CHAPTER 7: CALCULATION AND AGGREGATION OF PPPS

Introduction

7.1. Many methods have been developed to calculate and aggregate PPPs.[1] This chapter is limited to describing those methods that have immediate relevance to a Eurostat-OECD comparison. PPPs are calculated and aggregated in two stages and the chapter is divided into two parts accordingly. The first part explains the methods used to calculate PPPs for a basic heading. The second part explains the methods used to aggregate basic heading PPPs to obtain PPPs for each level of aggregation up to and including the level of GDP.

7.2. To calculate PPPs for a basic heading, Eurostat and the OECD use the Èltetö-Köves-Szulc or EKS method[2] (although, under certain conditions, a modified version of the EKS method – the EKS-S method[3] - may be used instead). The method has been used since the beginning of the Eurostat-OECD PPP Programme in the early 1980s. The principal alternative to the EKS method is the Country Product Dummy or CPD method[4]. It was used in the first round of the International Comparison Programme (ICP) in 1970 and has been used repeatedly in subsequent rounds of the ICP over the last three decades.[5] For this reason, the CPD method is described and compared with the EKS method in Part I of the chapter even though it is not used for Eurostat-OECD comparisons.

7.3. Eurostat and the OECD also use the EKS method to aggregate PPPs above the basic heading. It was first used for this purpose in the 1990 comparison. It has been used to provide official results for Eurostat-OECD comparisons ever since. Prior to 1990, the Geary Khamis or GK method[6] was used to compile official results. Eurostat and the OECD still employ the GK method, but the results produced are for analytical purposes only, they have no official status. The GK method has been used in all rounds of the ICP that have been conducted so far. The two aggregation methods are described and compared in Part II of the chapter.

7.4. The methods use to calculate and aggregate PPPs should provide price and volume indices that have certain properties. *Commensurability* is such a property. It is common to the four methods considered in the chapter. All produce results that are invariant to changes in the units of measurement for prices and quantities. The four methods are also *base country invariant*. They treat all participating countries symmetrically so that it makes no difference to the final results which country is chosen as the base. The country selected serves simply as the point of reference and its currency as the numéraire. Other properties of the methods – *transitivity*, *characteristicity*, *additivity* and the *Gerschenkron effect* - are discussed later in the chapter.

7.5. Worked examples illustrating how the EKS method is applied at the level of the basic heading and how the EKS and GK methods are applied above the basic heading level can be found in Annex V.

PART I: CALCULATION OF PPPS FOR A BASIC HEADING

Overview of the calculation procedure

7.6. A basic heading is defined as the lowest level of final expenditure on GDP for which explicit expenditure weights can be estimated. As expenditure weights are not available below the basic heading, the relative importance of the products priced for a basic heading cannot be determined by comparing their shares of total expenditure on the basic heading. Participating countries price not only products that are representative of their national market but also products that are representative of the national markets of others. Representative products usually have a lower price level than unrepresentative products. This has to be taken into account when calculating the PPPs for a basic heading otherwise the PPPs will be biased. To avoid this bias, implicit weights are employed to distinguish between representative and unrepresentative products.[7] So that implicit weights can be assigned, participating countries are required to indicate whether or not the products they priced are representative of their national markets when they report their prices. In this context, a product is said to be representative if it is sold in sufficient quantities for its price to be typical for that group of products in the national market.[8]

7.7. The information on representativeness, together with the prices to which it refers, is used to obtain unweighted PPPs at the basic heading level as follows:

- For each pair of countries, two binary PPPs are calculated. The first is the geometric mean of the price relatives for products representative of the first country – the *Laspeyres type PPP*. The second is the geometric mean of the price relatives for products representative of the second country – the *Paasche type PPP*. The geometric mean of these two PPPs is then taken to derive a single binary PPP between the two countries – the *Fisher type PPP*.[9]

- By following this procedure each basic heading is provided with a matrix of Fisher type PPPs. In some cases, the matrix is incomplete because it is not always possible to calculate a Fisher type PPP directly between each pair of countries. Moreover, the Fisher type PPPs in the matrix are intransitive.

- Gaps in the matrix are filled by taking the geometric mean of all the available indirect Fisher type PPPs bridging the pairs of countries for which direct Fisher type PPPs are missing.[10] This process does not always work. The circumstances for it not working and the alternatives

employed when it does not work are outlined at the end of Part II of the chapter under *Missing PPPs and reference PPPs*.

- The matrix is made transitive by applying the EKS procedure. Transitivity is attained by replacing the Fisher type PPP between each pair of countries by the geometric mean of itself squared and all the corresponding indirect Fisher type PPPs between the pair obtained using the other countries as a bridge.

Calculation of binary PPPs: Fisher type PPPs

7.8. Take two countries A and B and let p_{iA} denote the price of product i in country A and p_{iB} the price of the same product in country B. For a given basic heading, call the Laspeyres type PPP between country A and B, L_{AB}, the Paasche type PPP, P_{AB}, and the Fisher type PPP, F_{AB}. Further, let R_A be the set of those products that are representative for country A and suppose that there are n_A such products. Similarly, R_B stands for the set of products that are representative in country B, and the number of products in R_B is n_B. The three binary PPPs between countries A and B are then defined as:

$$L_{AB} = \prod_{i \in R_A} \left[\frac{p_{iB}}{p_{iA}} \right]^{\frac{1}{n_A}} \quad (1a)$$

$$P_{AB} = \prod_{i \in R_B} \left[\frac{p_{iB}}{p_{iA}} \right]^{\frac{1}{n_B}} \quad (1b)$$

$$F_{AB} = \sqrt{L_{AB} P_{AB}} \quad (1c)$$

7.9. L_{AB} and P_{AB} are given equal weight in calculating F_{AB}. Thus, equal weight is given to the average of the price relatives for each country's representative products, irrespective of the number of price relatives (or products) on which each average is based. This implies that, if there is a greater number of A's representative products than B's, each price relative for a representative product in A must carry a correspondingly smaller weight in F_{AB} than each of the price relatives for B's representative products. This prevents the Fisher type PPPs for the basic heading from being dominated by the price relatives of the country which has the larger number of representative products.

