

CAPITAL STOCKS, CAPITAL SERVICES AND DEPRECIATION¹

Peter Hill

Introduction

This paper is concerned with the measurement of capital stocks, capital services and depreciation in economic accounts. The values of these stocks and flows are all interdependent and they must be measured consistently within a coherent economic theoretic framework. For purposes of this paper, the capital stock consists of all the fixed assets such as machinery, equipment, buildings and other structures used by enterprises to provide inputs of capital services into processes of production.

The values of fixed assets to their owners depend on the flows of capital services they are capable of contributing as inputs into production over their service lives. As depreciation represents the decline in the value of a fixed asset over time, depreciation also depends on the service flows. Despite the central role played by capital services, however, they do not appear in the production account of the 1993 System of National Accounts, or SNA, or indeed anywhere in that system. This is a serious omission which greatly reduces the analytical usefulness of the accounts and may cause the data to be misinterpreted.

The values of stocks of fixed assets appear in the opening and closing balance sheets of the SNA while depreciation is recorded in the production and capital accounts. Capital services should also be recorded in the production account, but are not so at present. This paper describes the underlying economic theory linking all these stocks and flows together. It is shown that two concepts of economic depreciation exist. One is the traditional accounting concept, described here as time series depreciation, which measures the change in the value of an individual asset *over time*. The economic theory underlying time series depreciation dates back to Hotelling (1925) who defined depreciation as the rate of decrease of an asset's value with respect to time. The other concept, described here as cross section depreciation, measures the differences between the values of assets of different vintages *at the same point of time*. It is relevant when the vintages have to be aggregated to measure the capital stock for purposes of productivity analysis. The economic theory underlying cross section depreciation, which is based on the productive efficiency of the assets, has been developed over the last three decades.

The two concepts of depreciation do not generally coincide. It is shown in the paper that the two concepts differ when there is a foreseen revaluation of an asset due to obsolescence or other factors. Time series depreciation includes such a revaluation as an integral part of depreciation whereas cross section depreciation implicitly treats it as if it were some kind of capital loss. In practice, obsolescence may be at least as important as declining efficiency in determining depreciation over time, but its role has been neglected in the recent literature which has tended to focus on cross section depreciation. The two concepts serve different purposes. The time series concept is the

¹ This is an extended and more comprehensive version of an earlier draft originally intended to provide the theoretical section of the planned ECE Report on the capital stock in transition countries on which the author was working in 1998 while Statistical Adviser to the ECE. The present draft may be of interest to the 'Canberra Group'. In due course, it may also be of interest to the ISWG on National Accounts.

appropriate one for the capital account of the SNA whereas the cross section one is needed when estimating the capital stock at a point of time. Time series depreciation may also be recorded in the production account of the SNA, but it must not be interpreted as if it measured the value of the inputs of capital services into production².

The paper concludes by explaining the way in which capital services may, and should, be recorded in the production account of the SNA. For this purpose it is necessary replace the production account as defined in the 1993 SNA by that in the 1968 SNA which includes all inputs into production and not merely intermediate inputs. It is shown that by recording inputs of capital services, which cover the return on fixed assets as well as depreciation, the economic relevance and analytical usefulness of the production account, and of the SNA as a whole, can be greatly enhanced at little or no cost.

Capital services and asset values

The central feature of the relevant economic theory is the *interdependence* between the values of the various stocks and flows. Fixed assets are defined in the SNA (para. 10.33) as tangible or intangible produced goods (mainly structures, machinery and equipment) that are themselves repeatedly or continuously used in production over long periods of time. As fixed assets are acquired for the sole purpose of using them in production, their values at any point of time depend on the cumulative, capitalised value of the remaining flow of capital services that they are capable of providing.

Some notation needs to be defined. A new asset has a service life of n periods. The age of an asset at the start of period t is denoted by k , where $k = 0, 1, 2, \dots, n-1$.

- Let $v_t^k =$ the value at the start of period t of a fixed asset of age k at the start of period t ;
- $s_\tau^k =$ the value of the service provided in period τ by an asset of age k at the start of period t , where $\tau = t, t+1, t+2, \dots, t+n-k-1$;
- $r =$ the discount rate.

A new asset of age 0 at the start of period t provides a flow of services over the n periods from t to $t+n-1$. Its value at the start of period t is

$$v_t^0 = \sum_{\tau=t}^{\tau=t+n-1} \frac{s_\tau^0}{(1+r)^{\tau-t+1}} \quad (1)$$

In general, the value of an asset of age k at the start of period t whose remaining service life is $n-k$ periods is as follows:

² The need to distinguish depreciation from inputs of capital services has been stressed by Triplett (1996).

$$v_t^k = \sum_{\tau=t}^{\tau=t+n-k-1} \frac{s_{\tau}^k}{(1+r)^{\tau-t+1}} \quad (2)$$

When an asset is leased, the values of the services, s_{τ} , that it provides may be deduced from the market rentals payable to hire the assets for specified periods of time. The values of the services may be equated with the ‘basic’ rentals obtained by deducting the operating expenses incurred by the owners from the actual market rentals charged. Most assets, however, are probably purchased by the enterprises that use them so that the values of their services have to be imputed indirectly.

Capital services, like other goods and services, have their own prices and quantities. However, the prices and quantities cannot be treated symmetrically. By definition, *homogenous* assets all provide the same homogeneous type of service which is valued at the same price at the same point of time. This price may vary period to period, but not between different assets in the same time period. On the other hand, assets of different ages, i.e., different vintage of assets, may provide different quantities of services in the same period if the productive efficiency of that type of asset varies with age. This relationship, which is examined in more detail below, is described as the asset’s *efficiency profile*. The efficiency profiles for a set of assets which were homogeneous when new must, by definition, all be the same.

Thus, whereas the price of the service provided by an asset is a function of time, the quantity of the service is a function of the age of the asset. When decomposing the value of the service into its price and quantity components, the terms denoting the prices and the quantities have different subscripts.

