# International comparisons of levels of capital input and productivity

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Table of contents



# 1. Introduction

International comparisons of levels of labour and capital inputs, outputs and productivity tend to receive a great deal of attention because they respond directly to policy-makers' and analysts' interest in measuring competitiveness, economic well-being of countries' inhabitants and the intensity by which resources are used. Generally, such level comparisons are more difficult to put in place than comparisons of growth rates: data sources are more susceptible to problems of international comparability (Ahmad et al. 2004), and spatial price indices are required to account for differences in the levels of input or output prices.

While the OECD has a long tradition of measuring comparative levels of GDP and labour productivity by way of its purchasing power parity programme (OECD 2005), there has been much less work to compare levels of capital input, levels of capital productivity and capital intensity. There are three reasons for this:

- Even at the national level, and in terms of rates of change, data on capital input has been much scarcer than data on labour input or data on output – at the level of the total economy and even more so at the level of individual industries.
- The Eurostat/OECD PPP programme is primarily designed to produce currency conversion rates at the level of total GDP. Currency conversion rates for investment goods have played a secondary role, also because they tend to be less reliable than PPPs for other expenditure categories such as private final consumption.
- Analytical emphasis is more often on measures of labour productivity and GDP per capita than on capital input and capital productivity.

Some recent developments have changed this picture:

- The OECD Productivity Database[1] now features a set of capital service measures for 18 OECD countries that are as comparable internationally as the data situation permits.
- In some countries, measures of capital input and capital intensity are followed closely in the policy debate. This is, for example, the case for New Zealand where questions have been posed about the 'hollowing-out' of the New Zealand economy and where comparative measures of capital input are of significant interest to analysts to make an informed statement about the capital intensity of the New Zealand economy.
- Additional methodological work has been undertaken on comparisons of productivity levels in a number of places, including at the OECD with a forthcoming handbook on the subject (van Ark 2005).

[1] See data and descriptions of the *OECD Productivity Database* under www.oecd.org/statistics/productivity.

- One of the stated outputs of the EU-KLEMS project is the development of comparative measures of productivity levels across countries, and this will give additional impetus to conceptual and data work.
- Several studies with level comparisons of productivity and capital have been published in recent years, in particular Jorgenson (2003), O'Mahony and de Boer (2002).

The present paper is a contribution to these efforts. It pursues three objectives: (i) clear specification of methodology used in a spatial comparison. The methodology itself is not new and relies on well-established economic and index number concepts, but the paper discusses some of the links between spatial and temporal comparisons; (ii) provision of point estimates of relative productivity and capital services, based on the OECD Productivity Database and the Eurostat/OECD PPP programme. As a first step, the comparison relates to seven, mostly non-European OECD countries; (iii) discussion of the statistical uncertainties surrounding level comparisons and determination of likely error margins by way of a simple Monte-Carlo simulation. The paper is thus statistical in nature and makes no claim to deal with the analysis of comparative levels of international productivity.

## 2. Bilateral and multilateral comparisons: concepts for comparisons

There is a large body of literature on the international comparison of volumes and prices of output and GDP. The international comparison of the levels of capital input has been less prominent in the methodological literature and partly this is because the principles that apply on the output side are directly transferable to the input side. Also, data availability often forces the analyst to use highly simplified assumptions by which conceptual questions about international comparisons are more or less defined away. For example, when labour inputs are measured as undifferentiated hours worked, it is straight forward to compare them across countries. Such an easy comparison is, however, only possible because it is assumed that every hour worked has exactly the same productive properties, independent of the experience, the educational attainment or the skill of workers, and independent of the country or the industry where it is delivered. O'Mahony and de Boer (2002), Jorgenson (2003) and Colecchia (forthcoming) are exceptions to this rule – they derive international comparisons of labour input measures that take account of the compositional change of the labour force.

Comparisons of capital input suffer sometimes from a deficiency similar to comparisons of labour input when no account is taken of the composition of capital inputs. The following section describes how such compositional effects can be incorporated into level comparisons of capital input.

## 2.1 A quantity index of capital services

We start by re-stating the measurement of capital services over time within a country or within an industry. Capital services are the flow of services by which capital goods contribute to production. It is typically assumed that, for each type of capital goods, the flow of capital services is proportional to the productive stock of the same type of capital good. The productive stock (see Box 1) reflects the productive capacity embodied in the available stock. Proportionality between the productive stock and the flow of capital services implies that the rate of change of capital services equals the rate of change of the productive stock of each asset. An overall index of capital services is derived by weighting the flow of each asset's capital services by its marginal productivity. Marginal productivity cannot be observed directly, but the theory of production tell us that the marginal productivity of an asset relative to the overall marginal productivity of capital equals each asset's share in the overall user costs of capital. The latter can be measured as the price that the owner of a capital good would charge for renting it out during one period. This provides a handle for the derivation of conceptually correct weights in a capital services index. The theoretical foundations of capital services measures are largely due to Jorgenson (1963, 1965, 1967) and Jorgenson and Griliches (1967). The necessary theory of index numbers and aggregation has been developed by

Diewert (1978, 1980) and this literature forms the basis for most empirical studies in capital measurement.

Capital measures in the OECD Productivity Database[2] are also based on these theoretical foundations and time series of capital input between period t and period t-1 in country j are derived as a Törnqvist index:

(1)
$$\ln\left(\frac{K_t^j}{K_{t-1}^j}\right)=\sum_s^N \overline{w}_s \ln\left(\frac{K_{s,t}^j}{K_{s,t-1}^j}\right)$$
$$\overline{w}_{s,t}=\tfrac{1}{2}\left(w_{s,t}^j+w_{s,t-1}^j\right)$$
$$w_{s,t}^j\equiv\frac{u_{s,t}^j K_{s,t}^j}{\sum_s^N u_{s,t}^j K_{s,t}^j};$$

In (1), $K_{s,t}^j$ stands for the productive stock of asset type s in country j at the beginning of period t, $u_{s,t}^j$ is period t user cost per unit of the productive stock of type s. $K_{s,t}^j$ is itself constructed with the perpetual inventory method, i.e., by aggregating across volumes of investment in past periods and by weighting each of these investment flows with a factor that reflects productive efficiency and retirements (see Annex for a description).

## *2.2 Bilateral comparisons*

The temporal Törnqvist index of capital input above has a strong theoretical basis[3] and one can directly build on these properties to develop a similar index for spatial comparisons of capital input. In principle, all that needs to be done is to substitute the time periods t and t-1 with country indices A and B in expression (1). This is the

[2] For details see Schreyer et al. (2003).

