EUROSTAT - OECD PPP PROGRAMME 1999

THE CALCULATION AND AGGREGATION OF PARITIES

INTRODUCTION

The calculation and aggregation of parities requires from each country participating in a comparison, either bilateral or multilateral, the following:

- a set of national annual average prices;
- a breakdown of national expenditure.

The prices have to be consistent with those used to estimate the expenditures.

This note follows a worked example to show how the national annual average prices are converted into parities and how these parities are aggregated using national expenditure weights in Eurostat-OECD multilateral comparisons. The worked example is in two parts. The first part describes how parities are calculated for a basic heading. The second part explains how the parities for a basic heading are combined with those of other basic headings to provide weighted parities or PPPs for each level of aggregation up to the level of GDP.

THE CALCULATION OF PARITIES FOR A BASIC HEADING

Basic headings and representative products

Prices are collected and reported at the level of the basic heading. In principle, a basic heading consists of a group of similar well-defined commodities for which a sample of products can be selected that are representative both of their category and of the participating countries. In practice, a basic heading is defined by the lowest level of expenditure category for which explicit weights can be estimated. Thus, for example, cheese is a basic heading and cheddar, camembert, feta, gorgonzola, gouda, etc. are individual products within it. Expenditure on cheese is known, but expenditures on specific cheeses are not.

By definition, expenditure weights are not used below the basic heading level. Since, however, in Eurostat-OECD comparisons, countries price not only items that are representative of their national market but also items that are representative of the national markets of others, the representativeness of the goods and services priced needs to be taken into account when deriving parities at the basic heading level. Hence, when reporting prices, countries are required to indicate whether or not the products they priced are representative of their national markets.

In this context, a product is said to be representative if it is purchased in sufficient quantities for its price to be typical for that type of product in the national market. For instance, in the cheese example, cheddar is obviously representative of the United Kingdom, camembert of France, feta of Greece, gorgonzola of Italy and gouda of the Netherlands. However, cheddar is sold in sufficient quantities in France and the Netherlands that it is representative of these countries as well. Similarly, camembert is also representative of Germany, Norway and Sweden, and gouda of Greece, Spain and Portugal.

Method of calculation

There are five stages:

- Calculation of "Laspeyres" and "Paasche" price ratios.
- Calculation of "Fisher" price ratios.
- Completing the "Fisher" matrix.
- Building the EKS matrix of transitive parities.
- Standardising the EKS matrix of transitive parities.

Price matrix

The price matrix consists of national annual average prices in local currency. The representative products for each country are indicated by an asterisk. Each country has at least one representative product which is priced in at least one other country. Prices for products 2 and 3 are not available for country D and country A respectively; product 5 is not priced in either country A or country C.

Product	**Country**							
	A		B		C		D	
1	P_{1a}	3.43	P_{1b}	17.04*	P_{1c}	633	P_{1d}	9.57*
2	P_{2a}	1.27*	P_{2b}	15.67*	P_{2c}	588*	P_{2d}	- -
3	P_{3a}	- -	P_{3b}	27.27	P_{3c}	443*	P_{3d}	9.95*
4	P_{4a}	2.25	P_{4b}	20.93	P_{4c}	755	P_{4d}	10.22*
5	P_{5a}	- -	P_{5b}	15.75*	P_{5c}	- -	P_{5d}	11.32*

"Laspeyres" price ratio matrix

When calculating the "Laspeyres" price ratios only the representative products of the base country are involved irrespective of whether or not the products are representative of the partner country. Hence, when A is the base country, the ratios for product 2 are calculated; when B is the base country, the ratios for products 1, 2 and 5 are calculated; and so on. When a country has more than one representative product a simple geometric mean of the price ratios is taken.

A		B		C		D	
$L_{A/A}$	1.000	$L_{A/B}$	0.12773	$L_{A/C}$	0.00216	$L_{A/D}$	0.28090
$L_{B/A}$	12.339	$L_{B/B}$	1.000	$L_{B/C}$	0.04050	$L_{B/D}$	1.9310
$L_{C/A}$	462.99	$L_{C/B}$	37.335	$L_{C/C}$	1.000	$L_{C/D}$	60.144
$L_{D/A}$	- -	$L_{D/B}$	0.63534	$L_{D/C}$	0.02246	$L_{D/D}$	1.000

The elements of the "Laspeyres" price ratio matrix are calculated as follows:

Base A:

$L_{A/A}$ = P_{2a}/P_{2a} = 1.27/1.27 =1.000

$L_{B/A}$ = P_{2b}/P_{2a} = 15.67/1.27 = 12.339

$L_{C/A}$ = P_{2c}/P_{2a} = 588/1.27 = 462.99

Base B:

$L_{A/B}$ = $[(P_{1a}/P_{1b})(P_{2a}/P_{2b})]^{1/2} = [(3.43/17.04)(1.27/15.67)]^{1/2}$ = 0.12773

$L_{B/B}$ = $[(P_{1b}/P_{1b})(P_{2b}/P_{2b})(P_{5b}/P_{5b})]^{1/3} = [(17.04/17.04)(15.67/15.67)(15.75/15.75)]^{1/3}$ = 1.000

$L_{C/B}$ = $[(P_{1c}/P_{1b})(P_{2c}/P_{2b})]^{1/2} = [(633/17.04)(588/15.67)]^{1/2}$ = 37.335

$L_{D/B}$ = $[(P_{1d}/P_{1b})(P_{5d}/P_{5b})]^{1/2} = [(9.57/17.04)(11.32/15.75)]^{1/2}$ = 0.63534

Base C:

$L_{A/C}$ = P_{2a}/P_{2c} = 1.27/588 = 0.00216

$L_{B/C}$ = $[(P_{2b}/P_{2c})(P_{3b}/P_{3c})]^{1/2} = [(15.67/588)(27.27/443)]^{1/2}$ = 0.04050

