ANNEX V: CALCULATION AND AGGREGATION OF PPPS

Introduction

V.1. The calculation and aggregation of PPPs requires each country participating in a Eurostat-OECD comparison to provide a set of national annual prices for a selection of representative and comparable products chosen from a common basket of goods and services that covers the whole range of final expenditure on GDP and a detailed breakdown of final expenditure on GDP according to a common classification. Annex V – which should be read as a supplement to Chapter 7 - follows a worked example to show how the national annual prices are converted into PPPs and how these PPPs are aggregated using the final expenditures as weights. The worked example is in two parts. The first part describes how PPPs are calculated for a basic heading using the Èltetö-Köves-Szulc (EKS) method. The second part explains how the PPPs for a basic heading are combined with those of other basic headings to provide weighted PPPs for each level of aggregation up to the level of GDP. It covers two aggregation methods: the EKS method and the Geary-Khamis (GK) method.

PART I: CALCULATION OF PPPS FOR A BASIC HEADING

Basic headings, representative products and quasi expenditure weights

V.2. National annual prices are collected and reported at the level of the basic heading. A basic heading is the lowest level of disaggregation for which explicit expenditure weights can be estimated. For example, cheese is a basic heading and cheddar, camembert, feta, gorgonzola, gouda, etc. are individual products within it. Expenditure on cheese is known, but expenditures on specific cheese varieties are not. By definition, explicit expenditure weights can not be used below the basic heading level. Quasi expenditure weights are used instead as explained below.

V.3. Countries participating in Eurostat-OECD comparisons are required to price not only items that are representative of their national market but also items that are representative of the national markets of others. They are also required to indicate which of the products they have priced are representative of their national markets. A product is said to be representative if it is purchased in sufficient quantities for its price level to be typical for that type of product in the national market. In the cheese example above, cheddar is obviously representative of the United Kingdom, camembert of France, feta of Greece, gorgonzola of Italy and gouda of the Netherlands. But cheddar is sold in sufficient quantities in France and the Netherlands for it to be representative of these countries as well. Similarly, camembert is also representative of Germany, Norway and Sweden, and gouda of Greece, Spain and Portugal. Countries currently indicate representative products by an asterisk (*). Representative products are sometimes referred to as “asterisk products”

V.4. The representativity of the goods and services priced needs to be taken into account when calculating PPPs for a basic heading because the price levels of representative products are generally lower than the price levels of unrepresentative products. Failure to do so may result in the price level for the basic heading being underestimated or overestimated and the corresponding volume level being overestimated or underestimated. To avoid this bias, products that are representative – that is, the products identified by an asterisk - are assigned a quasi expenditure weight of “1” and products that are not representative – that is, the products with no asterisks - are given a quasi expenditure weight of “0”. The choice of “1” and “0” as quasi expenditure weights is an arbitrary convention. Weights of “2” and “1”, or any other similar combination, could also be used. It has been decided that for Eurostat-OECD comparisons it is preferable to exclude price relatives that are based on products that are unrepresentative of both countries when calculating PPPs between two countries for a basic heading.

Stages of calculation

V.5. There are six stages to the calculation of EKS PPPs for a basic heading:

- The calculation of a matrix of Laspeyres type PPPs.
- The calculation of a matrix of Paasche type PPPs.
- The calculation of a matrix of Fisher type PPPs.
- Completing the matrix of Fisher type PPPs.
- The calculation of the matrix of EKS PPPs.
- Standardising the matrix of EKS PPPs.

V.6. “Type” is used to qualify Laspeyres, Paasche and Fisher for two reasons. The first is that Laspeyres, Paasche and Fisher indexes are generally used for temporal comparisons rather than spatial comparisons. Traditional Laspeyres, Paasche and Fisher indexes have a “base period” and a “current period”. Laspeyres, Paasche and Fisher type PPPs have a “base country” and a “partner country”. The second reason is that a traditional Laspeyres index is a weighted arithmetic average and a traditional Paasche index is a weighted harmonic average. The Laspeyres and Paasche type PPPs calculated for a basic heading are quasi-weighted geometric averages. Note that this second reason is only valid for the Laspeyres and Paasche type PPPs calculated for a basic heading. The Laspeyres and Paasche type PPPs calculated for aggregates in Part IIA are, like traditional Laspeyres and Paasche indexes, weighted arithmetic and harmonic means respectively.

Table V.1: Matrix of national annual prices

Product	Country							
	A		B		C		D	
1	P_{1a}	3.43	P_{1b}	17.04*	P_{1c}	633	P_{1d}	9.57*
2	P_{2a}	1.27*	P_{2b}	15.67*	P_{2c}	588*	P_{2d}	- -
3	P_{3a}	- -	P_{3b}	27.27	P_{3c}	443*	P_{3d}	9.95*
4	P_{4a}	2.25	P_{4b}	20.93	P_{4c}	755	P_{4d}	10.22*
5	P_{5a}	- -	P_{5b}	15.75*	P_{5c}	- -	P_{5d}	11.32*

V.7. The starting point of the calculation is the price matrix for the basic heading such as that of Table V.1. The matrix contains each country's national annual prices in national currency for a selection of products covered by the basic heading. The representative products for each country are indicated by an asterisk (*). For example, product 1 is representative for countries B and D; product 2 is representative for countries A, B and C; and so on. Each country has at least one representative product which is priced in at least one other country. Prices for products 2 and 3 are not available for country D and country A respectively; product 5 is not priced by either country A or country C.

Table V.2: Matrix of Laspeyres type PPPs

A		B		C		D	
$L_{A/A}$	1.0000	$L_{A/B}$	0.12773	$L_{A/C}$	0.00216	$L_{A/D}$	0.28090
$L_{B/A}$	12.339	$L_{B/B}$	1.0000	$L_{B/C}$	0.04050	$L_{B/D}$	1.9310
$L_{C/A}$	462.99	$L_{C/B}$	37.335	$L_{C/C}$	1.0000	$L_{C/D}$	60.144
$L_{D/A}$	- -	$L_{D/B}$	0.63534	$L_{D/C}$	0.02246	$L_{D/D}$	1.0000

V.8. The Laspeyres type PPP for a basic heading between any pair of countries is defined as the quasi-weighted geometric mean of the price relatives between the two countries for the products that are representative of the base country. In other words, only products that are representative of the base country have a weight of "1" and products that are only representative of the partner country have a weight of "0". Hence, when A is the base country, the price relatives for product 2 are computed; when B is the base country, the price relatives for products 1, 2 and 5 are computed; and so on. When there is more than one representative product, a simple geometric average of the price relatives is taken.

V.9. The Laspeyres type PPPs of Table V.2 were calculated with the national annual prices and asterisks (the representative products) of Table V.1 as follows.

Base A:

$L_{A/A} = P_{2a}/P_{2a} = 1.27/1.27 = 1.0000$

$L_{B/A} = P_{2b}/P_{2a} = 15.67/1.27 = 12.339$

$L_{C/A} = P_{2c}/P_{2a} = 588/1.27 = 462.99$

Base B:

$L_{A/B} = [(P_{1a}/P_{1b})(P_{2a}/P_{2b})]^{1/2} = [(3.43/17.04)(1.27/15.67)]^{1/2} = 0.12773$

$L_{B/B} = [(P_{1b}/P_{1b})(P_{2b}/P_{2b})(P_{5b}/P_{5b})]^{1/3} = [(17.04/17.04)(15.67/15.67)(15.75/15.75)]^{1/3} = 1.0000$

$L_{C/B} = [(P_{1c}/P_{1b})(P_{2c}/P_{2b})]^{1/2} = [(633/17.04)(588/15.67)]^{1/2} = 37.335$

$L_{D/B} = [(P_{1d}/P_{1b})(P_{5d}/P_{5b})]^{1/2} = [(9.57/17.04)(11.32/15.75)]^{1/2} = 0.63534$

<u>Base C:</u>

$L_{A/C}$ $= P_{2a}/P_{2c} = 1.27/588 = 0.00216$

$L_{B/C}$ $= [(P_{2b}/P_{2c})(P_{3b}/P_{3c})]^{1/2} = [(15.67/588)(27.27/443)]^{1/2} = 0.04050$

$L_{C/C}$ $= [(P_{2c}/P_{2c})(P_{3c}/P_{3c})]^{1/2} = [(588/588)(443/443)]^{1/2} = 1.0000$

$L_{D/C}$ $= P_{3d}/P_{3c} = 9.95/443 = 0.02246$

<u>Base D:</u>

$L_{A/D}$ $= [(P_{1a}/P_{1d})(P_{4a}/P_{4d})]^{1/2} = [(3.43/9.57)(2.25/10.22)]^{1/2} = 0.28090$

$L_{B/D}$ $= [(P_{1b}/P_{1d})(P_{3b}/P_{3d})(P_{4b}/P_{4d})(P_{5b}/P_{5d})]^{1/4}$

$= [(17.04/9.57)(27.27/9.95)(20.93/10.22)(15.75/11.32)]^{1/4} = 1.9310$

$L_{C/D}$ $= [(P_{1c}/P_{1d})(P_{3c}/P_{3d}(P_{4c}/P_{4d})]^{1/3} = [(633/9.57)(443/9.95)(755/10.22)]^{1/3} = 60.144$

$L_{D/D}$ $= [(P_{1d}/P_{1d})(P_{3d}/P_{3d}(P_{4d}/P_{4d})(P_{5d}/P_{5d})]^{1/4}$

$= [(9.57/9.57)(9.95/9.95)(10.22/10.22)(11.32/11.32)]^{1/4} = 1.0000$

Table V.3: Matrix of Paasche type PPPs

A		B		C		D	
$P_{A/A}$	1.0000	$P_{A/B}$	0.08105	$P_{A/C}$	0.00216	$P_{A/D}$	- -
$P_{B/A}$	7.8293	$P_{B/B}$	1.0000	$P_{B/C}$	0.02678	$P_{B/D}$	1.5740
$P_{C/A}$	462.99	$P_{C/B}$	24.690	$P_{C/C}$	1.0000	$P_{C/D}$	44.523
$P_{D/A}$	3.5599	$P_{D/B}$	0.51785	$P_{D/C}$	0.01663	$P_{D/D}$	1.0000

V.10. The Paasche type PPP for a basic heading between any pair of countries is defined as the quasi-weighted geometric mean of the price relatives between the two countries for the products that are representative of the partner country. In other words, only products that are representative of the partner country have a weight of “1” and products that are only representative of the base country have a weight of “0”. Hence, when A is the partner country, the price relatives for product 2 are computed; when B is the partner country, the price relatives for products 1, 2 and 5 are computed; and so on. When there is more than one representative product, a simple geometric average of the price relatives is taken.