7.10. Even though there are no explicit weights in the calculation, there are implicit ones whose pattern depends on the relative numbers of representative products in the two countries and also on the relative size of the overlap between them – that is, the set of products that are representative in both countries. The pattern of weights can vary substantially between different pairs of countries or for different sets of prices. Quite complex systems of weights may be generated even though there is no explicit information about the expenditure on the products.

Achieving transitivity: EKS PPPs

7.11. The Fisher type PPPs derived above are not transitive. The EKS formula is used to make them transitive. To explain the procedure, it is first necessary to introduce the concept of an indirect PPP. An indirect PPP between two countries is one obtained by calculating it indirectly through a third country. Let the three countries be A, B and C. Denote the direct Fisher type PPP between A on B as F_{AB} and the indirect Fisher type PPP for A on B via C as ${}_C F_{AB}$. Then, ${}_C F_{AB}$ is defined as follows:

$${}_C F_{AB} \equiv F_{AC} / F_{BC} \quad (2)$$

F_{AC} is the direct Fisher type PPP between A on C and F_{BC} is the direct Fisher type PPP between B on C.

7.12. More generally, and for a larger set of countries N = {A, B, C, …}, transitivity requires that every indirect PPP, ${}_I PPP_{jk}$ (k, j, l ϵ N), should always equal the direct PPP, PPP_{jk}. Transitivity is regarded as a necessary property for a set of multilateral PPPs otherwise they would not be mutually consistent.

7.13. The EKS PPP is defined as the geometric mean of the direct PPP and all the indirect PPPs between a pair of countries, with the direct PPP having twice the weight of each indirect PPP. In the case of three countries A, B and C, the EKS PPP between country A and country B is:

$$EKS_{AB} = \left\{ F_{AB}^2 \times \frac{F_{AC}}{F_{BC}} \right\}^{\frac{1}{3}} \quad (3)$$

$$EKS_{AB} = \left\{ F_{AB}^2 \times_C F_{AB} \right\}^{\frac{1}{3}}$$

7.14. A similar expression can be derived for the EKS PPPs between countries A and C, and B and C. The expression in (3) can be generalised for the larger set of countries N = {A, B, C, …}. Suppose that the number of countries in N is n. Then, the EKS PPP between countries j and k is given by equation (4) where transitivity is achieved by estimating the PPP between any pair of countries as a geometric mean of direct Fisher type PPPs and indirect Fisher type PPPs:

$$EKS_{jk} = \left\{ F_{jk}^{2} \cdot \prod_{l \neq j,k} \frac{F_{jl}}{F_{kl}} \right\}^{\frac{1}{n}}$$

$$EKS_{jk} = \left\{ F_{jk}^{2} \cdot \prod_{l \neq j,k} {}_{l} F_{jk} \right\}^{\frac{1}{n}} \qquad (4)$$

$$j,k,l \in N$$

7.15. An important justification for the EKS method is that, in addition to being transitive, the resulting multilateral EKS PPPs differ as little as possible from the original binary Fisher type PPPs. More precisely, the EKS procedure minimises the expression $\Sigma_{j\epsilon N} \Sigma_{k\epsilon N} (\log EKS_{jk} - \log F_{jk})^2$ and satisfies characteristicity to the extent possible.[11]

7.16. *Characteristicity* is the property that requires the transitive multilateral comparisons between members of a group of countries to retain the essential features of the intransitive binary comparisons that existed between them before transitivity. A transitive multilateral comparison between a pair of countries is influenced by the price and quantity data of all other participating countries. Characteristicity requires that the impact of these influences should be kept to a minimum when they are introduced into the intransitive binary comparison. The extent to which the EKS PPP and the Fisher type PPP for a pair of countries differ depends on the degree of homogeneity among the price structures of the group of countries being compared.

A modified version of the Fisher type approach: EKS-S method

7.17. Unrepresentative products normally have higher price levels than representative products. Application of the EKS method as described above can lead to a bias in the Fisher type PPP when among the products priced by both countries being compared one country has a larger number of representative products than the other. There can be a downward bias in the Fisher type PPP for the country that provides prices for a larger number of representative products. Conversely, there can be an upward bias in the Fisher type PPP for the country that provides prices for a larger number of unrepresentative products. As explained below, the use of a "Laspeyres - Paasche" approach does not eliminate this possible bias from the Fisher type PPP.[12] A modified version of the EKS method - the EKS-S method - is designed to correct for this bias.

7.18. The EKS-S method starts from the observation that a binary Fisher type PPP, F_{AB}, can be regarded as a geometric average of three and not two PPPs - namely, those based on:

- products that are representative in both countries;
- products that are representative in A but not in B;
- products that are representative in B but not in A.

