



Optimal Retirement Payout Structures: Reconciling Theory and Practice

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Main Message:


- To **better link** the accumulation and payout phase of DC plans, we must develop a better understanding of the optimal trajectory of retirement income.
- In particular, I would like to focus on the **optimal withdrawal rate** at retirement with and without pension annuity income.

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 **Financial Economists and financial practitioners have very different views on “retirement income” strategies...**

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 **The Financial Planner Says Things Like:**

- Target 65% to 85% of your working income once you retire.
- Spend and/or withdraw between **4% and 5%** of your retirement nest egg and adjust that for inflation each year.
- You might live to **95** so make sure not to spend too much money at **65**.
- Buy a “guaranteed” (i.e. expensive) product, or you will starve to death on cat food...

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The media

- **Schwab.com** "...Shoot for a portfolio that is 25 times as large as your first-year withdrawal. This translates into a **4% withdrawal rate**..."
- **Money Magazine**: "...Withdraw **4% of your retirement savings** and adjust each year for inflation..."
- **Newsweek**: "...A rule of thumb holds that your money will last all the way through retirement if you take **4% of your savings** out the first year and then increase by inflation..."

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Financial economist

- *"...As far as I am aware, no one has challenged the view that if people were capable of it, they ought to plan their consumption, saving and retirement according to the principles enunciated by Modigliani and Brumberg in 1950s..."*
- **Professor Angus S. Deaton, Princeton University, 2005**

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The Financial Economist Says:

$$\max_c V(c) = \int_0^D e^{-\rho t} ({}_t p_x) u(c_t) dt$$

$$\dot{F}_t = vF_t - c_t + \pi_0$$

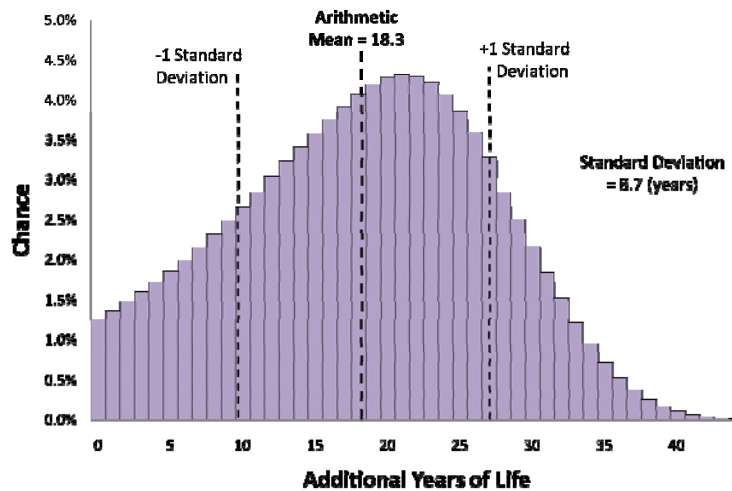
$$F_D = 0, F_0 = W$$

Smooth consumption, taking into account your “patience” and survival probabilities....

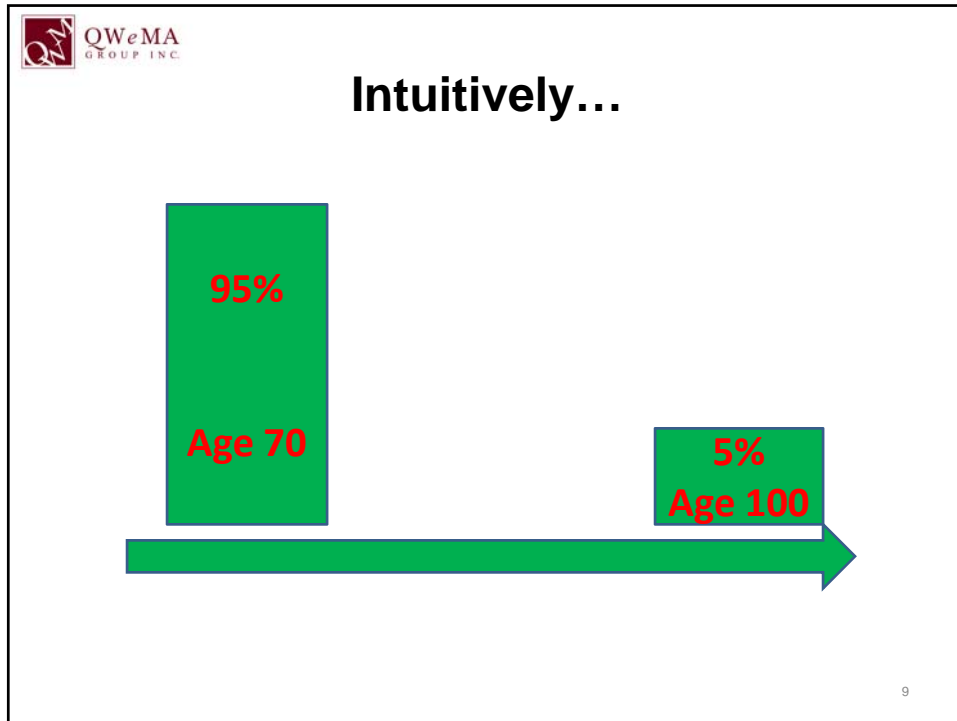
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Remaining Lifetime For 65 Year-Old Canadian



