073
Structural	3.975	14.305	4.293	4.826	10.894	10.761	3.816	4.437	7.086	4.133	6.099	5.112
Denton	3.974	14.809	4.291	4.821	10.954	10.621	3.830	4.517	7.086	4.191	6.258	5.116
Stram-Wei	4.146	14.388	4.509	5.129	10.581	10.790	3.964	4.561	7.828	4.443	6.905	5.259
H. Annual imports c.i.f. - Quarterly imports f.o.b.												
SUTSE	1.021	1.823	0.565	2.187	1.220	0.705	0.866	0.633	0.724	0.995	2.124	0.823
Chow-Lin	1.083	2.083	0.568	2.392	1.270	0.710	0.811	0.708	0.746	1.277	2.082	0.885
Fernández	1.051	1.968	0.580	2.371	1.277	0.712	0.822	0.688	0.731	1.365	2.093	0.823
Litterman	1.033	2.025	0.607	2.352	1.294	0.759	0.830	0.676	0.738	1.437	2.239	0.799
Denton	1.079	2.073	0.551	2.427	1.226	0.663	0.533	0.685	0.641	1.335	2.066	0.756
Structural	2.230	5.464	3.706	4.929	2.710	3.559	3.900	3.106	6.795	3.297	5.886	3.726
Denton	2.325	5.911	3.607	5.056	2.694	3.637	4.109	3.027	6.844	3.276	5.787	3.777
Stram-Wei	2.402	6.186	3.567	5.079	2.956	3.482	4.200	3.043	6.921	3.269	5.818	3.843

Table 2. Synthesis of the results obtained in terms of RMSPE

METHODS	SUTSE	Chow-Lin	Fernández	Litterman	Denton	Structural	Denton univ.	Stram-Wei
SUTSE	-	147.2	123.1	114.3	158.8	135.1	154.1	155.4
Chow-Lin	91.7	-	83.6	77.7	107.9	92.1	104.7	105.6
Fernández	86.5	26.0	-	92.9	129.0	110.1	125.2	126.2
Litterman	81.3	31.3	41.7	-	138.8	118.5	134.7	135.9
Denton	88.5	52.1	71.9	71.9	-	85.4	97.1	97.9
Structural	78.1	36.5	45.8	54.2	33.3	-	113.7	114.7
Denton univ.	82.3	41.7	57.3	62.5	36.5	76.0	-	100.8
Stram-Wei	83.3	43.8	59.4	65.6	36.5	79.2	59.4	-

NOTE: The Table shows, in the lower part, the percentage of success of the method in column over that in row, in the upper part the geometric average of the ratios of RMSPE of the method in column over the method in row. A geometric mean is used for its reciprocity properties.