7.19. In other words, as demonstrated in Box 7.1, the two sets of representative products introduced earlier, R_A and R_B, can be re-organised into three sets. The PPP based on the first set should provide an unbiased estimate of the basic heading PPP because representative products are being compared with representative products. The PPP for A relative to B based on the second set is likely to suffer from an upward bias, while the PPP based on the third set is likely to have a downward bias. If the second and the third set do not enter the calculation of the Fisher type PPP with the same weight, it can be argued that the result is a bias in the estimate of the PPP for the basic heading (see equation (4) in Box 7.1). In order to have an unbiased estimate, equal weight should be given to the PPPs for the second and third sets (see equation (5) in Box 7.1). In almost all cases, the weights for the second and third sets cannot be expected to be equal. And it can be argued that the EKS method is liable to produce biased results in general.

7.20. The procedure that the EKS-S method follows to avoid the bias is this:

- Divide the products and their PPPs into the three mutually exclusive sets defined above;
- Count each price relative in the first set twice on the grounds that PPPs between products that are representative in both countries are unbiased and likely to be more reliable;
- Adjust the total weights for the second and third sets to make them equal while keeping their combined weight unchanged (see equation (5) in Box 7.1)
- Take a weighted geometric mean of the PPPs for each of the three sets using the adjusted weights.

7.21. From a theoretical viewpoint, EKS-S seems to be marginally superior to the standard EKS method. While the two methods are likely to produce similar results in most cases, there may be exceptional cases in which they yield significantly different results. Both methods introduce differential weights for the price relatives that are by no means intuitively obvious, and which are liable to vary considerably depending on the relative sizes of each of three sets of products. These methods cannot be applied mechanically as other factors have to be taken into consideration, in particular the absolute number of products priced for the basic heading. Which method to use and when to use it has to be decided case by case. Not being able to specify the circumstances when each method should be used is clearly a disadvantage.[13]

Box 7.1: EKS weights and EKS-S weights

1. In the EKS method, Fisher type PPPs are calculated as the geometric mean of Laspeyres type PPPs and Paasche type PPPs. For a comparison with the EKS-S method, it is useful to present the formulas used for the EKS somewhat differently.

2. The formula for the Laspeyres type PPP can be broken down as follows:

$$(1)\ L_P = \left(\prod_{i=1}^{n_{11}} p_{i,11} \times \prod_{i=1}^{n_{10}} p_{i,10}\right)^{\frac{1}{n_{11}+n_{10}}} = \left(\prod_{i=1}^{n_{11}} p_{i,11}\right)^{\frac{1}{n_{11}+n_{10}}} \times \left(\prod_{i=1}^{n_{10}} p_{i,10}\right)^{\frac{1}{n_{11}+n_{10}}}$$

$$= \left(\left(\prod_{i=1}^{n_{11}} p_{i,11}\right)^{\frac{1}{n_{11}}}\right)^{\frac{n_{11}}{n_{11}+n_{10}}} \times \left(\left(\prod_{i=1}^{n_{10}} p_{i,10}\right)^{\frac{1}{n_{10}}}\right)^{\frac{n_{10}}{n_{11}+n_{10}}} = \tilde{p}_{11}^{\frac{n_{11}}{n_{11}+n_{10}}} \times \tilde{p}_{10}^{\frac{n_{10}}{n_{11}+n_{10}}}$$

where:

- $\boldsymbol{P}_{i,11}$ is the price relative for product i that is representative in both countries. n_{11} is the total number of these cases;
- $\boldsymbol{P}_{i,10}$ is the price relative for product i that is representative only in the first country. n_{10} is the total number of these cases;
- $\boldsymbol{P}_{i,01}$ is the price relative for product i that is representative only in the second country. n_{01} is the total number of these cases;
- $\tilde{p}_{11} \equiv \left(\prod_{i=1}^{n_{11}} p_{i,11}\right)^{\frac{1}{n_{11}}}$, $\tilde{p}_{10} \equiv \left(\prod_{i=1}^{n_{10}} p_{i,10}\right)^{\frac{1}{n_{10}}}$ and $\tilde{p}_{01} \equiv \left(\prod_{i=1}^{n_{01}} p_{i,01}\right)^{\frac{1}{n_{01}}}$ are geometric averages of the initial price relatives.

3. The formula for the Paasche type PPP can be broken down in a similar way:

$$(2)\ P_P = \tilde{p}_{11}^{\frac{n_{11}}{n_{11}+n_{01}}} \times \tilde{p}_{01}^{\frac{n_{01}}{n_{11}+n_{01}}}$$

4. The formula for the Fisher type PPP is then:

$$(3)\ F_P = \sqrt{L_P \times P_P} = \tilde{p}_{11}^{\frac{0.5\times n_{11}}{n_{11}+n_{10}}} \times \tilde{p}_{10}^{\frac{0.5\times n_{10}}{n_{11}+n_{10}}} \times \tilde{p}_{11}^{\frac{0.5\times n_{11}}{n_{11}+n_{01}}} \times \tilde{p}_{01}^{\frac{0.5\times n_{01}}{n_{11}+n_{01}}}$$

$$= \tilde{p}_{11}^{\left(\frac{0.5\times n_{11}}{n_{11}+n_{10}}+\frac{0.5\times n_{11}}{n_{11}+n_{01}}\right)} \times \tilde{p}_{10}^{\frac{0.5\times n_{10}}{n_{11}+n_{10}}} \times \tilde{p}_{01}^{\frac{0.5\times n_{01}}{n_{11}+n_{01}}}$$

5. Thus, the weighting scheme in the EKS method is:

$$(4)\ w_{11} = \frac{0.5\times n_{11}}{n_{11}+n_{10}} + \frac{0.5\times n_{11}}{n_{11}+n_{01}}\ ;\ w_{10} = \frac{0.5\times n_{10}}{n_{11}+n_{10}}\ ;\ w_{01} = \frac{0.5\times n_{01}}{n_{11}+n_{01}}$$

6. The comparison between w_{10} and w_{01} shows the possible asymmetry of the EKS method. The larger the difference between n_{10} and n_{01} the greater the asymmetry.