V.11. It can be seen from Table V.1 that $P_{B/A}$ – the Paasche type PPP when B is the partner country and A is the base country - is equal to $[(P_{1b}/P_{1a})(P_{2b}/P_{2a})]^{1/2}$. It can also be seen that $L_{A/B}$ - the Laspeyres type PPP when A is the partner country and B is the base country – is equal to $[(P_{1a}/P_{1b})(P_{2a}/P_{2b})]^{1/2}$. $L_{A/B}$ and $P_{B/A}$ are based on the same representative products. $L_{A/B}$ is the transpose of $P_{B/A}$ (and vice versa). Its reciprocal – $1/L_{A/B}$ – is equal to $[(P_{1b}/P_{1a})(P_{2b}/P_{2a})]^{1/2}$ which is equal to $P_{B/A}$.

V.12. Paasche type PPPs can be calculated either directly by following the procedure described in paragraph V.10 or indirectly by applying the identity established in paragraph V.11. The Paasche type PPPs of Table V.3 were obtained by transposing the matrix of Laspeyres type PPPs of Table V.2 and taking the reciprocals of the transposed PPPs as follows:

<u>Base A:</u>

$P_{A/A}$ $= P_{2a}/P_{2a} = 1/L_{A/A} = 1/1.0000 = 1.0000$

$P_{B/A}$ $= [(P_{1b}/P_{1a})(P_{2b}/P_{2a})]^{1/2} = 1/L_{A/B} = 1/0.12773 = 7.8293$

$P_{C/A}$ $= P_{2c}/P_{2a} = 1/L_{A/C} = 1/0.00216 = 462.99$

$P_{D/A}$ $= [(P_{1d}/P_{1a})(P_{4d}/P_{4a})]^{1/2} = 1/L_{A/D} = 1/0.28090 = 3.5599$

<u>Base B:</u>

$P_{A/B}$ $= P_{2a}/P_{2b} = 1/L_{B/A} = 1/12.339 = 0.08105$

$P_{B/B}$ $= [(P_{1b}/P_{1b})(P_{2b}/P_{2b})(P_{5b}/P_{5b})]^{1/3} = 1/L_{B/B} = 1/1.0000 = 1.0000$

$P_{C/B}$ $= [(P_{2c}/P_{2b})(P_{3c}/P_{3b})]^{1/2} = 1/L_{B/C} = 1/0.04050 = 24.690$

$P_{D/B}$ $= [(P_{1d}/P_{1b})(P_{3d}/P_{3b})(P_{4d}/P_{4b})(P_{5d}/P_{5b})]^{1/4} = 1/L_{B/D} = 1/1.9310 = 0.51785$

Base C:

$P_{A/C}$ $= P_{2a}/P_{2c} = 1/L_{C/A} = 1/462.99 = 0.00216$

$P_{B/C}$ $= [(P_{1b}/P_{1c})(P_{2b}/P_{2c})]^{1/2} = 1/L_{C/B} = 1/37.335 = 0.02678$

$P_{C/C}$ $= [(P_{2c}/P_{2c})(P_{3c}/P_{3c})]^{1/2} = 1/L_{C/C} = 1/1.0000 = 1.0000$

$P_{D/C}$ $= [(P_{1d}/P_{1c})(P_{3d}/P_{3c})(P_{4d}/P_{4c})]^{1/3} = 1/L_{C/D} = 1/60.144 = 0.01663$

Base D:

$P_{B/D}$ $= [(P_{1b}/P_{1d})(P_{5b}/P_{5d})]^{1/2} = 1/L_{D/B} = 1/0.63534 = 1.5740$

$P_{C/D}$ $= P_{3c}/P_{3d} = 1/L_{D/C} = 1/0.02246 = 44.523$

$P_{D/D}$ $= [(P_{1d}/P_{1d})(P_{3d}/P_{3d})(P_{4d}/P_{4d})(P_{5d}/P_{5d})]^{1/4} = 1/L_{D/D} = 1/1.0000 = 1.0000$

Table V.4: Matrix of Fisher type PPPs

A	B	C	D
$F_{A/A}$ 1.0000	$F_{A/B}$ 0.10174	$F_{A/C}$ 0.00216	$F_{A/D}$ - -
$F_{B/A}$ 9.8286	$F_{B/B}$ 1.0000	$F_{B/C}$ 0.03294	$F_{B/D}$ 1.7434
$F_{C/A}$ 462.99	$F_{C/B}$ 30.361	$F_{C/C}$ 1.0000	$F_{C/D}$ 51.747
$F_{D/A}$ - -	$F_{D/B}$ 0.57360	$F_{D/C}$ 0.01932	$F_{D/D}$ 1.0000

V.13. The Fisher type PPP for a basic heading between any pair of countries is defined as the unweighted geometric mean of their Laspeyres type PPP for the basic heading and their Paasche type PPP for the basic heading. Direct application of this definition would require the Fisher type PPPs of Table V.3 to be calculated using the Laspeyres type PPPs of Table V.2 and the corresponding Paasche type PPPs of Table V.3. But, because of the identity established in paragraph V.11, they were computed using just the Laspeyres type PPPs of Table V.2 as follows:

$F_{A/A}$ $= [L_{A/A}.P_{A/A}]^{1/2} = [L_{A/A}/L_{A/A}]^{1/2} = [1.0000/1.0000]^{1/2} = 1.0000$

$F_{B/A}$ $= [L_{B/A}.P_{B/A}]^{1/2} = [L_{B/A}/L_{A/B}]^{1/2} = [12.339/0.12773]^{1/2} = 9.8286$

$F_{A/B}$ $= [L_{A/B}.P_{A/B}]^{1/2} = [L_{A/B}/L_{B/A}]^{1/2} = [0.12773/12.339]^{1/2} = 0.10174$

$F_{C/A}$ $= [L_{C/A}.P_{C/A}]^{1/2} = [L_{C/A}/L_{A/C}]^{1/2} = [462.99/0.00216]^{1/2} = 462.99$

$F_{A/C}$ $= [L_{A/C}.P_{A/C}]^{1/2} = [L_{A/C}/L_{C/A}]^{1/2} = [0.00216/462.99]^{1/2} = 0.00216$

$F_{C/B}$ $= [L_{C/B}.P_{C/B}]^{1/2} = [L_{C/B}/L_{B/C}]^{1/2} = [37.335/0.04050]^{1/2} = 30.361$

$F_{B/C}$ $= [L_{B/C}.P_{B/C}]^{1/2} = [L_{B/C}/L_{C/B}]^{1/2} = [0.04050/37.335]^{1/2} = 0.03294$

$F_{D/B}$ $= [L_{D/B}.P_{D/B}]^{1/2} = [L_{D/B}/L_{B/D}]^{1/2} = [0.63534/1.9310]^{1/2} = 0.57360$

$F_{B/D}$ $= [L_{B/D}.P_{B/D}]^{1/2} = [L_{B/D}/L_{D/B}]^{1/2} = [1.9310/0.63534]^{1/2} = 1.7434$ ………… etc.

V.14. The Fisher type PPPs of Table V.4 satisfy the country reversal test – that is, $F_{B/A}.F_{A/B} = 1$; $F_{C/A}.F_{A/C} = 1$, etc. But they are not transitive – that is, $F_{B/A}/F_{C/A}$ =,/ $F_{B/C}$; $F_{A/B}/F_{C/B}$ =,/ $F_{A/C}$, etc. Also, the matrix is incomplete. There is no PPP for $F_{D/A}$ or $F_{A/D}$.

V.15. The matrix is incomplete because the Laspeyres type PPP - $L_{D/A}$ - and the Paasche type PPP - $P_{A/D}$ – could not be calculated and therefore the respective Fisher type PPPs - $F_{D/A}$ and $F_{A/D}$ – could not be calculated either. $L_{D/A}$ and $P_{A/D}$ could not be calculated because country D did not price any products that were representative of country A.

V.16. As the missing Fisher type PPPs could not be calculated, they were estimated instead. This was done by taking the geometric mean of all the indirect Fisher PPPs connecting - or bridging - the countries for which PPPs were missing as follows:

$F_{D/A}$ $= [(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/2} = [(0.5736/0.10174)(0.01932/0.00216)]^{1/2} = 7.1022$

$F_{A/D}$ $= [(F_{A/B}/F_{D/B})(F_{A/C}/F_{D/C})]^{1/2} = [(0.10174/0.5736)(0.00216/0.01932)]^{1/2} = 0.14080$

Table V.5: Completed matrix of Fisher type PPPs

A	B	C	D
$F_{A/A}$ 1.0000	$F_{A/B}$ 0.10174	$F_{A/C}$ 0.00216	$F_{A/D}$ 0.14080
$F_{B/A}$ 9.8286	$F_{B/B}$ 1.0000	$F_{B/C}$ 0.03294	$F_{B/D}$ 1.7434
$F_{C/A}$ 462.99	$F_{C/B}$ 30.361	$F_{C/C}$ 1.0000	$F_{C/D}$ 51.747
$F_{D/A}$ 7.1022	$F_{D/B}$ 0.57360	$F_{D/C}$ 0.01932	$F_{D/D}$ 1.0000

V.17. In the completed matrix of Fisher type PPPs of Table V.5, only the PPPs $F_{D/A}$ and $F_{A/D}$ are transitive. This is because of the way they were estimated. The other PPPs, which are the original Fisher type PPPs from Table V.4, are not transitive. Overall transitivity is obtained by applying the EKS method.