Table 3. Results and synthesis of the SUTSE models

COUNTRY	Can	Mex	Usa	Aus	Jap	Kor	Fra	Deu	Ita	Nld	Esp	Gbr
					A. Quarterly industrial production - Monthly deliveries							
n	495	195	507	72	507	147	327	135	255	375	86	507
Order	(2,0,1)	(2,1,2)	(2,0,1)	(2,0,1)	(2,1,2)	(2,1,2)	(2,0,2)	(1,1,1)	(2,0,1)	(2,0,1)	(2,1,0)	(2,1,2)
Seasonal	1	0	1	1	1	1	1	1	1	1	0	1
$\eta'(1,2)$	0.951	-	1.058	0.209	-	-	1.392	-	0.377	2.216	-	-
$\eta'(0,2)$	3.974	-	14.708	0.986	-	-	3.473	-	6.930	4.591	-	-
$\eta(1,2)$	-	0.062	-	-	0.107	0.153	-	0.046	-	-	0.223	0.143
$\eta(0,2)$	-	1.910	-	-	1.515	0.656	-	1.893	-	-	2.033	0.471
ζ	35.809	26.213	83.026	10.642	5.793	23.267	63.524	33.790	118.60	40.202	59.643	119.56
					B. Annual GDP - Quarterly industrial production							
n	85	88	169	108	88	53	97	45	129	101	89	169
Order	(2,1,0)	(2,0,0)	(0,2,0)	(1,1,1)	(1,1,2)	(2,0,0)	(2,0,1)	(2,0,0)	(2,0,0)	(1,1,1)	(0,2,0)	(2,1,0)
Cycle	×	×	2	×	×	×	×	×	×	×	2	×
$\eta'(1,2)$	-	0.418	-	-	-	0.044	-	0.044	0.031	-	-	-
$\eta'(0,2)$	-	6.978	-	-	-	0.525	-	1.390	0.190	-	-	-
$\eta(1,2)$	0.166	-	0.075	0.302	0.173	-	0.178	-	-	0.049	0.069	0.048
$\eta(0,2)$	1.283	-	1.706	7.317	0.645	-	0.406	-	-	0.916	3.218	1.290
ζ	4.889	52.679	28.471	170.03	12.326	-	4.357	2.478	5.163	5.361	55.941	43.925
					C. Quarterly consumer prices - Monthly producer prices							
n	508	256	508	133	388	160	508	508	256	316	508	508
Order	(2,2,0)	(2,2,1)	(2,2,0)	(2,1,0)	(2,0,0)	(2,0,0)	(0,2,0)	(2,0,1)	(2,0,0)	(2,2,1)	(2,2,1)	(2,0,2)
Seasonal	0	1	0	×	0	0	×	0	0	1	0	0
$\eta'(1,2)$	-	-	-	-	0.185	0.052	-	0.492	0.181	-	-	0.236
$\eta'(0,2)$	-	-	-	-	3.500	1.531	-	6.981	7.588	-	-	17.310
$\eta(1,2)$	0.411	0.294	0.577	0.132	-	-	0.098	-	-	0.250	0.752	-
$\eta(0,2)$	6.018	6.467	21.573	2.948	-	-	5.225	-	-	4.700	23.869	-
ζ	101.84	15.586	284.57	42.79	29.010	11.261	230.92	261.38	135.97	108.26	171.06	244.08
					D. Annual private consumption expenditures - Quarterly GDP							
n	85	88	169	168	88	129	97	45	128	101	89	169
Order	(2,1,2)	(2,0,0)	(2,1,2)	(2,1,1)	(2,0,0)	(2,0,0)	(2,0,0)	(2,0,0)	(2,0,0)	(2,1,0)	(2,1,1)	(2,1,0)
$\eta'(1,2)$	-	0.106	-	-	0.138	0.057	0.169	0.179	0.247	-	-	-
$\eta'(0,2)$	-	0.486	-	-	0.576	1.850	0.966	1.139	2.303	-	-	-
$\eta(1,2)$	0.228	-	0.061	0.050	-	-	-	-	-	0.089	0.109	0.061
$\eta(0,2)$	1.384	-	1.306	1.773	-	-	-	-	-	0.470	2.081	0.923
ζ	13.792	47.714	25.197	14.376	13.481	17.307	4.718	10.088	15.608	18.334	106.12	21.154

NOTE: n indicates the sample size. Order (l,s,i) indicates the rank of the level, slope and irregular disturbances respectively in the estimated SUTSE model: × states that the component is not present, − indicates that the test is not feasible. The same rule applies for seasonal and cyclical models. η' indicates the test of fixed against stochastic level when the slope is fixed. η and ζ indicate the test of fixed against stochastic slope when the level is respectively stochastic or null.