7. Using the same notation as above, the weights of various PPPs in the EKS-S method are the following:

$$(5)\ w_{11} = \frac{2\times n_{11}}{2\times n_{11}+n_{10}+n_{01}}\ ;\ w_{10} = w_{01} = 0.5\times\frac{n_{10}+n_{01}}{2\times n_{11}+n_{10}+n_{01}}$$

7.22. There can be difficulties in implementing either EKS or EKS-S if the absolute number of products in any of the three sets becomes very small or zero. For example, suppose there are seven products in the second set and one product in the third set. With EKS-S, the geometric mean for third set will be based on a single price relative. An average based on a single observation has to be erratic. It can be argued that it would not be optimal to reduce the seven price relatives in the second set to an average and then to give this average no more weight than the single price relative in the third set. If there are no products in the third set then the question arises of what use, if any, can be made of the price relatives in the second set when there are no counter-balancing price relatives for them in third set. The problem remains even if the standard EKS is used, although it is probably not as acute as it is for EKS-S. The inclusion of more products in an EKS calculation produces more robust results, but the bias persists if the imbalance between representative products and unrepresentative products is not addressed.

CPD method

7.23. The CPD method is the multilateral method used by the ICP to obtain transitive PPPs at the basic heading level through regression analysis. It is not used by Eurostat and the OECD, but it is the principal alternative to the EKS method, which is why it is discussed here.[14] The method treats the calculation of PPPs as a matter of statistical inference, an estimation problem rather than an index number problem.[15] Its underlying hypothesis is that, apart from random disturbance, the PPPs for individual products within a basic heading are all constant between any given pair of countries. In other words, it is assumed that the pattern of relative prices of the different products within a given basic heading is the same in all countries. It is also assumed that each country has its own overall price level for the basic heading and it is this price level which fixes the levels of absolute prices of the products in the basic heading for the country. By treating the prices observed in the countries for the basic heading as random samples, the PPPs between each pair of countries and the common pattern of relative prices can be estimated using classical least square methods.

7.24. More specifically, the CPD method is a statistical tool that can be used to derive the PPPs for a particular basic heading by regressing the logarithm of observed prices against a set of dummy variables that are defined with respect to the products priced and the participating countries. The procedure involves the model:

$$\ln p_{ij} = \eta_1 D_{1j} + \eta_2 D_{2j} + \ldots + \eta_m D_{mj} + \pi_1 D^*_{i1} + \pi_2 D^*_{i2} + \ldots + \pi_n D^*_{in} + u_{ij} \quad (5)$$

D_{ij} and D^*_{ij} (i = 1, 2, …, m; j = 1, 2, …, n) are, respectively, dummy variables for the m products in the basic heading and the n countries involved in the comparison. D_{ij} and D^*_{ij} are equal to 1 when product i is priced in country j, otherwise they equal 0. Once this regression equation is estimated, the PPP for currency of country k with country j as base can be obtained by the exponential of the difference in the estimates of π_j and π_k taken from the regression equation.

CPD and EKS methods compared

7.25. The EKS method views the calculation of PPPs as an index number problem. It starts with a binary approach and subsequently moves to a multilateral approach. The object is to maximise characteristicity – that is, to obtain multilateral transitive PPPs for the basic heading which are as close as possible to the binary intransitive PPPs initially calculated for the basic heading. The method provides greater flexibility by distinguishing between representative and unrepresentative products. It recognises that within a basic heading price differentials between products can vary from country to country.[16] But the Fisher type PPPs it produces are intransitive. Transitivity is imposed by applying the EKS procedure. Additionally, because of its binary approach, the EKS method is data demanding. Even so, in practice, there may be so few price matches between a single pair of countries that it is impossible to estimate the Fisher type PPP directly. And, as a consequence, price data will be ignored when the missing Fisher type PPPs are estimated indirectly.[17]

7.26. The CPD method considers the calculation of PPPs to be a question of statistical inference. It has a multilateral approach from the beginning. The aim is to calculate transitive PPPs for the basic heading with maximum use of the price data collected for the basic heading. The assumption underlying the method - that the pattern of relative prices within a basic heading is the same in all countries whether or not one or other, or both, of the products is representative - is unrealistic. It is also in direct conflict with the assumption underlying the EKS method. Later versions of the CPD method, for example the Country Product Representativity Dummy or CPRD method, include representativity as another dummy variable.[18] The CPD method produces PPPs that are immediately transitive. In this respect, it is considered to be more transparent than the EKS method. Moreover, missing prices are estimated using the regression coefficients of the respective dummy variables that are based on all the prices collected for the basic heading. The method also allows sampling errors to be estimated for the PPPs.

7.27. Neither method is thought to be absolutely better than the other. When all products are priced in all countries and representativity is not taken into account, the CPD and the EKS produce identical basic heading PPPs.[19] Experimental applications of the EKS, EKS-S, CPD and CPRD methods with ac-

tual data suggest that in practice the differences in results are usually not significant. The choice of method depends more on the circumstances of the comparison.