[3] The Törnqvist index is a superlative index number formula (Diewert 1976), i.e., an exact representation of a flexible aggregator function. In the present case, the underlying aggregator function is a cost function and the above index of capital input will be exact if we assume that the cost function is of the translog form, that capital markets are competitive and that producers minimise costs.

***translog bilateral input index*** $\gamma_t^{AB}$ as derived by Christensen et al. (1981) and by Caves et al. (1982):

$$\ln\gamma_t^{AB} = \sum_s^N \overline{w}_{s,t}^{AB} \ln\left(\frac{K_{s,t}^A}{K_{s,t}^B}\right)$$

(2) $$\overline{w}_{s,t}^{AB} = \tfrac{1}{2}\left(w_{s,t}^A + w_{s,t}^B\right)$$

$$w_{s,t}^j \equiv \frac{u_{s,t}^j K_{s,t}^j}{\sum_s^N u_{s,t}^j K_{s,t}^j};\ j = A, B.$$

**Box 1: Different measures of capital stock – an overview**

Production theory stipulates that there is a flow of productive services from the cumulative stock of past investments. This flow of productive services from a particular type of asset is called *capital services* and constitutes the conceptually correct measure of capital input for production and productivity analysis. Capital services reflect a quantity concept, not to be confused with the value, or price concept of capital. Productive services of an asset are typically taken as a proportion of the productive stock of the same asset where the *productive stock* reflects the productive capacity of capital. Empirically, the productive stock is obtained by cumulating investment flows of a particular type of asset, and correcting them for asset retirement and the loss in productive efficiency due to ageing.

Aggregation across different assets is obtained by valuing each type of asset with its user costs or rental prices. This valuation is designed to capture the marginal productivity of different assets when used in production.

Whereas measures of the productive stock are set up to capture the productive capacity of capital goods, and by implication the flow of capital services, the wealth (net) stock measures the market value of capital assets. Conceptually, the more familiar *net capital stock* is synonymous to the wealth capital stock. 'Wealth stock' is sometimes considered a more precise terminology, however, because there are other forms of 'net' stock, in particular the productive stock which is the gross stock 'net' of efficiency declines in productive assets. Empirically, the wealth stock is obtained by cumulating investment flows of a particular type of asset, and correcting them for asset retirement and depreciation, the loss in asset value due to ageing. When depreciation proceeds at a constant rate, the rate of depreciation in the wealth stock and the rate of efficiency loss in the productive stock coincide and the wealth stock equals the productive stock *at the level of individual types of assets but not at higher levels of aggregation.*

Aggregation of the wealth stock across different assets is obtained by valuing each type of asset with its current replacement value. This valuation is designed to capture the market value of assets that are used in production.

The *gross capital stock* is the cumulative flow of investments of a particular asset, corrected for asset retirement. The gross stock constitutes an intermediate step in the computation of the productive stock that takes account of the withdrawal of assets but does not correct the assets in operation for their loss in productive capacity due to ageing.

For further discussion and references on capital measures see OECD (2001).

The theoretical formulation in (2) implicitly assumes that at the level of individual assets, inputs are measured in physical units and that they can therefore be directly compared across countries. In practice, this is not the case and stocks of asset groups are expressed in national currency units of some base year such as 'constant 1995 dollars', reflecting the fact that that individual asset types are aggregations across similar sub-types of assets rather than truly homogenous investment goods that could be expressed in physical units. Thus, country A's productive stock of asset type s $K^{A}_{s,t}$ is measured in currency units of country A and consequently not comparable to $K^{B}_{s,t}$, expressed in currency units of country B. More specifically, the underlying valuation is in terms of investment goods prices of a base period. This base period for the underlying investment goods price index may or may not coincide with the year of the spatial comparison. We use the asset-specific price index and express each asset's productive stock at replacement costs of the comparison period. Finally, to make the productive stocks of countries A and B comparable, the purchasing power parity for investment good of type s, $q^{A}_{s,t}/q^{B}_{s,t}$ has to be applied to (2) to obtain:

(3) $$\ln \gamma^{AB}_{t} = \sum_{s}^{N} \overline{w}^{AB}_{s,t} \ln\left( \frac{K^{A}_{s,t}}{K^{B}_{s,t}} \frac{q^{B}_{s,t}}{q^{A}_{s,t}} \right).$$

The extension to an index of capital productivity is straightforward. We define a ***bilateral Törnqvist index of capital productivity***[4] as

(4) $$\ln \theta^{AB} = \ln \lambda^{AB} - \ln \gamma^{AB}$$

where $\lambda^{AB}$ is the volume of output in country A relative to country B. We skip the presentation of a theoretical Törnqvist quantity index of output here because in our

[4] The time subscript t has been dropped here to facilitate notation.

applications we use a readily-available indirect quantity index of GDP, obtained by dividing money values of GDP in the various countries by the OECD/Eurostat PPPs, i.e., by a spatial price index. This spatial deflation yields comparable volume indices of GDP (see also Box 2).

The index of capital productivity in (4), can be compared with an index of labour productivity. In principle, labour input should be gauged with a method that is exactly parallel to the measure of capital input, i.e., by aggregating across different types of labour taking into account the relative skills, qualifications and educational attainment of the labour force. Presently, the necessary data for such a differentiation is, however, not available and we have to content ourselves with a measure of labour input that reflects total but undifferentiated hours worked. Letting $H_t^A$ be the number of total hours in country A and period t and letting $H_t^B$ be the number of total hours in country B and period t, a ***bilateral index of labour input*** and a ***bilateral index of labour productivity*** are defined as:

(5) $$\ln h^{AB} = \ln\left(\frac{H^A}{H^B}\right)$$

(6) $$\ln \pi^{AB} = \ln \lambda^{AB} - \ln h^{AB}.$$

It is now a small step towards deriving an index of multifactor productivity (MFP). A bilateral index of MFP shows the difference in output between two countries that cannot be attributed to differences in the number of hours worked or to differences in capital input. Akin to the computation over time, MFP is a residual, obtained by weighting relative labour and capital inputs and adjusting relative outputs for relative inputs. Alternatively, MFP can be described as a weighted average of labour and capital productivity, where each of the two partial productivity measures are weighted by the respective share of labour and capital in total costs. For the purpose at hand, we shall choose the latter avenue and define a ***bilateral index of multifactor productivity*** $\mu^{AB}$ as:

(7)
$$\ln(\mu^{AB}) = \bar{v}^{AB} \ln \pi^{AB} + (1-\bar{v}^{AB}) \ln \theta^{AB}$$
$$\bar{v}^{AB} = \tfrac{1}{2}\left(v^{A} + v^{B}\right) \quad .$$
$$v^{j} = p_w^{j} H^{j} \Big/ \left(p_w^{j} H^{j} + \sum\nolimits_{s=1}^{N} u_s^{j} K_s^{j}\right) \quad j = A, B.$$

In (7), $v^A$ is the share of labour compensation $p_w^A H^A$ in the total compensation of labour and capital $p_w^A H^A + \sum_{s=1}^{N} u_s^A K_s^A$. Similarly, $v^B$ is country B's labour share and $\bar{v}^{AB}$ is the average share between the two countries.