Let p_{τ} = the price of the service in period τ ;

$q_{\tau-t+k}$ = the quantity of service provided in period τ by an asset of age k at the start of period t ;

then,

$$s_{\tau}^k = p_{\tau} q_{\tau-t+k} \quad (3)$$

The prices must be understood as the *real*, or *relative*, prices of the services. Similarly, the rate of interest used for discounting purposes in equations (1) and (2) must be a real rate. The p_{τ} ’s may vary over time in response to changes in the demand for, or supply of, the services of the asset due to factors such as technical progress. On the other hand, the quantity of service provided by given vintage of an asset does not vary from period to period. The quantities typically vary between different vintages in the same period.

When the service values are split into their price and quantity components, equations (1) and (2) may be rewritten as follows.

$$v_t^0 = \sum_{\tau=t}^{\tau=t+n-1} \frac{p_\tau q_{\tau-t+1}}{(1+r)^{\tau-t+1}} \quad (4)$$

$$v_t^k = \sum_{\tau=t}^{\tau=t+n-k-1} \frac{p_\tau q_{\tau-t+k}}{(1+r)^{\tau-t+1}} \quad (5)$$

When an asset is purchased new, market forces are assumed to ensure that its value to its owner as given by (1) or (4) equals the purchaser's price paid. For simplicity, it is assumed here that the discount rate remains constant over the life of the asset. In contrast to the p 's and q 's, there is no reason to assume that the discount rate varies in any systematic way over the life of the asset, and the assumption of a constant discount rate does not affect the substantive conclusions. For a given discount rate, r , equation (4) can be solved to obtain estimates of the prices and the values of the services, provided that the functional forms of the sequences of the p_τ 's and q_k 's over the life of the asset are known, or assumed. In other words, it is necessary to know, or specify, what the efficiency profile is for the asset and also how the price of the service is likely to vary over the life of the asset. The theoretical literature on capital stock measurement is mostly concerned with postulating what functional forms of these two relationships, especially the efficiency profile, are likely to take.

Service prices, asset prices and rentals

It should be noted that there are three distinct kinds of prices involved in the measurement of capital stocks and flows.

The first is the price of the service provided by the asset in period τ , namely p_τ .

The second price is the price of the asset itself. With efficient markets, the price of an asset of vintage k should equal the present value of its remaining services, v_t^k . Of course, the price of the asset and the service price are interdependent.

The third is the rental price, the price paid to rent the asset for one period of time. It is the actual or imputed amount payable to purchase the services provided by the asset in that period. The rental therefore equals the *value* of the services provided in a particular period, namely s_τ^k , as distinct from the price of the services, p_τ . When the efficiency of an asset declines as it ages, older vintages assets command lower rentals because they provide smaller quantities of services, not because their services have a lower prices. As already noted, a set of homogeneous assets all provide the same type of service at the same price in the same period of time.

Service price, quantity and value profiles

To make progress with the measurement of the capital stock and capital services, it is necessary to know, or make hypotheses about, the behaviour of the prices and the quantities of the services provided by assets over their service lives. The prices of certain types of capital service may be expected to fall over time if the demand for that type of service tends to fall as a result of progressive obsolescence created by technical progress. Similarly, the quantities of the services provided by many

assets may tend to fall as they become older as a result of wear and tear. The next two sections examine the price and quantity profiles in more detail.

Service price profiles

p_t as a function of time over the life of a particular asset is described as its *service price profile*. It is often assumed, either explicitly or implicitly, that the real price of the service remains constant over time. However, this assumption is not realistic when many assets continue to be used in production over very many years. Technical progress, advances in knowledge and changes in tastes inevitably take place which make many types of assets susceptible to obsolescence as they age. Technical progress tends to reduce the demand for the services of many kinds of existing assets as newer models are developed which are more efficient *relatively* to existing types of asset, or as new processes of production are developed which do not require the services of existing assets. Either way, the demand for, and the price of, the services of existing assets already in use will tend to fall.

If the price of the service provided by a particular type of asset continues to fall, production of new assets of that type will cease sooner or later. However, existing assets will continue to be used so long as there is still some demand for their services at lower prices. Assets of this kind which are still being used in production but are no longer produced themselves may be described as *partially obsolete*. They are only retired from production when they become completely obsolete : that is, when there is no demand for their services even at a zero price.

When assets are retired from production on grounds of obsolescence, they may not only still be capable of providing a flow of services but be functioning perfectly : for example, high technology assets such as computers. However, obsolescence is not confined to assets such as computers. Spectacular examples of large assets retired on grounds of obsolescence are provided by railways and ocean liners. Even many structures, including housing, are eventually retired because of obsolescence. Many fixed assets can continue to be used in production almost indefinitely if they are suitably maintained and serviced. The optimal level of repairs and maintenance will be influenced by the relative prices of other goods and services, especially labour services. In countries where labour is relatively cheap, structures, machinery and equipment may kept in use much longer than in other countries. The service life of an asset is by no means determined purely by the technical characteristics of the asset. For these reasons, many assets continue to be used in production in both developing countries and countries in transition long after they would have been scrapped in developed countries.

As the price of the service depends on the level of technology, it is a function of time rather than the age of the individual asset in question. The service price profiles for different vintages of the same type of asset are not necessarily all the same unless the rate of change in the price remains constant over time.

Service quantity, or efficiency profiles

The quantity of services provided by an asset is a measure of its productive efficiency. As already noted, the sequence of q_k 's as a function of the age of the asset is usually described as its *efficiency profile*. It depends mainly on the physical and technical characteristics of the asset. Different vintages of the same homogeneous asset must all have the same efficiency profile. The functional forms of the efficiency profiles of different kinds of fixed assets have been extensively

discussed in the literature, mainly because their importance for productivity analysis. It is generally assumed that because many fixed assets tend to deteriorate physically with usage the sequence of quantities will tend to fall, or at least to be non-increasing. In any case, the rate of decline need not be constant and may accelerate or decelerate as the asset ages.