Table V.6: Matrix of EKS PPPs

A	B	C	D
$EKS_{A/A}$ 1.0000	$EKS_{A/B}$ 0.08605	$EKS_{A/C}$ 0.00255	$EKS_{A/D}$ 0.14080
$EKS_{B/A}$ 11.621	$EKS_{B/B}$ 1.0000	$EKS_{B/C}$ 0.02968	$EKS_{B/D}$ 1.6363
$EKS_{C/A}$ 391.57	$EKS_{C/B}$ 33.694	$EKS_{C/C}$ 1.0000	$EKS_{C/D}$ 55.133
$EKS_{D/A}$ 7.1022	$EKS_{D/B}$ 0.61113	$EKS_{D/C}$ 0.01814	$EKS_{D/D}$ 1.0000

V.18. With EKS method, the transitive PPP for any two countries is derived by taking the unweighted geometric mean of the Fisher type PPP calculated between the pair directly and all the PPPs that can be calculated between the pair indirectly when each of the other countries is used as a bridge. The EKS PPPs of Table V.5 were computed following this procedure using the Fisher type PPPs in Table V.5 to provide the direct and indirect PPPs required:

$EKS_{A/A} = F_{A/A} = 1.0000$

$EKS_{B/A} = [(F_{B/A}/F_{A/A})(F_{B/B}/F_{A/B})(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(F_{B/A})^2(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(9.8286)^2(0.03294/0.00216)(1.7434/0.14080)]^{1/4} = 11.621$

$EKS_{C/A} = [(F_{C/A})^2(F_{C/B}/F_{A/B})(F_{C/D}/F_{A/D})]^{1/4}$

$= [(462.99)^2(30.361/0.10174)(51.747/0.14080)]^{1/4} = 391.57$

$EKS_{D/A} = [(F_{D/A})^2(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/4}$

$= [(7.1022)^2(0.57360/0.10174)(0.01932/0.00216)]^{1/4} = 7.1022$

$EKS_{A/B} = [(F_{A/B})^2(F_{A/C}/F_{B/C})(F_{A/D}/F_{B/D})]^{1/4}$

$= [(0.10174)^2(0.00216/0.03294)(0.14080/1.7434)]^{1/4} = 0.08605$

$EKS_{C/B} = [(F_{C/B})^2(F_{C/A}/F_{B/A})(F_{C/D}/F_{B/D})]^{1/4}$

$= [(30.361)^2(462.99/9.8286)(51.747/1.7434)]^{1/4} = 33.694$

$EKS_{D/B} = [(F_{D/B})^2(F_{D/A}/F_{B/A})(F_{D/C}/F_{B/C})]^{1/4}$

$= [(0.57360)^2(7.1022/9.8286)(0.01932/0.03294)]^{1/4} = 0.61113$ ………… etc.

V.19. Transitivity requires that the direct PPP between each pair of countries is equal to the indirect PPP derived via any third country. For example, $EKS_{B/A}$ should equal $EKS_{B/C}/EKS_{A/C}$ or $EKS_{B/D}/EKS_{A/D}$. That the EKS PPPs of Table V.6 meet this requirement is demonstrated below:

$EKS_{B/A} = EKS_{B/C} / EKS_{A/C} = 0.02968/0.00255 = 11.621$

$EKS_{B/A} = EKS_{B/D} / EKS_{A/D} = 1.6363/0.14080 = 11.621$

$EKS_{C/A} = EKS_{C/B} / EKS_{A/B} = 33.694 /0.08605 = 391.57$

$EKS_{C/A} = EKS_{C/D} / EKS_{A/D} = 55.133 /0.14080 = 391.57$

$EKS_{D/A} = EKS_{D/B} / EKS_{A/B} = 0.61113/0.08605 = 7.1022$

$EKS_{D/A} = EKS_{D/C} / EKS_{A/C} = 0.01814/0.00255 = 7.1022$

$EKS_{A/B} = EKS_{A/C} / EKS_{B/C} = 0.00255/0.02968 = 0.08605$

$EKS_{A/B} = EKS_{A/D} / EKS_{B/D} = 0.14080/1.6363 = 0.08605$

$EKS_{C/B} = EKS_{C/A} / EKS_{B/A} = 391.57/11.621 = 33.694$

$EKS_{C/B} = EKS_{C/D} / EKS_{B/D} = 55.133/1.6363 = 33.694$ ………… etc.

Table V.7: Matrix of standardised EKS PPPs

	A	B	C	D
EKS_A	0.0746	0.0746	0.0746	0.0746
EKS_B	0.8667	0.8667	0.8667	0.8667
EKS_C	29.204	29.204	29.204	29.204
EKS_D	0.5297	0.5297	0.5297	0.5297

V.20. In the matrix of EKS PPPs of Table V.6, the PPPs in each column are expressed with the corresponding country as a base. For example, in column A country A is the base - $EKS_{A/A}$, $EKS_{B/A}$, $EKS_{C/A}$ and $EKS_{D/A}$; in column B country B is the base - $EKS_{A/B}$, $EKS_{B/B}$, $EKS_{C/B}$ and $EKS_{D/B}$; and so on. In order to obtain a set of PPPs that has the group of countries as a base – thereby ensuring base country invariance - it is necessary to standardise the PPPs in the matrix. This is done by dividing each PPP by the geometric mean of the PPPs in its column. The standardised PPPs of Table V.7 were computed following this procedure:

$EKS_A = EKS_{A/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$

$= 1.000 / (1.000 \times 11.621 \times 391.57 \times 7.1022)^{1/4} = 0.0746$

$EKS_B = EKS_{B/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$

$= 11.621 / (1.000 \times 11.621 \times 391.57 \times 7.1022)^{1/4} = 0.8667$

$EKS_C = EKS_{C/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{¼}$

$= 391.56 / (1.000 \times 11.621 \times 391.57 \times 7.1022)^{1/4} = 29.204$

$EKS_D = EKS_{D/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{¼}$

$= 7.1022 / (1.000 \times 11.621 \times 391.57 \times 7.1022)^{1/4} = 0.5297$

$EKS_A = EKS_{A/B} / (EKS_{A/B} \times EKS_{B/B} \times EKS_{C/B} \times EKS_{D/B})^{1/4}$

$= 0.08605 / (0.08605 \times 1.000 \times 33.694 \times 0.61113)^{1/4} = 0.0746$

$EKS_B = EKS_{B/B} / (EKS_{A/B} \times EKS_{B/B} \times EKS_{C/B} \times EKS_{D/B})^{1/4}$

$= 1.000 / (0.08605 \times 1.000 \times 33.694 \times 0.61113)^{1/4} = 0.8667$

$EKS_C = EKS_{C/B} / (EKS_{A/B} \times EKS_{B/B} \times EKS_{C/B} \times EKS_{D/B})^{¼}$

$= 33.694 / (0.08605 \times 1.000 \times 33.694 \times 0.61113)^{1/4} = 29.204$

$EKS_D = EKS_{D/B} / (EKS_{A/B} \times EKS_{B/B} \times EKS_{C/B} \times EKS_{D/B})^{¼}$

$= 0.61113 / (0.08605 \times 1.000 \times 33.694 \times 0.61113)^{1/4} = 0.5297$ ………… etc.

V.21. The matrix of standardised EKS PPPs of Table V.8 reduces to the following vector of standardised EKS PPPs.

	A	B	C	D
EKS	0.0746	0.8667	29.204	0.5297

PART II A: AGGREGATION OF BASIC HEADING PARITIES: EKS METHOD

Stages of calculation

V.22. There are five stages to the calculation of EKS PPPs for an aggregate:

- The calculation of a matrix of Laspeyres type PPPs.
- The calculation of a matrix of Paasche type PPPs.
- The calculation of Fisher type PPPs.
- The calculation of the matrix of EKS PPPs.
- Standardising the matrix of EKS PPPs.

V.23. The starting points of the calculation are the matrix of basic heading EKS PPPs and the matrix of expenditures on the basic headings. For the worked example, the matrices cover five basic headings – v, w, x, y and z. The worked example shows how EKS PPPs are calculated for three aggregates, though only the calculations for aggregate 1 are described in detail. The three aggregates are:

- Aggregate 1 = v + w
- Aggregate 2 = x + y + z
- Aggregate 3 = v + w + x + y + z (the overall PPPs)

Table V.8: Matrix of basic heading EKS PPPs

Basic Heading	Country							
	A		B		C		D	
v	PPP_{va}	0.0746	PPP_{vb}	0.8667	PPP_{vc}	29.204	PPP_{vd}	0.5297
w	PPP_{wa}	0.0731	PPP_{wb}	0.9504	PPP_{wc}	20.725	PPP_{wd}	0.6945
x	PPP_{xa}	0.0739	PPP_{xb}	1.1382	PPP_{xc}	25.129	PPP_{xd}	0.4730
y	PPP_{ya}	0.0695	PPP_{yb}	0.8758	PPP_{yc}	27.803	PPP_{yd}	0.5908
z	PPP_{za}	0.0745	PPP_{zb}	0.7454	PPP_{zc}	26.833	PPP_{zd}	0.6708

V.24. The matrix of Table V.8 shows EKS PPPs by basic heading and by country. The PPPs have been calculated following the procedures described in Part I. The PPPs for each basic heading come from a separate vector of standardised EKS PPP. The PPPs for basic heading v are those from the vector of standardised EKS PPPs of paragraph V.21 in Part I.

Table V.9: Matrix of basic heading expenditures

Basic Heading	Country							
	A		B		C		D	
v	E_{va}	5	E_{vb}	110	E_{vc}	2000	E_{vd}	120
w	E_{wa}	20	E_{wb}	240	E_{wc}	5300	E_{wd}	180
x	E_{xa}	15	E_{xb}	300	E_{xc}	3500	E_{xd}	200
y	E_{ya}	35	E_{yb}	450	E_{yc}	10000	E_{yd}	250
z	E_{za}	25	E_{zb}	500	E_{zc}	6500	E_{zd}	250

V.25. The matrix of Table V.9 contains expenditure values in national currencies by basic heading and by country.