**Box 2: PPPs and direct volume indices**

In the present paper, comparisons of capital input are based on a quantity index specified in (2). An alternative to the (direct) quantity index is to develop a price index which is then used to deflate the values of productive stocks in the countries under consideration. The two methods are equivalent when Fisher Ideal index numbers are used and they are approximately equivalent (Diewert 1978) when Törnqvist indices are used.

Thus, an international comparison of capital inputs could also be carried out by defining an international price index of capital services, or a capital services PPP. This is the method that has for example been used by Jorgenson and Nishimizu (1978) or Jorgenson and Kuroda (1995). In the bilateral case, with countries A and B, such an index would take the form

$$\ln \rho^{AB} = \sum\nolimits_{s=1}^{N} \bar{w}^{AB} \ln\left(\frac{P_s^A}{P_s^B}\right)$$

where $P_s^A$ is the price of asset type s in country A, $P_s^B$ is the price of asset type s in country B and $\bar{w}_s^{AB}$ are the average weights of each asset in total user costs. Because asset prices are denominated in national currencies, the units of this price index are currency units such as dollars per euro. Thus, the international price index above has the properties of a currency conversion rate, or PPP. When applied to the ratio of nominal productive stocks or the value of capital services in each country ($\frac{\sum_{s=1}^{N} u_s^A K_s^A}{\sum_{s=1}^{N} u_s^B K_s^B}$), one obtains an indirect quantity index of capital input, approximately equal to the direct quantity index $\gamma^{AB}$ defined in the main text:

$$\tilde{\tilde{\gamma}}^{AB} = \frac{\sum_{s=1}^{N} u_s^A K_s^A}{\sum_{s=1}^{N} u_s^B K_s^B} \Big/ \rho^{AB} \approx \gamma^{AB}.$$

The same reasoning holds for the output side. Essentially, OECD/Eurostat PPPs are derived as Fisher Ideal price indices on the basis of detailed price comparisons in OECD countries. For any pair of countries, price relatives are weighted with the expenditure structure of each countries and a geometric mean is taken. Bilateral PPPs are transformed into transitive multilateral PPPs . When the PPP price index is applied to the ratio of current price GDP of two countries (a value index), this yields an indirect quantity or volume index that we apply throughout the present comparison.

## *2.3 Multilateral comparisons*

Bilateral comparisons, when applied to more than two countries, have the disadvantage of intransitivity, i.e., in general $\gamma^{AB} * \gamma^{BC} \neq \gamma^{AC}$. A number of techniques exist to obtain transitivity. For the purpose at hand, we apply the Caves et al. (1982) method: transitivity in a multilateral context is achieved by defining the capital input of country i relative to the capital input of all N countries as the geometric mean of the bilateral input comparisons between i and each of the countries:

(8) $\ln \overline{\gamma}^{i} = \frac{1}{N}\sum_{k=1}^{N} \ln \gamma^{ik}$

The ***multilateral Törnqvist index of capital inputs*** $\tilde{\gamma}^{ij}$ is defined as

(9) $\ln \tilde{\gamma}^{ij} = \ln \overline{\gamma}^{i} - \ln \overline{\gamma}^{j}$.

It is not difficult to verify that this index is transitive. If a spatial index of outputs had been constructed in the present exercise, the same method would have been applied to achieve transitivity. There is no need for a particular adjustment here, however, because the OECD/Eurostat PPPs that enter the calculations have already been made transitive by a similar procedure[5] to the one described in (8) and (9)

The ***multilateral Törnqvist index of capital productivity*** $\theta^{ij}$ is defined as

(10) $\ln \tilde{\theta}^{ij} = \ln \lambda^{ij} - \ln \tilde{\gamma}^{ij}$.

[5] The OECD/Eurostat Purchasing Power Parities Programme uses the Eltetö and Köves (1964) and Szulc (1964) "EKS" method to derive their spatial deflators. The EKS method reaches transitivity by a transformation that is identical to the one in equation (8), the only difference being that the EKS method uses a Fisher Ideal index number formula whereas we have used a Törnqvist formula.

Because the index of labour input is one-dimensional (the only unit are hours worked), no issue of transitivity arises and we can immediately define the ***multilateral index of labour productivity*** as:

(11) $\ln \pi^{ij} = \ln \lambda^{ij} - \ln h^{ij}$.

Finally, to compute a multilateral Törnqvist index of multi-factor productivity $\tilde{\mu}^{ij}$ we construct the geometric mean of the bilateral MFP comparisons between i and each of the N countries:

(12) $\ln \bar{\mu}^{i} = \frac{1}{N}\sum_{k=1}^{N} \ln \mu^{ik}$

The ***multilateral Törnqvist index of multi-factor productivity*** $\tilde{\mu}^{ij}$ is defined as

(13) $\ln \tilde{\mu}^{ij} = \ln \bar{\mu}^{i} - \ln \bar{\mu}^{j}$.

# 3. Results

The empirical productivity measures developed in the present paper all relate to the total economy. This reflects data constraints more than a choice. Preferably, computations would also single out the corporate or business sector as well as individual industries. However, neither capital input measures nor hours worked are easily available in such a sectoral breakdown and calculations remain at the aggregate level, in line with the data available from the *OECD Productivity Database*[6].

We start by reproducing the set of data on output and hours worked for 2002 that forms the basis for the measurement of relative labour productivity levels. Of the seven countries under consideration, only France exceeds the labour productivity level of the

[6] www.oecd.org/statistics/productivity

United States (see Pilat 2005 for a discussion on the link between labour productivity and GDP per capita).

Table 1: Levels of GDP, hours worked and labour productivity in 2002
USA = 100

| Multilateral index of: | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| GDP at 2002 PPPs | 5.4 | 8.9 | 16.5 | 33.2 | 0.8 | 16.5 | 100.0 |
| Hours worked | 6.8 | 10.9 | 15.3 | 46.8 | 1.4 | 19.2 | 100.0 |
| Labour productivity | 79.7 | 81.7 | 108.3 | 70.9 | 61.1 | 85.9 | 100.0 |

Source: *OECD Productivity Database* .

