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AGGREGATION METHODS ON THE BASIS OF STRUCTURAL INTERNATIONAL PRICES

Introduction

1. The current state of the ICP, its still unresolved problems, etc. are described in the following reports:

- Ahmad S. **„ICP at Cross Roads: Can We Choose The Right Path ?“**
- Heston A., Summers R., Nuxoll D. **„The Present State of ICP“**.

(Both papers were presented at the ICP expert group meeting in Philadelphia, January 1993).

- Roberts D. „**The ICP: Past, Present and Future**“ (Paper for International Statistical Institut meeting, 18-26 August 1997).
- Castles I. „**The Review of the Eurostat-OECD PPP program**“. OECD, Paris, 1997.
- Ryten J. „**The evaluation of the ICP“**. Cirencester and Ottawa, 1998.

2. In order to yield comparable indicators, international comparisons require solution to two groups of problems. First, aggregates used for comparisons have to be comparable from a point of view of the statistical methodology (inner comparability). Second, comparable aggregates expressed in national currency units have to be converted into a common currency (outer comparability).

3. The inner comparability of macroeconomic data is increasing due to harmonisation work by national statistical offices and international organisations resulting in standard definitions, classifications, meth ods of valuation, etc. This circumstance allows to take more attention to the computational procedures.

4. Since the beginning of the ICP remarkable efforts have been made in order to provide for an appropriate statistical treatment of the heterogeneities of the participating countries, e.g. the principle of graduality, the demand for sufficient representativity for all countries, the flexibility of methods applied to selected fields of the comparison. It should also be noticed that some of the problems addressed may be felt to be as old as ICP. Since phase IV (reference year 1980) the ICP has been organised on a regional basis with further generation of worldwide global figures by linking the results of regional comparisons. Regional sets of countries were considered as more homogeneous. It was, maybe, right in the past but the present situation is changed substantially in the 90^{th} – even the OECD and the Eurostat comparisons comprise heterogeneous sets of the countries for the moment.

5. Multilateral comparison is a price and volume comparison of more than two countries simultaneously that produces consistent relations (price and volume indices) among all pairs of countries involved. Multilateral methods are used at the basic heading level as well as at the more aggregate levels (aggregation methods). This paper focuses on the aggregation methods.

6. It is usually desired that the results of international comparisons (PPPs / Volume indices) possess some properties important from the analytical point of view. Detailed analysis of these properties are given by W. Diewert (1986) (1996), by B. Balk (1995) and by R. Hill (1997). The following major properties should be mentioned:

Characteristicity[1]. This property implies that the samples of items priced (or quantities) and weights (or common international prices) used in an international comparison are representative of all participating countries. This property is easier to satisfy in a bilateral comparison, especially if the two comparing countries are similar, than in a multilateral comparison when a wider group of countries is involved.

Base-country invariance**.** This property requires a symmetrical treatment of all countries, so that it makes no difference for the final results which country is chosen as the base. The country selected as the base serves simply as a numéraire (point of reference).

Transitivity (for multilateral comparisons only). This property requires that the indices (parities or volume ratios) between any pair of participating countries be the same whether derived from the direct comparison between them or from the comparison of each of the two with any third participating country. The property of transitivity is satisfied if $I_{j/k} = I_{j/l} / I_{k/l}$ (where I - indices between respective pair of countries; j,k,l are countries). Satisfying this property yields a unique cardinal scaling of countries with respect to volume ratios and parities.

Additivity[2] (for aggregation procedures only). This property, when satisfied, means that real values (comparable between all countries) for any country are directly comparable between categories or, in other words, countries' real values at any level of aggregation can be obtained as the sum of real values of lower level categories of a given aggregate. Additivity requires a method to compare all countries using a common vector of prices (a vector of international prices).

Commensurability. This property means that the results of the volume and price comparisons should be invariant to changes in the units of measurement for quantities and currencies.

7. Different methods have been proposed for making multilateral comparisons of price and volumes but no method of calculation can satisfy all the desired requirements. For example, there is always a "trade off" in a multilateral comparison between the characteristicity and transitivity properties. The importance attached to each of the properties may vary depending on the combination of comparing countries and the uses to which the results of the comparison are put. The comprehensive reviews of aggregation procedures using in multilateral comparisons are contained in the papers prepared by W. Diewert (1996), B. Balk (1996) and R. Hill (1997).

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[1] The term „Characteristicity" was introduced by L.Drechsler (1973) and it meant initially that the „best" bilateral index is obtained on the basis of input data of two countries in question only (without data of any 3rd countries). This term is used now very oft (incl. this paper) in some modified sense.

[2] The property of additivity is linked with the property of matrix consistency. The relationship between the additivity and matrix consistency is explained in detail in the paper: I.Sakuma, D.S.Prasada Rao, Y.Kurabayashi „*Additivity, Matrix Consistency and a new Method for International Comparisons of Real Income and Purchasing Power Parities*", 26th General Conference of the IARIW (Cracow, Poland, 27.08 – 02.09.2000). The categorization of additive methods is given in the paper of J.R.Cuthbert „*Categorisation of additive PPPs*" – The Review of Income and Wealth", No.2 (June), 1999.

8. A question about the choice of the best multilateral method was widely discussed on two special meetings of Experts Group in a field of ICP-methodology organised by EUROSTAT, OECD and Statistical Office of UN together: EUROSTAT, Luxembourg, June, 1988 and OECD, Paris, June, 1989. Majority of experts inclines to an opinion that no uniform method is and various methods are necessary used for various aims. For example, new version SNA (SNA -1993, p.397) indicates: "...*the methods used to compile statistics must be influenced by the purposes for which they are to be used*". This conclusion means in the practice that a set of possible multilateral methods should be broad and new ideas and proposals are desirable.

9. Mainly two kinds of multilateral aggregation methods are used in the present time:

A) **Averaging of bilateral indices** (e.g. the EKS method, etc.)
B) **Use of average international prices** – an averaging of national prices recalculated by PPPs into a common currency (the Geary-Khamis, Van IJzeren, Iklé methods, etc)

10. The methods of type B) are based usually on the simultaneous calculation of the international prices and overall PPPs or overall quantities within a system of equations, i.e. international prices are some functions from global PPPs (e.g. **Geary-Khamis, Van IJzeren, Rao** methods) or overall quantities (e.g. the **CKS** method – **C**ommensurable **K**urabyashi-**S**akuma) or even both (**Iklé** method) and simultaneously overall PPPs (overall quantities) are some functions from these international prices.

11. We would like to propose a new kind of the aggregation procedures of type B) which is based on the use of so called structural international prices. It is easy to show that volume index (IQ) for any pair of countries j and k can be calculated not only on the basis of a given set of prices (π) but also on the basis of their ratios (e.g. to a commodity M selected as a basis):

$$IQ^{j/k} = \frac{\sum_{i=1}^{M} \pi_i * q_{ij}}{\sum_{i=1}^{M} \pi_i * q_{ik}} = \frac{\sum_{i=1}^{M} (\pi_i / \pi_M)* q_{ij}}{\sum_{i=1}^{M} (\pi_i / \pi_M)* q_{ik}}$$

12. This circumstance allows to use an averaging of national price ratios instead of the averaging of national prices as it is done by the traditional methods of type B).

13. The structure of national prices for country j can be presented as the quadratic matrix of price ratios: P_{ij}/P_{lj} (i=1,2,...,M; l=1,2,...,M) or in a compressed form – as the vector of ratios "P_{ij}/P_{Mj}", where P_{Mj} is basic (numeraire) commodity M. The ratios "P_{ij}/P_{Mj}" are not depend on the national currencies and therefore are directly comparable between countries and they can be averaged without the use of PPPs. These average international price ratios can be named as **international structural prices**. The respective PPPs are derived indirectly during the next independent step as ratio between value for a given aggregate in national prices (national currency) and value measured by known international structural prices. In effect, the methods using structural prices are strictly additive.

14. It seems that this approach was not used yet directly in international comparisons. However is it easy to demonstrate that the well known Gerardi (Unit-Country-Weight = UCW) method can be presented in the form of structural international prices. Traditionally the Gerardi - UCW method (based on prices P_{ij} for N countries and M commodities) is presented in the form of average international prices (PM) which are obtained as simple geometric averages from national prices without any PPPs:

$$PM_i = (P_{i1} * P_{i2} ... * ... P_{ij} ... * ... P_{iN})^{1/N} \qquad i = 1,2,...,M$$
$$j = 1,2,..,N$$

15. This form is not very understandable from a economic point of view and this feature was sometimes a point for the criticism of the G-UCW method. Nevertheless the Gerardi formula presented above can be transformed to the form:

$$PMi = \left[\frac{Pi1}{PM1j} * \frac{Pi2}{PM2} * \ldots * \frac{Pij}{PMj} * \ldots * \frac{PiN}{PMN} \right]^{1/N} \qquad \begin{array}{l} i = 1,2,\ldots,M; \\ j = 1,2,..,N \end{array}$$

16. This formula shows clearly that the Gerardi method can be presented as an averaging of the structural national prices although the author of the UCW method (D.Gerardi) himself introduced this method from an other point of view.

17. So, main aim of this paper is to introduce a new type of international prices – structural international prices – into the PPP calculations. The outline of the paper is as follows: Section I contains a short analysis of the aggregation procedures presently used in ICP. Section II deals with the indicators measuring the similarity of price structures. The sections III and IV contain the descriptions of two methods based on the structural international prices which were elaborated by the author: Section III - the method of „Maximal possible characteristic prices“ (MPCP-method) and Section IV - the method of „Standardised Structure“ (SS-method). Section V contains some numerical illustrations on the basis of detailed data from the Eurostat 1997 and 1998 exercises.

I. Analysis of main aggregation procedures using in ICP

18. The most popular aggregation procedures presently using in ICP are the Geary-Khamis (GK) method (the block approach – averaging of national prices recalculated into a common currency) and the EKS method[3] (the averaging of bilateral results). Both of these methods are described in detail in the different reports[4]. Therefore, only a short analysis of advantages and drawbacks of the methods is briefly presented in this section.

[3] Some historical research found that the EKS method was first proposed by C.Gini in 1924 (Gini, 1931) and it was later rediscovered by three independent researchers in 1964: Ö. **E**lteto, P. **K**öves and B. **S**zulc, therefore the name **GEKS** is used in the last publications.

[4] Detailed description of the GK and the EKS methods and the analysis of its advantages and disadvantages are given in many publications: Kravis I. a.o. *A System of International Comparisons of GrossProduct and Purchasing Power*. Baltimore, 1975**;** Kravis I. a.o. *World Product and Income. Intenational Comparisons of Real Gross Product*. Baltimore,1982**;** Hill P. Multilateral *Measuremerents of Purchasing Power and Real GDP*. SOEC, 1982**;** *Handbook of ICP*. UN, N.Y.,1992**;** „International Comparison of Gross Domestic Product in Europe,1993. Results of the European Comparison Programme“. UN, Statistical Standarts and Studies - No.47. N.Y. and Geneva,1997**;** „*Comparison in real term of the aggregates of ESA. Results 1998*“. Luxembourg, Eurostat, 2000**;** „*PPP and Real expenditures. Results 1996*“, Paris, OECD, 1999. Short but very strong description of multilateral methods is there also in the new SNA-1993, chapter XYI, part F.

19. The Geary-Khamis method provides additivity[5], which is very desirable, if international comparisons are made at varying levels of aggregation (the comparisons of „ICP-type"). The main drawback of GK arises as a result of the fact that the GK common vector of international prices is obtained by taking a weighted average of the countries' price vectors.[6] Hence the vector of the international prices tend to be closer to the price vectors faced by large (or rich) countries than small (or poor) countries. It is well-known that the volume of a country tend to sink as the prices used in the comparison becomes relatively more closeness to its own national prices as compared with the prices of other countries, or, in other words, the more characteristic the common price vector is for a given country, the more its volume index will tend to be underestimated. This bias caused by unequal relative closeness of used prices is usually referred to in literature as the Engel-Gerschenkron effect[7]. The GK average prices calculated for a set of heterogeneous countries cannot be characteristic of outlying countries[8]. This effect may significantly distort the comparative real product levels (especially in the developing countries, which are more sensitive to choice of used methods).

20. The EKS method attempts to guarantee equi-characteristicity of results. It ignores the differences in the size of countries compared and permits avoiding "Engel-Gerschenkron" type of distortions in the results. Also, EKS results have another attractive property, namely that relationships between countries are only marginally influenced by the composition of the group of countries compared due to minimisation procedure applied. The main inconvenience of EKS is the lack of additivity. This means that the sum of real values obtained by EKS-type PPPs at the given aggregation level doesn't produce the EKS-type real value of higher level of aggregation. Consequently, the percentage distribution of these real values does not add up to 100 per cent. Therefore, the possibilities of structural analysis are limited. Moreover, the lack of additivity can lead to paradoxical results: the average index (or PPP) can be higher (or lower) as each of particular indices (this is the distortion so called „average test"). Below we would to give several examples from different exercises for the illustration[9]:

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5 The GK provides additivity in terms of international average prices only but not in terms of prices of any country. For example, the ICP 1975 used the GK method, the original PPPs to I$ were recalculated (and published) as PPPs to USD. In effect, the obtained PPPs to USD did not satisfy with the „average test". So, for Japan were obtained the following PPPs to USD: „Government-Total"- 323 Y/USD but both partial PPPs were lower: „Compensation"- 277 Y/USD and „Commodities" – 298 Y/USD; very unusual relations between average and partial PPPs were obtained for UK: „Recreation and Education" – 0.346 £/USD but both partial PPPs were higher: „Recreation" - 0.372 £/USD .and for „Education" - 0.361 £/USD.

6 For some comparisons this feature of GK method can be *vice versa* an advantage but not a drawback.

7 The Gerschenkron effect is duscussed in more detail in Nuxoll D. (1994) and Hill, R. (1995). Some authors believed that there is no a clear evidence that the GK method is influenced by the this effect or, at least, that the EKS method is not „free" from this effect also – see, for example, S.Khamis „Measurement of Real product: Some Index Number Aspects" (paper for the 25th General conference of the IARIW, Cambridge, 23-29.08.1998).

8 These outliers are usually the poor but sot sometimes the rich small countries involved in the comparison.

9 The scaling procedures for the EKS-PPPs are used within the Eurostat comparison (to Ecu/Euro) as well as within the OECD comparison (to USD). These procedures do not change the relations between countries within a Heading but they can change the internal relations of PPPs of different Headings within a country (see S.Sergueev „Scaling procedures used in the EU comparison: some proposals**".** - Paper for a special meeting of PPP compiliers (Eurostat, OECD, OeSTAT -LUX, 20.11.98). Therefore only the examples with Volume indices are „pure" examples of non-additivity.