$L_{C/C}$ = $[(P_{2c}/P_{2c})(P_{3c}/P_{3c})]^{1/2} = [(588/588)(443/443)]^{1/2}$ = 1.000

$L_{D/C}$ = P_{3d}/P_{3c} = 9.95/443 = 0.02246

Base D:

$L_{A/D}$ = $[(P_{1a}/P_{1d})(P_{4a}/P_{4d})]^{1/2} = [(3.43/9.57)(2.25/10.22)]^{1/2}$ = 0.28090

$L_{B/D}$ = $[(P_{1b}/P_{1d})(P_{3b}/P_{3d})(P_{4b}/P_{4d})(P_{5b}/P_{5d})]^{1/4}$

= $[(17.04/9.57)(27.27/9.95)(20.93/10.22)(15.75/11.32)]^{1/4}$ = 1.9310

$L_{C/D}$ = $[(P_{1c}/P_{1d})(P_{3c}/P_{3d}(P_{4c}/P_{4d})]^{1/3} = [(633/9.57)(443/9.95)(755/10.22)]^{1/3}$ = 60.144

$L_{D/D}$ = $[(P_{1d}/P_{1d})(P_{3d}/P_{3d}(P_{4d}/P_{4d})(P_{5d}/P_{5d})]^{1/4}$

= $[(9.57/9.57)(9.95/9.95)(10.22/10.22)(11.32/11.32)]^{1/4}$ = 1.000

“Paasche” price ratio matrix

A		B		C		D	
$P_{A/A}$	1.000	$P_{A/B}$	0.08104	$P_{A/C}$	0.00216	$P_{A/D}$	- -
$P_{B/A}$	7.8300	$P_{B/B}$	1.000	$P_{B/C}$	0.02678	$P_{B/D}$	1.5740
$P_{C/A}$	462.99	$P_{C/B}$	24.690	$P_{C/C}$	1.000	$P_{C/D}$	44.523
$P_{D/A}$	3.5600	$P_{D/B}$	0.51785	$P_{D/C}$	0.16626	$P_{D/D}$	1.000

The "Paasche" price ratios are calculated by taking only the representative products of the partner country regardless of whether these products are also representative in the base country. They are obtained by taking the reciprocals of the transposed "Laspeyres" price ratios, as follows:

Base A:

$P_{A/A} = P_{2a}/P_{2a} = 1/L_{A/A} = 1.000$

$P_{B/A} = [(P_{1b}/P_{1a})(P_{2b}/P_{2a})]^{1/2} = 1/L_{A/B} = 7.8300$

$P_{C/A} = P_{2c}/P_{2a} = 1/L_{A/C} = 462.99$

$P_{D/A} = [(P_{1d}/P_{1a})(P_{4d}/P_{4a})]^{1/2} = 1/L_{A/D} = 3.5600$

Base B:

$P_{A/B} = P_{2a}/P_{2b} = 1/L_{B/A} = 0.08104$

$P_{B/B} = [(P_{1b}/P_{1b})(P_{2b}/P_{2b})(P_{5b}/P_{5b})]^{1/3} = 1/L_{B/B} = 1.000$

$P_{C/B} = [(P_{2c}/P_{2b})(P_{3c}/P_{3b})]^{1/2} = 1/L_{B/C} = 24.690$

$P_{D/B} = [(P_{1d}/P_{1b})(P_{3d}/P_{3b})(P_{4d}/P_{4b})(P_{5d}/P_{5b})]^{1/4} = 1/L_{B/D} = 0.51785$

etc.

“Fisher” price ratio matrix

A "Fisher" price ratio is the unweighted geometric mean of the "Laspeyres" price ratio and the "Paasche" price ratio.

Hence:

$F_{A/A} = [L_{A/A}.P_{A/A}]^{1/2} = [L_{A/A}/L_{A/A}]^{1/2} = [1.000/1.000]^{1/2} = 1.000$

$F_{B/A} = [L_{B/A}.P_{B/A}]^{1/2} = [L_{B/A}/L_{A/B}]^{1/2} = [12.339/0.12773]^{1/2} = 9.8286$

$F_{A/B} = [L_{A/B}.P_{A/B}]^{1/2} = [L_{A/B}/L_{B/A}]^{1/2} = [0.12773/12.339]^{1/2} = 0.10174$

$F_{C/A} = [L_{C/A}.P_{C/A}]^{1/2} = [L_{C/A}/L_{A/C}]^{1/2} = [462.99/0.00216]^{1/2} = 462.98$

$F_{A/C} = [L_{A/C}.P_{A/C}]^{1/2} = [L_{A/C}/L_{C/A}]^{1/2} = [0.00216/462.99]^{1/2} = 0.00216$

$F_{C/B} = [L_{C/B}.P_{C/B}]^{1/2} = [L_{C/B}/L_{B/C}]^{1/2} = [37.335/0.04050]^{1/2} = 30.362$

$F_{B/C} = [L_{B/C}.P_{B/C}]^{1/2} = [L_{B/C}/L_{C/B}]^{1/2} = [0.04050/37.335]^{1/2} = 0.03294$

$F_{D/B} = [L_{D/B}.P_{D/B}]^{1/2} = [L_{D/B}/L_{B/D}]^{1/2} = [0.63534/1.9310]^{1/2} = 0.57360$

etc.