Table 2 below shows multilateral indices of capital services, capital intensity and capital productivity. As outlined in the methodological section, indices of capital services differ from indices based on net or gross capital stocks insofar as different assets are weighted with their share in total user costs. User costs are designed to capture the marginal productivity of assets so that high productivity assets receive larger weights. Typically, short-lived assets such as information and communication products fall under this category because short service lives and rapid price declines require high marginal productivity while such assets are in operation. Consequently, indices of capital services will tend to be higher for those countries whose investment and capital stock structure is biased towards high-productivity, short-lived capital goods relative to other countries. While the indices of capital services reflect also each country's size, indices of capital intensity and capital productivity are normalised by labour input and output. One notes considerable differences in capital intensity (i.e., capital services per hour worked) between countries.

Table 2: Levels of capital input, capital intensity and capital productivity in 2002
USA=100

| Multilateral index of: | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Capital services | 5.3 | 8.1 | 17.6 | 62.4 | 0.7 | 12.0 | 100.0 |
| Capital intensity | 77.7 | 74.3 | 115.6 | 133.3 | 48.6 | 62.6 | 100.0 |
| Capital productivity | 102.6 | 110.0 | 93.7 | 53.2 | 125.5 | 137.3 | 100.0 |

Source: *OECD Productivity Database and author's calculations* .

Checking the results against similar studies, we find that our results are in the same order of magnitude as O'Mahony and de Boer (2002) as far as the relative capital intensities between the United Kingdom, France and the United States are concerned[7] and considering that O'Mahony and de Boer use a measure of net capital stock, and it should be expected that they differ from capital input measures based on a concept of capital services. The bilateral results for Canada and the United States also seem to be roughly[8] in line with the relative capital intensity computed by Rao et al. (2003). Kondo et al. (2000) compare Japanese economy-wide levels of capital productivity and capital intensity with those of the United States and also find a sizeable gap in capital productivity between Japan and the United States, combined with a higher level of capital input per hour in Japan compared to the United States[9].

Jorgenson (2003) finds a much lower capital intensity for Japan and a significantly higher capital productivity measure than we do although it is difficult to assess to which extent differences in methods and scope could account for these

[7] With the UK=100, our capital per hour ratios are 159 for the USA, and 184 for France in the year 2002. This compares with O'Mahony's and de Boer (2002) values of 146 for the USA and 177 for France in the year 1999.

[8] Rao et al. (2003) compute relative a capital intensity of 95% for the year 2000 between Canada and the United States. However, their calculation relates to the business sector, and not to the total economy as in our study. Furthermore, the authors use a measure of the net stock in 1997 dollars, and so differ from the capital services concept used in the present study. If the USA has a relatively larger share in short-lived ICT capital, this would explain why our measure of capital intensity shows a relatively higher value for the USA than the measure obtained by Rao et al. (2003).

[9] Kondo et al. (2000) show a bilateral index of capital productivity between Japan and the United States of 61% for the year 1999 and a bilateral index of capital intensity of 113%.

differences in results[10]. We have also been unable to match our bilateral measures of capital intensity with those available from the Database of ICT Investment and Capital Stock Trends of the *Centre for the Study of Living Standards*[11] although there are also a number of differences in concept and scope. Nonetheless, some further investigation will be necessary to account for these differences.

Table 3 and Figure 1 exhibit multilateral indices of labour, capital and multifactor productivity for the year 2002. France comes out with the highest level of multifactor productivity, the main driver behind its high level of labour productivity. Japan shows up with a relatively low level of multifactor, labour and capital productivity. As will be argued in the section on robustness below, these figures should not be over-interpreted. Comparisons with other studies are difficult. Rao et al. (2003) find a similar MFP ratio for Canada vis-à-vis the United States but use a different concept of capital input. Jorgenson (2003) uses a constant quality measure of labour input whereas we use a simple measure of hours worked, which makes the comparison of the productivity residual difficult. O'Mahony and de Boer (2002) make a similar adjustment for labour composition as Jorgenson but do not adjust for capital composition. They find a larger productivity gap between the UK and the United States than we do and a smaller difference between the UK and France.

Table 3 Capital, labour and multifactor productivity in 2002

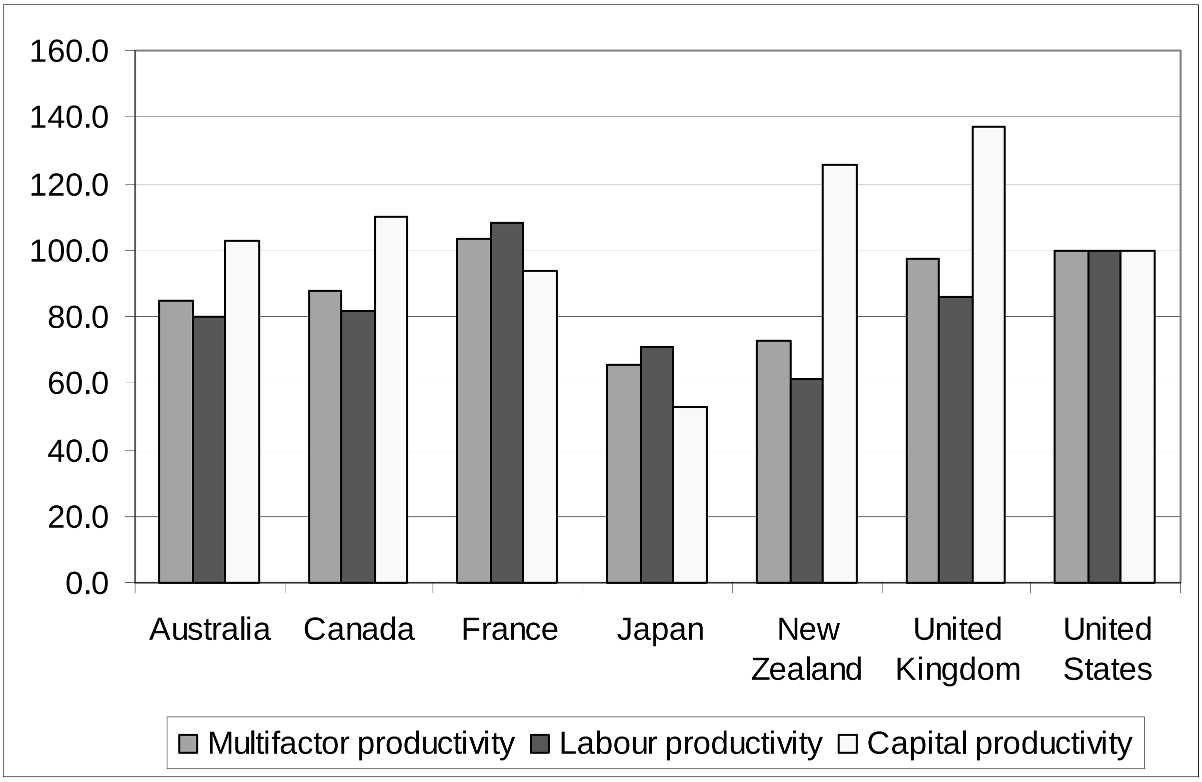
| Multilateral index of: | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Multifactor productivity | 85.0 | 88.0 | 103.5 | 65.5 | 72.8 | 97.2 | 100.0 |
| Labour productivity | 79.7 | 81.7 | 108.3 | 70.9 | 61.1 | 85.9 | 100.0 |
| Capital productivity | 102.6 | 110.0 | 93.7 | 53.2 | 125.5 | 137.3 | 100.0 |

Source: *OECD Productivity Database and author's calculation* .