Table V.10: Matrix of Laspeyres type PPPs for aggregate 1

A		B		C		D	
$L1_{A/A}$	1.0000	$L1_{A/B}$	0.07982	$L1_{A/C}$	0.00326	$L1_{A/D}$	0.11948
$L1_{B/A}$	12.725	$L1_{B/B}$	1.0000	$L1_{B/C}$	0.04141	$L1_{B/D}$	1.4756
$L1_{C/A}$	305.13	$L1_{C/B}$	25.543	$L1_{C/C}$	1.0000	$L1_{C/D}$	39.958
$L1_{D/A}$	9.0210	$L1_{D/B}$	0.69315	$L1_{D/C}$	0.02929	$L1_{D/D}$	1.0000

V.26. The Laspeyres type PPP for an aggregate between any pair of countries is defined as the weighted arithmetic average of the EKS PPPs between the two countries for the basic headings constituting the aggregate with the expenditures on the basic headings of the base country being used as weights. The Laspeyres type PPPs for aggregate 1 of Table V.10 were calculated using the EKS PPPs of Table V.8 and the expenditure values of Table V.9 for the basic headings v and w as follows:

Base A:

$L1_{A/A} = [(PPP_{va}/PPP_{va})E_{va} + (PPP_{wa}/PPP_{wa})E_{wa}] / (E_{va} + E_{wa})$

$= [(0.0746/0.0746)5 + (0.0731/0.0731)20] / (5 + 20) = 1.0000$

$L1_{B/A} = [(PPP_{vb}/PPP_{va})E_{va} + (PPP_{wb}/PPP_{wa})E_{wa}] / (E_{va} + E_{wa})$

$= [(0.8667/0.0746)5 + (0.9504/0.0731)20] / (5 + 20) = 12.725$

$L1_{C/A} = [(PPP_{vc}/PPP_{va})E_{va} + (PPP_{wc}/PPP_{wa})E_{wa}] / (E_{va} + E_{wa})$

$= [(29.204/0.0746)5 + (20.725/0.0731)20] / (5 + 20) = 305.13$

$L1_{D/A} = [(PPP_{vd}/PPP_{va})E_{va} + (PPP_{wd}/PPP_{wa})E_{wa}] / (E_{va} + E_{wa})$

$= [(0.5297/0.0746)5 + (0.6945/0.0731)20] / (5 + 20) = 9.0210$

Base B:

$L1_{A/B} = [(PPP_{va}/PPP_{vb})E_{vb} + (PPP_{wa}/PPP_{wb})E_{wb}] / (E_{vb} + E_{wb})$

$= [(0.0746/0.8667)110 + (0.0731/0.9504)240] / (110 + 240) = 0.07982$

$L1_{B/B} = [(PPP_{vb}/PPP_{vb})E_{vb} + (PPP_{wb}/PPP_{wb})E_{wb}]/(E_{vb} + E_{wb})$

$= [(0.8667/0.8667)110 + (0.9504/0.9504)240]/(110 + 240) = 1.0000$

$L1_{C/B} = [(PPP_{vc}/PPP_{vb})E_{vb} + (PPP_{wc}/PPP_{wb})E_{wb}] / (E_{vb} + E_{wb})$

$= [(29.204/0.8667)110 + (20.725/0.9504)240] / (110 + 240) = 25.543$

$L1_{D/B} = [(PPP_{vd}/PPP_{vb})E_{vb} + (PPP_{wd}/PPP_{wb})E_{wb}] / (E_{vb} + E_{wb})$

$= [(0.5297/0.8667)110 + (0.6945/0.9504)240] / (110 + 240) = 0.69315$

Base C:

$L1_{A/C} = [(PPP_{va}/PPP_{vc})E_{vc} + (PPP_{wa}/PPP_{wc})E_{wc}] / (E_{vc} + E_{wc})$

$= [(0.0746/29.204)2000 + (0.0731/20.725)5300] / (2000 + 5300) = 0.00326$

$L1_{B/C} = [(PPP_{vb}/PPP_{vc})E_{vc} + (PPP_{wb}/PPP_{wc})E_{wc}] / (E_{vc} + E_{wc})$

$= [(0.8667/29.204)2000 + (0.9504/20.725)5300] / (2000 + 5300) = 0.04141$

$L1_{C/C} = [(PPP_{vc}/PPP_{vc})E_{vc} + (PPP_{wc}/PPP_{wc})E_{wc}]/(E_{vc} + E_{wc})$

$= [(29.204/29.204)2000 + (20.725/20.725)5300]/(2000+5300) = 1.0000$

$L1_{D/C} = [(PPP_{vd}/PPP_{vc})E_{vc} + (PPP_{wd}/PPP_{wc})E_{wc}] / (E_{vc} + E_{wc})$

$= [(0.5297/29.204)2000 + (0.6945/20.725)5300] / (2000 + 5300) = 0.02929$

Base D:

$L1_{A/D} = [(PPP_{va}/PPP_{vd})E_{vd} + (PPP_{wa}/PPP_{wd})E_{wd}] / (E_{vd} + E_{wd})$

$= [(0.0746/0.5297)120 + (0.0731/0.6945)180] / (120 + 180) = 0.11948$

$L1_{B/D} = [(PPP_{vb}/PPP_{vd})E_{vd} + (PPP_{wb}/PPP_{wd})E_{wd}] / (E_{vd} + E_{wd})$

$= [(0.8667/0.5297)120 + (0.9504/0.6945)180] / (120 + 180) = 1.4756$

$L1_{C/D}$ = $[(PPP_{vc}/PPP_{vd})E_{vd} + (PPP_{wc}/PPP_{wd})E_{wd}] / (E_{vd} + E_{wd})$

= [(29.204/0.5297)120 + (20.725/0.6945)180] / (120 + 180) = 39.958

$L1_{D/D}$ = $[(PPP_{vd}/PPP_{vd})E_{vd} + (PPP_{wd}/PPP_{wd})E_{wd}]/(E_{vd} + E_{wd})$

= [(0.5297/0.5297)120 + (0.6945/0.6945)180] / (120 + 180) = 1.0000

Table V.11: Matrix of Laspeyres type PPPs for aggregate 2

A		B		C		D	
$L2_{A/A}$	1.0000	$L2_{A/B}$	0.08413	$L2_{A/C}$	0.00267	$L2_{A/D}$	0.12632
$L2_{B/A}$	12.296	$L2_{B/B}$	1.0000	$L2_{B/C}$	0.03270	$L2_{B/D}$	1.6138
$L2_{C/A}$	374.75	$L2_{C/B}$	31.126	$L2_{C/C}$	1.0000	$L2_{C/D}$	46.272
$L2_{D/A}$	8.2485	$L2_{D/B}$	0.70255	$L2_{D/C}$	0.02204	$L2_{D/D}$	1.0000

V.27. The Laspeyres type PPPs for aggregate 2 of Table V.11 were computed with the EKS PPPs of Table V.8 and the expenditure values of Table V.9 for the basic headings x, y and z following the procedure used for aggregate 1.

Table V.12: Matrix of Laspeyres type PPPs for aggregate 3

A		B		C		D	
$L3_{A/A}$	1.0000	$L3_{A/B}$	0.08319	$L3_{A/C}$	0.00283	$L3_{A/D}$	0.12426
$L3_{B/A}$	12.403	$L3_{B/B}$	1.0000	$L3_{B/C}$	0.03503	$L3_{B/D}$	1.5721
$L3_{C/A}$	357.35	$L3_{C/B}$	29.905	$L3_{C/C}$	1.0000	$L3_{C/D}$	44.378
$L3_{D/A}$	8.4416	$L3_{D/B}$	0.70056	$L3_{D/C}$	0.02398	$L3_{D/D}$	1.0000

V.28. The Laspeyres type PPPs for aggregate 3 of Table V.12 were computed using the EKS PPPs of Table V.8 and the expenditure values of Table V.9 for the basic headings v, w, x, y and z following the procedure described for aggregate 1. They could also have been calculated as the weighted averages of the PPPs for aggregate 1 of Table V.10 and the corresponding PPPs for aggregate 2 of Table V.11. For example:

$L3_{B/A}$ = $[L1_{B/A} (E_{va} + E_{wa}) + L2_{B/A} (E_{xa} + E_{ya} + E_{za})] / (E_{va} + E_{wa} + E_{xa} + E_{ya} + E_{za})$

= [12.725 (5 + 20) + 12.296 (15 + 35 + 25)] / (5 + 20 + 15 + 35 + 25) = 12.403

Table V.13: Matrix of Paasche type PPPs for aggregate 1

A		B		C		D	
$P1_{A/A}$	1.0000	$P1_{A/B}$	0.07860	$P1_{A/C}$	0.00328	$P1_{A/D}$	0.11085
$P1_{B/A}$	12.534	$P1_{B/B}$	1.0000	$P1_{B/C}$	0.03915	$P1_{B/D}$	1.4427
$P1_{C/A}$	306.70	$P1_{C/B}$	24.140	$P1_{C/C}$	1.0000	$P1_{C/D}$	34.131
$P1_{D/A}$	8.3689	$P1_{D/B}$	0.67779	$P1_{D/C}$	0.02502	$P1_{D/D}$	1.0000

V.29. The Paasche type PPP for an aggregate between any pair of countries is defined as the weighted harmonic average of the EKS PPPs between the two countries for the basic headings constituting the aggregate with the expenditures on the basic headings of the partner country being used as weights. But, for the reason given in paragraph V.11 in Part I, the Paasche type PPPs for aggregate 1 of Table V.13 were obtained by transposing the matrix of Laspeyres type PPPs of Table V.10 and taking the reciprocals of the transposed PPPs as follows:

Base A

$P1_{A/A}$ = $1/L1_{A/A}$ = 1/1.0000 = 1.0000

$P1_{B/A}$ = $1/L1_{A/B}$ = 1/0.07982 = 12.534

$P1_{C/A}$ = $1/L1_{A/C}$ = 1/0.00326 = 306.70

$P1_{D/A}$ = $1/L1_{A/D}$ = 1/0.11948 = 8.3698

Base B

$P1_{A/B}$ = $1/L1_{B/A}$ = 1/12.725 = 0.07860

$P1_{B/B}$ = $1/L1_{B/B}$ = 1/1.0000 = 1.0000

$P1_{C/B}$ = $1/L1_{B/C}$ = 1/0.04141 = 24.140

$P1_{D/B}$ = $1/L1_{B/D}$ = 1/1.4756 = 0.67779

Base C

$P1_{A/C}$ = $1/L1_{C/A}$ = 1/305.13 = 0.00328

$P1_{B/C}$ = $1/L1_{C/B}$ = 1/25.543 = 0.03915

$P1_{C/C}$ = $1/L1_{C/C}$ = 1/1.0000 = 1.0000

$P1_{D/C}$ = $1/L1_{C/D}$ = 1/39.958 = 0.02502

Base D

$P1_{A/D}$ = $1/L1_{D/A}$ = 1/9.0210 = 0.11085

$P1_{B/D}$ = $1/L1_{D/B}$ = 1/0.69315 = 1.4427

$P1_{C/D}$ = $1/L1_{D/C}$ = 1/0.02929 = 34.131

$P1_{D/D}$ = $1/L1_{D/D}$ = 1/1.0000 = 1.0000

Table V.14: Matrix of Paasche type PPPs for aggregate 2

A	B	C	D
$P2_{A/A}$ 1.0000	$P2_{A/B}$ 0.08133	$P2_{A/C}$ 0.00267	$P2_{A/D}$ 0.12123
$P2_{B/A}$ 11.886	$P2_{B/B}$ 1.0000	$P2_{B/C}$ 0.03213	$P2_{B/D}$ 1.4234
$P2_{C/A}$ 374.97	$P2_{C/B}$ 30.577	$P2_{C/C}$ 1.0000	$P2_{C/D}$ 45.365
$P2_{D/A}$ 7.9166	$P2_{D/B}$ 0.61965	$P2_{D/C}$ 0.02161	$P2_{D/D}$ 1.0000

Table V.15: Matrix of Paasche type PPPs for aggregate 3

A	B	C	D
$P3_{A/A}$ 1.0000	$P3_{A/B}$ 0.08063	$P3_{A/C}$ 0.00280	$P3_{A/D}$ 0.11846
$P3_{B/A}$ 12.022	$P3_{B/B}$ 1.0000	$P3_{B/C}$ 0.03344	$P3_{B/D}$ 1.42760
$P3_{C/A}$ 353.91	$P3_{C/B}$ 28.542	$P3_{C/C}$ 1.0000	$P3_{C/D}$ 41.695
$P3_{D/A}$ 8.0474	$P3_{D/B}$ 0.63559	$P3_{D/C}$ 0.02253	$P3_{D/D}$ 1.0000

V.30. The Paasche type PPPs for aggregate 2 of Table V.14 and the Paasche type PPPs for aggregate 3 of Table V.15 were obtained by transposing the Laspeyres type PPPs of Table V.11 and Table V.12 respectively and taking the reciprocals of the transposed PPPs.