PART II: AGGREGATION OF PPPS ABOVE THE BASIC HEADING

EKS Method

7.28. With the EKS method, the aggregation of basic heading PPPs is undertaken at each level of expenditure up to the level of GDP as follows:

- For each pair of countries, the basic heading EKS PPPs are weighted, summed and averaged using first the expenditures on the basic headings of the first country as weights and then the expenditures on the basic headings of the second country as weights. This gives two weighted PPPs: a *Laspeyres type PPP* and a *Paasche type PPP*.[20] The geometric mean of these two PPPs gives a single *Fisher type PPP* between the two countries.
- By following this procedure each level of aggregation is provided with a matrix of intransitive Fisher PPPs. Application of the EKS formula makes the matrix transitive.
- The EKS PPPs are then used to convert the national expenditures in national currencies to real final expenditures in a common currency.

7.29. The EKS method provides PPPs for each pair of countries in the comparison that are close to the PPPs that would be obtained if each pair of countries had been compared separately. This is because the EKS procedure in making the Fisher PPPs transitive minimises the differences between them and the resulting EKS PPPs. It also provides real final expenditures that are not additive. It is for this reason that PPPs have to be calculated for each level of aggregation. EKS real final expenditures are not subject to the Gerschenkron effect.

Additivity

7.30. The values of the final expenditure aggregates of participating countries are equal to the sum of the values of their components when both aggregates and components are valued at national prices. Additivity requires this identity to be preserved when the values of the aggregates and their components are valued at international prices. An aggregation method is additive if, for each country being compared, it provides real final expenditures for basic headings that sum to the real final expenditures of the aggregates of which they are components. An additive aggregation method provides volumes that satisfy the average test for volumes – that is, the average volume lies between the maximum and minimum volumes.[21]

Gerschenkron effect

7.31. The Gerschenkron effect applies to aggregation methods that use either a reference price structure – that is, each country's quantities are valued by a uniform set of prices - or a reference volume structure – that is, each country's prices are used to value a uniform set of quantities - to compare countries. With methods employing a reference price structure, a country's share of total GDP - that is, the total for the group of countries being compared - will rise as the reference price structure becomes less characteristic of its own price structure. With methods employing a reference volume structure, a country's share of total GDP will fall as the reference volume structure becomes less characteristic of its own volume structure.

7.32. The Gerschenkron effect arises because of the negative correlation between prices and volumes. In other words, expenditure patterns change in response to changes in relative prices because consumers switch their expenditure towards relatively cheap products. The EKS method does not use a reference price structure or a reference volume structure when estimating real final expenditures. The GK method uses a reference price structure.

GK method

7.33. When applied at the basic heading level, the GK method provides a way of calculating PPPs from observed price and quantity data. Let p_{ij} and q_{ij} denote the price and quantity of product i in country j. Also let P_i denote the international average price of product i. The GK method is defined through the system of interrelated equations below:

$$P_i = \frac{\sum_{j=1}^{M}(p_{ij} / PPP_j)q_{ij}}{\sum_{j=1}^{M} q_{ij}} = \frac{\sum_{j=1}^{M}(p_{ij}q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}}$$

and

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij}q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} = \frac{\sum_{i=1}^{N} p_{ij}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij} /(p_{ij} / P_i)} \quad (6)$$

PPPj is the PPP for an aggregate that includes the basic headings i = 1…n for a country j, N is the number of products and M is the number of countries. These equations are used for all the products and the currencies of the countries involved in the comparison. In practice, the input data are not prices and quantities for individual products, but notional prices and quantities for basic headings that comprise sets of products. The notional prices are the PPPs for the basic headings expressed as national currency units per unit of numéraire currency. The notional quantities are the expenditures on the basic headings expressed in terms of the numéraire currency – that is, national expenditures on the basic headings divided by the corresponding basic heading PPPs.

7.34. GK PPPs are transitive. They provide real final expenditures that are additive. As shown in the worked example in Annex V, it is sufficient to calculate GK PPPs and GK real final expenditures for basic headings only. The real final expenditure for an aggregate is obtained by summing the real final expenditures on its constituent basic headings. The PPPs for the aggregate is derived by dividing the national expenditure on the aggregate by the real final expenditure on the aggregate. GK real final expenditures are subject to the Gerschenkron effect.

EKS and GK methods compared

7.35. The EKS method treats participating countries as a set of independent units, each with an equal weight. It involves calculating a set of binary PPPs between each pair of countries. These binary PPPs are then made transitive by a procedure that minimises the differences between them and the multilateral PPPs it produces. The EKS method provides PPPs for each pair of countries that are “close” to the PPPs that would be obtained if each pair of countries had been compared separately. The extent to which the EKS PPP and the Fisher PPP for a pair of countries differ depends on the degree of homogeneity among the price and expenditure structures of the group of countries being compared. The real final expenditures derived using these PPPs are not additive; nor are they subject to the Gerschenkron effect. EKS PPPs and real final expenditures are considered to be better suited to comparisons across countries of the price and volume levels of individual aggregates, such as clothing and footwear, actual individual consumption, gross fixed capital formation or GDP.

7.36. The GK method treats countries as members of a group, each with a weight equal to its share of real GDP for the group. It entails valuing a matrix of quantities using a vector of prices. The vector is obtained by averaging prices across the group. One disadvantage of this is that a change in the composition of the group can change significantly the average prices as well as the relationships between countries. (This can also happen with the EKS method, but generally the changes are not significant.) Another more important disadvantage is that the real final expenditures are subject to the Gerschenkron effect which can be large.[22] The real final expenditures are additive. GK results are considered to be better suited to the analysis of price and volume structures across countries. Such analyses involve at least two aggregates: for example, clothing and footwear’s share of actual individual consumption or gross fixed capital formation’s price level relative to that of the GDP.