[10] Although Jorgenson (2003) does not directly report capital intensity and capital productivity ratios, it is straight forward to derive them from his summary tables. For the year 2001, one obtains a capital intensity index of about 55% for Japan relative to the United States and a capital productivity index of 125%. Allowance must, however, be made for the fact that the calculations in the present paper relate only to non-residential capital inputs whereas Jorgenson (2003) has a broader notion of capital encompassing also consumers' durables, land and residential buildings.

[11] Available from http://www.csls.ca/. In Table S32 of the database, the non-residential capital stock per worker in the Canadian business sector shows up with a 131% value over the corresponding U.S. figure.

Figure 1: Capital, labour and multifactor productivity in 2002
USA=100




# 4. Robustness of results

Many uncertainties prevail in the measurement of capital input, in the measurement of output and in the measurement of PPPs and the results shown above should be interpreted with a good deal of caution. For example, Ahmad et al. (2003) have estimated that level comparisons of GDP may well be subject to an error margin of several percentage points. OECD/Eurostat PPPs for GDP, while based on several thousand price observations, are nonetheless subject to statistical noise and a rule of thumb sets a 5 percentage point margin within which it may be difficult to make reliable statements about significant differences between countries' volume GDP per capita. Capital services measures, in particular when constructed at the international level, are also subject to error margins, partly because some of the underlying investment series may have been estimated, in particular for early periods. A particular issue in this context is the choice of an initial productive stock for non-residential structures: an assumed service life of 60 years would require investment series for structures from the 1920 to obtain an estimate of the productive stock in the mid-1980s. Such data are not available at the international level, and some rather simplifying assumptions have been made to establish a starting value for the stock of non-residential structures. Overall, then, there are good reasons to believe that level comparisons of capital and labour input are subject to measurement error and this part of the paper aims at establishing some bounds for such errors.

We proceed with a very simple Monte-Carlo simulation. Starting point is the assumption that the following variables are subject to measurement error: GDP, PPPs, hours actually worked, and capital services. More specifically, we assume that the observations of each of these variables are randomly distributed around their true value. Based on our point estimates for each variable, we generate a set of observations that enter the productivity level computations. For example, we generate $GDP_s^i$, i.e., an observation s for country i's GDP by the relationship $GDP_s^i = GDP^i(1+\varepsilon)$ where $GDP^i$ is the value for country i's GDP from the national accounts and $\varepsilon$ is an independently and normally distributed error variable with mean zero and a standard deviation of 0.02. In other words, we generate a set of data with the property that the GDP estimate from the national accounts is the most probable realisation and that there is a near 99% probability that the randomly generated observation for GDP lies within a range of 5% below and 5% above their mean, the observed value from the national accounts. A plus/minus 5% margin is generous and probably overstates the true likelihood of measurement errors. But we prefer to err on the high side than to evoke too optimistic a picture of precision in economic measurement.

Similar assumptions as for GDP levels are made for the other variables and a set of 100 artificial observations of GDP, PPPs, hours worked and productive stock for every asset and every country is generated for the year 2002.

For each of the 100 observations, we compute multilateral indices of capital services, labour and capital productivity, capital intensity and MFP. We compute the average and standard deviation of all observations to obtain statistical bounds[12] for the estimates at hand. Upper and lower bounds confine the area that contains 99% of all outcomes given the error structure that underlies the Monte Carlo experiment. They are shown in the following tables and graph.

Upper and lower bounds provide an order of magnitude for the uncertainties involved in the estimation of international indicators. The bracket for MFP estimates, for example, comprises up to 10 percentage points around the point estimate. In the case of the United Kingdom this means that on the basis of our data with their assumed observation error of +/-5%, there is a 99% probability that MFP relative to the United States may be situated somewhere between 89.6% and 103.3% (Table 7). Or labour productivity for France can be located somewhere between 99% and 114% of the U.S. level – a particularly large range. These boundaries once more show that precise rankings of countries may be difficult to obtain and in general should not be undertaken when countries are clustered around similar values of indices of productivity or per capita income – a point that has also been made in the context of the OECD/Eurostat PPP programme (OECD 2004).

Table 4: Upper and lower bounds for labour productivity in 2002
USA=100

| | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Upper bound | 85.5 | 87.7 | 115.5 | 76.0 | 64.9 | 92.0 | 100.0 |
| Point estimate | 79.7 | 81.7 | 108.3 | 70.9 | 61.1 | 85.9 | 100.0 |
| Lower bound | 73.7 | 76.1 | 101.0 | 65.6 | 56.9 | 79.7 | 100.0 |

Source: *OECD Productivity Database and author's calculation* .

[12] Given the various transformations that are necessary to obtain multilateral indices, it is not possible to demonstrate that, as a consequence of assuming normal distribution of the errors for the base data, the multilateral indices are also normally distributed. However, we apply a Jarque-Bera test to check for normality of the results generated by the Monte Carlo simulation and find that for virtually all variables the null hypothesis of a normal distribution cannot be rejected. This permits the construction of confidence intervals on the basis of normal distributions.

Table 5: Upper and lower bounds for capital intensity in 2002
USA=100

| | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Upper bound | 84.2 | 87.7 | 125.9 | 145.2 | 53.2 | 68.3 | 100.0 |
| Point estimate | 77.7 | 74.3 | 115.6 | 133.3 | 48.6 | 62.6 | 100.0 |
| Lower bound | 71.9 | 76.1 | 106.8 | 122.9 | 44.6 | 57.4 | 100.0 |

Source: *OECD Productivity Database and author's calculation* .