A		B		C		D	
$F_{A/A}$	1.000	$F_{A/B}$	0.10174	$F_{A/C}$	0.00216	$F_{A/D}$	- -
$F_{B/A}$	9.8286	$F_{B/B}$	1.000	$F_{B/C}$	0.03294	$F_{B/D}$	1.7434
$F_{C/A}$	462.98	$F_{C/B}$	30.362	$F_{C/C}$	1.000	$F_{C/D}$	51.748
$F_{D/A}$	- -	$F_{D/B}$	0.57360	$F_{D/C}$	0.01932	$F_{D/D}$	1.000

Note that $F_{B/A}.F_{A/B} = 1$, $F_{C/A}.F_{A/C} = 1$, etc; that is, the above "Fisher" price ratios satisfy the country reversal test.

The elements of the above matrix, however, are not transitive; that is, $F_{B/A}/F_{C/A} \neq F_{B/C}$, $F_{A/B}/F_{C/B} \neq F_{A/C}$, etc. Neither is the matrix complete.

Completed "Fisher" price ratio matrix

Since some prices are missing, some "Laspeyres" and some "Paasche" ratios cannot be calculated. Consequently, the respective "Fisher" ratios also cannot be produced. The missing "Fisher" ratios have therefore to be estimated by calculating the geometric mean of all the available indirect "Fisher" ratios connecting (or bridging) the countries for which the ratios are missing.

Hence:

$F_{D/A} = [(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/2} = [(0.5736/0.10174)(0.01932/0.00216)]^{1/2} = 7.1013$

$F_{A/D} = [(F_{A/B}/F_{D/B})(F_{A/C}/F_{D/C})]^{1/2} = [(0.10174/0.5736)(0.00216/0.01932)]^{1/2} = 0.14082$

A		B		C		D	
$F_{A/A}$	1.000	$F_{A/B}$	0.10174	$F_{A/C}$	0.00216	$F_{A/D}$	0.14082
$F_{B/A}$	9.8286	$F_{B/B}$	1.000	$F_{B/C}$	0.03294	$F_{B/D}$	1.7434
$F_{C/A}$	462.98	$F_{C/B}$	30.362	$F_{C/C}$	1.000	$F_{C/D}$	51.748
$F_{D/A}$	7.1013	$F_{D/B}$	0.57360	$F_{D/C}$	0.01932	$F_{D/D}$	1.000

EKS matrix of transitive parities

Although certain elements of the completed "Fisher" matrix are transitive because of the way they are estimated, the elements of the original "Fisher" matrix are not. In order to ensure overall transitivity, the EKS (Eltetö-Köves-Szulc) method is used to obtain the final balanced parities.

Transitivity is achieved by replacing each direct ratio by the geometric mean of itself and all the corresponding indirect ratios obtained by using each of the other countries as a bridge:

Hence:

$EKS_{A/A} = F_{A/A} = 1.000$

$EKS_{B/A} = [(F_{B/A}/F_{A/A})(F_{B/B}/F_{A/B})(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(F_{B/A})^2(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(9.8286)^2(0.03294/0.00216)(1.7434/0.14082)]^{1/4} = 11.621$

$EKS_{C/A} = [(F_{C/A})^2(F_{C/B}/F_{A/B})(F_{C/D}/F_{A/D})]^{1/4}$

$= [(462.98)^2(30.362/0.10174)(51.748/0.14082)]^{1/4} = 391.56$

$EKS_{D/A} = [(F_{D/A})^2(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/4}$

$= [(7.1013)^2(0.5736/0.10174)(0.01932/0.00216)]^{1/4} = 7.1013$

$EKS_{A/B} = [(F_{A/B})^2(F_{A/C}/F_{B/C})(F_{A/D}/F_{B/D})]^{1/4}$

$= [(0.10174)^2(0.00216/0.03294)(0.14082/1.7434)]^{1/4} = 0.08605$

$EKS_{C/B} = [(F_{C/B})^2(F_{C/A}/F_{B/A})(F_{C/D}/F_{B/D})]^{1/4}$

$= [(30.362)^2(462.98/9.8286)(51.748/1.7434)]^{1/4} = 33.692$

$EKS_{D/B} = [(F_{D/B})^2(F_{D/A}/F_{B/A})(F_{D/C}/F_{B/C})]^{1/4}$

$= [(0.57360)^2(7.1013/9.8286)(0.01932/0.03294)]^{1/4} = 0.61106$

etc.

A	B	C	D
$EKS_{A/A}$ 1.000	$EKS_{A/B}$ 0.08605	$EKS_{A/C}$ 0.00255	$EKS_{A/D}$ 0.14082
$EKS_{B/A}$ 11.621	$EKS_{B/B}$ 1.000	$EKS_{B/C}$ 0.02968	$EKS_{B/D}$ 1.6365
$EKS_{C/A}$ 391.56	$EKS_{C/B}$ 33.692	$EKS_{C/C}$ 1.000	$EKS_{C/D}$ 55.136
$EKS_{D/A}$ 7.1013	$EKS_{D/B}$ 0.61106	$EKS_{D/C}$ 0.01813	$EKS_{D/D}$ 1.000

It can be easily shown that the EKS matrix is transitive because the ratios between the corresponding items in each respective country (that is, the ratios of the respective elements between any given pair of columns) are all equivalent.

$EKS_{A/C}$ = $EKS_{A/B}$ / $EKS_{C/B}$ = 0.08605/33.692 = 0.00255

$EKS_{A/C}$ = $EKS_{A/D}$ / $EKS_{C/D}$ = 0.14082/55.136 = 0.00255

$EKS_{B/C}$ = $EKS_{B/A}$ / $EKS_{C/A}$ = 11.621/391.56 = 0.02968

$EKS_{B/C}$ = $EKS_{B/D}$ / $EKS_{C/D}$ = 1.6365/55.136 = 0.02968

$EKS_{D/C}$ = $EKS_{D/A}$ / $EKS_{C/A}$ = 7.1013/391.56 = 0.01813

$EKS_{D/C}$ = $EKS_{D/B}$ / $EKS_{C/B}$ = 0.61106/33.692 = 0.01813

etc.