As it is a matter of convenience what quantity units are used, the quantity of the service in the first period in which a new asset is used can be chosen as the unit. In other words, the efficiency profile may be normalised by setting the first term equal to unity. This implies that the price of the service is equal to the value of the service in the first period.

Efficiency profiles may take various forms but four special cases have attracted attention.

- One special case occurs when the asset provides a constant flow of services until it breaks down or is retired from production. This case is quaintly described as a “one-hoss shay”. Some kinds of assets, such as structures or electronic equipment, may provide a constant flow until they are retired on grounds of obsolescence even though they might not provide a constant flow indefinitely.
- The quantities of services may decline very slowly at first, the rate of decline gradually accelerating as the asset gets older and begins to deteriorate physically. Such a quantity, or efficiency, profile may be described by a hyperbolic function³ which, with the appropriate parameters, is concave from below.
- The quantities of services decline at a constant linear rate as the asset ages, either until they fall to zero or until the asset breaks down or is retired on grounds of obsolescence. This case is of interest because certain linear profiles give rise to straight-line depreciation, as shown later.
- The quantities of services decline at a constant exponential or geometric rate, either indefinitely or until the asset breaks down or is retired on grounds of obsolescence.

Service value profiles

The sequence of s_t 's as function of time may be called the service value profile. Its functional form is a combination of the service price and efficiency profiles. Beyond the fact that the service profile is almost bound to be non-increasing, it is difficult to generalise about its form except to note that obsolescence will tend to reinforce whatever downward trend there may be in the efficiency profile. For some assets, rapid obsolescence may be the dominant factor causing the service price to fall so quickly that the form of the efficiency profile is largely irrelevant. Conversely, for assets not subject to much obsolescence, the efficiency profile will be the dominant factor in determining the form of the service value profile.

Economic depreciation

Two concepts of economic depreciation are to be found in the literature on capital stock measurement. The first, and traditional concept in both economic theory and accounting, is that

³ Hyperbolic functions have been used extensively by the US Bureau of Labor Statistics in their studies of productivity. The hyperbolic function is a flexible one which, with the appropriate parameters, can also be used for the converse case in which the rate of decline decreases as the asset ages.

depreciation is the decline in the value of a fixed asset between the beginning and the end of the accounting period resulting from its use in production. Depreciation is defined this way in the 1993 SNA.⁴ This concept of depreciation will be called *time series depreciation* as it measures the change in the value of an asset between two points of time.

The second concept of depreciation, called *cross section depreciation* here, is defined as the difference in value between two successive vintages, of ages k and $k+1$, of the same homogenous type of asset at the same point of time. This concept of depreciation has been developed mainly in the context of capital stock measurement for purposes of productivity analysis. The two concepts are not the same and the resulting measures do not generally coincide. The difference between them is examined below.

Time series depreciation

The time series depreciation is defined as the decline in the value of an asset between the start and end of the period which is attributable to its use in production. It was proposed by Hotelling (1925). In the SNA it is defined as “the decline ... in the current value of the stock of fixed assets owned and used by a producer as a result of physical deterioration, normal obsolescence or normal accidental damage” (para. 6.179 of the 1993 SNA).

The service value profile for an individual asset extends into the future and is subject to uncertainty. It is necessary, however, to start by establishing what are the properties and behaviour of depreciation under the assumption of perfect foresight. This assumption will be relaxed later. When expectations are uncertain, capital gains and losses are liable to occur and these must be clearly differentiated from depreciation.

The value of an asset of age k at the start of period t is given by equation (5) above. The depreciation on such an asset between the start of period t and the start of period $t+1$, denoted here by d_t^k , is therefore defined as :

$$d_t^k = v_t^k - v_{t+1}^{k+1} \quad (6)$$

Substituting for the asset values using equation (5) it can be shown that

$$d_t^k = p_t q_k - r v_t^k \quad (7)$$

Or, alternatively,

$$p_t q_k = d_t^k + r v_t^k \quad (8)$$

⁴ The SNA has traditionally used its own expression ‘*consumption of fixed capital*’ to mean economic depreciation, perhaps to clarify that it only refers to depreciation on fixed assets or to differentiate it from depreciation at historic cost or depreciation allowances for tax purposes which may be quite arbitrary and devoid of economic significance. However, the use of different terminology normally implies some difference in concept and when there is none, it is likely to cause confusion. The shorter, and more familiar and convenient, term ‘depreciation’ is therefore used in this paper.

The sequence of asset values defined by equation (5) over the life of the asset may be described as the *asset price profile*. The first differences between successive terms in the asset price profile constitute depreciation. Obviously, their sum over the entire life of the asset must equal v_t^0 , the purchaser's price paid for the asset when it was new.

The sequence of d_t^k 's over the life of the asset may be described as the *depreciation profile*. Like the efficiency profile, the asset price profile can also be normalised by fixing the initial purchaser's price at unity. The depreciation profile derived from the normalised asset price profile is usually described as the *depreciation schedule*. The terms in the depreciation schedule must sum to unity. They may be converted into percentages, if preferred. Each percentage can then be interpreted as the percentage *depreciation rate* applying to an asset of that age.

Equations (7) and (8) describe basic relationships which have been widely used by both economists and accountants over the last century⁵. Equation (7) shows how the depreciation profile is related to the service value profile. It is common practice, however, in both business and economic accounting to depreciate assets using arbitrary formulae which are chosen for their convenience, simplicity or other reasons, such as tax avoidance. Of course, when such arbitrary formulae are used the resulting depreciation figures may be very different from economic depreciation as defined by (6), while the values of the assets shown in balance sheets are also incorrect. In principle, the appropriate procedure is first to consider what form the service value profile is likely to take over the life of the asset and then to use the depreciation schedule implied by that profile.

