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Linking countries and regions using chaining methods and spanning trees

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Linking Countries and Regions using Chaining Methods and Spanning Trees

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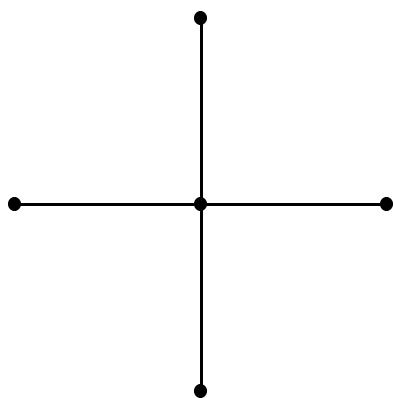
Although chaining is now widely used in time-series comparisons, it has not made much impact on the international comparisons literature. This is because an international comparison lacks a natural ordering of countries analogous to the chronological ordering of a time series. This paper proposes a methodology for linking countries together, so that international price and quantity indexes can be chained. Then it is shown how chaining can considerably simplify, and cut the cost of, multilateral international comparisons, while at the same time increasing characteristicity.

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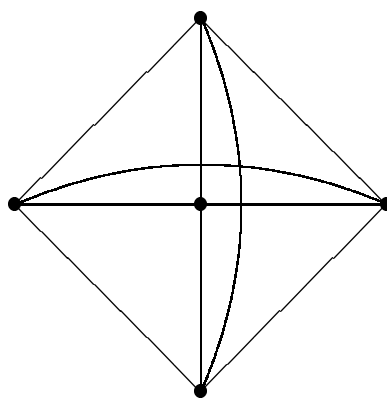
1. The Underlying Structure of Multilateral Methods for Computing International Price and Quantity Indexes

Graph Theory provides a useful framework for analyzing the underlying structure of multilateral methods for computing international price and quantity indexes. A graph consists of a collection of vertices linked by edges. In the context of international comparisons, each vertex represents one of the countries in the comparison, while each edge represents a bilateral comparison between a pair of countries. Two particularly important graphs are the *star graph* and the *complete graph*. Both are depicted in Figure 1, for the case of 5 vertices. The star graph has one central vertex to which all other vertices are connected. In contrast, in a complete graph, all pairs of vertices are connected.

FIGURE 1. — EXAMPLES OF GRAPHS



Star Graph



Complete Graph

Two broad classes of multilateral methods have been proposed in the index number literature.¹ The first class compares each country with an artificially constructed average country. By implication, the underlying structure of such methods is a star graph, with the artificial average country at the center of the star. For this reason, methods of

¹See Hill (1997) for a more general taxonomy of multilateral methods

this class are referred to here as *star methods*. Each bilateral comparison in the star is usually made using the Paasche price index formula, with the artificial country as the base. That is, the price index of country k , P_k , is calculated as follows:

$$P_k = P_{Xk}^P = \frac{p_k \cdot q_k}{p_X \cdot q_k}, \quad (1)$$

where p_k and q_k denote, respectively, the price and quantity vector of country k , while p_X denotes the price vector of the artificially constructed average country. Star methods that use the Paasche formula are sometimes also called *average price methods*.² The most widely used average price method is Geary (1958)-Khamis (1972). In particular, it has been used to make comparisons across the OECD countries and by the International Comparison Program (ICP) to construct the Penn World Table.³ The Geary-Khamis average price vector, p_X , is computed as follows:

$$p_{Xi} = \sum_{k=1}^K \left(\frac{q_{ki}}{\sum_{j=1}^K q_{ji}} \frac{p_{ki}}{P_{Xk}^P} \right) \quad \forall i = 1, \dots, N, \quad (2)$$

where $k = 1, \dots, K$ indexes the set of countries and $i = 1, \dots, N$ the set of headings over which the comparison is made. The average price vector, p_X , and Paasche price indexes, P_{Xk}^P , are obtained by solving the system of $N + K$ simultaneous equations in (1) and (2).⁴ Geary-Khamis has been criticized because it gives greater weight to the price vectors of larger countries when determining the reference price vector. Two equally weighted variants on the Geary-Khamis average price vector are the following:

$$p_{Xi} = \sum_{k=1}^K \left(\frac{v_{ki}}{\sum_{j=1}^K v_{ji}} \frac{p_{ki}}{P_{Xk}^P} \right) \quad \forall i = 1, \dots, N, \quad (3)$$

$$p_{Xi} = \sum_{k=1}^K \left(\frac{q_{ki}/Q_{Xk}^L}{\sum_{j=1}^K (q_{ji}/Q_{Xj}^L)} \frac{p_{ki}}{P_{Xk}^P} \right) \quad \forall i = 1, \dots, N. \quad (4)$$

In equation (3), v_{ki} denotes the expenditure share of good i in country k , i.e., $v_{ki} = \frac{p_{ki}q_{ki}}{\sum_{i=1}^N p_{ki}q_{ki}}$. In equation (4), $Q_{Xk}^L = \frac{(p_X \cdot q_k)}{(p_X \cdot q_X)}$ denotes a Laspeyres

²An attractive feature of such methods is that they generate quantity indexes, when expressed in value terms, that literally add up over different levels of aggregation. This additivity property is particularly useful in national accounts comparisons.

³See, for example, OECD (1996), Summers and Heston (1991) and World Bank (1993).

⁴Khamis (1972) proves existence and uniqueness for the Geary-Khamis system.

quantity index. It is not necessary to define q_X since the $(p_X \cdot q_X)$ term cancels out of (4). The formula in (4) was first proposed by Iklè (1972).

The second class, which includes EKS (see Eltetö and Köves, 1964, and Szulc, 1964) and CCD (see Caves, Christensen and Diewert, 1982), makes bilateral comparisons between all possible pairs of countries. This means that the underlying structure of such methods is a complete graph. Hence methods of this class are referred to here as complete-graph methods. However, to obtain an internally consistent set of multilateral price indexes from a complete graph, the bilateral price indexes must be transitivized using a formula first proposed by Gini (1931). The price index of country k , P_k is calculated as follows:

$$P_k = \prod_{j=1}^K (P_{jk}^{1/K}),$$

where P_{jk} denotes the result of a bilateral comparison between countries j and k . The EKS and CCD methods use the Fisher and Törnqvist formulae respectively to make each bilateral comparison. The EKS method is the most widely used complete-graph method. In particular, it is used by the OECD and Eurostat.⁵

Both approaches become increasingly problematic as the sample of countries in a comparison rises, since they require all countries to provide price and expenditure data on the same set of basic headings. The more diverse the set of countries, the harder it becomes to construct a list of commodities within each basic heading that are representative of all the countries in the comparison.

A related problem faced by average-price star methods is that the price vector of the artificial country at the center of the star becomes increasingly unrepresentative of the prices faced by many of the countries in the comparison for very heterogeneous groupings of countries such as those included in the Penn World Table. This can cause substitution bias which may seriously distort estimates of both per capita income differentials at a point in time and convergence rates over time (see Nuxoll, 1994, Dowrick and Quiggin, 1997, and Hill 2000). Geary-Khamis, in particular, will tend to underestimate per capita income differentials across countries, since its average price vector usually approximates more closely the price vectors of the richer countries in the

⁵See OECD (1995) and Eurostat (1983).

comparison. Hence the substitution bias tends to be larger for poorer countries. This tendency is sometimes referred to as the *Gerschenkron effect* (see Gerschenkron, 1951). Equally weighted variants on Geary-Khamis, such as Iklé, are just as much affected by substitution bias as Geary-Khamis. However, for these methods it is less obvious exactly how the results are distorted.