Table V.16: Matrix of Fisher type PPPs for aggregate 1

A	B	C	D
$F1_{A/A}$ 1.0000	$F1_{A/B}$ 0.07921	$F1_{A/C}$ 0.00327	$F1_{A/D}$ 0.11508
$F1_{B/A}$ 12.629	$F1_{B/B}$ 1.0000	$F1_{B/C}$ 0.04025	$F1_{B/D}$ 1.4590
$F1_{C/A}$ 305.91	$F1_{C/B}$ 24.831	$F1_{C/C}$ 1.0000	$F1_{C/D}$ 36.930
$F1_{D/A}$ 8.6894	$F1_{D/B}$ 0.68538	$F1_{D/C}$ 0.02708	$F1_{D/D}$ 1.0000

V.31. The Fisher type PPP for an aggregate between any pair of countries is defined as the unweighted geometric mean of their Laspeyres type PPP for the aggregate and their Paasche type PPP for the aggregate. But, as explained in paragraph V.13 in Part I, the Fisher type PPPs for aggregate 1 of Table V.16 were not calculated directly using the Laspeyres type PPPs of Table V.10 and the corresponding Paasche type PPPs of Table V.13. Instead, they were computed using just the Laspeyres type PPPs of Table V.10 as follows:

$F_{A/A} = [L_{A/A}.P_{A/A}]^{1/2} = [L_{A/A}/L_{A/A}]^{1/2} = [1.0000/1.0000]^{1/2} = 1.0000$

$F_{B/A} = [L_{B/A}.P_{B/A}]^{1/2} = [L_{B/A}/L_{A/B}]^{1/2} = [12.725/0.07982]^{1/2} = 12.629$

$F_{A/B} = [L_{A/B}.P_{A/B}]^{1/2} = [L_{A/B}/L_{B/A}]^{1/2} = [0.07982/12.725]^{1/2} = 0.07921$

$F_{C/A} = [L_{C/A}.P_{C/A}]^{1/2} = [L_{C/A}/L_{A/C}]^{1/2} = [305.13/0.00326]^{1/2} = 305.91$

$F_{A/C} = [L_{A/C}.P_{A/C}]^{1/2} = [L_{A/C}/L_{C/A}]^{1/2} = [0.00326/305.13]^{1/2} = 0.00327$

$F_{D/A} = [L_{D/A}.P_{D/A}]^{1/2} = [L_{D/A}/L_{D/A}]^{1/2} = [9.0210/0.11948]^{1/2} = 8.6894$

$F_{A/D} = [L_{A/D}.P_{A/D}]^{1/2} = [L_{A/D}/L_{D/A}]^{1/2} = [0.11948/9.0210]^{1/2} = 0.11508$

$F_{C/B} = [L_{C/B}.P_{C/B}]^{1/2} = [L_{C/B}/L_{B/C}]^{1/2} = [25.543/0.04141]^{1/2} = 24.831$

$F_{B/C} = [L_{B/C}.P_{B/C}]^{1/2} = [L_{B/C}/L_{C/B}]^{1/2} = [0.04141/25.543]^{1/2} = 0.04025$ ………… etc.

Table V.17: Matrix of Fisher Type PPPs for aggregate 2

A		B		C		D	
$F2_{A/A}$	1.0000	$F2_{A/B}$	0.08272	$F2_{A/C}$	0.00267	$F2_{A/D}$	0.12375
$F2_{B/A}$	12.090	$F2_{B/B}$	1.0000	$F2_{B/C}$	0.03241	$F2_{B/D}$	1.5156
$F2_{C/A}$	374.86	$F2_{C/B}$	30.850	$F2_{C/C}$	1.0000	$F2_{C/D}$	45.816
$F2_{D/A}$	8.0808	$F2_{D/B}$	0.65980	$F2_{D/C}$	0.02183	$F2_{D/D}$	1.0000

Table V.18: Matrix of Fisher type PPPs for aggregate 3

A		B		C		D	
$F3_{A/A}$	1.0000	$F3_{A/B}$	0.08189	$F3_{A/C}$	0.00281	$F3_{A/D}$	0.12133
$F3_{B/A}$	12.211	$F3_{B/B}$	1.0000	$F3_{B/C}$	0.03422	$F3_{B/D}$	1.4980
$F3_{C/A}$	355.62	$F3_{C/B}$	29.215	$F3_{C/C}$	1.0000	$F3_{C/D}$	43.016
$F3_{D/A}$	8.2421	$F3_{D/B}$	0.66755	$F3_{D/C}$	0.02325	$F3_{D/D}$	1.0000

V.32. The Fisher type PPPs for aggregate 2 of Table V.17 and the Fisher type PPPs for aggregate 3 of Table V.18 were obtained using the Laspeyres type PPP of Table V.11 and Table V.12 respectively following the same procedure as that used for aggregate 1.

V.33. The Fisher type PPPs of Tables V.16, V.17 and V.18 satisfy the country reversal test – that is, $F_{B/A}.F_{A/B} = 1$; $F_{C/A}.F_{A/C} = 1$, etc. But they are not transitive – that is $F_{B/A}/F_{C/A}$ =,/ $F_{B/C}$; $F_{A/B}/F_{C/B}$ =,/ $F_{A/C}$, etc. Transitivity is obtained by applying the EKS method.

Table V.19: Matrix of EKS PPPs for aggregate 1

A		B		C		D	
$EKS1_{A/A}$	1.0000	$EKS1_{A/B}$	0.07963	$EKS1_{A/C}$	0.00321	$EKS1_{A/D}$	0.11655
$EKS1_{B/A}$	12.563	$EKS1_{B/B}$	1.0000	$EKS1_{B/C}$	0.04031	$EKS1_{B/D}$	1.4646
$EKS1_{C/A}$	311.52	$EKS1_{C/B}$	24.796	$EKS1_{C/C}$	1.0000	$EKS1_{C/D}$	36.317
$EKS1_{D/A}$	8.5778	$EKS1_{D/B}$	0.68277	$EKS1_{D/C}$	0.02753	$EKS1_{D/D}$	1.0000

V.34. With EKS method, the transitive PPP for any two countries is obtained by taking the unweighted geometric mean of the Fisher type PPP calculated between the pair directly and all PPPs that can be calculated between the pair indirectly when each of the other countries is used as a bridge. The EKS PPPs for aggregate 1 of Table V.19 were computed following this procedure using the Fisher type PPPs in Table V.16 to provide the direct and indirect PPPs required:

$EKS1_{A/A}$ = $F_{A/A}$ = 1.00000

$EKS1_{B/A}$ = $[(F_{B/A}/F_{A/A})(F_{B/B}/F_{A/B})(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

= $[(F_{B/A})^2(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

= $[(12.62913)^2(0.04025/0.00327)(1.45905/0.11508)]^{1/4}$ = 12.563

$EKS1_{C/A}$ = $[(F_{C/A})^2(F_{C/B}/F_{A/B})(F_{C/D}/F_{A/D})]^{1/4}$

= $[(305.9139)^2(24.83137/0.07921)(36.93008/0.11508)]^{1/4}$ = 311.52

$EKS1_{D/A}$ = $[(F_{D/A})^2(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/4}$

= $[(8.68941)^2(0.68538/0.07921)(0.02707/0.00326)]^{1/4}$ = 8.5778

$EKS1_{A/B}$ = $[(F_{A/B})^2(F_{A/C}/F_{B/C})(F_{A/D}/F_{B/D})]^{1/4}$

= $[(0.07921)^2(0.00326/0.04025)(0.11508/1.45905)]^{1/4}$ = 0.07963

$EKS1_{C/B}$ = $[(F_{C/B})^2(F_{C/A}/F_{B/A})(F_{C/D}/F_{B/D})]^{1/4}$

= $[(24.83137)^2(305.9135/12.62913)(36.93008/1.45905)]^{1/4}$ = 24.796

$EKS1_{D/B}$ = $[(F_{D/B})^2(F_{D/A}/F_{B/A})(F_{D/C}/F_{B/C})]^{1/4}$

= $[(0.68538)^2(8.68941/12.62913)(0.02707/0.04025)]^{1/4}$ = 0.68277 ………… etc.

V.35. Transitivity requires that the direct PPP between each pair of countries is equal to the indirect PPP derived via any third country. For example, $EKS1_{B/A}$ should equal $EKS1_{B/C}$ / $EKS1_{A/C}$ or $EKS1_{B/D}$ / $EKS1_{A/D}$. That the EKS PPPs for aggregate 1 of Table V.19 meet this requirement is demonstrated below:

$EKS1_{B/A}$ = $EKS1_{B/C}$ / $EKS1_{A/C}$ = 0.04031/0.00321 = 12.563

$EKS1_{B/A}$ = $EKS1_{B/D}$ / $EKS1_{A/D}$ = 1.4646/0.11655 = 12.563

$EKS1_{C/A}$ = $EKS1_{C/B}$ / $EKS1_{A/B}$ = 24.796/0.07963 = 311.52

$EKS1_{C/A}$ = $EKS1_{C/D}$ / $EKS1_{A/D}$ = 36.317/0.11655 = 311.52

$EKS1_{D/A}$ = $EKS1_{D/B}$ / $EKS1_{A/B}$ = 0.68277/0.07963 = 8.5778

$EKS1_{D/A}$ = $EKS1_{D/C}$ / $EKS1_{A/C}$ = 0.02753/0.00321 = 8.5778

$EKS1_{A/B}$ = $EKS1_{A/C}$ / $EKS1_{B/C}$ = 0.00321/0.04031 = 0.07963

$EKS1_{A/B}$ = $EKS1_{A/D}$ / $EKS1_{B/D}$ = 0.11655/1.4646 = 0.07963

$EKS1_{C/B}$ = $EKS1_{C/A}$ / $EKS1_{B/A}$ = 311.52/12.563 = 24.796

$EKS1_{C/B}$ = $EKS1_{C/D}$ / $EKS1_{B/D}$ = 36.317/1.4646 = 24.796 ………… etc.