Missing PPPs and reference PPPs

7.37. Any method of aggregation requires the matrix of basic heading PPPs to be complete. This is not always the case and there are basic headings for which PPPs cannot be calculated for a country. Either the country has not priced a representative product or, if it has, other countries have not priced its representative product or, if they have, it has not priced their representative products. Consequently, no direct binary PPP can be calculated between it and any other country. In such cases, the PPPs for the countries and basic headings are taken either from a comparable basic heading - such as beef for veal - or from the next level of aggregation - such as meat for pork.

7.38. For a number of basic headings no prices are collected because, for various reasons, it is difficult to specify and to price products that are comparable across countries for them. PPPs based on price data that have been collected for other basic headings are used for these basic headings. Such PPPs are called “reference PPPs”. They serve as proxies for the PPPs that would have been calculated had prices been collected for the basic headings for which no prices were collected.

7.39 The basic headings affected and the reference PPPs selected for them are listed in Box 7.2. From the Box, it will be seen that the reference PPPs are either for highly aggregated expenditure components, such as household final consumption expenditure, or for goods and services that are similar to the goods and services for which no prices were collected. Exchange rates are used for net purchases abroad and for balance of exports and imports.

Box 7.2: Reference PPPs by basic heading

Basic heading	Reference PPP
INDIVIDUAL CONSUMPTION EXPENDITURE BY HOUSEHOLDS	
11.02.31.1 Narcotics	PPPs for household final consumption expenditure on the domestic market (excluding all basic headings under health and education and all basic headings with reference PPPs)
11.04.42.1 Miscellaneous services relating to the dwelling	PPPs for water supply or actual rent
11 06.31 1 Hospital services	PPPs for production of health services by government (without receipts from sales)
11.07.14.1 Animal drawn vehicles	PPPs for household final consumption expenditure on the domestic market (excluding all basic headings under health and education and all basic headings with reference PPPs)
11.07.35.1 Combined passenger transport	PPP for transport services
11.08.21.1 Telephone and telefax equipment 11.09.21.1 Major durables for outdoor recreation 11.09.22.1 Musical instruments and major durables for indoor recreation	PPPs for durable goods
11.09.43.1 Games of chance	PPPs for household final consumption expenditure on the domestic market (excluding all basic headings under health and education and all basic headings with reference PPPs)
11.09.61.1 Package holidays	Weighted average of PPPs for transport services and PPPs for restaurants and hotels
11.10.11.1 Pre-primary and primary education 11.10.21.1 Secondary education 11.10.31.1 Post-secondary non-tertiary education 11.10.41.1 Tertiary education	PPPs for production of education services by government (without receipts from sales)
11.11.12.1 Canteens	PPPs calculated from selected prices for catering services
11.12.21.1 Prostitution	PPPs for household final consumption expenditure on the domestic market (excluding all basic headings under health and education and all basic headings with reference PPPs)
11.12.41.1 Social protection	PPPs for individual consumption expenditure by government (excluding social protection, recreation and culture, and receipts from sales)
11.12.51.1 Insurance 11.12.61.1 FISIM	PPPs for household final consumption expenditure on the domestic market (excluding all basic headings under health and education and all basic headings with reference PPPs)
11.13.11.1 Purchases by residential households in the rest of the world 11.13.11.2 Purchases by non-residential households in the economic territory of the country	Exchange rates
INDIVIDUAL CONSUMPTION EXPENDITURE BY NPISHs	
12.01.11.1 Housing	PPPs for actual rents
12.02.11.1 Health	PPPs for production of health services by government (without receipts from sales)
12.03.11.1 Recreation and culture	PPPs for individual consumption expenditure by government (excluding social protection, recreation and culture, and receipts from sales)
12.04.11.1 Education	PPPs for production of education services by government (without receipts from sales)
12.05.11.1 Social protection 12.06.11.1 Other services	PPPs for individual consumption expenditure by government (excluding social protection, recreation and culture, and receipts from sales)

Box 7.2: (contd.)