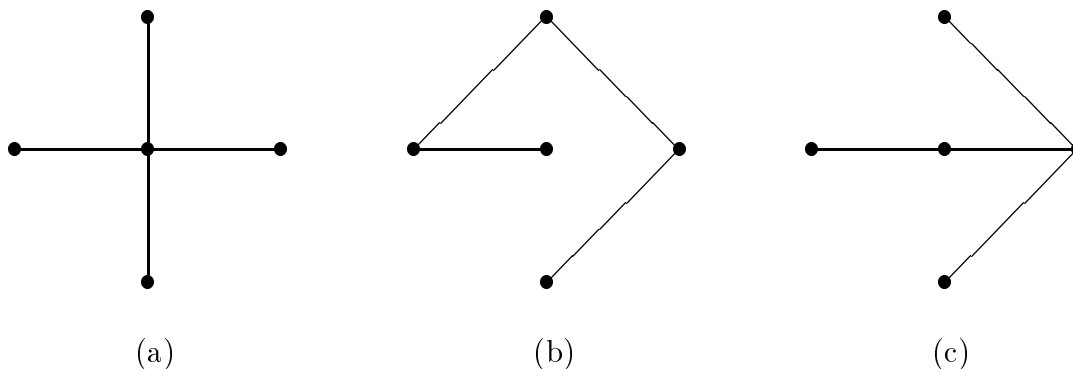
The main problem with complete-graph methods is that, in a comparison between K countries, there are $K(K - 1)/2$ possible bilateral comparisons. This means that, as the number of countries rises, the number of bilateral comparisons that must be made soon becomes rather large. For example, the World Bank and ICP are currently making a comparison over 115 countries. If made using the EKS method, this would require 6555 bilateral comparisons. Inevitably, some of these comparisons are more reliable than others. For example, there may be very little overlap between the goods and services consumed in the United States and Tanzania, making a meaningful direct comparison very difficult. Also, as noted above, a comparison between the United States and Canada will not be as good as it could be if the commodities within each basic heading, and the basic headings themselves, have to be the same as those used in all other 6554 bilateral comparisons. Thus it is tempting to conclude that the overall results could be improved by excluding bilateral comparisons between countries with very different consumption patterns. Later sections of this paper discuss some ways in which this can be done.

2. Spanning Tree Methods

The two classes of multilateral methods outlined in the previous section are not the only options. In fact, a multilateral comparison between K countries can be made by simply chaining together $K - 1$ bilateral comparisons, as long as the underlying graph is a *spanning tree*. A spanning tree is a connected graph that does not contain any cycles. In other words, any pair of vertices in the graph are connected by one and only one path of edges. The reason why there must be no cycles in the graph is to ensure that the multilateral price indexes are transitive and hence internally consistent. Otherwise, the

Gini transitivization formula must be used. A total of K^{K-2} different spanning trees are defined on a set of K vertices. Three examples of spanning trees defined on the set of 5 vertices are shown in Figure 2.

FIGURE 2. — EXAMPLES OF SPANNING TREES



The resulting set of multilateral price indexes depends both on the choice of formula used for making the bilateral comparisons and on the choice of spanning tree. The bilateral comparisons should be made using a superlative formula such as Fisher or Törnqvist. (Superlative indexes are discussed in more detail in the next section.) Since superlative formulae satisfy the country reversal test (i.e., $P_{jk} = 1/P_{kj}$), there is no need for directional arrows on the edges in the spanning tree to identify the base country in each bilateral comparison.

The choice of spanning tree is more problematic. A criterion is needed for deciding which edges (bilateral comparisons) to include and which to exclude. Ideally, we should use whichever bilateral comparisons are most reliable. Reliability in this context is measured by the sensitivity of a bilateral comparison to the choice of index number formula. The less sensitive a bilateral comparison is to the choice of formula, the more confidence we can have in the result. Three possible criteria for measuring sensitivity are discussed in the next section.

3. Measuring the Sensitivity of a Bilateral Comparison to the Choice of Index Number Formula

(i) Relative Price Variability

One possible metric for measuring the distance, D_{jk} , between the price vectors of countries j and k is the standard deviation of the logarithm of the price relatives, p_{ki}/p_{ji} , defined over the set of goods and services indexed by $i = 1, \dots, N$.⁶

$$D_{jk} = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\ln \left(\frac{p_{ki}}{p_{ji}} \right) - \frac{1}{N} \sum_{i=1}^N \ln \left(\frac{p_{ki}}{p_{ji}} \right) \right]^2} \quad (5)$$

There is no index number problem if the price data of the two countries satisfy the conditions for Hicks's (1946) composite commodity theorem (i.e., $p_k = \lambda p_j$), since, under this scenario, all bilateral price index formulae will give the same answer λ . In this case, the distance metric $D_{jk} = 0$. Conversely, if the data are not consistent with Hicks's composite commodity theorem, then $D_{jk} > 0$. This is the basis for the claim that the distance metric, D_{jk} , measures the sensitivity of a bilateral comparison to the choice of index number formula. A larger value of D_{jk} implies greater sensitivity.

(ii) Paasche-Laspeyres Spread

The Paasche and Laspeyres price indexes are defined as follows:

$$\text{Paasche : } P_{jk}^P = \frac{p_k \cdot q_k}{p_j \cdot q_k}, \quad \text{Laspeyres : } P_{jk}^L = \frac{p_k \cdot q_j}{p_j \cdot q_j}.$$

Following Hill (1999), the Paasche-Laspeyres spread (PLS) between countries j and k is defined here as

$$PLS_{jk} = \left| \ln \left(\frac{P_{jk}^L}{P_{jk}^P} \right) \right|.$$

Like the distance metric, D_{jk} , the Paasche-Laspeyres spread, PLS_{jk} , equals zero if the price data satisfy the conditions for Hicks's composite commodity theorem (i.e., $p_k = \lambda p_j$). However, unlike D_{jk} , the Paasche-Laspeyres spread also equals zero if $q_k = \mu q_j$. In this case, the quantity data satisfy the conditions for Leontief's (1936)

⁶This distance metric is also used by Allen and Diewert (1981), Debelle and Lamont (1997) and Hill (2000) to measure relative price variability.

aggregation theorem, and hence all quantity index formulae will give the same answer, μ . Even though our focus here is price indexes, Leontief's aggregation theorem is still directly relevant. This is because price and quantity indexes are not independent of each other. A price index, P_{jk} , and quantity index, Q_{jk} , are linked by the *factor reversal test* as follows:

$$P_{jk}Q_{jk} = \frac{p_k \cdot q_k}{p_j \cdot q_j}.$$

Using the factor reversal test, a price index can be derived implicitly from a quantity index. In other words, if all quantity indexes give the same answer, then so should all price indexes, i.e.,

$$Q_{jk} = \mu \Rightarrow P_{jk} = \frac{p_k \cdot q_k}{\mu(p_j \cdot q_j)}.$$

For this reason, PLS_{jk} is probably a better measure than D_{jk} of the sensitivity of a bilateral comparison to the choice of index number formula, since it is consistent with both Hicks's and Leontief's aggregation theorems.