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The figure contains the text "The optimal (total) consumption rate over time will satisfy the following equation:" followed by a mathematical equation. The equation is
$$c_0^* e^{\left(\frac{v-\rho}{\gamma}\right)t} ({}_t p_x)^{1/\gamma}$$
. In the top-left corner of the slide area, there is a logo for "QWeMA GROUP INC." and the number "10" is located in the bottom-right corner.



You can derive a closed-form analytic expression for the initial consumption rate at retirement....

$$c_0^* = \frac{\left(W + \frac{\pi_0}{r}\right)e^{r\tau} - \pi_0/r}{a_x^\tau(r-k, m^*, b)e^{r\tau}}.$$

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Ok. Very elegant...

Are there policy implications from this?

I think yes and will offer **seven lessons.**

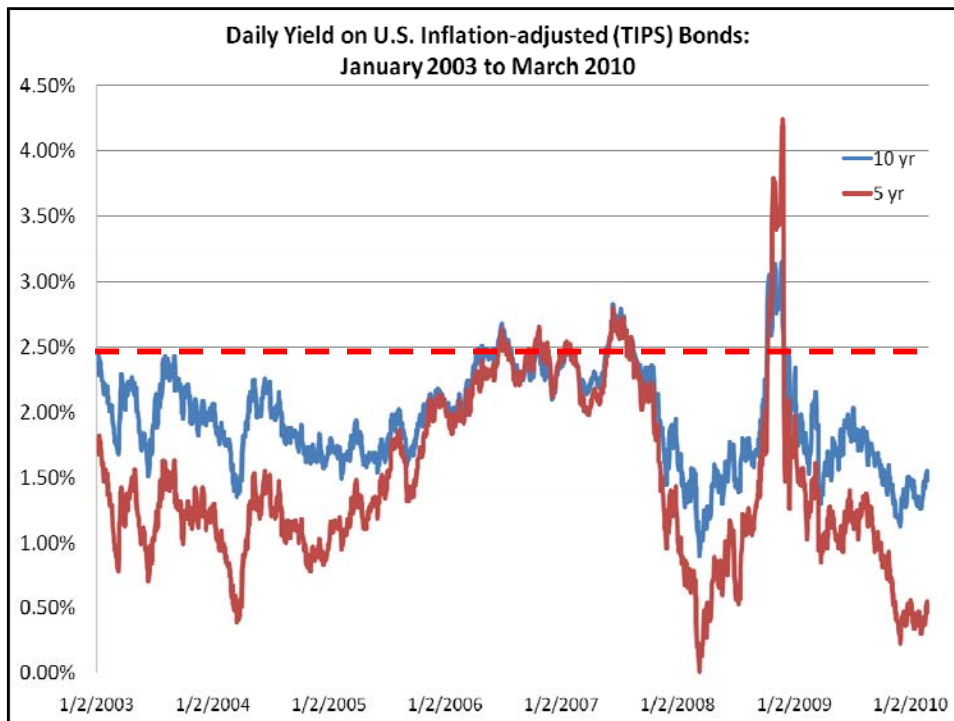
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Assume we live in a world in which the only source of uncertainty is longevity...

How does Mr. Spock (i.e. the rational consumer) behave if his entire portfolio is invested in real-return bonds?