1) OECD 1993 comparison:

Volume indices per capita (OECD = 100) for Belgium:

- Clothing and footwear = = 94
  - Clothing = 93
  - Footwear = 92

2) OECD 1993 comparison:

Volume indices per capita (OECD = 100) for EU12 :

- Gross rent, fuel & power = = 89
  - Gross rent = 88
  - Fuel & power = 88

3) Overall ECP 1993 comparison:

Volume indices per capita (Austria = 100) for Switzerland:

- Gross rent, fuel & power = = 140.0
  - Gross rent = 134.8
  - Fuel & power = 135.4

4) Overall ECP 1993 comparison:

Volume indices per capita (Austria = 100) for Ireland:

- Gross rent, fuel & power = = 64.2
  - Gross rent = 64.8
  - Fuel & power = 67.9

5) EUROSTAT 1994 comparison:

Volume indices (EU15 = 100) for Poland:

- Beverages-Total = 3.43
  - Non-alcoholic = 3.28
  - Alcoholic = 3.42

6) EUROSTAT 1995 comparison:

PPPs „Italian Lira/PPS“ for Italy:

- Clothing and footwear = = 1784
  - Clothing = 1787
  - Footwear = 1789

7) EUROSTAT 1995 comparison:

PPPs „Escudo/PPS“ for Portugal:

- Clothing and footwear = = 194.6
  - Clothing = 195.1
  - Footwear = 196.1

8) EUROSTAT 1997 comparison:

PPPs „Drachma/PPS“ for Greece:

- Beverages = 304.4
  - Non-alcoholic = 300.5
  - Alcoholic = 304.2

9) EUROSTAT 1997 comparison:

PPPs „BF/PPS“ for Belgium:

- Gross rents, fuel, etc. = = 41.09
  - Gross rents = 41.54
  - Fuel, etc. = 41.25

10) EUROSTAT 1998 comparison:

PPPs „Drachma/PPS“ for Greece:

- Gross rents, fuel, etc. = = 234.5
  - Gross rents = 237.5
  - Fuel, etc. =237.3

21. P.Hill gave the comments about multilateral methods that take the Fisher indices as the starting point (EKS and the like) in an aphoristic form: „*The construction of a multilateral set of measurements at a later stage has then to be regarded as a process whereby an initial set of perfectly good binary measures has to be distorted, rather in the manner practised by Procrustes, in the interests of securing transitivity*" (see Kravis I. a.o. „World Product and Income. Intentional Comparisons of Real Gross Product". Baltimore,1982., p.77).

22. S.Khamis commented the use of the EKS-method within EUROSTAT-comparison with the following words: „In the opinion of the author, the adoption of the EKS(F) results by EUROSTAT is a retrogressive step in comparison with their excellent earlier comparisons including those of 1975 based on the Gerardi UCW method" (see, S. Khamis. „On some aspects of the measurement of Purchasing Power Parities". Reports of ISI Session, Florence, August 1993).

23. The ICP consists of the set of very heterogeneous countries. It is useful to investigate which impact has the choice of concrete method on the results. The calculations for different parts of ICP (EU, OECD, ESCAP comparison) show also that the choice an aggregation procedure has a sufficient impact on the results.[10] The differences among the results calculated by different methods for several countries and aggregates are 10-20%.

24. An extreme example was found within the provisional results of the 1993 ESCAP comparison [see, Hill (1997b)]:

	Volume index per capita	
	Japan	Singapore
GK	100	177
EKS	100	83

25. Which are reasons for such extreme numerical differences ? It seems that the main reason is the different structures of countries' prices. Japan as well as Singapore are rich countries but Singapore is very small country (Japan - a large country). Therefore GK prices are much more close to Japanese prices than to prices of Singapore. As effect the Japanese result by GK method is much lower than by EKS method.

26. To combine the advantages and to decrease the disadvantages, it was decided to use in the framework of EUROSTAT-OECD PPP Programme both methods in parallel: the EKS-method for volume comparisons (official results) and the Geary-Khamis method for structural analyses (for analytical purposes). Obviously, such mechanical combination is not very appropriate for the concrete analytical works. The use of different outcomes (EKS results and GK results) can lead to some irritations. These circumstances are a reason for further investigations of multilateral methods. The GK method provides additivity but avoids characteristicity, the EKS method - *vice versa.* **Is it possible to combine advantages of both methods within an unique method?**

[10] The last detailed investigations about the influence of the choice of multilateral method on the results of ICP are the following: Y. Dikhanov „*Sensitivity of PPP-Based Income Estimates to Choice of aggregation Procedures*" (IARIW session on international comparison; St.Andrews, New Brunswick, Canada on 21-27 August 1994); R. Hill „*Comparing Price Levels and Living Standards across the ESCAP Countries Using Spanning Trees and other Aggregation Methods*" (Paper for a UN Seminar on comparison within the ESCAP, Bejing, 16 - 20.06.1997).

27. S. Ahmad (World Bank) in the paper „*Reduced Information initiative*“ [see, Ahmad (1994), page 2] concerning the problems of ICP expressed the following considerations about the problem of the choice of aggregation procedure:

„*To reduce the Gerschenkron effect and at the same time retain matrix consistency (additivity) of the results, it is proposed that an unweighted or equal weighted) Geary - Khamis be used. Tests show that the results are very similar to EKS but with an added advantage of additivity.* ***Should another aggregation method such as Iklé be used instead ?*** “.

28. The following methods were proposed for this aim:

1) The **„Implicit price“ method** (IP method) developed by EUROSTAT (see, the materials of Paris- ICP meeting, June 1989);
2) The **Iklé-method** (see, D. Iklé.“ A new Approach to the Index Number Problem“. Quarterly Journal of Economics, No.2, 1972). This method was analysed in detail by Y.Dichanov [1994].
3) The **Minimum Spanning Tree** method (see, R.J. Hill „International Comparisons using Spanning Trees“, in „ International and Interarea Comparisons of Prices, Income and Output“, NBER, 1997).
4) **Weighted EKS** method and **Generalised CPD** method (see, D.S.Prasada Rao „Aggregation Methods for International Comparison of PPPs and Real Income: Analytical Issues and Some Recent Developments“; ISI Session, Istanbul, 18-26 August 1997)
5) **Generalised GK** method (see, J.Cuthbert „Categorisation of additive PPPs“ – The Review of Income and Wealth“, No.2 / June, 1999).
6) **CKS method** (**C**ommensurable **K**urabayashi-**S**akuma method) – see I.Sakuma, D.S. Prasada Rao, Y. Kurabayashi „Additivity, Matrix Consistency and a new Method for International Comparisons of Real Income and Purchasing Power Parities“, 26th General Conference of the IARIW (Cracow, Poland, 27.08 – 02.09.2000).

29. These methods have some advantages contrary the EKS and the GK but they do not solve directly the task concerning the obtaining of simultaneously additive and characteristic (for all comparing countries) results of multilateral comparison. It seems that to obtain characteristic and simultaneously additive results over all countries involved in a multilateral comparison it is necessary to use a vector of international prices, which are maximally possible characteristic for all countries involved.

30. So the EKS-method was selected as the official method for Eurostat/OECD comparison and for the ECP since 1990 because the imaginary prices of the EKS method[11] were claimed by a expert group to be more neutral in relation to national price structures than those of the other alternative [see, OECD (1995), p.4]. However, to examine the hypothesis about this neutrality it is necessary to have the method for measuring of similarity (dissimilarity) of price structures. Below we propose a new procedure for measuring of similarity of price structures[12].

[11] In reality, the EKS method does not use any vector of international average prices. Therefore it is more correct to say about imaginary prices by EKS calculation [see, Köves (1995)]. The recent OECD PPP publication [OECD (1999)] indicated on this feature (see page 25) but the titles of Tables 7-11 were not changed - „... at international prices“).

[12] The main ideas of this method were suggested by S. Sergueev in the Ph. D. Dissertation „Multilateral Methods for International Comparisons“. The Researcher Institute of Central Statistical Committee of Soviet Union, Moscow, 1982 (in Russian).

II. Countries' price structures and measure of their similarity

31. There are several well-known methods for measuring of similarity (dissimilarity) of national price structures:

1) On the basis of the ratios of Laspeyres and Paasche indices (so called Laspeyres - Paasche spread = LPS[13]). See, for example, the following publications:

- P. Köves. *Index Theory and Economic Reality*. (§ 8.3.3). Budapest, 1983.
- M. Martini. *The multilateral comparison in the axiomatic* approach.
- B. Zavanella. *Comparison of consumption among EEC countries: Prices, Quantity and Values.*

(Both latter reports were prepared for International Statistical Institut Session: Florence, August 1993)

- R.Hill. *Chained PPPs and Minimum Spanning Trees*. (Conference of Income and Wealth, March 1996, Washington) – so called, Hill's Distance, which is a version of L/P ratio in logarithmic terms - | Log(L/P) |.

32. The LPS is an useful „tool" for analysing of the similarity of price and volume structures but it seems that the LPS indicator is not enough sensitive to big differences in the price structures. The countries with very different price structures can have Laspeyres/Paasche ratio not very far from 1. One imaginary numeric example is given below for an illustration:

Table II.1
Calculation of Laspeyres/Paasche ratio

Country A				Country B			
Price	Quantity	Pa*Qa	Pb*Qa	Price	Quantity	Pb*Qb	Pa*Qb
1	10	10	30	3	5	15	5
2	15	30	30	2	10	20	20
3	20	60	20	1	15	15	45
Total Values		100	80	Total Values		50	70

Volume Index - Laspeyres 'A/B' =	1.6000
Volume Index - Paasche 'A/B' =	1.4286
Ratio 'L/P' =	**1.1199**

33. The countries A and B have very different price structures although L/P ratio is close to 1 (= 1.1199). This is possible to explain: the L/P ratio depends on both sides of input data: differences in prices and differences in quantities and therefore the L/P ratio is not an appropriate tool for the measuring of „pure" price similarities.

[13] Sometimes an inverse indicator „Paasche – Laspeyres spread" (PLS) is used.

2) A more sophisticated method was used in the earlier phases of ICP:

„...the measure of similarity between the vectors of national prices is their weighted raw-correlation coefficient between the ratios domestic price and international prices referring to any pair of countries"

(see, Kravis I. et.al. World Product and Income. International Comparisons of Real Gross Product. Baltimore,1982; p.348).[14]

34. In our opinion the latter method is more preferable than the former. However, there are two disadvantages: the use of this method is possible only after the calculation the international prices (a) and the use the international prices as a bridge-vector can bring the effect of the overestimation (so called 'false' correlation because of common factor - international prices) of the similarity (b).

3) The thoughtful method was proposed by R.Allen and W.Diewert [see, Allen and Diewert (1981)]. The Allen-Diewert distance (D) is calculated as the dispersion of individual price ratios (PPPs) between countries (in logarithmic terms). The larger the value of D, the less similar the price structures in given countries. The Allen-Diewert measure lies between zero and infinity, i.e. it has a lower bound only. This is main disadvantage of the method.

4) J.Cuthbert (2000) proposed to calculate a summary statistic of price structure in terms of the root-mean-square deviation from 1 of the ratios of actual prices recalculated by overall PPP into common currency to average international prices (IP). The similarity of price structure between the countries is calculated as a correlation coefficient. This indicator is a useful analytical tool but there are two drawbacks: the proposed indicator is not symmetric about 1 and this is too sensible for extreme values of IP. In effect, the values of log(IP) were actually used and values of IP below 0.333 and above 3 were excluded from the analysis. Obviously an arbitrary exclusion of some products from the analysis is a weakness of the method.

35. Due to the drawbacks of the methods mentioned above we would like to propose a new method for measuring of similarity (dissimilarity) of price structures.

36. As it was indicated in the section „Introduction", the structure of national prices for country "j" can be presented as a quadratic matrix of price ratios: P_{ij}/P_{lj} (i=1,2,...,M; l=1,2,...,M) or in compressed form – as a vector of scalars **"P_{ij}/P_{Mj}"**, where P_{Mj} is price of basic (numeraire) commodity M. If the price data of countries **j** and **k** satisfy the conditions for Hicks's composite commodity theorem - $P_{ij} = \lambda * P_{ik}$ – then the price structures are identical. In effect, the coefficient of similarity of price structures can be concluded in a similar way with the deduction a coefficient of linear correlation as a calculation a geometric mean from the two regression coefficients)[15]. The minimisation of the sum of squares of relative differences in the both directions is used to obtain the measure, which is invariant to changes in the units of measurement. So, the coefficient of the price similarity can be calculate by the following steps:

a) First step

Let $P_{ij} = c1 * P_{ik}$ - regression line I.

$$\sum_i \{(P_{ij} - c1 * P_{ik})/P_{ij}\}^2 \rightarrow \min;$$

$$c1 = \sum_i (P_{ik}/P_{ij}) / \sum_i (P_{ik}/P_{ij})^2. \qquad i= 1,2,...,M$$

b) Second step

Let $P_{ik} = c2 * P_{ij}$ - regression Line II.

[14] This method is used presently by the OECD.

[15] Coefficient of similarity of national quantity structures can be obtained in the same way. The formula will be the same as (II.1) but the price symbols must be replaced by quantity symbols.

$\sum_i \{(P_{ik} - c2 * P_{ij})/P_{ik}\}^2 \rightarrow \min;$

$c2 = \sum_i (P_{ij}/P_{ik}) / \sum_i (P_{ij}/P_{ik})^2$ i= 1,2,...,MThird step

$\tau_{jk} = (c1 * c2)^{1/2}$;

37. The indicator τ_{jk} is a certain analogy (but not identical) with a coefficient of correlation[16] and has some analogous attractive properties:

$\tau_{jk} = \tau_{kj}$;

$0 < \tau_{jk} \leq 1$;

- if $\tau_{jk} = 1$ - price structures in the countries "j" and "k" are identical;
- if $\tau_{jk} \rightarrow 0$ - price structures are very different.