Two depreciation formulae are so widely used that it is worth considering exactly what they imply for the associated service value profiles.

Straight-line depreciation

With the straight-line depreciation the initial cost of the asset is distributed evenly over the expected service life of the asset so that depreciation is constant from period to period. Equation (7) shows that depreciation is equal to the value of the service provided in the period *less* the return on the value of the asset at the start of the period. If depreciation is constant, the value of the asset must decline by a constant amount each period so that the second term in equation (7), the return on the asset, must also decline at a linear rate. In order for depreciation to remain constant, therefore, the value of the service must also decline at exactly the same linear rate. The value of the service is at a minimum when it equals depreciation, at which point the asset is retired. Thus, the service value profile required for straight-line depreciation is a truncated declining arithmetic progression.

This general kind of functional form for the service value profile is not implausible for many assets as it could be generated by obsolescence, or declining efficiency, or a combination of the two. While it may be a mere coincidence for economic depreciation to be precisely constant, there may be many types of assets for which straight-line depreciation may provide an acceptable approximation to the true economic depreciation. The service value profile consistent with straight-line depreciation is concave from below and could sometimes serve as a linear approximation to the kind of hyperbolic function preferred by some analysts.

⁵ An early reference is Canning (1929), p. 300. Other examples are Wright (1964), p. 84, Hill (1979), p. 27 and Hartwick and Hageman (1993) who discuss equation (7) at some length.

It is worth noting in passing that equation (7) shows that if the service profile itself is constant, depreciation must increase from period to period as the asset ages.

Geometric depreciation

If the service values decline by a constant percentage from period to period, then the value of the asset must decline at the same rate. It follows that the depreciation *rate* is also constant and equal to the same percentage. This case is attractive for its analytic simplicity and mathematical convenience and is widely used in the theoretical literature.⁶

This kind of functional form for the service value profile could be generated by progressive obsolescence causing the price of the service to decline by a constant percentage from period to period. It could also be generated by declining efficiency, although some analysts consider that the shape of the efficiency profile is more likely to be concave to the origin rather than the convex form implied by geometrically declining efficiency.

Equation (8) shows that the value of the service provided in period t is equal to the depreciation in period t plus the return on the value of the asset at the start of period t . As explained in a later section, this is the appropriate value for the input of capital services to be recorded in the production account.

Cross section depreciation

Instead of tracking the price of an individual asset as it ages, namely the *asset price profile* as defined above, a *vintage price function* may be defined. This function expresses the values, or prices, of different vintages of the same homogeneous type of asset at the same point of time as a function of their ages. Cross section depreciation may then be defined as the first differences of the vintage price function.

Cross section depreciation is needed when aggregating the values of different vintages of assets to estimate the value of the capital stock at a point of time. As explained further below, a two stage estimation procedure is normally used. First, all assets of a given type, whatever their age, are valued at the price of a new asset at that point of time. Second, the values of the older vintages are then scaled down by deducting the accumulated depreciation on them. This is the approach adopted when the capital stock is estimated using the perpetual inventory method, for example. Using this type of methodology, the relevant depreciation schedule is the cross section one.

The cross section depreciation in period t , defined as the difference between the value of an asset aged k and one aged $k+1$, will be denoted by δ_t^k . By definition,

$$\delta_t^k = v_t^k - v_t^{k+1} \quad (9)$$

Substituting for the asset values using equation (5) it can be shown that

⁶ Exponentially declining depreciation was described in the earlier literature on economic growth as “depreciation by evaporation”: see, for example, Meade (1961), chapter 9, ‘Depreciation and Replacement’

$$\delta_t^k = \sum_{\tau=t}^{\tau=t+n-k-1} \frac{p_\tau \Delta q_{\tau-t+k}}{(1+r)^{\tau-t+1}} \quad (10) \quad \text{where} \quad \Delta q_{\tau-t+k} = q_{\tau-t+k} - q_{\tau-t+k+1}$$

The Δq 's are the first differences of the efficiency profile. They measure the reduction in efficiency between an asset aged k and one aged $k+1$. The cross section and the time series efficiency schedules must be identical for a set of homogeneous assets. Jorgenson (1989) describes the sequence of Δq 's over the entire life of the asset as its "mortality distribution". The Δq 's for a normalised efficiency profile must sum to unity. In the case of a one-hoss shay, all the Δq 's are zero except for the last term which is unity.

It can be seen from (10) that cross section depreciation δ_t^k as defined by (9) is equal to the present value of the Δq 's *as calculated at time t* using the vector of prices p_t to $p_{t+n-k+1}$. When the price varies with time, however, the present value of the of an asset of vintage k at time $t+1$ is not the same as at time t because the vector p_{t+1} to $p_{t+n-k+2}$ is not the same as the vector p_t to $p_{t+n-k+1}$. Although the Δq 's are the same at times t and $t+1$, their price weights may have shifted.

When calculating the aggregate value of a stock of assets of different vintages at a particular point of time, the cross section concept of depreciation is the relevant one. For this reason, many economists, for example, Hulten and Wycoff in their 1996 survey article on economic depreciation, specifically choose to define depreciation in this way. Equation (9) in the Hulten and Wycoff paper is basically similar to equation (10) above. They observe (p. 14) that the rate of depreciation "is the percentage difference in the price of capital between two successive ages in the *same year* [their italics]." They continue: "This is the definition of economic depreciation ... , and it is seen to be equal to the present value of the *shift* in asset efficiency from one age to the next." Other examples are provided by Jorgenson (1989) and Triplett (1996) who use the same concept, their equations (11) and (3a) being virtually the same as equation (10) above apart from differences in notation.