(iii) Superlative Spread

A consensus has emerged in the index number literature that bilateral comparisons should be made using a superlative index number formula.⁷ It is natural, therefore, to restrict the range of index number formulae under consideration to the family of superlative indexes. However, measuring the spread between the superlatives is not straightforward since Diewert (1976) showed that there are an infinite number of them. This is not an issue that has received much attention in the index number literature, since there are only three superlatives that are actually used in practice – Fisher, Törnqvist and Walsh. The spread between these indices is typically very small and, as a result, it is widely believed that the spread between superlatives will always be small.⁸ However, Hill (2001)

⁷Diewert (1976) defines an index as superlative if it exactly equals the cost of living index when the underlying cost function is *flexible* (i.e., it can approximate an arbitrary linearly homogenous expenditure function to the second order). All direct superlative price indexes coincide when the price data satisfy the conditions for Hicks's composite commodity theorem. Similarly, all implicit superlative price indexes coincide when the quantity data satisfy the conditions for Leontief's aggregation theorem.

⁸Also, Diewert (1978) shows that all known superlative price indexes approximate each other to the second order at a point where $p_2 = \lambda p_1$ and $q_2 = \mu q_1$.

shows that this is not true. In fact, in most cases the spread between the largest and smallest superlative indexes, out of the infinite set of known superlative indexes, is usually larger than the spread between Paasche and Laspeyres, and can vary considerably from one bilateral comparison to another. Hence users face a problem. According to the economic approach to index numbers, all superlative indexes have equally attractive properties. And yet the results can differ quite significantly depending on which is used (in extreme cases by as much as a factor of four in Hill's (2001) comparison of 43 OECD and former Eastern block countries in 1996). Hence, the smaller the spread between the superlatives the more confidence we can have in the result of a bilateral comparison.

(iv) Paasche-Laspeyres Spreads and Superlative Spreads

The correlation between Paasche-Laspeyres spreads and superlative spreads is investigated in Hill (2001) using the 1996 OECD data set containing 43 countries. A total of 903 different bilateral comparisons can be made for this data set. Over these 903 observations, the correlation coefficient varies between 0.46 and 0.57 depending on the range of superlative indexes considered. Hence, the PLS_{jk} measure is a reasonably good indicator of the sensitivity of a bilateral comparison to the choice of superlative index number formula, while at the same time being much easier to compute than the superlative spread.

4. The Minimum-Spanning Tree Method

A total of $K(K - 1)/2$ distinct bilateral comparisons can be made over a set of K countries. The minimum-spanning tree method for computing multilateral price indexes requires a weight to be placed on each of these bilateral comparisons.⁹ The weights could be given by D_{jk} , PLS_{jk} , or the superlative spread, or indeed some other measure of the sensitivity of a bilateral comparison to the choice of formula. However, for the reasons discussed in the previous section, here we will use the Paasche-Laspeyres spreads, PLS_{jk} , as weights. The minimum-spanning tree is the spanning tree with the smallest

⁹See Hill (1999) for a more detailed analysis of the minimum-spanning tree method.

sum of weights on its edges. (Each edge corresponds to a bilateral comparison.) The minimum-spanning tree can be computed using Kruskal's algorithm.¹⁰ Kruskal's algorithm selects sequentially the edges (bilateral comparisons) with the smallest weights, subject to the constraint that adding each edge does not create a cycle. The program terminates once $K - 1$ edges have been selected, since at this point it is no longer possible to select any more edges without creating a cycle. The resulting graph is the minimum-spanning tree.¹¹

If the Paasche-Laspeyres spreads are used as weights, a reasonable case can be made for arguing that the resulting minimum-spanning tree is the spanning tree that minimizes the sensitivity of the *multilateral* price indexes to the choice of bilateral index number formula. This is because it is constructed from the bilateral comparisons that are least sensitive to the choice of formula.

The minimum-spanning tree (MST) method is illustrated below using four cross-sections of data. The first two cross-sections are drawn from the ICP database used to construct the Penn World Table, and cover the years 1980 and 1985. The first cross-section consists of price and quantity data for 30 countries defined over 151 goods and services headings. The second cross-section consists of price and quantity data for the same 30 countries defined over 139 goods and services headings.¹² The countries in the sample consist of 10 from Western Europe, 3 from Eastern Europe, 2 from North America, 7 from Asia and 8 from Africa.¹³

The second two cross-sections use OECD data for 1993 and 1996, respectively. The 1993 cross-section consists of price and quantity data for 34 countries defined over 147 goods and services headings. The 1996 cross-section consists of price and quantity data for the same 34 countries defined over 162 goods and services headings. The countries

¹⁰More detailed explanations of Kruskal's algorithm and the concept of a minimum-spanning tree can be found in any introductory book on Graph Theory. For example, see Wilson (1985).

¹¹A proof of this result can be found in Wilson (1985).

¹²Price and quantity data were unavailable for some headings in some countries. See the Appendix for a discussion of how these missing observations were handled.

¹³A total of 42 countries were present in both the 1980 and 1985 ICP data sets. To make the regional groupings of countries more balanced, 5 smaller European countries and 7 African countries were removed from the sample.

included are the European countries and some former Soviet republics.

The minimum spanning trees for 1980 and 1985 are depicted respectively in Figures 3 and 4.¹⁴ The minimum spanning trees for 1993 and 1996 are depicted respectively in Figures 5 and 6.¹⁵ Multilateral (transitive) price indexes are obtained by chaining Fisher or Törnqvist price indexes across the minimum-spanning tree. The resulting per capita income rankings are discussed below.

5. An Empirical Comparison of Multilateral Methods

Tables 1-4 show the per capita income rankings for each cross-section obtained using five different multilateral methods. In Tables 1 and 2, PWT denotes the Penn World Table rankings which were computed using Geary-Khamis for 1980, and separate regional Geary-Khamis comparisons in 1985 which were then linked by bilateral comparisons between core countries.¹⁶ The MST80 rankings are obtained by chaining Fisher price indexes across the 1980 minimum-spanning tree depicted in Figure 3. Likewise, the MST85 rankings are obtained by chaining Fisher price indexes across the 1985 minimum-spanning tree depicted in Figure 4. (Note that we use the 1980 minimum-spanning tree to compute multilateral price indexes in 1985 as well as 1980, and the 1985 minimum-spanning tree to compute multilateral price indexes in 1980 as

¹⁴The country codes in Figures 3 and 4 are DEU - Germany, DNK - Denmark, ESP - Spain, FIN - Finland, FRA - France, GBR - Great Britain, GRC - Greece, HUN - Hungary, ITA - Italy, NOR - Norway, POL - Poland, PRT - Portugal, YUG - Yugoslavia, CAN - Canada, USA - United States of America, HKG - Hong Kong, IND - India, JPN - Japan, KOR - Korea, LKA - Sri Lanka, PAK - Pakistan, PHL - Philippines, CIV - Ivory Coast, CMR - Cameroon, KEN - Kenya, MAR - Morocco, NGA - Nigeria, TUN - Tunisia, TZA - Tanzania, ZWE - Zimbabwe.