Table 6: Upper and lower bounds for capital productivity in 2002
USA=100

| | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Upper bound | 113.7 | 121.0 | 102.7 | 58.9 | 137.9 | 152.2 | 100.0 |
| Point estimate | 102.6 | 110.0 | 93.7 | 53.2 | 125.5 | 137.3 | 100.0 |
| Lower bound | 90.5 | 98.9 | 83.6 | 46.9 | 111.6 | 121.4 | 100.0 |

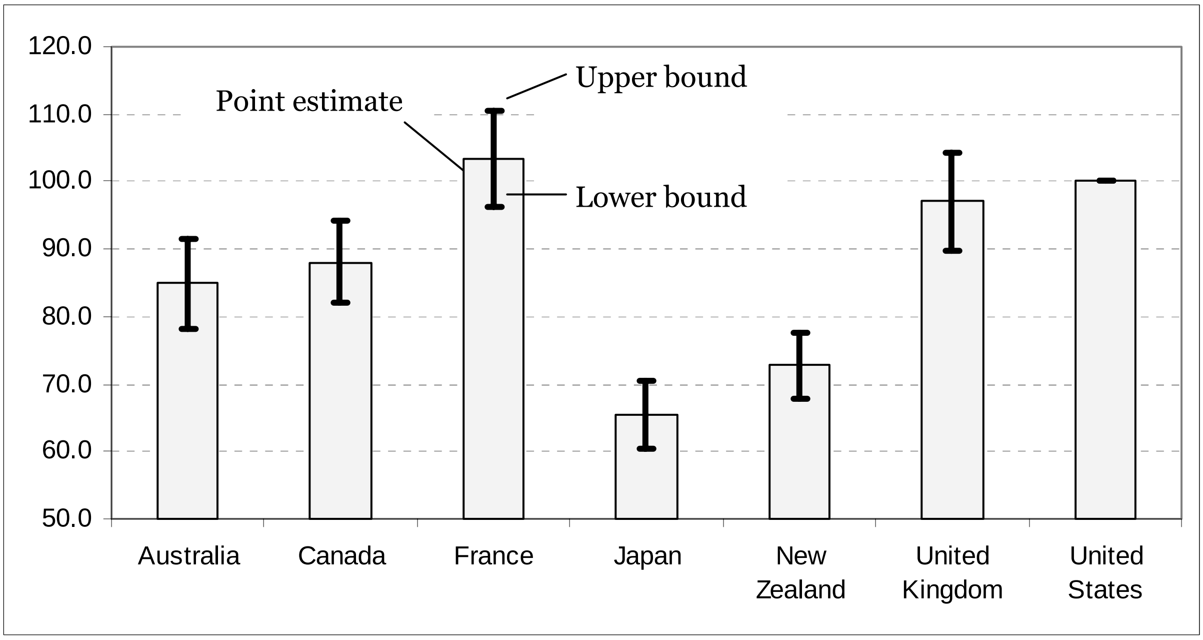
Source: *OECD Productivity Database and author's calculation* .

Table 7: Upper and lower bounds for multi-factor productivity in 2002
USA=100

| | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Upper bound | 91.5 | 94.3 | 110.5 | 70.3 | 77.4 | 104.4 | 100.0 |
| Point estimate | 85.0 | 88.0 | 103.5 | 65.5 | 72.8 | 97.2 | 100.0 |
| Lower bound | 78.1 | 82.0 | 96.3 | 60.3 | 67.7 | 89.8 | 100.0 |

Source: *OECD Productivity Database and author's calculation* .

Figure 2 Upper and lower bounds for multi-factor productivity in 2002
USA=100




## 5. Time-space comparisons

To this point, all comparisons have related to a single point in time, the year 2002. It is, however, of considerable interest to combine spatial and temporal comparisons to make statements about patterns of convergence or divergence of partial and multi-factor productivity. Combined temporal-spatial comparisons raise additional methodological issues. Probably the most apparent issue is the inconsistency that typically arises in general between the growth rates of capital input (or output or any other variable for that matter) that are implied by the comparisons of two spatial indices in time and the growth rates of productivity when computed directly.

For example, let the capital input of country A relative to country B be 100% in year t and 102% in year t+1, where these values have been computed in line with equation (2). The implicit relative growth of capital input in country A over country B would be computed as 2%. There is no reason to expect that the rate of change of capital input, computed directly over time axis, in line with equation (1) should yield the same 2% result. The reason for the difference lies in the weighting pattern. A look at equations

(1) and (2) shows that the former has the average user cost shares of country A and B as weights for the spatial index whereas the latter has the average user cost shares of country A in two periods as weights. Thus, country A's direct temporal index is only governed by weights that relate to country A. In contrast, country A's temporal index that is implicit in two spatial comparisons, is also dependent on the weights in country B. Consequently, the two temporal results may differ.

Differences will tend to become more important, if there are large shifts in spatial weights over time and/or when points of comparisons are far apart[13]. In practice, results from on spatial comparison are often extrapolated backwards or forwards by using direct temporal rates of change and so force consistency in time and space. This comes, however, at the cost of a possible bias because spatial weights will have been kept fixed at the benchmark year and it is well-known that fixed weights may create substitution biases in index number formulae.

We present two sets of results of multilateral multi-factor productivity measures for 1995: (i) one set that has been computed with the 1995 spatial weights; (ii) another set that has been computed by extrapolating 2002 indices to 1995, using the temporal rates of productivity change. Differences play out most visibly in the case of France where the level of MFP, computed directly with asset shares of the year 1990 shows a productivity level for France that exceeds that for the United States by 7%. On the other hand, the productivity level for 1990, obtained by applying the growth rate of MFP 1990-02 to France's MFP level of 2002, yields a value that is only one percent above the value of the United States. The implication is that the structure of user costs of different assets in France vis-à-vis other countries was quite different in 1990 from the structure in 2002.