38. So, we propose to measure the level of similarity of national price structures between country **j** and country **k** by help of the following coefficient (indicator) τ_{jk}:

$$\tau_{jk} = \left[\frac{\sum_i (P_{ij}/P_{ik}) * \sum_i (P_{ik}/P_{ij})}{\sum_i (P_{ij}/P_{ik})^2 * \sum_i (P_{ik}/P_{ij})^2} \right]^{1/2} \qquad \text{(II.1)}$$

$j = 1,2,..,N; \quad k = 1,2,...,N$

$i = 1,2,...,M$

39. An example (see imaginary data in Table II.1) is given below for the illustration:

Table II.2
Calculation of coefficient of similarity of price structures

Pa/Pb	$(Pa/Pb)^2$	Pb/Pa	$(Pb/Pa)^2$
0.3333	0.1111	3.0000	9.0000
1.0000	1.0000	1.0000	1.0000
3.0000	9.0000	0.3333	0.1111
4.3333	10.1111	4.3333	10.1111
TauCoeff =		**0.4286**	

40. As we can see the proposed indicator τ (TauCoeff = 0.4286) showed the significant differences in the price structures between countries A and B. The indicator τ measures the „pure“ similarity of price structures (without influence of differences in quantities) and it is a preference relatively the Laspeyres/ Paasche ratio which reflects simultaneously the differences (and correlation)[17] between the price and quantity structures.

41. Indicator τ has a some similarity with the Allen-Diewert distance. But τ coefficient has an important preference because it has a lower bound as well as an upper bound (it lies between zero and 1), while the Allan-Diewert measure lies between 0 (zero) and infinity, i.e. it has only a lower bound but not upper bound.

[16] The coeffcient of the correlation itself is not very appropriate for the measuring of price similarities because it reflects the correlation in the form Y = c*X + b but we need the form Y = c*X.

[17] The negative correlation exists usually between cross-country price quantity ratios [see Bortkiewitz (1922, 1924)].

42. The different products have different shares in the GDP (or in an other related macroeconomic indicator). Additionally, in actual comparisons, input data are not quite those originally envisioned: prices and quantities for individual commodities. Usually PPPs ('National currency/Numeraire currency') for primary groups are used as „notional" (fictitious) prices and a set of „notional" (fictitious) quantities, each obtained as ratio of nominal value (in national currency) to corresponding PPP. The PPP for concrete basic heading represents a set of products. Therefore it is preferable to use the weighting in this situation.[18]

43. Weighted version of indicator τ can be proposed in the following form:

$$(II.2)\quad \tau_{jk} = \left[\frac{\sum_i \{(P_{ij}/P_{ik}) * d_i^{jk}\} * \sum_i \{(P_{ik}/P_{ij}) * d_i^{jk}\}}{\sum_i \{(P_{ij}/P_{ik})^2 * d_i^{jk}\} * \sum_i \{(P_{ik}/P_{ij})^2 * d_i^{jk}\}} \right]^{1/2} \qquad \begin{array}{l} j = 1,2,..,N;\ \ k = 1,2,...,N \\ i = 1,2,...,M \end{array}$$

where

$d_i^{jk} = (d_i^j + d_i^k)/2$ - average weight for basic heading i between countries j and k,

$d_i^j = w_{ij} / W_j$; $d_i^k = w_{ik} / W_k$ - weights for basic heading i in country j and country k,

$w_{ij} = p_{ij} * |q_{ij}|$ - absolute[19] nominal value for ith basic heading in the jth country (in national currency);

$W_j = \sum_{i=1}^{M} w_{ij}$ - total absolute value of the aggregate in question for country j at national prices (w_{ik} and W_k are defined analogously).

44. To illustrate the application of proposed method, the calculations were carried out on the basis of Group II data from the ECP'90/II, the ECP'93/II and the ECP'96/II by the weighted version of indicator (II.2). These calculations were made on the basis of basic heading PPP data, i.e. the BH-PPPs were used as implicit prices. Input data for the ECP'90/II contain PPPs „National currency/Austrian Shilling" for 300 basic headings, for the ECP'93/II - for 295 basic headings and for the ECP'96/II - for 266 basic headings. Obtained coefficients of similarity of price structures with Austria for three rounds of the ECP/Group II are shown in the Table II.3:

[18] The use of non-weighted version can lead to the distortion of the overall results even if one product (basic heading) only has an extreme anomal value (see, eg, J.Cuthbert (2000), page 433 – an example for New Zeland in the OECD 1993 comparison: due to an anomal price for dried vegetables only, the respective indicator was fully distorted).

[19] The reason of using of absolute values is the necessity of the obtaining a „correct" structure, i.e. the individual shares must lay in the limits [0;1].

Table II.3

Coefficients of price similarity (weighted) (Austria = 1)
(different rounds of the ECP / Group II)

	GDP-Total		
	1990	**1993**	**1996**
Austria	1.0000	1.0000	1.0000
Poland	0.4394	0.5385	---
Czech Rep. *)	0.4299	0.4787	---
Hungary	0.4897	0.6476	---
Russia *)	0.3229	0.1724	0.3317
Romania	0.3877	0.3608	0.2550
Belarus *)	0.3229	0.0875	0.2457
Bulgaria	---	0.4086	0.2229
Croatia *)	0.3525	0.5275	0.5491
Slovak Rep. *)	0.4229	0.3630	---
Slovenia *)	0.3525	0.6390	0.7493
Ukraine *)	0.3229	0.0755	0.2358
Moldova	---	---	0.1188
Estonia	---	---	0.4154
Latvia	---	---	0.3129
Lithuania	---	---	0.3511
Albania	---	---	0.2219
Macedonia	---	---	0.4767
Former CSFR	0.4299	---	---
Former SU	0.3229	---	---
Former YUG	0.3525	---	---

*) The coefficients of similarity for former united CSFR, SU and Yugoslavia are shown for 1990.

**) Czech Rep., Hungary, Poland, Slovak Rep. participated in the ECP'96 within Group I (OECD comparison)

45. It seems that proposed indicator reflects the differences in the price structures correctly enough. So, Hungary and Poland had the most high similarity of price structures with Austrian prices (τ for Hungary - 0.4897, for Poland = 0,4394,) in 1990. These results have a clear explanation: Hungary and Poland were the most advanced from the transition countries in 1990 (the liberalisation of prices and external trade, etc. have been done in these countries). On the other hand, the former USSR had lowest similarity with the Austrian price structure: τ(SU, AUT) = 0.3229. This is also understandable because the price liberalisation, etc. were started in the USSR after 1990.

46. The coefficients of similarity can be used also for the analysis of price dynamics as one of the characteristics of transition process. Table II.3 shows that the countries which have had more progress between 1990 and 1993 in the transition process (Czech Rep., Hungary, Poland, Slovenia) had more similar price structures with Austria (a developed country) than „non-advanced" countries (Russia, Belarus, Ukraine). Czech Rep., Hungary, Poland, Croatia, Slovenia had in 1993 higher coefficients of price similarity with Austria than in 1990. Slovenia and Croatia Slovenia had in 1996 higher coefficients than in 1993. It reflects obviously remarkable progress in transition process. Probably the real similarities in 1996 are some higher but the Eurostat methodology was used for the ECP'96/II for the Non-market services (without Productivity Adjustments) and in effect the PPPs for the NMS are more far from the PPPs for market categories than earlier. Romania and Bulgaria had in 1996 lower coefficients than in 1993. It reflects the existing economic problems (especially difficult economic situation in Bulgaria during 1996).

47. *Vice versa* Russia and especially Belarus and Ukraine had in 1993 very low coefficients of price similarity with Austria and even much lower than the former SU in 1990. It reflects very chaotic movements of prices during the beginning of the transition process. But the coefficients of price similarity for these countries in 1996 were much higher than in 1993. It reflects obviously some progress in the transition process (price stabilisation, etc.).

48. Similar calculations were carried out also on the basis of information from the Eurostat 1997 and 1998 comparisons (see Section V) which also are shown a good efficiency of proposed method for the measure of similarity of price structures.

A possible collateral use

49. It seems that the indicator τ can be used also for an other purpose. The L/P ratio or its modification – Hill‘s distance [see R.Hill, (1999)] are presently used often as the indicators of the reliability of bilateral comparisons. The comparisons with smaller differences between Laspeyres and Paasche indices are regarded as more reliable exercises because these are based on data of the countries with more similar structures. In general, this postulate can be accepted but there is an additional circumstance, which should be taken into account.

50. So, Hill's distance is a symmetrical measure: for example, the situations L/P = 1.25 and L/P = 0.8 (=1/1.25) bring the same distances between countries. However, if the situations with L/P > 1 can be considered as an usual "normal" case then all situations with L/P < 1 should be considered as problematic cases. The ratio L/P <1 (P/L > 1) shows that the comparing countries are not very comparable (have different price and quantity structures). The publication about the ICP 1975 (KHS, 1982) contains the following indication (Chapter 7, page 228, footnote 4): *"A Paasche-Laspeyres spread (PLS) greater than 1 indicates that price and quantity structures are different...*". Obviously, the countries with different structures are less comparable and in effect the bilateral results are less reliable. Therefore a selective method is used sometimes in the comparisons by the EKS method: original direct F-PPPs with L/P ratios which are greater than some limit (usually 1.5) and lower than 1 are replaced by indirect PPPs obtained via 3rd countries.

51. We would like to illustrate this thesis with an example from the exercise of Rao, Timmer (2000) which used the Hill’s distance: there were 3 pairs of countries with L/P ratios lower than 1 (P/L >1): "Germany - Japan", "Indonesia - Japan", "Indonesia- Taiwan". Because the Hill‘s distance is a symmetrical indicator then the reliability (weight) for "Japan - Australia" (4.75) was given lower than the reliability (weight) for "Japan - Indonesia" (7.63). It means that the comparison "Japan - Indonesia" with a very unusual ratio L/P <1 (= 0.877!) was regarded as much more reliable than the comparison "Japan - Australia" with a "normal" L/P ratio = 1.23. It seems that the measure of reliability on the basis of the L/P ratios should take into account the direction of the L/P ratios: the L/P ratios are lower than 1 should be considered as less reliable and should have lower weights.

52. As rule, all "anomalous" L/P ratios (L/P < 1) are due to the fact that price-quantity structures are very different in the comparing countries. For example, the former CPE countries had many fixed tariffs or fixed prices without any connection with the economical factors, i.e. normal price structures were distorted. In effect, many comparisons between the CPE and Western countries brought the "anomalous" L/P ratios. Obviously, the L/P ratios being smaller than 1 can be explained or, in other words, some economic reasons for this phenomenon can be found but these are two different directions of the analysis: an "explanation" of L/P ratios smaller than 1 and a reliability of the results for the countries with L/P ratios smaller than 1.

53. R.Hill found (Hill, 1999) that only **5** pairs of countries from **870** binary comparisons of 30 countries for two benchmark ICP years (1980 and 1985) had L/P ratios smaller than 1, i.e. the distribution of L/P ratios (L/P < 1 and L/P > 1) is non-symmetrical. The L/P ratios < 1 are very rare cases and they are obtained by very unusual circumstances (distorted structures, etc.).

54. In this situation, an other indicator of reliability should be tested. For example, the coefficient of similarity of price structures can be used for this purpose. It can believe that the coefficient of similarity of price structures between a pair of given countries will show the lower similarity for the countries with L/P ratios < 1 than for the countries with the same deviation of L an P indices but with L/P > 1. It would be interesting to calculate the measure (**II.2**) for the pairs "Japan - Indonesia" and "Japan - Australia" from the example above. These pairs of countries have approx. the same L/P ratios but with different directions. It can believe Indonesia had more extensive agricultural protection and subsidies and therefore the Indonesian price structure had more "distortions" than Japan or Australia. Probably, the measure of price similarities (**II.2**) will show that "Japan - Australia" had much higher similarity of price structures than "Japan - Indonesia", i.e. "Japan - Australia" should receive a higher weight than "Japan - Indonesia".

55. Additionally, the Hill's measure is too sensitive for the L/P ratios, which are very close to 1. The reliability is inversely proportional to the Hill's distance the comparisons and the pairs of countries with the L/P ratios close to 1 receive extraordinary high weights: for example, the distance "Australia - Korea" was **1496.83** (see Rao, Timmer (2000, table 3.8, p.23) and all other distances were between **1.92** and **118.66** only. The measure (**II.2**) can be applied in a direct form - the larger the value of this measure, the greater the similarity (weight) and the greater the reliability of bilateral indices – in effect, the problem of sensitivity is partly avoided.

56. An analysis of the results for the different phases and the parts of the ICP showed that the significant differences are kept in the price structures between the countries. Therefore the elaboration of the multilateral methods which could take into account (with neutralisation of respective negative effect) the existing differences in the price structures, is an actual task.

III. Method of "Maximal Possible Characteristic Prices" (MPCP - method)

57. A new aggregation multilateral method producing additive and simultaneously the most characteristic results was elaborated on the basis of the use of the similarities of price structures described in the previous section. This method was named as the **MPCP**-method – the method of „**M**aximal **P**ossible **C**haracteristic **P**rices".

58. Suppose we have to carry out a multilateral comparison among N countries of an aggregate which contains M commodities (or basic headings). The matrix of national prices **Pij** (i=1,2,...,M; j=1, 2,...,N) and the matrix of corresponding quantities **qij** (i=1,2,...,M; j=1,2,...,N) are used as input data. The aim is to obtain transitive, additive and most possible characteristic (for each country) results.

59. Suppose there is a positive vector of international prices $\mathbf{PM}_i$ (i=1,2,...,M; $PM_i > 0$). We can measure the level of similarity between the national price structure of each country and the structure of international prices analogously (II.1):

$$\text{(III.1)} \qquad \tau_j = \left[\frac{\sum_i (P_{ij}/PM_i) * \sum_i (PM_i/P_{ij})}{\sum_i (P_{ij}/PM_i)^2 * \sum_i (PM_i/P_{ij})^2} \right]^{1/2} \qquad \begin{matrix} j = 1,2,...,N; \\ i = 1,2,...,M \end{matrix}$$

60. The term τ_j can be considered as the degree of characteristicity of given international prices $\mathbf{PM}_i$ for country j.

61. As it has been mentioned already in the section II, it is preferable to use the weighting in actual comparisons. Weighted version of (II.2) can be proposed in the following form:

$$(III.2)\qquad \tau_j = \left\lceil \frac{\sum_i \{(P_{ij}/PM_i)*d_i\} * \sum_i \{(PM_i/P_{ij}) * d_i\}}{\sum_i \{(P_{ij}/PM_i)^2 * d_i\} * \sum_i \{(PM_i/P_{ij})^2 * d_i\}} \right\rfloor^{1/2} \qquad \begin{matrix} j = 1, 2,...,N \\ i = 1, 2,...,M \end{matrix}$$

where

$d_i = \sum_{k=1}^{N} d_i^k / N$ - average weight for basic heading i across all countries (d_i^k are defined earlier in the section II).