¹⁵The country codes in Figures 5 and 6 are AUT - Austria, BEL - Belgium, DNK - Denmark, FRA - France, FIN - Finland, GER - Germany, GRC - Greece, IRE - Ireland, ITA - Italy, LUX - Luxembourg, NLD - Netherlands, PRT - Portugal, SPA - Spain, SWE - Sweden, UK - United Kingdom, ICE - Iceland, NOR - Norway, POL - Poland, SWI - Switzerland, CZE - Czech Republic, HUN - Hungary, TUR - Turkey, SLK - Slovak Republic, RUS - Russia, ROM - Romania, BLR - Belarus, BGR - Bulgaria, HRV - Croatia, SVN - Slovenia, UKR - Ukraine, MDA - Moldova, EST - Estonia, LVA - Latvia, LTU - Lithuania.

¹⁶See World Bank (1993) for more details.

FIGURE 3. — MINIMUM SPANNING TREE FOR 1980

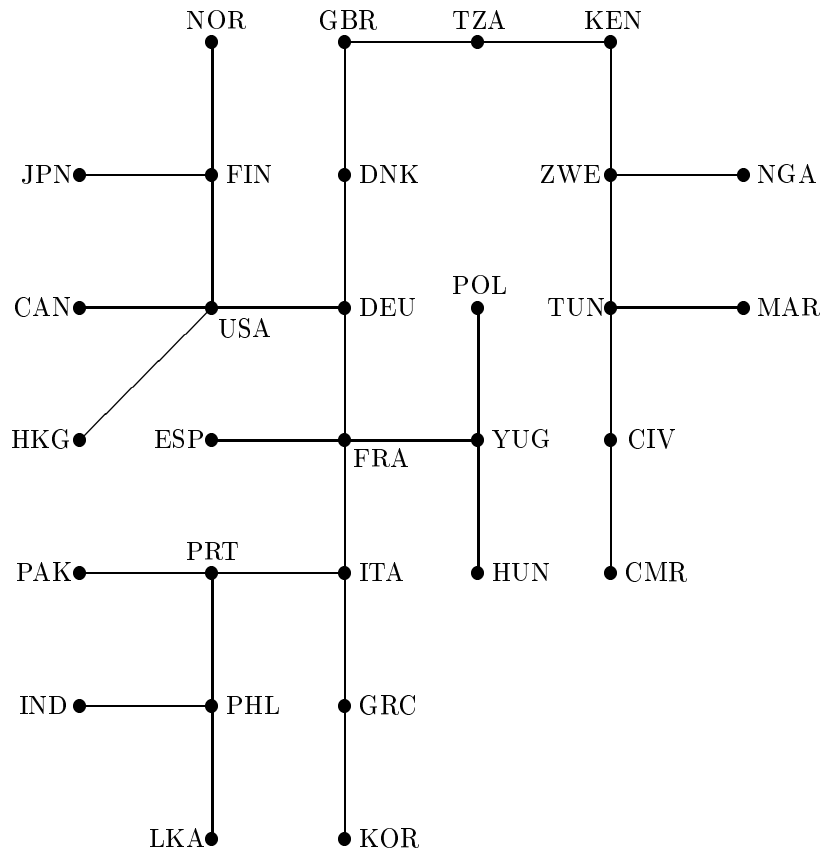


FIGURE 4. — MINIMUM SPANNING TREE FOR 1985

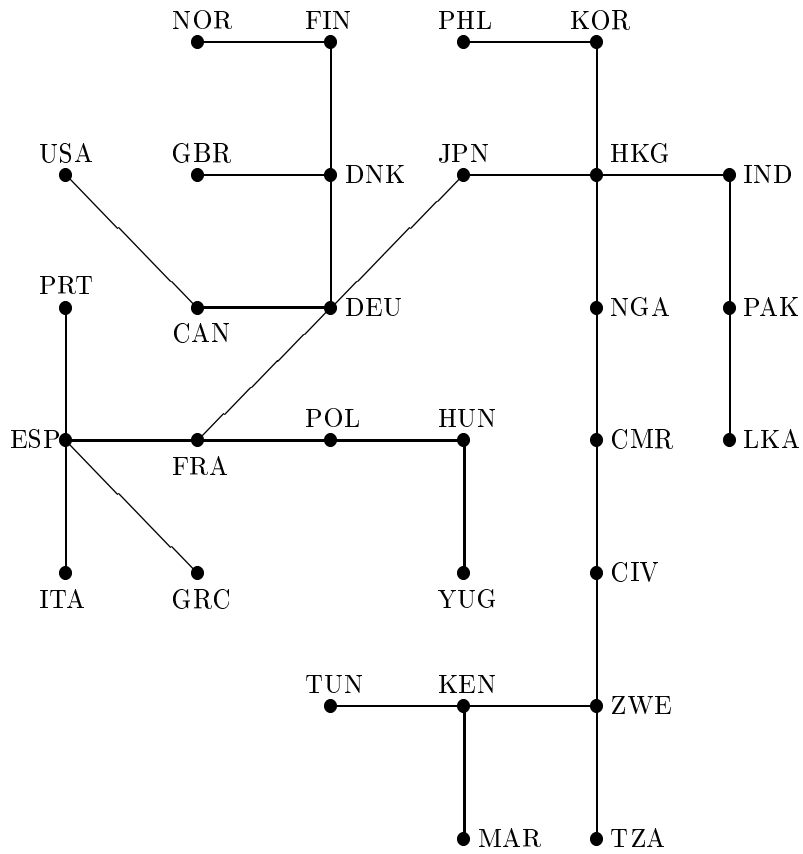
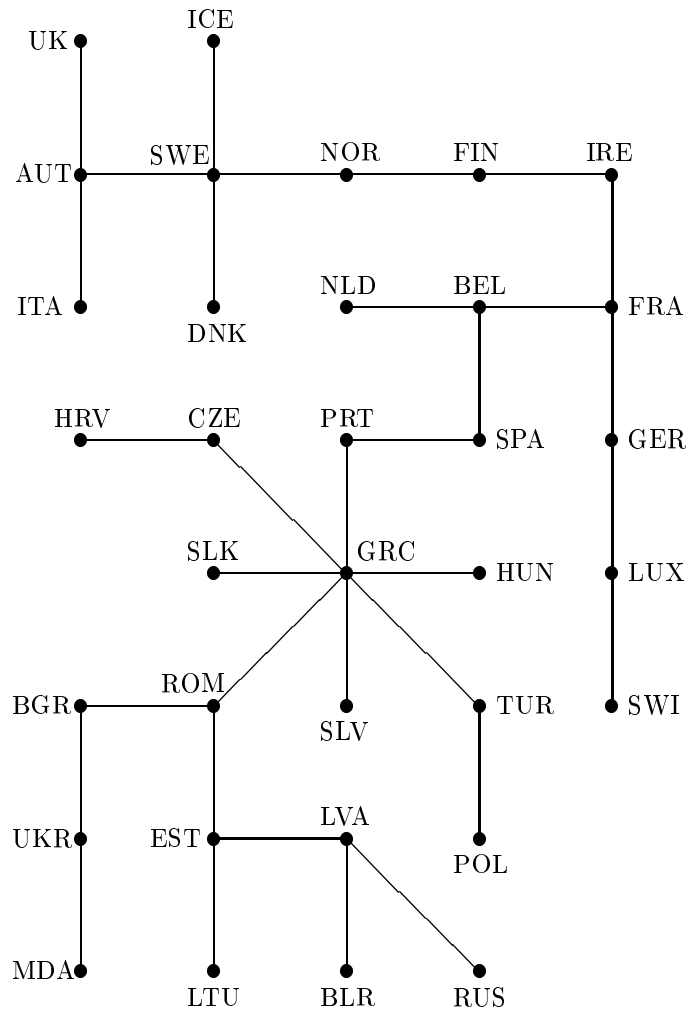