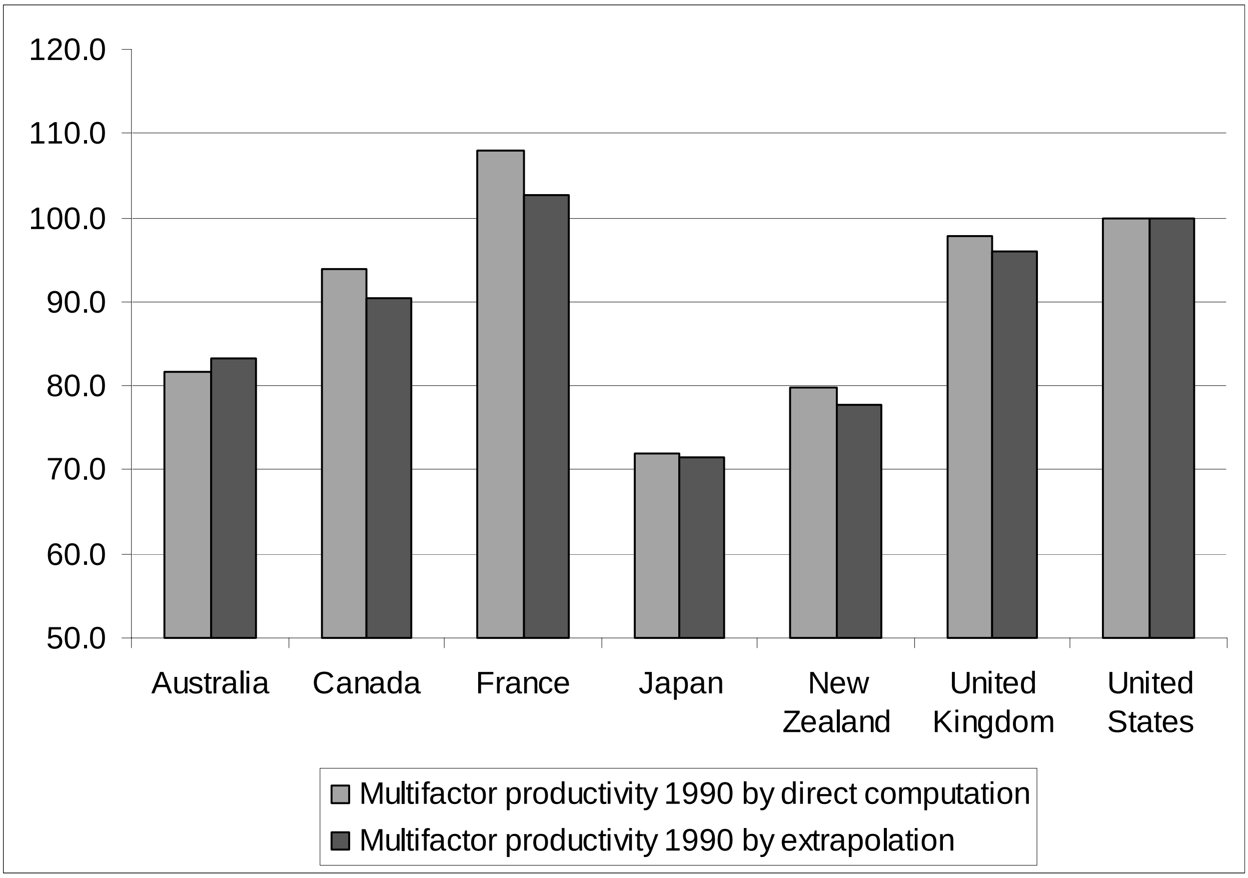
[13] It is possible to construct a fully consistent system of space-time index numbers if every comparison over time and in space is based on the same average weighting pattern. For example, a spatial comparison between two countries that uses weights that are averaged over the two countries and over two periods in time produces the same indirect temporal growth rates as a direct temporal comparison that uses the same four-fold average as its weights. The consequence is of course that every country's growth rate depends on other countries' investment and expenditure patterns and that every comparison in time and space is liable to change as soon as an extra observation is added – by way of an additional country or an additional period.

Table 8 Spatial-temporal comparisons of MFP

| Multilateral index of: | Australia | Canada | France | Japan | New Zealand | United Kingdom | United States |
|---|---|---|---|---|---|---|---|
| Multifactor productivity 1990 by direct computation | 81.6 | 93.9 | 108.0 | 72.0 | 79.8 | 97.8 | 100.0 |
| Multifactor productivity 1990 by extrapolation | 83.3 | 90.5 | 102.6 | 71.5 | 77.7 | 96.0 | 100.0 |
| Multifactor productivity 1990 extrapolation, USA 2002 = 100 | 72.1 | 78.4 | 88.9 | 61.9 | 67.3 | 83.1 | 86.6 |
| Multifactor productivity 2002 | 85.0 | 88.0 | 103.5 | 65.5 | 72.8 | 97.2 | 100.0 |

Source: *OECD Productivity Database and author's calculation* .

Figure 3: Multi-factor productivity in 1990 – alternative methods for computation USA=100



## 6. Conclusions

The present study provides a first set of partial and multi-factor productivity measures for seven OECD countries. The paper focuses on the statistical aspects of these indicators without embarking on productivity analysis. Three main conclusions arise from this work:

- Level estimates of capital and multi-factor productivity are feasible and provide a useful complement to the labour productivity estimates that have been an integral part of the OECD productivity estimates for several years.
- Methodology matters – the choice of the conceptually correct measure of capital input shapes results as does the choice of index number formulae when comparisons along both the time and spatial dimension are undertaken.
- Many statistical uncertainties remain and results have to be interpreted with a good deal of caution. We provide Monte Carlo estimates to examine the effects of measurement errors in the base data and these simulations showed that boundaries for the resulting indicators can be important.

This paper only marks the beginning of a longer project towards developing multilateral indices for levels of capital input and multi-factor productivity in the context of the OECD Productivity Database. It is planned to extend the range of countries to be included in the comparison and to further review the underlying statistics.

# Annex: Data Sources

## *Measuring output*

The starting point for the measure of output is GDP for the year 2002 at current national prices. A multilateral index of output is computed by applying GDP-level PPPs as published by the OECD (OECD 2004). While this is a convenient measure of output, readily available and consistent with labour productivity levels as presently published in the OECD productivity database, two modifications are desirable and efforts will be made to implement them in a future revision of the present work:

- Rather than measuring GDP at purchasers' prices, value-added should be measured at basic prices, i.e., excluding taxes on products and including subsidies on products, because this valuation constitutes the economically relevant variable from a producer perspective, the relevant perspective for productivity analysis.

- An adjustment to aggregate value-added is required to maintain consistency between input and output data: capital input in the *OECD Productivity Database* is limited to non-residential, fixed assets in scope and consequently, the value-added produced with residential assets should be excluded from productivity calculations. Thus, total value-added should be corrected for the production of owner-occupiers and for that part of the real estate industry that produces housing services. Absent such a correction, the assumption has to be made that the share of housing services (owner-occupied and provided on the market) in total value-added is similar between the countries considered.

## *Measuring labour input*

Labour input is measured as total hours worked in the economy – a difficult task in particular at the international level. Even so, this remains an imperfect measure: no account is taken of labour quality as hours of persons with skills and experience are simply added up. A more appropriate index of labour input would weight different types of hours worked with their corresponding share in overall compensation. The most important measurement issues are described in a note available on the site of the *OECD Productivity Database*.