Thus, when all the elements in column A, for example, are multiplied by 0.08605 ($EKS_{A/B}$), they turn out to be just the same as the respective elements in column B (subject to minor rounding errors). Similarly, when the elements in column B are multiplied by 0.02968 ($EKS_{B/C}$) and the elements in column C are multiplied by 55.136 ($EKS_{C/D}$), they give rise to the equivalent elements in column C and D respectively.

$EKS_{A/B}$ = $EKS_{A/A}$ x $EKS_{A/B}$ = 1.000 x 0.08605 = 0.08605

$EKS_{B/B}$ = $EKS_{B/A}$ x $EKS_{A/B}$ = 11.621 x 0.08605 = 1.000

$EKS_{C/B}$ = $EKS_{C/A}$ x $EKS_{A/B}$ = 391.56 x 0.08605 = 33.692

$EKS_{D/B}$ = $EKS_{D/A}$ x $EKS_{A/B}$ = 7.1013 x 0.08605 = 0.61106

$EKS_{A/C}$ = $EKS_{A/B}$ x $EKS_{B/C}$ = 0.08605 x 0.02968 = 0.00255

$EKS_{B/C}$ = $EKS_{B/B}$ x $EKS_{B/C}$ = 1.000 x 0.02968 = 0.02968

$EKS_{C/C}$ = $EKS_{C/B}$ x $EKS_{B/C}$ = 33.692 x 0.02968 = 1.000

$EKS_{D/C}$ = $EKS_{D/B}$ x $EKS_{B/C}$ = 0.61106 x 0.02968 = 0.01813

etc.

Moreover, the factors used to convert B to C and C to D can be used indirectly to convert B to D and generate the same results as those obtained by using the direct coefficient.

$EKS_{A/D}$ = $EKS_{A/B}$ x $EKS_{B/D}$ = 0.08605 x 1.6365 = 0.14082

$EKS_{B/D}$ = $EKS_{B/B}$ x $EKS_{B/D}$ = 1.000 x 1.6365 = 1.6365

$EKS_{C/D}$ = $EKS_{C/B}$ x $EKS_{B/D}$ = 33.692 x 1.6365 = 55.136

$EKS_{D/D}$ = $EKS_{D/B}$ x $EKS_{B/D}$ = 0.61106 x 1.6365 = 1.000

etc.

EKS matrix of standardised parities

In the EKS matrix of transitive parities, the parities in each column are expressed with the corresponding country as a base. To obtain a set of standardised parities -- that is with the group of countries as a base -- each element of the matrix is divided by the geometric mean of its column's elements.

Hence:

$EKS_A = EKS_{A/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$
$= 1.000 / (1.000 \times 11.621 \times 391.56 \times 7.013)^{1/4} = 0.0746$

$EKS_B = EKS_{B/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$
$= 11.621 / (1.000 \times 11.621 \times 391.56 \times 7.013)^{1/4} = 0.8657$

$EKS_C = EKS_{C/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$
$= 391.56 / (1.000 \times 11.621 \times 391.56 \times 7.013)^{1/4} = 29.2159$

$EKS_D = EKS_{D/A} / (EKS_{A/A} \times EKS_{B/A} \times EKS_{C/A} \times EKS_{D/A})^{1/4}$
$= 7.1013 / (1.000 \times 11.621 \times 391.56 \times 7.013)^{1/4} = 0.5298$

etc.

	A	B	C	D
EKS_A	0.0746	0.0746	0.0746	0.0746
EKS_B	0.8657	0.8657	0.8657	0.8657
EKS_C	29.2159	29.2159	29.2159	29.2159
EKS_D	0.5298	0.5298	0.5298	0.5298

This provides the following vector of standardised parities.

	A	B	C	D
EKS	0.0746	0.8657	29.2159	0.5298

THE AGGREGATION OF BASIC HEADING PARITIES

Method of calculation

There are four stages:

- Calculation of Laspeyres and Paasche PPPs.
- Calculation of Fisher PPPs.
- Building the EKS matrix of transitive PPPs.
- Standardising the EKS matrix.

Parity matrix and value matrix

The parity matrix consist of the parities calculated by the EKS method for each of the basic headings as described in Part One. Each row comes from a separate standardised EKS vector. The first row comes from the standardised EKS vector in Part One.

Basic Heading	**Country**							
	A		B		C		D	
v	P_{va}	0.0746	P_{vb}	0.8657	P_{vc}	29.2159	P_{vd}	0.5298
w	P_{wa}	0.0731	P_{wb}	0.9504	P_{wc}	20.7252	P_{wd}	0.6945
x	P_{xa}	0.0739	P_{xb}	1.1382	P_{xc}	25.1295	P_{xd}	0.4730
y	P_{ya}	0.0695	P_{yb}	0.8758	P_{yc}	27.8034	P_{yd}	0.5908
z	P_{za}	0.0745	P_{zb}	0.7454	P_{zc}	26.8328	P_{zd}	0.6708

The value matrix contains expenditure values in national currencies by basic heading and by country.