FIGURE 6. — MINIMUM SPANNING TREE FOR 1996



well as 1985.) Finally the EKS and ExR results were computed, respectively, using the EKS method and market exchange rates. In Tables 3 and 4, GK denotes Geary-Khamis, while MST93 and MST96 rankings are obtained, again, by chaining Fisher price indexes across the respective minimum spanning tree.¹⁷

One important reason for making international comparisons is to determine the level of inequality across countries. This can be measured by the standard deviation of the logarithm of per capita income, x_k , across the set of countries $k = 1, \dots, K$.

$$I = \sqrt{\frac{1}{K} \sum_{k=1}^K \left[\ln(x_k) - \frac{1}{K} \sum_{k=1}^K \ln(x_k) \right]^2} \quad (6)$$

As a measure of inequality, I has the advantage that it is invariant to changes in the currency units in which per capita income is measured. The results are shown in Table 5.

The most striking result that emerges from Table 5 is that exchange rates clearly overestimate inequality across countries.¹⁸ The fact that the Penn World Table (based on Geary-Khamis) in 1980 and 1985 and Geary-Khamis in 1993 and 1996 generate the smallest per capita income differentials is consistent with the so-called Gerschenkron effect. In contrast, EKS, and the minimum-spanning tree results should tend neither to overestimate nor underestimate inequality across countries, since they are all constructed from superlative Fisher indexes. It is not surprising, therefore, that these methods generate similar per capita income differentials.

Determining whether or not income levels are converging or diverging across countries has, in the last decade, become a research industry in the Economics profession (for example, see Barro and Sala-i-Martin, 1992, and Quah, 1996). The results in Table 5 for the period 1980-1985 provide a sobering insight into the measurement problems faced by this literature. According to three of the five methods, some convergence occurred over this period, while according to the other two there was divergence. However,

¹⁷For technical reasons, mainly due to the difficulty of comparing public consumption between OECD countries and former Eastern-bloc countries, a few basic headings were not available in 1993. Hence the 1993 per capita incomes in Table 3 do not cover the whole of GDP and the rankings may be somewhat distorted.

¹⁸For an explanation of this result, see for example Kravis and Lipsey (1983).

TABLE 1.— PER CAPITA INCOME RANKINGS IN 1980 (US DOLLARS)

PWT		MST80		MST85		EKS		ExR	
CAN	11615	USA	11448	USA	11448	USA	11448	NOR	14110
USA	11448	CAN	10838	CAN	10837	CAN	10708	DEU	13306
NOR	11325	NOR	10251	DEU	10178	DEU	9858	DNK	12964
DEU	10200	DEU	9961	NOR	10066	NOR	9822	FRA	12137
DNK	9831	DNK	9696	DNK	9906	FRA	9637	USA	11448
FRA	9780	FRA	9652	FRA	9861	DNK	9447	CAN	10817
FIN	8641	JPN	8787	JPN	8661	GBR	8008	FIN	10479
JPN	8414	GBR	8311	GBR	8491	JPN	7999	GBR	9345
GBR	8253	FIN	8124	FIN	7977	FIN	7665	JPN	8910
ITA	7788	ITA	7476	ITA	7527	ITA	7545	ITA	6899
HKG	7136	HKG	6294	HKG	6428	HKG	6171	ESP	5635
ESP	6353	ESP	5918	ESP	6046	ESP	6043	HKG	5445
GRC	5097	HUN	4273	GRC	4277	GRC	4337	GRC	4192
HUN	4632	GRC	4255	HUN	3771	YUG	3782	YUG	3150
POL	4322	POL	3940	POL	3542	HUN	3767	PRT	2429
YUG	4042	YUG	3760	YUG	3319	POL	3616	POL	2247
PRT	3932	PRT	3188	PRT	3184	PRT	3403	HUN	1917
KOR	2583	KOR	1966	KOR	2140	KOR	2030	KOR	1528
TUN	1993	TUN	1614	TUN	1988	TUN	1596	TUN	1366
PHL	1740	PHL	1324	PHL	1461	PHL	1387	CIV	1282
CIV	1368	CIV	1056	CIV	1276	CIV	1013	NGA	997
LKA	1226	MAR	959	MAR	1118	MAR	900	MAR	878
MAR	1200	LKA	911	PAK	1007	LKA	890	CMR	830
PAK	1097	PAK	809	LKA	968	PAK	850	PHL	736
CMR	911	ZWE	769	ZWE	933	ZWE	773	ZWE	724
ZWE	894	CMR	734	CMR	887	CMR	734	KEN	423
NGA	894	NGA	716	NGA	780	NGA	675	PAK	347
KEN	637	KEN	573	KEN	694	KEN	571	LKA	280
IND	570	IND	479	IND	544	IND	497	TZA	257
TZA	361	TZA	312	TZA	359	TZA	303	IND	244

TABLE 2.— PER CAPITA INCOME RANKINGS IN 1985 (US DOLLARS)

PWT		MST80		MST85		EKS		ExR	
USA	16494	USA	16494	USA	16494	USA	16494	USA	16494
CAN	15258	CAN	14753	CAN	14753	CAN	14540	NOR	14066
NOR	13913	NOR	13322	NOR	13558	NOR	13453	CAN	13634
DNK	12234	DNK	12201	DNK	11995	DNK	12097	DNK	11404
DEU	12169	DEU	11887	DEU	11687	DEU	11929	FIN	11125
JPN	11795	FRA	11432	ITA	11262	FRA	11353	JPN	11064
FIN	11464	ITA	11379	FRA	11239	ITA	11059	DEU	10334
FRA	11434	FIN	10987	FIN	11181	FIN	10782	FRA	9333
GBR	10905	GBR	10760	GBR	10578	GBR	10495	GBR	8020
ITA	10827	JPN	10169	JPN	9995	JPN	10146	ITA	7431
HKG	10204	HKG	9701	HKG	9859	HKG	9768	HKG	6130
ESP	7589	ESP	6602	ESP	6489	ESP	6558	ESP	4275
GRC	5861	GRC	5791	GRC	5693	GRC	5605	GRC	3271
PRT	5568	PRT	4413	PRT	4130	PRT	4199	PRT	2160
HUN	5140	HUN	4018	HUN	3867	YUG	4096	KOR	2102
YUG	4810	YUG	4007	YUG	3856	HUN	3984	YUG	2021
POL	4039	POL	3410	KOR	3396	POL	3360	POL	1928
KOR	3979	KOR	3209	POL	3248	KOR	3284	HUN	1788
TUN	3446	TUN	2220	TUN	2628	TUN	2400	TUN	1138
CMR	2724	CMR	1895	CMR	1931	CMR	2043	CMR	989
MAR	2371	PHL	1733	PHL	1657	PHL	1611	NGA	948
LKA	1852	LKA	1415	MAR	1447	MAR	1481	CIV	712
PHL	1791	CIV	1401	CIV	1428	CIV	1447	PHL	602
CIV	1710	MAR	1368	ZWE	1343	LKA	1405	ZWE	600
ZWE	1684	ZWE	1267	LKA	1295	ZWE	1303	MAR	542
PAK	1342	PAK	1100	PAK	1136	PAK	1020	LKA	360
KEN	995	NGA	806	NGA	856	NGA	870	PAK	317
NGA	980	IND	795	KEN	688	IND	652	TZA	295
IND	749	KEN	649	IND	681	KEN	640	KEN	294
TZA	418	TZA	331	TZA	302	TZA	314	IND	283