## *Measuring capital input*

Capital inputs are derived on the basis of the perpetual inventory method. The estimation of capital service flows starts with identifying those assets that correspond to the breakdown currently available from the OECD/Eurostat National Accounts questionnaire, augmented by information on information and communication technology assets. Only non-residential gross fixed capital formation is considered, and in particular, six types of assets or products:

| **Type of product/asset** |
|---|
| Products of agriculture, metal products and machinery |
| Of which:<br>IT Hardware<br>Communications equipment<br>Other |
| Transport equipment |
| Non-residential construction |
| Software |

***Investment***. For each type of asset, a time series of current-price investment expenditure and a time series of corresponding price indices is established, starting with the year 1960. A description of the various sources for investment data can be found on the website of the *OECD Productivity Database*.

Price indices should be constant quality deflators that reflect price changes for a given investment good. This is particularly important for those items that have seen rapid quality change, in particular information and communication technology assets. There, observed price changes of 'computer boxes' have to be quality-adjusted for comparison of different vintages. Wyckoff (1995) was one of the first to point out that the large differences that could be observed between computer price indices in OECD countries were likely much more a reflection of differences in statistical methodology than true differences in price changes. In particular, those countries that employ hedonic methods to construct ICT deflators tend to register a larger drop in ICT prices than countries that do not. Schreyer (2000) used a set of 'harmonised' deflators to control for some of the differences in methodology. We follow this approach and assume that the ratios between ICT and non-ICT asset prices evolve in a similar manner across countries, using the United States as the benchmark. Although no claim is made that the 'harmonised' deflator is necessarily the correct price index for a given country, the possible error due to using a harmonised price index is smaller than the bias arising from comparing capital services based on national deflators[14].

***Productive stocks***. Given price and volume series for investment goods, for each type of asset, a productive stock $K_{s,t}$ has been constructed as follows:

$$K_{s,t} = \sum_{\tau=1}^{T^s} I_{s,t-\tau} h_{s,\tau} F_{s,\tau} \qquad , s=1,...6.$$

In this expression, the productive stock of asset s at the beginning of period t is the sum over all past investments $I_{s,t-\tau}$ in this asset, where current price investment in past periods has been deflated with the purchase price index of new capital goods. $T^s$ represents the maximum service life of asset type s.

[14] See Schreyer et al. (2003) for details. There is a difficulty with the harmonised deflator that should be noted. From an accounting perspective, adjusting the price index for investment goods for any country implies an adjustment of the volume index of output. In most cases, such an adjustment would increase the measured rate of volume output change. At the same time, effects on the economy-wide rate of GDP growth appear to be relatively small (see Schreyer (2002) for a discussion).

Because past vintages of capital goods are less efficient than new ones, an age efficiency function $h_{\tau}^{s}$ has been applied. It describes the efficiency time profile of an asset, conditional on its survival and is defined as a hyperbolic function of the form used by the United States Bureau of Labor Statistics (BLS 1983) $h_{s,\tau} = (T^{s} - \tau)/(T^{s} - \beta\tau)$.

Capital goods of the same type purchased in the same year do not generally retire at the same moment. More likely, there is a retirement distribution around a mean service life. In the present calculations, a normal distribution with a standard deviation of 25% of the average service life is chosen to represent probability of retirement. The distribution was truncated at an assumed maximum service life of 1.5 times the average service life. The parameter $F_{s,\tau}$ is the cumulative value of this distribution, describing the probability of survival over the cohort's life span. The following average service lives are assumed for the different assets: 7 years for IT equipment, 15 years for communications equipment, other equipment and transport equipment, 60 years for non-residential structures, and 3 years for software. The parameter β in the age-efficiency function was set to 0.75 for structures and 0.5 for other capital goods. Service lives and parameter values follow BLS practice.

***User costs of capital***. In a fully functioning asset market, the purchase price of an asset will equal the discounted flow of the value of services that the asset is expected to generate in the future. This equilibrium condition is used to derive the rental price or user cost expression for assets:

$$u_{s,t}^{i} = q_{s,t-1}^{i}\left(r_{t}^{i} + d_{s,t}^{i} - \zeta_{s,t}^{i} + d_{s,t}^{i}\zeta_{s,t}^{i}\right), \text{ where}$$

$u_{s,t}^{i}$ is the user cost for a new asset of type s in period t and country i;

$q_{s,t}^{i}$ is the purchase price of a new capital good s during period t in country i;

$r_{t}^{i}$ is the nominal net rate of return, expected at the beginning of period t to prevail during the period;

$d^i_{s,t}$ is the rate of depreciation of asset s, defined as $d^i_{s,t} \equiv 1 - q^i_{s,t,1} / q^i_{s,t}$;

$q^i_{s,t,1}$ is the purchase price of a one-year old capital good of type s during period t and in county i;

$\zeta^i_{s,t}$ is the expected price change of a new asset of type s during period t in country i, defined as $\zeta^i_{s,t} \equiv q^i_{s,t} / q^i_{s,t-1} - 1$.

***Exogenous net rate of return***. To compute the net rate of return, we follow a suggestion by Diewert (2001) and use as a starting point a constant value for the expected real interest rate rr. The constant real rate is computed by taking a series of annual nominal rates (un-weighted average of interest rate with different maturities[15]) that have been deflated by the consumer price index. The resulting series of real interest rates is averaged over the period (1980-2000) to yield a value for rr. The expected nominal interest rate for every year is then computed as $r_t = (rr + 1)(1 + p_t) - 1$ where $p_t$ is the expected value of the consumer price index. Schreyer (2004) discusses the implications of an exogenous rate of return for productivity measurement.

To obtain a measure for $p_t$, we construct a 5-year centred moving average of the rate of change of the consumer price index $p_t = \sum_{s=-2}^{+2} CPI_{t-s} / 5$ where $CPI_t$ is the annual percentage change of the consumer price index. This yields the expected rate of overall price change and, by implication, the nominal net rate of return.

***Expected asset price changes***, another element in the user cost equation, are derived as a smoothed series of actual asset price change: a simple 5-year centred moving average serves as a filter.

***Depreciation rates*** have been computed using the definition given above $d^i_{s,t} \equiv 1 - q^i_{s,t,1} / q^i_{s,t}$: the rate of depreciation for a new asset equals one minus the ratio of

[15] These are the average bank rate, the bank rate on prime loans, long-term government bond yields, short-term government bond yields, the interest rate on a 90 day bank fixed deposit and the treasury bill rate.

the market price for a one-year old asset over the market price for a new asset. While the market price for a new asset can be observed directly, the vintage price for a one-year old asset has to be computed, using the asset market equilibrium condition, the age-efficiency function h and the discount rate.

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