Basic Heading	**Country**							
	A		B		C		D	
v	V_{va}	5	V_{vb}	110	V_{vc}	2000	V_{vd}	120
w	V_{wa}	20	V_{wb}	240	V_{wc}	5300	V_{wd}	180
x	V_{xa}	15	V_{xb}	300	V_{xc}	3500	V_{xd}	200
y	V_{ya}	35	V_{yb}	450	V_{yc}	10000	V_{yd}	250
z	V_{za}	25	V_{zb}	500	V_{zc}	6500	V_{zd}	250

The following shows how PPPs are calculated for three aggregates:

- Aggregate 1 = v + w
- Aggregate 2 = x + y + z
- Aggregate 3 = v + w + x + y + z (overall PPPs)

Laspeyres PPPs

The Laspeyres PPPs for Aggregate 1 are calculated as follows:

Base A:

$L1_{A/A} = [(P_{va}/P_{va})V_{va} + (P_{wa}/P_{wa})V_{wa}] / (V_{va} + V_{wa})$
$= [(0.0746/0.0746)5 + (0.0731/0.0731)20] / (5 + 20) = 1.000$

$L1_{B/A} = [(P_{vb}/P_{va})V_{va} + (P_{wb}/P_{wa})V_{wa}] / (V_{va} + V_{wa})$
$= [(0.8657/0.0746)5 + (0.9504/0.0731)20] / (5 + 20) = 12.722$

$L1_{C/A} = [(P_{vc}/P_{va})V_{va} + (P_{wc}/P_{wa})V_{wa}] / (V_{va} + V_{wa})$
$= [(29.2159/0.0746)5 + (20.7252/0.0731)20] / (5 + 20) = 305.12$

$L1_{D/A} = [(P_{vd}/P_{va})V_{va} + (P_{wd}/P_{wa})V_{wa}] / (V_{va} + V_{wa})$
$= [(0.5298/0.0746)5 + (0.6945/0.0731)20] / (5 + 20) = 9.021$

Base B:

$L1_{A/B} = [(P_{va}/P_{vb})V_{vb} + (P_{wa}/P_{wb})V_{wb}] / (V_{vb} + V_{wb})$
$= [(0.0746/0.8657)110 + (0.0731/0.9504)240] / (110 + 240) = 0.07982$

$L1_{B/B} = [(P_{vb}/P_{vb})V_{vb} + (P_{wb}/P_{wb})V_{wb}]/(V_{vb} + V_{wb})$
$= [(0.8657/0.8657)110 + (0.9504/0.9504)240]/(110 + 240) = 1.000$

$L1_{C/B} = [(P_{vc}/P_{vb})V_{vb} + (P_{wc}/P_{wb})V_{wb}] / (V_{vb} + V_{wb})$
$= [(29.2159/0.8657)110 + (20.7252/0.9504)240] / (110 + 240) = 25.560$

$L1_{D/B} = [(P_{vd}/P_{vb})V_{vb} + (P_{wd}/P_{wb})V_{wb}] / (V_{vb} + V_{wb})$
$= [(0.5298/0.8657)110 + (0.6945/0.9504)240] / (110 + 240) = 0.69342$

Base C:

$L1_{A/C} = [(P_{va}/P_{vc})V_{vc} + (P_{wa}/P_{wc})V_{wc}] / (V_{vc} + V_{wc})$
$= [(0.0746/29.2159)2000 + (0.0731/20.7252)5300] / (2000 + 5300) = 0.00326$

$L1_{B/C} = [(P_{vb}/P_{vc})V_{vc} + (P_{wb}/P_{wc})V_{wc}] / (V_{vc} + V_{wc})$
$= [(0.8657/29.2159)2000 + (0.9504/20.7252)5300] / (2000 + 5300) = 0.04141$

$L1_{C/C} = [(P_{vc}/P_{vc})V_{vc} + (P_{wc}/P_{wc})V_{wc}]/(V_{vc} + V_{wc})$
$= [(29.2159/29.2159)2000 + (20.7252/20.7252)5300]/(2000+5300) = 1.000$

$L1_{D/C} = [(P_{vd}/P_{vc})V_{vc} + (P_{wd}/P_{wc})V_{wc}] / (V_{vc} + V_{wc})$
$= [(0.5298/29.2159)2000 + (0.6945/20.7252)5300] / (2000 + 5300) = 0.02929$

Base D:

$L1_{A/D} = [(P_{va}/P_{vd})V_{vd} + (P_{wa}/P_{wd})V_{wd}] / (V_{vd} + V_{wd})$

$= [(0.0746/0.5298)120 + (0.0731/0.6945)180] / (120 + 180) = 0.11947$

$L1_{B/D} = [(P_{vb}/P_{vd})V_{vd} + (P_{wb}/P_{wd})V_{wd}] / (V_{vd} + V_{wd})$

$= [(0.8657/0.5298)120 + (0.9504/0.6945)180] / (120 + 180) = 1.47468$

$L1_{C/D} = [(P_{vc}/P_{vd})V_{vd} + (P_{wc}/P_{wd})V_{wd}] / (V_{vd} + V_{wd})$

$= [(29.2159/0.5298)120 + (20.7252/0.6945)180] / (120 + 180) = 39.9632$

$L1_{D/D} = [(P_{vd}/P_{vd})V_{vd} + (P_{wd}/P_{wd})V_{wd}]/(V_{vd} + V_{wd})$

$= [(0.5298/0.5298)120 + (0.6945/0.6945)180]/(120 + 180) = 1.000$

Aggregate 1							
A		B		C		D	
$L_{A/A}$	1.00000	$L_{A/B}$	0.07982	$L_{A/C}$	0.00326	$L_{A/D}$	0.11947
$L_{B/A}$	12.72200	$L_{B/B}$	1.00000	$L_{B/C}$	0.04141	$L_{B/D}$	1.47468
$L_{C/A}$	305.14160	$L_{C/B}$	25.55985	$L_{C/C}$	1.00000	$L_{C/D}$	39.96319
$L_{D/A}$	9.02092	$L_{D/B}$	0.69342	$L_{D/C}$	0.02929	$L_{D/D}$	1.00000

The Laspeyres PPPs for Aggregates 2 and 3 are similarly calculated.