There is a set of values: $\tau_1, \tau_2, \tau_3, \ldots, \tau_N$ in the given multilateral comparison. Let the indicator τ_{min} is the minimal value among τ_j (j=1,N):

$$(III.3)\qquad \tau_{min} = \min (\tau_1, \tau_2, \tau_3, \ldots, \tau_N)$$

62. The indicator τ_{min} can be considered as the degree of general characteristicity of a given international prices **PM** for the set of all comparing countries (j=1,N).

63. The MPCP method consists of searching for the vector of scalars **PM** in such way that the indicator τ_{min} must have maximal (highest) possible value (τ_{opt}), which we can obtain by our primary data:

$$(III.4)\qquad \tau_{opt} = \max_{J=1,2,...,N}(\min_{J=1,2,...,N} \tau_j) = \max \left\{ \min \left\lceil \frac{\sum_i (P_{ij}/PM_i)*d_i * \sum_i (PM_i/P_{ij})*d_i}{\sum_i (P_{ij}/PM_i)^2*d_i * \sum_i (PM_i/P_{ij})^2 * d_i} \right\rfloor^{1/2} \right\}$$

$$PM_i > 0 \qquad (i = 1, 2, 3,...,M)$$

64. Obviously, if a vector **PM** is a solution of (III.4) then each vector •***PM** is also a solution. Therefore we can find the prices with accuracy up to scalar. The structural (relative) prices are very appropriate for this purpose: a base product (e.g. product B) is selected, its price is set as 1 ($PM_B=1$) and all other prices are measured to the relation to the price of base product.

65. The term (III.4) is a non-linear function. The method of configurations (R.Hooke's - T.Jeeves's method) or J.Nelder's - R.Mead's method can be used for solving of (III.4). Latter method is more preferable for practical calculations with significant number of unknown variables PM_i.

66. The values PM_i are searched by help of numeric iterative procedure starting from certain initial vector. Several tests showed that the best approximation for PM_i brings the following initial vector **PM**:

$$(III.5)\qquad PM_i = [\min_j (P_{ij}/P_{Bj}) * \max_j (P_{ij}/P_{Bj})]^{1/2}; \quad i=1, 2,..., M; \quad PM_B=1$$

where

67. P_{Bj} - the price for basic commodity B ($1 \le B \le M$) in 'j'th country (the indicators like τ are invariant to the commodity selected as basis).

68. International prices PM_i obtained from (III.4) have the highest possible degree of characteristicity for all comparing countries (j=1,N). The results of volume comparisons on the basis of vector PM_i from (III.4) are transitive, additive and they have the highest possible degree of characteristicity for all countries.

69. **Two main new ideas** of proposed method are the following:

1) Using the indicator of similarity of price structures instead of using a distance between values of different kinds of indices. For example, „IP" method uses the distance between IP-indices and Fisher's (or EKS) indices; EKS method uses the distance between EKS indices and Fisher's indices (or other type of bilateral indices).
2) Using the principle of maximum[20] of similarity of price structures for searching of international price PMi instead of functions of following types:
   - the minimisation of the total sum of distances (for all countries in question) between different kinds of indices (as it is made in EKS or IP methods)

   or
   - the maximisation of the total sum of coefficients τj (for all countries in question), i.e. $\sum \tau j \rightarrow \max$.

70. The latter (2) is very important because there are situations when total sum of similarity coefficients is high but some countries have very high τ_j and another countries have very small τ_j.

For example: let us have 3 countries:

1st variant (for given PM^1): $\tau_1 = 0.90$; $\tau_2 = 0.95$; $\tau_3 = 0.40$.
$\min(\tau_j) = 0.40$; $\sum \tau_j = 2.25$.

2nd variant (for given PM^2): $\tau_1 = 0.70$; $\tau_2 = 0.80$; $\tau_3 = 0.60$.
$\min(\tau_j) = 0.60$; $\sum \tau_j = 2.10$.

71. It can believe that the 2nd variant is preferable from the point of view of obtaining the maximal possible characteristic results for all countries.

72. A practical example, from the Eurostat 1997 comparison (see Table V.1): average coefficient of similarity between international prices by the GK method and national prices is some higher (= 0.8980) than by the MPCP method (= 0.8664) but minimal value obtained by the MPCP is much higher (= 0.7661, CH) than by the GK method (= 0.6146, Poland).

73. We would like to illustrate the principal features of the proposed method by help one simple imaginary example. Let us have 3 countries (A, B, C) and an aggregate which consists of two commodities only. Let us start from the following price data:

- for country A : $P_{1A} = 2$, $P_{2A} = 1$;
- for country B : $P_{1B} = 8$, $P_{2B} = 1$;
- for country C : $P_{1C} = 10$, $P_{2C} = 1$.

[20] We think that it is a certain analogy with Pareto's optimality prinziple.

74. The unweighted version (III.1) is used for the simplicity. If there are only two products then the formula (III.1) can be transformed to a more easy form:

(III.6) $$\tau_j = \sqrt{P_{1j} * PM_1} * \frac{P_{1j} + PM_1}{P_{1j}^2 + PM_1^2} \quad ; \quad j = A, B, C$$

because there is only one unknown variable $\mathbf{PM}_1$ ($\mathbf{PM}_2 = 1$).

75. The functions τ_j (III.6) for countries A, B, C are shown on the Chart 1. The function 'max(min τ_j)' for the countries A, B, C is shown separately on the Chart 2.

Chart 1

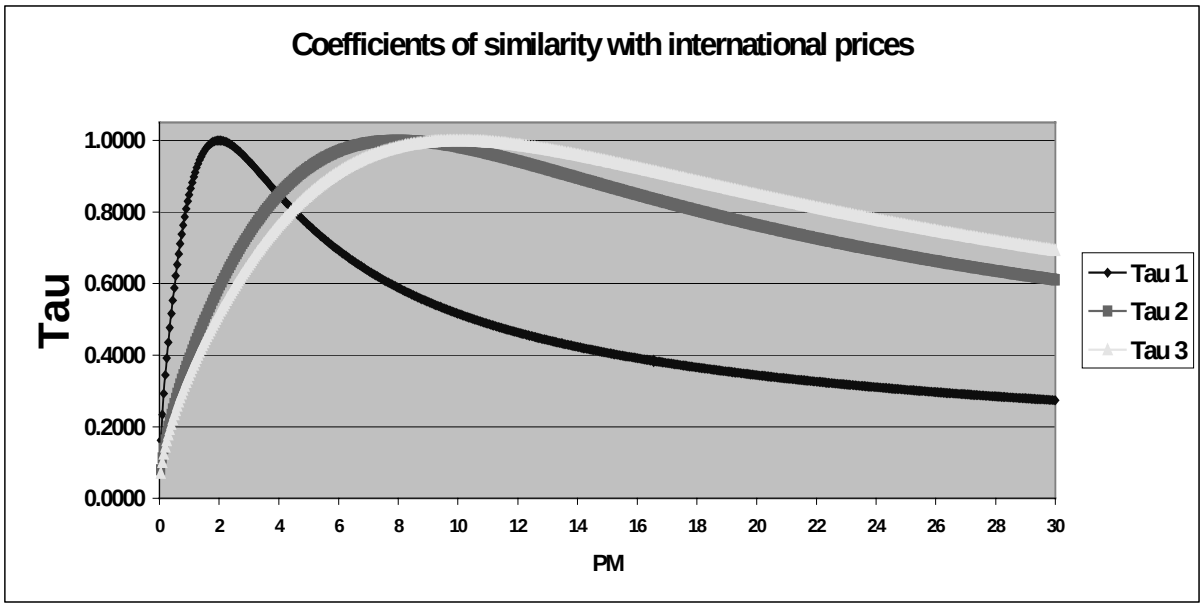
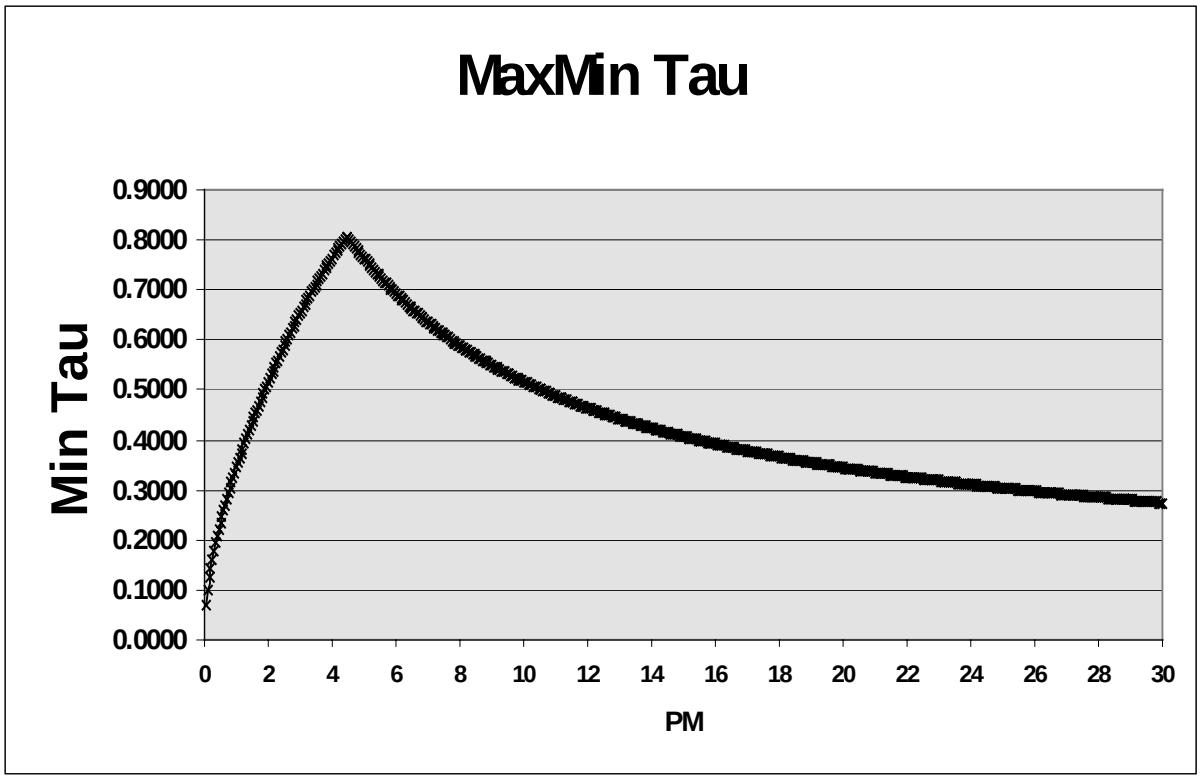


Chart 2



76. The max(min τ_j) for given input data is 0.8065 (τ_A = 0.8065; τ_B = 0.8881; τ_C = 0.8065) which is obtained with international price $\mathbf{PM}_1$ = 4.47 ($\mathbf{PM}_2$ = 1). To the point, in case of only two products the initial approximation for PM_1 calculated by help (III.1) will be exactly equal of optimal solution: $4.47 = (2*10)^{1/2}$.

77. A „pure" MPCP method was described above. The practical application of the MPCP method in actual comparisons can be problematic when a comparing aggregate includes very great number of commodities (about 200-300 items / basic headings). The searching of vector **PM** from (III.4) is very difficult task for the calculations in this case. To do this task more easy and to use some modifications of proposed method.

78. One possible version is the following procedure:

(III.7) $$PM_i = [\min_j (P_{ij} / f_j) * \max_j (P_{ij} / f_j)]^{1/2}; \quad i=1, 2,..., M;$$

where

f_j - global PPP for a given aggregate between the currency of country 'j' (national prices) and implicit common currency (international prices) obtained from (III.4).

79. In this case we have only (N-1) unknown variables f_j (f_N = 1) which will be used in function (III.4) instead of unknown variables PM_i. The values f_j obtained from (III.4) by help (III.7) can be used for the calculation of the international prices.

80. The experiments showed that the „best“ initial values for the variables f_j can be obtained in the following way:

a) calculation of initial values of standardised international prices by (III.7)
$$PM_i = [\min_j (P_{ij}/P_{Mj}) * \max_j (P_{ij}/P_{Mj})]^{1/2} ; \quad i=1, 2,..., M .$$

b) calculation of initial values for PPP (fj) as ratio between total real values in international prices and total nominal values in the national currency.

81. It is possible to use also a combination of GK method and proposed method. In accordance with the GK method the international prices are quantity-weighted average of the PPPs-adjusted national prices:

(III.8) $$PM_i = \sum_{j=1}^{N} (P_{ij} / f_j) * q_{ij} / \sum_{j=1}^{N} q_{ij} ; \quad i = 1,2,...,M$$

82. The values f_j obtained from (III.4) are a basis for calculation of the international prices. Latter modification of searching of international prices is a compromise between depending of international prices by weighting procedure (averaging the national prices) which means their gravitation to the prices of large (rich) countries and their characteristicity for all countries.

83. Due to this modification there is a possibility to use only one set of results of multilateral comparison instead of two sets of results as in EUROSTAT - OECD PPP Programme (EKS - for volume comparisons / the request of characteristicity has the preference and GK - for structural comparisons / the request of additive has the preference). The MPCP method produces additive as well as highest possible results. Hence the MPCP results can be used for structural analysis as well as for volume comparison (due to elimination of Engel-Gerschenkron effect).

84. The MPCP method described above can be used also for linking results of several comparisons (e.g. to make up overall ICP from ICP-Group results or world comparison from a set of regional comparisons).

85. Suppose we have a set of results from ‘L’ regional comparisons. Each regional comparison is based on its own vector of regional international prices $\mathbf{PMR}_k$ (k=1,L). (III.4) can be solved for these regional price sets. In effect we will have the set of world international prices which will be equi-characteristic for each region.

86. The first experimental calculations by the MPCP-method were carried out on the basis of aggregated data for 39 analytical categories from the OECD-90 comparison. The non-weighted version of (III.1) was applied for these first provisional experiments due to high aggregated input data.

87. We can see (Table III.1) that the MPCP-method produced a set of international prices PM_i obtained from (III.1) which have the high degree of characteristicity for all comparing countries. So, obtained values of indicator τ_j lay between 0.8552 (Turkey) and 0.9245 (Greece). Therefore, the results of volume comparisons by help of vector PM_i from (III.1) are practically **equi-characteristic for all OECD countries**.