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Longevity Risk Aversion vs. Financial Risk Aversion

Coefficient of Relative Risk Aversion (CRRA)	Allocation to "Stocks" in Asset Allocation model
$\gamma = 1$	150%
$\gamma = 2$	80%
$\gamma = 4$	40%
$\gamma = 8$	20%


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- ### Economic Question #1:
- What is the optimal initial portfolio withdrawal and consumption rate during retirement?
 - How do pre-existing pensions and/or risk aversion impact these spending rates?
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 **Net-Withdrawal Rates from \$100 at age 65**
Realistic Investment Assumption: $v = 2.5\%$

	Increasing Degree of Risk Aversion....			
Pre-Existing Pension Annuity	$\gamma = 1$	$\gamma = 2$	$\gamma = 4$	$\gamma = 8$
$\pi = \$0$	6.33%	5.30%	4.60%	4.12%
$\pi = \$1$	6.80%	5.65%	4.87%	4.32%
$\pi = \$2$	7.16%	5.92%	5.08%	4.48%
$\pi = \$5$	8.02%	6.55%	5.55%	4.83%

Note: Assumes 5% Survival to Age 100, 25% Survival to Age 93 and 50% to Age 87.
Subjective Discount Rate (ρ) assumed equivalent to real investment rate.



Ok, what if I am more optimistic about the rate of return my portfolio can earn?

What if the REAL investment assumption is increased by 100 basis points?

Answer: Increase spending rates by about 40 to 80 basis points.



Takeaway #1

- Optimal consumption rates (total) and withdrawal rates (from portfolio) depend on **longevity risk aversion** and pre-existing pension income in addition to other “background” variables like mortality assumptions.
- Rational rates can not be determined based only on the age at retirement and/or size of the nest egg.

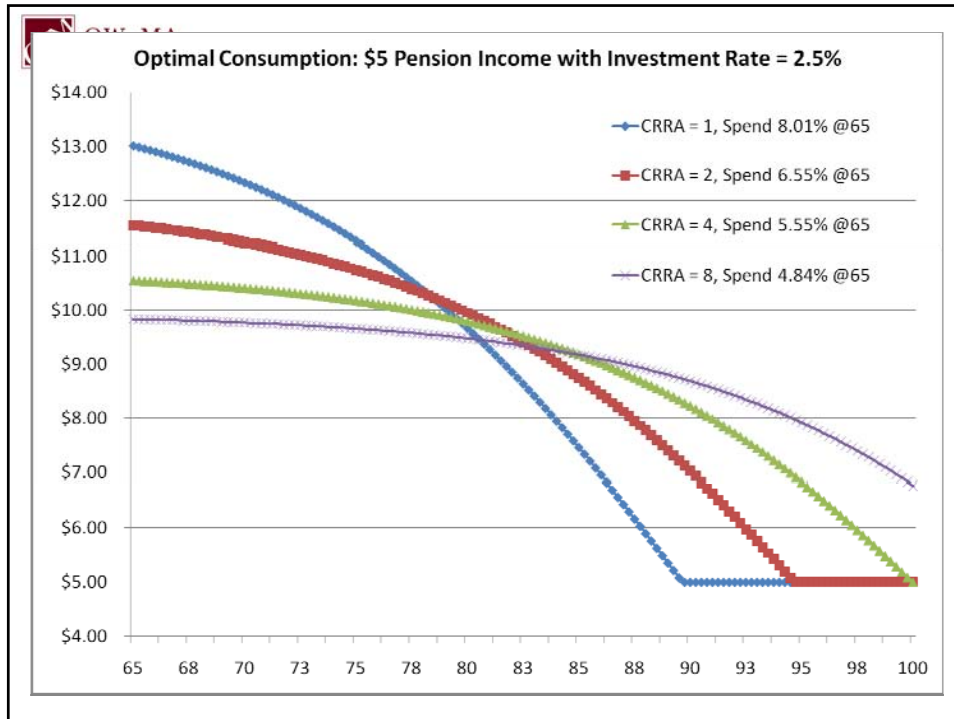
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Economic Question #2:

- How does the “optimal” consumption rate evolve over time?
- Does Mr. Spock withdraw the same amount each year (adjusted for inflation) forever?

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Irving Fisher (1930) *The Theory of Interest*

...The shortness of life thus tends powerfully to increase the degree of impatience or rate of time preference beyond what it otherwise might be...

...Everyone at some point in his life doubtless changes his degree of impatience for income..."

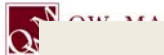
...He expects to die and he thinks: Instead of piling up for the remote future, why shouldn't I enjoy myself during the few years that remain...



Takeaway #2

- The optimal consumption rate declines as you age and progress thru retirement.
- The greater your level of longevity risk aversion, the slower the rate of decline because you are afraid you might live to an advanced age.
- New way to think about longevity risk aversion: People who are worried they are younger than they really are.