Aggregate 2							
A		B		C		D	
$L_{A/A}$	1.00000	$L_{A/B}$	0.08412	$L_{A/C}$	0.00266	$L_{A/D}$	0.12631
$L_{B/A}$	12.29617	$L_{B/B}$	1.00000	$L_{B/C}$	0.03270	$L_{B/D}$	1.61381
$L_{C/A}$	374.75620	$L_{C/B}$	31.12659	$L_{C/C}$	1.00000	$L_{C/D}$	46.27289
$L_{D/A}$	8.24845	$L_{D/B}$	0.70255	$L_{D/C}$	0.02204	$L_{D/D}$	1.00000

Aggregate 3							
A		B		C		D	
$L_{A/A}$	1.00000	$L_{A/B}$	0.08318	$L_{A/C}$	0.00282	$L_{A/D}$	0.12426
$L_{B/A}$	12.40263	$L_{B/B}$	1.00000	$L_{B/C}$	0.03503	$L_{B/D}$	1.57207
$L_{C/A}$	357.35260	$L_{C/B}$	29.90886	$L_{C/C}$	1.00000	$L_{C/D}$	44.37999
$L_{D/A}$	8.44157	$L_{D/B}$	0.70055	$L_{D/C}$	0.02398	$L_{D/D}$	1.00000

Paasche PPPs

The Paasche PPPs are obtained by taking the reciprocals of the transposed Laspeyres indices:

Hence for Aggregate 1:

$P1_{A/A}$ = $1/L1_{A/A}$ = 1/1.0000 = 1.0000
$P1_{B/A}$ = $1/L1_{A/B}$ = 1/0.07982 = 12.52818
$P1_{C/A}$ = $1/L1_{A/C}$ = 1/0.00326 = 306.74850
$P1_{D/A}$ = $1/L1_{A/D}$ = 1/0.11947 = 8.37030

etc.

Aggregate 1							
A		B		C		D	
$P_{A/A}$	1.00000	$P_{A/B}$	0.07860	$P_{A/C}$	0.00327	$P_{A/D}$	0.11085
$P_{B/A}$	12.52818	$P_{B/B}$	1.00000	$P_{B/C}$	0.03912	$P_{B/D}$	1.44212
$P_{C/A}$	306.74850	$P_{C/B}$	24.14875	$P_{C/C}$	1.00000	$P_{C/D}$	34.14134
$P_{D/A}$	8.37030	$P_{D/B}$	0.67811	$P_{D/C}$	0.02502	$P_{D/D}$	1.00000

The Paasche PPPs for Aggregates 2 and 3 are similarly calculated.

Aggregate 2							
A		B		C		D	
$P_{A/A}$	1.00000	$P_{A/B}$	0.08132	$P_{A/C}$	0.00266	$P_{A/D}$	0.12123
$P_{B/A}$	11.88777	$P_{B/B}$	1.000	$P_{B/C}$	0.03212	$P_{B/D}$	1.42338
$P_{C/A}$	375.93990	$P_{C/B}$	30.58104	$P_{C/C}$	1.00000	$P_{C/D}$	45.37205
$P_{D/A}$	7.91514	$P_{D/B}$	0.61968	$P_{D/C}$	0.02161	$P_{D/D}$	1.00000

Aggregate 3							
A		B		C		D	
$P_{A/A}$	1.00000	$P_{A/B}$	0.08062	$P_{A/C}$	0.00279	$P_{A/D}$	0.11846
$P_{B/A}$	12.02212	$P_{B/B}$	1.00000	$P_{B/C}$	0.03343	$P_{B/D}$	1.42744
$P_{C/A}$	354.60990	$P_{C/B}$	28.54695	$P_{C/C}$	1.00000	$P_{C/D}$	41.70141
$P_{D/A}$	8.04764	$P_{D/B}$	0.63610	$P_{D/C}$	0.02253	$P_{D/D}$	1.00000

Fisher PPPs

The Fisher PPPs are the unweighted geometric means of the Laspeyres and Paasche PPPs.

Hence, for Aggregate 1:

$F1_{A/A}$ $= [L1_{A/A}.P1_{A/A}]^{1/2} = [L1_{B/A}/L1_{A/B}]^{1/2} = [1.0000/1.0000]^{1/2} = 1.0000$
$F1_{B/A}$ $= [L1_{B/A}.P1_{B/A}]^{1/2} = [L1_{B/A}/L1_{A/B}]^{1/2} = [12.7222/0.07982]^{1/2} = 12.6247$
$F1_{C/A}$ $= [L1_{C/A}.P1_{C/A}]^{1/2} = [L1_{C/A}/L1_{A/C}]^{1/2} = [305.14/0.00326]^{1/2} = 305.944$
$F1_{D/C}$ $= [L1_{D/A}.P1_{D/A}]^{1/2} = [L1_{D/A}/L1_{D/A}]^{1/2} = [9.02092/0.11947]^{1/2} = 8.68952$

etc.

Aggregate 1							
A		B		C		D	
$F_{A/A}$	1.00000	$F_{A/B}$	0.07921	$F_{A/C}$	0.00326	$F_{A/D}$	0.11508
$F_{B/A}$	12.62472	$F_{B/B}$	1.00000	$F_{B/C}$	0.04025	$F_{B/D}$	1.45831
$F_{C/A}$	305.94400	$F_{C/B}$	24.84428	$F_{C/C}$	1.00000	$F_{C/D}$	36.93774
$F_{D/A}$	8.68952	$F_{D/B}$	0.68572	$F_{D/C}$	0.02707	$F_{D/D}$	1.00000

The Fisher PPPs for Aggregates 2 and 3 are similarly calculated.