Table V.20: Matrix of EKS PPPs for aggregate 2

A		B		C		D	
$EKS2_{A/A}$	1.0000	$EKS2_{A/B}$	0.08234	$EKS2_{A/C}$	0.00268	$EKS2_{A/D}$	0.12377
$EKS2_{B/A}$	12.144	$EKS2_{B/B}$	1.0000	$EKS2_{B/C}$	0.03254	$EKS2_{B/D}$	1.5030
$EKS2_{C/A}$	373.23	$EKS2_{C/B}$	30.733	$EKS2_{C/C}$	1.0000	$EKS2_{C/D}$	46.193
$EKS2_{D/A}$	8.0797	$EKS2_{D/B}$	0.66531	$EKS2_{D/C}$	0.02165	$EKS2_{D/D}$	1.0000

Table V.21: Matrix of EKS PPPs for aggregate 3

A		B		C		D	
$EKS3_{A/A}$	1.0000	$EKS3_{A/B}$	0.08174	$EKS3_{A/C}$	0.00281	$EKS3_{A/D}$	0.12157
$EKS3_{B/A}$	12.236	$EKS3_{B/B}$	1.0000	$EKS3_{B/C}$	0.03440	$EKS3_{B/D}$	1.4876
$EKS3_{C/A}$	355.64	$EKS3_{C/B}$	29.065	$EKS3_{C/C}$	1.0000	$EKS3_{C/D}$	43.236
$EKS3_{D/A}$	8.2254	$EKS3_{D/B}$	0.67224	$EKS3_{D/C}$	0.02313	$EKS3_{D/D}$	1.0000

V.36. The EKS PPPs for aggregate 2 of Table V.20 and the EKS PPPs of Table V.21 were obtained following the procedure described in paragraph V.35 and using the Fisher type PPPs in Table V.17 and Table V.18 to provide the direct and indirect PPPs required.

Table V.22: Matrix of standardised EKS PPPs for aggregate 1

	A	B	C	D
$EKS1_A$	0.0739	0.0739	0.0739	0.0739
$EKS1_B$	0.9281	0.9281	0.9281	0.9281
$EKS1_C$	23.01	23.01	23.01	23.01
$EKS1_D$	0.6337	0.6337	0.6337	0.6337

V.37. In the matrix of EKS PPPs for aggregate 1 of Table V.19, the PPPs in each column are expressed with the corresponding country as a base. For example, in column A country A is the base – $EKS1_{A/A}$, $EKS1_{B/A}$, $EKS1_{C/A}$ and $EKS1_{D/A}$; in column B country B is the base – $EKS1_{A/B}$, $EKS1_{B/B}$, $EKS1_{C/B}$ and $EKS1_{D/B}$; and so on. In order to obtain a set of PPPs that has the group of countries as a base – thereby ensuring base country invariance - it is necessary to standardise the PPPs in the matrix. This is done by dividing each PPP by the geometric mean of the PPPs in its column. The standardised EKS PPPs for aggregate 1 of Table V.22 were computed following this procedure:

$EKS1_A = EKS1_{A/A} / (EKS1_{A/A} \times EKS1_{B/A} \times EKS1_{C/A} \times EKS1_{D/A})^{1/4}$

$= 1.000 / (1.000 \times 12.563 \times 311.52 \times 8.5778)^{1/4} = 0.0739$

$EKS1_B = EKS1_{B/A} / (EKS1_{A/A} \times EKS1_{B/A} \times EKS1_{C/A} \times EKS1_{D/A})^{1/4}$

$= 12.563 / (1.000 \times 12.563 \times 311.52 \times 8.5778)^{1/4} = 0.9281$

$EKS1_C = EKS1_{C/A} / (EKS1_{A/A} \times EKS1_{B/A} \times EKS1_{C/A} \times EKS1_{D/A})^{1/4}$

$= 311.52 / (1.000 \times 12.563 \times 311.52 \times 8.5778)^{1/4} = 23.01$

$EKS1_D = EKS1_{D/A} / (EKS1_{A/A} \times EKS1_{B/A} \times EKS1_{C/A} \times EKS1_{D/A})^{1/4}$

$= 8.5778 / (1.000 \times 12.563 \times 311.52 \times 8.5778)^{1/4} = 0.6337$ ………… etc.

Table V.23: Matrix of standardised EKS PPPs for aggregate 2

	A	B	C	D
$EKS2_A$	0.0723	0.0723	0.0723	0.0723
$EKS2_B$	0.8779	0.8779	0.8779	0.8779
$EKS2_C$	26.98	26.98	26.98	26.98
$EKS2_D$	0.5841	0.5841	0.5841	0.5841

Table V.24: Matrix of standardised EKS PPPs for aggregate 3

	A	B	C	D
$EKS3_A$	0.0727	0.0727	0.0727	0.0727
$EKS3_B$	0.8896	0.8896	0.8896	0.8896
$EKS3_C$	25.86	25.86	25.86	25.86
$EKS3_D$	0.5980	0.5980	0.5980	0.5980

V.38. The standardised EKS PPPs of Table V.23 and V.24 were obtained by standardising the EKS PPPs Tables V.20 and V.21 respectively following the procedure described in paragraph V.37.

V.39. The three matrices of standardised EKS reduce to three vectors of standardised EKS PPPs:

	A	B	C	D
Aggregate 1	0.0739	0.9281	23.01	0.6337
Aggregate 2	0.0723	0.8779	26.98	0.5841
Aggregate 3	0.0727	0.8896	25.86	0.5980

PART II B: AGGREGATION OF BASIC HEADING PARITIES: GK METHOD

Introduction

V.40. The Geary-Khamis (GK) method requires that a set of theoretical prices - one average price for each basic heading - and a set of overall PPPs – one average PPP for each country - are calculated simultaneously. To do this, the round-iteration system of computation is followed in the worked example. Starting with an initial set of arbitrary determined overall PPPs, an initial set of theoretical prices is derived. The initial set of theoretical prices is then used to obtain a new – or second - set of overall PPPs, which, in turn, is used to derive a new – or second - set of theoretical prices. The process is repeated. The second set of theoretical prices is then used to obtain a new – or third - set of overall PPPs, which, in turn, is used to derive a new - or third - set of theoretical prices. The process continues until the sets of overall PPPs and theoretical prices no longer change. The number of rounds required before convergence occurs depends on the number of countries and the initial set of overall PPPs used to start the process.

V.41. The starting points of the calculation are the matrix of basic heading EKS PPPs, the matrix of expenditures on the basic headings and the matrix of notional quantities. For the worked example, the matrices cover five basic headings – v, w, x, y and z. The worked example shows how GK PPPs are calculated for the same three aggregates that were covered in Part IIA: aggregate 1 (v + w), aggregate 2 (x + y + z) and aggregate 3 (v + w + x + y + z). Unlike EKS PPPs, GK PPPs for aggregates 1 and 2 cannot be determined independently of the overall GK PPPs for aggregate 3. The worked example focuses on the calculation of the overall PPPs.

Table V.25: Matrix of basic heading EKS PPPs

Basic Heading	Country			
	A	B	C	D
v	PPP_{va} 0.0746	PPP_{vb} 0.8667	PPP_{vc} 29.204	PPP_{vd} 0.5297
w	PPP_{wa} 0.0731	PPP_{wb} 0.9504	PPP_{wc} 20.725	PPP_{wd} 0.6945
x	PPP_{xa} 0.0739	PPP_{xb} 1.1382	PPP_{xc} 25.129	PPP_{xd} 0.4730
y	PPP_{ya} 0.0695	PPP_{yb} 0.8758	PPP_{yc} 27.803	PPP_{yd} 0.5908
z	PPP_{za} 0.0745	PPP_{zb} 0.7454	PPP_{zc} 26.833	PPP_{zd} 0.6708

Overall PPPs	$PPP1_A$ 0.0731	$PPP1_B$0.9151	$PPP1_C$ 25.941	$PPP1_D$ 0.5918

V.42. The matrix of Table V.25 shows EKS PPPs by basic heading and by country. These are the same PPPs that were used for Part IIA (see Table V.8 and paragraph V.24). The Table also shows the initial vector of overall PPPs – $PPP1_A$, $PPP1_B$, $PPP1_C$ and $PPP1_D$ - that was used to kick start the process of iteration for the worked example. These overall PPPs are simple arithmetic averages of the basic heading PPPs. But they did not need to be. There is no fixed rule. Exchanges rates or another set of figures, such as a series of “1s”, could have been used instead. The round-iteration method would have yielded identical final results for the overall PPPs and the same structure of final theoretical prices if the initial overall PPPs for the worked example had been assigned other values.

Table V.26: Matrix of basic heading expenditures

Basic Heading	Country							
	A		B		C		D	
v	E_{va}	5	E_{vb}	110	E_{vc}	2000	E_{vd}	120
w	E_{wa}	20	E_{wb}	240	E_{wc}	5300	E_{wd}	180
x	E_{xa}	15	E_{xb}	300	E_{xc}	3500	E_{xd}	200
y	E_{ya}	35	E_{yb}	450	E_{yc}	10000	E_{yd}	250
z	E_{za}	25	E_{zb}	500	E_{zc}	6500	E_{zd}	250
Total	E_A	100	E_B	1600	E_C	27300	E_D	1000

V.43. The matrix of Table V.26 contains expenditure values in national currencies by basic heading and by country. These expenditure values are the same as those used for Part IIA (see Table V.9). The matrix also shows each country's total expenditure on all five basic headings – E_A, E_B, E_C and E_D.