Time series and cross section depreciation compared

As cross section depreciation, as given in equation (10), appears to be very different from the corresponding expression for time series depreciation given in (7), it is necessary to clarify the difference between them. Subtracting cross section depreciation from the corresponding time series term, we have :

$$d_t^k - \delta_t^k = (v_t^k - v_{t+1}^{k+1}) - (v_t^k - v_t^{k+1}) = v_t^{k+1} - v_{t+1}^{k+1} \quad (11)$$

The two terms on the right side of (11) denote the value of an asset of age $k+1$ in period t and an asset of age $k+1$ in period $t+1$. The difference between them is equivalent to the revaluation of an asset aged $k+1$ between t and $t+1$. Denote this revaluation by Δv_t^{k+1} : it can be shown

$$v_t^{k+1} - v_{t+1}^{k+1} = \Delta v_t^{k+1} = \sum_{\tau=t}^{\tau=t+n-k-1} \frac{\Delta p_\tau q_{\tau-t+k}}{(1+r)^{\tau-t+1}} \quad (12) \quad \text{where} \quad \Delta p_\tau = p_\tau - p_{\tau+1}$$

The Δp 's are the first differences of the service price profile. They measure changes in the price of the service from one time period to the next, due to factors such as technical progress. The

vector of p 's associated with the remaining q 's for an asset of vintage k in one period may well be different from those associated with assets of vintage k in preceding and subsequent periods.

Equations (11) and (12) shows that the difference between time series and cross section depreciation is equal to the present value of the remaining sequence of service quantities valued by the Δp 's. If the asset is prone to obsolescence, the service price tends to fall over time so that some or all of the Δp 's are likely to be positive. In that case, expression (12) is positive. As expected, obsolescence causes time series depreciation from period to period to exceed the corresponding cross section depreciation between successive vintages in the same period.

If the service price remains constant from period, expression (12) is zero. In this special case, time series and cross section depreciation coincide. However, there is no justification for assuming the service price is generally constant when obsolescence is often the principal factor determining both service life of an asset and its rate of depreciation.

A different expression for time series depreciation can be obtained by rearranging terms in equations (11) and (12). Substituting for δ_t^k in (11) using equation (10) we obtain :

$$d_t^k = v_t^k - v_{t+1}^{k+1} = \delta_t^k + \Delta v_t^{k+1} = \sum_{\tau=t}^{\tau=t+n-k-1} \left\{ \frac{p_\tau \Delta q_{\tau-t+k} + \Delta p_\tau q_{\tau-t+k}}{(1+r)^{\tau-t+1}} \right\} \quad (13)$$

It can be seen from (13) that time series depreciation consists of two components : cross section depreciation and the revaluation term. As already noted, cross section depreciation can be interpreted as the loss in value attributable to the aging of the asset and the consequential loss in productive potential which is often loosely described as 'wear and tear'. The revaluation term will capture the loss in value attributable to a declining service price resulting from obsolescence. Thus, (13) effectively decomposes time series depreciation into its two traditional components, wear and tear and obsolescence.

Time series and cross section depreciation are different concepts which serve rather different purposes. The conceptual difference between them hinges on the interpretation of the revaluation term. The time series concept of depreciation includes any foreseen revaluation due to obsolescence as an integral part of the depreciation itself, whereas the cross section concept implicitly treats it as if it were a capital gain or loss.

Service values, imputed rentals and user costs

User cost is the imputed cost incurred by the owner of an asset that is equivalent to the rental payable when an asset is leased. It is equal to the value of the service provided in period τ , namely s_t^k or $p_t q_k$. Given that the values of the services of assets owned by their users cannot be observed, it may be possible to exploit equation (8) to impute them. Equation (8) shows that the value of the service is equal to time series depreciation, d_t^k , plus the return on the asset, rv_t^k . If it is possible to make a direct, and economically realistic, estimate of depreciation, equation (8) can be used to work back from depreciation and the estimated return on the asset to derive an estimate of the value of the service. This kind of approach could be used in the SNA, as explained in more detail below.

Suppose, however, that a direct estimate of cross section depreciation is available (say from the prices of used assets) instead of time series depreciation. Equation (13) shows that time series depreciation is equal to cross section depreciation *plus* the revaluation term. Substituting for d_t^k in equation (8) using equation (13), it follows that

$$p_t q_k = r v_t^k + \delta_t^k + \Delta v_t^{k+1} \quad (14)$$

This is the alternative version of equation (8) which uses cross section instead of time series depreciation. It includes the revaluation term Δv_t^{k+1} as well as depreciation and the return on the asset. Although it bears a superficial resemblance to the Hall-Jorgensen (1967) “user cost” formula, the revaluation term must not be interpreted as a capital gain or loss.

These equations are all derived on the assumption of perfect foresight so that capital gains or losses cannot occur. From a conceptual point of view, the interesting feature of (14) is that the revaluation term measures the effect of *foreseen* changes in the price of the service, such as those associated with foreseen obsolescence, and not some kind of capital gain or loss. Time series depreciation treats foreseen price reductions due to obsolescence, or other factors, in exactly the same way as foreseen reductions in the stream of services that an asset is capable of providing due to shortening asset lives or declining efficiency. The progressive downward revaluation of an asset due to foreseen obsolescence is an integral part of time series depreciation. The revaluation term, together with cross section depreciation, *is* a component of time series depreciation.

One important consequence of the distinction between cross section and time series depreciation is that the market prices of used assets at a particular point of time are liable to provide biased estimates of time series depreciation. If the asset markets are efficient, the prices of used assets must, by the definition of cross section depreciation, provide good estimates of cross section depreciation. However, cross section depreciation will tend to underestimate time series depreciation for assets subject to obsolescence.

*Capital gains, depreciation and obsolescence*⁷

Up to this point it has been assumed that the entrepreneur undertaking the investment in a fixed asset has perfect foresight of its service price and efficiency profiles. On this assumption, capital gains or losses cannot occur. Whatever changes occur in the price or quantities of the services provided by an asset over its entire life are fully incorporated in the present value of the asset at each and every stage in its life. The actual change in the value of an asset between two points of time must coincide with the expected change so that capital gains or losses cannot drive a wedge between the actual *ex post* change and time series depreciation.