TABLE 3.— PER CAPITA INCOME RANKINGS IN 1993 (DEUTSCHMARKS)

GK		MST93		MST96		EKS	
LUX	54742	LUX	49957	LUX	49957	LUX	52233
SWI	44531	SWI	45883	SWI	45883	SWI	44597
AUT	35802	FRA	35180	BEL	35306	BEL	35751
ICE	35318	AUT	34833	ICE	35068	DNK	34201
BEL	35154	ICE	34774	FRA	34826	ICE	34190
GER	34149	BEL	34657	AUT	34593	AUT	34156
NOR	33998	GER	34149	GER	34149	GER	34149
FRA	33318	NOR	34111	DNK	33626	NOR	34044
DNK	32994	DNK	33860	NOR	32958	FRA	33393
ITA	32889	ITA	32747	ITA	32521	ITA	32550
NLD	30435	NLD	30355	NLD	30995	NLD	31237
UK	29178	SWE	29583	SWE	29379	SWE	29045
SWE	28650	UK	29475	UK	28900	UK	28308
FIN	27301	FIN	28032	FIN	27750	FIN	27443
IRE	25260	IRE	26354	IRE	26089	IRE	26457
SPA	24475	SPA	23717	SPA	23877	SPA	22755
PRT	21906	PRT	20968	PRT	21110	PRT	19481
GRC	20463	GRC	18878	GRC	19859	GRC	17726
SVN	17597	SVN	17614	SVN	17695	SVN	16265
CZE	14960	CZE	15760	CZE	16691	CZE	14516
HUN	12123	BLR	10552	HUN	11551	HUN	10069
BLR	11484	HUN	10402	SLK	11386	SLK	9809
SLK	11276	SLK	9920	TUR	10155	TUR	9092
TUR	10341	RUS	9754	RUS	9651	RUS	8340
RUS	9723	TUR	9254	POL	9329	POL	8153
BGR	9528	POL	8328	BGR	8183	BLR	7529
POL	9336	BGR	7982	ROM	7756	BGR	7380
LTU	9314	LTU	7665	BLR	7560	ROM	6714
ROM	8377	ROM	7565	EST	7419	EST	6634
EST	8176	EST	6963	LTU	7296	LTU	6565
UKR	7971	UKR	6778	HRV	6996	HRV	6306
HRV	7120	HRV	6742	UKR	6898	UKR	5741
MDA	5981	LVA	5612	LVA	5980	LVA	5338
LVA	5617	MDA	4666	MDA	4267	MDA	3743

TABLE 4.— PER CAPITA INCOME RANKINGS IN 1996 (DEUTSCHMARKS)

GK		MST93		MST96		EKS		ExR	
LUX	64204	LUX	63656	LUX	63656	LUX	66193	SWI	62623
NOR	52114	SWI	51610	NOR	52585	NOR	53806	LUX	61456
SWI	51005	NOR	51493	SWI	51610	SWI	50441	NOR	54208
ICE	45541	DNK	47568	DNK	48678	DNK	47190	DNK	50008
DNK	45013	BEL	44928	ICE	47247	ICE	45820	SWE	43292
AUT	43484	ICE	44885	AUT	45875	BEL	45513	GER	43235
GER	43235	AUT	44828	BEL	45302	AUT	44553	AUT	42716
BEL	42952	GER	43235	GER	43235	NLD	44030	ICE	40598
NLD	41554	NLD	42369	ITA	43075	GER	43235	BEL	39737
FRA	39952	ITA	42093	NLD	42599	ITA	42110	FRA	39419
ITA	39863	FRA	41723	FRA	41761	FRA	41135	NLD	38407
UK	38700	SWE	40171	SWE	41109	SWE	40539	FIN	36741
SWE	38670	UK	39400	UK	40634	UK	39618	ITA	31848
FIN	36883	FIN	39141	FIN	39177	FIN	39273	UK	29518
IRE	34082	IRE	36868	IRE	36901	IRE	38129	IRE	29300
SPA	30066	SPA	31366	SPA	31130	SPA	30636	SPA	22424
PRT	28374	PRT	28567	CZE	29306	PRT	27630	GRC	17664
CZE	27308	CZE	28101	PRT	28352	SVN	25948	PRT	16493
GRC	26669	SVN	28032	SVN	27477	CZE	25842	SVN	14255
SVN	25558	GRC	24926	GRC	27146	GRC	25308	CZE	8238
HUN	20276	SLK	19979	HUN	21369	HUN	18937	HUN	6668
SLK	19945	HUN	19031	SLK	21291	SLK	18222	HRV	6227
EST	15354	POL	13831	POL	15269	POL	13805	SLK	5352
POL	14474	EST	13075	RUS	14767	EST	13547	POL	5245
ROM	14147	RUS	13017	ROM	14625	RUS	12905	EST	4475
RUS	14026	ROM	13000	EST	14554	TUR	12254	RUS	4377
HRV	13214	TUR	12683	TUR	13451	ROM	12035	TUR	4356
LTU	13146	HRV	11828	HRV	13008	HRV	12001	LTU	3203
TUR	13053	LTU	11528	LTU	12593	LTU	11261	LVA	3106
LVA	12562	BLR	11437	BLR	11277	LVA	10434	ROM	2339
BLR	12452	LVA	9961	LVA	11088	BLR	9739	BLR	1962
BGR	11714	BGR	9855	BGR	11086	BGR	9550	BGR	1769
UKR	8805	UKR	6367	UKR	7512	UKR	6466	UKR	1312
MDA	5262	MDA	3437	MDA	3839	MDA	3493	MDA	579

TABLE 5.— Average Per Capita Income Differentials

	1980	1985		1993	1996
PWT	1.058	1.015	GK	0.6505	0.6064
MST80	1.129	1.100	MST93	0.7041	0.7017
MST85	1.060	1.102	MST96	0.6999	0.6631
EKS	1.121	1.105	EKS	0.7524	0.7111
ExR	1.332	1.364	ExR	—	1.8621

TABLE 6.— SIMILARITY OF PER CAPITA INCOME LEVELS IN 1980

	PWT	MST80	MST85	EKS	ExR
PWT	0.0000	0.0982	0.0877	0.0921	0.4287
MST80	0.0982	0.0000	0.0962	0.0432	0.3734
MST85	0.0877	0.0962	0.0000	0.0905	0.3902
EKS	0.0921	0.0432	0.0905	0.0000	0.3818
ExR	0.4287	0.3734	0.3902	0.3818	0.0000

over the period 1993-1996 all four methods show some convergence, although this result should not be taken too seriously given the problems with the underlying data for 1993 alluded to earlier.