88. Comparing with GK results it is possible to give the following conclusion. Respective GK prices have high similarity with prices of USA (0.9743) and other rich countries but they have sufficiently lower similarity with the prices of outliers, e.g. Turkey (0.6558). As consequence of Engel-Gerschenkron effect, the volume index per capita for Turkey by the GK method (33%, OECD=100) was substantial higher than one by the EKS method (27%, OECD=100). The Table III.2 contains the volume indices per capita (USA=100) obtained by different methods (GK, EKS and MPCP).

Table III.1
Coefficients of similarity between national and international prices
(data from OECD - 1990 comparison for 39 analytical categories)

Country	by G-K method	by MPCP
Belgium	0.9740	0.9118
Denmark	0.9354	0.9191
France	0.9569	0.8902
Germany	0.9711	0.8707
Greece	0.8460	0.9245
Ireland	0.9165	0.8949
Italy	0.9322	0.8820
Luxembourg	0.9574	0.9001
Netherlands	0.9610	0.9128
Portugal	0.7917	0.8654
Spain	0.9187	0.9208
United Kingdom	0.9453	0.8781
Austria	0.9717	0.9179
Switzerland	0.9599	0.8789
Finland	0.8825	0.8872
Iceland	0.8566	0.8694
Norway	0.9049	0.8717
Sweden	0.9266	0.8908
Turkey	0.6558	0.8552
Australia	0.9605	0.9017
New Zealand	0.9026	0.8699
Japan	0.9028	0.8552
Canada	0.9561	0.8630
USA	0.9743	0.8554

Table III.2
Volume indices per capita for GDP (USA = 100)
(data from OECD-1990 comparison)

Country	Volume indices per capita for GDP (USA=100)		
	by G-K method	by EKS	by MPCP
Belgium	78.7	76.8	75.6
Denmark	84.4	78.4	76.5
France	83.6	81.6	80.3
Germany	86.9	84.8	84.3
Greece	36.9	34.4	34.5
Ireland	50.0	49.6	47.8
Italy	76.2	74.4	73.5
Luxembourg	91.0	90.4	91.5
Netherlands	76.2	73.6	71.7
Portugal	45.9	40.8	42.4
Spain	56.6	55.2	53.6
United Kingdom	76.2	74.4	73.2
Austria	77.9	77.6	75.6
Switzerland	99.2	97.6	97.6
Finland	78.7	76.8	75.9
Iceland	80.3	77.6	77.0
Norway	79.5	75.2	75.5
Sweden	82.8	80.0	78.7
Turkey	27.0	21.6	22.7
Australia	77.0	75.2	73.0
New Zealand	65.6	63.2	62.1
Japan	86.9	82.4	82.6
Canada	91.8	89.6	89.7
USA	100.0	100.0	100.0

The calculations were carried out also for Group II countries on the basis of detailed information from the ECP'93/II (see Table III.3) and the ECP'96/II (see Table III.4). The weighted version and the variant (III.7) were used for these calculations.

Table III.3

Volume indices per capita for GDP (AUT = 100)

The results within ECP'93/IIA calculated by different methods

COUNTRY	Bilateral (Fisher's index)	Official EKS-12 procedure	MPCP-method (weighted version)	Coeff. of price similarity with international prices
POLAND	24.6	23.6	22.3	0.6737
CZECH Rep.	44.1	45.2	40.6	0.7147
HUNGARY	31.7	29.8	26.9	0.6803
RUSSIAN Fed.	24.4	26.8	25.5	0.4732
ROMANIA	19.5	19.4	18.3	**0.7602**
BELARUS	22.2	25.0	26.6	0.4349
BULGARIA	22.6	21.1	20.4	0.5688
CROATIA	20.2	19.5	18.5	0.6770
SLOVAK Rep.	29.4	29.1	26.6	0.6539
SLOVENIA	48.3	47.0	44.5	0.5540
UKRAINE	17.0	17.0	16.0	0.5730
AUSTRIA	100.0	100.0	100.0	**0.4335**

89. As we can see, the volume indices p.c. (Austria = 100) by the MPCP method are usually lower than one by the EKS method. This indicates that the implicit (imaginary) set of international prices indirectly used by the EKS method is closer to the Austrian prices than to prices of the transition countries. The MPCP method cannot guarantee the equi-characteristicity for all countries too but the MPCP method guarantees the maximum possible characteristicity for all countries. Two countries (Austria = the most „market" country and Belarus = the most „non-market" country) had within the ECP'93/II the most different (from other countries) price structures. The set of structural MPCP prices reproduces the national structures of prices in the maximum possible degree: the diapason of coefficients of similarity is 0.4335 (Austria) and 0.7602 (Romania). It means that this set of prices is not very well for Austria but all other sets are worse for the Group II as a whole.

Table III.4
Volume indices per capita for GDP (AUT = 100)
The results within ECP'96/II calculated by different methods

COUNTRY	Bilateral (Fisher's) index	**Official** EKS-14 procedure	MPCP-method (weighted version)	Coeff. of price similarity with int.prices
RUSSIA	28.7	30.5	25.5	0.7948
ROMANIA	26.8	29.8	25.3	0.7463
BELARUS	22.2	23.3	21.2	0.6835
BULGARIA	22.0	22.5	19.1	0.5896
CROATIA	29.3	28.4	23.4	**0.8761**
SLOVENIA	57.7	59.7	52.7	0.7126
UKRAINE	15.4	15.0	13.5	0.7254
MOLDOVA	10.1	9.5	9.1	0.4974
ESTONIA	31.5	30.0	25.5	0.8757
LATVIA	24.9	22.8	19.3	0.8388
LITHUANIA	26.4	26.0	22.0	0.8490
ALBANIA	12.2	13.0	11.3	0.7060
MACEDONIA	19.3	18.8	15.8	0.8677
AUSTRIA	100.0	100.0	100.0	**0.4809**

90. Two countries (Austria = the most „market" country and Moldova = the most „non-market" country) had the most different (from other countries) price structures within the ECP'96/II. The set of structural MPCP prices reproduces the national structures of prices in the maximum possible level: the diapason of coefficients of similarity is following: 0.4809 (Austria) and 0.8761 (Croatia). It means that this set of prices is not very well for Austria (as it was already within the ECP'93/II) but all other sets are worse for the Group II as a whole.

91. It was mentioned above that within the ECP'96/II all Non-market Services (NMS) were treated without productivity adjustment from organisational reason (formal identity with Eurostat methodology due to the presence within the ECP'96/II of several EU Candidate countries). Group II is one part of overall ECP (1) and six Group II countries (Bulgaria, Estonia, Latvia, Lithuania, Romania and Slovenia) are in a status of Candidate Countries (CC) to the EU. Therefore Group II has to produce official results strictly comparable with the results obtained within Eurostat comparison and other parts of the overall ECP.

92. This circumstance decreases the level of dissimilarity of national price structures (especially between Austria and the transition countries). As we can see, the volume indices p.c. (Austria = 100) by the MPCP method are in the most cases substantially lower than one by the EKS method. Probably, the set of structural international prices obtained by MPCP method reduces the influence of high non-similarity of prices for the NMS without productivity adjustment and obtained volume indices per capita seem as more plausible and realistic than official results (without productivity adjustment). Practically the volume indices per capita (Austria = 100) for NMS areas by MPCP are lower for the most of countries than by the official EKS method. This fact does not mean that the MPCP method can automatically produce in a magic way the „true" results on the basis of distorted data, but this means that the MPCP method can reduce this negative impact.

IV. Method of Standardised Structure (SS-method)

93. Obtaining meaningful international prices by the MPCP method is already quite difficult. Some considerations of Mr.Cuthbert concerning the IP algorithm [see Cuthbert (1997) and a paper to the Asia-Pacific Seminar on the use of PPP prepared by Mr.Sakuma together with Mr.Kurabayashi; pages 166-168; 26-30.11.90, Niigata] - the absence of uniqueness and possible instability - are valid, in principle, for the MPCP method too. Both methods (IP and MPCP) are not very operational: the search of an optimum of a complicated function in multidimensional space is very difficult task from a computational point of view and, respectively, the obtaining of meaningful international price vector by an iterative method depends partly on the selection of a good initial approximation.

94. Therefore an attempt is done in this paper to elaborate a new aggregation multilateral method producing additive and simultaneously characteristic results on the basis of some traditional elements of standard elementary indices. This method was named by the author as the method of **Standardised Structure** (**SS** method).

95. The application of an average structure has been used by some methods, e.g. the Tornkqvist-method or the Walsh-method. These methods use the averaging of individual indices between countries. To obtain the invariant results the geometric mean is used by these methods. However this leads simultaneously to the non-additivity of the results because geometric mean is additive in logarithmic terms but not in usual linear terms. To use the average structure with the obtaining of additive results, it is useful to look on the standard quantity Indices of Laspeyres and Paasche from an another point of view than it does usually.

Non-traditional presentation of Laspeyres and Paasche indices

96. Let us have two countries A and B. Input data for an aggregate are sets of national prices (p_A and p_B) and quantities (q_A and q_B).

97. Paasche Quantity index ($P_q^{A/B}$) – an index on the basis of prices of country A can be presented in the following form (the sequence no. of individual items are omitted for the simplicity):

$$\textbf{(IV.1)} \qquad P_q^{A/B} = \frac{\Sigma\, p_A * q_A}{\Sigma\, p_A * q_B} = \frac{\Sigma\, [q_A / (q_A+q_B)] * [p_A * (q_A+q_B)]}{\Sigma\, [q_B / (q_A+q_B)] * [p_A * (q_A+q_B)]} = \frac{\Sigma\, dQ_A * wT_A}{\Sigma\, dQ_B * wT_A} =$$

$$= \frac{\Sigma\, dQ_A * [wT_A / \Sigma\, wT_A]}{\Sigma\, dQ_B * [wT_A / \Sigma\, wT_A]} = \frac{\Sigma\, dQ_A * dwT_A}{\Sigma\, dQ_B * dwT_A}$$

$Q = (q_A+q_B)$ – total quantity for a concrete item for countries A and B

dQ_A,= q_A/Q; $dQ_B = q_B/Q$ – the shares of countries A and B in the total quantity for a concrete item;

$wT_A = p_A * Q$ – total value for a concrete item at the prices of country A

$dwT_A = wT_A / \Sigma wT_A$ – the share of a concrete item in the total value ($\Sigma dwT_A = 1$)

98. Laspeyres Quantity index ($L_q^{A/B}$) – an index on the basis of prices of country B can be presented in a similar form:

$$(IV.2)\quad L_q^{A/B} = \frac{\Sigma p_B * q_A}{\Sigma p_B * q_B} = \frac{\Sigma [q_A / (q_A+q_B)] * [p_B * (q_A+q_B)]}{\Sigma [q_B / (q_A+q_B)] * [p_B * (q_A+q_B)]} = \frac{\Sigma dQ_A * wT_B}{\Sigma dQ_B * wT_B} =$$

$$= \frac{\Sigma dQ_A * [wT_B / \Sigma wT_B]}{\Sigma dQ_B * [wT_B / \Sigma wT_B]} = \frac{\Sigma dQ_A * dwT_B}{\Sigma dQ_B * dwT_B}$$

$Q = (q_A+q_B)$ - – total quantity for a concrete item for countries A and B

dQ_A,= q_A/Q; $dQ_B = q_B/Q$ - – the shares of countries A and B in the total quantity for a concrete item;

$wT_B = p_B * Q$ – total value for a concrete item at the prices of country B;

$dwT_B = wT_B / \Sigma wT_B$ - the share of a concrete item in the total value $(\Sigma dwT_B = 1)$

99. As we can see, two main new features are included in the consideration relatively the traditional presentation of Laspeyres and Paasche indices:

1) the use of the shares of quantities (dQ) instead of the quantity indices
2) the use of general weights dwT (calculated on the basis of set of Quantities –Total) instead of the use on national structures

100. The shares dwT_A reflect the structure of prices of country A, the shares dwT_B – the structure of prices of country B (the vector of quantities is the same in both cases). An arithmetic average of the shares dwT_A and dwT_B will be equi-characteristic for both countries (the distances between the average structure and the structures on the basis of national prices reflect will be the same):

$$(IV.3)\quad \overline{dwT} = (dwT_A + dwT_B)/2 \qquad (\Sigma \overline{dwT} = 1)$$

101. We named this method as the method of standardised structure (**SS method**).

102. In effect, the quantity index between countries A and B by the method of standardised structure ($IQ_{SS}^{A/B}$) can be obtained as:

$$(IV.4)\quad IQ_{SS}^{A/B} = \frac{\Sigma dQ_A * \overline{dwT}}{\Sigma dQ_B * \overline{dwT}}$$

103. It is easy to show that taking into account that [$L_q^{A/B} = 1 / P_q^{B/A}$] the term (IV.4) can be presented as :

$$(IV.5)\qquad IQ_{SS}^{A/B} = \frac{\Sigma\, dQ_A * \overline{dwT}}{\Sigma\, dQ_B * \overline{dwT}} = \frac{\Sigma\, dQ_A * (dwT_A + dwT_B)/2}{\Sigma\, dQ_B * (dwT_A + dwT_B)/2} =$$

$$= \frac{\{\Sigma p_A q_A * [\Sigma p_B * (q_A + q_B)] + \Sigma p_B q_A * [\Sigma p_A * (q_A + q_B)]\} / (\Sigma p_A q_A * \Sigma p_B q_B)}{\{\Sigma p_A q_B * [\Sigma p_B * (q_A + q_B)] + \Sigma p_B q_B * [\Sigma p_A * (q_A + q_B)]\} / (\Sigma p_A q_A * \Sigma p_B q_B)} =$$

$$= \frac{(1 + L_q^{A/B}) + L_q^{A/B} + L_q^{A/B} * L_q^{B/A}}{L_q^{A/B} * L_q^{B/A} + L_q^{B/A} + (1 + L_q^{B/A})} = \frac{L_q^{A/B} * (1 + P_q^{A/B}) + P_q^{A/B} * (1 + L_q^{A/B})}{(1 + P_q^{A/B}) + (1 + L_q^{A/B})}$$

104. The SS method has some similar general features with the CKS method[21]:

a) the PPPs by both methods are not defined directly from the method, rather indirectly (dividing the expenditure ratio by the quantity ratio)

b) both methods use the total values of national quantities as a basis for the standardisation (normalisation)

105. These common features have a general character, the concrete algorithms are different. Some comparative analysis is given below.