In the real world where expectations are uncertain, unexpected events outside the control of the entrepreneur may occur which lead to periodic revisions of expectations about the present values of the remaining flows of services from an asset. Whenever expectations and asset values are revised in this way capital gains or losses will occur. When expectations are revised for any reason during period t it follows that

⁷ This section draws heavily on the paper by Hill and Hill (1999) on capital gains and depletion. ‘Capital gains’ are used here to cover both the SNA’s ‘holding gains’ resulting from unexpected changes in the price of the asset and ‘other volume changes’ resulting from unexpected changes to the quantities of services it is capable of providing.

$$E_{t+1}v_t^k \neq E_t v_t^k \quad (15)$$

where E_t denotes the expectation at the start of period t . At the end of the period with the benefit of hindsight, the value of the asset at the start of the period may be known to have been different from what it was expected to be at the start of the period. Depreciation can only be defined with reference to a single set of expectations held at a particular point of time which determine a single service value profile with its associated service price and quantity, or efficiency, profiles. The use of different service value profiles at the start and the end of the period in question would generate an internally inconsistent measure of depreciation. Any theoretically satisfactory definition of depreciation requires the asset to be valued consistently at both times on the basis of the *same* information and expectations. If depreciation is to be calculated *ex post*, the best estimate must be based on the information and expectations held at the end of the period in question because they supersede those held at the beginning of the period. The appropriate *ex post* estimate of depreciation at the end of the period is therefore

$$E_{t+1}d_t^k = E_{t+1}v_t^k - E_{t+1}v_{t+1}^{k+1} \quad (16)$$

This is, in fact, the precise *ex post* definition of depreciation proposed by Hicks (1942)⁸. Once uncertainty is allowed for, the values of all the variables concerned including depreciation itself, must be prefixed by an operator specifying the time at which the expectations are held and the measurement takes place.

The difference between the values of the asset as actually estimated at the start and the end of the period and recorded in the opening and closing balance sheets may be denoted by ΔB .

$$\Delta B = E_t v_t^k - E_{t+1} v_{t+1}^{t+1} \quad (17)$$

Note that ΔB mixes expectations held two different points of time. Subtracting *ex post* depreciation from ΔB , that is subtracting (16) from (17), we have

$$\Delta B - E_{t+1}d_t^k = E_{t+1}v_t^k - E_t v_t^k = (E_{t+1} - E_t)v_t^k = E_{t+1}G_t \quad (18)$$

where $E_{t+1}G_t$ is the capital gain or loss in period t as estimated at the start of period $t+1$. It measures the amount by which the opening value of an asset of vintage k , as estimated at the start of period t , has to be revised *in retrospect* as a result of the revised expectations based on the additional information and knowledge gained during the course of period t .

Thus, in principle, *ex post* time series depreciation might be estimated by subtracting an estimate of the capital gain or loss on the asset, $E_{t+1}G_t$ from the difference between the balance sheet values, ΔB . However, this is unlikely to provide an alternative way of estimating depreciation in practice. In order for this to be a viable method, both the balance sheet values and the capital gains must be estimated satisfactorily. It is doubtful whether it is any easier to estimate capital gains directly than depreciation itself. Moreover, actual balance sheet figures tend to be very unreliable,

⁸ "Let us then define the depreciation of the original stock of capital as the difference between the total value of the goods comprising that stock as it is at the end of the year and the value which would have been put upon the initial stock at the beginning of the year if the events of the year had been correctly foreseen, including among those events the capital value at the end of the year." (Hicks, 1942, p.177, italics in the original)

those in business accounts being valued quite inappropriately. If the balance sheet figures are unreliable, the difference between them, ΔB , and the estimate of depreciation derived from it, are both likely to be highly sensitive to error.

In any case, because of the interdependence between the various stocks and flows involved, the balance sheet values themselves may depend on prior estimates of depreciation, as when the perpetual inventory method is used, in which case subtracting estimated capital gains or losses from the difference between the balance sheet values would not provide an independent estimate of depreciation. The best estimate of *ex post* depreciation is obtained by utilising the service value profile based on expectations held at the end of the period.

Capital stock estimation

Time series depreciation is the relevant concept in the flow accounts of the SNA. In both economic and business accounting, depreciation has always been understood as the decline in the value of an asset with the passage of time, namely the accounting period. It has therefore always been defined to include foreseen obsolescence as well as wear and tear. On the other hand, cross section depreciation has been developed in order to estimate the capital stock at a particular point of time by the perpetual inventory or similar method, usually for purposes of productivity analysis.

The capital stock may be estimated by a two stage procedure. First, all the fixed assets held by producers for purposes of production, whatever their age or physical condition, are valued as if new at current market prices. The aggregate value of all assets valued in this way is described as the 'gross capital stock'. Second, the value of each asset is then scaled down by deducting the cumulative depreciation for an asset of that age calculated with reference to the current price of a new asset of that type on the market. As the object is to estimate the market value of each vintage of the asset at the same moment of time it follows, by definition, that the appropriate concept of depreciation is cross section and not time series. The value of the asset after deducting the cumulative cross section depreciation is described as the 'net' value. This is the appropriate value to record in the balance sheets of the SNA in which all assets are valued at their current market prices.

This summary description of the perpetual inventory method may create the impression that depreciation due to obsolescence has been omitted from the calculation of the net values, but this is not the case. Obsolescence is a function of time and not of the age of an asset. All vintages of the same type of asset, including new assets, must be equally obsolete at the same moment of time.