Table V.27: Matrix of notional quantities

Basic Heading	Country									
	A		B		C		D		Total	
v	Q_{va}	67.0	Q_{vb}	127.1	Q_{vc}	68.5	Q_{vd}	226.5	Q_V	489.0
w	Q_{wa}	273.6	Q_{wb}	252.5	Q_{wc}	255.7	Q_{wd}	259.2	Q_W	1041.0
x	Q_{xa}	203.0	Q_{xb}	263.6	Q_{xc}	139.3	Q_{xd}	422.8	Q_X	1028.7
y	Q_{ya}	503.6	Q_{yb}	513.8	Q_{yc}	359.7	Q_{yd}	423.2	Q_Y	1800.2
z	Q_{za}	335.6	Q_{zb}	670.8	Q_{zc}	242.2	Q_{zd}	372.7	Q_Z	1621.3

V.44. The notional quantities of Table V.27 were obtained by dividing the basic heading expenditures of Table V.26 by the corresponding EKS PPPs of Table V.25 as follows:

Q_{va} $= E_{va}/PPP_{va} = 5/0.0746 = 67.0$

Q_{wa} $= E_{wa}/PPP_{wa} = 20/0.0731 = 273.6$

Q_{xa} $= E_{xa}/PPP_{xa} = 15/0.0739 = 203.0$

Q_{ya} $= E_{ya}/PPP_{ya} = 35/0.0695 = 503.6$

Q_{za} $= E_{za}/PPP_{za} = 25/0.0745 = 335.6$

Q_{vb} $= E_{vb}/PPP_{vb} = 110/0.8667 = 127.1$

Q_{wb} $= E_{wb}/PPP_{wb} = 240/0.9504 = 252.5$

Q_{yb} $= E_{yb}/PPP_{yb} = 300/1.1382 = 263.6$ ………… etc.

V.45. The quantities are notional because they are without units, though for each basic heading the "unit of quantity" is the same. The quantities are not commensurate across basic headings – that is, they cannot be sum down rows to provide a meaningful total by country. But they are commensurate across countries and can be summed across columns to provide a total quantity for each basic heading – Q_V, Q_W, Q_X, Q_Y and Q_Z.

Progression of a round

V.46. Each round starts with a vector of overall PPPs from the previous round – or, in the case of the opening round, a vector of overall PPPs that have been arbitrary determined (see paragraph V.42). The overall PPPs are used to produce a matrix of intermediate real expenditures which in turn is used to estimate a vector of theoretical prices. The theoretical prices are subsequently used to produce a second matrix of intermediate real expenditures which in turn is used to estimate the vector of overall PPPs for the next round. The rounds continue until the vectors of overall PPPs and theoretical prices no longer change. Note that the difference between the first matrix of intermediate real expenditures and the second matrix of real expenditure is that in the first matrix the real expenditures are valued using overall PPPs while in the second they are valued using theoretical prices.

ROUND ONE

Table V.28: Matrix of intermediate real expenditures 1

Basic Heading	Country									
	A		B		C		D		Total	
v	$V1_{va}$	68.4	$V1_{vb}$	120.2	$V1_{vc}$	77.1	$V1_{vd}$	202.8	$V1_V$	468.5
w	$V1_{wa}$	273.5	$V1_{wb}$	262.3	$V1_{wc}$	204.3	$V1_{wd}$	304.2	$V1_W$	1044.3
x	$V1_{xa}$	205.1	$V1_{xb}$	327.8	$V1_{xc}$	134.9	$V1_{xd}$	338.0	$V1_X$	1005.9
y	$V1_{ya}$	478.7	$V1_{yb}$	491.7	$V1_{yc}$	385.5	$V1_{yd}$	422.5	$V1_Y$	1778.4
z	$V1_{za}$	341.9	$V1_{zb}$	546.4	$V1_{zc}$	250.6	$V1_{zd}$	422.5	$V1_Z$	1561.3
Total	$V1_A$	1367.6	$V1_B$	1748.4	$V1_C$	1052.4	$V1_D$	1689.8	$V1_{TOT}$	5858.2

V.47. The intermediate real expenditures of Table V.28 were calculated by dividing the matrix of basic heading expenditures of Table V.26 by the vector of overall PPPs of Table V.25 as follows:

$V1_{va}$ $= E_{va}/PPP_A = 5/0.0731 = 68.4$

$V1_{vb}$ $= E_{vb}/PPP_B = 110/0.9151 = 120.2$

$V1_{vc}$ $= E_{vc}/PPP_C = 2000/25.941 = 77.1$

$V1_{vd}$ $= E_{vd}/PPP_D = 120/0.5918 = 202.8$

$V1_V$ $= V1_{va} + V1_{vb} + V1_{vc} + V1_{vd} = 68.4 + 120.2 + 77.1 + 202.8 = 468.5$

$V1_{wa}$ $= E_{wa}/PPP_A = 20/0.0731 = 273.5$

$V1_{wb}$ $= E_{wb}/PPP_B = 240/0.9151 = 262.3$

$V1_{wc}$ $= E_{wc}/PPP_C = 5300/25.941 = 204.3$ ………… etc.

V.48. In other words, the vector of overall PPPs has been used to convert the matrix of basic heading expenditures that is valued in national currencies into a matrix of intermediate real expenditures that is valued at a common but unspecified currency. The matrix can be summed across columns and down rows to give totals by basic heading and by country. The vector - $V1_V$, $V1_W$, $V1_X$, $V1_Y$ and $V1_Z$ - is use in the next step of the round: the estimation of theoretical prices.

Table V.29: Theoretical prices 1

Basic heading	Total intermediate real expenditures 1		Total notional quantities		Theoretical prices 1	
v	$V1_V$	468.5	Q_V	489.0	$P1_V$	0.958
w	$V1_W$	1044.3	Q_W	1041.0	$P1_W$	1.003
x	$V1_X$	1005.9	Q_X	1028.7	$P1_X$	0.978
y	$V1_Y$	1778.4	Q_Y	1800.2	$P1_Y$	0.988
z	$V1_Z$	1561.3	Q_Z	1621.3	$P1_Z$	0.963

V.49. The theoretical prices of Table V.29 were obtained by dividing the vector of total intermediate real expenditures by the vector of total notional quantities. For example, $P1_V = V1_V/Q_V = 468.5/489.0 = 0.958$. The intermediate real expenditures were taken from the "total column" of Table 28 and the notional quantities from the "total column" of Table V.27. The theoretical prices are expressed in the common unspecified currency of Table V.28. The vector - $P1_V$, $P1_W$, $P1_X$, $P1_Y$ and $P1_Z$ - is used in the next step of the round: the calculation of the second matrix of intermediate real expenditure.

Table V.30: Matrix of intermediate real expenditures 2

Basic Heading	Country									
	A		B		C		D		Total	
v	$V2_{va}$	64.2	$V2_{vb}$	121.7	$V2_{vc}$	65.6	$V2_{vd}$	217.0	$V2_{V}$	468.5
w	$V2_{wa}$	274.4	$V2_{wb}$	253.3	$V2_{wc}$	256.5	$V2_{wd}$	260.0	$V2_{W}$	1044.3
x	$V2_{xa}$	198.5	$V2_{xb}$	257.7	$V2_{xc}$	136.2	$V2_{xd}$	413.5	$V2_{X}$	1005.9
y	$V2_{ya}$	497.5	$V2_{yb}$	507.6	$V2_{yc}$	355.3	$V2_{yd}$	418.0	$V2_{Y}$	1778.4
z	$V2_{za}$	323.2	$V2_{zb}$	646.0	$V2_{zc}$	233.3	$V2_{zd}$	358.9	$V2_{Z}$	1561.3
Total	$V2_{A}$	1357.8	$V2_{B}$	1786.3	$V2_{C}$	1046.9	$V2_{D}$	1667.3	$V2_{TOT}$	5858.2

V.50. The intermediate real expenditures of Table V.30 were calculated by multiplying the matrix of notional quantities of Table V.27 by the vector of theoretical prices of Table V.29 as follows:

$V2_{va}$ $= Q_{va}.P1_{V} = 67.0 \times 0.958 = 64.2$

$V2_{wa}$ $= Q_{wa}.P1_{W} = 273.6 \times 1.003 = 274.4$

$V2_{xa}$ $= Q_{xa}.P1_{X} = 203.0 \times 0.978 = 198.5$

$V2_{ya}$ $= Q_{ya}.P1_{Y} = 503.6 \times 0.988 = 497.5$

$V2_{za}$ $= Q_{za}.P1_{Z} = 335.6 \times 0.963 = 323.2$

$V2_{A}$ $= V2_{va} + V2_{wa} + V2_{xa} + V2_{ya} + V2_{za} = 64.2 + 274.4 + 198.5 + 497.5 + 323.2 = 1357.8$

$V2_{vb}$ $= Q_{vb}.P1_{V} = 127.1 \times 0.958 = 121.7$

$V2_{wb}$ $= Q_{wb}.P1_{W} = 252.5 \times 1.003 = 253.3$

$V2_{xb}$ $= Q_{xb}.P1_{X} = 263.6 \times 0.978 = 257.7$ ………… etc.

V.51. In other words, the vector of theoretical prices has been used to convert the matrix of notional quantities into a matrix of intermediate real expenditures valued in the common unspecified currency of Table V.28. The matrix can be summed across columns and down rows to give totals by basic heading and by country. The vector - $V2_{A}$, $V2_{B}$, $V2_{C}$ and $V2_{D}$ - is used in the next step of the round: the estimation of overall PPPs.

Table 31: Overall PPPs 2

Country	Total basic heading expenditures		Total intermediate real expenditures 2		Overall PPPs 2	
A	E_{A}	100	$V2_{A}$	1357.8	$PPP2_{A}$	0.0737
B	E_{B}	1600	$V2_{B}$	1786.3	$PPP2_{B}$	0.8957
C	E_{C}	27300	$V2_{C}$	1046.9	$PPP2_{C}$	26.078
D	E_{D}	1000	$V2_{D}$	1667.3	$PPP2_{D}$	0.5998

V.52. The overall PPPs of Table V.31 were obtained by dividing the vector of total basic heading expenditures by the vector of total intermediate real expenditures. For example, $PPP2_{A} = E_{A}/V2_{B} = 100/1357.8 = 0.0737$. The basic heading expenditures were taken from the "total row" of Table 26 and the intermediate real expenditures from the "total row" of Table V.30. The vector – $PPP2_{A}$, $PPP2_{B}$, $PPP2_{C}$ and $PPP2_{D}$ – is carried over to the next round to use in the calculation of the first matrix of intermediate real expenditures for that round.