Table 3 (follows). Results and synthesis of SUTSE models

COUNTRY	Can	Mex	Usa	Aus	Jap	Kor	Fra	Deu	Ita	Nld	Esp	Gbr
						E. Annual GDP deflator - Quarterly consumer prices						
n	85	88	169	168	68	52	169	45	128	100	89	169
Order	(2,1,0)	(1,1,1)	(1,2,2)	(1,2,1)	(1,1,0)	(2,0,0)	(2,0,1)	(1,1,0)	(2,1,0)	(2,1,2)	(1,1,1)	(1,1,0)
$\eta'(1,2)$	-	-	-	-	-	0.228	0.128	-	-	-	-	-
$\eta'(0,2)$	-	-	-	-	-	0.976	5.309	-	-	-	-	-
$\eta(1,2)$	0.042	0.090	0.606	0.208	0.361	-	-	0.154	0.164	0.079	0.162	0.051
$\eta(0,2)$	2.999	2.116	9.831	5.805	1.637	-	-	2.331	6.426	3.482	1.161	7.043
ζ	13.671	13.577	80.889	9.732	46.684	3.344	51.806	42.456	153.237	58.942	17.545	52.433
						F. Quarterly broad money supply - Monthly narrow money supply						
n	409	293	506	499	506	505	250	466	286	190	444	235
Order	(2,2,2)	(2,1,2)	(2,1,0)	(2,2,0)	(2,2,0)	(0,2,0)	(1,2,1)	(2,2,1)	(2,1,1)	(2,2,1)	(2,2,1)	(2,2,1)
$\eta'(1,2)$	-	-	-	-	-	-	-	-	-	-	-	-
$\eta'(0,2)$	-	-	-	-	-	-	-	-	-	-	-	-
$\eta(1,2)$	0.286	0.015	0.295	0.114	0.159	0.096	0.495	0.246	1.268	0.183	0.036	0.114
$\eta(1,2)$	7.987	5.350	4.011	12.268	22.921	1.982	4.611	8.042	9.436	4.252	9.304	2.182
ζ	137.71	9.377	93.648	43.788	137.89	23.191	17.988	29.237	73.583	49.518	32.994	57.166
						G. Annual short-term interest rates - Monthly long-term interest rates						
n	497	136	389	388	160	136	348	108	84	156	132	232
Order	(2,0,0)	(2,0,0)	(2,0,1)	(2,0,1)	(2,0,1)	(2,0,0)	(2,0,0)	(0,2,0)	(2,×,0)	(2,1,0)	(2,×,0)	(2,0,0)
$\eta'(1,2)$	0.105	0.098	0.100	0.059	0.334	0.076	0.101	0.073	-	0.059	-	0.155
$\eta'(0,2)$	0.596	2.478	0.425	0.902	2.988	0.487	1.608	0.716	-	0.520	-	1.997
$\eta(1,2)$	-	-	-	-	-	-	-	0.079	0.119*	0.090	0.100*	-
$\eta(0,2)$	-	-	-	-	-	-	-	1.117	0.988*	0.476	1.428*	-
ζ	5.634	45.946	4.150	5.578	42.048	15.968	15.968	26.638	7.873	3.578	2.791	34.634
						H. Annual imports c.i.f. - Quarterly imports f.o.b.						
n	168	89	168	169	69	49	29	125	56	65	48	168
Order	(2,1,0)	(2,1,2)	(2,0,1)	(2,0,1)	(1,1,2)	(0,0,2)	(2,0,0)	(1,1,1)	(1,1,0)	(0,1,2)	(2,0,0)	(2,0,0)
Seasonal	×	1	1	1	0	×	×	1	1	1	1	×
$\eta'(1,2)$	-	-	0.050	0.242	-	0.060	0.107	-	-	-	0.091	0.095
$\eta'(0,2)$	-	-	1.943	3.169	-	0.169	1.178	-	-	-	0.877	3.726
$\eta(1,2)$	0.035	0.065	-	-	0.090	-	-	0.102	0.063	0.056	-	-
$\eta(0,2)$	2.136	1.155	-	-	1.907	-	-	0.311	0.191	0.518	-	-
ζ	14.201	20.511	4.556	11.491	34.407	1.448	14.551	1.521	1.941	2.464	15.560	48.515

NOTE: * indicates that the test is referred to the LL model.