106. The CKS quantity index ($IQ_{CKS}^{A/B}$) can be presented as follows:

$$(IV.6)\qquad IQ_{CKS}^{A/B} = L_q^{A/B} * \frac{1 + P_q^{A/B}}{1 + L_q^{A/B}}$$

107. The Edgeworth-Marshall quantity index[22] ($IQ_{EM}^{A/B}$) can be presented in a very similar form:

$$(IV.7)\qquad IQ_{EM}^{A/B} = P_q^{A/B} * \frac{1 + L_q^{A/B}}{1 + P_q^{A/B}}$$

108. As we can see the product of CKS and EM indices (it is related the price indices as well as quantity indices) is Fisher index in quadrat:

$$IQ_{CKS}^{A/B} * IQ_{EM}^{A/B} = (F_q^{A/B})^2,$$

109. ie. Fisher's index ($F_q^{A/B}$) can be presented as a geometric unweighted average from the CKS and EM indices:

$$F_q^{A/B} = (IQ_{CKS}^{A/B} * IQ_{EM}^{A/B})^{1/2}$$

[21] Commensurable Kurabayashi-Sakuma method – see I.Sakuma, D.S.Prasada Rao, Y.Kurabayashi (2000)

[22] The Edgeworth-Marshall method focuses on the PPP calculation. Here the Edgeworth-Marshall quantity index is the index obtained as value index divided by EM-PPP.

110. It is easy to show that the $IQ_{SS}^{A/B}$ index is an arithmetic average from $IQ_{CKS}^{A/B}$ and $IQ_{EM}^{A/B}$ indices with the specific weighs $(1 + L_q^{A/B})$ and $*(1+ P_q^{A/B})$:

$$IQ_{SS}^{A/B} = \frac{IQ_{CKS}^{A/B} *(1 + L_q^{A/B}) + IQ_{EM}^{A/B} *(1+ P_q^{A/B})}{(1 + L_q^{A/B}) + (1 + P_q^{A/B})}$$

111. So, both volume indices: $F_q^{A/B}$ and $IQ_{SS}^{A/B}$ are some averages from the CKS and EM indices[23]. In effect, it can believe that the $IQ_{SS}^{A/B}$ index will be in the most cases (if differences between L- and P-indices are note very high) very close to the Fisher's index. Numerous imitations on the basis of simple numerical examples confirmed this hypotheses. The bilateral versions of the following methods were tested: Edgeworth-Marshall (EM), Geary-Khamis (GK), Gerardi (G), Tornqvist (T), Commensurable Kurabayshi-Sakuma (CKS) and Standardised Structure (SS). The SS method produced in the most cases the closest results to the Fisher index. The Fisher's index is regarded usually as an „ideal" („best") index in a bilateral case. The $IQ_{SS}^{A/B}$ index possesses all properties of the Fisher's index and simultaneously this is an additive index.

Multilateral version of the SS method

112. A bilateral case was considered above. The proposed method can be applied in a multilateral case also[24]. Let us have a multilateral comparison with N countries for an aggregate with M primary groups (basic headings = BH). The standard sets (matrices of size M x N) of input data are the following:

p_{ij} is actual or „notional" price of ith item in the jth country (expressed in the units of national currency or as PPP for primary group i);

q_{ij} is actual (physical units) or „notional" quantity (values in a common currency) for ith item in the jth country.

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[23] It is interesting to note that the bilateral Fisher, EM, CKS and SS indices lay strictly between Laspeyres and Paasche indices. The bilateral Tornqvist, Gerardi and Geary-Khamis indices does not possess this property in general case.

[24] The main idea of the SS method was proposed firstly by S. Sergueev in „*Methods for the multilateral comparisons*". - Ph.D.Dissertation. Research Institut of Central Statistical Office of SU. Moscow, 1982 (in Russian). This method was described also in some later publications of the author of this paper (in Russian):
„*The making of the indices for international comparisons*". - Journal "Economic and mathematical methods", vol.XIX, No.6, 1983. USSR Academy of Sciences, Central Economic Mathematical Institute. (in co-autorship).
„*An improvement of the methods of making of inter-space indices.*" - Journal of the Central State Statistical Commitee of USSR "Westnik statistiki" (Herold of Statistics), Moscow, N 2, 1986.
„*An improvement of international comparisons of main macroeconomic indicators of CMEA-countries.*" - In the book "Statistical cooperation the CMEA-countries", Vol.5. Moscow, CMEA, 1989. (in co-autorship).
„*International comparisons of macroeconomic economic indicators.*" - Journal "Economic cooperation of the CMEA-countries", CMEA, N 11, 1989. (in co-autorship).
„*Modern tendencies in the cross-country comparison of the macroeconomic indicators.*" - Journal "Economic and mathematical methods", vol.XXYI, No.4, 1990. USSR Academy of Sciences, Central Economic Mathematical Institute. (in co-autorship).

113. The general computational algorithm of the multilateral SS method is the same as in a bilateral case described above. The following intermediate indicators should be calculated (i = 1,2, ...,m; j = 1, 2, ...,N):

$Q_i = \sum_{j=1}^{N} q_{ij}$ - total quantity for all countries of the ith basic heading,

$dQ_{ij} = q_{ij} / \sum_{j=1}^{N} q_{ij}$ - share of country j in total Quantity of BH i

$dwT_{ij} = (p_{ij} * Q_i) / \sum_{l=1}^{M} (p_{lj} * Q_l)$ - share of BH i in total Value at prices of country j.

114. The average standardised structure is calculated on the basis of the sets of standardised structures at the national prices of all participating countries:

(IV.8) $$\overline{dwT_i} = \sum_{j=1}^{N} dwT_{ij}/N \qquad i = 1, 2, ..., M$$

115. The quantity (volume) index between each pair of countries j and k for the aggregate in question can be obtained as follows:

(IV.9) $$IQ^{j/k} = \frac{\sum_{i=1}^{M} dQ_{ij} * \overline{dwT_i}}{\sum_{i=1}^{M} dQ_{ik} * \overline{dwT_i}}$$

116. The formula (IV.9) allows to calculate the volume indices for each sub-aggregate but it is much more appropriate to obtain the detailed results (the results at the lower aggregate levels) by help of the respective common set of international prices.

117. The SS method is based on a set of shares ($\overline{dwT_i}$) and a set of respective quantities (Q_i). Therefore, a set of respective prices (we named these prices as structural prices or the prices of standardised structure) can be obtained:

(IV.10) $$\overline{dwT_i} = \frac{\pi_i * Q_i}{\sum_{l=1}^{M} (\pi_l * Q_l)} \qquad i = 1, 2, ..., M$$

where

π_i is „International structural price“ of the ith item.

118. The system of linear equations (IV.10) consists of M linear equations in M unknowns, one is redundant. This system is homogeneous. Obviously, we need for the comparison the prices with accuracy up to scalar, i.e. we need relative prices. We can select a base product (e.g. product M), set its price as 1 (π_M) and to measure all other prices to the relation to the price of base product. By dropping one equation (e.g. last equation) and setting $\pi_M = \mathbf{1}$ we become modified system which is no longer homogeneous because everything is now standardised on the product M.

119. The modified system has (M - 1) equations and (M - 1) unknowns variables π_i:

(IV.11) $$(\mathbf{Q_i} - \mathbf{Q_k}\overline{dwT_i}) * (\pi_i) = \mathbf{Q_M}\overline{dwT_i}$$

where

$\mathbf{Q_i}$ - a diagonal matrix of order (M-1) by (M-1), diagonal elements are Q_i values;

$\mathbf{Q_k}\ \overline{dwT_i}$ - a matrix of order (M-1) by (M-1), elements are Q_k * $\overline{dwT_i}$ values (i –a sequence No. of equations; k – a sequence no. of unknown variable);

$\mathbf{Q_M}\ \overline{dwT_i}$ - a vector of order (M-1), elements are Q_M * $\overline{dwT_i}$ values (i –a sequence No. of equation);

π_i **– a** vector of order (M-1), elements are prices of standardised structure.

120. There is no problem to solve the system of linear equations (11) by the standard methods of linear algebra. However it is possible to obtain an explicit term for the prices of standardised structure by help of an elementary procedure taking into account that the ratio of two shares is equal to product of price and quantity ratios:

(IV.12) $$\overline{dwT_i} : \overline{dwT_M} = \frac{\pi_i * Q_i}{\sum_{l=1}^{M}(\pi_l * Q_l)} : \frac{\pi_M * Q_M}{\sum_{l=1}^{M}(\pi_l * Q_l)} = (\pi_i : \pi_M) * (Q_i : Q_M)$$

121. Selecting the product (basic heading) M as a basis we represent all other prices in the relation to the price of the selected product up to a factor of proportionality:

(IV.13) $$\pi_i = (\overline{dwT_i} : \overline{dwT_M}) / (Q_i : Q_M); \qquad i = 1, 2, ..., M \qquad \pi_M = 1$$

122. The volume index for each pair of countries (j and k) and for each level of aggregation (for each sub-aggregate - assume that this sub-aggregate include basic headings with the sequences No. from l1 to l2: $1 \le l1 < l2 \le M$) can be obtained on the basis of the prices (13) of standardised structure (SS-prices) in an usual way:

(IV.14) $$IQ^{j/k} = \frac{\sum_{i=l1}^{l2} \pi_i * q_{ij}}{\sum_{i=l1}^{l2} \pi_i * q_{ik}}$$

123. The purchasing power parity of given aggregate (GDP, etc.) for the jth country (PPP_j - national currency to common currency) can be obtained in an indirect way as by the following formula:

$$\text{(IV.15)} \qquad PPP_j = \sum_{i=1}^{12} (p_{ij} * q_{ij}) / \sum_{i=1}^{12} (\pi_i * q_{ij}); \qquad j = 1,2,...,N$$

124. The general approach was described above. However it should be indicated here yet one particular problem which is important for practical calculations for all aggregation procedures. There are so-called „Problem categories", i.e. primary groups that sometimes have negative nominal values (and correspondingly- negative „notional" quantities) – „Changes in stocks", „Net export", „Net expenditures of residents abroad", etc. A direct use of negative values can lead to meaningless results - as, for example, a negative international price for primary group or even a negative global PPP for GDP. Problem categories are usually excluded from the aggregation calculations and for these categories are made some special calculations after the main aggregation procedure (sometimes very complicated).

125. A simple method was elaborated by the author to include all categories in general calculations. The „balancing Items" (Net exports", „Changes in stocks", etc.) were included by the SS aggregation procedure by the use of absolute quantities for the calculation of common structural prices (IV.13)[25]. Modified method guarantees the obtaining of positive meaningful structural prices. Of course, the absolute quantities are used for the calculations of structural prices only. For calculations of volume indices (IV.14) the actual quantities (values) based upon fact (with sign) have to be used.

126. In this section we provide an empirical illustration of the new method, termed the method of standardised structure (SS-method), described in the previous section. The illustration uses data drawn from the international comparison undertaken at the OECD for the benchmark year 1996 for all 28 OECD-members.

127. Since the results shown are for purposes of illustration only, the aggregation undertaken here considers data on an aggregate level – 40 analytical categories (see Table IV.2). These categories represent broad commodity groups, PPPs ('National currency/Numeraire currency') for groups are used as „notional" (fictitious) prices and a set of „notional" (fictitious) quantities, each obtained as ratio of nominal value (in national currency) to corresponding PPP. The results (Volume indices per capita, USA = 100) from the SS method and the official results calculated by the EKS method are presented in the Table IV.1.

[25] This modification is applicable to other aggregation procedures (GK, EKS, CKS, etc.) also. See S.Sergueev „*Treatment of basic headings with negative nominal values within the aggregation procedures*" - Eurostat, Meeting of the Working Group on Purchasing Power Parities (LUX, 23th - 25th May 2000).

Table IV.1
OECD 1996 comparison: Volume indices per capita (USA = 100)

	Official results by EKS method	Experimental res. by SS method	Differences
	1	2	3 = 1 - 2
AUT	79.5	77.3	2.2
BEL	79.8	79.2	0.6
FIN	68.5	68.4	0.1
FRA	73.3	71.8	1.5
GER	76.6	76.4	0.3
IRE	65.6	65.1	0.5
ITA	72.8	71.9	0.9
LUX	114.4	119.4	-5.0
NLD	75.6	75.5	0.1
PRT	49.6	49.3	0.3
SPA	54.8	53.0	1.8
DNK	83.1	83.0	0.2
GRC	47.5	46.0	1.5
SWE	71.6	71.8	-0.2
UK	70.2	68.3	1.8
ICE	84.3	82.7	1.6
NOR	91.6	93.9	-2.3
CHE	89.6	91.2	-1.6
TUR	21.6	22.8	-1.2
AUS	76.4	74.9	1.4
NZL	63.1	61.8	1.3
JAP	86.2	87.6	-1.4
CAN	81.7	81.0	0.7
USA	100.0	100.0	0.0
CZE	45.7	48.7	-3.1
HUN	33.5	36.2	-2.7
POL	24.7	27.2	-2.5
MEX	25.8	26.1	-0.3
OECD28	71.4	71.4	-0.024

128. In fact the SS results obtained are quite close to the official results obtained by the EKS method and additionally the SS method produced additive results. The prices of standardised structure were calculated on the basis of formula (IV.13) – see Table IV.2 below. These prices allow to calculate the results for each sub-aggregate.

Table IV.2

OECD 1996 comparison: structural prices by the SS method

	SNA analytical categories	Prices by SS method		SNA analytical categories	Prices by SS method
1	Bread and cereals	**0.8513**	21	Operation transport equipment	**1.0357**
2	Meat	**1.0321**	22	Transport services	**0.8812**
3	Fish	**0.7383**	23	Communication	**0.8790**
4	Milk, cheese and eggs	**0.9244**	24	Recreational equip. & repairs	**1.2049**
5	Oils and fats	**0.9904**	25	Recreational & cultural services	**0.8105**
6	Fruit, vegetables and potatoes	**0.9565**	26	Books, magazines, newspapers	**1.0002**
7	Other food	**0.9917**	27	Education	**0.7644**
8	Non-alcoholic beverages	**1.0953**	28	Restaurants, cafes and hotels	**1.1247**
9	Alcoholic beverages	**1.0509**	29	Other goods and services	**0.8193**
10	Tobacco	**1.0409**	30	Net purchases abroad	**1.0000**
11	Clothing including repairs	**1.0447**	31	Final government consumption	**0.7956**
12	Footwear including repairs	**1.1823**	32	Residential buildings	**0.8305**
13	Gross rent and water charges	**0.7712**	33	Non-residential buildings	**0.8512**
14	Fuel and power	**0.9943**	34	Civil engineering works	**0.7178**
15	Furniture, floor coverings	**0.9163**	35	Transport equipment	**1.1587**
16	Household textiles and repairs	**1.1082**	36	Non-electrical equipment	**0.9336**
17	Household appliances & repairs	**0.9785**	37	Electrical equipment	**0.9908**
18	Other goods & services	**0.9227**	38	Other products	**0.8662**
19	Medical and health care	**0.7351**	39	Increase in stocks	**1.1220**
20	Personal transport equipment	**1.2767**	40	Balance of exports & imports	**1.0000**

129. Of course, this our illustration is based on the aggregated data. The official OECD comparison 1996 includes 213 primary groups (basic headings) and the differences by more detailed input data can be some higher.