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Economic Question #3

- How exactly does risk aversion interact with optimal retirement spending rates?

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Mathematically Speaking:

$$({}_t p_x)^{1/\gamma} = \exp((x+t-m^*)/b)/b$$

$$m^* = m + b \ln[\gamma]$$

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Risk-Adjusted Survival Probability: $p^{(1/\gamma)}$ It makes you feel younger

	Survive to Age 80	Survive to Age 90	Survive to Age 100	Survive to Age 115
True Probability	74%	37%	5%	4×10^{-7}
$\gamma = 4$	92%	78%	47%	2%
$\gamma = 8$	96%	88%	69%	15%

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Takeaway #3

- The relevant probability of survival is intertwined with longevity risk aversion.
- You shouldn't tell people (ok, Vulcan's) to worry about living to age 100 (i.e. they should behave as if they are very risk averse) and then advise them to invest in the stock market (which is only suitable for people with low risk aversion.)

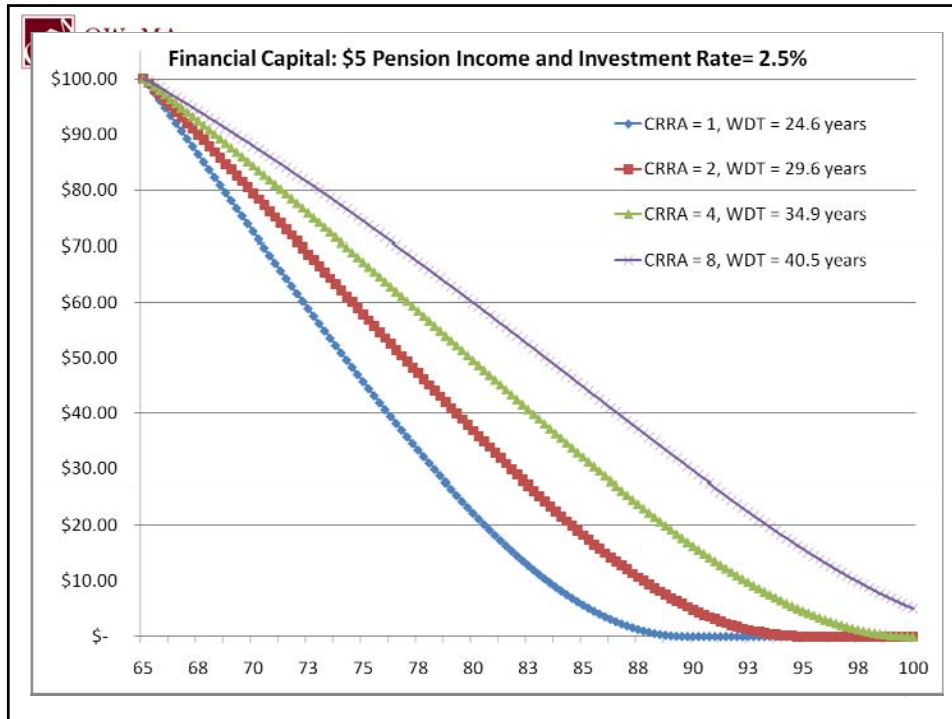
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Economic Question #4

- What is the optimal trajectory of wealth over the entire retirement horizon, if I maximize my utility of consumption?

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Takeaway #4

- There are bag-ladies on Planet Vulcan.
- It is optimal to spend down your wealth and eventually only consume your pension income (if you have any.)
- The greater your risk aversion the more wealth you have at any point during retirement.
- Kids love risk-averse parents, because they leave more leftovers.

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Economic Question #5

- What is the optimal reaction to shocks and unforeseen market events? By how much do you adjust your consumption rate (total) and withdrawal rate (from the portfolio)?

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**Five years later (age 70) unexpected things happen...
 By how much do you adjust portfolio spending?**

	-30% Shock	-15% Shock	+15% Shock	+30% Shock
No Pension Income	-30.0%	-15.0%	+15.0%	+30.0%
\$2 Pension	-27.1%	-13.5%	+13.5%	+27.1%
\$5 Pension	-23.5%	-11.7%	+11.7%	+23.5%

Notes: Assumes a high degree of risk-aversion ($\gamma=8$). The investment rate is equal to the subjective discount rate (SDR) which is assumed to be 2.5% inflation-adjusted.