Basic heading	Reference PPP
INDIVIDUAL CONSUMPTION EXPENDITURE BY GOVERNMENT	
13.01.11.1 Housing	PPPs for actual rentals
Health benefits & reimbursements	
13.02.12.4 Hospital services	PPPs for production of health services by government (without receipts from sales)
Production of health services	
Intermediate consumption: ▪ 13.02.22.1 Pharmaceutical products ▪ 13.02.22.2 Other medical goods ▪ 13.02.22.3 Therapeutic appliances and equipment	PPPs for household final consumption expenditure on: ▪ Pharmaceutical products ▪ Other medical products ▪ Therapeutic appliances and equipment
13.03.22.4 Intermediate consumption n.e.c.	PPPs for individual consumption expenditure by households on the domestic market (excluding all basic headings with reference PPPs)
13.02.23.1 Gross operating surplus	PPPs for gross fixed capital formation
13.02.24.1 Net taxes on production	PPPs for production of health services by government (without net taxes on production and receipts from sales)
13.02.25.1 Receipts from sales	PPPs for production of health services by government (without receipts from sales)
Recreation and culture	
13.03.11.1 Recreation and culture	PPPs for individual consumption expenditure by government (excluding social protection, recreation and culture, and receipts from sales)
Education benefits & reimbursements	
13.04.11.1 Education benefits & reimbursements	PPPs for production of education services by government (without receipts from sales)
Production of education services	
13.04.22.1 Intermediate consumption	PPPs for individual consumption expenditure by households on the domestic market (excluding all basic headings with reference PPPs)
13.04.23.1 Gross operating surplus	PPPs for gross fixed capital formation
13.04.24.1 Net taxes on production	PPPs for production of education services by government (without net taxes on production and receipts from sales)
13.04.25.1 Receipts from sales	PPPs for production of education services by government (without receipts from sales)
Social protection	
13.05.11.1 Social protection	PPPs for individual consumption expenditure by government (excluding social protection, recreation and culture, and receipts from sales)
COLLECTIVE CONSUMPTION EXPENDITURE BY GOVERNMENT	
14.01.12.1 Intermediate consumption (collective services relating to defence)	PPPs for gross fixed capital formation in machinery and equipment
14.01.12.2 Intermediate consumption (collective services other than defence)	PPPs for individual consumption expenditure by households on the domestic market (excluding all basic headings with reference PPPs)
14.01.13.1 Gross operating surplus	PPPs for gross fixed capital formation
14.01.14.1 Net taxes on production	PPPs for production of collective services by government (without net taxes on production and receipts from sales)
14.01.15.1 Receipts from sales	PPPs for production of collective services by government (without receipts from sales)

Box 7.2: (contd.)

Basic heading	Reference PPP
GROSS FIXED CAPITAL FORMATION	
15.01.22.1 Ships, boats, steamers, tugs, floating platforms, rigs 15.01.22.2 Locomotives and rolling stock	PPPs for gross fixed capital formation in machinery and equipment
15.01.22.3 Aircraft, helicopters and other aeronautical equipment	Exchange rates
15.03 11.1 Products of agriculture, forestry, fisheries and aquaculture 15.03.13.1 Other products n.e.c.	PPPs for gross fixed capital formation
OTHER	
16.01.11.1 Change in inventories	Weighted average of the PPPs for consumer goods and the PPPs for equipment goods
16.02.11.1 Acquisition less disposals of valuables	PPPs for jewellery, clocks and watches
17.01.11.1 Exports of goods and services 17.01.11.2 Imports of goods and service	Exchange rates.

[1] See, for example, "A taxonomy of multilateral methods for making international comparisons of prices and quantities", R. Hill, *The Review of Income and Wealth*, March 1997.

[2] The EKS method is named after the three individuals who independently advocated its use: Èltetö, Köves and Szulc. It was called "EKS" by Lazlo Drechsler in "Weighting of index numbers in multilateral comparisons", *The Review of Income and Wealth*, March 1973. The formula was actually proposed 40 years earlier by Gini in "On the circular test of index numbers", *International Review of Statistics*, Vol. 9, No. 2, 1931. "EKS" refers to a procedure whereby any set of intransitive binary index numbers are made transitive. The procedure is independent of the method used to calculate the intransitive binary indices. But, as used in this chapter and in most literature on the subject, "EKS" covers both the way the intransitive binary PPPs are calculated and the procedure to make them transitive and multilateral.

[3] The second "S" stands for Sergey Sergeev who proposed the modification in *Equi-representativity and Some Modifications of the EKS Method at the Basic Heading Level* at the Joint Consultation on the European Comparison Programme, ECE, Geneva, 2003.

[4] The CPD method was proposed by Robert Summers in "International comparisons with incomplete data", *The Review of Income and Wealth*, March 1973.

[5] See *A System of International Comparisons of Gross Product and Purchasing Power*, I. Kravis. Z. Kenessey, A. Heston and R. Summers, The John Hopkins University Press, Baltimore, 1975; *International Comparisons of Real Product and Purchasing Power*, I. Kravis, A. Heston and R. Summers, The John Hopkins University Press, Baltimore, 1978; and *World Product and Income, International Comparisons of Real Product and Purchasing Power*, I. Kravis, A. Heston and R. Summers, The John Hopkins University Press, Baltimore, 1982.

[6] The GK method was proposed by R. C. Geary in "A note on the comparison of exchange rates and the purchasing power between countries", *Journal of the Royal Statistical Society*, Series A, Vol. 121, 1958. Practical application was developed by S. H. Khamis in: "Properties and conditions for the existence of a new type of index numbers", *Sankhya*, Series , Vol. 2, 1970; "A new system of index numbers for national and international purposes", *Journal of the Royal Statistical Society*, Series A, Vol. 135, 1972; and "On aggregation methods for international comparisons", *The Review of Income and Wealth*, Vol.30, 1984.

[7] Eurostat and the OECD assign a weight of "1" to products that are representative and a weight of "0" to products that are not representative. In this way the price relatives that are based on products that are unrepresentative of both countries are excluded when calculating PPPs between two countries for a basic heading. Even so, the choice of "1" and "0" as weights is arbitrary. Weights of "2" and "1", or any other similar combination, could also be used.

[8] For a more complete explanation of representativity and the assigning of representativity indicators see paragraphs 4.57 to 4.62 of Chapter 4.