V. Empirical Illustrations on the basis of data from actual comparisons

130. To examine the feasibility of the new methods and the numerical differences in the results by the traditional methods within the actual comparisons on the basis of detailed input data, some experimental calculation were made on the basis of input data from official Eurostat comparisons for 1997 and for 1998. The Eurostat comparisons 1997 and 1998 included 270 primary groups (basic headings) for 20 countries.

131. Tables V.1 and V.2 contain the detailed matrices of the coefficients of price similarity. The comparative results (Volume indices per capita) obtained by different methods are placed in the Tables V.3 (year 1997) and Table V.4 (year 1998). To obtain the full comparability with the official results the two-stage procedure was applied by all methods to hold the fixity of the results for the 15 EU Members[26] (a standard principle used within the Eurostat comparisons due to official reasons).

132. Tables V.3 and V.4 shows that the SS method works enough efficiently within the actual comparisons on the basis of detailed data. The SS method can be recommended for the practical applications due to its simplicity. In fact the SS results are quite close to the official Eurostat results obtained by the EKS method, i.e. these SS results posess the property of characteristicity and additionally the SS method produced additive results.

[26] It means that the methods were applied fistly to the 15 EU countries, secondly – to all 20 participating countries and the results (Volume indices) for 15 EU Members from the 1st calculation were integrated in the results obtained during the 2nd calculation. The answer on the question: What should be a object for the fixity by the methods like the EKS – Volume indices or PPPs? is not unambigious. This problem is described in the S.Sergueev „Calculation of fixed results within the EU comparison: some considerations“ - Paper for a special meeting of PPP compiliers (Eurostat, OECD, OeSTAT) -LUX, 20.11.98).

Table V.1

Coefficients of similarity of price structures: Eurostat 1997 comparison (part 1)

(minimal and maximal values for countries in columns are highlighted)

		D	F	I	NL	B	L	*UK*	IRL	*DK*	*EL*	E	P	A	*S*	FIN
1	**D**	**1.0000**	0.9009	0.8308	0.9226	0.9252	0.9157	0.8706	0.8281	0.8885	0.8150	0.8273	0.6316	0.9104	0.8885	0.8687
2	**F**	0.9009	**1.0000**	0.8508	0.9261	0.8727	0.8781	0.8831	0.8711	0.8987	0.8231	0.8830	0.7266	0.9098	0.9058	0.8279
3	**I**	0.8308	0.8508	**1.0000**	0.8565	0.8840	0.8501	0.8607	0.8900	0.8576	0.8625	0.9254	0.7717	0.9004	0.8574	0.8366
4	**NL**	0.9226	0.9261	0.8565	**1.0000**	0.9393	0.8923	0.8956	0.8811	0.9132	0.8385	0.8702	0.7122	0.9191	0.8972	0.8760
5	**B**	0.9252	0.8727	0.8840	0.9393	**1.0000**	0.9101	0.8996	0.8947	0.8977	0.8661	0.8834	0.7067	0.9205	0.8792	0.8526
6	**L**	0.9157	0.8781	0.8501	0.8923	0.9101	**1.0000**	0.8245	0.8172	0.8468	0.8060	0.8548	0.6044	0.8851	0.8614	0.8253
7	***UK***	0.8706	0.8831	0.8607	0.8956	0.8996	0.8245	**1.0000**	0.9142	0.8735	0.8508	0.8740	0.7688	0.9121	0.9100	0.9015
8	**IRL**	0.8281	0.8711	0.8900	0.8811	0.8947	0.8172	0.9142	**1.0000**	0.8625	0.8429	0.8746	0.8064	0.8965	0.8738	0.8445
9	***DK***	0.8885	0.8987	0.8576	0.9132	0.8977	0.8468	0.8735	0.8625	**1.0000**	0.8326	0.8604	0.7420	0.9326	0.9292	0.9009
10	***EL***	0.8150	0.8231	0.8625	0.8385	0.8661	0.8060	0.8508	0.8429	0.8326	**1.0000**	0.8752	0.7982	0.8633	0.8049	0.8516
11	**E**	0.8273	0.8830	0.9254	0.8702	0.8834	0.8548	0.8740	0.8746	0.8604	0.8752	**1.0000**	0.8384	0.9061	0.8552	0.8242
12	**P**	0.6316	0.7266	0.7717	0.7122	0.7067	0.6044	0.7688	0.8064	0.7420	0.7982	0.8384	**1.0000**	0.7715	0.6979	0.7261
13	**A**	0.9104	0.9098	0.9004	0.9191	0.9205	0.8851	0.9121	0.8965	0.9326	0.8633	0.9061	0.7715	**1.0000**	0.9316	0.9017
14	***S***	0.8885	0.9058	0.8574	0.8972	0.8792	0.8614	0.9100	0.8738	0.9292	0.8049	0.8552	0.6979	0.9316	**1.0000**	0.9480
15	**FIN**	0.8687	0.8279	0.8366	0.8760	0.8526	0.8253	0.9015	0.8445	0.9009	0.8516	0.8242	0.7261	0.9017	0.9480	**1.0000**
16	**CH**	0.9024	0.8416	0.8072	0.8478	0.8413	0.8934	0.8155	0.7834	0.8477	0.7441	0.7932	0.5766	0.8771	0.8839	0.8850
17	**IS**	0.7913	0.7917	0.7431	0.7851	0.7480	0.7302	0.8305	0.8236	0.7198	0.7943	0.7767	0.7085	0.8099	0.8304	0.8458
18	**NO**	0.8472	0.8568	0.8592	0.8769	0.8708	0.8119	0.9096	0.8869	0.9187	0.8207	0.8342	0.7226	0.9120	0.9433	0.8900
19	**PL**	0.4607	0.5520	0.6155	0.5661	0.5580	0.4837	0.6424	0.6220	0.5756	0.6758	0.6598	0.8046	0.5928	0.5397	0.5687
20	**CY**	0.7995	0.6990	0.8323	0.8158	0.8313	0.7946	0.8119	0.8209	0.8141	0.8570	0.8250	0.7386	0.8167	0.8071	0.8510
Average - Total		**0.8329**	**0.8368**	**0.8364**	**0.8543**	**0.8516**	**0.8150**	**0.8552**	**0.8439**	**0.8480**	**0.8222**	**0.8443**	**0.7291**	**0.8721**	**0.8550**	**0.8435**
Average - EU15		**0.8589**	**0.8684**	**0.8596**	**0.8814**	**0.8808**	**0.8408**	**0.8742**	**0.8641**	**0.8740**	**0.8379**	**0.8680**	**0.7359**	**0.8972**	**0.8743**	**0.8561**
Average - Euro11		**0.8561**	**0.8647**	**0.8596**	**0.8796**	**0.8789**	**0.8433**	***0.8732***	**0.8604**	***0.8728***	***0.8402***	**0.8687**	**0.7296**	**0.8921**	***0.8724***	**0.8384**

Coefficients of similarity of price structures: Eurostat 1997 comparison (part 2)

		CH	IS	NO	PL	CY	SS prices		MPCP prices		GUCW prices		GK prices	
							EUR20	EU15	EUR20	EU15	EUR20	EU15	EUR20	EU15
1	**D**	0.9024	0.7913	0.8472	0.4607	0.7995	0.9243	0.9309	0.8418	0.8802	0.9248	0.9330	0.9437	0.9499
2	**F**	0.8416	0.7917	0.8568	0.5520	0.6990	0.9262	0.9423	0.8888	0.9022	0.9218	0.9381	0.9313	0.9383
3	**I**	0.8072	0.7431	0.8592	0.6155	0.8323	0.9338	0.9354	0.8975	0.9277	0.9340	0.9358	0.9295	0.9262
4	**NL**	0.8478	0.7851	0.8769	0.5661	0.8158	0.9500	0.9558	0.8968	0.9179	0.9524	0.9588	0.9578	0.9594
5	**B**	0.8413	0.7480	0.8708	0.5580	0.8313	0.9477	0.9544	0.8847	0.9274	0.9502	0.9582	0.9558	0.9583
6	**L**	0.8934	0.7302	0.8119	0.4837	0.7946	0.8979	0.9063	0.8119	0.8771	0.8964	0.9072	0.9087	0.9139
7	***UK***	0.8155	0.8305	0.9096	0.6424	0.8119	0.9535	0.9520	0.9138	0.9101	0.9539	0.9526	0.9514	0.9492
8	**IRL**	0.7834	0.8236	0.8869	0.6220	0.8209	0.9370	0.9349	0.9017	0.9144	0.9359	0.9349	0.9200	0.9202
9	***DK***	0.8477	0.7198	0.9187	0.5756	0.8141	0.9472	0.9488	0.8828	0.8968	0.9434	0.9463	0.9300	0.9299
10	***EL***	0.7441	0.7943	0.8207	0.6758	0.8570	0.9114	0.9088	0.9009	0.9218	0.9155	0.9127	0.9044	0.8999
11	**E**	0.7932	0.7767	0.8342	0.6598	0.8250	0.9429	0.9458	0.9173	0.9530	0.9453	0.9468	0.9387	0.9341
12	**P**	0.5766	0.7085	0.7226	0.8046	0.7386	0.8039	**0.7956**	0.8477	**0.8600**	0.8080	**0.7970**	0.7756	**0.7592**
13	**A**	0.8771	0.8099	0.9120	0.5928	0.8167	0.9684	0.9744	0.9112	0.9346	0.9679	0.9736	0.9612	0.9628
14	***S***	0.8839	0.8304	0.9433	0.5397	0.8071	0.9455	0.9460	0.8681	0.8752	0.9431	0.9437	0.9306	0.9318
15	**FIN**	0.8850	0.8458	0.8900	0.5687	0.8510	0.9327	0.9254	0.8517	0.8815	0.9316	0.9256	0.9084	0.9084
16	**CH**	**1.0000**	0.7444	0.8296	0.4125	0.7864	0.8724	0.8741	**0.7661**	0.8107	0.8685	0.8725	0.8743	0.8806
17	**IS**	0.7444	**1.0000**	0.8911	0.6494	0.7978	0.8773	0.8483	0.8703	0.8139	0.8793	0.8521	0.8578	0.8498
18	**NO**	0.8296	0.8911	**1.0000**	0.6086	0.8024	0.9362	0.9250	0.8768	0.8559	0.9353	0.9250	0.9161	0.9131
19	**PL**	0.4125	0.6494	0.6086	**1.0000**	0.6485	**0.6332**	0.6147	0.7679	0.6574	**0.6419**	0.6183	**0.6146**	0.5946
20	**CY**	0.7864	0.7978	0.8024	0.6485	**1.0000**	0.8744	0.8525	0.8308	0.8486	0.8753	0.8557	0.8497	0.8429
Average - Total		**0.7954**	**0.7796**	**0.8470**	**0.5914**	**0.7974**	**0.9058**	**0.9036**	**0.8664**	**0.8783**	**0.9062**	**0.9044**	**0.8980**	**0.8961**
Average - EU15		**0.8227**	**0.7819**	**0.8641**	**0.5945**	**0.8077**	**0.9282**	**0.9304**	**0.8811**	**0.9053**	**0.9283**	**0.9310**	**0.9231**	**0.9228**
Average - Euro11		**0.8226**	**0.7776**	**0.8517**	**0.5895**	**0.8023**	**0.9241**	**0.9274**	**0.8774**	**0.9069**	**0.9244**	**0.9281**	**0.9210**	**0.9210**

Table V.2
Coefficients of similarity of price structures: Eurostat 1998 comparison (part 1)

(minimal and maximal values for countries in columns are highlighted)