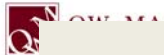
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Takeaway #5

- When unexpected things happen (to your portfolio) you should adjust your spending plans by less than the change in wealth assuming you have pension income.
- With no pension income it is one-for-one.
- Vulcan's don't panic and then reduce spending by 30% because their portfolio declined by 30%.


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Economic Question #6

- What is the utility/value of pension (income) annuities exactly and why do financial economist like them so much?
- Is it worthwhile to spend some of my \$100 nest egg to purchase a pension annuity? Why not take my chances?

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How Does “Pensionization” Impact Retirement Consumption at age 65?

Percent of \$100 Pensionized	Medium Risk Aversion (CRRA = 4)	High Risk Aversion (CRRA = 8)
0%	\$4.605	\$4.121
20%	\$5.263	\$4.801
40%	\$5.795	\$5.385
60%	\$6.227	\$5.937
100%	\$6.330	\$6.330

Note: Cost of \$1 lifetime income annuity is \$15.791 at age 65, assuming a real pricing rate of 2.5% per annum.

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Subtle but important...

- If the retiree’s **subjective** mortality rate is much higher than the **objective** rates being used to price life annuities, and the retiree is **longevity-risk tolerant**...then it might be optimal to drawdown most of wealth and annuitize only a fraction of the nest egg at retirement, even in a world in which real (inflation-adjusted) annuities are available.

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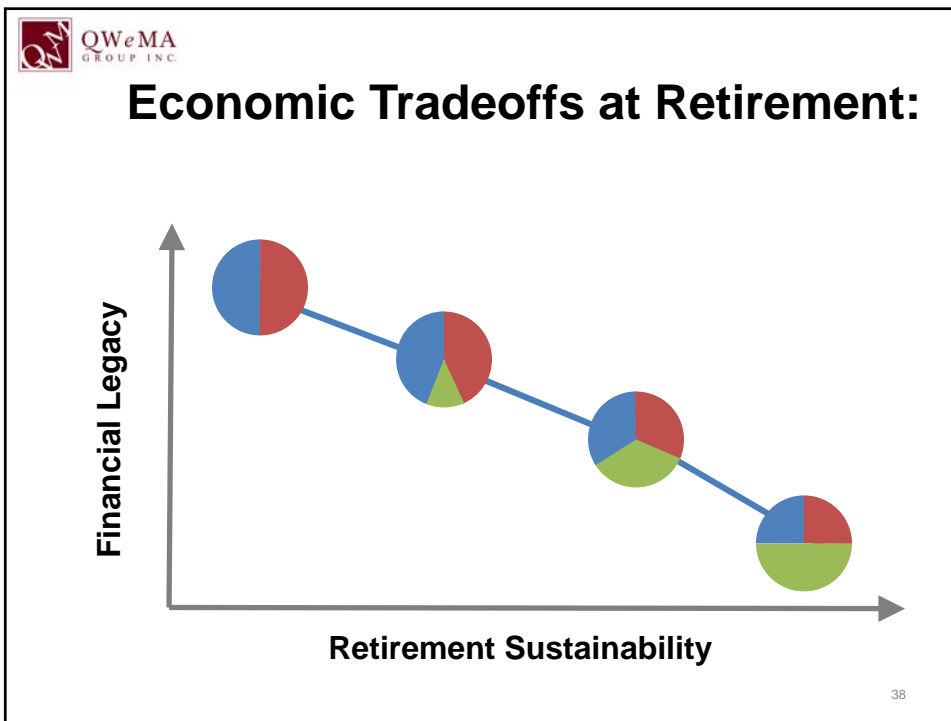
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Example: Optimal Pensionization @ 65

	Longevity risk tolerant $\gamma = 1$	Average risk aversion $\gamma = 3$	Longevity risk averse $\gamma = 8$
Poor Health (MRL = 10 years)	7.5%	42.5%	72.5%
Average Health (MRL = 15 years)	40%	75%	87.5%
Excellent Health (MRL = 20 years)	80%	95%	100%

Note: 5% loading on Real SPIA with MRL = 25 years, under 2.5% pricing rate.

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Takeaway #6

- Pension annuities are valuable because they don't "waste" investment returns.
- Don't confuse Yaari's (1965) actuarial notes with real-world life annuities.
- Pensionizing (a portion of) the retirement nest egg will reduce the financial capital and hence legacy/bequest.
- The optimal amount depends on pricing and **longevity risk aversion.**

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Economic Question #7

- If retirees will eventually seek to annuitize (pensionize) a portion of their nest egg, then why not start this process earlier?