[9] The qualifier "type" is used for two reasons. The first is that Laspeyres, Paasche and Fisher indexes are generally used for temporal comparisons rather than spatial comparisons. Standard Laspeyres, Paasche and Fisher indexes have a "base period" and a "current period", whereas Laspeyres, Paasche and Fisher type PPPs have a "base country" and a "partner country". The second reason is that, whereas a standard Laspeyres index is a weighted arithmetic mean and a standard Paasche index is a weighted harmonic mean, the Laspeyres and Paasche type PPPs calculated for a basic heading are quasi-weighted geometric means. In this respect, the terminology is misleading. It would be more accurate and simpler to refer to the Laspeyres and Paasche type PPPs as "Jevons type PPPs" and to refer to the Fisher type PPPs as "Törnqvist type PPPs". This terminology has recently been introduced in Chapter 10, "The estimation of PPPs for basic headings", revision 1, February 2005, of the *ICP 2003-2006 Handbook* at www.worldbank.org/data/.

[10] The process is iterative. During the first round only the original Fisher type PPPs are used. If the matrix remains incomplete, there is a second round using both the original Fisher type PPPs and the new Fisher type PPPs derived indirectly during the first round. The process is continued until either the matrix is complete or it becomes clear that the matrix cannot be completed.

[11] The EKS procedure is applied to intransitive binary Fisher type PPPs in Eurostat-OECD comparisons. But, as already mentioned in footnote 3, the procedure can be applied to other sorts of intransitive binary PPPs such as Törnqvist, Marshall-Edgeworth, etc.

[12] This is for practical rather than theoretical reasons. In Eurostat-OECD comparisons the Fisher type PPP between any pair of participating countries is based on a set of commonly-priced products selected from a product list designed to facilitate a multilateral comparison and not from a product list that has been tailored specifically to make a bilateral comparison between the two countries in question. Hence, the numbers of representative and unrepresentative products among the commonly-priced products depend on what the two countries have priced. And what the two countries have priced is determined by factors other than ensuring that the numbers of representative and unrepresentative products are appropriately balanced for the bilateral comparison between them or with any other participating country.

[13] Not being able to specify the circumstances under which different methods should be used is not specific to the EKS and the EKS-S. For example, when to use the EKS method and when to use the CPD method is still being debated.

[14] Or "was" the principal alternative. See paragraph 7.26 and footnote 18.

[15] In *Multilateral Measurement of Purchasing Power and Real GDP* (Eurostat, 1982), Peter Hill asks "whether or not it is legitimate to by-pass index number problems in this way by falling back on the somewhat unfashionable concept of price level, even at the very detailed level of disaggregation of a basic heading" though he concedes that "there is much more justification for associating specific price levels with countries when dealing with individual basic headings containing small numbers of fairly homogeneous items" than "at a much higher levels of aggregation with larger and more heterogeneous groups of goods and services whose relative prices have much greater scope for variation from country to country". In Annex 2 of *Equi-representativity and Some Modifications of the EKS Method at the Basic Heading Level* (Joint Consultation on the European Comparison Programme, ECE, Geneva, 2003), Sergey Sergeev points out that the economics of the regression equation are not obvious making the CPD appear as a mathematical exercise. He also points out that the stochastic assumptions for the regression procedure are not realistic in practice when the number of products priced per basic heading is small. This leads him to propose a modification to the CPD method that makes possible its presentation as an index number method.

[16] In practice, with the use of "1" and "0" as weights, there is an implicit assumption that products countries nominate as representative are equally representative and that products they nominate as unrepresentative are equally unrepresentative. In other words, the price differential between representative and unrepresentative products is the same for all countries.

[17] This is not the only example of price data being ignored. As explained in footnote 7, when calculating the Laspeyres and Paasche type PPPs between a pair of countries, Eurostat and the OECD assign weights of "1" and "0" to representative and unrepresenta-

tive products respectively. As a consequence, no account is taken of the prices of products that are unrepresentative of both countries and which both have priced.

[18] The inclusion of a variable for representativity in the CPD was first suggested by James and Margaret Cuthbert in "On aggregation methods of purchasing power parities", *OECD Department of Economic Statistics Working Papers*, No.56, Paris, November 1988. It is very likely that the CPRD, and not the CPD, will be used for ICP 2005. The CPRD is explained in Chapter 10, "The estimation of PPPs for basic headings", revision 1, February 2005, of the *ICP 2003-2006 Handbook* at www.worldbank.org/data/.

[19] When first applied, the EKS method did not take into account the representativity of the products priced. This version of the EKS is sometimes referred to as the "classic" EKS. Similarly, the original version of the CPD, which also does not take representativity into account, is sometimes referred as the "classic" or "traditional" CPD.

[20] Unlike their namesakes in Part I which are quasi-weighted geometric means, these Laspeyres and Paasche type PPPs are like standard Laspeyres and Paasche indexes being, respectively, weighted arithmetic means and weighted harmonic means.

[21] Additive aggregation methods generally use a reference price structure to value national expenditures on basic headings at international prices. The reference price structure consists of an international price for each basic heading. An international price for a basic heading is defined as the average of the national prices for the basic heading prevailing in participating countries. The average may be weighted or unweighted, PPP adjusted or PPP-unadjusted. It may be an average of prices or an average of price structures. For example, in the GK method which is discussed later, the average is defined as a quantity-weighted arithmetic average of the national prices adjusted by the global PPPs across all countries.

[22] For example, countries significantly affected by the Gerschenkron effect in the GK calculation for the 1999 comparison were: Turkey, Romania, FYROM, Estonia, and Lithuania - all over 15 per cent; and the Russian Federation, Latvia, Bulgaria and Ukraine - all over 20 per cent. The percentage shows the difference between the EKS and GK real final expenditure on GDP for a country expressed as a percentage of its EKS real final expenditure on GDP.