		D	F	I	NL	B	L	*UK*	IRL	*DK*	*EL*	E	P	A	*S*	FIN
1	**D**	**1.0000**	0.9075	0.8456	0.9251	0.9337	0.9389	0.8861	0.7524	0.8967	0.8204	0.8462	0.6510	0.9236	0.9037	0.8878
2	**F**	0.9075	**1.0000**	0.8946	0.9355	0.8739	0.8902	0.8849	0.8275	0.8986	0.8231	0.8904	0.7258	0.9146	0.9102	0.8269
3	**I**	0.8456	0.8946	**1.0000**	0.8975	0.8805	0.8322	0.8646	0.8706	0.8795	0.8532	0.9203	0.8442	0.9102	0.8787	0.8377
4	**NL**	0.9251	0.9355	0.8975	**1.0000**	0.9453	0.8913	0.9017	0.8428	0.9191	0.8607	0.8749	0.7476	0.9206	0.8845	0.8749
5	**B**	0.9337	0.8739	0.8805	0.9453	**1.0000**	0.9152	0.9001	0.8232	0.9031	0.8664	0.8831	0.7147	0.9249	0.8797	0.8650
6	**L**	0.9389	0.8902	0.8322	0.8913	0.9152	**1.0000**	0.8228	0.8386	0.8606	0.7914	0.8617	0.6341	0.8941	0.8761	0.8455
7	***UK***	0.8861	0.8849	0.8646	0.9017	0.9001	0.8228	**1.0000**	0.7616	0.8642	0.8495	0.8742	0.7665	0.9115	0.8985	0.9005
8	**IRL**	0.7524	0.8275	0.8706	0.8428	0.8232	0.8386	0.7616	**1.0000**	0.8202	0.8066	0.8657	0.7459	0.7663	0.9017	0.8106
9	***DK***	0.8967	0.8986	0.8795	0.9191	0.9031	0.8606	0.8642	0.8202	**1.0000**	0.8271	0.8774	0.7468	0.9325	0.9187	0.9020
10	***EL***	0.8204	0.8231	0.8532	0.8607	0.8664	0.7914	0.8495	0.8066	0.8271	**1.0000**	0.8791	0.8208	0.8606	0.8158	0.8461
11	**E**	0.8462	0.8904	0.9203	0.8749	0.8831	0.8617	0.8742	0.8657	0.8774	0.8791	**1.0000**	0.8507	0.9207	0.8658	0.8453
12	**P**	0.6510	0.7258	0.8442	0.7476	0.7147	0.6341	0.7665	0.7459	0.7468	0.8208	0.8507	**1.0000**	0.7886	0.6730	0.6884
13	**A**	0.9236	0.9146	0.9102	0.9206	0.9249	0.8941	0.9115	0.7663	0.9325	0.8606	0.9207	0.7886	**1.0000**	0.9390	0.9115
14	***S***	0.9037	0.9102	0.8787	0.8845	0.8797	0.8761	0.8985	0.9017	0.9187	0.8158	0.8658	0.6730	0.9390	**1.0000**	0.9488
15	**FIN**	0.8878	0.8269	0.8377	0.8749	0.8650	0.8455	0.9005	0.8106	0.9020	0.8461	0.8453	0.6884	0.9115	0.9488	**1.0000**
16	**CH**	0.9102	0.8509	0.7998	0.8374	0.8453	0.9032	0.8198	0.8232	0.8488	0.7372	0.8101	0.5975	0.8859	0.8898	0.9022
17	**IS**	0.8020	0.7999	0.7938	0.8285	0.8092	0.7680	0.8404	0.8468	0.8072	0.8217	0.8303	0.7572	0.8435	0.8870	0.8796
18	**NO**	0.8465	0.8461	0.8608	0.8868	0.8616	0.8185	0.8996	0.8622	0.8982	0.8262	0.8451	0.7278	0.9117	0.9363	0.8860
19	**PL**	0.5305	0.5730	0.6323	0.6293	0.5945	0.4785	0.6493	0.5959	0.6049	0.6957	0.6460	0.7924	0.6167	0.5753	0.5793
20	**CY**	0.8022	0.7032	0.8382	0.8176	0.8322	0.7978	0.8123	0.8094	0.8118	0.8541	0.8231	0.7707	0.8171	0.8158	0.8654
Average - Total		**0.8426**	**0.8409**	**0.8492**	**0.8643**	**0.8553**	**0.8241**	**0.8478**	**0.8090**	**0.8535**	**0.8240**	**0.8532**	**0.7391**	**0.8733**	**0.8631**	**0.8476**
Average - EU15		**0.8656**	**0.8717**	**0.8721**	**0.8872**	**0.8792**	**0.8495**	**0.8633**	**0.8167**	**0.8747**	**0.8372**	**0.8754**	**0.7427**	**0.8942**	**0.8781**	**0.8565**
Average - Euro11		**0.8612**	**0.8687**	**0.8733**	**0.8855**	**0.8760**	**0.8542**	***0.8613***	**0.8143**	***0.8760***	***0.8389***	**0.8759**	**0.7391**	**0.8875**	***0.8783***	**0.8394**

Coefficients of similarity of price structures: Eurostat 1998 comparison (part 2)

		CH	IS	NO	PL	CY	SS prices		MPCP prices		GUCW prices		GK prices	
							EUR20	EU15	EUR20	EU15	EUR20	EU15	EUR20	EU15
1	**D**	0.9102	0.8020	0.8465	0.5305	0.8022	0.9284	0.9354	0.8429	0.8768	0.9278	0.9372	0.9446	0.9529
2	**F**	0.8509	0.7999	0.8461	0.5730	0.7032	0.9290	0.9462	0.8856	0.9234	0.9242	0.9421	0.9410	0.9501
3	**I**	0.7998	0.7938	0.8608	0.6323	0.8382	0.9466	0.9462	0.9184	0.9632	0.9470	0.9471	0.9425	0.9388
4	**NL**	0.8374	0.8285	0.8868	0.6293	0.8176	0.9548	0.9595	0.9125	0.9361	0.9567	0.9625	0.9627	0.9640
5	**B**	0.8453	0.8092	0.8616	0.5945	0.8322	0.9486	0.9546	0.8918	0.9302	0.9502	0.9581	0.9571	0.9600
6	**L**	0.9032	0.7680	0.8185	0.4785	0.7978	0.9038	0.9121	0.8052	**0.8662**	0.9023	0.9139	0.9143	0.9205
7	***UK***	0.8198	0.8404	0.8996	0.6493	0.8123	0.9496	0.9492	0.9052	0.9285	0.9494	0.9498	0.9502	0.9496
8	**IRL**	0.8232	0.8468	0.8622	0.5959	0.8094	0.8746	0.8547	0.8709	0.8816	0.8862	0.8742	0.8566	0.8484
9	***DK***	0.8488	0.8072	0.8982	0.6049	0.8118	0.9480	0.9496	0.8849	0.9083	0.9440	0.9466	0.9350	0.9338
10	***EL***	0.7372	0.8217	0.8262	0.6957	0.8541	0.9100	0.9072	0.9127	0.9044	0.9134	0.9103	0.9033	0.8980
11	**E**	0.8101	0.8303	0.8451	0.6460	0.8231	0.9437	0.9458	0.9154	0.9472	0.9456	0.9476	0.9385	0.9345
12	**P**	0.5975	0.7572	0.7278	0.7924	0.7707	0.8060	**0.7943**	0.8666	0.8704	0.8111	**0.7969**	0.7839	**0.7638**
13	**A**	0.8859	0.8435	0.9117	0.6167	0.8171	0.9719	0.9762	0.9102	0.9477	0.9705	0.9749	0.9660	0.9685
14	***S***	0.8898	0.8870	0.9363	0.5753	0.8158	0.9475	0.9473	0.8778	0.8957	0.9452	0.9450	0.9330	0.9340
15	**FIN**	0.9022	0.8796	0.8860	0.5793	0.8654	0.9350	0.9283	0.8675	0.8790	0.9336	0.9282	0.9094	0.9091
16	**CH**	**1.0000**	0.7743	0.8264	0.4508	0.7861	0.8761	0.8789	**0.7691**	0.8151	0.8727	0.8780	0.8742	0.8815
17	**IS**	0.7743	**1.0000**	0.9170	0.6588	0.8000	0.8875	0.8643	0.8803	0.8411	0.8931	0.8696	0.8668	0.8572
18	**NO**	0.8264	0.9170	**1.0000**	0.6263	0.8104	0.9379	0.9269	0.8920	0.8977	0.9380	0.9273	0.9164	0.9129
19	**PL**	0.4508	0.6588	0.6263	**1.0000**	0.6450	**0.6456**	0.6278	0.7695	0.6715	**0.6525**	0.6308	**0.6311**	0.6150
20	**CY**	0.7861	0.8000	0.8104	0.6450	**1.0000**	0.8729	0.8515	0.8518	0.8389	0.8745	0.8550	0.8489	0.8413
Average - Total		**0.8052**	**0.8140**	**0.8470**	**0.6092**	**0.8007**	**0.9059**	**0.9028**	**0.8715**	**0.8861**	**0.9069**	**0.9048**	**0.8988**	**0.8967**
Average - EU15		**0.8307**	**0.8210**	**0.8609**	**0.6129**	**0.8114**	**0.9265**	**0.9271**	**0.8845**	**0.9106**	**0.9272**	**0.9290**	**0.9225**	**0.9217**
Average - Euro11		**0.8332**	**0.8144**	**0.8503**	**0.6062**	**0.8070**	**0.9220**	**0.9230**	**0.8806**	**0.9111**	**0.9232**	**0.9257**	**0.9197**	**0.9191**

Table V.3
The Eurostat 1997 results by different methods
Volume indices per capita (EU15 = 100; with fixity for EU15*)

Country	EKS official	SS method	MPCP	G-UCW	GK
D	108.5	108.4	109.5	108.4	107.6
F	101.9	101.7	101.6	101.7	101.8
I	100.0	100.0	101.0	100.1	99.6
NL	109.7	108.7	108.8	108.6	109.1
B	112.2	112.0	112.6	111.6	111.3
L	166.8	170.9	175.5	171.1	172.2
UK	101.5	100.6	100.0	100.6	100.9
IRL	101.4	101.7	102.8	101.9	103.3
DK	122.7	122.5	123.1	122.7	125.1
EL	66.3	68.7	66.6	68.4	69.0
E	79.1	79.7	76.8	79.7	79.8
P	72.6	77.6	75.0	77.5	78.6
A	113.0	111.7	112.4	111.8	112.3
S	102.4	102.7	105.3	102.9	104.7
FIN	98.8	98.5	100.7	98.8	100.1
CH	129.4	130.8	131.9	130.9	131.1
IS	116.8	119.1	118.3	119.0	120.1
NO	130.4	132.5	135.3	132.8	135.9
PL	35.8	39.4	37.4	39.2	40.1
CY	79.9	84.4	88.7	84.5	83.6

*) Official 1997 EKS results were calculated with two levels of fixity: EU15 and EEA18

TableV.4
The Eurostat 1998 results by different methods
Volume indices per capita (EU15 = 100; with fixity for EU15*)

Country	EKS official	SS method	MPCP	G-UCW	GK
D	106.2	106.2	107.3	106.2	105.8
F	98.8	98.5	98.9	98.4	98.7
I	104.0	104.9	104.4	104.9	104.1
NL	114.3	113.4	114.5	113.4	113.4
B	111.6	111.2	110.9	110.7	110.8
L	176.1	178.7	183.1	178.9	180.3
UK	102.2	101.7	100.5	101.7	102.0
IRL	112.0	112.5	112.8	112.6	113.6
DK	121.1	120.9	121.5	121.0	122.6
EL	67.7	70.0	69.7	69.7	70.3
E	79.0	78.2	77.3	78.3	78.4
P	72.4	76.4	75.0	76.4	77.4
A	110.0	108.5	109.0	108.6	108.8
S	102.4	102.6	103.7	102.9	104.1
FIN	100.8	100.5	101.8	100.8	101.8
CH	128.2	129.7	131.2	129.8	130.1
IS	121.1	122.0	120.7	122.2	123.1
NO	123.3	124.2	125.0	124.1	126.7
PL	38.3	42.6	40.3	42.3	42.1
CY	80.7	84.9	87.4	85.0	85.1

*) Official 1998 EKS results were calculated with one level of fixity: EU15.

5. Conclusions

133. The multilateral methods are applied within the international comparisons many years. The analysis showed once again that there is no uniform method and various methods are necessary used for various aims. It is very desirable to have methods which combine the advantages and eliminate the disadvantages of different methods. This conclusion means in the practice that a set of possible multilateral methods should be broad and new ideas and proposals are desirable.

134. Mainly two kinds of multilateral aggregation methods are used in the present time:

A) **Averaging of bilateral indices** (e.g. the EKS method, etc.)
B) **Use of average international prices** – an averaging of national prices recalculated by PPPs into a common currency (the Geary-Khamis, Van IJzeren, Iklé methods, etc)

135. The methods of type **B** are based usually on the simultaneous calculation of the international prices and global PPPs or global quantities within a system of equations, i.e. international prices are some functions from global PPPs (e.g. **Geary-Khamis, Van IJzeren, Rao** methods) or global quantities (e.g. the **CKS** method) or even both (**Iklé** method) and simultaneously global PPPs (global quantities) are some functions from these international prices.

136. A new kind of the aggregation procedures based on the use of so called structural international prices was proposed in this paper. It was shown that a volume index (IQ) for any pair of countries j and k can be calculated not only on the basis of a set of common prices but also on the basis of their ratios (e.g. to a commodity M selected as a basis / numeraire). This circumstance allows to use an averaging of national price ratios instead of the averaging of national prices as it is done by the methods of type **B**.

137. The structure of national prices for country j can be presented as the quadratic matrix of price ratios or in a compressed form – as a vector of price ratios to price of basic commodity M (P_{Mj}). These internal (within a country) price ratios are not depend on the national currencies, i.e. they are directly comparable between countries and can be averaged without the use of PPPs. These average international price ratios can be named as **international structural prices**. The respective PPPs are derived indirectly during the next independent step by known international structural prices as ratio between value for a given aggregate in national prices (national currency) and value measured by known international prices. In effect, the methods using structural prices are strictly additive.

138. Two new methods of the basis of international structural prices were proposed in the paper.

139. The **MPCP method** (**M**aximal **P**ossible **C**haracteristic **P**rices) is an additive method which reflects in maximal possible degree the price structures of all participating countries, i.e. allows to obtain the maximally possible characteristic results. Therefore this method can be fruitfully used for volume comparisons as well as for structural comparisons for heterogeneous set of countries. It is very important for the overall ICP which consists of very heterogeneous set of countries.

140. An attempt was done in this paper to elaborate a new aggregation multilateral method producing additive and simultaneously characteristic results on the basis of the use of some traditional elements of standard elementary indices like Lasypeyres and Paasche. The standard quantity Indices of Laspeyres and Paasche are presented from an another point of view than it did usually. Two main new features were included in the consideration relatively the traditional presentation of Laspeyres and Paasche indices:

1) the use of the shares of quantities (dQ) instead of the quantity indices
2) the use of general weights dwT (calculated on the basis of set of Quantities –Total) instead of the use on national structures

141. Taking into account these new features in the bilateral case, a new multilateral method was proposed on the basis of the use of an average **S**tandardised **S**tructure for N participating countries (therefore this method was named as the **SS method** – method of **S**tandardised **S**tructure). It was

demonstrated that a set of respective common international prices (these prices were named as structural prices or the prices of standardised structure) can be obtained in a very simple way on the basis of the standardised structure and a set of respective common quantities.

142. Several empirical illustrations of the new methods were provided. The 1st illustration used the aggregated data (40 analytical categories) drawn from the OECD comparisons for the benchmark years 1990 (24 OECD-Members) and for 1996 (28 OECD-Members). The more detailed experiments were done to examine the feasibility of the new methods within an actual comparison on the basis of detailed input data: Eurostat comparison for 1997 and 1998 (20 countries; 270 primary groups).

143. The experiments showed that the methods on the basis of international structural prices work enough efficiently within the actual comparisons on the basis of detailed data. The SS method can be recommended for the practical applications due to its simplicity. The experiments showed, that, in fact, the SS results are quite close to the official Eurostat/OECD results obtained by the EKS method, i.e. these SS results posess the property of characteristicity and additionally the SS method produced additive results.

144. Further investigations and comparative analysis of advantages and disadvantages of different methods allows to improve aggregation multilateral methods for ICP-purposes.

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