It is also informative to compare the similarity of the per capita income levels generated by the five methods. An ideal measure of similarity between the results of two methods a and b must also be invariant to changes in the currency units in which per capita income is measured. One such measure is the standard deviation of the logarithm of the per capita income relatives, x_{ak}/x_{bk} , across the set of countries $k = 1, \dots, K$.

$$S_{jk} = \sqrt{\frac{1}{K} \sum_{k=1}^K \left[\ln \left(\frac{x_{bk}}{p_{ak}} \right) - \frac{1}{K} \sum_{k=1}^K \ln \left(\frac{x_{bk}}{x_{ak}} \right) \right]^2} \quad (7)$$

The results are shown in Tables 6-9.

Not surprisingly, all the results obtained using price indexes are much closer to

TABLE 7.— SIMILARITY OF PER CAPITA INCOME LEVELS IN 1985

	PWT	MST80	MST85	EKS	ExR
PWT	0.0000	0.1526	0.1351	0.1315	0.4864
MST80	0.1526	0.0000	0.0581	0.0532	0.4037
MST85	0.1351	0.0581	0.0000	0.0413	0.3993
EKS	0.1315	0.0532	0.0413	0.0000	0.3935
ExR	0.4864	0.4037	0.3993	0.3935	0.0000

TABLE 8.— SIMILARITY OF PER CAPITA INCOME LEVELS IN 1993

	GK	MST93	MST96	EKS
GK	0.0000	0.0785	0.1061	0.1313
MST93	0.0785	0.0000	0.0630	0.0676
MST96	0.1061	0.0630	0.0000	0.0643
EKS	0.1313	0.0676	0.0643	0.0000

TABLE 9.— SIMILARITY OF PER CAPITA INCOME LEVELS IN 1996

	GK	MST93	MST96	EKS	ExR
GK	0.0000	0.1138	0.0806	0.1151	1.4265
MST93	0.1128	0.0000	0.0510	0.0441	1.3883
MST96	0.0806	0.0510	0.0000	0.0621	1.4063
EKS	0.1151	0.0441	0.0621	0.0000	1.3760
ExR	1.4265	1.3883	1.4063	1.3760	0.0000

each other than they are to the results of an exchange rates comparison. Also, with the exception of the 1980, the minimum-spanning tree and EKS results are rather closer to each other than to the Penn World Table/Geary-Khamis results. These results have significant implications. A spanning tree comparison requires only $K - 1$ bilateral comparisons. In contrast, EKS requires $K(K - 1)/2$. In other words, EKS requires a factor of $K/2$ more bilateral comparisons. For the case of $K = 34$ considered here for 1993 and 1996, the factor is 17. Inspection of Tables 6-9 strongly suggests that switching from EKS to a minimum-spanning tree would dramatically simplify the process and reduce the cost of making a multilateral comparison without having much impact on the price index estimates. Furthermore, by no longer requiring all countries to provide price and expenditure data on the same set of headings, the characteristicity of each bilateral comparison in the spanning tree could be improved.

6. The Problem of Robustness

The minimum-spanning trees in Figures 3 and 4 each have 29 edges. Only 9 edges are common to both spanning trees. The minimum-spanning trees in Figures 5 and 6 have 33 edges of which only 10 are common to both. Hence clearly the minimum-spanning tree is not stable over time. Neither is it likely to be robust to slight changes in the data. This can be seen from Kruskal's algorithm. Any change in the ranking of the PLS_{jk} measures may alter the minimum-spanning tree. The PLS_{jk} measures whose rankings are most likely to change, if the data are perturbed, are the smallest ones since they will tend to differ only very slightly. At the same time, as far as Kruskal's algorithm is concerned, these are also the most important PLS_{jk} measures, since the algorithm selects sequentially edges with the smallest weights (subject to the constraint that adding each additional edge does not create a cycle). By implication, the minimum-spanning tree will tend not to be very robust.

Nevertheless, both pairs of minimum-spanning trees generate similar clusters of countries. For 1980 and 1985, the clusters coincide closely with the regional clusters used by Summers and Heston in the 1985 ICP comparison. Notably, in both spanning

trees, the 8 African countries form a cluster, as do the 3 Eastern European and 5 Latin European countries. Also, in both minimum-spanning trees, France forms a bridge between Northern, Southern and Eastern Europe, while the United States and Canada are linked to Europe through Germany. The remaining countries are somewhat dispersed in Figure 3. However, in Figure 4, the Asian and Northern European countries also form separate clusters. Hence when viewed from the perspective of clusters of countries, the minimum-spanning tree is quite stable over the period 1980-5.

In the 1993 and 1996 minimum-spanning trees, the countries are arranged in two easily discernible clusters. The first consists of the member countries of the European Community (excluding Greece), while the second consists of the countries of Eastern Europe and former Soviet Republics. In both cases, the link country for the EC is Portugal. In 1993 the link country for the second cluster is Romania, while in 1996 it is Greece. Indeed, Greece plays a pivotal role in the 1996 minimum-spanning tree, as does Romania in 1993. In principle, the second cluster in 1996 could itself be subdivided into two parts, the first of which would consist of primarily central European countries (Czech Republic, Slovakia, Slovenia, Croatia, Hungary, Poland, Greece and Turkey). Again, although the links within clusters are not robust, the clusters themselves are. The only country that switches cluster from 1993 to 1996 is Turkey, which in 1993 is linked to Portugal, while in 1996 it is linked to Greece.

The maximum-spanning tree for each data set is also of interest, since it provides the upper bound on the summed PLS measures. The maximum-spanning tree is the spanning tree whose multilateral price indexes will be most sensitive to the choice of bilateral index number formula. In this sense it is the worst possible way of linking countries. The maximum spanning trees for both ICP data sets are remarkably similar. Both closely resemble the star spanning tree with Sri Lanka at the center. For the 1980 data set, 25 out of 29 countries are linked directly to Sri Lanka, while for the 1985 data set, 20 countries are linked directly to Sri Lanka. The structure of the maximum-spanning tree is somewhat more complicated for the OECD data sets. In 1993, it consists of four star spanning trees linked together. The countries at the centers of the stars are Luxembourg, Belarus, Norway and Lithuania, which are linked together in

TABLE 10.— SUMMED PLS MEASURES

	MinST 1980	MinST 1985	MaxST 1980	MaxST 1985
1980 data	5.288	8.609	31.513	27.808
1985 data	6.780	4.447	30.633	32.893
	MinST 1993	MinST 1996	MaxST 1993	MaxST 1996
1993 data	1.326	3.610	30.531	20.923
1996 data	4.251	1.506	26.317	36.671

that order. Former Eastern bloc countries are linked to either Luxembourg or Norway, while EC countries are linked to either Belarus or Lithuania. In 1996, the maximum-spanning tree consists of two star spanning trees linked together. The countries at the center of the stars are Luxembourg and Moldova. All the EC countries are linked to Moldova, while the former Eastern-bloc countries are linked to Luxembourg, with the exception of Croatia which is linked to Switzerland.