Aggregate 2							
A		B		C		D	
$F_{A/A}$	1.00000	$F_{A/B}$	0.08271	$F_{A/C}$	0.00266	$F_{A/D}$	0.12374
$F_{B/A}$	12.08670	$F_{B/B}$	1.000	$F_{B/C}$	0.03241	$F_{B/D}$	1.51561
$F_{C/A}$	375.34750	$F_{C/B}$	30.85261	$F_{C/C}$	1.00000	$F_{C/D}$	45.82027
$F_{D/A}$	8.08103	$F_{D/B}$	0.65980	$F_{D/C}$	0.02182	$F_{D/D}$	1.00000

Aggregate 3							
A		B		C		D	
$F_{A/A}$	1.00000	$F_{A/B}$	0.08189	$F_{A/C}$	0.00280	$F_{A/D}$	0.12133
$F_{B/A}$	12.21089	$F_{B/B}$	1.00000	$F_{B/C}$	0.03422	$F_{B/D}$	1.49801
$F_{C/A}$	355.97870	$F_{C/B}$	29.21996	$F_{C/C}$	1.00000	$F_{C/D}$	43.01985
$F_{D/A}$	8.24225	$F_{D/B}$	0.66755	$F_{D/C}$	0.02324	$F_{D/D}$	1.00000

Note that $F_{B/A}.F_{A/B} = 1$, $F_{C/A}.F_{A/C} = 1$, etc; that is, the above Fisher PPPs satisfy the country reversal test.

The elements of the above matrices, however, are not transitive; that is, $F_{B/A}/F_{C/A} \neq F_{B/C}$, $F_{A/B}/F_{C/B} \neq F_{A/C}$, etc. In order to ensure transitivity, the EKS method is used to obtain the final PPPs.

EKS PPPs

Transitivity is achieved by replacing each PPP by the geometric mean of itself and all the corresponding PPPs obtained by using each of the other countries as a bridge as follows:

Hence for Aggregate 1:

$EKS1_{A/A} = F_{A/A} = 1.00000$

$EKS1_{B/A} = [(F_{B/A}/F_{A/A})(F_{B/B}/F_{A/B})(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(F_{B/A})^2(F_{B/C}/F_{A/C})(F_{B/D}/F_{A/D})]^{1/4}$

$= [(12.62472)^2(0.04025/0.00326)(1.45831/0.11508)]^{1/4} = 12.56158$

$EKS1_{C/A} = [(F_{C/A})^2(F_{C/B}/F_{A/B})(F_{C/D}/F_{A/D})]^{1/4}$

$= [(305.944)^2(24.84428/0.07921)(36.93777/0.11508)]^{1/4} = 311.65390$

$EKS1_{D/A} = [(F_{D/A})^2(F_{D/B}/F_{A/B})(F_{D/C}/F_{A/C})]^{1/4}$

$= [(8.68952)^2(0.68572/0.07921)(0.02707/0.00326)]^{1/4} = 8.58025$

$EKS1_{A/B} = [(F_{A/B})^2(F_{A/C}/F_{B/C})(F_{A/D}/F_{B/D})]^{1/4}$

$= [(0.07921)^2(0.00326/0.04025)(0.11508/1.45831)]^{1/4} = 0.07961$

$EKS1_{C/B} = [(F_{C/B})^2(F_{C/A}/F_{B/A})(F_{C/D}/F_{B/D})]^{1/4}$

$= [(24.84428)^2(305.944/12.62472)(36.93774/1.45831)]^{1/4} = 24.81012$

$EKS1_{D/B} = [(F_{D/B})^2(F_{D/A}/F_{B/A})(F_{D/C}/F_{B/C})]^{1/4}$

$= [(0.68572)^2(8.68952/12.62472)(0.02707/0.04025)]^{1/4} = 0.68306$

etc.

Aggregate 1			
A	B	C	D
$EKS_{A/A}$ 1.00000	$EKS_{A/B}$ 0.07961	$EKS_{A/C}$ 0.00321	$EKS_{A/D}$ 0.11655
$EKS_{B/A}$ 12.56158	$EKS_{B/B}$ 1.00000	$EKS_{B/C}$ 0.04031	$EKS_{B/D}$ 1.46401
$EKS_{C/A}$ 311.65390	$EKS_{C/B}$ 24.81012	$EKS_{C/C}$ 1.00000	$EKS_{C/D}$ 36.32226
$EKS_{D/A}$ 8.58025	$EKS_{D/B}$ 0.68306	$EKS_{D/C}$ 0.02753	$EKS_{D/D}$ 1.00000

It can be easily shown that the EKS matrix is transitive because the ratios between the corresponding items in each respective country (that is, the ratios of the respective elements between any given pair of columns) are all equivalent.

$EKS_{A/C}$ = $EKS_{A/B}/EKS_{C/B}$ = 0.07961/24.81012 = 0.00321

$EKS_{A/C}$ = $EKS_{A/D}/EKS_{C/D}$ = 0.11655/36.32226 = 0.00321

$EKS_{B/C}$ = $EKS_{B/D}/EKS_{C/D}$ = 1.46401/36.322 = 0.04031

$EKS_{B/C}$ = $EKS_{B/A}/EKS_{C/A}$ = 12.56158/311.654 = 0.04031

$EKS_{D/C}$ = $EKS_{D/A}/EKS_{C/A}$ = 8.58025/311.654 = 0.02753

$EKS_{D/C}$ = $EKS_{D/B}/EKS_{C/B}$ = 0.68306/24.811 = 0.02753

etc.

Thus, when all the elements in column A, for example, are multiplied by 0.07961 ($EKS_{A/B}$), they turn out to be just the same as the respective elements in column B (subject to minor rounding errors). Similarly, when the elements in column B are multiplied by 0.04031 ($EKS_{B/C}$) and the elements in column C are multiplied by 36.32226 ($EKS_{C/D}$), they give rise to the equivalent elements in column C and D respectively.