ROUND TWO

Table V.32: Matrix of intermediate real expenditures 3

Basic Heading	Country				
	A	B	C	D	Total
v	$V3_{va}$ 67.9	$V3_{vb}$ 122.8	$V3_{vc}$ 76.7	$V3_{vd}$ 200.1	$V3_{V}$ 467.5
w	$V3_{wa}$ 271.6	$V3_{wb}$ 267.9	$V3_{wc}$ 203.2	$V3_{wd}$ 300.1	$V3_{W}$ 1042.9
x	$V3_{xa}$ 203.7	$V3_{xb}$ 334.9	$V3_{xc}$ 134.2	$V3_{xd}$ 333.5	$V3_{X}$ 1006.3
y	$V3_{ya}$ 475.2	$V3_{yb}$ 502.4	$V3_{yc}$ 383.5	$V3_{yd}$ 416.8	$V3_{Z}$ 1777.9
z	$V3_{za}$ 339.4	$V3_{zb}$ 558.2	$V3_{zc}$ 249.3	$V3_{zd}$ 416.8	$V3_{D}$ 1563.7
Total	$V3_{A}$ 1357.8	$V3_{B}$ 1786.3	$V3_{C}$ 1046.9	$V3_{D}$ 1667.3	$V3_{TOT}$ 5858.2

V.53. The intermediate real expenditures of Table V.32 were calculated by dividing the matrix of basic heading expenditures of Table V.26 by the vector of overall PPPs of Table V.31.

Table V.33: Theoretical prices 2

Basic heading	Total intermediate real expenditures 3	Total notional quantities	Theoretical prices 2
v	$V3_{V}$ 467.5	Q_{V} 489.0	$P2_{V}$ 0.956
w	$V3_{W}$ 1042.9	Q_{W} 1041.0	$P2_{W}$ 1.002
x	$V3_{X}$ 1006.3	Q_{X} 1028.7	$P2_{X}$ 0.978
y	$V3_{Y}$ 1777.9	Q_{Y} 1800.2	$P2_{Y}$ 0.988
z	$V3_{Z}$ 1563.7	Q_{Z} 1621.3	$P2_{Z}$ 0.965

V.54. The theoretical prices of Table V.33 were obtained by dividing the vector of total intermediate real expenditures of Table V.32 by the vector of total notional quantities of Table V.27.

Table V.34: Matrix of intermediate real expenditures 4

Basic Heading	Country				
	A	B	C	D	Total
v	$V4_{va}$ 64.1	$V4_{vb}$ 121.5	$V4_{vc}$ 65.4	$V4_{vd}$ 65.4	$V4_{V}$ 467.5
w	$V4_{wa}$ 274.1	$V4_{wb}$ 253.0	$V4_{wc}$ 256.2	$V4_{wd}$ 259.6	$V4_{W}$ 1042.9
x	$V4_{xa}$ 198.6	$V4_{xb}$ 257.8	$V4_{xc}$ 136.2	$V4_{xd}$ 413.6	$V4_{X}$ 1006.3
y	$V4_{ya}$ 497.4	$V4_{yb}$ 507.4	$V4_{yc}$ 355.2	$V4_{yd}$ 417.9	$V4_{Y}$ 1777.9
z	$V4_{za}$ 323.7	$V4_{zb}$ 647.0	$V4_{zc}$ 233.6	$V4_{zd}$ 359.5	$V4_{Z}$ 1563.7
Total	$V4_{A}$ 1357.7	$V4_{B}$ 1786.7	$V4_{C}$ 1046.7	$V4_{D}$ 1667.1	$V4_{TOT}$ 5858.2

V.55. The intermediate real expenditures of Table V.34 were calculated by multiplying the matrix of notional quantities of Table V.27 by the vector of theoretical prices of Table V.33.

Table V.35: Overall PPPs 3

Country	Total expenditures		Total intermediate real expenditures 4		Overall PPPs 3	
A	E_A	100	$V4_A$	1357.7	$PPP3_A$	0.0737
B	E_B	1600	$V4_B$	1786.7	$PPP3_B$	0.8955
C	E_C	27300	$V4_C$	1046.7	$PPP3_C$	26.082
D	E_D	1000	$V4_D$	1667.1	$PPP3_D$	0.5998

V.56. The overall PPPs of Table V.35 were obtained by dividing the vector of total expenditures of Table V.26 by the vector of total real expenditures of Table V.34.

ROUND THREE

Table V.36: Matrix of intermediate real expenditures 5

Basic Heading	Country									
	A		B		C		D		Total	
v	$V5_{va}$	67.9	$V5_{vb}$	122.8	$V5_{vc}$	76.7	$V5_{vd}$	200.1	$V5_v$	467.5
w	$V5_{wa}$	271.5	$V5_{wb}$	268.0	$V5_{wc}$	203.2	$V5_{wd}$	300.1	$V5_w$	1042.8
x	$V5_{xa}$	203.7	$V5_{xb}$	335.0	$V5_{xc}$	134.2	$V5_{xd}$	333.4	$V5_x$	1006.3
y	$V5_{ya}$	475.2	$V5_{yb}$	502.5	$V5_{yc}$	383.4	$V5_{yd}$	416.8	$V5_y$	1777.9
z	$V5_{za}$	339.4	$V5_{zb}$	558.3	$V5_{zc}$	249.2	$V5_{zd}$	416.8	$V5_z$	1563.8
Total	$V5_a$	1357.7	$V5_b$	1786.7	$V5_c$	1046.7	$V5_d$	1667.1	$V5_{tot}$	5858.2

V.57. The intermediate real expenditures of Table V.36 were calculated by dividing the matrix of basic heading expenditures of Table V.26 by the vector of overall PPPs of Table V.35.

Table V.37: Theoretical prices 3

Basic heading	Total intermediate real expenditures 5		Total notional quantities		Theoretical prices 3	
v	$V5_V$	467.5	Q_V	489.0	$P3_V$	0.956
w	$V5_W$	1042.8	Q_W	1041.0	$P3_W$	1.002
x	$V5_X$	1006.3	Q_X	1028.7	$P3_X$	0.978
y	$V5_Y$	1777.9	Q_Y	1800.2	$P3_Y$	0.988
z	$V5_Z$	1563.8	Q_Z	1621.3	$P3_Z$	0.965

V.58. The theoretical prices of Table V.37 were obtained by dividing the vector of total intermediate real expenditures of Table V.36 by the vector of total notional quantities of Table V.27. They are the same as the theoretical prices of the previous round (see Table V.33).

Table V.38: Matrix of intermediate real expenditures 6

Basic Heading	Country									
	A		B		C		D		Total	
v	$V6_{va}$	64.1	$V6_{vb}$	121.5	$V6_{vc}$	65.4	$V6_{vd}$	216.5	$V6_{V}$	467.5
w	$V6_{wa}$	274.1	$V6_{wb}$	253.0	$V6_{wc}$	256.2	$V6_{wd}$	259.6	$V6_{W}$	1042.8
x	$V6_{xa}$	198.6	$V6_{xb}$	257.8	$V6_{xc}$	136.2	$V6_{xd}$	413.6	$V6_{X}$	1006.3
y	$V6_{ya}$	497.3	$V6_{yb}$	507.4	$V6_{yc}$	355.2	$V6_{yd}$	417.9	$V6_{Y}$	1777.9
z	$V6_{za}$	323.7	$V6_{zb}$	647.0	$V6_{zc}$	233.6	$V6_{zd}$	359.5	$V6_{Z}$	1563.8
Total	$V6_{A}$	1357.7	$V6_{B}$	1786.7	$V6_{C}$	1046.7	$V6_{D}$	1667.1	$V6_{TOT}$	5858.2

V.59. The intermediate real expenditures of Table V.38 were calculated by multiplying the matrix of notional quantities of Table V.27 by the vector of theoretical prices of Table V.37.

Table V.39: Overall PPPs 4

Country	Total expenditures		Total intermediate real expenditures 6		Overall PPPs 4	
A	E_A	100	$V6_A$	1357.7	$PPP3_A$	0.0737
B	E_B	1600	$V6_B$	1786.7	$PPP3_B$	0.8955
C	E_C	27300	$V6_C$	1046.7	$PPP3_C$	26.082
D	E_D	1000	$V7_D$	1667.1	$PPP3_D$	0.5998

V.60. The overall PPPs of Table V.39 were obtained by dividing the vector of total expenditures of Table V.26 by the vector of total real expenditures of Table V.38. They are the same as the overall PPPs of the previous round (see Table V.35).

V.61. Both theoretical prices and the overall PPPs of round three are the same as those in round two and the iteration procedure is completed. The theoretical prices of Table V.37 and the overall PPPs of Table V.39 are final. Similarly, the intermediate real expenditures of Table V.38 are not intermediate, but final.

V.62. These final real expenditures are additive. It is therefore possible to calculate the PPP for any given aggregate by dividing the sum of the expenditures on the constituent basic headings from Table V.26 by the sum of the corresponding final real expenditures from Table V.38 as follows:

Aggregate 1 (v + w)

$PPP1_A = (E_{va} + E_{wa})/(V6_{va} + V6_{wa}) = (5 + 20)/(64.1 + 274.1) = 0.0739$

$PPP1_B = (E_{vb} + E_{wb})/(V6_{vb} + V6_{wb}) = (110 + 240)/(121.5 + 253.0) = 0.9346$

$PPP1_C = (E_{vc} + E_{wc})/(V6_{vc} + V6_{wc}) = (2000 + 5300)/(65.4 + 256.2) = 22.699$

$PPP1_D = (E_{vd} + E_{wd})/(V6_{vd} + V6_{wd}) = (120 + 180)/(216.5 + 259.6) = 0.6322$

Aggregate 2 (x + y + z)

$PPP1_A = (E_{xa} + E_{ya} + E_{za})/(V6_{xa} + V6_{ya} + V6_{za}) = (15 + 35 + 25)/(198.6 + 497.3 + 323.7) = 0.0736$

$PPP1_B = (E_{xb} + E_{yb} + E_{zb})/(V6_{xb} + V6_{yb} + V6_{zb}) = (300 + 450 + 500)/(257.8 + 507.4 + 647.0) = 0.8851$

$PPP1_C = (E_{xc} + E_{yc} + E_{zc})/(V6_{xc} + V6_{yc} + V6_{zc}) = (3500 + 10000 + 6500)/(136.2 + 355.2 + 233.6) = 27.586$

$PPP1_D = (E_{xd} + E_{yd} + E_{zd})/(V6_{xd} + V6_{yd} + V6_{zd}) = (200 + 250 + 250)/(413.6 + 417.9 + 359.5) = 0.5877$

V.63. The PPPs for aggregate 3 (v + w + x + y + z) – that is, the overall PPPs – have been calculated already. They are the overall PPPs of Table V.39.