The summed PLS measures of the minimum and maximum-spanning trees for all four data sets are shown in Table 10. The summed PLS measures of the minimum and maximum-spanning trees provide a useful point of reference when evaluating the performance of other spanning trees. The results in Table 10 indicate that MinST85 performs reasonably well in 1980, as does MinST80 in 1985 (i.e., the summed PLS measures of both are far closer to the lower bound than to the upper bound). Likewise, MinST93 and MinST96 also perform reasonably well in 1996 and 1993, respectively. Hence, once a minimum-spanning tree has been constructed, it is not unreasonable to use it again in subsequent comparisons.

Of course, with regard to robustness, what really matters is the robustness of the multilateral price indexes, not the robustness of the minimum-spanning tree. The results in Tables 6-9 indicate that the multilateral price indexes are rather less sensitive to the choice of spanning tree than one might suspect (at least as long as the Fisher index is used to make the bilateral comparisons). For example, in Tables 8 and 9, the similarity measure S_{jk} between MST93 and MST96 is less than half that between

Geary-Khamis and EKS. In other words, multilateral price indexes constructed using a spanning tree are less sensitive to the choice of spanning tree than multilateral price indexes constructed using more traditional methods are to the choice of multilateral formula.

7. A Pragmatic Approach to Chaining Price and Quantity Indexes Across Countries

(i) Choosing a Spanning Tree

Using the same spanning tree for a number of years would dramatically simplify multilateral international comparisons. Each country would only have to compare itself with its immediate neighbors in the spanning tree, thus reducing the cost and increasing the timeliness of international comparisons. Furthermore, by construction, each country's immediate neighbors in the minimum spanning tree will tend to have similar consumption patterns. This may substantially increase the characteristicity of the comparisons. Geary-Khamis by contrast, compares all countries using a single average price vector. In a comparison over rich and poor countries the average price vector may bear little resemblance to the actual price vectors of many of the countries in the comparison. Conversely, EKS uses all possible combinations of bilateral comparisons. This also requires all countries to provide price and expenditure data on the same set of basic headings, thus reducing the characteristicity of each comparison.

Of course, to compute a minimum-spanning tree also requires all countries to provide price and expenditure data on the same set of basic headings. Therefore, the minimum-spanning tree method is only useful if the same spanning tree is used for more than one period. This is why determining the robustness of the minimum-spanning tree over time is important. Although it is robust over clusters of countries, the fact that it is not at the level of individual countries implies that minimum-spanning tree method should not be used blindly. This conclusion is reinforced by the fact that, in the real world, there are other considerations that must be taken into account apart from the sensitivity of the results to the choice of bilateral index number formula. In particular,

some countries have better resourced national statistical offices than others. It would make little sense to put a country with an under-resourced national statistical office at the center of a regional star even if so specified by the minimum-spanning tree.

What is required is some way of combining prior knowledge with the minimum-spanning tree methodology. It turns out that prior restrictions can easily be imposed on the minimum-spanning tree. Suppose, for example, in Figure 4 we do not want India to be linked directly with Hong Kong. This restriction can be imposed by replacing the PLS measure between India and Hong Kong, in the $K \times K$ PLS matrix, by a large dummy value, i.e., by a value larger than the largest PLS measure in the matrix. This ensures that the corresponding edge is not selected by Kruskal's algorithm. Similarly, suppose we want Korea to be linked directly with Japan. This restriction can be imposed by replacing the PLS measure between Korea and Japan with a small dummy value, i.e., a value smaller than the smallest PLS measure in the matrix (excluding the terms on the lead diagonal), but still strictly positive. This ensures that the corresponding edge is selected.

One approach for choosing a spanning tree would be to first compute the minimum-spanning tree. Then one or more prior restrictions of either variety are placed on the minimum-spanning tree, which is then recomputed. If necessary, subsequent rounds of prior restrictions are imposed. After each round, the impact of the prior restrictions on the performance of the restricted minimum-spanning tree can be checked by computing the sum of its PLS measures. Ultimately the decision as to whether or not to include a particular prior restriction involves a trade-off between the conviction with which the prior belief is held and its impact on the summed PLS measures. Inevitably, this means that the final spanning tree is somewhat subjective. Nevertheless, the presence of an underlying methodology, should make it easier to justify to users.

(ii) Hybrid Methods

The minimum-spanning tree method is very flexible. In principle, it could be combined with other methods such as Geary-Khamis or EKS. For example, separate comparisons could be made for each region using, say, the EKS method. These regions

could then be combined using the minimum-spanning tree method. This could be done by imposing prior restrictions on the matrix of PLS measures, so that the restricted minimum-spanning tree forms the desired clusters. In this context, the spanning-tree structure of each region, which is set *a priori*, is irrelevant. The minimum-spanning tree is simply used to determine how each region is linked. For example, suppose a set of 28 countries are divided into four regional groupings of seven countries, and that comparisons within each group will be made using the EKS method. To link the four regions, three bilateral comparisons must be selected. Although one might think that this leaves the minimum-spanning tree method with very little to do, even with these prior restrictions, the method still has 10,588,410 different spanning trees to choose from. If necessary, prior restrictions can also be imposed on the links between regions. One attraction of such an approach is that it allows methods such as Geary-Khamis and EKS to be used where they work best. That is, in comparisons over a reasonably homogeneous group of countries.

The whole laborious process of making international comparisons would be vastly simplified if the world were broken up into regional blocs, and then separate comparisons were made for each bloc. The minimum-spanning tree method could be used to guide the choice of blocs, the way comparisons are made within each bloc, and the way the blocs are linked.

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Appendix

One problem with the ICP data set is that the matrices of price and quantity data have gaps; i.e., prices and/or quantities are unavailable for some basic headings for certain countries. Hungary, Poland, and Yugoslavia, in particular, are a problem. For these countries, 26 out of 151 headings in 1980 are missing, while, in 1985, 11 out of the 139 headings are missing. It is not clear how these missing observations should be handled. Simply ignoring them may seriously bias the results. For example, a Paasche price index between Hungary and Germany, which compares prices using Germany's quantities as weights, will be too large, since Hungary's 26 missing headings will receive implicit prices of zero. In contrast, there is no such bias in the Laspeyres formula, since, when prices are compared using Hungary's quantities as weights, these 26 headings are absent. Hence if a country has many missing headings, then its PLS measures will tend to be too small. This problem can be resolved by removing from each bilateral comparison all headings that are not priced by both countries, or, alternatively, by removing Hungary, Poland, and Yugoslavia from the comparison. Both solutions, however, have the disadvantage of throwing away useful information. To avoid throwing away data, this paper sets the missing quantity headings equal to zero, and specifies dummy values for the missing price headings. The ICP price data are expressed in real terms, i.e., the price of each heading for each country has been divided by its corresponding exchange rate, and then all the prices for the United States have been normalized to one. Therefore, if Hungary has a price entry of 0.5 for cereals, this means that the real price of cereals in Hungary is half that in the United States. The spanning trees in Figures 3 and 4 were obtained by setting all the missing quantity headings equal to zero, and the missing prices equal to one (a relatively high price in most instances, since the cost of living is higher in the United States than in Eastern Europe). This solution is consistent with the consumer substitution effect; i.e., a heading with a relatively low quantity should have a relatively high price.