The fact that partially obsolete assets are no longer being produced so that there are no current market prices for new assets of that type creates serious valuation problems. Economic theory suggests two options, namely demand reservation prices or supply reservation prices. The demand reservation price is the highest price that a purchaser - here an entrepreneur contemplating investing in the asset - would be prepared to pay to purchase a new asset. The supply reservation price is the lowest price that would induce a supplier to produce and market a new asset.⁹ When the supply reservation price exceeds the demand reservation price no new assets are produced, existing assets becoming partially obsolete. In SNA terminology, the demand reservation price can be interpreted as a hypothetical purchaser's price and the supply reservation price as a hypothetical producer's price. Whereas for assets actually produced and sold, the former exceeds the latter by the trade and transport margins, in the case of partially obsolete assets no longer in production, the hypothetical purchaser's price may lie

⁹ For a further elaboration of demand and supply reservation prices, see Fisher and Shell, (1998), pp. 34-39.

well below the hypothetical producer's price. The general SNA rule to value goods for which there are no observable prices by their estimated market prices is of no help for partially obsolete assets.

As the value of a fixed asset is given by the present value of the flow of services it contributes to production, it follows that the estimated hypothetical price for a new partially obsolete asset must be the demand reservation price. The price of the services of a partially obsolete asset cannot have fallen to zero if it continues to be used in production even though it must have fallen relatively to other prices. In these circumstances, there could even be a market rental to be observed as partially obsolete assets may well be rented out at reduced rates in practice. The present value of the services that would be provided by a new asset estimated using the lower service price must be the demand reservation price. The fact that it is below the supply reservation price is the defining characteristic of a partially obsolete asset. There may be considerable practical difficulties involved in estimating demand reservation prices for partially obsolete assets, but if satisfactory estimates cannot be made, the value of the capital stock itself cannot be estimated, at least using a method such as the perpetual inventory method.

One conclusion to be drawn is that the value of new partially obsolete assets must not be estimated on the basis of their current replacement costs. The current replacement cost of an asset is its supply reservation price and this is bound to overvalue new partially obsolete assets. Valuation at estimated current market prices is not equivalent to valuation at current replacement costs. Current replacement cost may provide a useful point of departure for estimating the current market price because it at least places an upper bound on the latter. In general, it is likely to be easier to estimate the current replacement cost, assuming the technology of producing the asset has not changed much.

When a two stage procedure of the kind outlined above is used to estimate the capital stock obsolescence is taken into account in the first stage and declining productive potential due to aging and wear and tear in the second stage. Although the two effects are estimated sequentially, they take place simultaneously in the real world. Because obsolescence affects all vintages of an asset, including new assets, equally the gross capital stock already takes account of the extent to which certain types of asset have become obsolete as compared with earlier periods of time. When a particular kind of asset is becoming obsolete, it carries a progressively smaller weight in the capital stock the later the year whose prices are used to value the gross stock. The gross value of the capital stock is no more than an interim *hypothetical* valuation when the stock is made up of assets which were in fact new at very different points of time.

Finally, it is precisely because obsolescence has already been taken into account in estimating the gross capital stock that cross section depreciation depends only on the efficiency profile of the asset and its associated mortality function. As all the different vintages of the same asset are obsolete to the same extent at the same point of time, obsolescence does not affect their relative prices.

Capital services in the SNA

Despite the fact that capital services are inputs into processes of production and are instrumental in determining the value of fixed assets and depreciation they do not figure anywhere in the SNA. This is a serious omission and in consequence the SNA's production account¹⁰ does not

¹⁰ The 1993 SNA splits the traditional production account (as given in the 1968 SNA, p. 157) into two quite separate accounts, the 'production account' with gross value added as its balancing item and a second account, the 'generation of

provide the kind of economic information that is needed for purposes of economic analysis, especially productivity analysis. It is out of touch with economic theory and needs to be updated. It will be shown in this section that this can be accomplished very easily and at little cost.

Primary inputs of labour and capital should be treated symmetrically. The value of labour services is recorded in the production account of the SNA by compensation of employees. The value of the flow of capital services should be recorded in a similar way. Although depreciation has always been recorded in the production account, it is clear from the main part of this paper that depreciation does not measure the value of the services contributed by fixed assets to production.¹¹

When assets are leased, the rental payments are treated as purchases of services in the SNA and recorded under intermediate consumption. In these circumstances, the full values of the capital services are recorded as inputs into production by the enterprises renting the assets. However, most assets are purchased, not rented, by the units that use them in production, in which case the capital services are produced for own consumption and their values have to be estimated or imputed. In any case, leasing merely shifts the problem rather than eliminating it, as the enterprises that make a business of renting out assets are themselves engaged in production and the values of the capital services they consume remain to be estimated.

It was shown in equation (8) above that the value of the capital service consumed in any period is equal to depreciation plus the return on the asset. This provides a way to estimate the value of the service if it is possible to arrive at an acceptable estimate of depreciation. The theoretical framework outlined in this paper shows that the crucial issue is to try to determine what the functional form of service value profile of the asset is likely to be, taking into account both obsolescence and the efficiency profile of the asset. Once a view is taken about the service value profile, it is possible to deduce what form the depreciation schedule of the asset is likely to take. The depreciation schedule can then be applied to the actual or estimated value of a new asset to determine both the depreciation in the current period and the net value of the asset at the start of the period. Estimating depreciation remains a difficult exercise, but by utilising the theory underlying its determination it should be possible to make a rational and theoretically defensible estimate of depreciation instead taking a purely arbitrary decision about its value which may have no economic justification. In any case, a theoretically defensible estimate of depreciation is needed even in the present SNA, given the prominent role played by depreciation, or consumption of fixed capital, in the SNA.

Assuming that the net value of the asset at the start of the period can be estimated, it possible to estimate the return on the asset in the current period, namely rv_t in equation (8), by multiplying the net value of the asset by an appropriate interest rate. The value of the capital services is then given by the sum of depreciation and the return on the asset. A simple numerical example may be given. Suppose a new asset costs 1000 and depreciation is 100 per year over 10 years. Suppose also the rate of return is 8 per cent. After three years, for example, the net value of the asset is 700 and the return on the asset

income account', in which the primary incomes payable out of gross value added are recorded. While there are advantages in showing value added as a separate balancing item, this could have been done by splitting the production account into two sub-accounts instead of creating two separate accounts. For purposes of production and productivity analysis, the labour and capital inputs have to be recorded in the production account. The production account has to be understood here as being that in the 1968 SNA and in economics generally, i.e., it records inputs of labour and capital as well as intermediate inputs. Its balancing item is the operating surplus and not value added.