$EKS_{A/B}$ = $EKS_{A/A}$ x $EKS_{A/B}$ = 1.00000 x 0.07961 = 0.07961

$EKS_{B/B}$ = $EKS_{B/A}$ x $EKS_{A/B}$ = 12.56158 x 0.07961 = 1.00000

$EKS_{C/B}$ = $EKS_{C/A}$ x $EKS_{A/B}$ = 311.6539 x 0.07961 = 24.81012

$EKS_{D/A}$ = $EKS_{D/A}$ x $EKS_{A/B}$ = 8.58025 x 0.07961 = 0.68306

$EKS_{A/C}$ = $EKS_{A/B}$ x $EKS_{B/C}$ = 0.07961 x 0.04031 = 0.00321

$EKS_{B/C}$ = $EKS_{B/B}$ x $EKS_{B/C}$ = 1.00000 x 0.04031 = 0.04031

$EKS_{C/C}$ = $EKS_{C/B}$ x $EKS_{B/C}$ = 24.81012 x 0.04031 = 1.00000

$EKS_{D/C}$ = $EKS_{D/B}$ x $EKS_{B/C}$ = 0.68306 x 0.04031 = 0.02753

etc.

Moreover, the factors used to convert B to C and C to D can be used indirectly to convert B to D and generate the same results as those obtained by using the direct coefficient.

$EKS_{A/D}$ = $EKS_{A/B}$ x $EKS_{B/D}$ = 0.07961 x 1.46401 = 0.11655

$EKS_{B/D}$ = $EKS_{B/B}$ x $EKS_{B/D}$ = 1.00000 x 1.46401 = 1.46401

$EKS_{C/D}$ = $EKS_{C/B}$ x $EKS_{B/D}$ = 24.81012 x 1.46401 = 36.32226

$EKS_{D/D}$ = $EKS_{D/B}$ x $EKS_{B/D}$ = 0.68306 x 1.46401 = 1.00000

etc.

The EKS matrices for Aggregates 2 and 3 are similarly calculated.

Aggregate 2							
A		B		C		D	
$EKS_{A/A}$	1.00000	$EKS_{A/B}$	0.08228	$EKS_{A/C}$	0.00268	$EKS_{A/D}$	0.12368
$EKS_{B/A}$	12.15297	$EKS_{B/B}$	1.00000	$EKS_{B/C}$	0.03253	$EKS_{B/D}$	1.50303
$EKS_{C/A}$	373.64330	$EKS_{C/B}$	30.74503	$EKS_{C/C}$	1.00000	$EKS_{C/D}$	46.21071
$EKS_{D/A}$	8.08565	$EKS_{D/B}$	0.66532	$EKS_{D/C}$	0.02164	$EKS_{D/D}$	1.00000

Aggregate 3							
A		B		C		D	
$EKS_{A/A}$	1.00000	$EKS_{A/B}$	0.08168	$EKS_{A/C}$	0.00281	$EKS_{A/D}$	0.12150
$EKS_{B/A}$	12.24222	$EKS_{B/B}$	1.00000	$EKS_{B/C}$	0.03439	$EKS_{B/D}$	1.48741
$EKS_{C/A}$	355.97210	$EKS_{C/B}$	29.07738	$EKS_{C/C}$	1.00000	$EKS_{C/D}$	43.24987
$EKS_{D/A}$	8.23058	$EKS_{D/B}$	0.67231	$EKS_{D/C}$	0.02312	$EKS_{D/D}$	1.00000

Standardised EKS PPPs

In the three EKS matrices, the PPPs in each column are expressed with the corresponding country as a base. To obtain a set of standardised PPPs -- that is, with the group of countries as a base -- each element of the matrices is divided by the geometric average of its column's elements.

Hence for Aggregate 1:

$EKS1_A = EKS1_{A/A} / (EKS1_{A/A} \text{ x } EKS1_{B/A} \text{ x } EKS1_{C/A} \text{ x } EKS1_{D/A})^{1/4}$

$= 1.000 / (1.000 \text{ x } 12.56158 \text{ x } 311.6539 \text{ x } 8.58025)^{1/4} = 0.0738$

$EKS1_B = EKS1_{B/A} / (EKS1_{A/A} \text{ x } EKS1_{B/A} \text{ x } EKS1_{C/A} \text{ x } EKS1_{D/A})^{1/4}$

$= 12.56158 / (1.000 \text{ x } 12.56158 \text{ x } 311.6539 \text{ x } 8.58025)^{1/4} = 0.9278$

etc.

Aggregate 1				
	A	B	C	D
EKS_A	0.0738	0.0738	0.0738	0.0738
EKS_B	0.9278	0.9278	0.9278	0.9278
EKS_C	23.0207	23.0207	23.0207	23.0207
EKS_D	0.6337	0.6337	0.6337	0.6337

The calculation is repeated for Aggregates 2 and 3.

Aggregate 2				
	A	B	C	D
EKS_A	0.0722	0.0722	0.0722	0.0722
EKS_B	0.8779	0.8779	0.8779	0.8779
EKS_C	26.9925	26.9925	26.9925	26.9925
EKS_D	0.5841	0.5841	0.5841	0.5841

Aggregate 3				
	A	B	C	D
EKS_A	0.0726	0.0726	0.0726	0.0726
EKS_B	0.8895	0.8895	0.8895	0.8895
EKS_C	25.8665	25.8665	25.8665	25.8665
EKS_D	0.5980	0.5980	0.5980	0.5980

The three standardised EKS matrices obtained can be reduced to three EKS vectors of standardised parities.

	A	B	C	D
Aggregate 1	0.0738	0.9278	23.0207	0.6337
Aggregate 2	0.0722	0.8779	26.9925	0.5841
Aggregate 3	0.0726	0.8895	25.8665	0.5980