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Linking the Beginning and the End

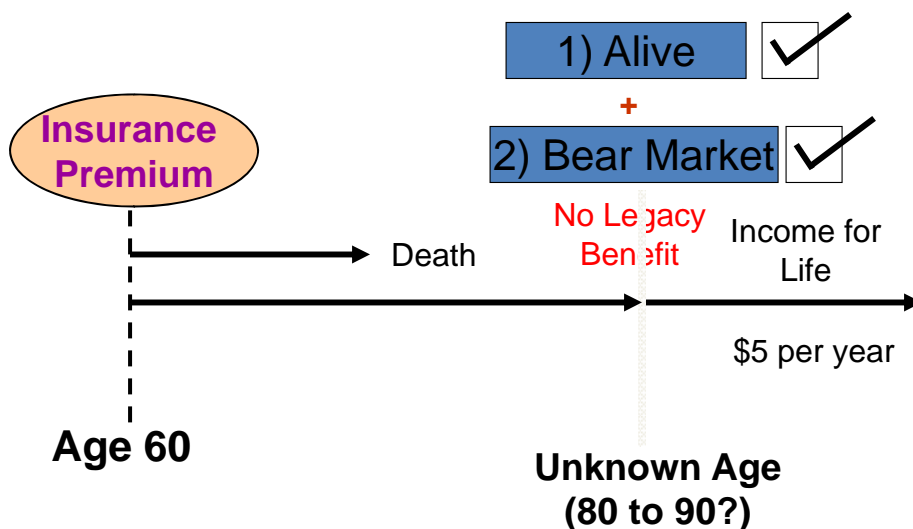
- There is a strong argument to be made for having individuals purchase longevity protection early-on in the lifecycle...
- In a DC system there is a huge sensitivity to investment rates and realized returns around the retirement age.
- Perhaps this “timing risk” should be hedged earlier as opposed to mitigated in expectation (only).

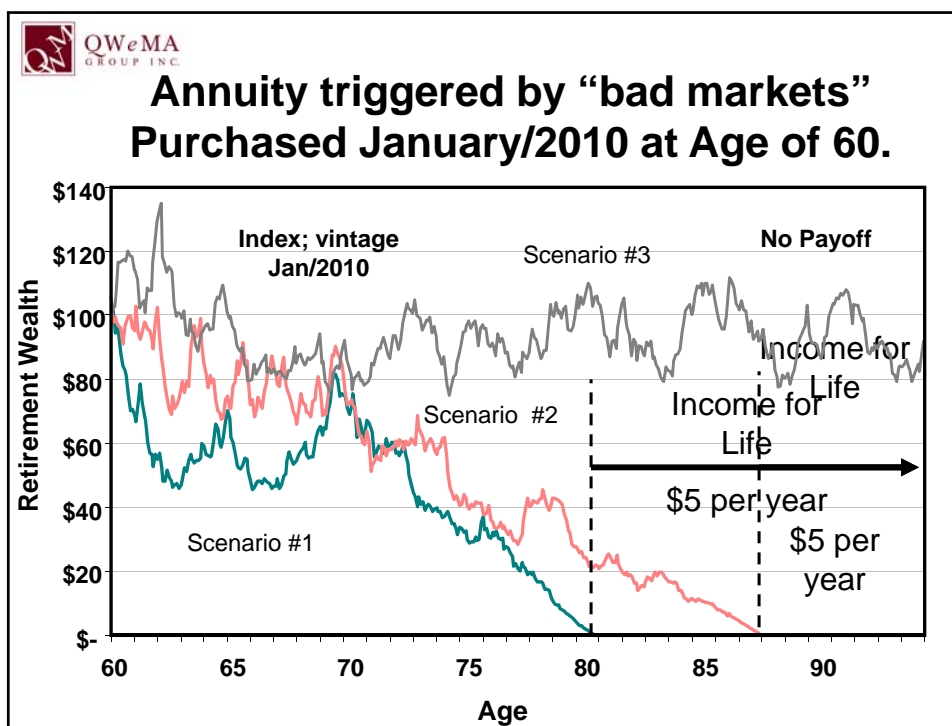
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Return Contingent Life Annuity (RCLA) “Conceptual Idea”





Concluding Remarks

- **Longevity risk aversion** leads to reduced spending and withdrawal rates, just like financial risk aversion leads to reduced equity in the investment asset allocation.
- Pension **annuities** are valuable because they allow you to withdraw from the portfolio at a greater rate, all else being equal.
- Must retain **flexibility** in system, to allow for personal preferences around risk.

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