¹¹ As already noted, depreciation is described as "consumption of fixed capital" in the SNA. However, consumption of fixed capital should not be interpreted as measuring the consumption of capital services.

in the fourth year is 56. The value of the services in that year would then be 156. The value of the services falls from 180 in the first year to 108 in the last year.

The present SNA records depreciation but not the return on the asset. If the value of the capital service is recorded instead depreciation, the balancing item of the production account, the net operating surplus, is reduced by an amount equal to the return on the asset. Both the reduced net operating surplus and the return on the asset are carried forward into the next account as primary incomes of the owner of the enterprise. As their sum is identical with the existing net operating surplus in the 1993 SNA, it follows that the total primary of the owner of the enterprise is unaffected. The consequences for the SNA of recording capital services in the production account are minimal from a purely technical accounting point of view. In effect, it is equivalent to appropriating part of the existing net operating surplus as defined in the 1993 SNA and reclassifying it as a different kind of primary income, namely the return to the enterprise in its capacity as the owner of the fixed asset.

From an economic point of view, however, the changes are fundamental. The theoretical innovation is the recognition that the owner of an enterprise typically acts in two different capacities, as an entrepreneur and also as the owner of some or all of the fixed assets used in the enterprise. While the entrepreneur and the owner may be one and the same unit they need not be. The entrepreneur can always rent some or all of the fixed assets.

It is perfectly feasible for all the fixed assets to be rented. In such a case, there is no more justification for attributing the operating profits to the fixed assets than to any other inputs, whether intermediate or labour. The rentals have to be paid to the owners of fixed assets used in production, irrespectively of how profitable the production process has been, just as all the other inputs of goods and services, including labour services, must be paid for. The remaining operating profits accrue to the owner of the enterprise as a return to entrepreneurship.

If, on the other hand, the owner of the enterprise chooses to buy rather than rent its fixed assets, basic rentals should be imputed just as the services of rented assets have to be paid for. After charging the imputed rentals the resulting net operating surplus can then be seen as the return to entrepreneurship. It measures the return to the owner of the enterprise in respect of factors such as initiative, risk taking, innovation and managerial skill.

Inputs of capital services should be treated consistently with intermediate and labour inputs. The costs of the inputs charged against production should reflect the opportunity costs involved. These do not vary according to the overall profitability of the particular production process in which the resources happen to be used. However, many studies of factor productivity and growth accounting measure the values of the inputs of capital services by an *accounting residual*, the operating surplus, which does reflect the overall profitability of the production process. This implies that the value of the input is dependent on the value of the output. However, the value of an input cannot depend on output in practice. The value of an input is determined before it enters the production process and reflects the resources required to provide that input. Costs must be measurable before they are incurred. Inputs of capital services are no different from intermediate inputs and inputs of labour services in this respect.

There is no identity between the total values of the inputs and the outputs of a production process, just as there is no identity between the real growth of total inputs and total output over time. Market forces may prevent the value of the output from diverging much from the total value of the inputs and outputs, at least under competitive conditions, but an equilibrium represents a balance, or equality, between two independent and separately measurable forces. An identity, such as an

accounting identity, is not an equilibrium. Only by inventing another ‘input’, such as entrepreneurship, separate from both labour and capital services, whose value is defined residually by the net operating surplus can an identity between the values of the ‘inputs’ and outputs be achieved. However, the value of an ‘input’ which is indeterminate before it enters the production process cannot be treated as a cost of production.

In estimating, or imputing, the value of the capital services provided by the fixed asset owned by an enterprise, an estimate of the return on the asset is needed. This is essentially a form of property income accruing to the owner of the asset who chooses to invest in a fixed asset rather than in other assets, such as land or financial assets. This suggests that the rate of return on the asset may be estimated by using some long term market rate of interest. However, such a procedure does *not* imply that interest, whether explicit or implicit, is introduced into the production account. Although the estimated return on the asset may equal the interest cost incurred by the owner, it constitutes a form of property income which is conceptually quite different from interest. A new category of property income needs to be defined for the purpose alongside land rent and interest.

If the asset has been purchased out of loan capital the actual interest charges payable continue to be recorded in the primary income account of the 1993 SNA. As already noted, the owner’s balance of primary income and disposable income are unchanged by the treatment proposed here. One supposed advantage of the existing SNA production account is that the net operating surplus is invariant to whether the fixed assets owned by an enterprise are financed out of loan or equity capital. The new treatment proposed here preserves this invariance.

The change in method of recording in the production account of the SNA is summarised in the following table.

<u>Existing SNA</u>	<u>Revised treatment</u>
Consumption of fixed capital	Capital services
Net operating surplus,	of which (<i>implicitly</i>)
of which (<i>implicitly</i>)	Consumption of fixed capital
Net return on fixed asset(s)	Net return on fixed asset(s)
<i>Revised</i> net operating surplus	<i>Revised</i> net operating surplus

The main advantage of the revised treatment is that users of the accounts interested in production functions and productivity analysis are provided with more meaningful and useful data which are consistent with the underlying economic theory of production. Estimates of depreciation, or consumption of fixed capital, still have to be compiled as they are needed for the accumulation accounts of the SNA and not just in the production account. However, the estimates of depreciation must be consistent with the overall theoretical framework outlined above and not made on the basis of some arbitrary formulae chosen on grounds of convenience. As the estimates of depreciation and capital services are so interdependent, one cannot be estimated satisfactorily without the other. Conversely, if one can be estimated so can the other. The revised treatment proposed above is therefore not inherently more difficult on practical grounds than the present SNA. If countries are unable to estimate capital services, they cannot estimate depreciation either.

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